Naturalness, supersymmetry and dark matter

Peter Athron, Csaba Balázs, Ben Farmer, Doyoun Kim
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conventional knowledge:

new physics models with $O(\text{TeV})$ dark matter possess unacceptably high electroweak fine tuning
example:

the Minimal Supersymmetric Standard Model is pushed by the LHC to parameter regions which are fine tuned at worse than percent level
this talk:

electroweak fine tuning is overestimated by the conventional measures

$O(\text{TeV})$ dark matter can be natural
Hmmm… huh?

Naturalness ???

Now they tell me!

Gosh!!!

Hey!

Hmmm…

Huh?
LET'S ...

... GET ...

... NATURAL
HELP!

HA-HA-HA...
the electroweak naturalness problem

physical phenomena characterized by disparate
(energy or length) scales are separated:
their governing laws can be understood largely
independently from each other

when viewed as an effective theory
the Standard Model Higgs mass receives
quantum corrections from the Planck scale

\[ m_H = 125 \text{ GeV} \Rightarrow \text{the Standard Model is unnatural} \]
What is natural?

naturalness is quantified by a fine-tuning measure
Barbieri-Ellis-Giudice measure

the most used, misused and abused electroweak fine tuning measure

\[ \frac{\partial (\text{electroweak observable})}{\partial (\text{theory parameters})} \]

measures the "sensitivity" of the "weak scale" to the change in the "theory parameter(s)"
Barbieri-Ellis-Giudice measure: example

Constrained Minimal Supersymmetric Standard Model

theory parameters: $M_0, M_{1/2}, A_0, \tan \beta, \text{sign}(\mu)$

dominant fine tuning term: $\frac{\partial m_Z}{\partial \mu}$
Barbieri-Ellis-Giudice measure: example

electroweak symmetry breaking condition:

\[
\frac{m_Z^2}{2} = \frac{M_{H_u}^2 - M_{H_d}^2 \tan^2 \beta}{\tan^2 \beta - 1} - \mu^2
\]

fixes \( \mu \) by connecting it to \( m_Z \)

this justifies \( \frac{\partial m_Z}{\partial \mu} \)
Bayesian evidence

\[ \mathcal{E} = \int \mathcal{L}(\mu) \pi(\mu) \, d\mu \]

for a theory with a single parameter \( \mu \).

\( \mathcal{E} \) quantifies the plausibility of the theory.
the theory predicts $m_Z(\mu)$
so $m_Z(\mu)$ is invertible

$$m_Z(\mu) \Rightarrow \mu(m_Z)$$
write $\mathcal{E}$ as an integral over $m_Z$

\[
\mathcal{E} = \int \mathcal{L}(m_Z) \pi(m_Z) \frac{d\mu}{dm_Z} dm_Z
\]

\[
\mathcal{E} = \int \mathcal{L}(m_Z) \pi(m_Z) \left(\frac{dm_Z}{d\mu}\right)^{-1} dm_Z
\]

Allanach, Hooper JHEP 0810:071 (2008)
Bayesian evidence $\sim$ inverse of fine-tuning measure!

$$\mathcal{E} = \int \mathcal{L}(m_Z) \pi(m_Z) \left( \frac{dm_Z}{d\mu} \right)^{-1} dm_Z$$

fixing $\mu$ using $m_Z$ automatically induces $\frac{dm_Z}{d\mu}$ in $\mathcal{E}$

$\mathcal{E} \sim$ plausibility that the theory correctly predicts $m_Z$

this is Occam’s razor at work!
MSSM

trade \{m_Z, m_t, \tan\beta\}
for \{\mu, y_t, B_0\}

\[
\text{naturalness prior} = \begin{vmatrix}
\frac{\partial m_Z}{\partial \mu} & \frac{\partial m_t}{\partial \mu} & \frac{\partial \tan\beta}{\partial \mu} \\
\frac{\partial m_Z}{\partial y_t} & \frac{\partial m_t}{\partial y_t} & \frac{\partial \tan\beta}{\partial y_t} \\
\frac{\partial m_Z}{\partial B_0} & \frac{\partial m_t}{\partial B_0} & \frac{\partial \tan\beta}{\partial B_0}
\end{vmatrix}
\]

Cabrera, Casas, deAustri JHEP 0903:075,2009
Barbieri-Ellis-Giudice EW FT measure:
\[
\frac{\partial (\text{electroweak observable})}{\partial (\text{theory parameters})}
\]

- evidence = integral over parameters
  observables can be used to eliminate parameters
- evidence ratios have clear normalization scale
- Observables: subjective choice!
- Parameters: subjective choice!
- Form: depends on prior!
- Expression: determinant!
- What does \( \frac{\partial m_Z}{\partial \mu} \) have to do with \( \Delta m_H, \mu, m_{\tilde{t}}, m_{\tilde{g}}, \ldots \)?
- Beyond MSSM, NMSSM, SUSY: evidence can be defined!
NMSSM

trade \{\ln m_Z^2, \ln \lambda, \ln \tan \beta\}

for \{\ln m_S^2, \ln \lambda_0, \ln \kappa_0\}

naturalness prior =

\begin{bmatrix}
\frac{\partial \ln m_Z^2}{\partial \ln m_S^2} & \frac{\partial \ln \lambda}{\partial \ln m_S^2} & \frac{\partial \ln \tan \beta}{\partial \ln m_S^2} \\
\frac{\partial \ln m_Z^2}{\partial \ln \lambda_0} & \frac{\partial \ln \lambda}{\partial \ln \lambda_0} & \frac{\partial \ln \tan \beta}{\partial \ln \lambda_0} \\
\frac{\partial \ln m_Z^2}{\partial \ln \kappa_0} & \frac{\partial \ln \lambda}{\partial \ln \kappa_0} & \frac{\partial \ln \tan \beta}{\partial \ln \kappa_0}
\end{bmatrix}

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naturalness prior map in the CMSSM

$$A_0 = -2.5 \text{ TeV}, \tan \beta = 10$$

Ahron, Balazs, Farmer, Kim PRD90 5 055008 (2014)
naturalness prior map in the CNMSSM

\[ A_0 = -2.5 \text{ TeV}, \tan\beta = 10 \]

Ahron, Balazs, Farmer, Kim PRD90 5 055008 (2014)
naturalness prior map in NMSSM-11

\[ A_0 = -2.5 \text{ TeV}, \tan\beta = 10 \]

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naturalness prior agrees with EW FT measure

\[ A_0 = -2.5 \text{ TeV}, \quad \epsilon_{\epsilon' \beta} = 10 \]
conclusions

naturalness is a robust principle
it can be quantified within the Bayesian framework
fine tuning is measured by the naturalness prior
according to the naturalness prior $O(\text{TeV})$ dark matter can be perfectly natural