Inflationary universe in a conformally-invariant two-scalar-field theory with an R^2 term

Reference: Eur. Phys. J. C 75, 344 (2015)
[arXiv:1505.00854 [hep-th]]

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Planck 2015 results

[Ade *et al*. [Planck Collaboration], arXiv:1502.02114]

(1) **Spectral index of power spectrum of the curvature perturbations**

$$n_s = 0.968 \pm 0.006 \quad (68\% \text{ CL})$$

(2) **Tensor-to-scalar ratio**

$$r < 0.11 \, (95\% \text{ CL})$$

Keck Array and BICEP2 constraints

$$r_{0.05} < 0.09 \; (0.07) \quad (95\% \text{ CL})$$

(Combined results with the Planck analysis)

[Ade *et al*. [Keck Array and BICEP2 Collaborations], arXiv:1510.09217]

Planck 2015 results (2)

R^2 (Starobinsky) inflation is supported.

[Starobinsky, Phys. Lett. B <u>91</u>, 99 (1980)]

R : Scalar curvature

R^2 (Starobinsky) inflation

Action: $$S = \int d^4x \sqrt{-g} \frac{1}{2\kappa^2} \left(R + \frac{1}{6M^2} R^2 \right)$$

[Starobinsky, Phys. Lett. B 91, 99 (1980)]

g : Determinant of the metric $g_{\mu\nu}$

$\kappa^2 \equiv 8\pi G_{\mathrm{N}}$

R : Scalar curvature, M : Constant , G_{N} : Gravitatioanl constant

- $N_e = 60$ ⇨ $n_{\mathrm{s}} = 0.967$, $r = 3.33 \times 10^{-3}$

N_e : Number of *e*-folds during inflation

Cf. **[Hinshaw *et al*., Astrophys. J. Suppl. 208, 19 (2013)]**

Motivations and Purposes

→ **We explore the possibility of the theoretical extension of R^2 (Starobinsky) inflation.**

- **Extension of the gravitational term**

  [Sebastiani, Cognola, Myrzakulov, Odintsov and Zerbini, Phys. Rev. D 89, 023518 (2014)]

- **Quantum anomaly effect**

  [KB, Myrzakulov, Odintsov and Sebastiani, Phys. Rev. D 90, 043505 (2014)]

  Cf. [KB and Odintsov, Symmetry 7, 220 (2015)]

⇨ **Two-scalar-field theory plus an R^2 term**

→ **We show that the values of n_{S} and r can be consistent with the Planck results.**

II. Model

$$S = \int d^4x\sqrt{-g}\left\{\underline{\frac{\alpha}{2}R^2} + \underline{\frac{s}{2}\left[\frac{(\phi^2-u^2)}{6}R + (\nabla\phi)^2 - (\nabla u)^2\right]}\right.$$

$$\left. - (\phi^2-u^2)^2\underline{J(y)}\right\}$$

$2\kappa^2 = 1$

$s = \pm 1$

$\phi,\ u$: Scalar fields

$\alpha(\neq 0)$: Constant, $y \equiv u/\phi$, $J(y)$: Function of y

∇ : Covariant derivative

(i) If there is no R^2 term: Conformal invariance

(ii) Scale invariance

Action

We introduce an auxiliary field Φ.

$$S = \int d^4x\sqrt{-g}\left\{\left[\Phi + \frac{s}{12}(\phi^2 - u^2)\right]R - \frac{\Phi^2}{2\alpha}\right.$$

$$\left. + \frac{s}{2}\left[(\nabla\phi)^2 - (\nabla u)^2\right] - (\phi^2 - u^2)^2 J(y)\right\}$$

$$\Phi + \frac{s}{12}(\phi^2 - u^2) = 1$$

Action (2)

Potential V

$$S = \int d^4x \sqrt{-g} \left\{ R + \frac{s}{2} \left[(\nabla\phi)^2 - (\nabla u)^2 \right] - V(\phi, u, J) \right\}$$

$$V(\phi, u, J) \equiv \frac{1}{2\alpha} \left[1 - \frac{s}{12} (\phi^2 - u^2) \right]^2 + (\phi^2 - u^2)^2 J(y)$$

$$y \equiv u/\phi$$

- Einstein-Hilbert term

- The scale invariance is broken.

Equation of motion

Gravitational field equation

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \frac{s}{2} \left(\nabla_\mu \phi \nabla_\nu \phi - \nabla_\mu u \nabla_\nu u \right)$$

$$+ \frac{1}{2} g_{\mu\nu} \left\{ V - \frac{s}{2} \left[(\nabla \phi)^2 - (\nabla u)^2 \right] \right\} = 0$$

Scalar field equation

$$s \Box \phi + V_\phi = 0 \,, \quad s \Box u - V_u = 0$$

$$V_\phi \equiv \partial V / \partial \phi \,, \quad V_u \equiv \partial V / \partial u$$

$\Box \equiv g^{\mu\nu} \nabla_\mu \nabla_\nu$: Covariant d'Alembertian operator

Background space-time

Flat Friedmann-Lemaitre-Robertson-Walker (FLRW) space-time

$$ds^2 = -dt^2 + a^2(t) \sum_{i=1,2,3} \left(dx^i\right)^2$$

$a(t)$: Scale factor

$H = \dot{a}/a$: Hubble parameter

* The dot shows the time derivative.

Field equations in the FLRW space-time

Gravitational field equations

$$3H^2 + \frac{s}{4}(\dot{\phi}^2 - \dot{u}^2) - \frac{1}{2}V = 0$$

$$2\dot{H} + 3H^2 - \frac{s}{4}(\dot{\phi}^2 - \dot{u}^2) - \frac{1}{2}V = 0$$

Scalar field equations

$$s(\ddot{\phi} + 3H\dot{\phi}) - V_{\phi} = 0$$

$$s(\ddot{u} + 3H\dot{u}) + V_u = 0$$

ϕ : Dynamical inflaton field

$\longrightarrow$ $u = u_0$ = Constant

- **Example:** $u_0 = 0\,, \quad J = 1$

Effective potential

$$V_{\rm eff}(\phi) = \frac{1}{2\alpha}\left(1 - \frac{s}{12}\phi^2\right)^2 + C\phi^4$$

C : Constant

Slow-roll inflation

Number of *e*-folds during inflation

$$N_e(\phi) = \int_{\phi}^{\phi_{\rm f}} H(\hat{\phi}) \frac{d\hat{\phi}}{\dot{\hat{\phi}}} = -\frac{s}{2} \int_{\phi_{\rm f}}^{\phi} \frac{V(u,\hat{\phi},J)}{V_\phi(u,\hat{\phi},J)} d\hat{\phi}$$

Slow-roll parameters

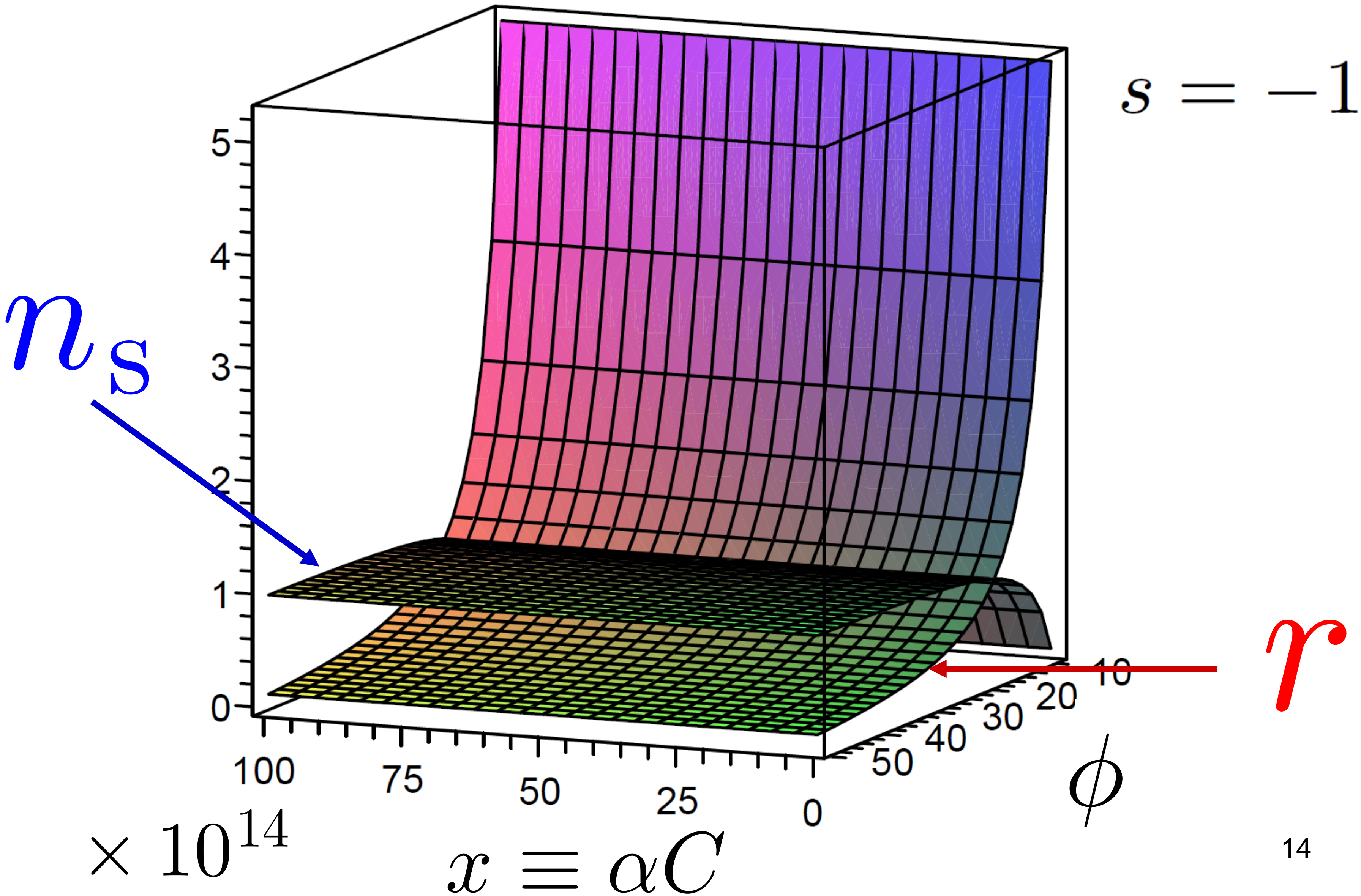
$$\epsilon \equiv -\frac{\dot{H}}{H^2}\,, \quad \eta \equiv \epsilon - \frac{\ddot{H}}{2H\dot{H}}$$

$\phi_{\rm f}$: Value of ϕ at the end of inflation $t_{\rm f}$.

$$n_{\rm s} - 1 = -6\epsilon + 2\eta$$

$$r = 16\epsilon$$

Plots of $n_{\rm S}$ and r



Values of n_{s} and r

- $N_e = 55$, $\phi \approx 34.7$, $x \equiv \alpha C \approx 2.8 \times 10^9$

⇨ $n_{\mathrm{s}} = 0.9603, \ r \approx 0.212$

III. Case that two scalar fields are dynamical

- **Slow-roll conditions**

$$\ddot{u} \ll \dot{u}H\,, \quad \ddot{\phi} \ll \dot{\phi}H, \quad \dot{\phi}^2 - \dot{u}^2 \ll H^2$$

- **In the flat FLRW space-time**

Friedmann equation $$H^2 = \frac{1}{6}V$$

Scalar field equations

$$3sH\dot{\phi} = V_\phi\,, \quad 3sH\dot{u} = -V_u$$

Case that two scalar fields are dynamical (2)

Number of *e*-folds

$$N_e \equiv \int_{a_\mathrm{i}}^{a_\mathrm{f}} d\ln a = \int_{t_\mathrm{i}}^{t_\mathrm{f}} H\,dt$$

$$= \frac{s}{2}\int_{\phi,u}^{\phi_\mathrm{f},u_\mathrm{f}} \frac{V\left(V_u du + V_\phi d\phi\right)}{V_\phi^2 - V_u^2}$$

a_i, a_f, : Value of a at the initial time t_i of inflatin and its end t_f

Slow-roll parameters

u_f : Value of u at t_f

$$\epsilon = -\frac{\dot{V}}{2HV} = \frac{V_u^2 - V_\phi^2}{sV^2}$$

$$\eta = -\frac{1}{4HV\dot{V}}\left(\dot{V}^2 + 2\ddot{V}V\right) = -\frac{2\left(V_\phi^2 V_{\phi\phi} + V_u^2 V_{uu}\right)}{sV\left(V_\phi^2 - V_u^2\right)}$$

$$V_{\phi\phi} \equiv \partial^2 V/\partial\phi^2, \qquad V_{uu} \equiv \partial^2 V/\partial u^2$$

Values of n_{S} and r

- $N_e = 60$, $x \equiv \alpha C = 1.0 \times 10^{25}$,

  $\phi^2 - u^2 = 2317$, $s = -1$, $J = C$

⇒ $n_{\mathrm{s}} \approx 0.9793, \quad r \simeq 0.11$

IV. Einstein frame

∗ The bar denote the quantities in the Einstein frame.

Jordan frame

$g_{\mu\nu} = \Lambda \bar{g}_{\mu\nu}$ **: Conformal transformation**

Einstein frame

- $\Lambda\left[\Phi + \frac{s}{12}\left(\phi^2 - u^2\right)\right] = 1$
- $\Lambda \equiv \mathrm{e}^{\lambda}$

λ : Scalar field

$$S = \int d^4x \sqrt{-\bar{g}} \left\{ \bar{R} - \frac{3}{2}(\bar{\nabla}\lambda)^2 + \frac{s}{2}\mathrm{e}^{\lambda}\left\{(\bar{\nabla}\phi)^2 - (\bar{\nabla}u)^2\right\} - V(\lambda, \phi, u, J) \right\}$$

$$V(\lambda, \phi, u, J) = \frac{1}{2\alpha}\left[1 - \frac{s}{12}\mathrm{e}^{\lambda}(\phi^2 - u^2)\right]^2 + \mathrm{e}^{2\lambda}(\phi^2 - u^2)^2 J(y)$$

Equation of motion

Gravitational field equation

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \frac{s}{2}\mathrm{e}^{\lambda}(\nabla_\mu\phi\nabla_\nu\phi - \nabla_\mu u\nabla_\nu u) - \frac{3}{2}\nabla_\mu\lambda\nabla_\nu\lambda$$
$$+ \frac{1}{2}g_{\mu\nu}\left\{V - \frac{s}{2}\mathrm{e}^{\lambda}\left[(\nabla\phi)^2 - (\nabla u)^2\right] + \frac{3}{2}(\nabla\lambda)^2\right\} = 0$$

Scalar field equations

$$3\Box\lambda + \frac{s}{2}\mathrm{e}^{\lambda}[(\nabla\phi)^2 - (\nabla u)^2] - V_\lambda = 0$$

$$s\mathrm{e}^{\lambda}\Box\phi + s\mathrm{e}^{\lambda}\nabla^\mu\lambda\nabla_\mu\phi + V_\phi = 0$$

$$s\mathrm{e}^{\lambda}\Box u + s\mathrm{e}^{\lambda}\nabla^\mu\lambda\nabla_\mu u - V_u = 0$$

$$V_\lambda \equiv \partial V/\partial\lambda$$

Field equations in the FLRW space-time

Gravitational field equations

$$3H^2 + \frac{s}{4}\mathrm{e}^{\lambda}(\dot{\phi}^2 - \dot{u}^2) - \frac{3}{4}\dot{\lambda}^2 - \frac{1}{2}V = 0$$

$$2\dot{H} + 3H^2 - \frac{s}{4}\mathrm{e}^{\lambda}(\dot{\phi}^2 - \dot{u}^2) + \frac{3}{4}\dot{\lambda}^2 - \frac{1}{2}V = 0$$

Scalar field equations

$$3\ddot{\lambda} + 9H\dot{\lambda} + \frac{s}{2}\mathrm{e}^{\lambda}(\dot{\phi}^2 - \dot{u}^2) + V_{\lambda} = 0$$

$$s\mathrm{e}^{\lambda}\ddot{\phi} + 3s\mathrm{e}^{\lambda}H\dot{\phi} + s\mathrm{e}^{\lambda}\dot{\lambda}\dot{\phi} - V_{\phi} = 0$$

$$s\mathrm{e}^{\lambda}\ddot{u} + 3s\mathrm{e}^{\lambda}H\dot{u} + s\mathrm{e}^{\lambda}\dot{\lambda}\dot{u} + V_{u} = 0$$

λ : Dynamical inflaton field

- $\phi = \phi_0$ (= Constant $\neq 0$)
- $u = u_0$ (= Constant $\neq 0$)

$$u_0 \neq \pm\phi_0$$

Effective potential

$$V_{\text{eff}}(\lambda) = \frac{1}{2\alpha}\left(1 - \frac{\zeta}{12}\mathrm{e}^{\lambda}\right)^2$$

$$\zeta \equiv s(\phi_0^2 - u_0^2)$$

Number of *e*-folds $\quad N_e = \frac{3}{4}\lambda + \frac{9}{\zeta \mathrm{e}^{\lambda}}$

Values of n_{s} and r

$$\longrightarrow \quad n_{\mathrm{s}} = \frac{432 - 5\zeta^2 \mathrm{e}^{2\lambda} - 168\zeta \mathrm{e}^{\lambda}}{3(\zeta \mathrm{e}^{\lambda} - 12)^2}$$

$$r = \frac{64\zeta^2 \mathrm{e}^{2\lambda}}{3(\zeta \mathrm{e}^{\lambda} - 12)^2}$$

- **For** $N_e = 60$ **and** $\zeta = 0.10$,

⇨ $n_{\mathrm{s}} = 0.9652\,, \quad r = 4.0 \times 10^{-3}$

These are compatible with the Planck analysis.

IV. Graceful exit from inflation

Perturbation of the de Sitter solution

$$H = H_{\mathrm{inf}} \left(1 + \underline{\delta(t)}\right)$$

$|\delta(t)| \ll 1$

$H_{\mathrm{inf}} (> 0)$

: Hubble parameter at the inflationary stage

Gravitational field equation

$$\longrightarrow \quad 2\ddot{H} + 6H\dot{H} + \frac{3}{2}\dot{\lambda}\ddot{\lambda} + \frac{\zeta}{24\alpha}\left(1 - \frac{\zeta}{12}\mathrm{e}^{\lambda}\right)\mathrm{e}^{\lambda}\dot{\lambda} = 0$$

Field equation of λ

$$3\ddot{\lambda} + 9H\dot{\lambda} - \frac{\zeta}{12\alpha}\left(1 - \frac{\zeta}{12}\mathrm{e}^{\lambda}\right)\mathrm{e}^{\lambda} = 0$$

Instability of the de Sitter solutions

Perturbation: $\delta(t) = \exp(\beta t)$

β : Constant

$$\longrightarrow \quad 2H_{\rm inf}\beta^2 + 6H_{\rm inf}^2\beta + \frac{81}{2}H_{\rm inf}^3 = 0$$

Solution: $$\beta_{\pm} = \frac{3\left(-1 \pm \sqrt{10}\right)H_{\rm inf}}{2}$$

$\longrightarrow$ **We can have the solution of** $\beta = \beta_+ > 0$ **.**

⇨ **There can exist an unstabel de Sitter solution and hence the universe can gracefully exit from inflation.**

V. Conclusions

- **We have considered inflationary cosmology in a theory where there exist two scalar fields and an R^2 term.**

$\longrightarrow$ **We have investigated slow-roll inflation driven by the potential of the conformal scalar field in the Einstein frame.**

$\Downarrow$

- **We have found that the values of n_{s} and r can be compatible with the Planck results.**

- **We have demonstrated that the graceful exit from inflation can be realized.**

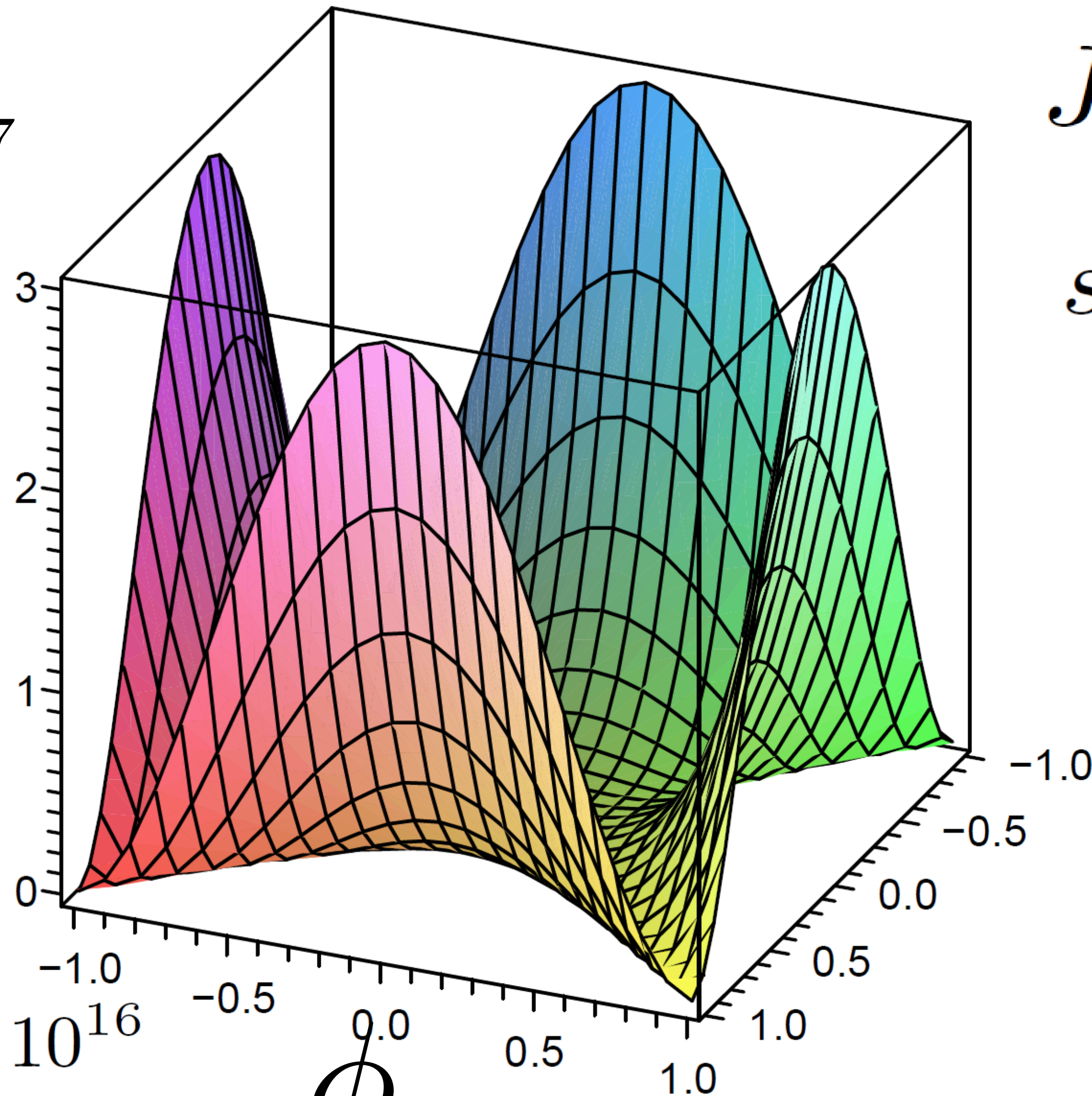
Backup slides

ポテンシャル $V(\phi, u, J)$

$\times 10^{-7}$

$J = C$

$s = -1$



u

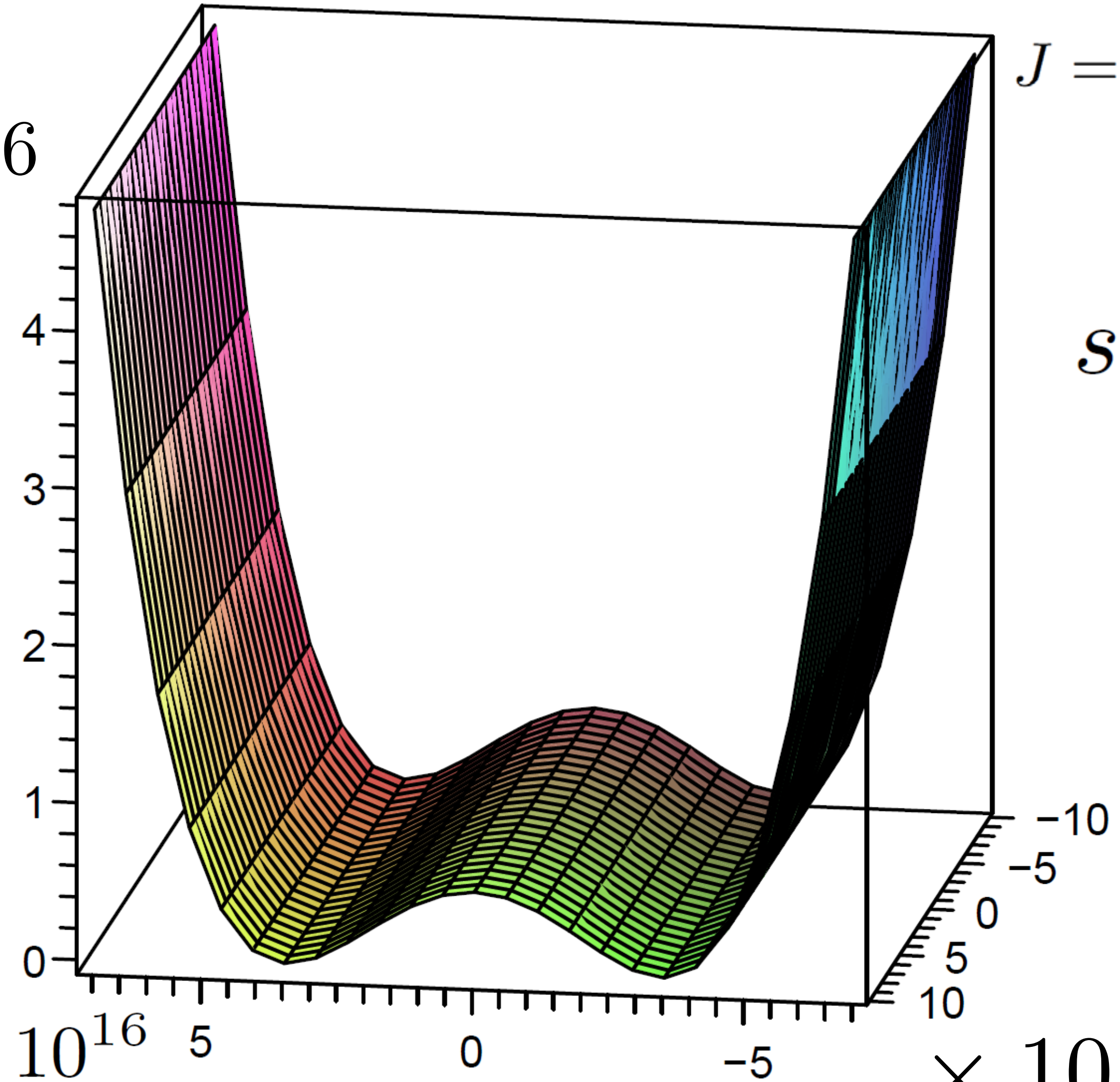
ϕ

$\alpha = 1.0 \times 10^{16}$

$C = 3.0 \times 10^{-7}$

ポテンシャル $V(\phi, u, J)$

$J = C(y^2 - 1)^{-2}$



$s = -1$

$\alpha = 1.0 \times 10^{16}$

$C = 3.0 \times 10^{-7}$

スローロールインフレーション

スローロールパラメーター: $(\epsilon,\ \eta,\ \xi^2)$

$$\epsilon \equiv \frac{1}{2\kappa^2}\left(\frac{V'(\phi)}{V(\phi)}\right)^2, \quad \eta \equiv \frac{1}{\kappa^2}\frac{V''(\phi)}{V(\phi)}, \quad \xi^2 \equiv \frac{1}{\kappa^4}\frac{V'(\phi)V'''(\phi)}{(V(\phi))^2}$$

* プライム: 各関数の引数に関する微分を表す。
（ $V'(\phi) \equiv \partial V(\phi)/\partial\phi$ ）

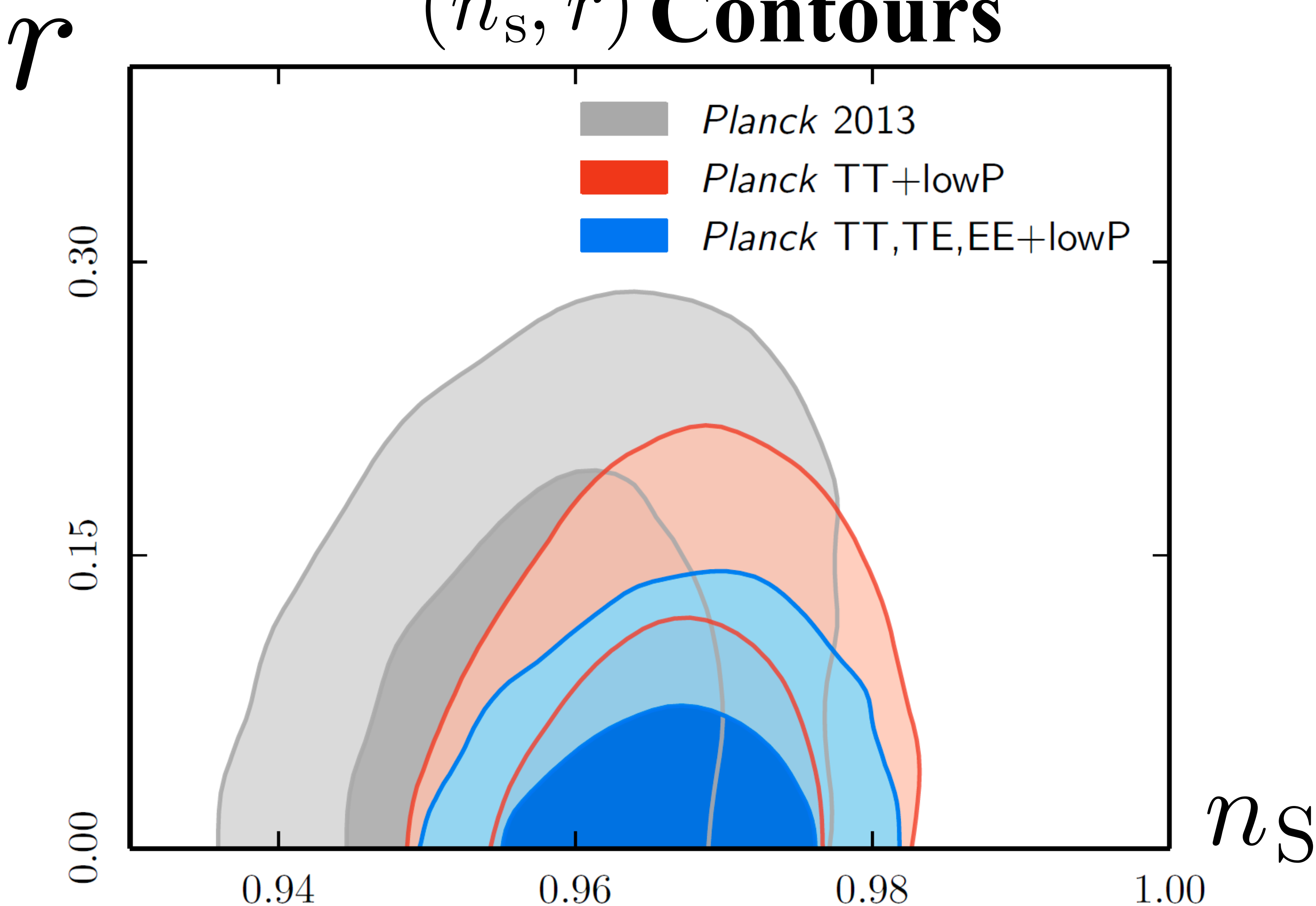
インフレーションに関する観測量: $(n_{\mathrm{S}}, r, \alpha_{\mathrm{S}})$

$$n_{\mathrm{s}} - 1 \sim -6\epsilon + 2\eta\ , \qquad r = 16\epsilon$$

k : 波数

$$\alpha_{\mathrm{s}} \equiv \frac{dn_{\mathrm{s}}}{d\ln k} \sim 16\epsilon\eta - 24\epsilon^2 - 2\xi^2$$

(n_{S}, r) Contours

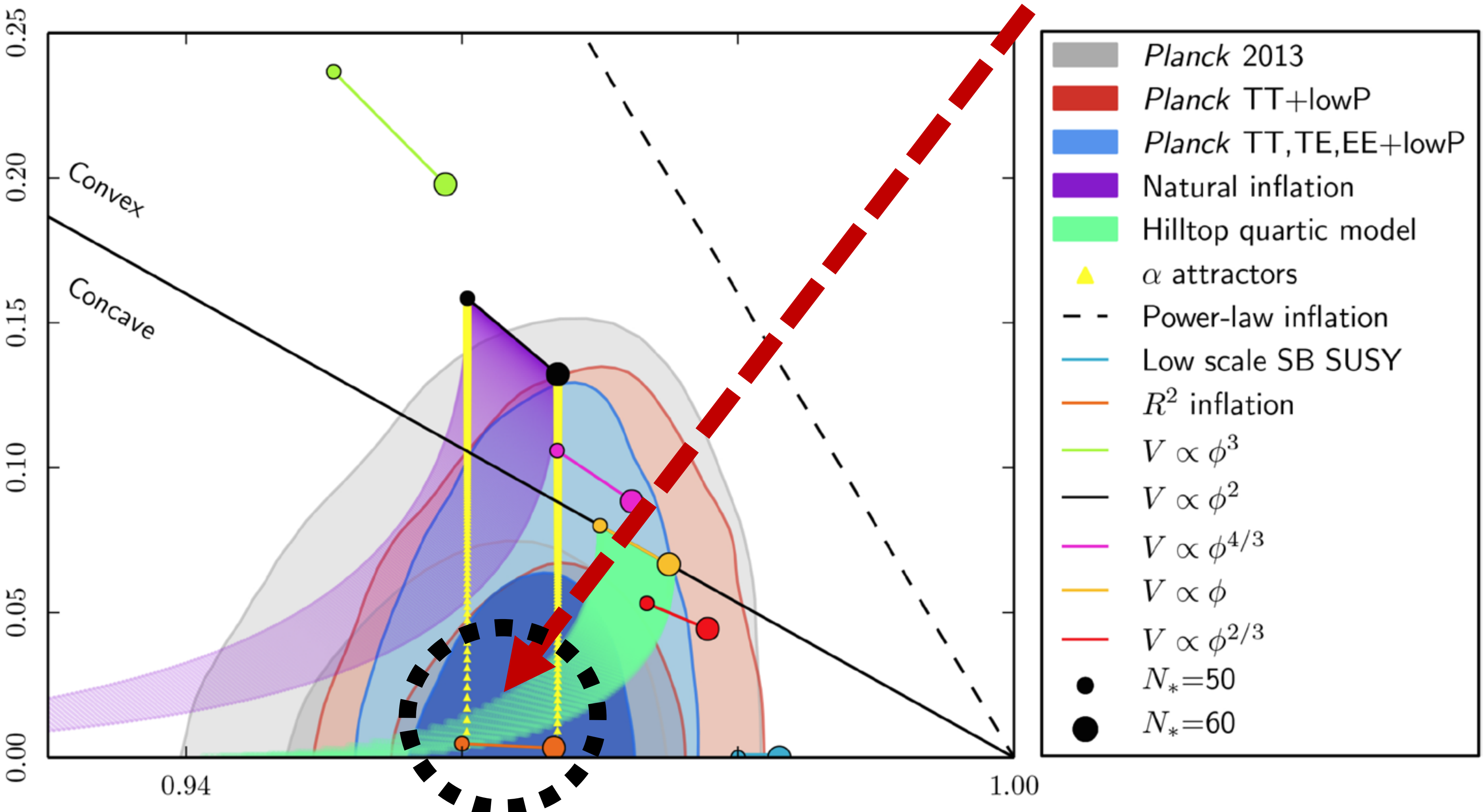


[Ade *et al*. [Planck Collaboration], arXiv:1502.02114]より転載。

インフレーションモデルへの制限

$r_{0.002}$ $(k = 0.002\ \mathrm{Mpc}^{-1})$

R^2インフレーション



n_S

[Ade *et al.* [Planck Collaboration], arXiv:1502.02114]より転載。

IV. ２つの動的なスカラー場の理論

→ ϕ, u がスローロール条件を満たす場合

- $N_e = 60$ ， $x \equiv \alpha C = 1.0 \times 10^{25}$,

$\phi^2 - u^2 = 2317$, $s = -1$, $J = C$

⇒ $n_{\mathrm{s}} \approx 0.9793 \quad r \simeq 0.11$

$n_{\rm s}$ 及び r の値

$$N_e = 60\,, \quad x = 10^{25}\,, \quad \phi^2 - u^2 = 2317$$

$$s = -1\,, \quad J = C$$

⇩

$$n_{\rm s} = 5.0 \times 10^{-9}\mathcal{G}^4 + 0.9793$$

$$\approx 0.9793$$

$$\mathcal{G}^4 \equiv u^2\phi^2$$

$$r \simeq 0.11$$

n_{s} 及び r の値

- $N_e = 55$, $\phi \approx 34.7$, $x \equiv \alpha C \approx 2.8 \times 10^9$

$$n_{\mathrm{s}} = 0.9603 \, , \quad r \approx 0.212$$

$$V_{\mathrm{eff}} \sim \frac{1.0 \times 10^{15}}{\alpha}$$

$$C < 3.0 \times 10^{-6} \; , \quad \alpha > 1.0 \times 10^{15}$$

n_s 及び r の値

$$n_\mathrm{s} = \frac{432 - 5\zeta^2 \mathrm{e}^{2\lambda} - 168\zeta\mathrm{e}^{\lambda}}{3(\zeta\mathrm{e}^{\lambda} - 12)^2}, \quad r = \frac{64\zeta^2\mathrm{e}^{2\lambda}}{3(\zeta\mathrm{e}^{\lambda} - 12)^2}$$

- $N_e = 60$ ， $\zeta = 0.10$ の場合

$\longrightarrow$ $n_\mathrm{s} = 0.9652\,, \quad r = 4.0 \times 10^{-3}$

Planck衛星の観測と整合する値が得られる。

$$V_\mathrm{eff} \approx \frac{0.7}{2\alpha}, \quad s/\,(6\alpha) \ll 1$$

ϕ が動的なインフラトンの場合

$\longrightarrow$ $u = u_0$ = 一定

$$V_u = \frac{s}{6\alpha} u \left[1 - \frac{s}{12}(\phi^2 - u^2) \right]$$

$$- 4u(\phi^2 - u^2)J(y) + \frac{1}{\phi}(\phi^2 - u^2)^2 J'(y) = 0$$

$$u_0 = 0, \quad J = 1$$

$$J'(y) \equiv dJ/dy$$

C : 定数

$$V_{\rm eff}(\phi) = \frac{1}{2\alpha}\left(1 - \frac{s}{12}\phi^2\right)^2 + C\phi^4$$

スローロールインフレーション

$$N_e(\phi) = \int_{\phi}^{\phi_{\rm f}} H(\hat{\phi}) \frac{d\hat{\phi}}{\dot{\hat{\phi}}} = -\frac{s}{2} \int_{\phi_{\rm f}}^{\phi} \frac{V(u,\hat{\phi},J)}{V_{\phi}(u,\hat{\phi},J)} d\hat{\phi}$$

スローロールパラメーター

$$\epsilon \equiv -\frac{\dot{H}}{H^2}, \quad \eta \equiv \epsilon - \frac{\ddot{H}}{2H\dot{H}}$$

⇨
$$n_{\rm s} = 1 - 6\epsilon + 2\eta$$
$$r = 16\epsilon$$

λ が動的なインフラトンの場合

$\phi = \phi_0$（＝一定 $\neq 0$）

$u = u_0$（＝一定 $\neq 0$）

$u_0 \neq \pm\phi_0$

$J'(y) \equiv dJ/dy$

$$V_\phi = \underline{-\frac{s}{6\alpha}\left[1 - \frac{s}{12}\mathrm{e}^\lambda(\phi_0^2 - u_0^2)\right]\mathrm{e}^\lambda\phi_0}$$

$$+ 4\mathrm{e}^{2\lambda}(\phi_0^2 - u_0^2)\phi_0 J(y_0) - \mathrm{e}^{2\lambda}(\phi_0^2 - u_0^2)^2 J'(y_0)\frac{u_0}{\phi_0^2} = 0$$

$$V_u = \underline{\frac{s}{6\alpha}\left[1 - \frac{s}{12}\mathrm{e}^\lambda(\phi_0^2 - u_0^2)\right]\mathrm{e}^\lambda u_0}$$

$$- 4\mathrm{e}^{2\lambda}(\phi_0^2 - u_0^2)u_0 J(y_0) + \mathrm{e}^{2\lambda}(\phi_0^2 - u_0^2)^2 J'(y_0)\frac{1}{\phi_0} = 0$$

λ が動的なインフラトンの場合

- $\phi = \phi_0$（＝一定 $\neq 0$）
- $u = u_0$（＝一定 $\neq 0$）

$$u_0 \neq \pm\phi_0$$

$$J'(y) \equiv dJ/dy$$

⇨
$$V_\phi = -\frac{s}{6\alpha}\left[1 - \frac{s}{12}\mathrm{e}^\lambda(\phi_0^2 - u_0^2)\right]\mathrm{e}^\lambda\phi_0$$

$$+\,4\mathrm{e}^{2\lambda}(\phi_0^2 - u_0^2)\phi_0 J(y_0) - \mathrm{e}^{2\lambda}(\phi_0^2 - u_0^2)^2 J'(y_0)\frac{u_0}{\phi_0^2} = 0$$

$$V_u = \frac{s}{6\alpha}\left[1 - \frac{s}{12}\mathrm{e}^\lambda(\phi_0^2 - u_0^2)\right]\mathrm{e}^\lambda u_0$$

$$-\,4\mathrm{e}^{2\lambda}(\phi_0^2 - u_0^2)u_0 J(y_0) + \mathrm{e}^{2\lambda}(\phi_0^2 - u_0^2)^2 J'(y_0)\frac{1}{\phi_0} = 0$$

$s/(6\alpha) \ll 1$ の場合

$$\longrightarrow \quad J(y_0) = 0, \quad J'(y_0) = 0$$

例: $J(y) = C(y - y_0)^q \qquad q \geq 2$

有効ポテンシャル

$$V_{\mathrm{eff}}(\lambda) = \frac{1}{2\alpha}\left(1 - \frac{\zeta}{12}\mathrm{e}^{\lambda}\right)^2 \qquad \zeta \equiv s(\phi_0^2 - u_0^2)$$

$$\longrightarrow \quad N_e(\phi) = \frac{3}{4}\lambda + \frac{9}{\zeta \mathrm{e}^{\lambda}}$$

$$N_e = 60$$

$$\zeta = 10^{-1},\ 10^{-2},\ 10^{-3},\ 10^{-4},\ 10^{-5},\ 10^{-6},$$

$$n_{\rm s} = 0.9652,\ 0.9641,\ 0.9629,\ 0.9617,\ 0.9604,\ 0.9589$$

$$r = 4.0 \times 10^{-3}$$

$$N_e = 50$$

$$\zeta = 10^{-1},\ 10^{-2},\ 10^{-3},\ 10^{-4},\ 10^{-5},\ 10^{-6},$$

$$n_{\mathrm{s}} = 0.9577,\ 0.9561,\ 0.9543,\ 0.9524,\ 0.9504,\ 0.9481$$

$$r = (5.0 - 8.0) \times 10^{-3}$$

Case 1: $K \equiv (s/12)\,\mathrm{e}^{\lambda}(\phi_0^2 - u_0^2) \gg 1$

$$\longrightarrow \quad \left[1 - \frac{s}{12}\mathrm{e}^{\lambda}(\phi_0^2 - u_0^2)\right] \approx -\frac{s}{12}\mathrm{e}^{\lambda}(\phi_0^2 - u_0^2)$$

$$J'(y_0) = 0\,, \qquad J(y_0) = -s^2/(288\alpha)$$

$$\Rightarrow \quad V_{\mathrm{eff}}(\lambda) = \frac{1}{2\alpha}\left[1 - \frac{s}{6}\mathrm{e}^{\lambda}(\phi_0^2 - u_0^2)\right]$$

$$N_e(\lambda) = \int_{\lambda}^{\lambda_{\mathrm{f}}} H(\lambda)\frac{d\lambda}{\dot{\lambda}} = \frac{3}{2}\int_{\lambda_{\mathrm{f}}}^{\lambda}\frac{V}{V_{\lambda}}d\lambda \approx \frac{3}{2}\lambda$$

スローロールインフレーション

スローロールパラメーター

$$\epsilon = \frac{1}{3}\left(\frac{1}{V_{\text{eff}}(\lambda)}\frac{dV_{\text{eff}}(\lambda)}{d\lambda}\right)^2$$

$$\eta = \frac{2}{3}\left(\frac{1}{V_{\text{eff}}(\lambda)}\frac{d^2V_{\text{eff}}(\lambda)}{d\lambda^2}\right)$$

$$\Rightarrow \quad n_{\text{s}} = \frac{\zeta^2 \mathrm{e}^{2\lambda} - 60\zeta \mathrm{e}^{\lambda} + 108}{3\left(\zeta \mathrm{e}^{\lambda} - 6\right)^2}, \quad r = \frac{16\zeta^2 \mathrm{e}^{2\lambda}}{\left(\zeta \mathrm{e}^{\lambda} - 6\right)^2}$$

$$\zeta \equiv s(\phi_0^2 - u_0^2)$$

n_{s} 及び r の値

$$N_e = 60\,, \qquad \zeta = 0.01$$

$$\longrightarrow \quad \underline{n_{\mathrm{s}} = 0.9617\,, \quad r = 4.0 \times 10^{-3}}$$

$$K = 0.013\ (\ll 1)$$

$$N_e = 50 \quad x = 10^{30} \quad J = C$$

$$\phi^2 - u^2 = 2593$$

$$\Downarrow$$

$$n_{\rm s} = 3.7 \times 10^{-9} \mathcal{G}^4 + 0.9815$$

$$r \simeq 0.1$$

Jordan系からEinstein系への共形変換

Jordan 系

作用 $S = \int d^4x\sqrt{-g}\,\frac{F(R)}{2\kappa^2}$ $\kappa^2 \equiv 8\pi G_{\mathrm{N}}$

$F(R)$:スカラー曲率Rの関数 G_{N} :重力定数

g :計量テンソル $g_{\mu\nu}$ の行列式

共形変換

$$\hat{g}_{\mu\nu} = \Omega^{-2} g_{\mu\nu}\,, \quad \Omega^2 \equiv F_R\,, \quad F_R(R) \equiv dF(R)/dR$$

$$\varphi \equiv -\sqrt{3/2}\,(1/\kappa)\ln F_R$$

[Maeda, Phys. Rev. D <u>39</u>, 3159 (1989)]
[Fujii and Maeda, *The Scalar-Tensor Theory of Gravitation* (Cambridge University Press, Cambridge, United Kingdom, 2003)]

Einstein系での作用

$$S_{\mathrm{E}} = \int d^4x \sqrt{-\hat{g}} \left(\frac{\hat{R}}{2\kappa^2} - \frac{1}{2}\hat{g}^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - V(\varphi) \right)$$

$$V(\varphi) = \frac{F_R\hat{R} - F}{2\kappa^2 (F_R)^2} \qquad F_R = \exp\left(-\sqrt{2/3}\kappa\varphi\right)$$

＊ハットはEinstein系での量を現す。

→ **スカラー場 φ をスローロールインフレーションを引き起こすインフラトン場と考える。**

・**インフレーション期での*e*-folds数**

$$N \equiv \ln\left(\frac{a_{\mathrm{f}}}{a_{\mathrm{i}}}\right)$$

$a_{\mathrm{i}}, a_{\mathrm{f}}$ ：インフレーションの最初と最後でのスケールファクター a の値

(3) スペクトル指数のランニング

$$\alpha_{\mathrm{S}} = -0.003 \pm 0.007 \ (68\% \ \mathrm{CL})$$

Starobinsky (R^2)インフレーション

作用積分 $S = \int d^4x\sqrt{-g}\frac{1}{2\kappa^2}(R + \alpha_{\mathrm{S}}R^2)$

[Starobinsky, Phys. Lett. B 91, 99 (1980)]

$\kappa^2 \equiv 8\pi/{M_{\mathrm{Pl}}}^2$ M_{Pl} :プランク質量 α_{S} :定数

- $N_e = 50 \longrightarrow n_{\mathrm{S}} = 0.960, \quad r = 4.80 \times 10^{-3}$
- $N_e = 60 \longrightarrow n_{\mathrm{S}} = 0.967, \quad r = 3.33 \times 10^{-3}$

N_e :インフレーション期のe-folds数

Cf. [Hinshaw *et al.*, Astrophys. J. Suppl. 208, 19 (2013)]

運動方程式

重力場

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \frac{s}{2}(\nabla_\mu\phi\nabla_\nu\phi - \nabla_\mu u\nabla_\nu u)$$
$$+ \frac{1}{2}g_{\mu\nu}\left\{\frac{1}{2\alpha}\left[1 - \frac{s}{12}(\phi^2 - u^2)\right]^2\right.$$
$$\left. - \frac{s}{2}\left[(\nabla\phi)^2 - (\nabla u)^2\right] + (\phi^2 - u^2)^2 J(y)\right\} = 0$$

スカラー場

$$s\Box\phi - \frac{s}{6\alpha}\phi\left[1 - \frac{s}{12}(\phi^2 - u^2)\right]$$
$$+ 4\phi(\phi^2 - u^2)J(y) - \frac{u}{\phi^2}(\phi^2 - u^2)^2 J'(y) = 0$$

$$s\Box u - \frac{s}{6\alpha}u\left[1 - \frac{s}{12}(\phi^2 - u^2)\right]$$
$$+ 4u(\phi^2 - u^2)J(y) - \frac{1}{\phi}(\phi^2 - u^2)^2 J'(y) = 0$$

$\Box \equiv g^{\mu\nu}\nabla_\mu\nabla_\nu$:共変ダランベルシャン $J'(y) \equiv dJ/dy$

II. スカラー場の理論

$$N_e(\phi) = \int_{\phi}^{\phi_{\rm f}} H(\hat{\phi}) \frac{d\hat{\phi}}{\dot{\hat{\phi}}} = -\frac{s}{2}\int_{\phi_{\rm f}}^{\phi} \frac{V(u,\hat{\phi},J)}{V_{\phi}(u,\hat{\phi},J)} d\hat{\phi}$$

↓

$\phi \gg \phi_{\rm f}$ ：インフレーション終了時の ϕ の値

$$N_e(\phi) = -\frac{s}{2}\left[\frac{\phi^2}{8} + \frac{432\alpha C \ln(s^2\phi^2 + 288\alpha C\phi^2 - 12s)}{s\,(s^2 + 288\alpha C)} - \frac{3\ln\phi}{s}\right]$$

$$\epsilon = -\frac{1}{s}\left(\frac{1}{V_{\rm eff}(\phi)}\frac{dV_{\rm eff}(\phi)}{d\phi}\right)^2 = -\frac{16\phi^2}{s}\left[\frac{(s^2 + 288\alpha C)\phi^2 - 12s}{288\alpha C\phi^4 + (12 - s\phi^2)^2}\right]^2$$

$$\eta = -\frac{2}{s}\frac{1}{V_{\rm eff}(\phi)}\frac{d^2V_{\rm eff}(\phi)}{d\phi^2} = -\frac{24}{s}\frac{(s^2 + 288\alpha C)\phi^2 - 4s}{288\alpha C\phi^4 + (12 - s\phi^2)^2}$$

$$S = \int d^4x\sqrt{-g}\left\{R - \frac{1}{2\alpha}\left[1 - \frac{s}{12}(\phi^2 - u^2)\right]^2 + \frac{s}{2}\left[(\nabla\phi)^2 - (\nabla u)^2\right] - (\phi^2 - u^2)^2 J(y)\right\}$$

$$N_e = \frac{s}{16}(u^2 - \phi^2) + \frac{3}{2}\ln(\phi^2 - u^2) - \frac{432x}{288x + s^2}\ln\left[\left(288x + s^2\right)\left(\phi^2 - u^2\right) - 12s\right]$$

$$\epsilon = -\frac{16(\phi^2 - u^2)}{s}\left\{\frac{(288x + s^2)(\phi^2 - u^2) - 12s}{288x(\phi^2 - u^2)^2 + [12 - s(\phi^2 - u^2)]^2}\right\}^2$$

$$\eta = -\left[\frac{8}{s\left(\phi^2 - u^2\right)}\right]\frac{\left(288x + s^2\right)\left(3\phi^4 - 2\phi^2u^2 + 3u^4\right) - 12s\left(\phi^2 - u^2\right)}{288x\left(\phi^2 - u^2\right)^2 + [12 - s(\phi^2 - u^2)]^2}$$

$$R = \Lambda^{-1}\left[\bar{R} - 3\Lambda^{-1}\bar{\Box}\Lambda + \frac{3}{2}\Lambda^{-2}(\bar{\nabla}\Lambda)^2\right] \qquad \kappa^2 = 8\pi G$$

G :重力定数

$$J = J(y^2) \qquad J(y) = \frac{C}{(y^2-1)^2}$$

$$S = \int d^4x\sqrt{-\bar{g}}\left\{\bar{R} - \frac{3}{2}\Lambda^{-2}(\bar{\nabla}\Lambda)^2 - \frac{1}{2\alpha} + \frac{s}{12\alpha}\Lambda(\phi^2 - u^2)\right.$$
$$\left. + \frac{s}{2}\Lambda\left[(\bar{\nabla}\phi)^2 - (\bar{\nabla}u)^2\right] - \Lambda^2(\phi^2-u^2)^2 J(y)\right\}$$

$$S = \int d^4x\sqrt{-\bar{g}}\left\{\bar{R} - \frac{3}{2}(\bar{\nabla}\lambda)^2 - \frac{1}{2\alpha}\left[1 - \frac{s}{12}e^{\lambda}(\phi^2-u^2)\right]^2\right.$$
$$\left. + \frac{s}{2}e^{\lambda}\left[(\bar{\nabla}\phi)^2 - (\bar{\nabla}u)^2\right] - e^{2\lambda}(\phi^2-u^2)^2 J(y)\right\}$$

II. スカラー場の理論

$$\lambda \gg \lambda_{\mathrm{f}}$$

$$N_e(\lambda) = \frac{3}{2}\left[\lambda + \frac{6}{s(\phi_0^2 - u_0^2)\mathrm{e}^{\lambda}}\right]$$

$$\Lambda = \mathrm{e}^{\lambda}$$

$$\epsilon = \frac{1}{3}\left(\frac{1}{V_{\mathrm{eff}}(\lambda)}\frac{dV_{\mathrm{eff}}(\lambda)}{d\lambda}\right)^2$$

$$H \approx 1/(3t)$$

$$\eta = \frac{2}{3}\left(\frac{1}{V_{\mathrm{eff}}(\lambda)}\frac{d^2V_{\mathrm{eff}}(\lambda)}{d\lambda^2}\right)$$

$$\lambda \approx \ln(Ht)$$

$$n_{\mathrm{s}} = \frac{\zeta^2\mathrm{e}^{2\lambda} - 60\zeta\mathrm{e}^{\lambda} + 108}{3\left(\zeta\mathrm{e}^{\lambda} - 6\right)^2}$$

$$r = \frac{16\zeta^2\mathrm{e}^{2\lambda}}{\left(\zeta\mathrm{e}^{\lambda} - 6\right)^2}$$

$$\zeta \equiv s(\phi_0^2 - u_0^2)$$

II. スカラー場の理論

$$N_e = \frac{s}{2}\int_{\phi,u}^{\phi_{\rm f},u_{\rm f}} \frac{V\left(V_u du + V_\phi d\phi\right)}{V_\phi^2 - V_u^2}$$

$$\epsilon = -\frac{\dot{V}}{2HV} = \frac{V_u^2 - V_\phi^2}{sV^2}$$

$$\eta = -\frac{1}{4HV\dot{V}}\left(\dot{V}^2 + 2\ddot{V}V\right) = -\frac{2\left(V_\phi^2 V_{\phi\phi} + V_u^2 V_{uu}\right)}{sV\left(V_\phi^2 - V_u^2\right)}$$

$$\phi \approx u \qquad\qquad J = C$$

$$dV = V_u du + V_\phi d\phi = V_u \dot{u} dt + V_\phi \dot{\phi} dt$$