

Realizing the relaxion from multiple axions  
and its **UV completion** with **high scale supersymmetry**

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# Outline

- Review of the relaxion mechanism  
Huge relaxion excursion
- Relaxion from multiple axions  
Clockwork mechanism
- A UV completion with high scale SUSY
- Summary

# Relaxion mechanism

Graham, Kaplan, Rajendran (2015)

Relaxion-dependent  
Higgs mass

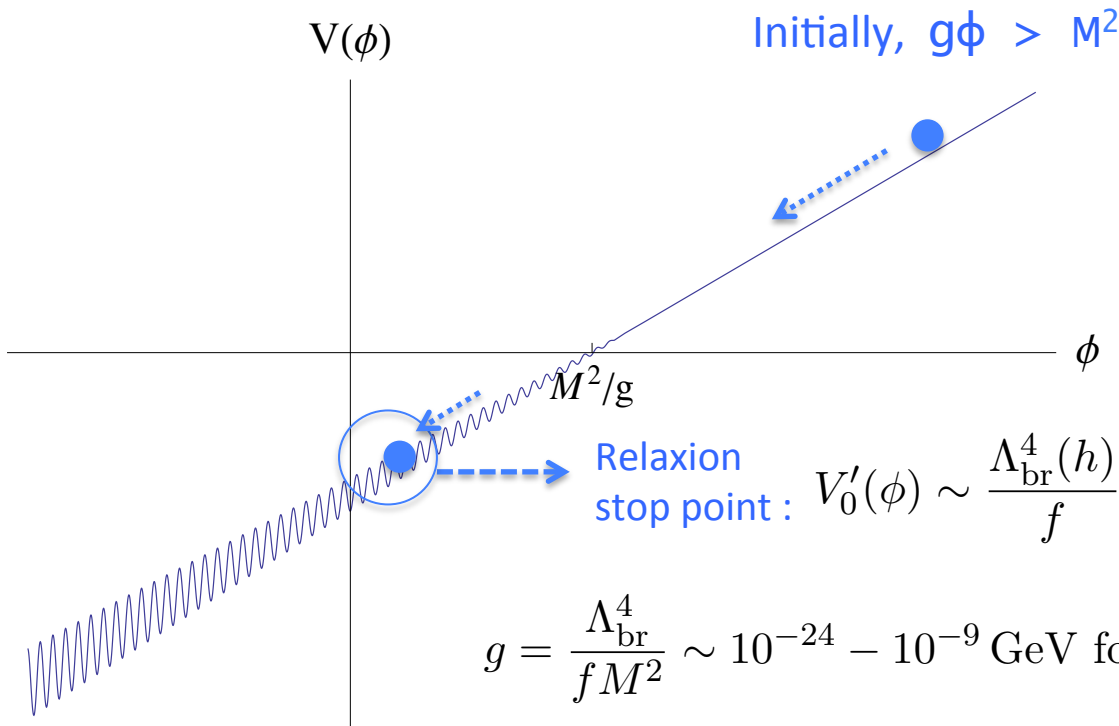
Relaxion rolling  
potential

Back reaction  
sector

$$V(\phi, h) = \boxed{(-M^2 + g\phi) h^2} + \boxed{V_0(g\phi)} + \boxed{\frac{1}{32\pi^2} \frac{\phi}{f} \tilde{G}^{\mu\nu} G_{\mu\nu}}$$

$g$ : small shift-symmetry ( $\phi \rightarrow \phi + \alpha$ )  
breaking parameter

$$\rightarrow (-M^2 + g\phi) h^2 + (gM^2\phi + g^2\phi^2 + \dots) + \boxed{\Lambda_{\text{br}}^4(h) \cos\left(\frac{\phi}{f}\right)}$$



$$\Lambda_{\text{br}}^4(h) \sim \begin{cases} y_u \Lambda_{\text{QCD}}^3 h \\ \Lambda_{\text{HC}}^2 h^2 \end{cases}$$

$$\left(\frac{\langle h \rangle}{v}\right)^{1,2} = g \frac{M^2 f}{\Lambda_{\text{br}}^4(h=v)}$$

$$g = \frac{\Lambda_{\text{br}}^4}{fM^2} \sim 10^{-24} - 10^{-9} \text{ GeV for } M = 100 \text{ TeV} \quad \langle h \rangle = v$$

# Long field excursion of the relaxion

- Without fine-tuning of the initial position of the relaxion,

$$(-M^2 + g\phi)h^2 \rightarrow g\Delta\phi \gtrsim M^2$$



$$\Delta\phi \gtrsim \frac{M^2}{g} = f \left( \frac{M}{\Lambda_{\text{br}}} \right)^4 = 10^{22} \text{ GeV} \left( \frac{f}{10^{10} \text{ GeV}} \right) \left( \frac{M}{100 \text{ TeV}} \right)^4 \left( \frac{100 \text{ GeV}}{\Lambda_{\text{br}}} \right)^4$$

- Hierarchy conversion :

$$M/v \gg 1 \quad \rightarrow \quad \Delta\phi/f \gtrsim M^4/v^4$$

- A sensible EFT with a UV completion ?



# Multiple axions : overall picture

$$V(\phi, h) = \boxed{\mu_h^2(\phi)|h|^2} + \boxed{V_0(\phi)} + \boxed{V_{\text{br}}(\phi, h)}$$

Relaxion-dependent  
Higgs mass
Relaxion rolling  
potential
Back reaction  
potential

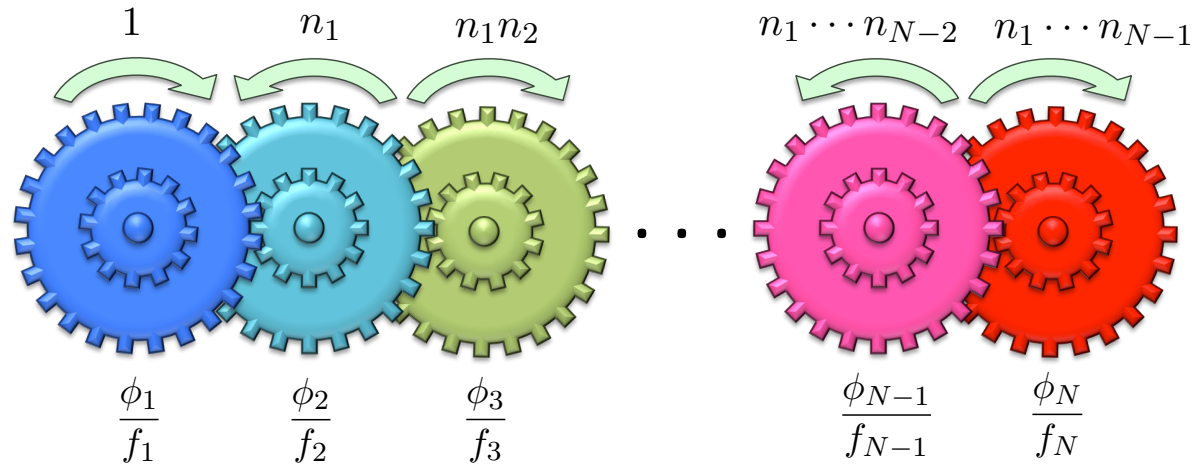
$$\begin{aligned}
 \mu_h^2(\phi) &= -M^2 + g\phi + \dots & g &\sim \frac{M^2}{f_{\text{eff}}} & \mu_h^2(\phi) &= -c_1 M^2 + c_2 M^2 \cos\left(\frac{\phi}{f_{\text{eff}}} + \delta_h\right) \\
 V_0(\phi) &= gM^2\phi + g^2\phi^2 + \dots & & & V_0(\phi) &= c_3 M^4 \cos\left(\frac{\phi}{f_{\text{eff}}} + \delta_0\right) \\
 V_{\text{br}}(\phi, h) &= \Lambda_{\text{br}}^4(h) \cos\left(\frac{\phi}{f}\right) & & & V_{\text{br}}(\phi, h) &= \Lambda_{\text{br}}^4(h) \cos\left(\frac{\phi}{f} + \delta_{\text{br}}\right)
 \end{aligned}$$

$$\text{with } f_{\text{eff}} \sim e^{\xi N} f \quad (\xi \sim \mathcal{O}(1))$$

$N$ : number of axions

# Clockwork mechanism for exponentially large $f_{\text{eff}}$

K. Choi, S.H. Im (1511.00132); Kaplan, Rattazzi (1511.01827)



$$V_{\text{clock}} = \Lambda_1^4 \cos\left(n_1 \frac{\phi_1}{f_1} + \frac{\phi_2}{f_2}\right) + \Lambda_2^4 \cos\left(n_2 \frac{\phi_2}{f_2} + \frac{\phi_3}{f_3}\right) + \dots + \Lambda_{N-1}^4 \cos\left(n_{N-1} \frac{\phi_{N-1}}{f_{N-1}} + \frac{\phi_N}{f_N}\right)$$



$$\left(\frac{\Delta\phi_1}{f_1}, \frac{\Delta\phi_2}{f_2}, \frac{\Delta\phi_3}{f_3}, \dots, \frac{\Delta\phi_N}{f_N}\right) = (1, -n_1, n_1 n_2, \dots, (-1)^{N-1} n_1 \dots n_{N-1})$$

Flat direction



$$f_{\text{eff}} = \sqrt{\sum_{k=1}^N n_1^2 \dots n_{k-1}^2 f_k^2} \sim n_1 \dots n_{N-1} f \sim e^N f$$

# Relaxion potentials from the “end axions”

$$\mathcal{L} = \frac{1}{2} \sum_{i=1}^N (\partial_\mu \phi_i)^2 + \Lambda_1^4 \cos \left( n_1 \frac{\phi_1}{f_1} + \frac{\phi_2}{f_2} \right) + \Lambda_2^4 \cos \left( n_2 \frac{\phi_2}{f_2} + \frac{\phi_3}{f_3} \right) + \dots + \Lambda_{N-1}^4 \cos \left( n_{N-1} \frac{\phi_{N-1}}{f_{N-1}} + \frac{\phi_N}{f_N} \right) + \epsilon_1 \Lambda^4 \cos \left( \frac{\phi_1}{f_1} \right) + \epsilon_N \Lambda^4 \cos \left( \frac{\phi_N}{f_N} \right)$$

$V_{\text{clock}}$

defines flat direction

$\epsilon_1, \epsilon_N \ll 1$

Integrating out the heavy axions,

$$\mathcal{L}_{\text{eff}} = \frac{1}{2} (\partial_\mu \phi_{\text{rel}})^2 + \epsilon_1 \Lambda^4 \cos \left( \frac{\phi_{\text{rel}}}{f_{\text{eff}}} \right) + \epsilon_N \Lambda^4 \cos \left( \frac{n_1 \cdots n_{N-1}}{f_{\text{eff}}} \phi_{\text{rel}} \right) \sim \frac{\phi_{\text{rel}}}{f}$$

**Long periodic potential for relaxion rolling & Higgs mass scanning**

**Short periodic potential for back reaction**

$f_{\text{eff}} \sim n_1 \cdots n_{N-1} f$   
 $\sim e^N f$

# A UV model : Need for SUSY

$$V(\phi_k, h) = \sum_{k=1}^N \Lambda_k^4 \cos\left(n_k \frac{\phi_k}{f_k} + \frac{\phi_{k+1}}{f_{k+1}}\right) \text{ defines flat direction (clockwork)}$$

$$+ \left[ -M_h^2 + c_1 M_h^2 \cos\left(\frac{\phi_1}{f_1} + \delta_1\right) \right] h^2 + c'_1 M_h^4 \cos\left(\frac{\phi_1}{f_1} + \delta'_1\right) + \Lambda_{\text{br}}^4(h) \cos\left(\frac{\phi_N}{f_N} + \delta_N\right)$$

$M_h$  : Higgs mass cutoff  $c_1 > 1$

Relaxion-dependent Higgs mass Relaxion rolling potential Back reaction potential

Radiative correction from Higgs loop  $c'_1 > 1$

- In order not to spoil the approximate flat direction determined by the clockwork potential,

$$\Lambda_{\text{cut-off}}^4 > \Lambda_k^4 \gg c'_1 M_h^4 > M_h^4$$

- The Higgs mass cutoff must be well below the cut-off scale of the theory.  
 → SUSY !

# $N$ axions from $N$ $U(1)$ 's

$$U(1)_i : \quad X_i \rightarrow e^{i\beta_i} X_i, \quad Y_i \rightarrow e^{-3i\beta_i} Y_i \quad (i = 1, 2, \dots, N)$$



$X_i, Y_i$  : gauge-singlet chiral superfields

Murayama, Suzuki, Yanagida (1992)

Choi, Chun, Kim (1996)

$$W_1 = \sum_i \frac{X_i^3 Y_i}{M_*}$$

$M_*$  : Cut-off scale such as the Planck or GUT scale



$$V(X_i, Y_i) = m_{X_i}^2 |X_i|^2 + m_{Y_i}^2 |Y_i|^2 + \left( A_i \frac{X_i^3 Y_i}{M_*} + \text{h.c.} \right) + \frac{|X_i|^6}{M_*^2} + 9 \frac{|X_i|^4 |Y_i|^2}{M_*^2}$$

$$m_{X_i}^2 < 0, \quad m_{Y_i}^2 > 0 \quad \rightarrow \quad X_i \sim Y_i \sim \sqrt{m_{\text{SUSY}} M_*}$$



$$X_i \propto e^{i\phi_i/f_i}, \quad Y_i \propto e^{-3i\phi_i/f_i}$$

$N$  axions  $\phi_i$  ( $i = 1 \dots N$ ) with  $f_i \sim \sqrt{m_{\text{SUSY}} M_*}$

# Clockwork from a hidden sector dynamics

- Introduce  $(N - 1)$  hidden Yang-Mills sectors.

Gauge group  $G = \prod_{i=1}^{N-1} SU(p_i)$

Vector-like charged matter fields  $\Psi_{ia} + \Psi_{ia}^c, \quad \Phi_i + \Phi_i^c \quad (i = 1, 2, \dots, N - 1; a = 1, 2, \dots, n_i)$

- The previous  $U(1)_i$  charged field  $X_i$  couples to these matter fields through

$$W_2 = (X_1 \Psi_{1a} \Psi_{1a}^c + X_2 \Phi_1 \Phi_1^c) + (X_2 \Psi_{2a} \Psi_{2a}^c + X_3 \Phi_2 \Phi_2^c) + \dots + (X_{N-1} \Psi_{N-1a} \Psi_{N-1a}^c + X_N \Phi_{N-1} \Phi_{N-1}^c)$$



$U(1)_{i,i+1} \times SU(p_i) \times SU(p_i)$  anomalies

Axion-dependent holomorphic gauge kinetic function of  $SU(p_i)$

$$\tau_i = \frac{1}{g_i^2} + \frac{i}{8\pi^2} \left( n_i \frac{\phi_i}{f_i} + \frac{\phi_{i+1}}{f_{i+1}} \right) + \theta^2 M_{\lambda_i}$$

# Clockwork from a hidden sector dynamics

confining scale  
of  $SU(p_i)$

$$\Lambda_i \gg m_{\text{SUSY}}$$



gaugino  
condensation

$$W_{\text{np}} \sim \langle \lambda_i \lambda_i \rangle \propto \left( \exp(-8\pi^2 \tau_i) \right)^{1/p_i}$$



$$V_{\text{clock}} = \sum_{i=1}^{N-1} \frac{8\pi^2}{p_i} M_{\lambda_i} \Lambda_i^3 \cos \left( \frac{1}{p_i} \left( n_i \frac{\phi_i}{f_i} + \frac{\phi_{i+1}}{f_{i+1}} \right) \right)$$

Note  $V_{\text{clock}} \sim m_{\text{SUSY}} \Lambda_i^3 \gg M_h^4 \sim m_{\text{SUSY}}^4$



The flat direction is stable against radiative correction.

# Relaxion-dependent Higgs mass

Superpotential to generate the MSSM  $\mu$ -term

$$W_3 = \left( \frac{X_1^2}{M_*} + \frac{X_2^2}{M_*} \right) H_u H_d$$

Kim, Nilles (1984)

$$\begin{aligned} \mu_{\text{eff}} &= \mu_1 \exp\left(i2 \frac{\phi_{\text{rel}}}{f_{\text{eff}}}\right) + \mu_2 \exp\left(-i2n_1 \frac{\phi_{\text{rel}}}{f_{\text{eff}}}\right), \\ B\mu_{\text{eff}} &= b_1 \exp\left(i2 \frac{\phi_{\text{rel}}}{f_{\text{eff}}}\right) + b_2 \exp\left(-i2n_1 \frac{\phi_{\text{rel}}}{f_{\text{eff}}}\right) \end{aligned}$$

$$\begin{aligned} f_{\text{eff}} &\sim n_1 \cdots n_{N-1} f \\ &\sim e^N f \end{aligned}$$



where

$$|\mu_1| \sim |\mu_2| \sim \frac{f^2}{M_*} \sim m_{\text{SUSY}}, \quad |b_1| \sim |b_2| \sim m_{\text{SUSY}}^2$$



Naturally  
 $\mu_{\text{eff}} \sim m_{\text{SUSY}}$



$$\begin{aligned} |\mu_{\text{eff}}|^2 &= |\mu_1|^2 + |\mu_2|^2 + 2|\mu_1\mu_2| \cos\left(2(n_1 + 1) \frac{\phi_{\text{rel}}}{f_{\text{eff}}} + \delta_{\mu_1} - \delta_{\mu_2}\right), \\ |B\mu_{\text{eff}}|^2 &= |b_1|^2 + |b_2|^2 + 2|b_1b_2| \cos\left(2(n_1 + 1) \frac{\phi_{\text{rel}}}{f_{\text{eff}}} + \delta_{b_1} - \delta_{b_2}\right) \end{aligned}$$

$$\mathcal{D} = (m_{H_u}^2 + |\mu_{\text{eff}}|^2)(m_{H_d}^2 + |\mu_{\text{eff}}|^2) - |B\mu_{\text{eff}}|^2$$

Higgs mass  
determinant

For an appropriate range of  $\delta_\mu$  and  $\delta_b$ , the determinant can change its sign from positive to negative during  $\Delta\phi_{\text{rel}} = 0(f_{\text{eff}})$ .



# Relaxion rolling potential

$$W_3 = \left( \frac{X_1^2}{M_*} + \frac{X_2^2}{M_*} \right) H_u H_d$$



Radiative  
correction

$$\Delta K = \frac{X_1^2 X_2^{\dagger 2}}{M_*^2} + \text{h.c.}$$



$$V_0(\phi_{\text{rel}}) = m_0^4 \cos \left( 2(n_1 + 1) \frac{\phi_{\text{rel}}}{f_{\text{eff}}} + \delta \right)$$

$$m_0^4 \sim \frac{f_1^2 f_2^2}{M_*^2} m_{\text{SUSY}}^2 \sim m_{\text{SUSY}}^4$$

# Back reaction potential

Graham, Kaplan, Rajendran (2015)

## I. QCD

$$W_{\text{br}} = X_N Q Q Q^c \longrightarrow \frac{1}{32\pi^2} \frac{\phi_N}{f_N} G \tilde{G} \longrightarrow \frac{1}{32\pi^2} \frac{\phi_{\text{rel}}}{f} G \tilde{G} \quad f = \frac{f_{\text{eff}}}{\left(\prod_{i=1}^{N-1} n_i\right)} \sim f_N$$

$$V_{\text{br}}(h, \phi_{\text{rel}}) \approx y_u \Lambda_{\text{QCD}}^3 h \cos\left(\frac{\phi_{\text{rel}}}{f} + \delta_{\text{br}}\right)$$

## II. Hidden Color

$$W_{\text{br}} = \kappa_N \frac{X_N^2}{M_*} L L^c + \kappa_u H_u L N^c + \kappa_d H_d L^c N$$

$\underbrace{L + L^c}_{\text{SU}(2)_L \text{ doublet}}$  **Vector-like hidden colored matter fields**  
 $\underbrace{N + N^c}_{\text{SU}(2)_L \times \text{U}(1)_Y \text{ singlet}}$

Integrating out  $L + L^c$

$$V_{\text{br}}(h, \phi_{\text{rel}}) \approx \frac{\kappa_u \kappa_d \sin(2\beta)}{m_L} \Lambda_{\text{HC}}^3 h h^\dagger \cos\left(2\frac{\phi_{\text{rel}}}{f} + \delta_{\text{br}}\right)$$

$\Lambda_{\text{HC}} < m_L \sim \kappa_N m_{\text{SUSY}} < 4\pi v$   
 to prevent the Higgs loop  $> v^2$

# Summary

- The relaxion mechanism requires a **huge field excursion** (typically trans-Planckian) due to the hierarchy between a Higgs mass cutoff scale and the weak scale.
- Our **clockwork mechanism** can yield an **exponentially long relaxion** direction  $\sim e^N f$  within the compact field space of  $N$  periodic axions with their decay constants  $\sim f$  well below the Planck scale.
- Our scheme finds a **natural UV completion in high scale SUSY scenario** in order to preserve the approximate flat relaxion direction against radiative correction.
- The required **relaxion potential** is generated by a superpotential term providing a **natural solution to the MSSM  $\mu$ -problem**.
- The back reaction sector is restricted to make the (radiatively induced) Higgs independent part be subdominant.