# CMB probes on the correlated axion isocurvature peturbations 

Kenji Kadota<br>IBS Center for Theoretical Physics of the Universe(CTPU), Institute for Basic Science(IBS), S. Korea

## Based on:

$>$ "CMB probes on the correlated axion isocurvature perturbation" (arXiv:1411.3974) KK, Jinn-Ouk Gong (APCTP), Kiyotomo Ichiki (Nagoya), Takahiro Matsubara (Nagoya)
$>$ "Axion inflation with cross-correlated axion isocurvature perturbations"(arXiv:1509.04523) KK, Tatsuo Kobayashi (Hokkaido), Hajime Otsuka (Waseda).
> Motivations for cross correlations:

Parameter precision
> Concrete example (Analytical formula for cross-correlation)
> Conclusion

$$
P=P_{R}+P_{I}+P_{C}
$$

Planck (2015) 95\% CL
Uncorrelated isocurvature mode:

$$
\frac{P_{I}}{P_{R}+P_{I}}<0.038
$$

Cross-correlated isocurvature mode:

$$
0.034<\frac{P_{I}}{P_{R}+P_{I}}<0.28
$$



## Cross correlation between curvature and isocurvature perturbation

( Polarski, Starobinsky (1994), Pierpaoli, Garcia-Bellido, Borgani(1999), Enqvist, Kurki-Suonio (2000),Bucher,Noodley, Turok(2001), Amendola, Gordon, Wands, Sasaki (2002), ...)

$$
\begin{gathered}
\boldsymbol{P}=\boldsymbol{P}_{\boldsymbol{R}}+\boldsymbol{P}_{\boldsymbol{I}}+\boldsymbol{P}_{\boldsymbol{C}} \\
\mathcal{P}_{X}=A_{X}\left(k_{0}\right)\left(\frac{k}{k_{0}}\right)^{n_{X}-1} \\
A_{X}=\left(\begin{array}{cc}
A_{R} & A_{C} \\
A_{C} & A_{I}
\end{array}\right) \quad n_{X}=\left(\begin{array}{cc}
n_{R} & n_{C} \\
n_{C} & n_{I}
\end{array}\right)
\end{gathered}
$$

$\chi$ : inflaton
$a$ : energetically subdominant field (e.g. axion)

$$
\begin{aligned}
& C_{R} \sim \int d^{3} k T_{\chi}(k) T_{\chi}(k)\langle\delta \chi(k) \delta \chi(k)\rangle \\
& C_{I} \sim \int d^{3} k T_{a}(k) T_{a}(k)\langle\delta a(k) \delta a(k)\rangle \\
& \underset{\text { kenji kadota (crip, }}{C_{C}} \boldsymbol{\int} d^{3} k T_{\chi}(k) T_{a}(k)\langle\delta \chi(k) \delta a(k)\rangle
\end{aligned}
$$

Two independent solutions for the acoustic wave equation
Adiabatic initial condition
 - $-\infty \leq k \neq 27$ ace

Isocurvature initial condition


Kurki-Suonio et al (2004)


Robustness of the base $\wedge$ CDM model against different assumptions on initial conditions

Planck (2015)



> Motivations for cross correlations:
> Parameter precision
> Concrete example: Axion (analytical formula for cross-correlation)
> Conclusion


How large does $\beta c$ have to be, for the isocurvature parameters to be determined precisely by Planck? $\beta c \geq 0.1$

|  | $T$ | $T E$ | $T L$ | Joint |
| :---: | :---: | :---: | :---: | :---: |
| $\beta_{\mathcal{C}}=1$ |  |  |  |  |
| $\sigma\left(n_{\mathcal{I}}\right) / n_{\mathcal{I}}$ | 33 | 13 | 21 | 12 |
| $\sigma\left(A_{\mathcal{I}}\right) / A_{\mathcal{I}}$ | 240 | 81 | 220 | 80 |
| $\sigma\left(A_{\mathcal{C}}\right) / A_{\mathcal{C}}$ | 65 | 11 | 20 | 11 |
| $\beta_{\mathcal{C}}=0.1$ |  |  |  |  |
| $\sigma\left(n_{\mathcal{I}}\right) / n_{\mathcal{I}}$ | 110 | 39 | 65 | 38 |
| $\sigma\left(A_{\mathcal{I}}\right) / A_{\mathcal{I}}$ | 260 | 100 | 260 | 100 |
| $\sigma\left(A_{\mathcal{C}}\right) / A_{\mathcal{C}}$ | 230 | 76 | 170 | 74 |

Polarization Important!



How much does $\beta c$ affect the $\Lambda$ CDM parameter estimations? $10 \%$ or more.

|  | $\Omega_{\Lambda}$ | $\Omega_{m} h^{2}$ | $\Omega_{b} h^{2}$ | $n_{\mathcal{R}}$ | $A_{\mathcal{R}}$ | $\tau$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\beta_{\mathcal{C}}=1$ | 1.1 | 1.1 | 1.0 | 1.4 | 0.97 | 0.94 |
| $\beta_{\mathcal{C}}=0.1$ | 1.1 | 1.1 | 1.0 | 1.4 | 1.1 | 1.1 |
| No correlation | 1.0 | 1.0 | 1.0 | 1.1 | 1.1 | 1.1 |

Normalized error $\sigma / \sigma_{\text {no }}$ iso

isocurvature
> Motivations for cross correlations:
> Parameter precision
> Concrete example: Axion (analytical formula for cross-correlation)
> Conclusion

Example: Nambu-Goldstone boson


$$
\phi=\frac{r e^{i \theta}}{\sqrt{2}}, a=f_{a} \theta
$$

## Analytical Formula for $\beta c$ in terms of axion parameters

1) Upper bound on $\beta c$ in terms of axion parameters. $B C \sim 0.1$ possible.
2) Implications: Spontaneous Symmetry breaking scale $\sim \mathrm{Mp}$ is desired

$$
\begin{aligned}
& \text { e.g. I: } V \sim g \frac{\chi \phi^{4}}{m_{p l}} \quad \frac{n_{R}+n_{I}}{2}=n_{C} \\
& \beta_{C} \equiv \frac{P_{C}}{\sqrt{P_{R} P_{I}}} \sim g \sin \left(4 \theta_{0}\right)\left(\frac{f_{a}}{m_{p l}}\right)^{3}\left(\frac{m_{p l}}{H}\right)^{2} \\
& e . g . \mathrm{II}: V_{\mathrm{int}} \sim g \frac{\chi^{m} \phi^{n}}{m_{p l}^{m+n-4}} \\
& \beta_{C} \equiv \frac{P_{C}}{\sqrt{P_{R} P_{I}}} \sim g \sin \left(n \theta_{\mathrm{o}}\right)\left(\frac{\chi_{\mathrm{o}}}{m_{p l}}\right)^{m-1}\left(\frac{f_{a}}{m_{p l}}\right)^{n-1}\left(\frac{m_{p l}}{H}\right)^{2}
\end{aligned}
$$

## Model discrimination



Planck (2015), KK, Kobayashi, Otsuka(2015)

Concrete Example: Natural inflation (Freese, Frieman, Olinto (1990))
Adiabatic fluctuations:
Isocurvature fluctuations:

$$
\begin{aligned}
& V_{\mathrm{inf}}=\Lambda_{1}^{4}\left(1-\cos \frac{\phi}{f}\right) \\
& V_{\mathrm{int}}=\Lambda_{2}^{4}\left(1-\cos \left(\frac{\phi}{g_{1}}+\frac{\chi}{g_{2}}\right)\right)
\end{aligned}
$$



DSU Kyoto 2015
KK, Kobayashi, Otsuka (2015)

Concrete Example：
Axion monodromy inflation（McAllister，Silverstein，Westphal $(2008,2010)$ ）

$$
\begin{gathered}
V=\mu_{1}^{4-p} \phi^{p}+\mu_{2}^{4} \cos \left(\frac{\phi}{g_{1}}+\frac{\chi}{g_{2}}\right) \\
\frac{P_{1}}{P_{R}}-\left(\frac{\Omega_{a}}{\Omega_{m}}\right)^{2}\left(\frac{1}{\xi_{1} \theta_{0}}\right)^{2}\left(\frac{P}{\phi_{0}}\right)^{2}
\end{gathered}
$$



KK，Kobayashi，Otsuka（2015）

| $p$ | $N$ | 91 | $\mathrm{g}_{2}$ | $\mu_{2}^{4-p} / H^{2}$ | $\Omega_{a} / \Omega_{m}$ | $\cos \left(\psi_{0}+\theta_{0}\right)$ | $\theta_{0}$ | $\beta_{c}$ | $\beta_{\text {iso }}$ | $n_{s}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2 | 55 | $10^{-2}$ | $10^{-2}$ | $6 \times 10^{-7}$ | 0.03 | 1／2 | 2 | 0.002 | 0.14 | 0.964 |
| 4／3 | 55 | 10 | 10 | $3 \times$ | 0.03 | 1／2 | 2 | 0.001 | 0.1 | 0.97 |
| 1 | 55 | 10 | 10 | $4 \times$ | 0.03 | $1 /$ | 2 | 0.001 | 0.08 | 0.973 |
| Reple | 㐌勿曻 | ${ }^{10} 10-2$ | $10^{-2}$ | $4 \times 10^{-7}$ | 20.03 | 15 1／2 | 2 | 0.001 | 0.05 | 0.976 |



ㄴ. $95 \% \mathrm{CL}$ ㄴI $68 \% \mathrm{CL}$

- $\mathrm{C}=50: \phi^{2}$
- $N=60$
- $N=50$
.. $C=50: \phi^{4 / 3}$
- $\mathrm{C}=50: \phi$
- C=50: $\phi^{2 / 3}$
- $c=10: \phi_{4 / 3}^{2}$
$\cdots c=10: \phi^{4 / 3}$
- $C=10: \phi$
- $c=10: \phi^{2 / 3}$
$-c=1: \phi^{2}$
$-C=1: \phi^{2}$
$\cdots \quad c=1: \phi^{4 / 3}$
- $c=1: \phi$
- $C=1: \phi^{2 / 3}$


Conclusion:

CMB measurement perspectives:
Polarization is very powerful to constrain the isocurvature cross-correlation.
^CDM parameter estimation can be affected by $10 \%$ or more.
> Cross-correlated isocurvature modes through a concrete example: Axion

Inflation model discrimination:
preferred for the monodromy inflation, disfavored for natural inflation.

