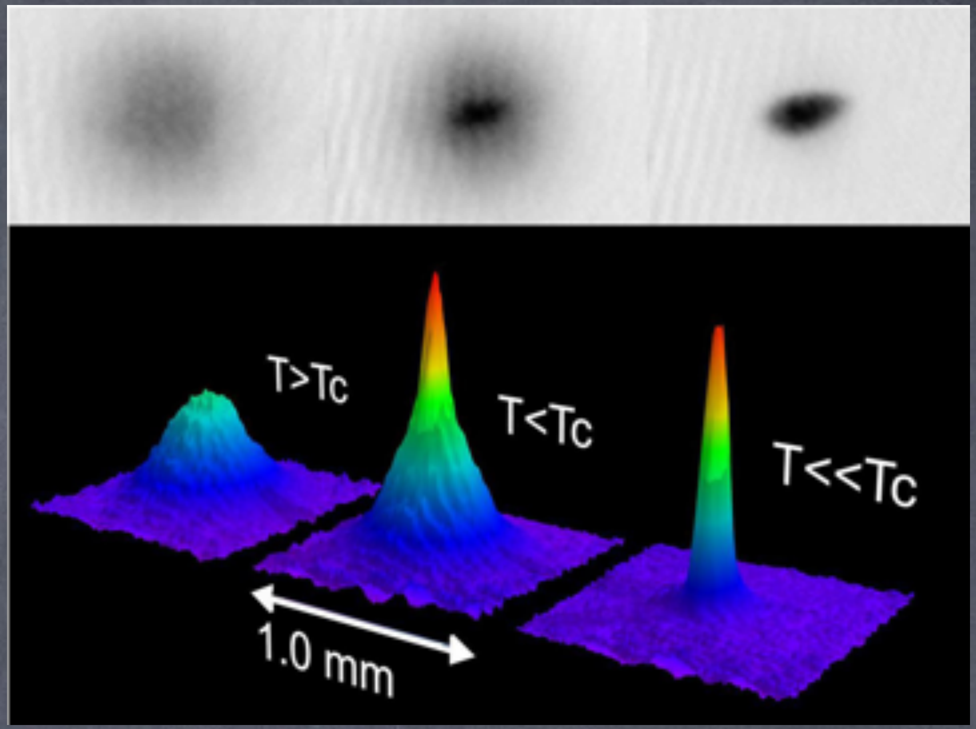


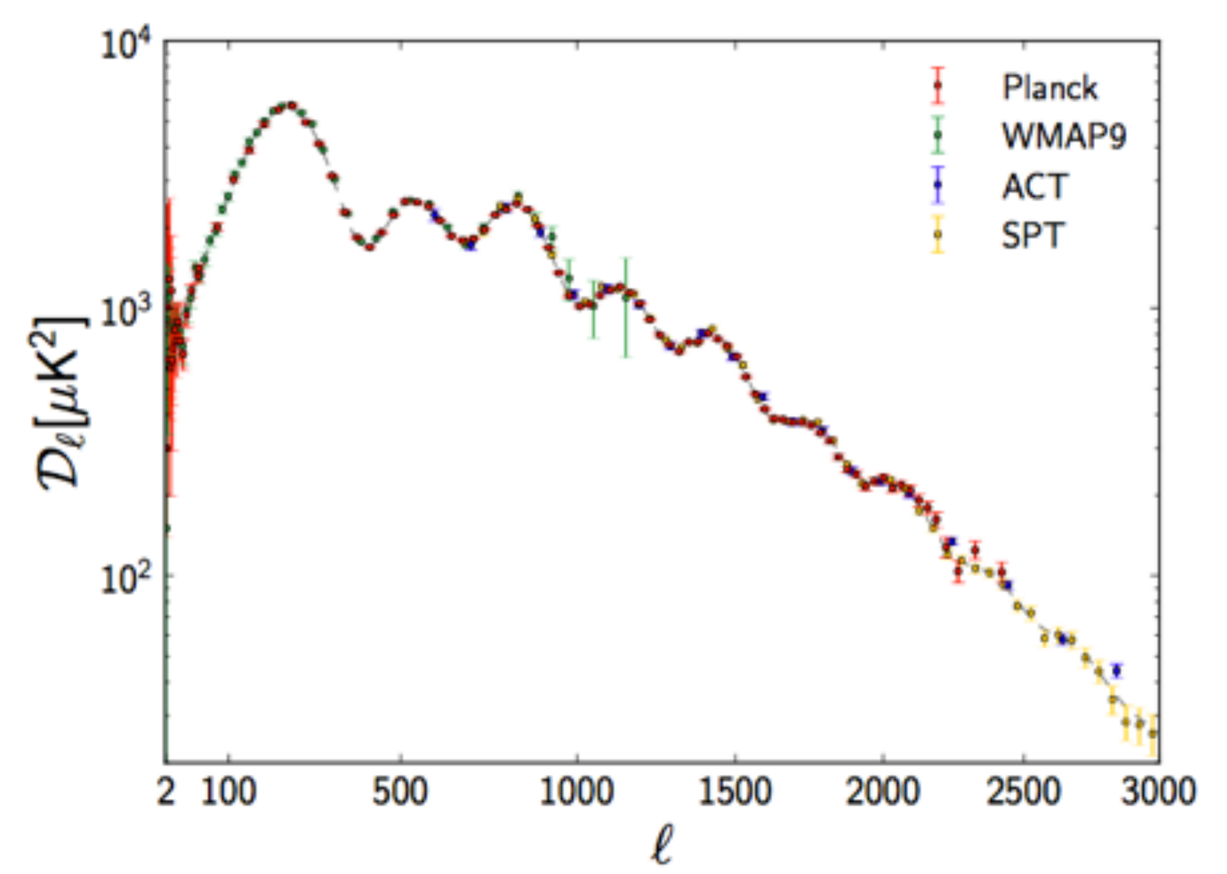
Dark Matter Superfluidity

Justin Khoury (U. Penn)

JK, 1409.0012
L. Berezhiani & JK, 1506.07877 + 1507.01019

Ongoing work with
B. Famaey, T. Lubensky, V. Miranda, A. Sharma, A. Solomon, J. Wang

The coarse-grained evidence



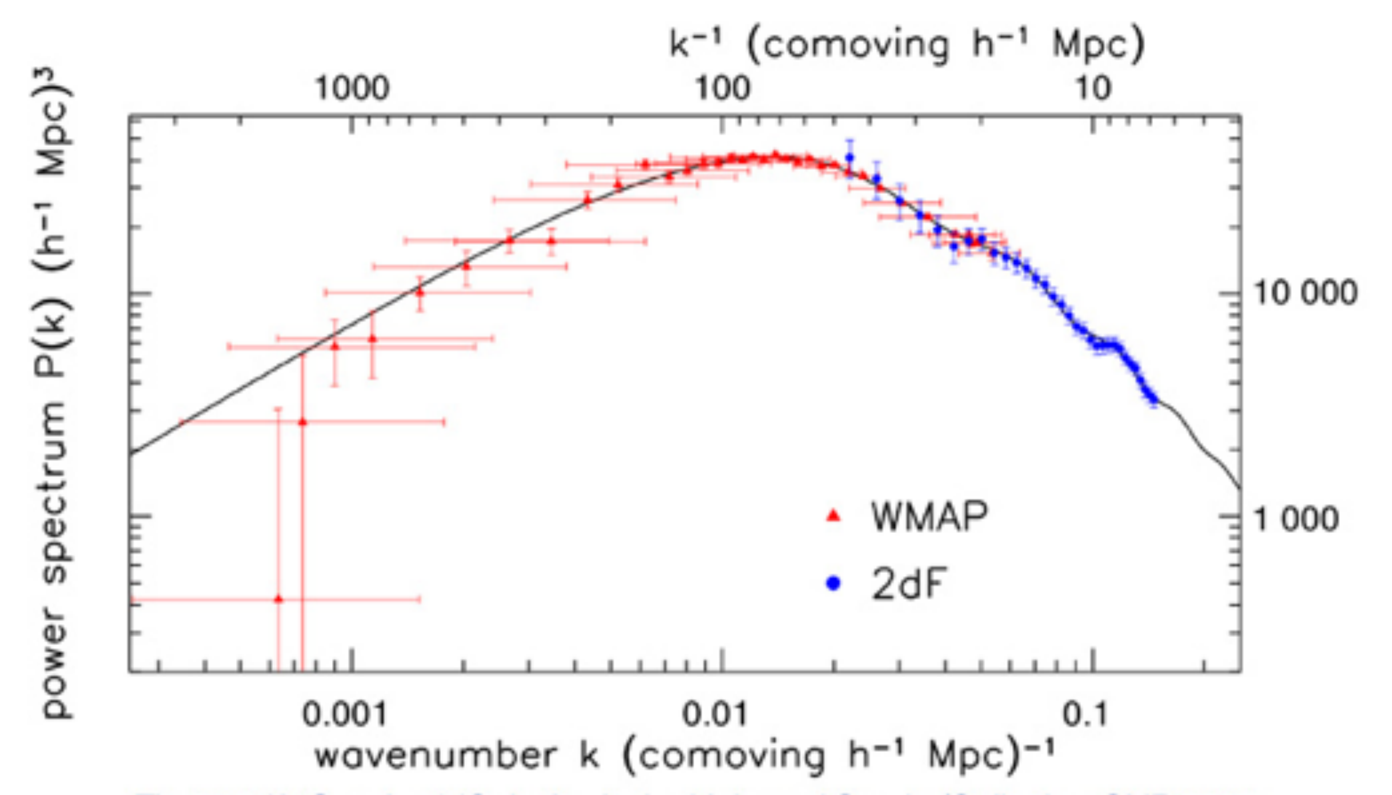
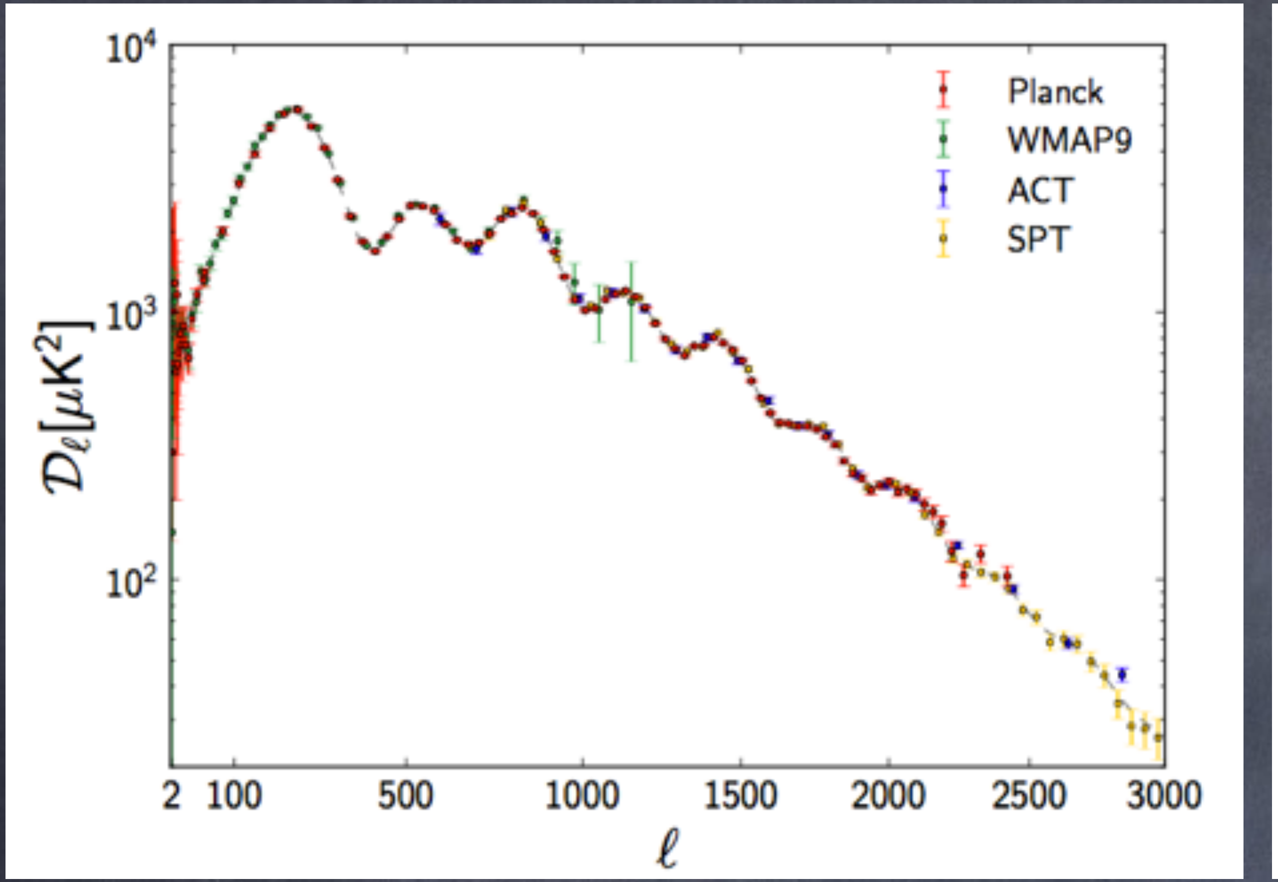
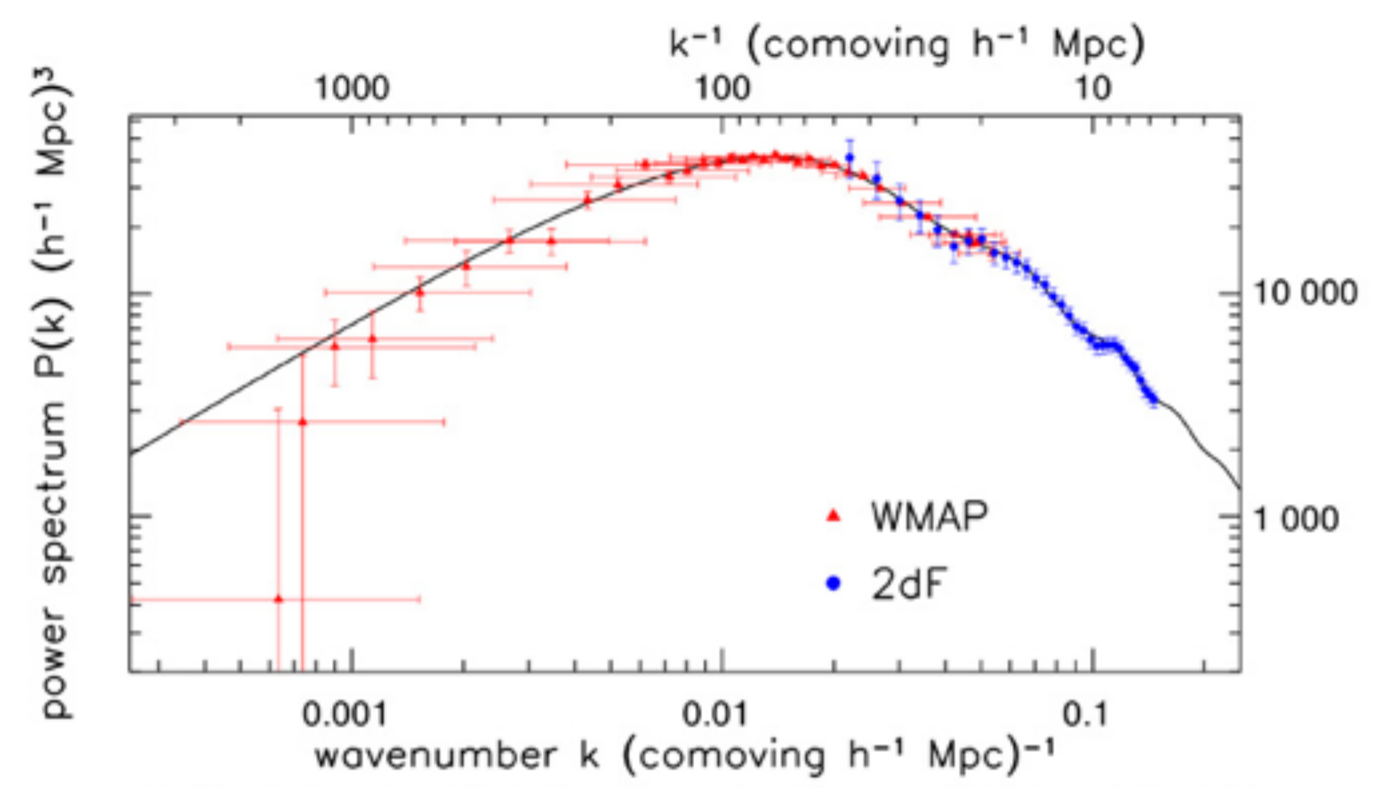


Fig 8.17 (A. Sanchez) 'Galaxies in the Universe' Sparke/Gallagher CUP 2007

- Most clear-cut evidence for DM comes from large (linear) scales

The coarse-grained evidence





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- Most clear-cut evidence for DM comes from large (linear) scales
- On these scales, only use the hydrodynamical limit of DM

$$\boxed{T_{\mu\nu} = (\rho + P)u_\mu u_\nu + P g_{\mu\nu}}$$

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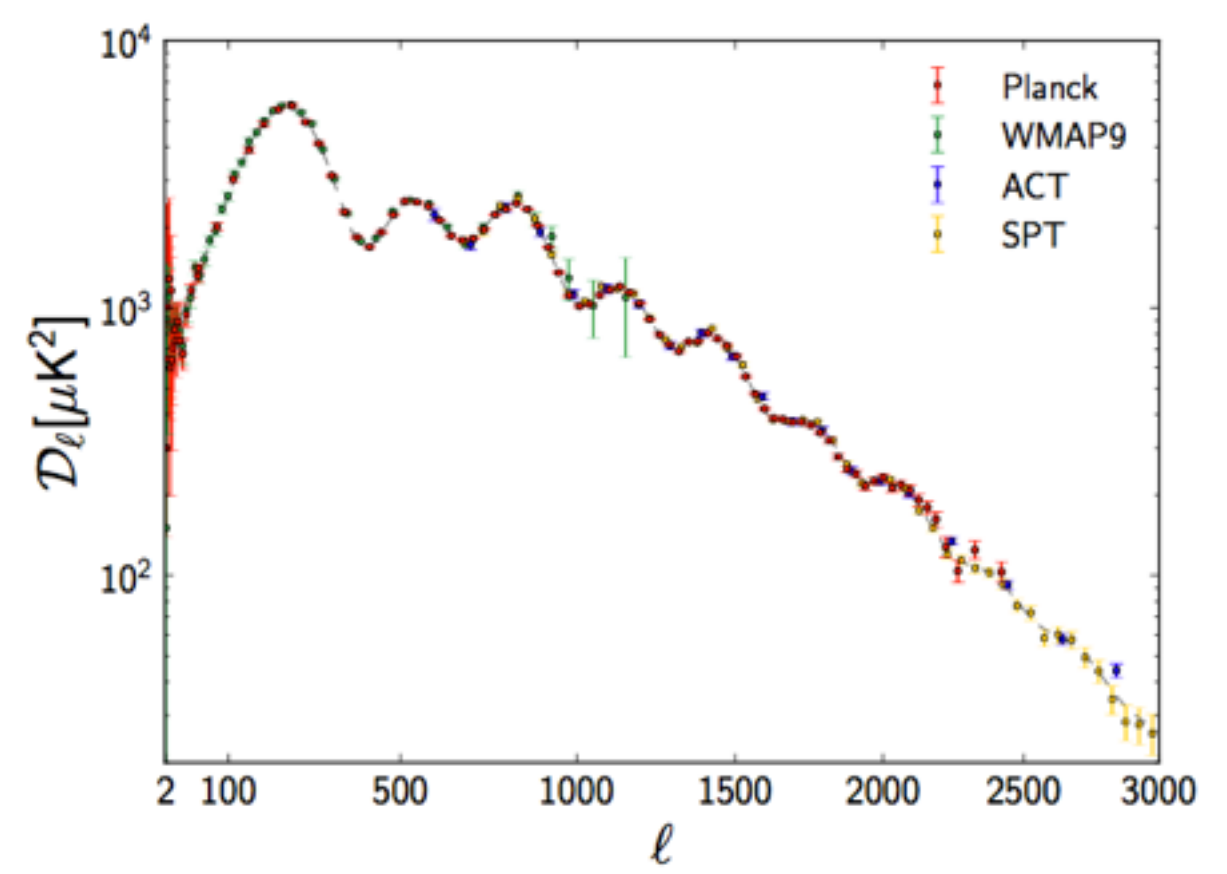

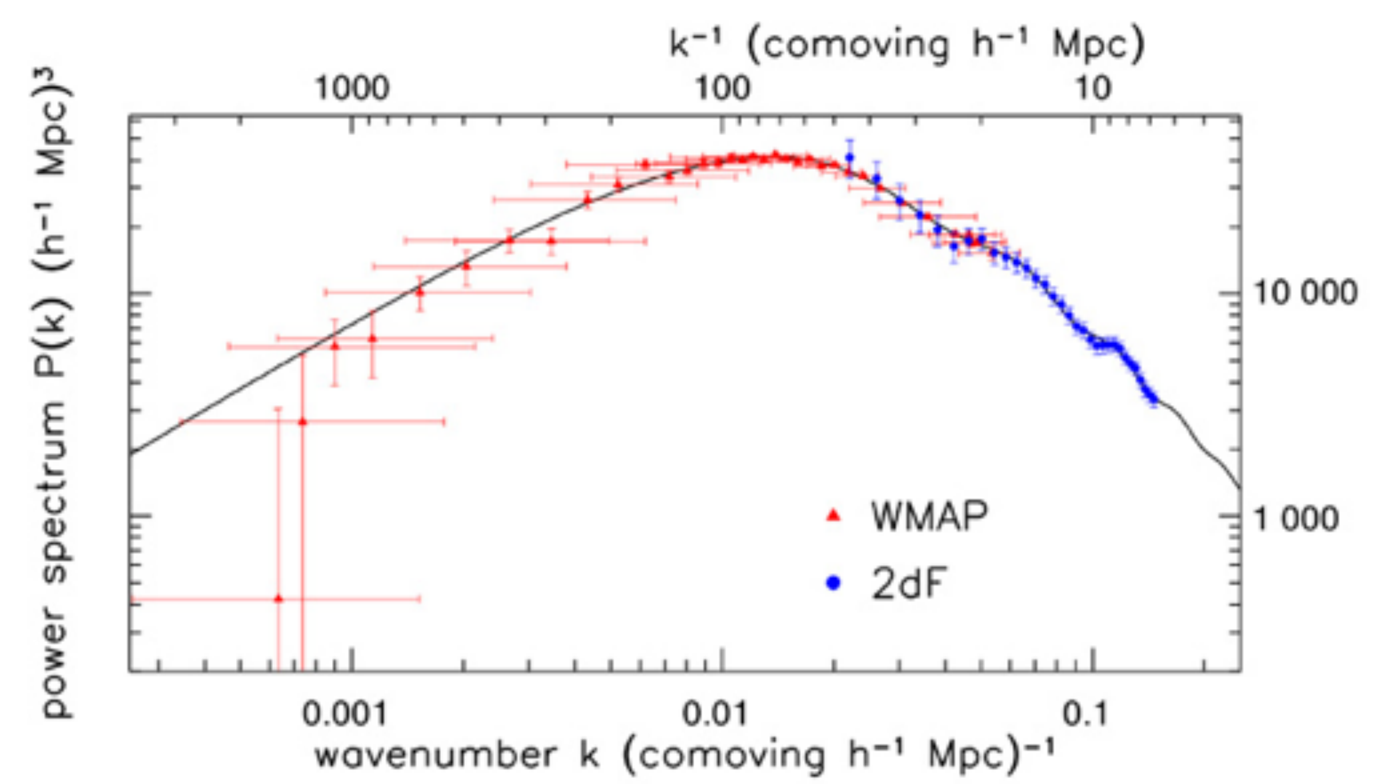

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- On these scales, only use the hydrodynamical limit of DM

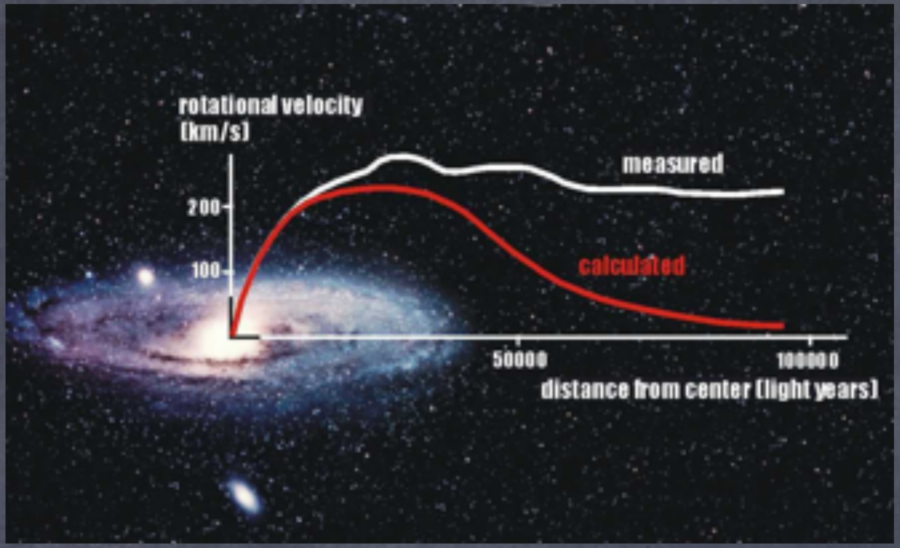
$$\boxed{T_{\mu\nu} = (\rho + P)u_\mu u_\nu + P g_{\mu\nu}}$$

$\Longrightarrow$ Any perfect fluid with $P \simeq 0$ and $c_s \simeq 0$ does the job.

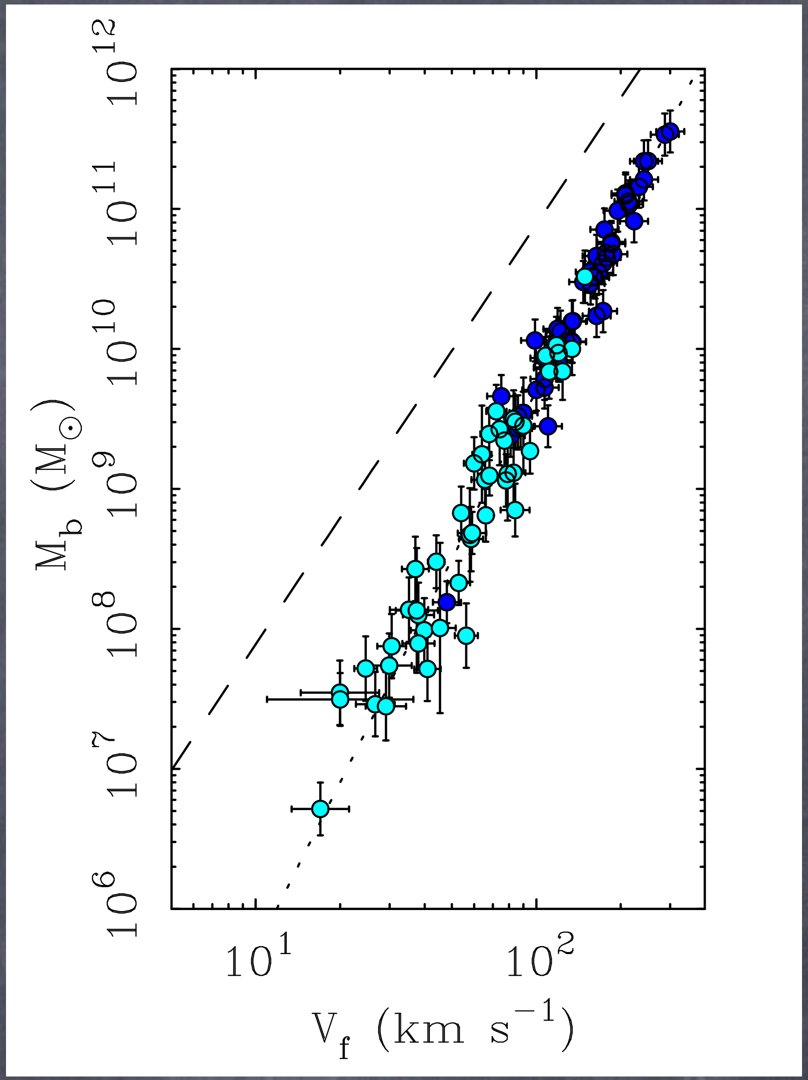
Actual galaxies are remarkably regular

- Baryonic Tully-Fisher relation

McGaugh (2011)



$$\boxed{M_\mathrm{b} \sim v_c^4}$$



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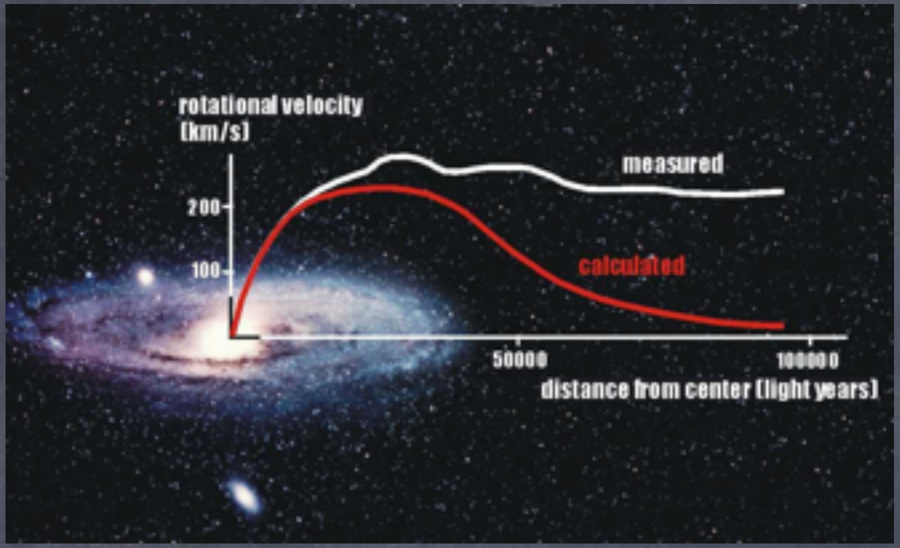

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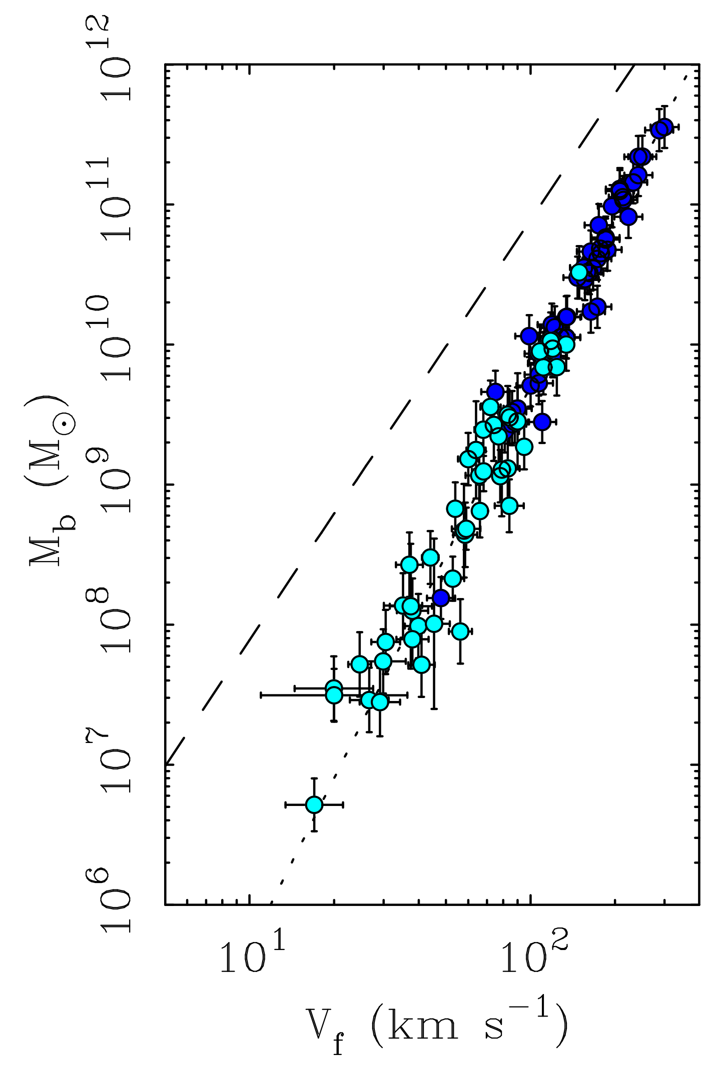
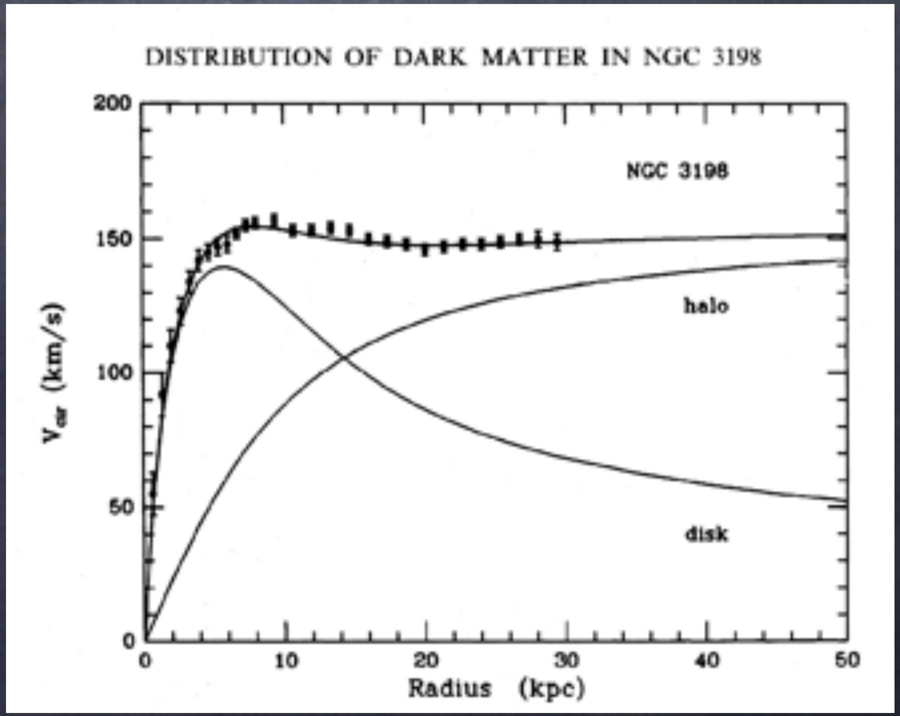


- Flat rotation curves

$$\boxed{\rho(r) \sim \frac{1}{r^2}}$$

isothermal



N-body sims: $$\rho_{\rm NFW}(r) = \frac{\rho_s}{\frac{r}{r_s}\left(1+\frac{r}{r_s}\right)^2}$$

Navarro, Frenk & White (1996)

Hydro simulations

- Star formation model
- Stellar evolution
- Mass and metal return
- Supernovae rates
- Gas enrichment
- Cooling and heating rates
- Self-shielding
- Stellar feedback
- Local and non-local SNII feedback
- Black hole and AGN feedback

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How can these feedback processes, which are inherently stochastic, result in tight correlation displayed in BTFR?

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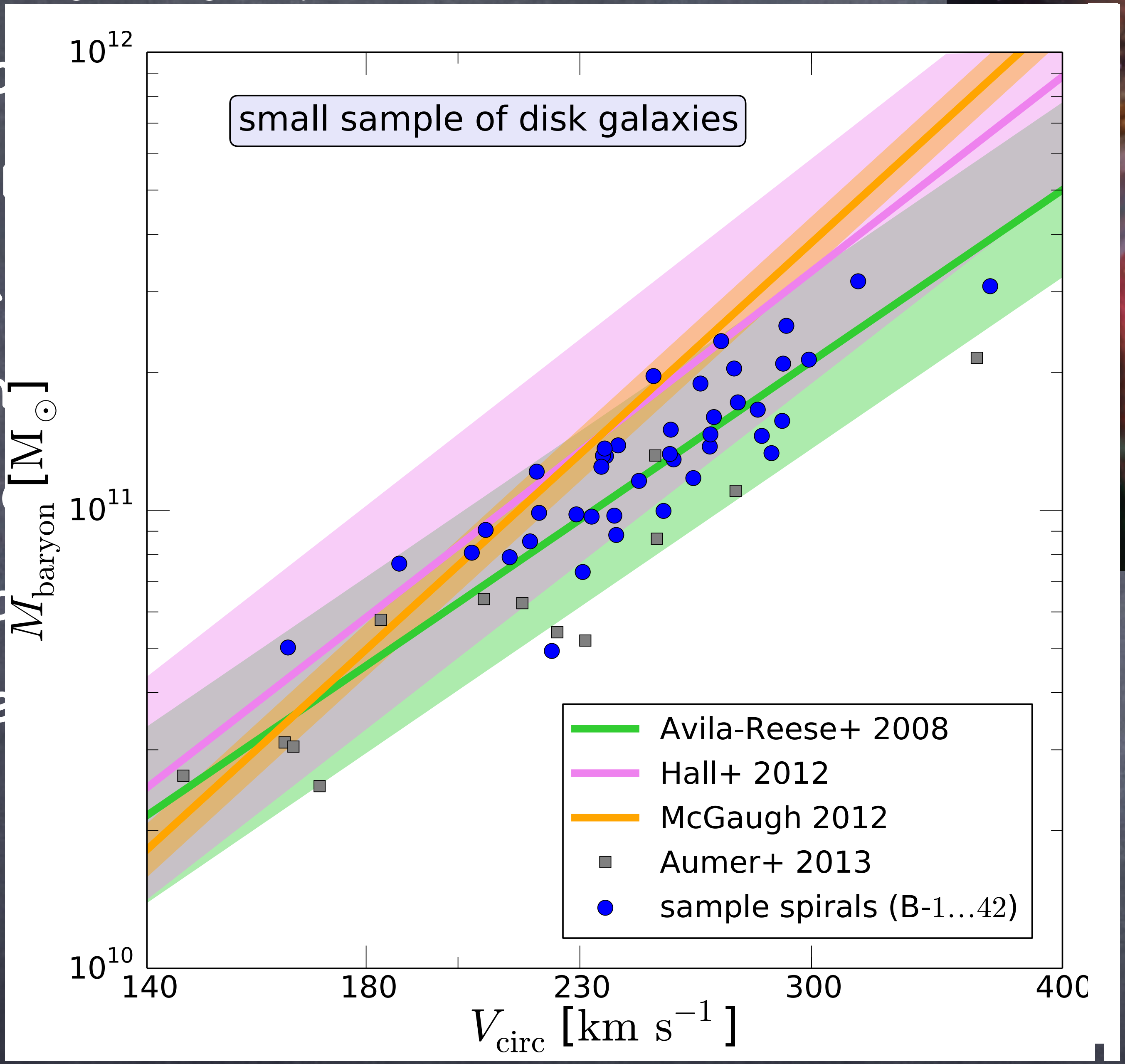

Vogelsberger et al. (2014)

How can these feedback processes, which are inherently stochastic, result in tight correlation displayed in BTFR?

Modified Newtonian Dynamics (MOND)

Milgrom (1983)

$$\boxed{a = \begin{cases} a_{\mathrm{N}} & a_{\mathrm{N}} \gg a_0 \\ \sqrt{a_{\mathrm{N}} a_0} & a_{\mathrm{N}} \ll a_0 \end{cases}}$$

$$a_{\mathrm{N}} = \frac{G_{\mathrm{N}} M_{\mathrm{b}}(r)}{r^2}$$

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Consider test mass orbiting galaxy in MOND regime,

$$\frac{v^2}{r} = \sqrt{\frac{G_{\mathrm{N}} M_{\mathrm{b}} a_0}{r^2}}$$

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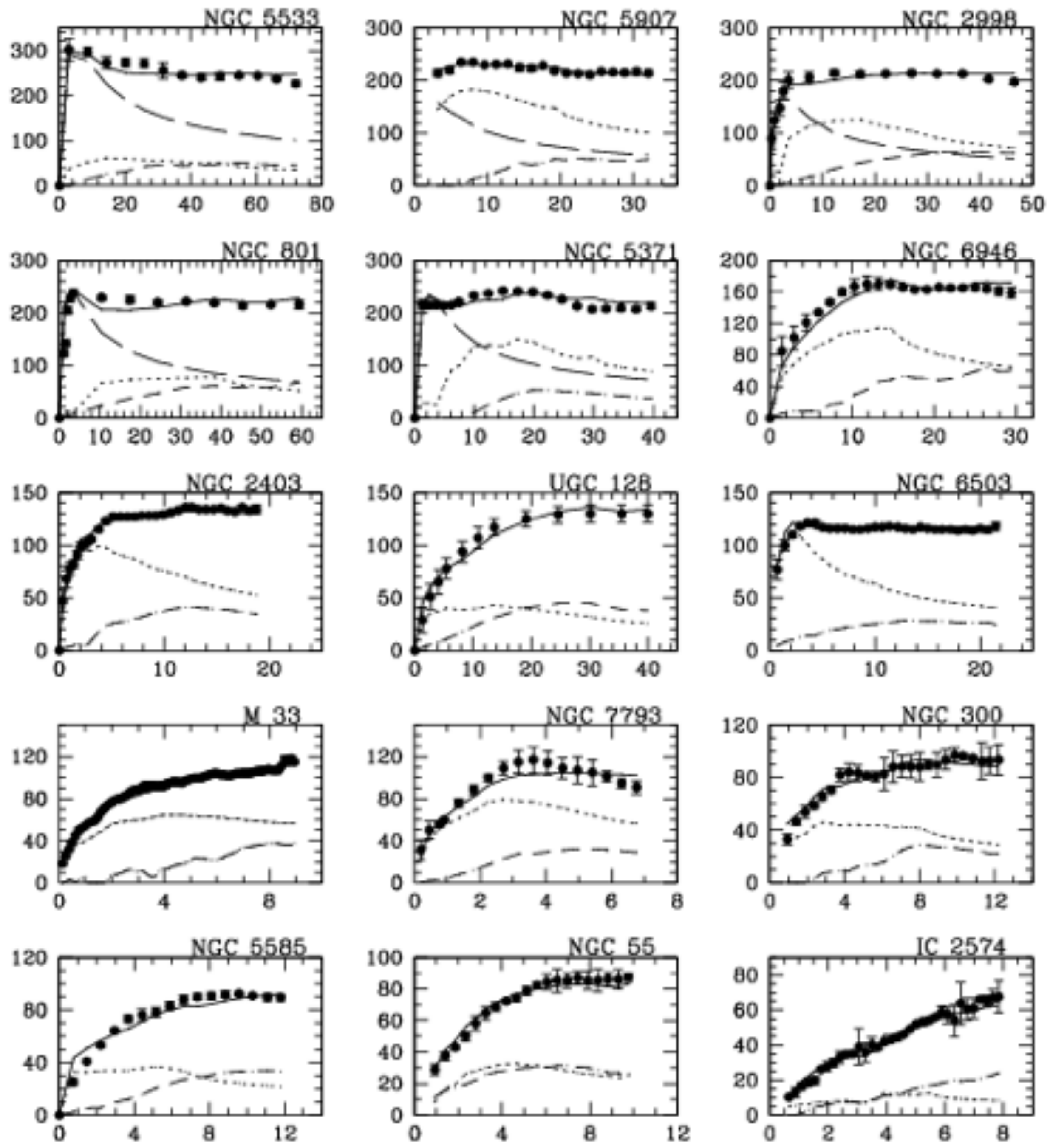
$\Longrightarrow$ Flat rotation curve

$\Longrightarrow$ $\boxed{v^4 = G_{\rm N} M_{\rm b} a_0}$ Baryonic Tully-Fisher



$$a_0 \simeq \frac{1}{6} H_0 \simeq 1.2 \times 10^{-8}\ \mathrm{cm/s^2}$$

Scalar MOND

Bekenstein & Milgrom (1984)

The MOND regime is described by the effective theory:

$$\boxed{\mathcal{L}_{\rm MOND} = -\frac{2M_{\rm Pl}^2}{3a_0}\left((\partial\phi)^2\right)^{3/2} + \frac{\phi}{M_{\rm Pl}}\rho_{\rm b}}$$

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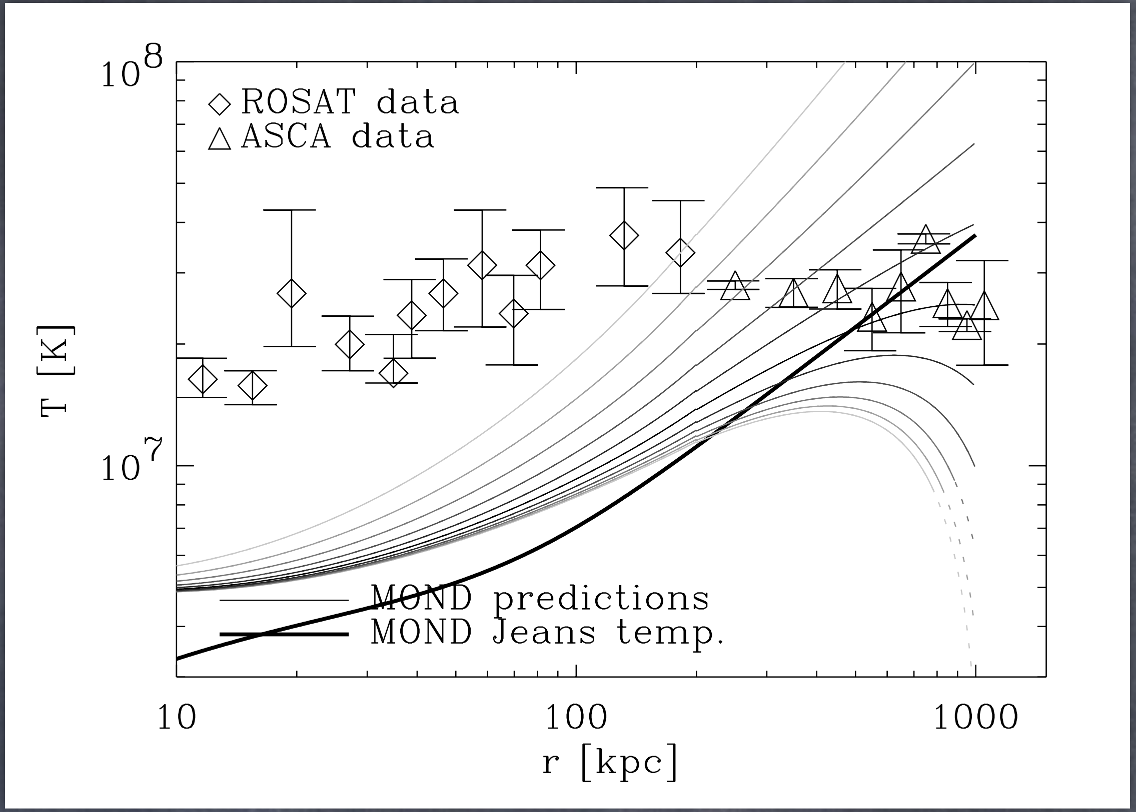
MOND? For static, spherically-symmetric source,

$$\vec{\nabla}\cdot\left(\frac{|\vec{\nabla}\phi|}{a_0}\vec{\nabla}\phi\right) = 4\pi G_{\rm N}\rho$$

$$\Longrightarrow \quad \boxed{\phi' = \sqrt{a_0\frac{G_{\rm N}M(r)}{r^2}} = \sqrt{a_0 a_{\rm N}}}$$

Poor fit to galaxy clusters:

Aguirre (2001)



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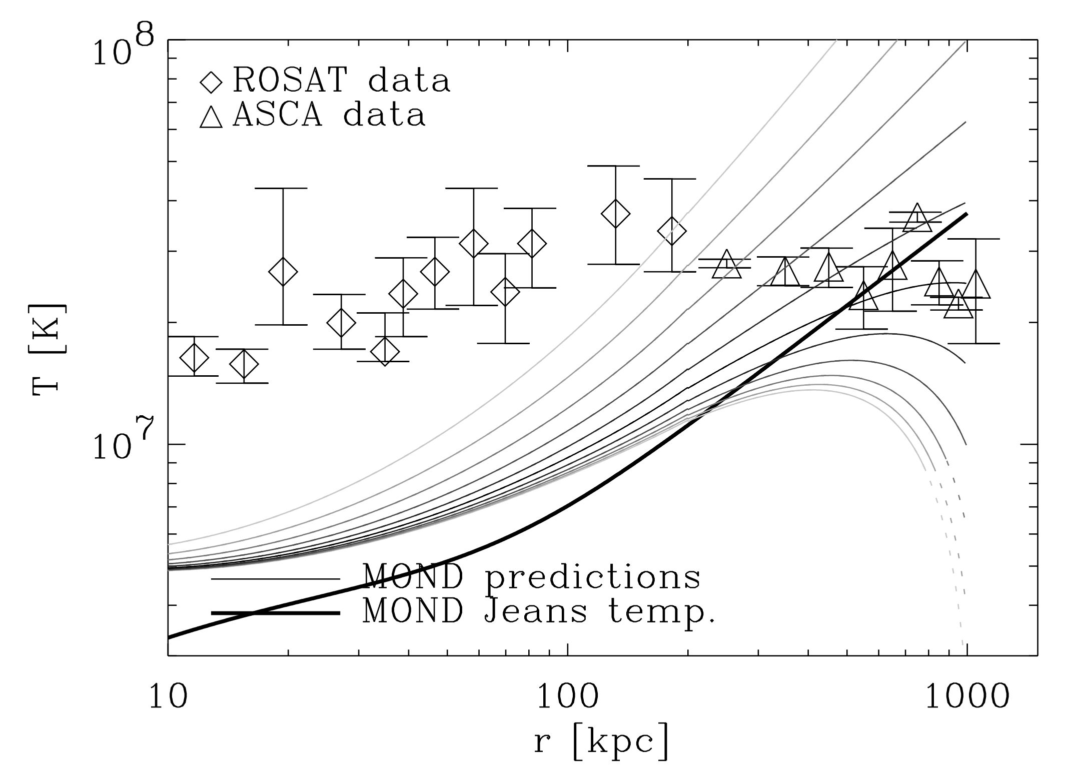

Relativistic extensions are rather frightening...

$$\mathcal{L} = -\frac{1}{2}\left[\sigma^2 h^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi + \frac{G_\mathrm{N}}{2\ell^2}\sigma^4 F(kG_\mathrm{N}\sigma^2)\right]$$
$$- \frac{K}{32\pi G_\mathrm{N}}\left[g^{\alpha\beta}g^{\mu\nu}B_{\alpha\mu}B_{\beta\nu} + \frac{2\lambda}{K}\left(g^{\mu\nu}u_\mu u_\nu - 1\right)\right]$$
$$+ S_\mathrm{matter}\left[\psi_\mathrm{m}, e^{2\phi}g^{\mu\nu} - 2u^\mu u^\nu \sinh(2\phi)\right]$$

$$B_{\mu\nu} = \partial_\mu u_\nu - \partial_\nu u_\mu \qquad h_{\alpha\beta} = g_{\alpha\beta} - u_\alpha u_\beta$$

Bekenstein (2004)

DM-MOND hybrids

Blanchet (2006); Bruneton et al. (2008); Ho, Minic & Ng (2009); JK (2014)...

How about MOND and DM together?

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- Occam's razor?
- Common origin?

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How about MOND and DM together?

- Occam's razor?
- Common origin?
- Different regimes

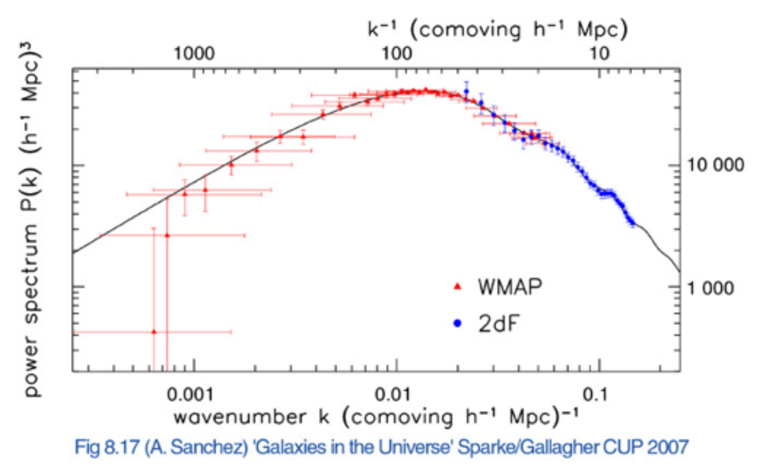
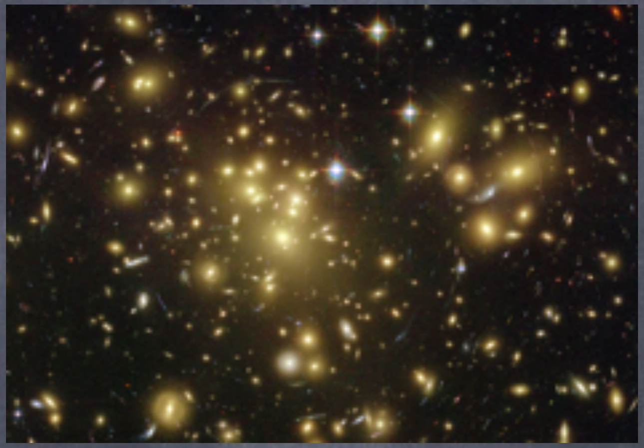


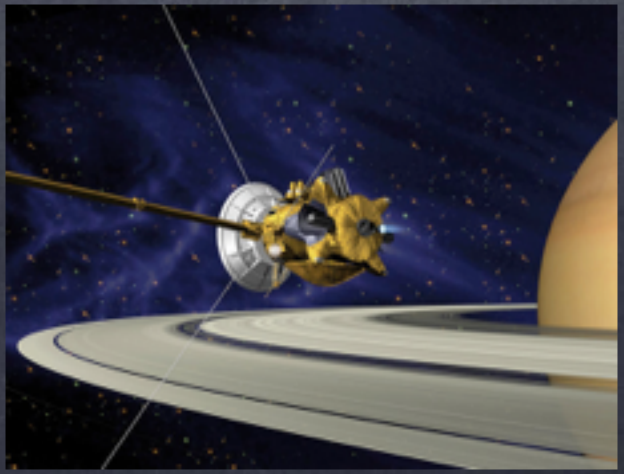
Fig 8.17 (A. Sanchez) 'Galaxies in the Universe' Sparke/Gallagher CUP 2007

Mostly DM



Mostly DM

Mostly MOND



No MOND

Unified approach:

MOND phenomenon from DM superfluidity

2 Conditions for DM Condensation

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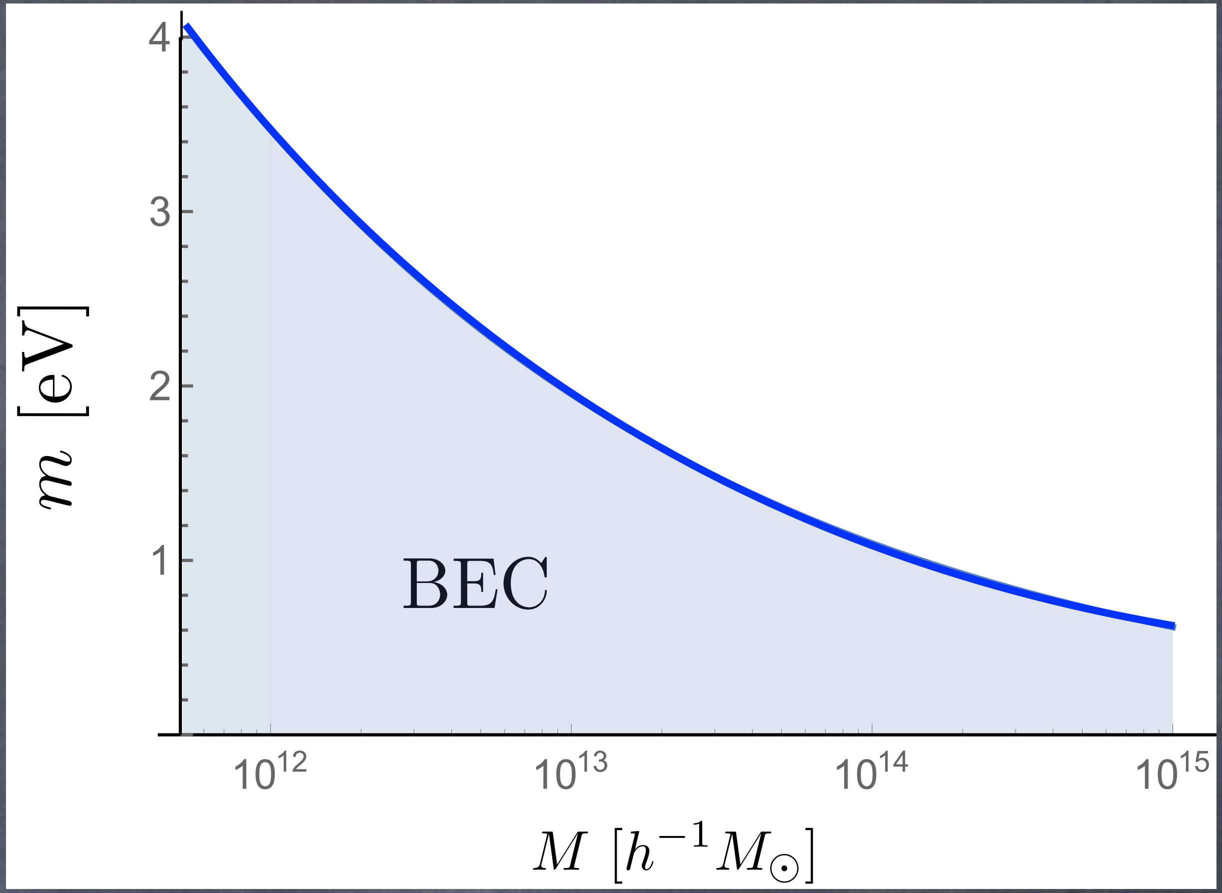
- Overlapping de Broglie wavelength

$$\lambda_{\rm dB} \sim \frac{1}{mv} \quad \gtrsim \quad \ell \sim \left(\frac{m}{\rho_{\rm vir}}\right)^{1/3}$$

$$\Longrightarrow \quad \boxed{m \lesssim 2 \text{ eV}}$$



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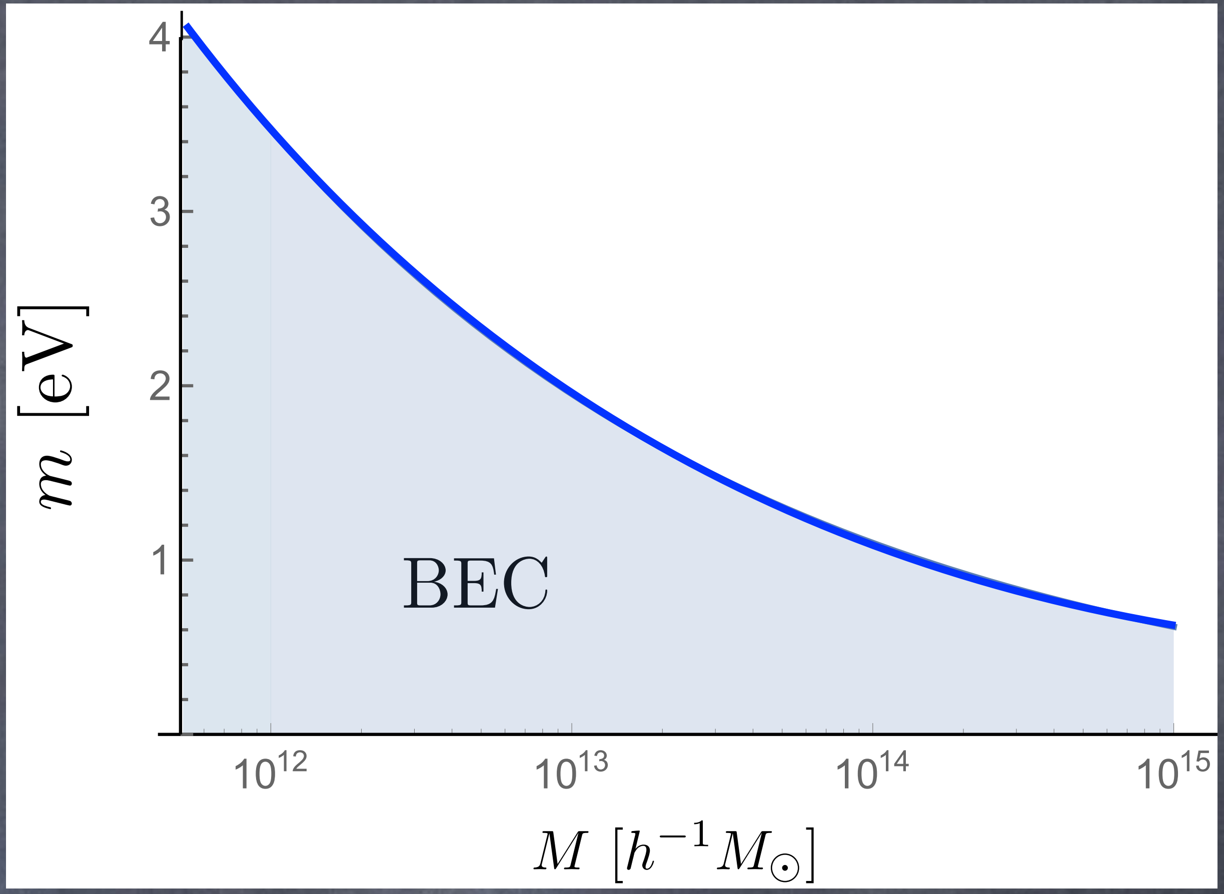


- Thermal equilibrium

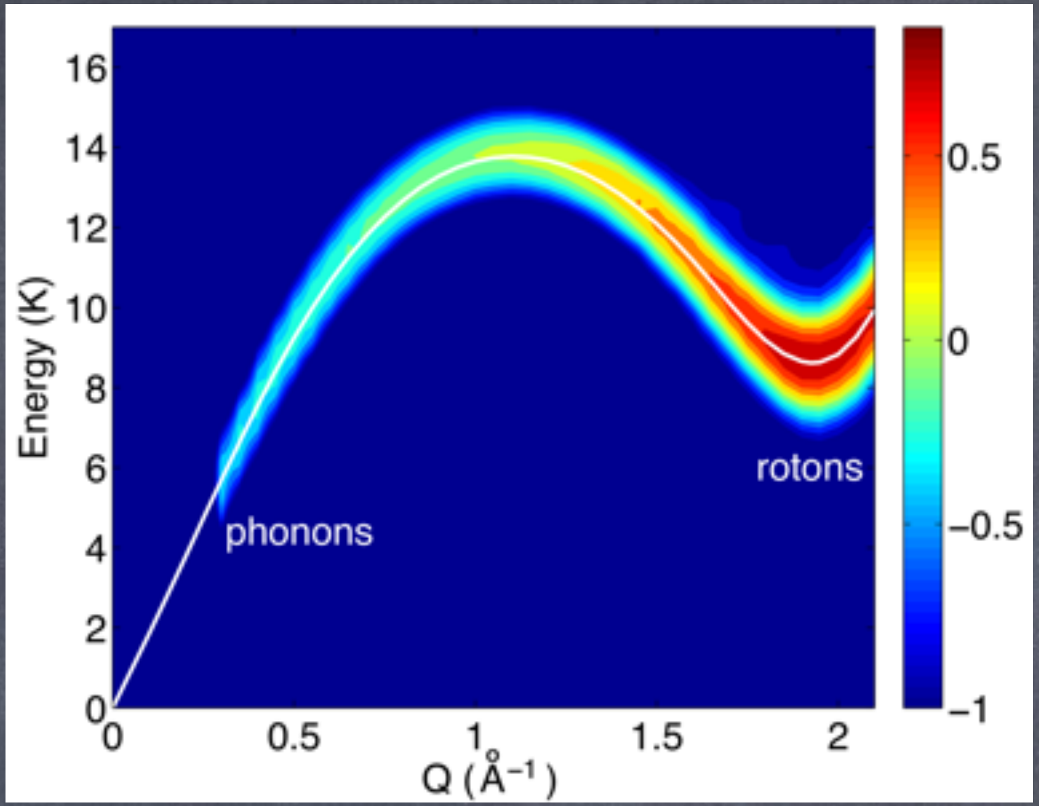
$$\Gamma \sim N v \sigma \frac{\rho_{\rm vir}}{m} \gtrsim t_{\rm dyn}^{-1} \quad \Longrightarrow \quad \boxed{\frac{\sigma}{m} \gtrsim \left(\frac{m}{\rm eV}\right)^4 \frac{\rm cm^2}{g}}$$

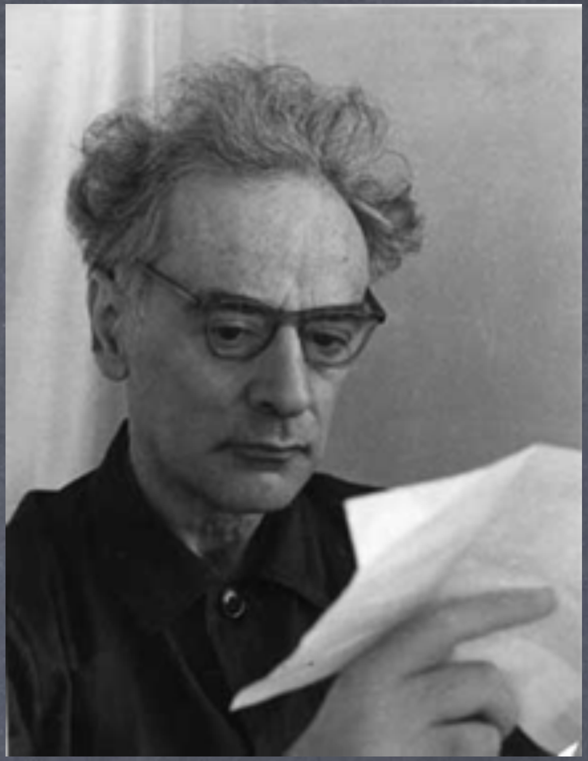
DM is quite cold:

$$T_{\rm c} = 6.5 \left(\frac{\rm eV}{m}\right)^{5/3} (1+z_{\rm vir})^2\ {\rm mK}$$

(^{7}Li atoms $\Longrightarrow$ $T_{\rm c} \sim 0.2$ mK)

Two-fluid model





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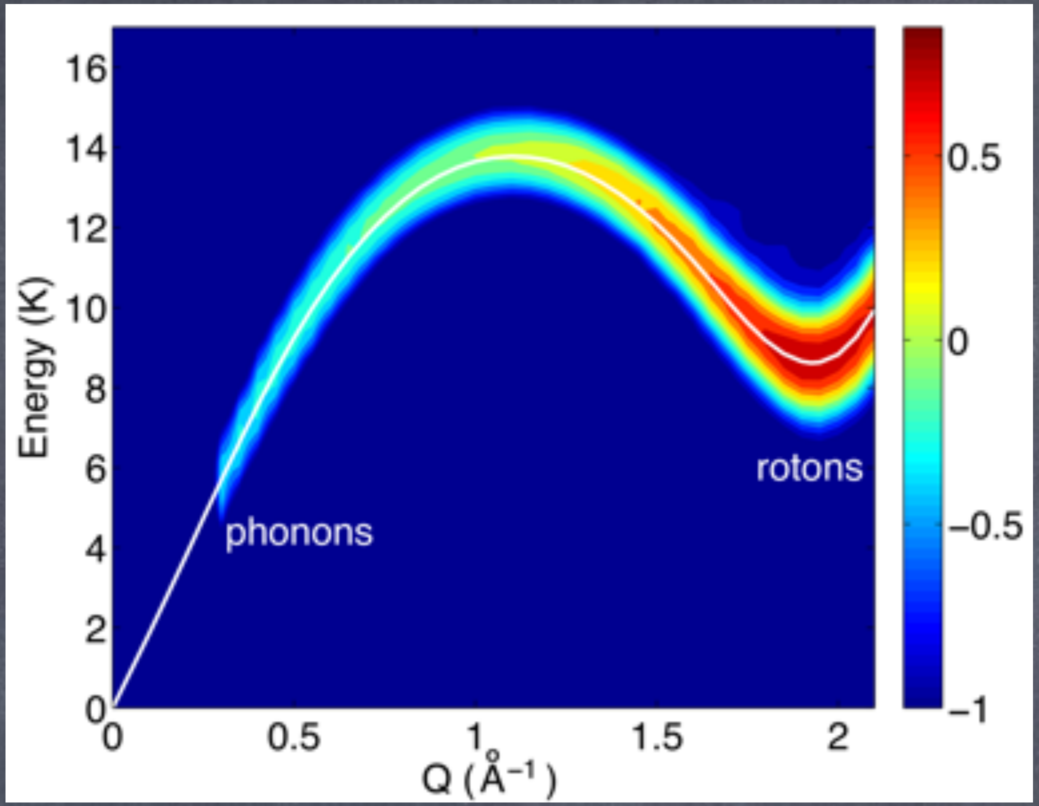
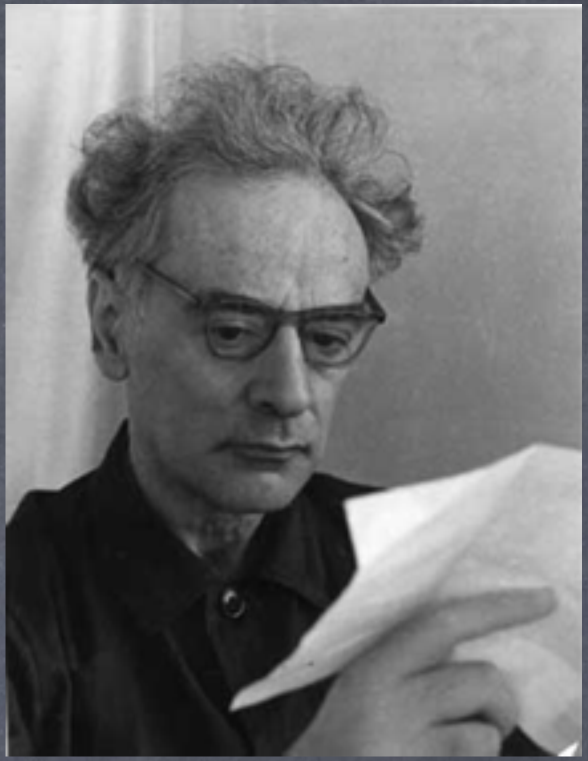
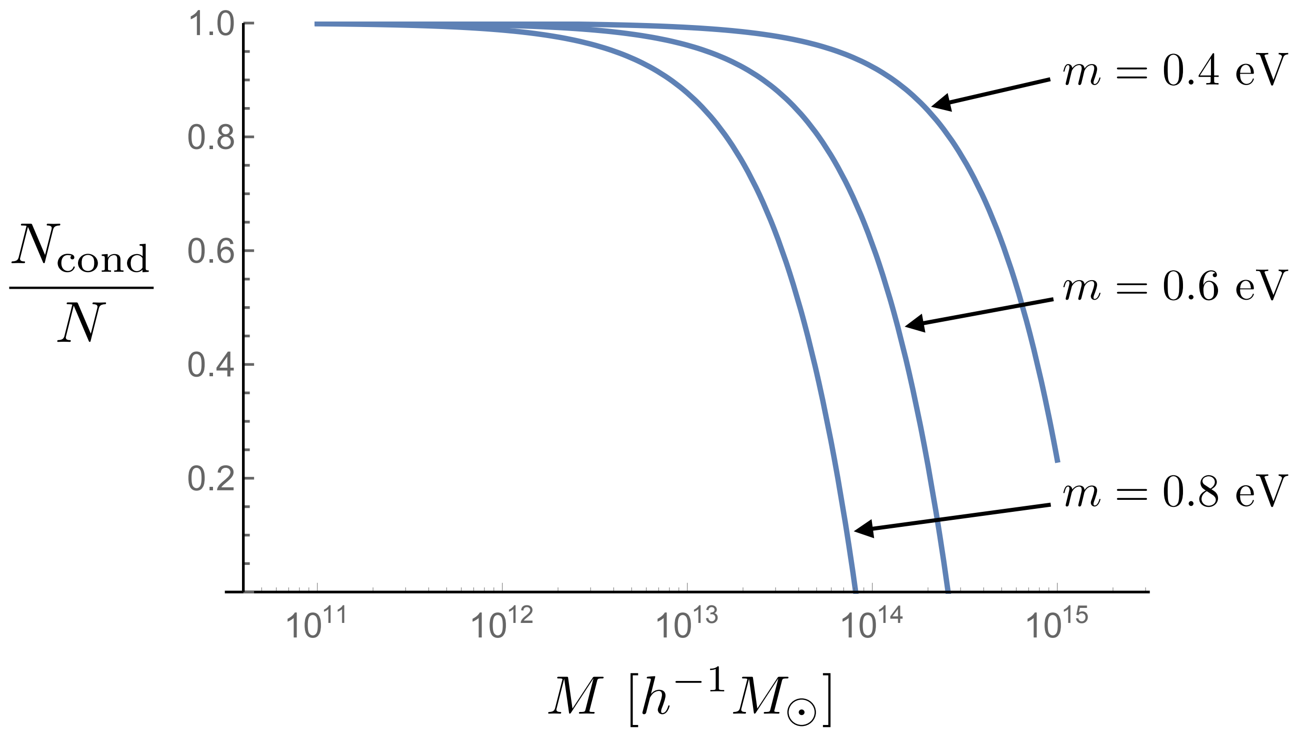




Free bose gas:

$$\frac{N_{\rm cond}}{N} = 1 - \left(\frac{T}{T_{\rm c}}\right)^{3/2}$$

- Galaxies are mostly condensed
- Galaxy clusters are in mixed phase



Can generalize to include interactions.

Khoury, Lubensky, Miranda & Sharma (to appear)

Effective Description of Superfluids

Greiter, Wilczek & Witten (1989)

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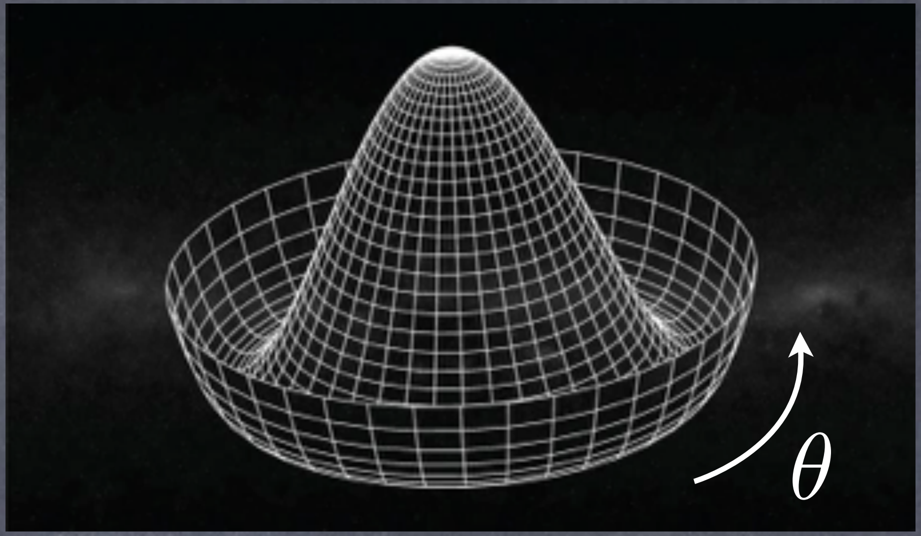

A superfluid phase is defined as:

- Global U(1) symmetry, spontaneously broken

$\Longrightarrow$ Goldstone boson $\theta \to \theta + c$

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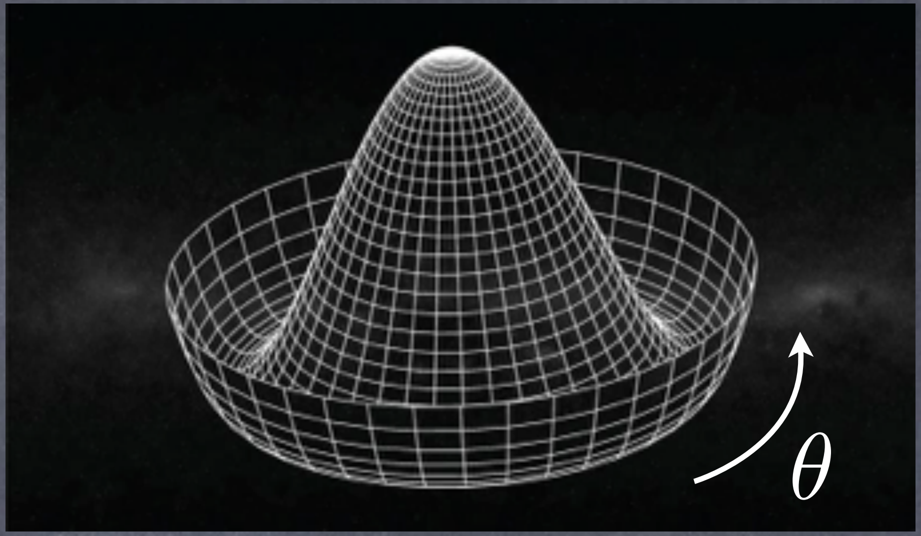

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- State has finite charge density, $\langle J^0 \rangle \sim \langle \dot{\theta} \rangle \neq 0$

  By redefining field, can set

$$\theta = \underbrace{\mu}_{\text{chemical potential}} t + \underbrace{\phi}_{\text{phonons}}$$

Effective Description of Superfluids

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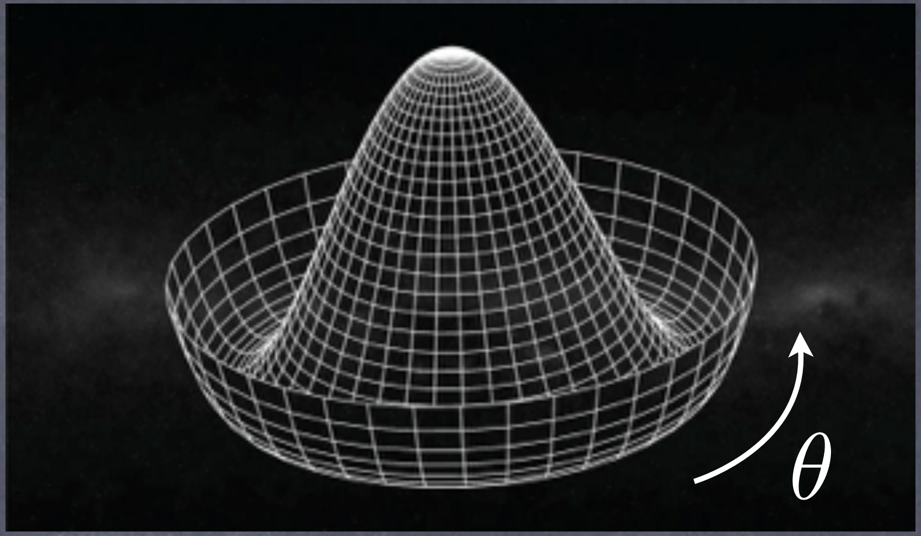

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$$\theta = \mu t + \phi$$

chemical potential

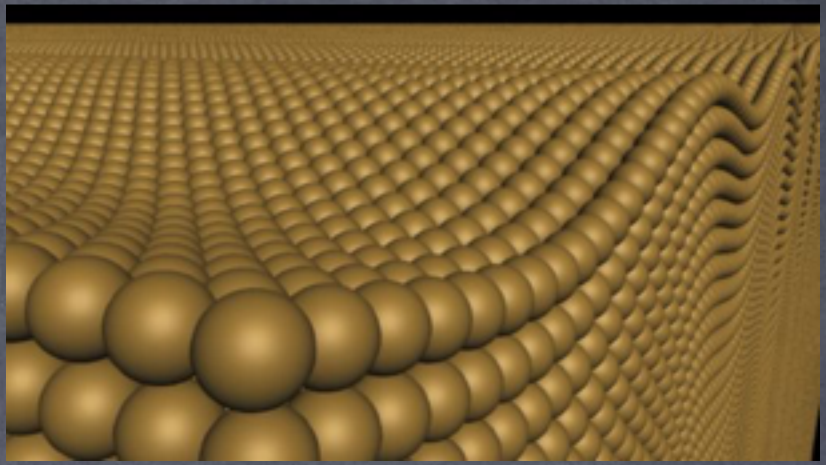
phonons

Hence, at lowest order in derivatives the EFT of phonons is

$$\boxed{\mathcal{L} = P(X)\,; \qquad X = \mu + \dot{\phi} - \frac{(\vec{\nabla}\phi)^2}{2m}}$$

Phonons

At lowest order in derivatives, the zero temperature effective action is



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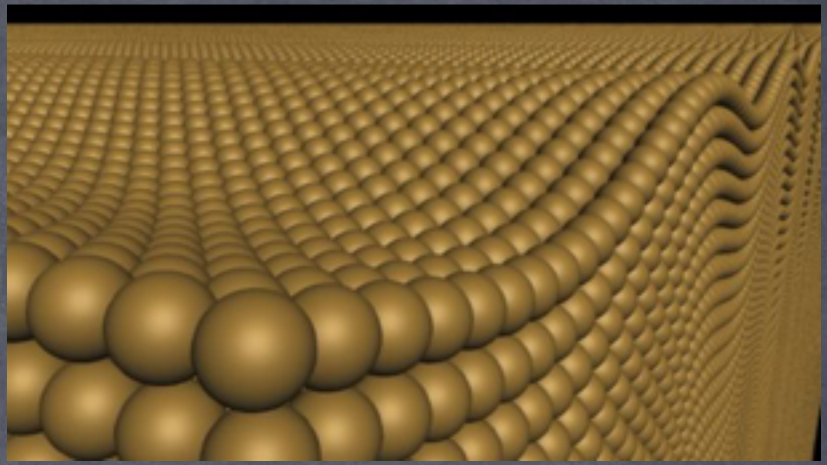
Greiter, Wilczek & Witten (1989); Son and Wingate (2005)

Phonons



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Greiter, Wilczek & Witten (1989); Son and Wingate (2005)

Conjecture: DM superfluid phonons are governed by MOND action

$$\boxed{P_{\rm MOND}(X) = \frac{2\Lambda(2m)^{3/2}}{3} X\sqrt{|X|}}$$

Phonons couple to baryons:

$$\mathcal{L}_{\rm coupling} = -\frac{\Lambda}{M_{\rm Pl}}\phi\rho_{\rm b}$$

$$\Lambda = \sqrt{a_0 M_{\rm Pl}} \simeq 0.8 \text{ meV}$$

(Match to MOND scale)

- Weyl symmetry Milgrom (2008)

$$\mathcal{L}_{\mathrm{MOND}} \sim \sqrt{h}\left(h^{ij}\partial_i\phi\partial_j\phi\right)^{3/2}$$

invariant under $h_{ij} \to \Omega^2(x)h_{ij}$. Symmetry group is $SO(4,1)$.

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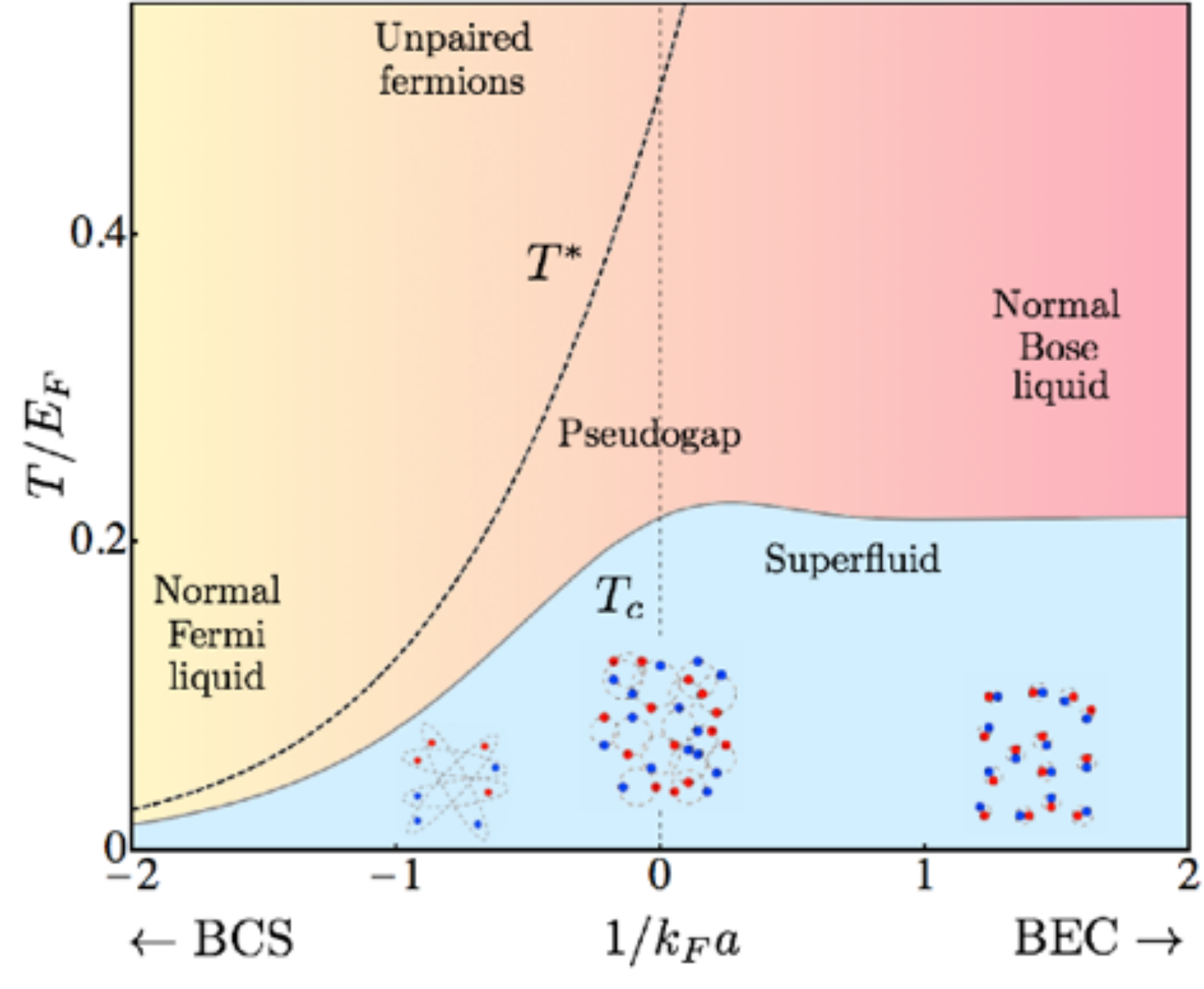
- Unitary Fermi Gas

$$\mathcal{L}_{\rm UFG} \sim m^{3/2}X^{5/2}$$

Son & Wingate (2005)



Condensate properties

Action uniquely fixes properties of the condensate through standard thermodynamics

- Pressure: $$P_{\text{cond}} = \frac{2\Lambda}{3}(2m\mu)^{3/2}$$
- Number density: $$n_{\text{cond}} = \frac{\partial P_{\text{cond}}}{\partial \mu} = \Lambda(2m)^{3/2}\mu^{1/2}$$

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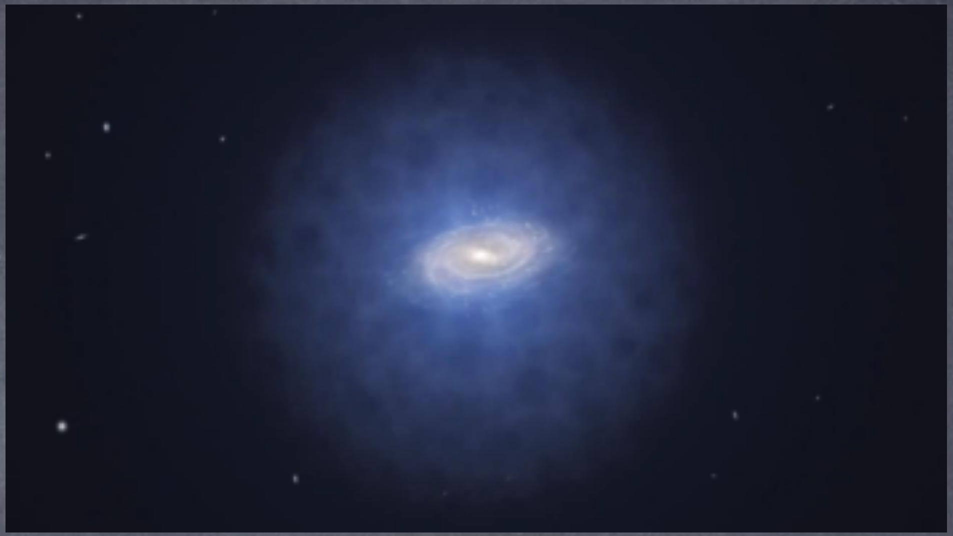
In the non-relativistic approx'n, $\rho_{\rm cond} = m n_{\rm cond}$, therefore:

$$\boxed{P_{\rm cond} = \frac{\rho^3_{\rm cond}}{12\Lambda^2 m^6}}$$

- Polytropic equation of state, with index $n = 1/2$
- Different than BEC DM, where $P_{\rm cond} \sim \rho^2_{\rm cond}$

Sin (1994), Goodman (2000), Peebles (2000), Boehmer & Harko (2007)

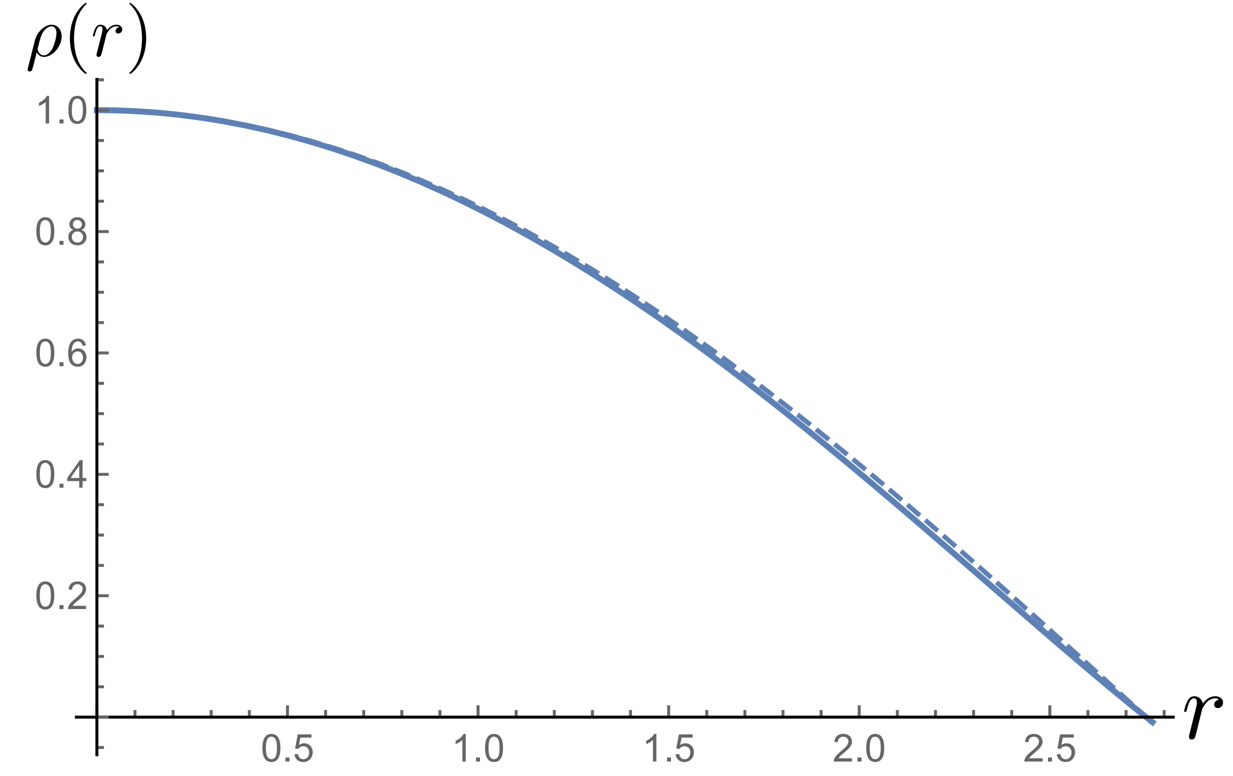
Density profile



Assuming hydrostatic equilibrium,

$$\frac{1}{\rho_{\rm cond}(r)}\frac{{\rm d}P_{\rm cond}(r)}{{\rm d}r} = -\frac{4\pi G_{\rm N}}{r^2}\int_0^r {\rm d}r' r'^2 \rho(r')$$

Using equation of state $P_{\rm cond} \sim \rho_{\rm cond}^3$, find:

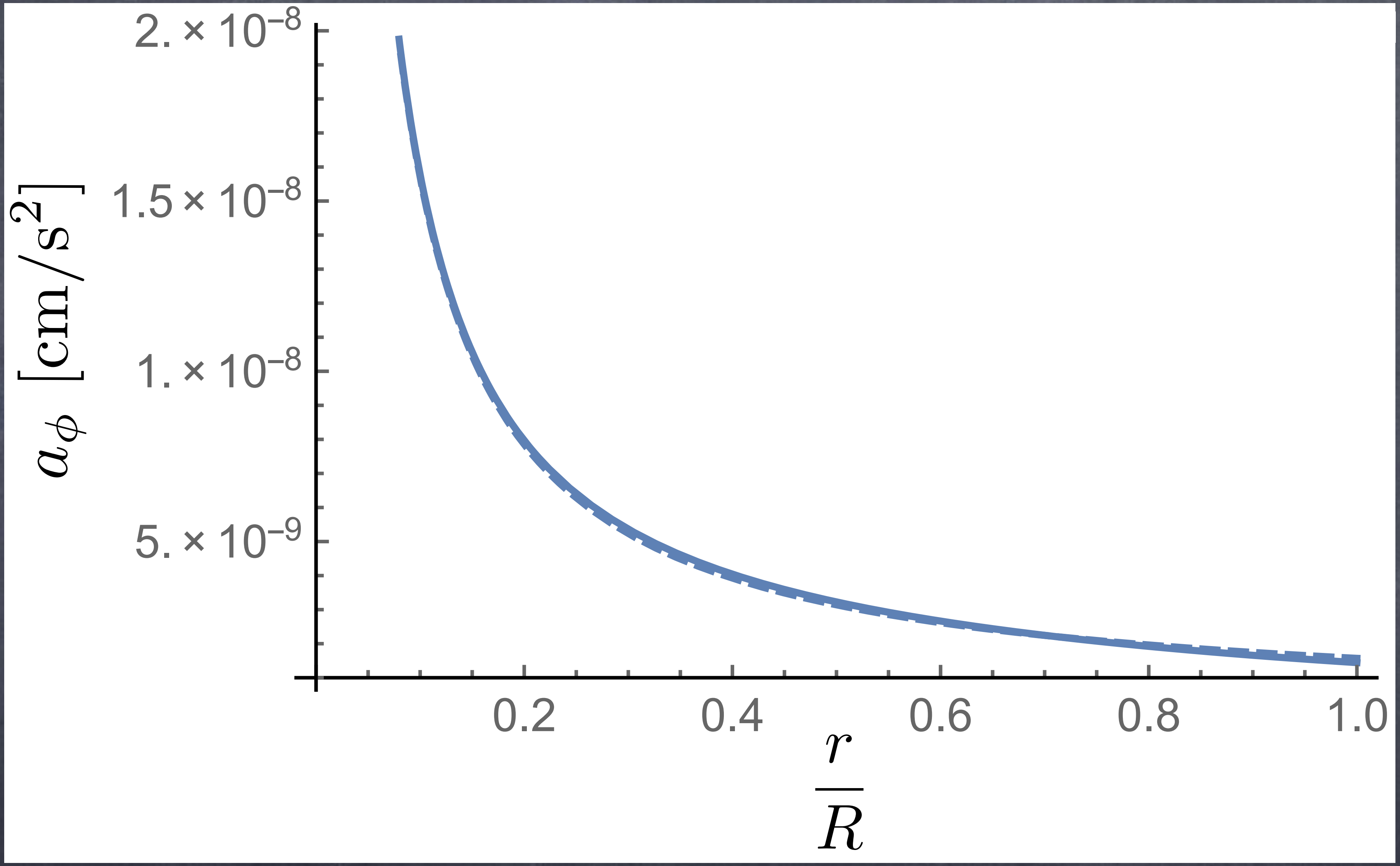


Cored density profile

$$R \simeq \left(\frac{M_{\rm DM}}{10^{12}M_\odot}\right)^{1/5}\left(\frac{\rm eV}{m}\right)^{6/5}\left(\frac{\rm meV}{\Lambda}\right)^{2/5} 36\ {\rm kpc}$$

Remarkably, have realistic-size halos with $m \sim {\rm eV}$ and $\Lambda \sim {\rm meV}$!

Phonon force is indistinguishable from MOND…



… but there is also dark matter.

$$\frac{a_{\rm DM}}{a_\phi} \simeq \begin{cases} 0.4\,\frac{r}{r_\star} & (r \ll r_\star) \\ 0.5\,\left(\frac{r}{r_\star}\right)^2 & (r \gg r_\star) \end{cases}$$

rotation curves

lensing

$$r_\star \simeq \left(\frac{M_{\rm b}}{10^{11}\,M_\odot}\right)^{1/10}\left(\frac{M_{\rm DM}}{M_{\rm b}}\right)^{-2/5}\left(\frac{m}{\rm eV}\right)^{-8/5}\left(\frac{\Lambda}{\rm meV}\right)^{-8/15} 28~{\rm kpc}$$

Validity of effective theory

$$\boxed{v_{\rm s} = \frac{|\nabla\phi|}{m} < v_{\rm c} \sim \left(\frac{\rho}{m^4}\right)^{1/3}}$$

Satisfied for $r \gtrsim \mathrm{kpc}$

$\Longrightarrow$ Quasi-particle production (DM-like behavior) in inner regions of galaxies

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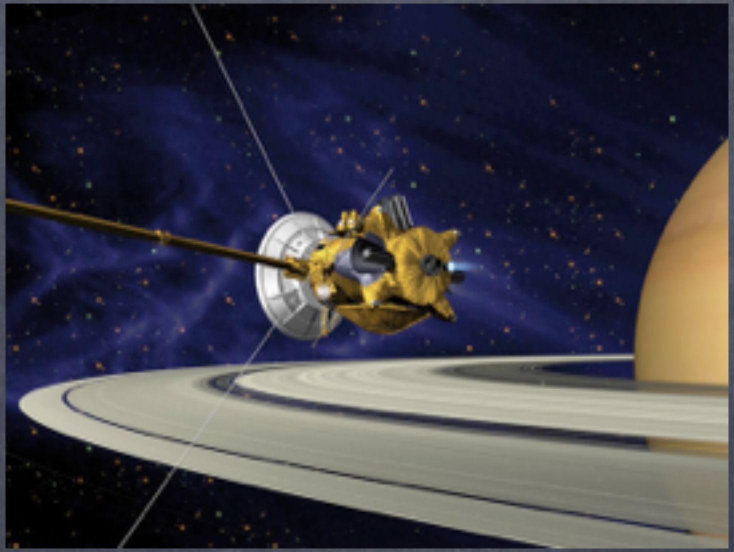
$\Longrightarrow$ Quasi-particle production (DM-like behavior) in inner regions of galaxies

Solar system



A MOND scalar acc'n, $\frac{\Delta a}{a_{\rm N}} = \sqrt{\frac{a_0}{a_{\rm N}}}$, albeit small in the solar system, is ruled out.

$\Longrightarrow$ must we complicate the theory?

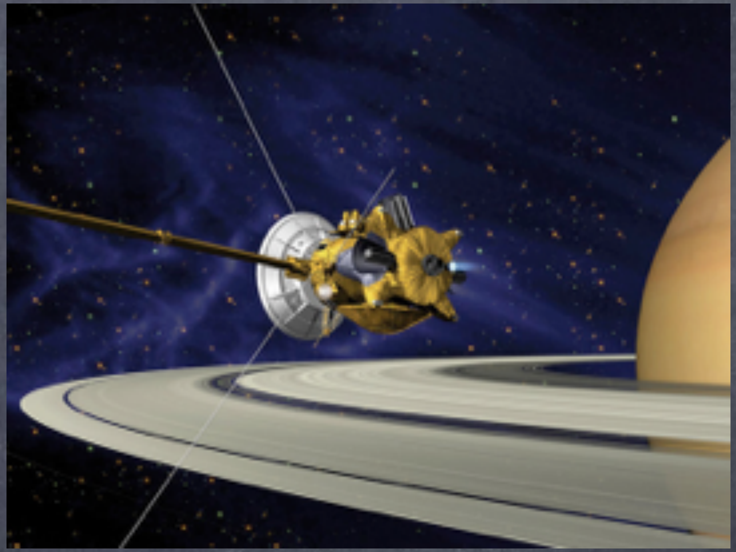
Validity of effective theory

$$\boxed{v_{\rm s} = \frac{|\nabla\phi|}{m} < v_{\rm c} \sim \left(\frac{\rho}{m^4}\right)^{1/3}}$$

Satisfied for $r \gtrsim \mathrm{kpc}$

$\Longrightarrow$ Quasi-particle production (DM-like behavior) in inner regions of galaxies

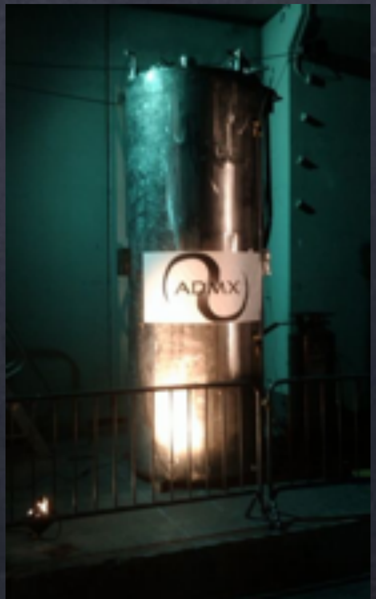
Solar system



A MOND scalar acc'n, $\frac{\Delta a}{a_{\rm N}} = \sqrt{\frac{a_0}{a_{\rm N}}}$, albeit small in the solar system, is ruled out.

$\Longrightarrow$ must we complicate the theory?

No need to! Above criterion is satisfied only for $r \gtrsim 1000 \text{ AU}$



$\Longrightarrow$ superfluid description breaks down in solar system. DM behaves as ALPs.

$\Longrightarrow$ Good news for ALP searches.

Gravitational Lensing without DM

Claim: Conformal coupling $\tilde{g}_{\mu\nu} = e^{-2\phi} g_{\mu\nu}$ not enough.

Proof: Null geodesics are invariant under Weyl transf'ns, hence photons are oblivious to ϕ .

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TeVeS solution:

Saunders (1997)
Bekenstein (2004)

- Introduce unit, time-like vector A^{μ}
- Couple to matter in a very specific way

$$\boxed{g_{\mu\nu}^{\text{TeVeS}} = e^{-2\phi} g_{\mu\nu} - 2A_{\mu}A_{\nu}\sinh 2\phi}$$

$\Longrightarrow$ Lensing mass estimates = Dynamical estimates

Gravitational Lensing (cont'd)

Our case is much simpler:

- Normal DM component already provides a time-like vector

$$u^\mu$$

Relative velocity between normal and superfluid components

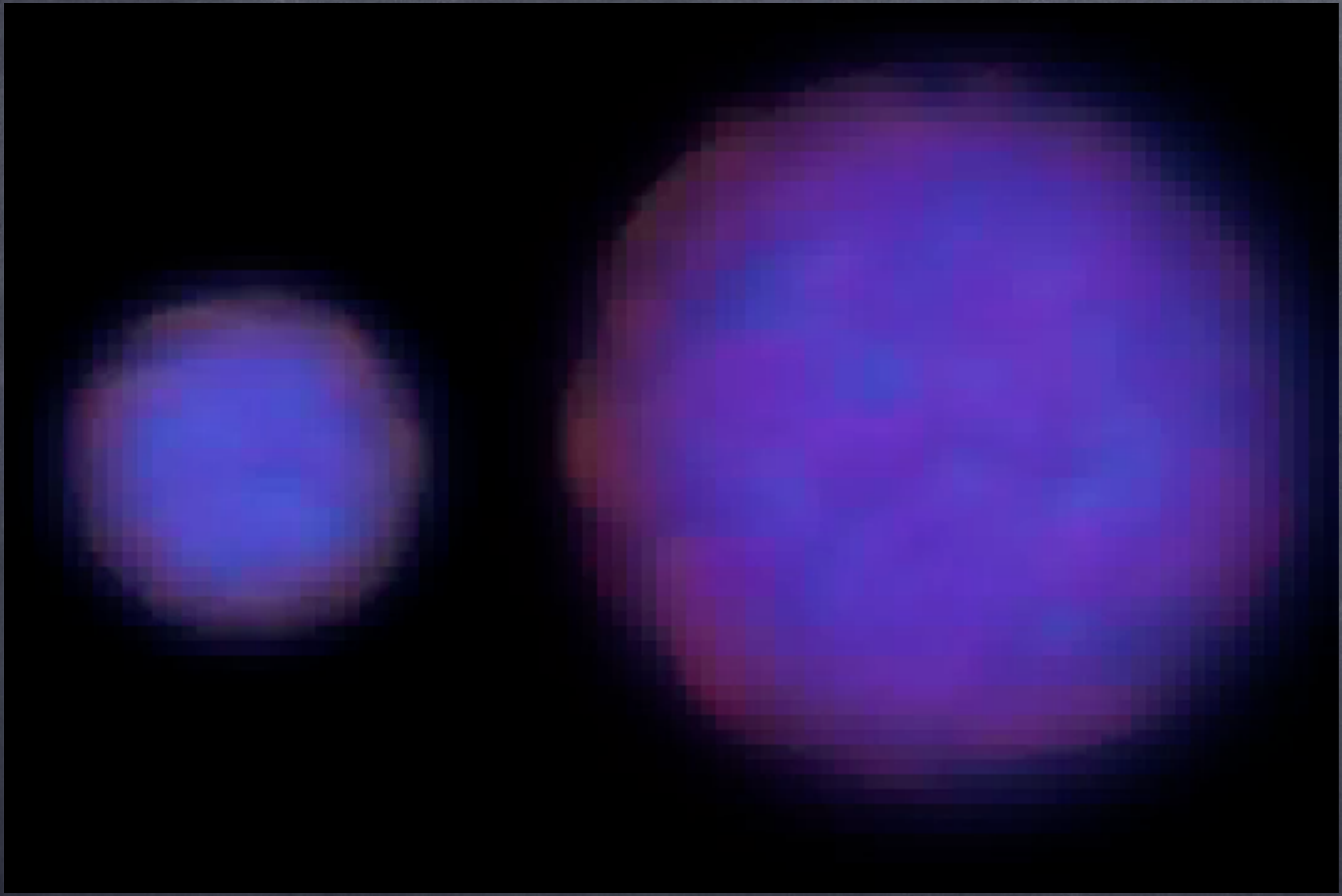
- DM contributes to lensing, can consider more general metric

$$\tilde{g}_{\mu\nu} \simeq g_{\mu\nu} - 2\phi\Big(\gamma g_{\mu\nu} + (1+\gamma)u_\mu u_\nu\Big)$$

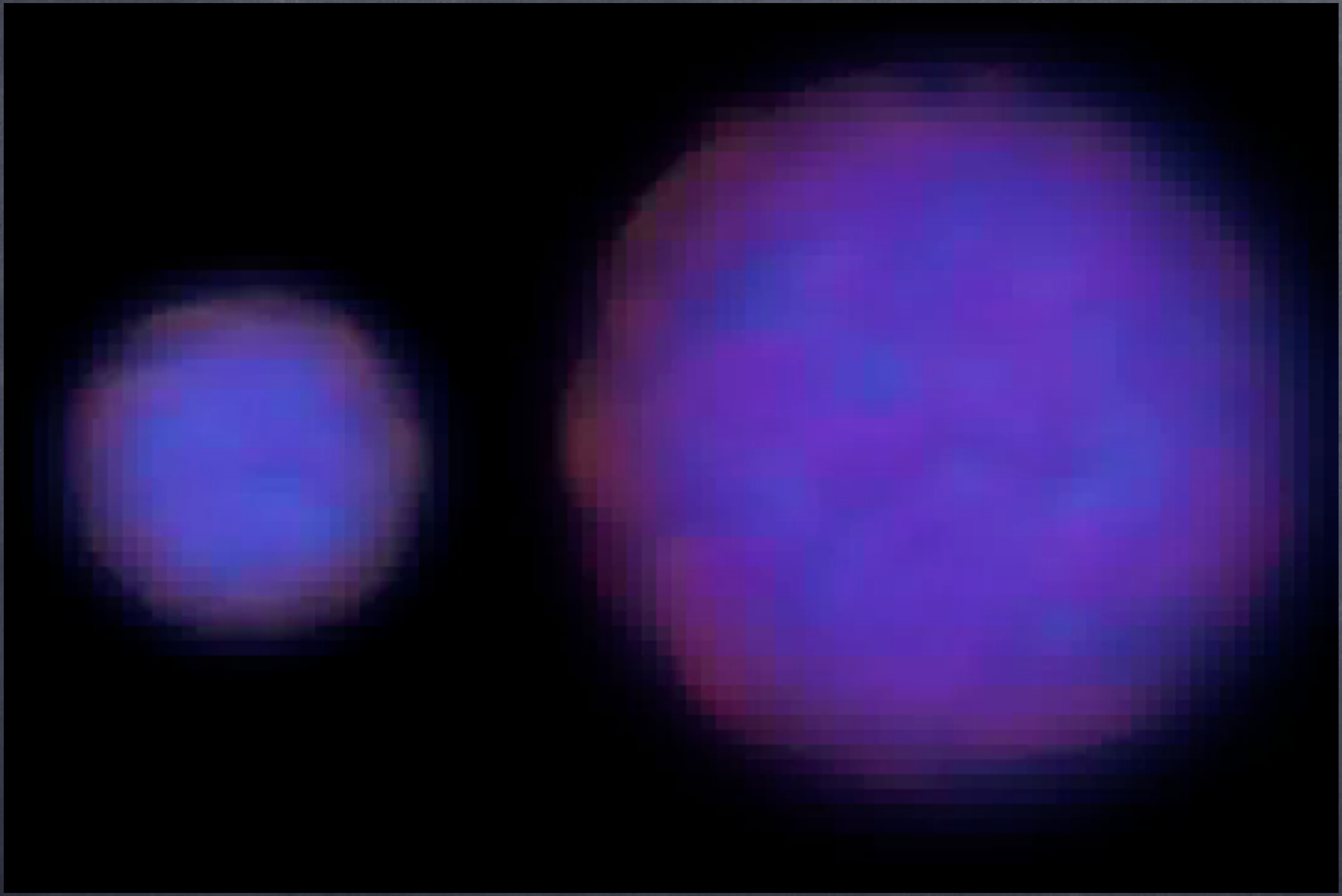
Maybe even conformal coupling ($\gamma = -1$) is allowed?

Observational Signatures

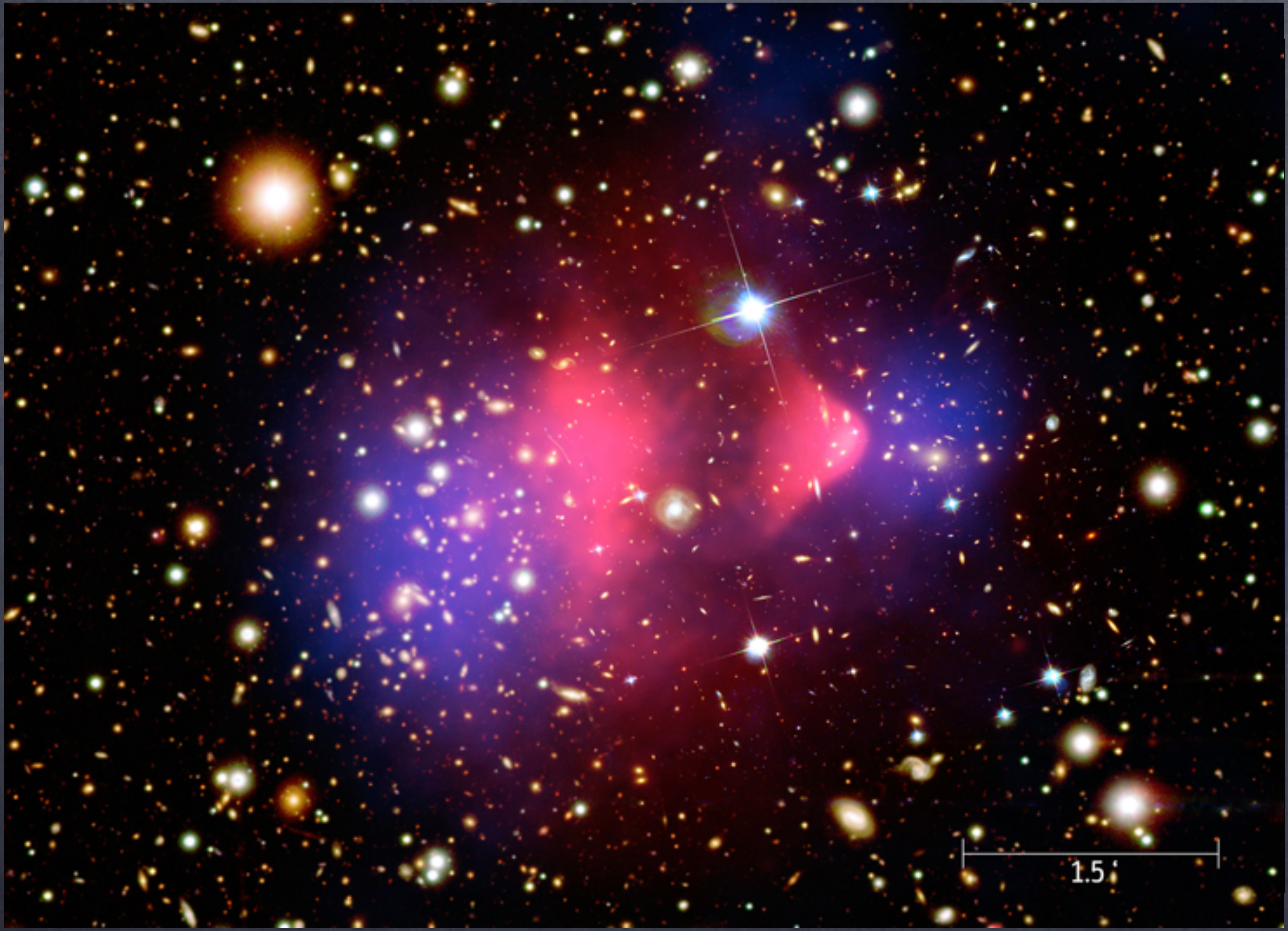
Bullet-Like Clusters



Bullet-Like Clusters



Bullet-Like Clusters



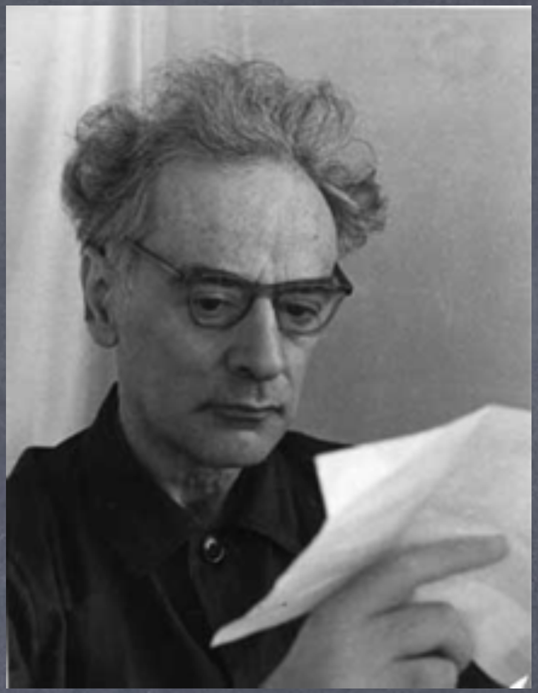
$$\frac{\sigma}{m} \lesssim 0.5\,\frac{\mathrm{cm}^2}{g}$$

Harvey et al. (2015)

Superfluid cores should pass through each other with negligible dissipation if

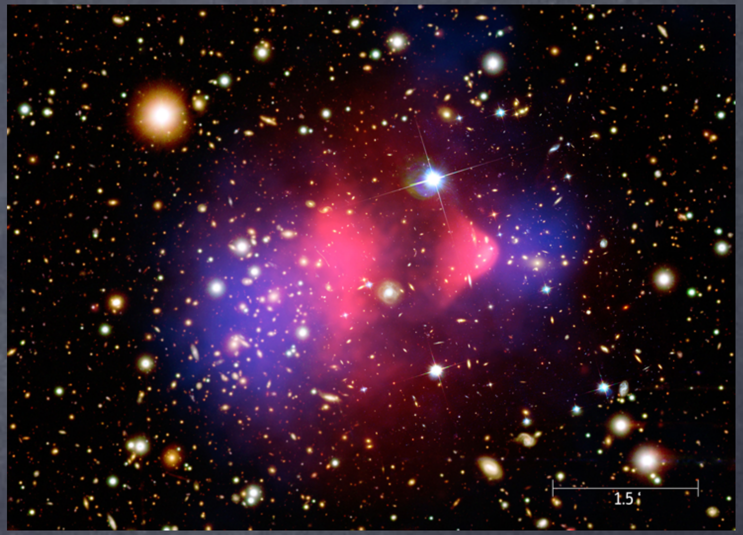
$$v_{\rm infall} \lesssim c_s$$

(Landau's criterion)



We find:

- Sub-cluster: $c_s \simeq 1400\ {\rm km/s}$
- Main cluster: $c_s \simeq 3500\ {\rm km/s}$

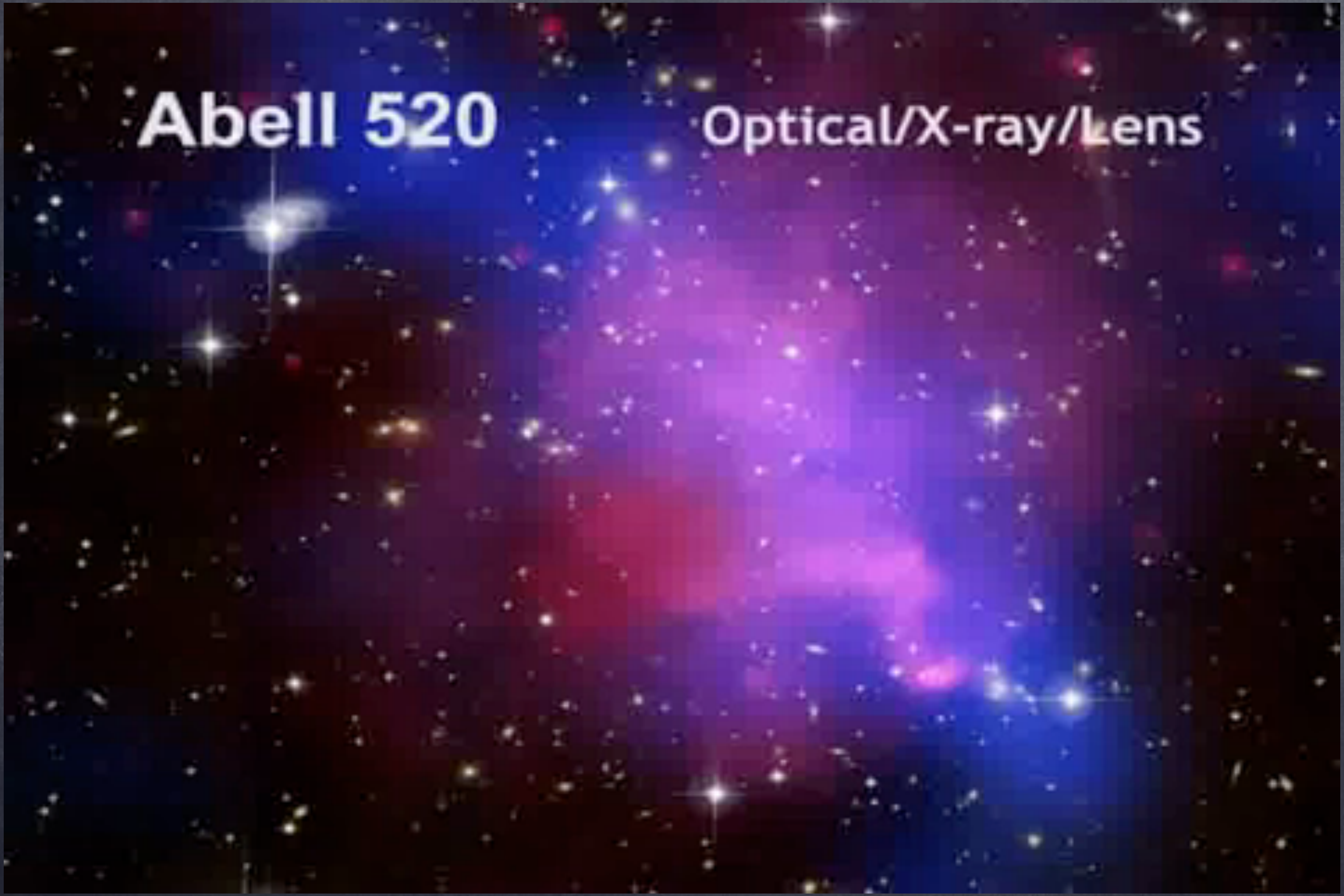


i.e., comparable to the infall velocity: $v_{\rm infall} \simeq 2700\ {\rm km/s}$

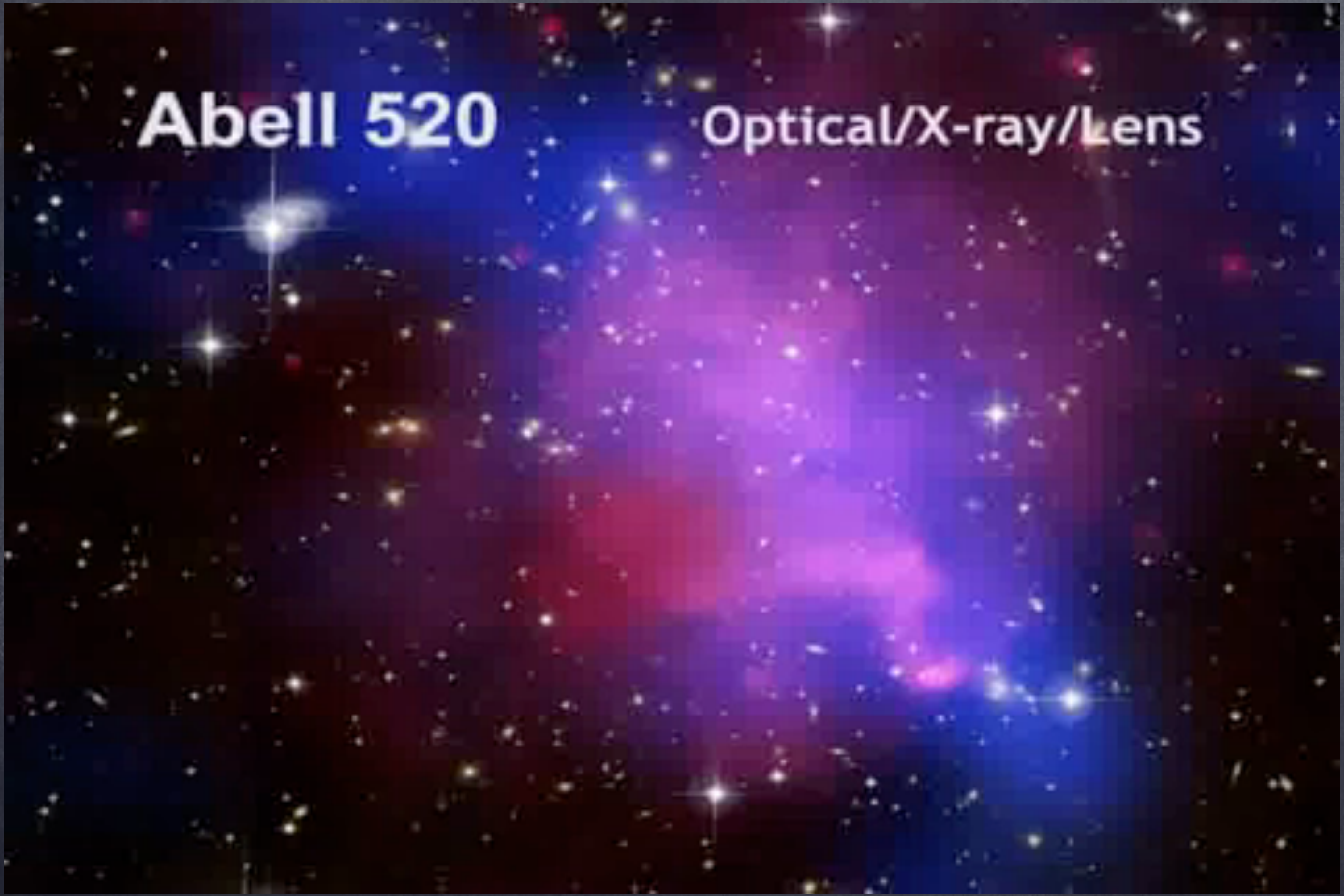
Springel & Farrar (2007)

$\Longrightarrow$ Dissipative processes between superfluid cores should be suppressed

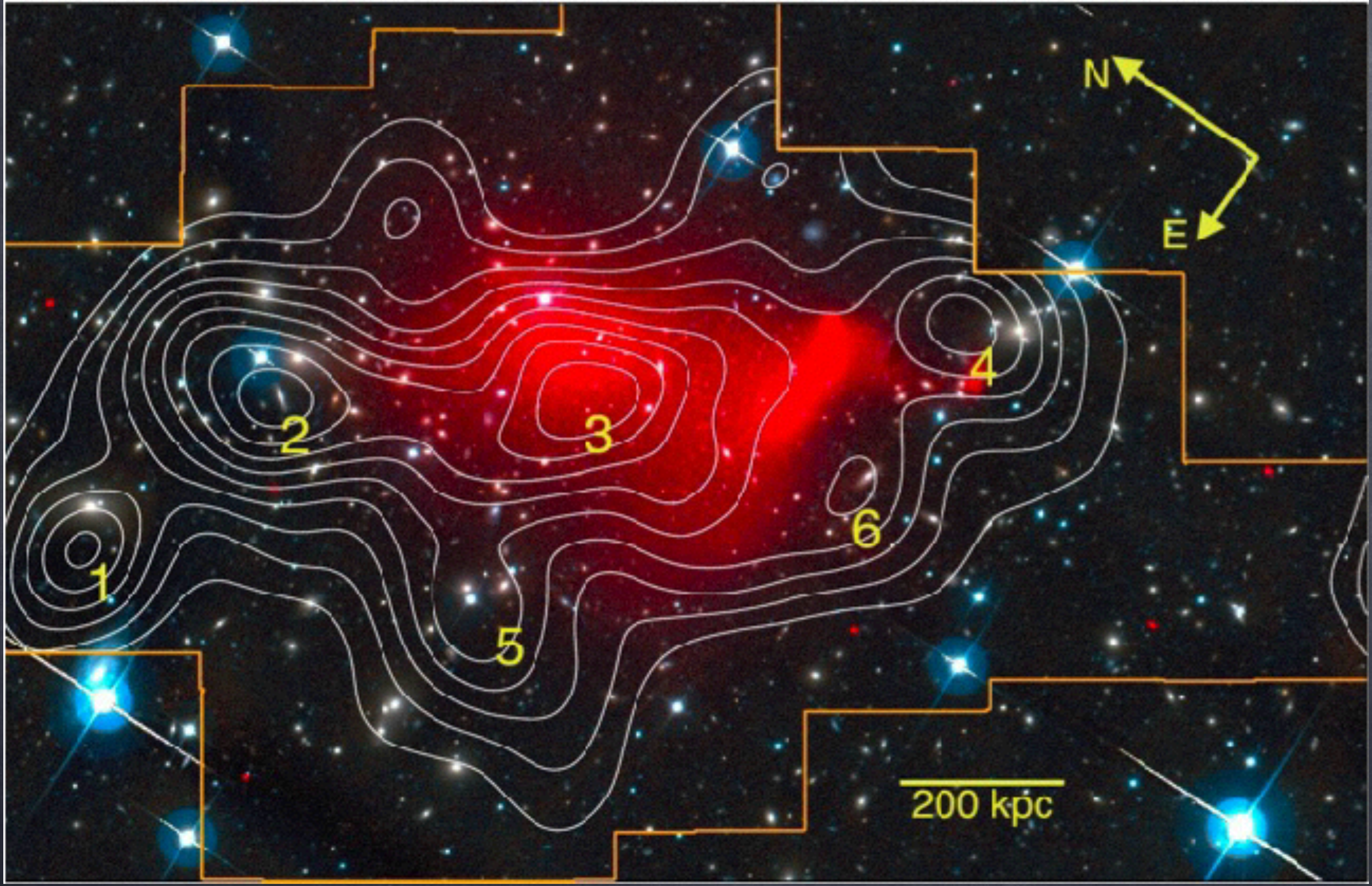
The Counter-Bullet



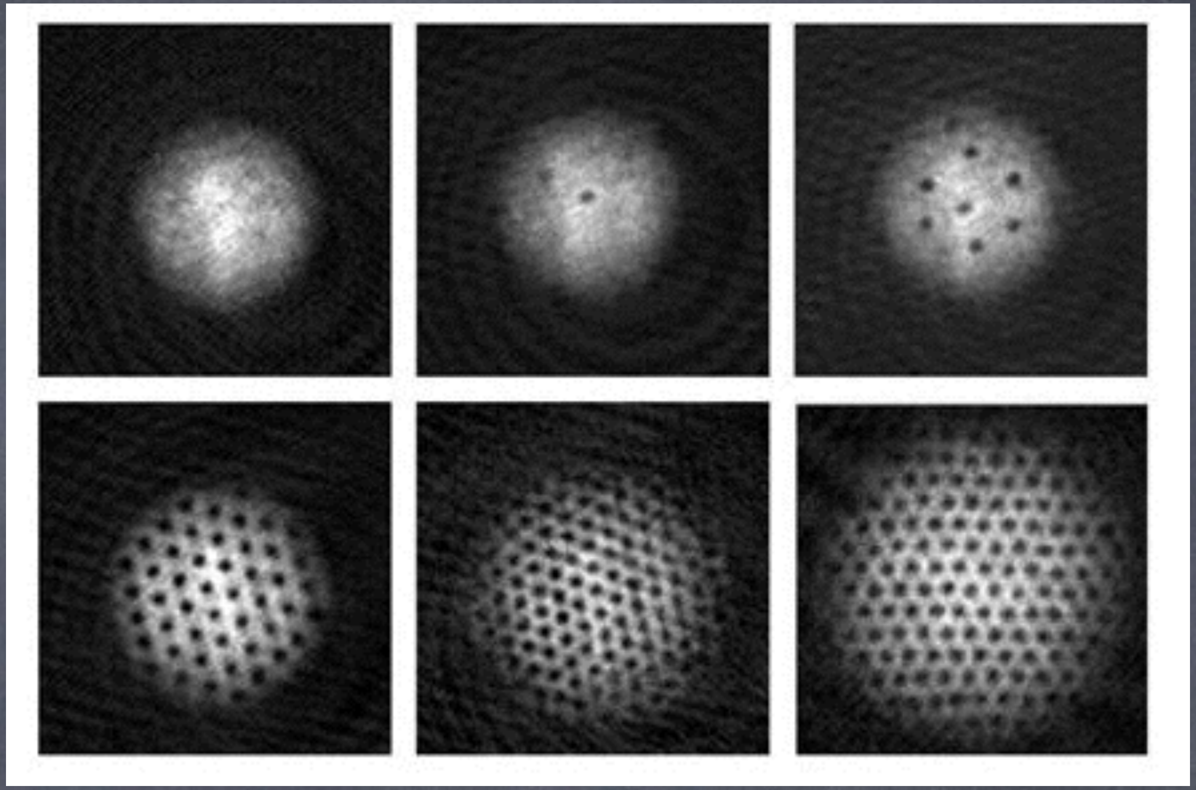
The Counter-Bullet



The Counter-Bullet



Vortices



When spun faster than critical velocity, superfluid develops vortices.

$$\omega_{\rm cr} \sim \frac{1}{mR^2} \sim 10^{-41}{\rm s}^{-1}$$

For a halo of density ρ ,

$$\omega \sim \lambda\sqrt{G_{\rm N}\rho} \sim 10^{-18}\lambda\,{\rm s}^{-1}\,; \qquad 0.01 < \lambda < 0.1$$

$\Longrightarrow$ Vortex formation is unavoidable

Line density: $\boxed{\sigma_{\rm v} \sim m\omega \sim 10^2\lambda\ {\rm AU}^{-2}}$

Observational consequences?

cf. Silverman & Mallett (2002);
Rindler-Daller & Shapiro (2012)

Galaxy mergers

JK, Mota & Winther, in progress

- Force between galaxies same as in CDM (MOND confined to galaxies)

$\Longrightarrow$ "Encounter rate" as in CDM

What happens then?

- If $v_{\rm infall} < c_s \sim 200\ {\rm km/s}$, then negligible dynamical friction between superfluids

$\Longrightarrow$ Longer merger time scale + multiple encounters

- If $v_{\rm infall} > c_s$, then encounter will excite DM particles out of the condensate, which will result in dynamical friction

$\Longrightarrow$ Merged halo thermalize and settle back to condensate

Galaxy mergers

JK, Mota & Winther, in progress

- Force between galaxies same as in CDM (MOND

$\Longrightarrow$ "Encount

What happens

- If $v_{\rm infall} < c$

dynamical fricti

$\Longrightarrow$ Longer s

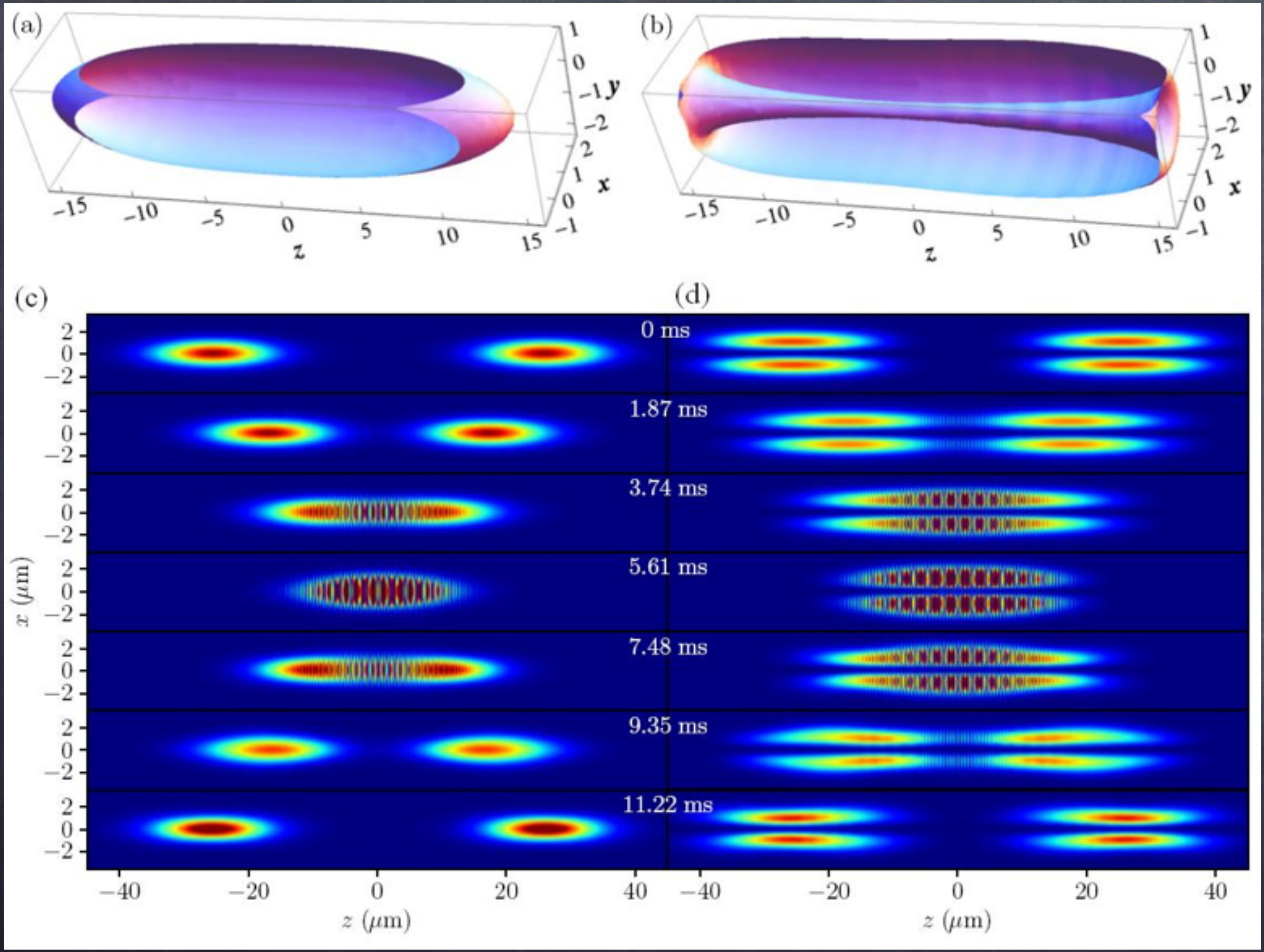
- If $v_{\rm infall} >$ es

out of the cond ion

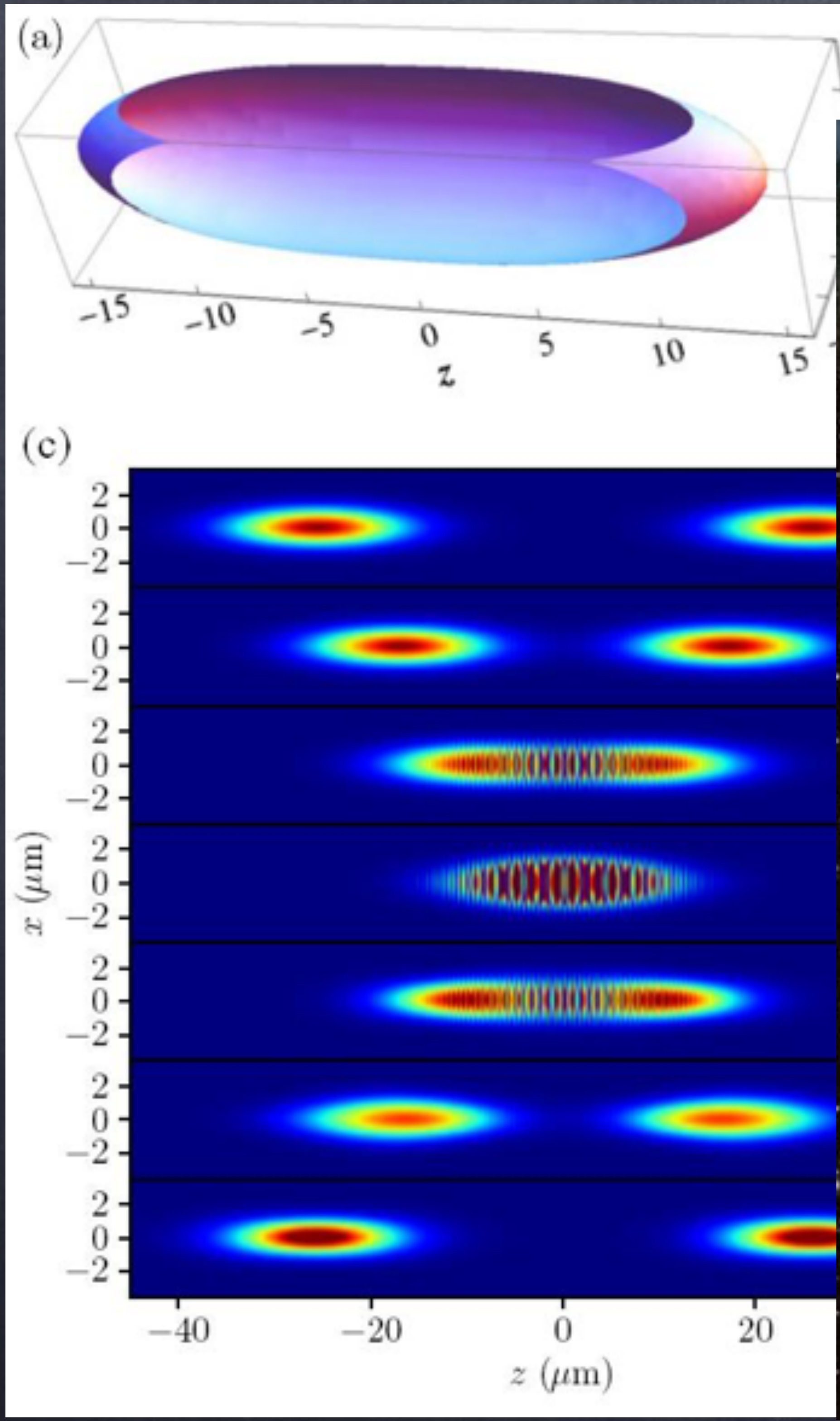
$\Longrightarrow$ Merged ndensate

Fornax

When superfluids collide

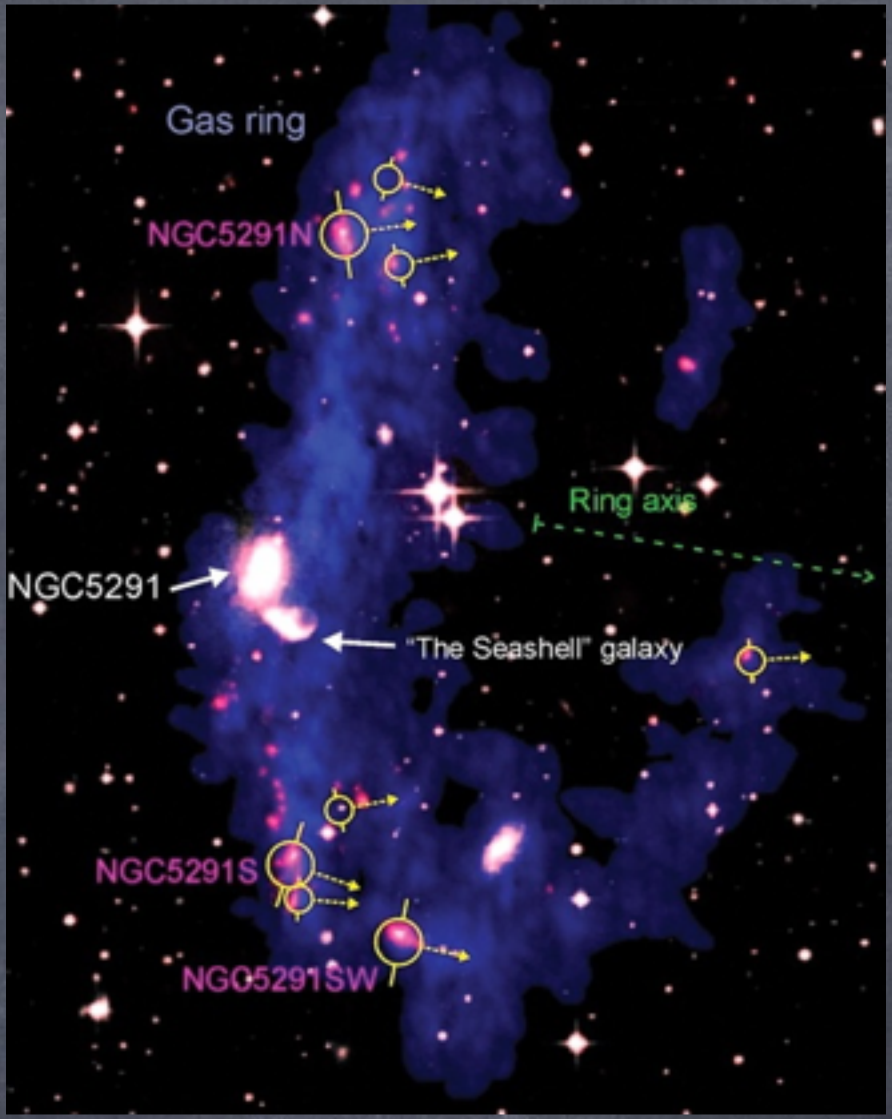


When superfluids collide



No DM ⟹ No MOND

Globular clusters

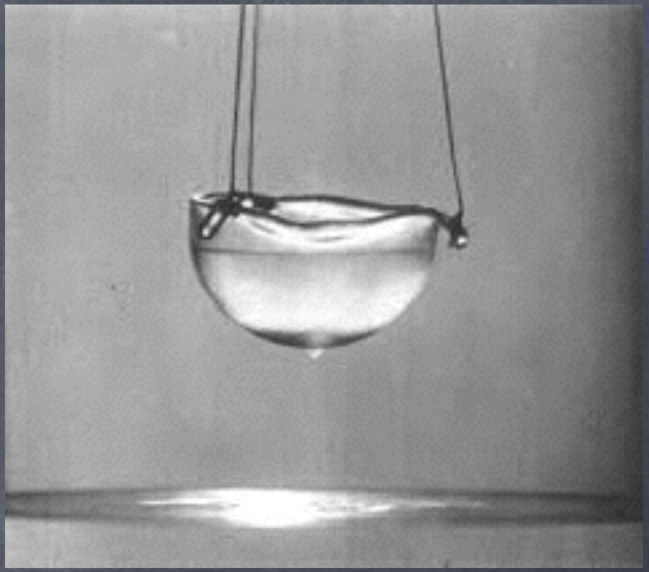


Tidal dwarf galaxies

Conclusions

- Small scales present greatest challenge to $\Lambda\mathrm{CDM}$

- DM superfluidity:
  - DM and MOND as different phases of same substance
  - All scales are comparable: $m \sim \mathrm{eV} \qquad \Lambda \sim \mathrm{meV}$



- Open questions:
  - Can we find a precise CM analogue?

$$P \sim \rho^3 \implies$$ 3-body interactions

  - No DM $\implies$ no MOND. Helpful?
  - How does dark energy fit into this picture?