Novel potentials for string axion inflation

Tatsuo Kobayashi

1. Introduction
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Abe, T.K., Otsuka, arXiv:1411.4768

T.K. Oikawa, Otsuka, arXiv:1510.08768

1. Introduction

In superstring theory on 6D compact space,
there are lots of axions as well as moduli,
which have a flat potential.

The shift symmetry controls
a flat potential of axions,
and some effects violate it
and generate axion potential.

Thus, axions are one of good candidates for the inflaton
in superstring theory.

Axion inflation

Indeed, lots of studies have been done.

Famous axion inflations

natural inflation

Freese, Frieman, Olinto, '90

$$V = C(1 - \cos(a/f))$$

monodromy inflation

Silverstein, Westphal, 08

$$V = Ca^r \qquad r = 2,\ 1,\ 2/3, \ldots$$

Modulation

Sub-leading corrections provide modulation terms

natural inflation

$$V = C(1-\cos(a/f)) + C'(1-\cos(a/f'))$$

monodromy inflation

$$V = Ca^r + C'(1-\cos(a/f))$$

Kobayashi, Takahashi, '11,

Czerny, Kobayashi, Takahashi, '14

T.K., Seto, Yamaguchi, '14

Higaki, T.K., Seto, Yamaguchi, '14

Modulation can change observables, ns and r

Modulation is important.

Axion inflation

Are there other forms of potentials for axion inflation in superstring theory ?

We would like to discuss it today.

2. Thereshold corections: Dedekind eta function

Non-perturbative effect on string moduli

We have to generate their potential by non-peturbative effects.

$$T = t + ia$$

$$W = Ae^{-cT} + \ldots.$$

→ axion potential

$$V = \Lambda(1 - \cos(ca)) + \ldots$$

potential for natural inflation

Freese, Frieman, Olinto, '90

Gaugino condensation

SYM in the hidden sector

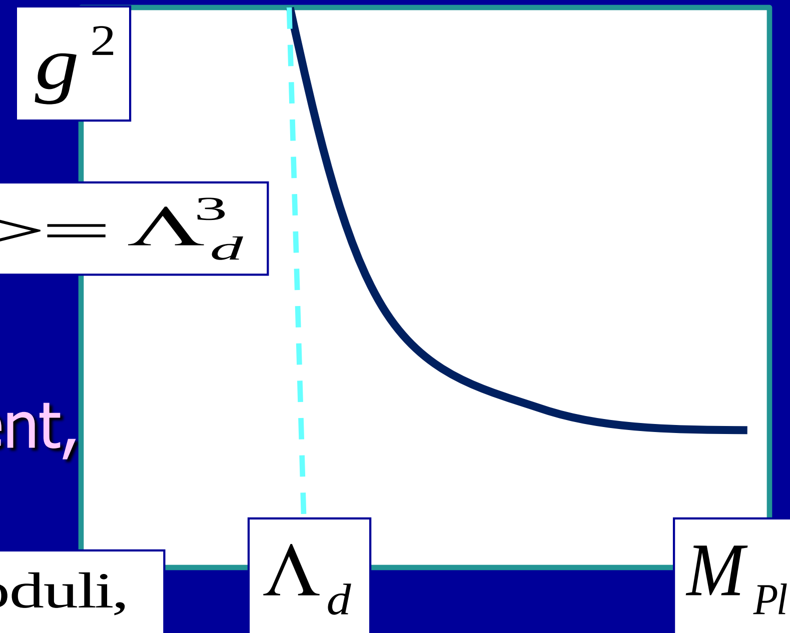
Strong coupling dynamics

→

gaugino condensation

$$< \lambda\lambda > = \Lambda_d^3$$

$$\Lambda_d = M_{Pl} \exp[-8\pi^2/(bg^2)]$$



b is one-loop beta function coefficient,

e.g. b=3N for SU(N) SYM.

$1/g^2$ is given by real part of some moduli,

e.g. $1/g^2 = T/(2\pi)$

the compact space volume,

where this gauge field lives.

$$W = < \lambda\lambda > = Ae^{-cT} + \ldots$$

$$c = 12\pi/b,$$

e.g. $c = 4\pi/N$ for SYM

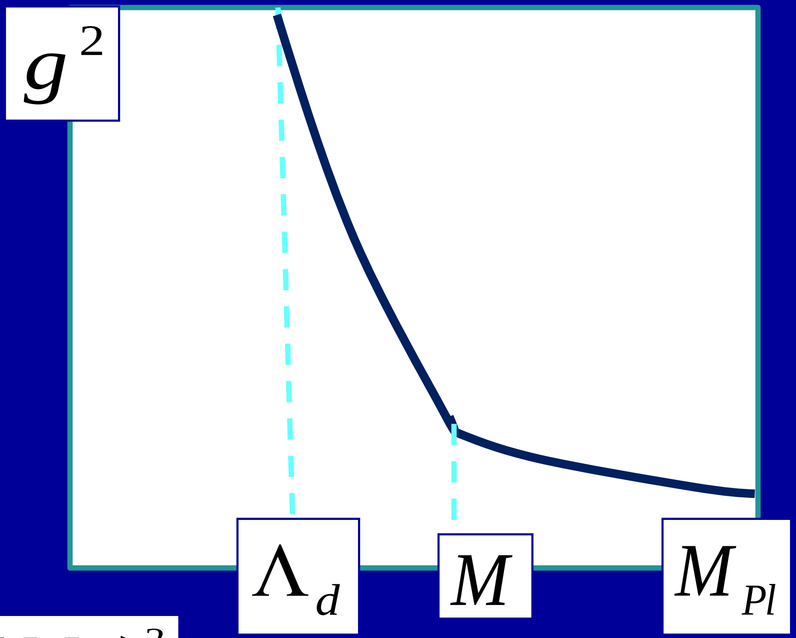
c = O(1) or larger

unless b >> O(10), eg. SU(100)

$$V = \Lambda(1 - \cos(ca)) + \ldots$$

For O(1) of Kahler metric of a, decay constatant is small.

Threshold corrections



some modes have the mass M.
They decouple at M.

The beta-function coefficient changes at M.

$$1/g^2(M) = T + (\Delta b/16\pi^2)\ln(M/M_{st})^2$$

Δb is beta-function coefficient contribution due to massive modes

Threshold corrections

Threshold corrections

some mass scale → correction

$$1/g^2(M) = T + (\Delta b/16\pi^2)\ln(M/M_{st})^2$$

For example, KK-modes affect the gauge couplings their masses depend on the moduli.

$$M_k = k/R$$

R: modulus (size of compact space)

Threshold corrections $= (\Delta b/8\pi^2)\Sigma \ln(k/RM_{st})$

$$\approx (\Delta b/8\pi^2)\int dk \ln(k/RM_{st})$$

$$= (\Delta b/8\pi^2)[-RM_{st} + \ln(RM_{st}) - 1]$$

Moduli-dependent term appears

Threshold corrections

some mass scale → correction

$$1/g^2(M) = T + (\Delta b/16\pi^2)\ln(M/M_{st})^2$$

KK-modes affect the gauge couplings

their masses depend on the moduli.

$$1/g^2(M) = T + \Delta(\text{Moduli})/16\pi^2$$

gaugino condensation $< \lambda\lambda >= \Lambda_d^3$

$$\Lambda_d = M_{Pl}\exp[-8\pi^2/(bg^2)] = M_{Pl}\exp[-(8\pi^2/b)(\Delta/16\pi^2) + \ldots]$$

$$W = < \lambda\lambda >= Ae^{-C} + \ldots.$$

$$C = (24\pi^2/b)(\Delta/16\pi^2)$$

$$V = \Lambda(1 - \cos(ca)) + \ldots$$

A new factor of O(100) appears

→ c <O(1), large decay constant, f > O(1)

Threshold corrections

S: dilaton, T: Kahler moduli, U: complex structure moduli

Heterotic string theory on orbifold: certain twisted sector

$$1/g^2(M) = S + \Delta/16\pi^2, \quad \Delta = 4b_{N=2} \ln \eta(T)$$

$$\eta(U) = \exp[-\pi U/12] \prod_m (1 - e^{-2\pi m U})$$

Dixon, Kaplunovsky, Louis, '91

Type IIA intersecting D-brane models: certain parallel D-brane

$$1/g^2(M) = aS + bU + \Delta/16\pi^2, \quad \Delta = 4b_{N=2} \ln \eta(T)$$

Lust, Stieberger, '07

T-dual

Type IIB D-brane models

D3/D7-branes or D5/D9-branes

$$1/g^2(M) = aS + bT + \Delta/16\pi^2, \quad \Delta = 4b_{N=2} \ln \eta(U)$$

Maybe also some correction in Hetero. Type IIA/B on CY

Eta function Inflation potential

$$1/\mathrm{g}^2(M) = aS + bT + \Delta/16\pi^2, \qquad \Delta = 4b_{N=2}\ln\eta(U)$$

$$\mathrm{W} = \mathrm{A}[\eta(U)]^{-3b_{N=2}/(b\pi)} + \ldots.$$

We assume that all of the moduli are stabilized except the axion = Im(U).

$$\eta(U) = \exp[-\pi U/12]\prod_m (1 - e^{-2\pi m U})$$

For a large Re(U), the leading term is dominant.

$$\text{For a large Re(U), e.g. Re(U)} = 1.3, \qquad \ln\eta(U) = -\pi U/12$$

Gaugino condensation

$$V = \Lambda(1 - \cos(ca)) + \ldots$$

$$c = b_{N=2}\pi/(6dL), \quad d = \mathrm{Re}(U) \quad \text{for SU(L)}$$

$$\text{decay constant} \quad f = 6dL/(b_{N=2}\pi)$$

the decay constant can be large, f> 1.

Eta function inflation potential

$$V = \Lambda(1 - \cos(ca)) + \ldots$$

$$c = b_{N=2}\pi/(6dL), \quad d = \mathrm{Re}(U) \quad \text{for SU(L)}$$

$$\text{decay constant} \quad f = 6dL/(b_{N=2}\pi)$$

Example

$$\mathrm{Re(U)} = 1.3, \qquad b_{N=2} = 1, \quad L = 4$$

Observables

$$n_s = 0.96, \quad r = 0.11, \quad N = 50, \quad \alpha_S = -10^{-3}$$

Eta function inflation potential

$$1/g^2(M) = aS + bT + \Delta/16\pi^2, \qquad \Delta = 4b_{N=2}\ln\eta(U)$$

Smaller Re(U) (leading + subleading)

$$\eta(U) = \exp[-\pi U/12](1-e^{-2\pi U}) + \ldots$$

$$V = \Lambda(1-\cos(ca)) + \Lambda_2\cos(c_2 a) + \ldots$$

Gaugino condensation $\Lambda_2 = \Lambda_1(2b_{N=2}/L)\exp[-(2\pi + b_{N=2}/(6L))\mathrm{Re}U]$

$$c = b_{N=2}\pi/(6dL), \;\; c_2 = (2\pi + b_{N=2}\pi/(6dL))/d, \;\; d = \mathrm{Re}(U) \quad \text{for SU(L)}$$

decay constant $f = 6dL/(b_{N=2}\pi)$

modulatin term is important.

Kobayashi, Takahashi, '11,

Czerny, Kobayashi, Takahashi, '14

T.K., Seto, Yamaguchi, '14

Higaki, T.K., Seto, Yamaguchi, '14

Natural inflation in Type IIB

$$\eta(U) = \exp[-\pi U/12](1-e^{-2\pi U}) + \ldots$$

$$V = \Lambda(1-\cos(ca)) + \Lambda_2 \cos(c_2 a) + \ldots$$

$$\Lambda_2 = \Lambda_1 (2b_{N=2}/L)\exp[-(2\pi + b_{N=2}/(6L))\,\mathrm{Re}\,U]$$

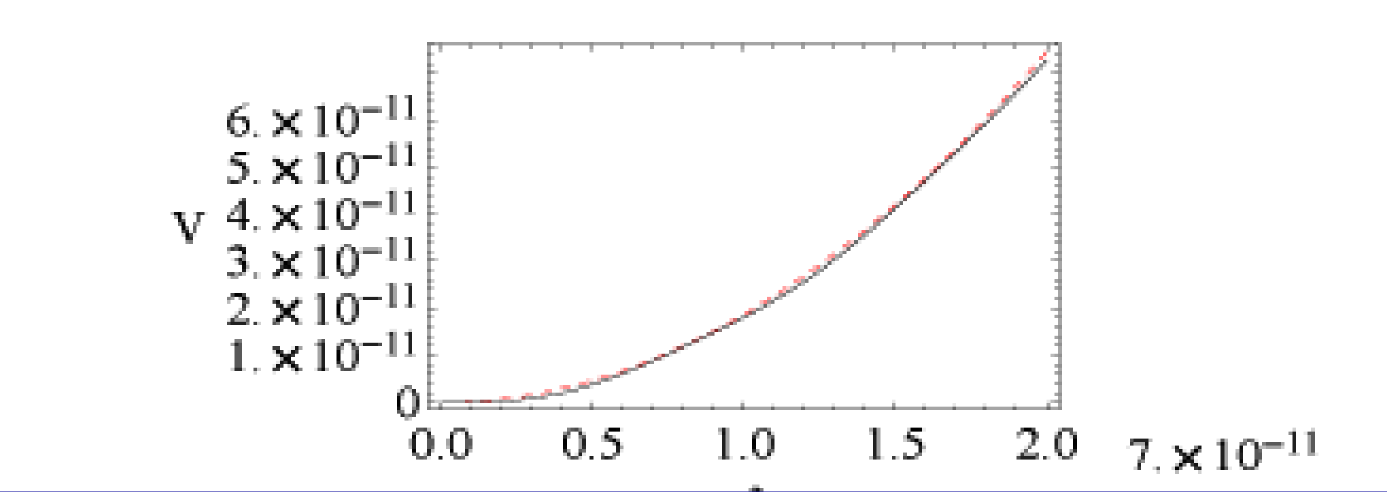
$$c = b_{N=2}\pi/(6dL),\ \ c_2 = (2\pi + b_{N=2}\pi/(6dL))/d,\ \ d = \mathrm{Re}(U) \quad \text{for SU(L)}$$

Example

$$\text{decay constant} \quad f = 6dL/(b_{N=2}\pi)$$

$$\mathrm{Re(U)} = 1.2, \qquad b_{N=2}/L = 1/5$$

Potential



Natural inflation in Type IIB

$$\eta(U) = \exp[-\pi U/12](1-e^{-2\pi U})$$

$$V = \Lambda(1-\cos(ca)) + \Lambda_2 \cos(c_2 a) + \ldots$$

$$\Lambda_2 = \Lambda_1(2b_{N=2}/L)\exp[-(2\pi + b_{N=2}/(6L))\,\mathrm{Re}\,U]$$

$$c = b_{N=2}\pi/(6dL),\ c_2 = (2\pi + b_{N=2}\pi/(6dL))/d,\ d = \mathrm{Re}(U) \quad \text{for SU(L)}$$

$$\text{decay constant} \quad f = 6dL/(b_{N=2}\pi)$$

Examples

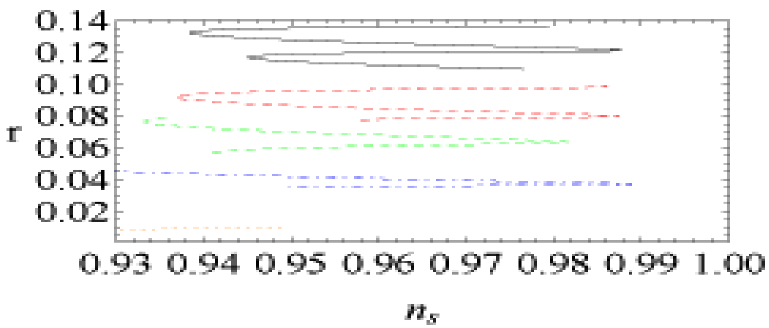


Figure 1: Predictions of (n_s, r) in the range of e-folding number, $50 \leq N_e \leq 60$. For the universal value of $\langle \mathrm{Re}\, U^2 \rangle = 1$, black-solid, red-dashed, green-dashed, blue-dotdashed and orange-dotted lines correspond to the fixed ratios $b/L = 1/10, 1/5, 1/4, 1/3, 1/2$, respectively.

Natural inflation in Type IIB

$$V = \Lambda(1-\cos(ca)) + \Lambda_2 \cos(c_2 a) + \ldots$$

$$c = b_{N=2}\pi/(6dL),\ c_2 = (2\pi + b_{N=2}\pi/(6dL))/d,\ d = \mathrm{Re}(U) \quad \text{for SU(L)}$$

b/L	$\mathrm{Re}\,U$	N	n_s	r	α_s
1/10	1.3	57	0.96	0.12	-0.0008
1/5	1.2	60	0.96	0.08	-0.001
1/4	1.2	58	0.96	0.06	-0.001
1/3	1.1	60	0.96	0.04	-0.001
1/2	1.1	50	0.95	0.01	-0.0003

Little effect of modulation in the first case

3. Quantum corrected period vector

In type IIB superstring theory, non-vanishing 3-form fluxes induce a potential for complex structure moduli and dilaton.

$$W_{GVW}(S,U) = \int (F_3 - iSH_3)\Lambda\Omega(U)$$
$$= \sum (N_F - iSN_H)^{\alpha}\Pi_{\alpha}(U)$$

The period vector $\Pi_{\alpha}(U)$ is polynomial functions of U at the tree level.

Some or all of complex moduli and dilaton can be stabilized.

Quantum corrected period vector

Quantum corrections have been computed.

Hosono, Klemm, Theisen, Yau, '94

$$\Pi_\alpha = \begin{pmatrix} 1 \\ U^i \\ 2F - U^i \partial_i F \\ \partial_i F \end{pmatrix}$$

The prepotential

$$F = -\frac{1}{3!}\kappa_{ijk} U^i U^j U^k - \frac{1}{2}\kappa_{ij} U^i U^j + \kappa_i U^i + \frac{1}{2}\kappa_0 - \frac{1}{(2\pi i)^3} \sum n_\beta \mathrm{Li}_3\left(e^{2\pi i d_i U^i}\right)$$

The polylogarithm function

$$\mathrm{Li}_3(z) = \sum \frac{z^n}{n^3}$$

Quantum corrected period vector

Superpotential

$$W_{GVW}(S,U) = \sum (N_F - iSN_H)^{\alpha} \Pi_{\alpha}$$

Kahler potential

$$e^{-K} = i(2(F - \overline{F}) - (U^i - \overline{U}^i)(F_i + \overline{F}_i))$$

Superpotential

Superpotential at tree level

$$W(S,U) = g_0(z) + g_1(z)(U_2 + NU_1)$$

z are other moduli

massive

massless

$$\Psi = U_2 + NU_1$$

$$\Phi = U_2$$

Quantum correction

$$\begin{aligned}\Delta W &= (g_2 + g_3 U_1)e^{-2\pi U_1} \\ &= (g_2 + (g_3/N)(\Psi - \Phi))e^{-2\pi(\Psi-\Phi)/N}\end{aligned}$$

Superpotential for inflation

T.K. Oikawa, Otsuka, 1510.0876

Superpotential

$$W = w_0 + (c + c'\Phi)e^{-2\pi\Phi/N}$$

inflaton

$$\phi = \mathrm{Im}\,\Phi$$

We also assume uplifting such that $V(\phi_{\min}) = 0$

Inflation potential

$$V \approx \Lambda_4\phi^2 + \Lambda_5\phi\sin(\phi/M_3) + \Lambda_6(1-\cos(\phi/M_3))$$

$$M_3 = Nk/(4\pi)$$

$$\Lambda_4, \Lambda_5, \Lambda_6 = O(e^{-4\pi\,\mathrm{Re}(U_1)})$$

Inflation potential

T.K. Oikawa, Otsuka, 1510.0876

Inflation potential

$$V \approx \Lambda_4 \phi^2 + \Lambda_5 \phi \sin(\phi / M_3) + \Lambda_6 (1 - \cos(\phi / M_3))$$

$$M_3 = Nk /(4\pi)$$

Hebecker, et.al. arXiv:1503.07912

studied a similar model with limited 3-form flux

$$V \approx \Lambda_6 (1 - \cos(\phi / M_3))$$

but we introduce the full 3-form flux

Inflation potential

T.K. Oikawa, Otsuka, 1510.0876

Inflation potential

$$V \approx \Lambda_4 \phi^2 + \Lambda_5 \phi \sin(\phi / M_3) + \Lambda_6 (1 - \cos(\phi / M_3))$$

$$M_3 = Nk / (4\pi)$$

In some case

$$V \approx \Lambda_4 \phi^2 + \Lambda_6 (1 - \cos(\phi / M_3))$$

another case

$$V \approx \Lambda_4 \phi^2$$

Numerical study

T.K. Oikawa, Otsuka, 1510.0876

$$V \approx \Lambda_4 \phi^2 + \Lambda_5 \phi \sin(\phi / M_3) + \Lambda_6 (1 - \cos(\phi / M_3))$$

$$M_3 = Nk/(4\pi)$$

M_3	Λ_4/Λ_6	Λ_4/Λ_6	N	r	n_s	α_s
5	1	5	55	0.06	0.965	-0.0007
3	1	3	60	0.008	0.97	0.0009
3	1	3	55	0.0097	0.964	0.0009
5	1/5	1	60	0.05	0.968	-0.0007

Large-field inflation

Small-field inflation is possible.

Small-field inflation

Kadota, T.K. Oikawa, Omoto,Otsuka, Tatsuishi, in progress

$$V \approx \Lambda_4 \phi^2 + \Lambda_5 \phi \sin(\phi / M_3) + \Lambda_6 (1 - \cos(\phi / M_3))$$

$$M_3 = Nk/(4\pi)$$

When we tune parameters,
we can realize

$$\mathrm{r} = \mathrm{O}(0.01) \;\; \text{or much smaller}$$

Variation of field is small during inflation, but
its field-value is large such as the Planck scale.

Summary

We have studied new types of axion inflation potential within the framework of superstring theory.

Threshold corrections lead to the eta function inflation potential.

It can enhance the axion decay constant by the inverse of loop factor.

Stringy threshold corrections also include modulation terms, which are interesting effects on observables.

Summary

Quantum corrections on period vector
can lead a new type of axion inflation potential,
which is a mixture of polynomial functions
and sinusoidal functions.
That can lead to both large tensor-to-scalar ratio
and small r.

Further work

Many models with the SM gauge group and three chiral generations have been constructed in type IIA (intersecting) D-brane models and type IIB magnetized D-brane models as well as heterotic orbifold models.

It is important to combine such models as the visible sector and the present inflation sector to realize the complete theory.

We know the coupling between axion/inflaton and SM fields. Then, we study details.