# f (R) Cosmology and Dark Matter

**Dark side of the Universe**

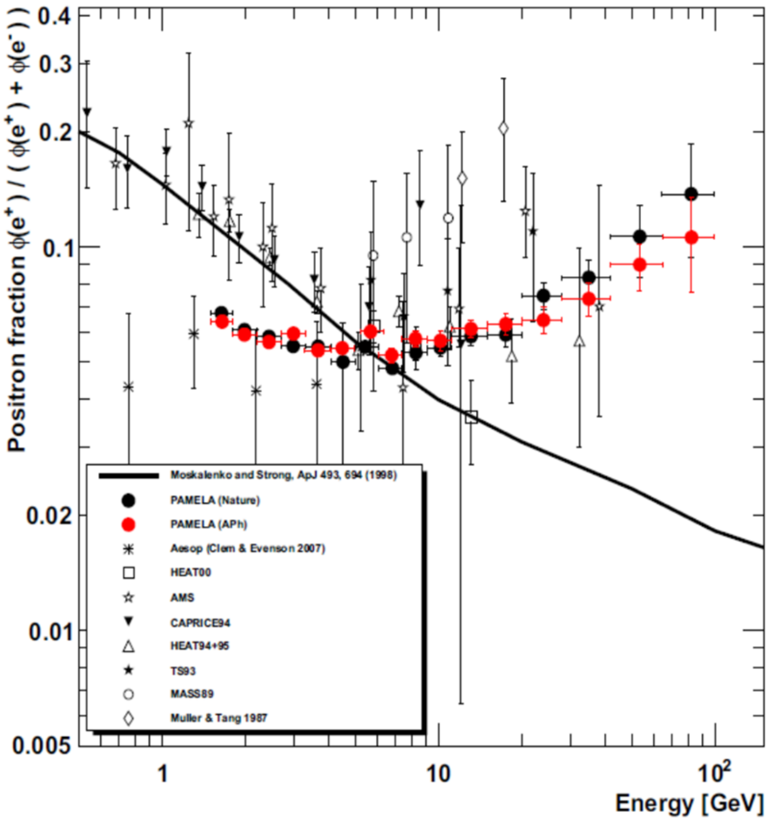
***Kyoto, December 14-18, 2015***

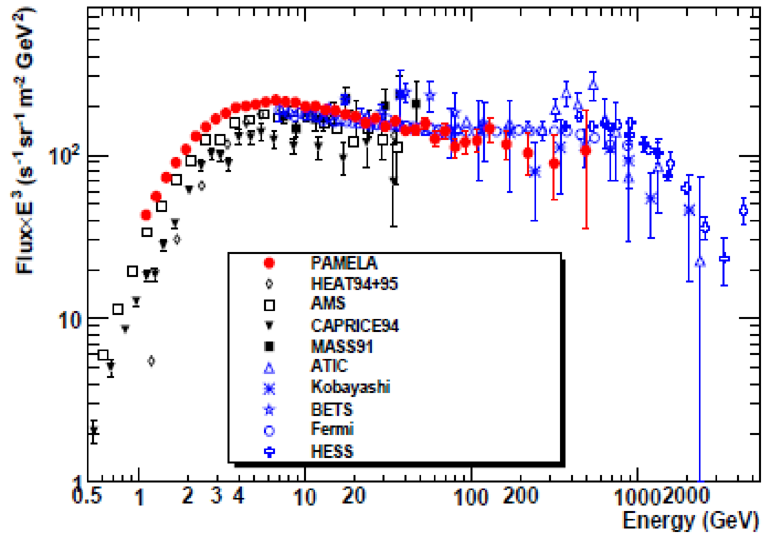
**G. Lambiase**
Università di Salerno
Dipartimento di Fisica E.R. Caianiello
& INFN - Naples
Italy

in collaboration with Capozziello, Galluzzi, Pizza (2015)

# Outlook

- PAMELA experiment and Dark Matter relic abundance (WIMPs)
- Extended theories of gravity (f(R) theories)
- Conclusions and perspectives




PAMELA is a dedicated satellite experiment conceived to study the anti–particle component of the cosmic radiation.

1. Significant deviation of the positron flux between theoretical and measurement expectation above 10 GeV

2. The fact that the positron fraction increases with the energy suggests the presence of primary positrons.

3. Electrons lose energy via inverse Compton scattering (with the highest energy particles coming from the closest distance)

# Astrophysical / DM explanation

The unidentified source of primary electrons/positrons must be local and capable to generate highly energetic leptons

**Astrophysics** [1]

**Non baryonic Dark Matter** [2] **(WIMPs)**

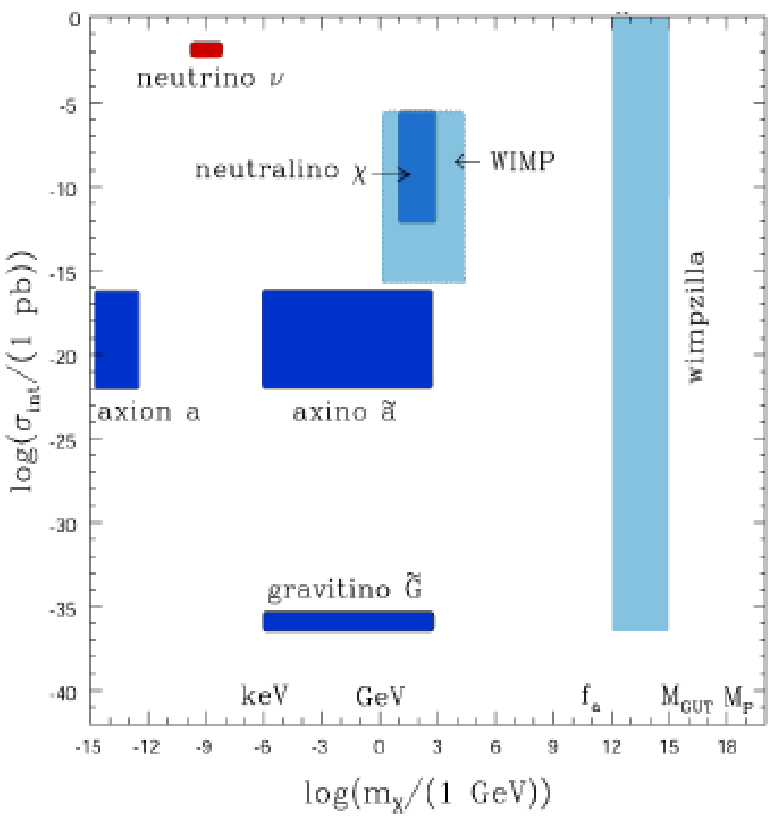
**WIMPs - $f(R)$ Cosmology**

(1) See for example:P. Blasi, Phys. Rev. Lett. (2009); P. Blasi and P.D. Serpico, Phys. Rev. Lett. (2009)
D. Gaggero, L. Maccione, D. Grasso, G. di Bernardo, C. Evolo, Phys. Rev. D (2014)
P.D. Serpico, Astrophysical models for the origin of positron excess, Astr.op. Phys. 2012

(2) See for example: M. Schelke, R. Catena, N. Fornengo, A. Masiero, M. Pietroni, Phys. Rev. D (2006)
F. Donato, N. Fornengo, M. Schelke (2007).
N. Okada, O. Seto, (2004),

# Dark Matter: WIMPs (miracle)

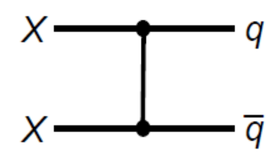
Weakly Interacting Massive Particles are the favorite candidate of DM because they **can be produced via thermal freezout with an abundance compatible with observations**




$m_{WIMPs} \sim (10 - 10^4)$GeV

- Assume a new (heavy) particle *X* is initially in thermal equilibrium
- Its relic density is

$$\Omega_X \propto \frac{1}{\langle \sigma v \rangle} \sim \frac{m_X^2}{g_X^4}$$




- $m_X \sim 100$ GeV, $g_X \sim 0.6 \rightarrow \Omega_X \sim 0.1$

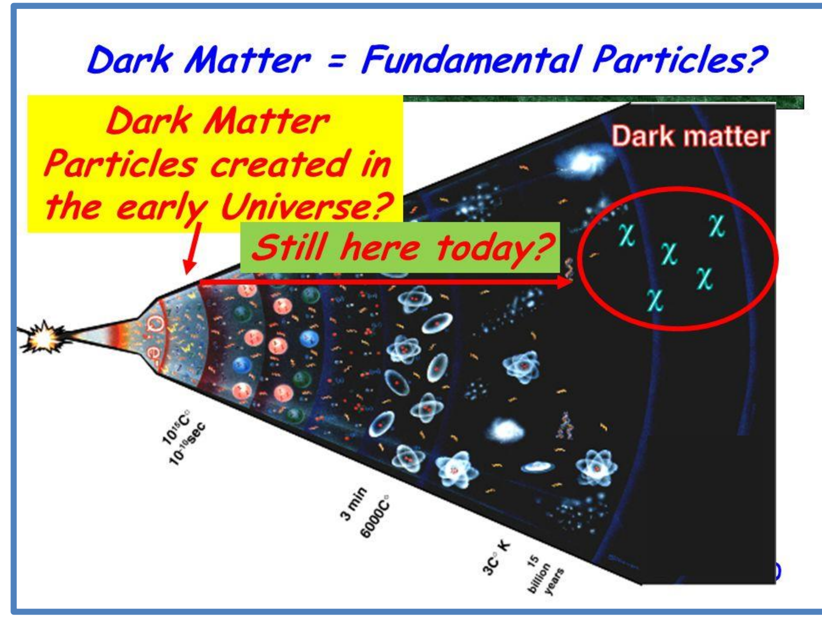
Remarkable coincidence:
**Particle Physics predicts particles with the right density to be Dark Matter**

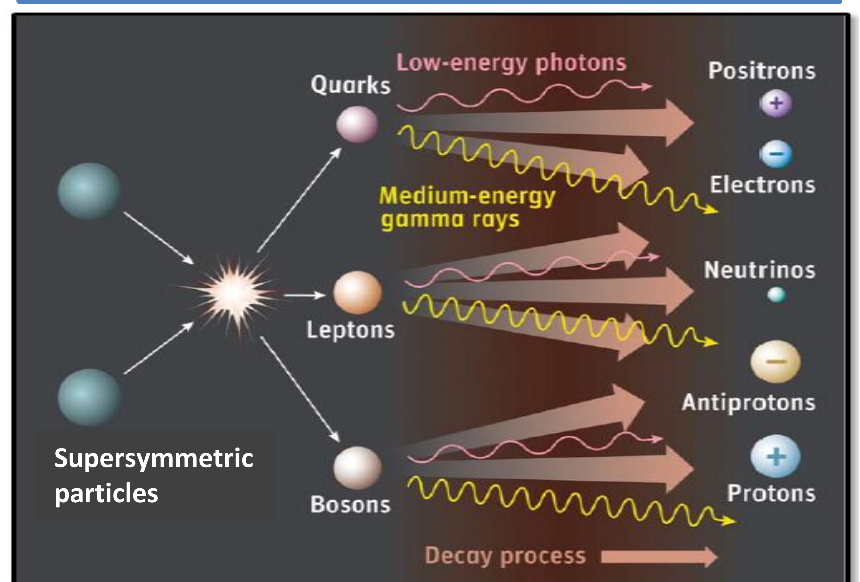
# DM in Galaxy (Indirect detection)

Dark Matter in our Galactic neighborhood may annihilate to different particles:

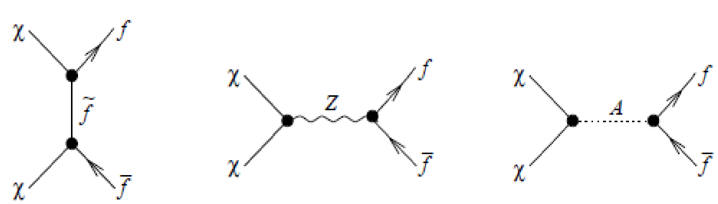
$\chi + \chi \longrightarrow$

- Photons
- Neutrinos
- Positrons
- Antiprotons
- Antideuterons




Late Dark Matter annihilation/decay processes might be observable today by looking for annihilation/decay products in Cosmic Radiation in the form of anomalous abundance



# Relic density of WIMPs

The relic density of WIMPs in the standard cosmological scenario is described by **Boltzmann equation**

Dilution from expansion

The thermal average of the annihilation cross section σ multiplied with the relative velocity $v$ of the two annihilating χ particles

$$\frac{dn}{dt} = -3\underbrace{H}_{GR}n - \langle \sigma v \rangle \left[ n^2 - n_{\rm eq}^2 \right]$$

$Y = \frac{n}{s}$ (relic DM abundance)

$\chi\chi \rightarrow f\bar{f}$

$f\bar{f} \rightarrow \chi\chi$

These terms on RHS represent the decrease (increase) of the number density due to annihilation into (production from) lighter particles

Review PAMELA exp and DM relic abundance
In the framework of $f(R)$ cosmology

# Alternative/Modified cosmology (f(R))
# DM relic abundance and PAMELA experiment

Alternative/Modified cosmologies enter into Boltzmann equation trough the expansion rate $H$

$$\frac{dn}{dt} = -3Hn - \langle \sigma v \rangle \left[ n^2 - n_{\rm eq}^2 \right]$$

1. Alternative cosmologies yield a modified thermal histories in the early Universe during the pre BBN (an epoch not directly constrained by cosmological observations)

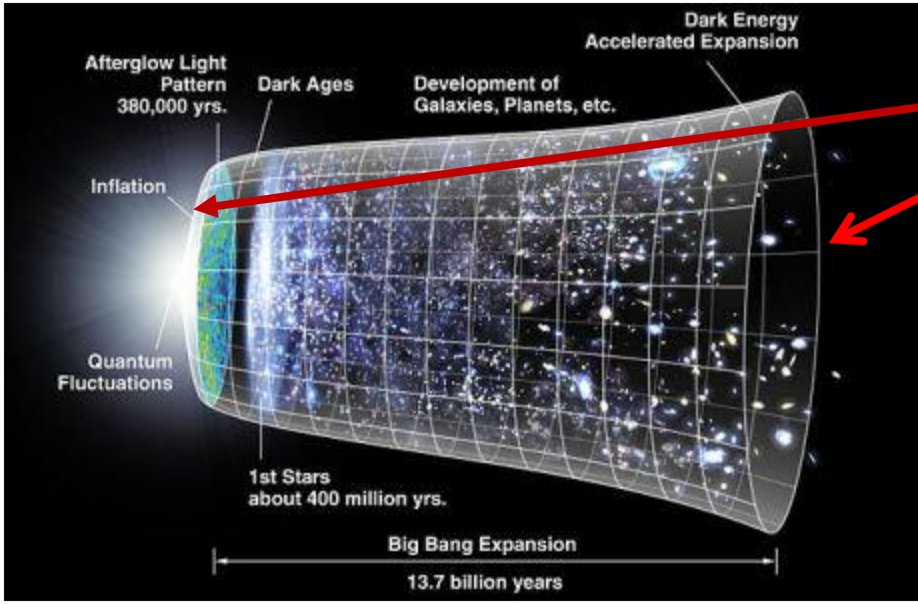
2. In particular when the expansion rate is enhanced with respect to GR

$$H(T) = A(T) H_{GR}(T)$$

thermal relics decouple with larger relic abundance

3. This effect may have an impact on SUSY candidates for DM (WIMPs)

4. We propose an interplay between *DM physics* and $f(R)$ cosmology, *rather than replace DM with alternative cosmology*




Extended theories of gravity

$f(R)$ - gravity

Generalization of the Hilbert-Einstein action to a generic (unknown) *f(R)* theory of gravity

$f(R) = R$ → Hilbert-Einstein action

$$A = \int d^4x \sqrt{-g}[f(R) + L_{matter}]$$

Copeland-Sami-Tsujikawa (2006). Nojiri & Odintsov (2011). Capozziello & Lambiase (2013), Joyce,Jain,Khoury, Trodden (2015)

Available models, particularly interesting for the cosmology, are

$$f(R) = R - \mu R_c (R/R_c)^n , \quad 0 < n < 1 , \quad \mu, R_c > 0 ,$$ Tsujikawa et al.

$$f(R) = R - \mu R_c \frac{(R/R_c)^{2n}}{(R/R_c)^{2n} + 1} , \quad n, \mu, R_c > 0 ,$$ Hu-Sawicky

$$f(R) = R - \mu R_c \left[1 - (1 + R^2/R_c^2)^{-n}\right] , \quad n, \mu, R_c > 0 ,$$ Starobinsky

$$f(R) = R - \mu R_c \tanh(R/R_c) , \quad \mu, R_c > 0 .$$ Tsujikawa

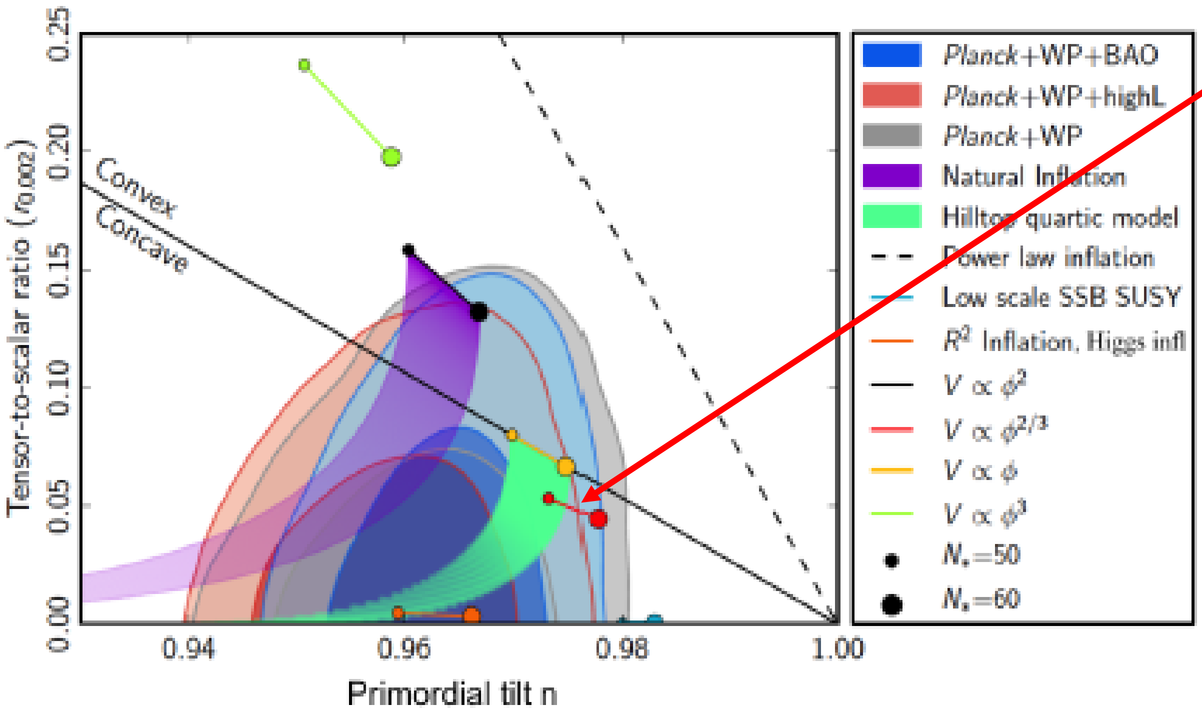
$$R_c \sim H_0^2 \sim \frac{8}{3} \frac{\rho_c}{m_P^2} \simeq 10^{-84} \mathrm{GeV}^2 , \quad \mu \simeq \mathcal{O}(1)$$

We shall consider the model $$f(R) = R + \alpha R^n$$

# $f(R)$ cosmology - $f(R) = R + \alpha R^n$

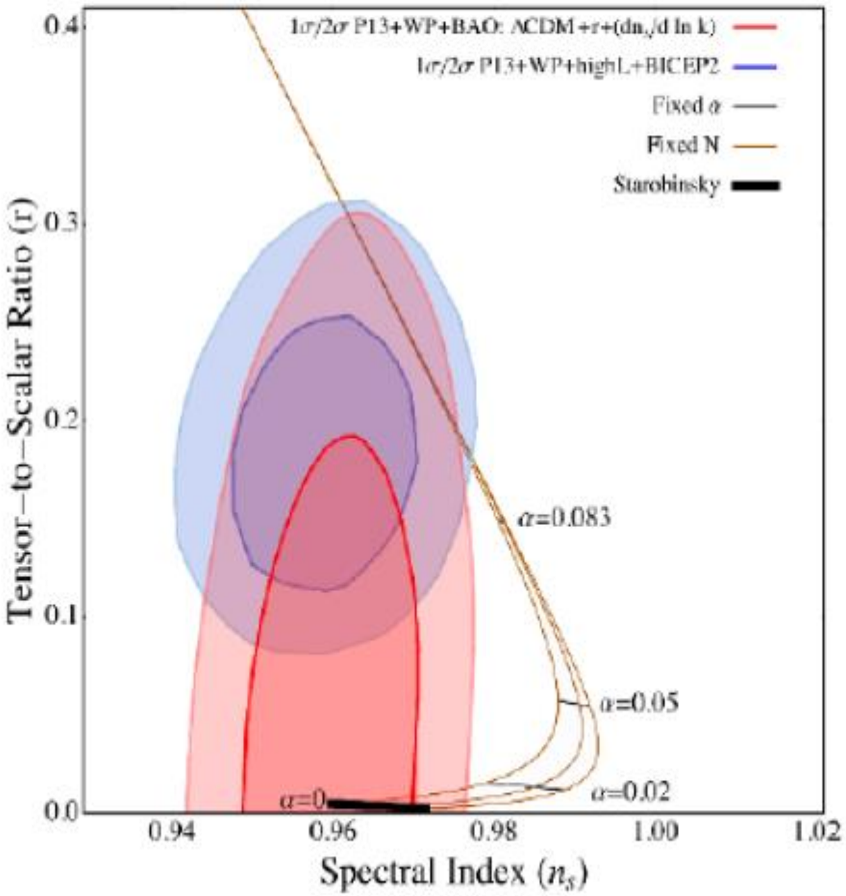
Starobinsky's model

$f(R) = R + \alpha R^2$




n-r plane with various inflation models for 50-60 e-foldings of inflation*

*Ade et al. [Planck Collaboration], Astron. Astrophys. 571, A22 (2014)




$$f(R) = R + \alpha R^n$$

$$\alpha R^n \gg R$$

Sizeble primordial tensor modes can be generated in *marginally deformed* models provided the constraints on the parameter$^{(1,2)}$

$$1 < n \leq 2$$

(by a direct comparison with the combined fit of BICEP2 and Planck data$^{(3)}$)

(1) F. Sannino et al. JHEP 02, 050 (2015)
(2) -$R + \alpha R^n$ model can be derived from SUGRA: S. Ferrara, A. Kehagias, and A. Riotto, Fortsch. Phys. 62, 573 (2014); Fortsch. Phys. 63, 2 (2015). J. Ellis, N.E. Mavromatos, D.V. Nanopoulos, Phys.Lett. B 732, 380 (2014).
(3) BICEP2 Collaboration, P.A.R. Ade et al., Detection of B-mode polarization at degree angular scales by BICEP2, Phys. Rev. Lett. 112 (2014) 241101 [arXiv:1403.3985] [INSPIRE]

$$f(R) = R + \alpha R^n$$

$$\delta(S_{f(R)} + S_m) = 0 \Rightarrow \quad f' R_{\mu\nu} - \frac{f}{2} g_{\mu\nu} - \nabla_\mu \nabla_\nu f' + g_{\mu\nu} \Box f' = \kappa^2 T^m_{\mu\nu}$$

the field equations

$$3\Box f' + f'R - 2f = \kappa^2 T^m, \qquad T^m = \rho - 3p$$

$$\nabla^\mu G^c_{\mu\nu} = 0, \qquad \nabla_\mu T^{m\,\mu\nu} = 0$$

**Bianchi's identities**

Spatially flat Friedman-Robertson-Walker (FRW) metric

$$ds^2 = dt^2 - a^2(t)[dx^2 + dy^2 + dz^2]$$

$$G^{c\,0}_{\ 0} = f' R^0_0 - \frac{1}{2} f + 3H \dot{f}',$$

$$G^{c\,j}_{\ i} = f' R^j_i - \frac{f}{2} \delta^j_i + \left( \ddot{f}' + 4H \dot{f}' \right) \delta^j_i,$$

$\alpha R^n > R \qquad (n > 1)$

**Radiation dominated era**

$$\rho = \frac{\pi^3 g_*}{30} T^4$$

$$a(t) = a_0 t^\beta .$$

Field equations

$$\alpha \Omega_{\beta,n} R^n = \kappa^2 \rho ,$$
$$\alpha \Gamma_{\beta,n} R^n = \kappa^2 T^m ,$$

$$\Omega_{\beta,n} \equiv \frac{1}{2}\left[\frac{n(\beta+2n-3)}{2\beta-1} - 1\right],$$

$$\Gamma_{\beta,n} \equiv n - 2 - \frac{n(n-1)(2n-1)}{\beta(2\beta-1)} + \frac{3n(n-1)}{2\beta-1},$$

Trace equation ⟶ $T^m = \rho - 3p = 0 \quad \rightarrow \quad \beta = \frac{n}{2}$

Friedmann equation ⟶ $\Omega_n > 0 \quad \rightarrow \quad n \geq 1.3$

$t_*$ (or $T_*$) the transition time (temperature)

$$\begin{pmatrix} \text{Universe described} \\ \text{by } f(R) \text{ cosmology} \end{pmatrix} \xrightarrow{t_* (\text{or} T_*)} \begin{pmatrix} \text{Universe described} \\ \text{by standard cosmology} \end{pmatrix}$$

Pre-BBN

$$\alpha \Omega_{\beta,n} R^n(t_*) = H_{GR}^2(t_*)\,.$$

After BBN

$$t_* = \left[4\tilde{\alpha}\Omega_{\beta,n}[6|\beta(2\beta-1)|]^n\right]^{\frac{1}{2(n-1)}} M_{Pl}^{-1}\,,$$

$$T_* \equiv M_{Pl}[6(|\beta(1-2\beta)|)]^{-\frac{n}{4(n-1)}} \left[\frac{15}{16\pi^3 g_*}\right]^{\frac{1}{4}} [4\tilde{\alpha}\Omega_{\beta,n}]^{-\frac{1}{4(n-1)}}\,,$$

$$t = t_* \left(\frac{T}{T_*}\right)^{-\frac{2}{n}}\,.$$

$$T_* = T_*(n, \alpha, \beta = n/2)$$

# Modified expansion rate in f(R) cosmology

$$H(T) = A(T) H_{GR}(T)\,,$$

Modified expansion rate with respect to General Relativity

$$A(T) = \eta \left(\frac{T}{T_f}\right)^{\nu}$$

$$\eta = \sqrt{3}\, n \left(\frac{T_f}{T_*}\right)^{\nu}$$

$$\nu = \frac{2}{n} - 2$$

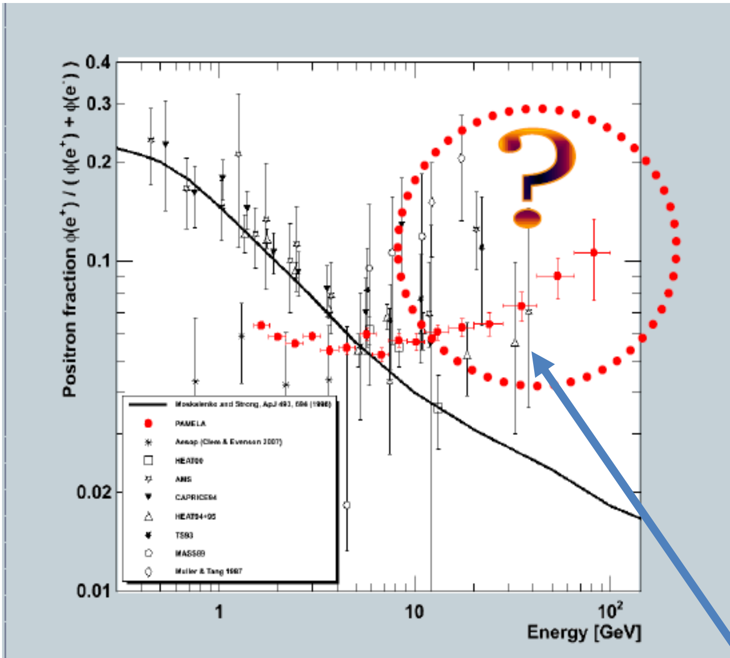
$$T_* \equiv M_{Pl}[6(|\beta(1-2\beta)|)]^{-\frac{n}{4(n-1)}} \left[\frac{15}{16\pi^3 g_*}\right]^{\frac{1}{4}} [4\tilde{\alpha}\Omega_{\beta,n}]^{-\frac{1}{4(n-1)}}\,,$$

$$\Gamma(T_f) = H(T_f)$$

$T_f$ = freeze-out temperature at which WIMPs decouple

The transition temperature $T_*$ is fixed around the BBN temperature

| $n$ | $T_*$(MeV) | $\alpha$ |
|---|---|---|
| 1.3 | 1 | $10^{14}\mathrm{GeV}^{-0.6}$ |
| | $10^2$ | $10^{12}\mathrm{GeV}^{-0.6}$ |
| 2 | 1 | $10^{44}\mathrm{GeV}^{-2}$ |
| | $10^2$ | $10^{36}\mathrm{GeV}^{-2}$ |




Adriani et al. –AP 34 (2010) 1

# PAMELA Exp & DM abundance

To explain the PAMELA results, taking into account for the precise WMAP measurements of DM relic abundance require the study of the Boltzmann equation for relic DM particle Y

$Y = \frac{n_\chi}{s}$ = Dark Matter abundance

$$x = \frac{m_\chi}{T}$$

$$\chi + \chi \to f + f$$

$$H(T) = A(T) H_{GR}(T)$$

The enhancement of the expansion rate implies that thermal relics decouple with larger relic abundance

$$\frac{1}{Y_0} = \frac{1}{Y_f} + \sqrt{\frac{\pi M_{Pl}^2}{45}} \int_{x_f}^{\infty} \frac{g_*(x) < \sigma_{ann} v >}{\sqrt{g_*(x)} A(x) x^2} dx$$

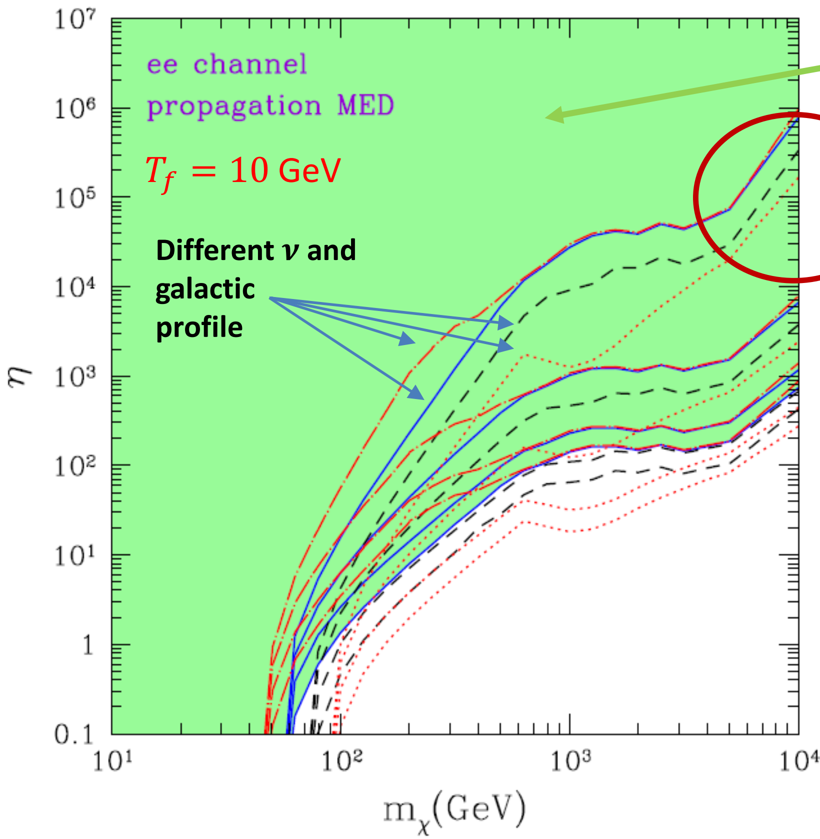
Relic density $\Omega_\chi h^2 = \frac{m_\chi s_0 Y_0}{\rho_c}$, WMAP Coll.: $\Omega_\chi h^2 = 0.11$

$$H(T) = A(T) H_{GR}(T) \,,$$

$$A(T) = \eta \left( \frac{T}{T_*} \right)^{\nu}$$

Similar expressions are obtained in different frameworks:

- $\nu = 2$ in Randall-Sundrum type II brane cosmology
- $\nu = 1$ in kination models
- $\nu = 0$ (boosts of the expansion rate)
- $\nu = -1$ in scalar-tensor cosmology




Excluded region

$\nu = -1,0$

$\eta \geq 0.1$

$m_\chi \sim (10^2 - 10^4) GeV$

$$f(R) = R + \alpha R^n \qquad 1.3 \leq n \leq 2$$

$$\nu = \frac{2}{n} - 2 \quad \rightarrow \quad \nu = [-1, -0.46]$$

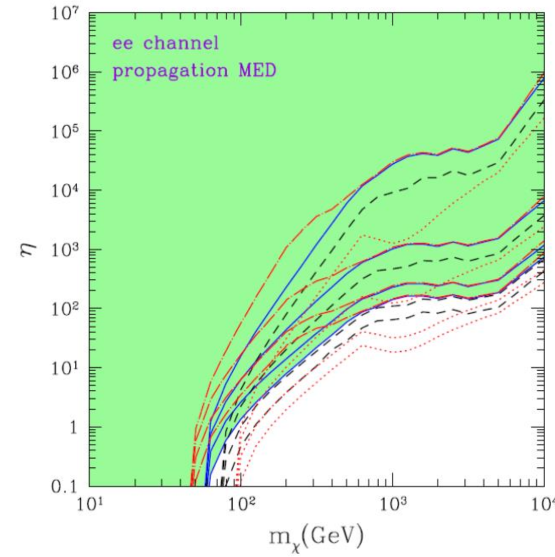
Catena, Fornengo, Masiero, Pati, Pieri, PRD 81, 123522 (2010)

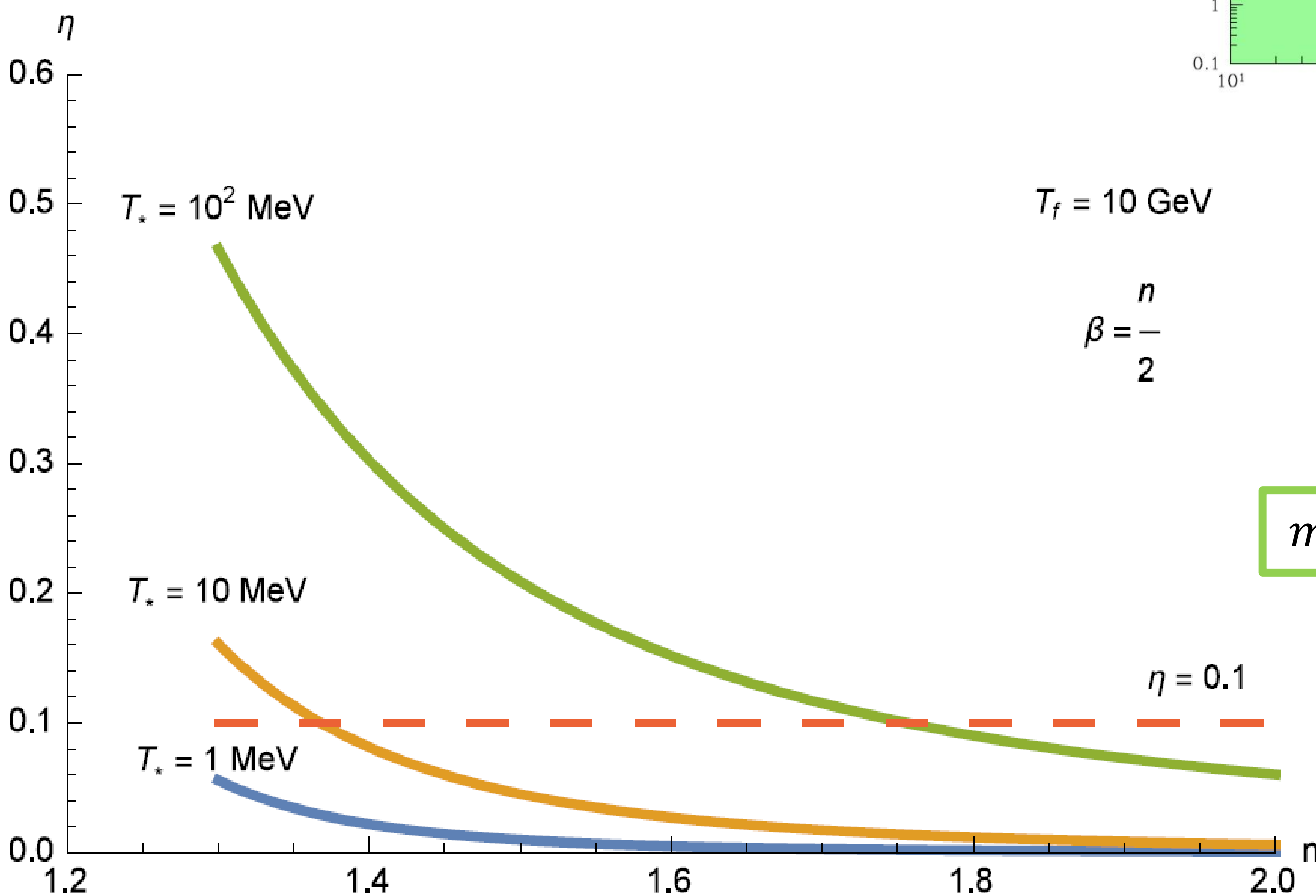
| $n$ | $T_*$(MeV) | $\alpha$ |
|---|---|---|
| 1.3 | 1 | $10^{14}\mathrm{GeV}^{-0.6}$ |
| | $10^2$ | $10^{12}\mathrm{GeV}^{-0.6}$ |
| 2 | 1 | $10^{44}\mathrm{GeV}^{-2}$ |
| | $10^2$ | $10^{36}\mathrm{GeV}^{-2}$ |

$$A(T) = \eta\left(\frac{T}{T_f}\right)^{\nu}$$

$$\eta = \sqrt{3}\,n\left(\frac{T_f}{T_*}\right)^{\nu}$$

$$\nu = \frac{2}{n} - 2$$




$$m_\chi \sim (10^2 - 10^4)GeV$$

# CONCLUSIONS

We have discussed the possibility to explain both DM relic abundance and PAMELA experiment in the framework of f(R) cosmology

Results are based on the fact that cosmic evolution of the early Universe gets modified in Modified cosmology

We have considered f(R) model of the form

$f(R) = R + \alpha R^n$ $\quad\quad$ $1 < n \leq 2$ $\quad\quad$ WMAP –BICEP2-Planck data

The model is consistent for DM candidates (WIMPs) with mass $m_\chi \sim (10^2 - 10^4) GeV$

***Improvement of the model:***

**1.** Find a more general solution (with respect to the power law solution $a(t) = a_0\, t^{\beta}$)

The trace eq. $T^m = 0$ implies $\beta = {}^{n}/_{2}$ (i.e. $\beta$ is fixed), and $\alpha$ is fixed by imposing $T_* \sim T_{BBN}$

A possibility is to consider the case $T^m \neq 0$ $i.e.$ $\beta$ *and* $n$ *independent*

$$P_{\mathcal{R}} \simeq \frac{N^2}{3\pi}\frac{1}{\alpha M_{Pl}^2} \quad \text{(amplitude curvature perturbation)}$$

$$N = 50-60 \quad \text{(e-folding number)}$$

$$P_{\mathcal{R}} \sim 10^{-9} \quad \text{(WMAP data)}$$

$$\alpha_{WMAP} \simeq 10^{-23}\text{GeV}^{-2}$$

$$f(R) = R + \alpha R^n$$

$$n = 2$$

$f(R)$ cosmology → Standard cosmology

$T_*(\beta)$

a) Bulk viscosity effects:

$$T_\alpha^\beta = (\rho + p + \Pi)u_\alpha u^\beta - (p + \Pi)\delta_\alpha^\beta \quad \text{->} \quad T^m = -3\Pi$$

$\Pi$ = viscosity

b) Trace anomaly of gauge fields $T \propto \beta(g) F_{\alpha\beta} F^{\alpha\beta} \sim \varsigma \ (\varsigma = 1 - 3w)$

$$\varsigma = \frac{5}{18\pi^2} \frac{g^4}{(4\pi)^2} \frac{\left(N_C + \frac{5}{4}N_f\right)\left(\frac{11}{3}N_C - \frac{2}{3}N_f\right)}{2 + \frac{7}{2}\frac{N_C N_f}{N_C^2 - 1}} + O(g^5)$$

c) Gravitational trace anomaly in f(R) gravity

$$T = \langle T_\alpha^\alpha \rangle_{GR} = k_3 \left(\frac{R^2}{3} - R_{\alpha\beta}R^{\alpha\beta}\right) - 6k_1 \boxdot R$$

**2.** Determine an **upper/lower bound** on Dark Matter masses by considering different cosmologcal models or models based on curvature invariants

$$L = L(R, R_{\alpha\beta}R^{\alpha\beta}, \boxdot R, \boxdot^l R, \ldots)$$

**Thank you for your attention**