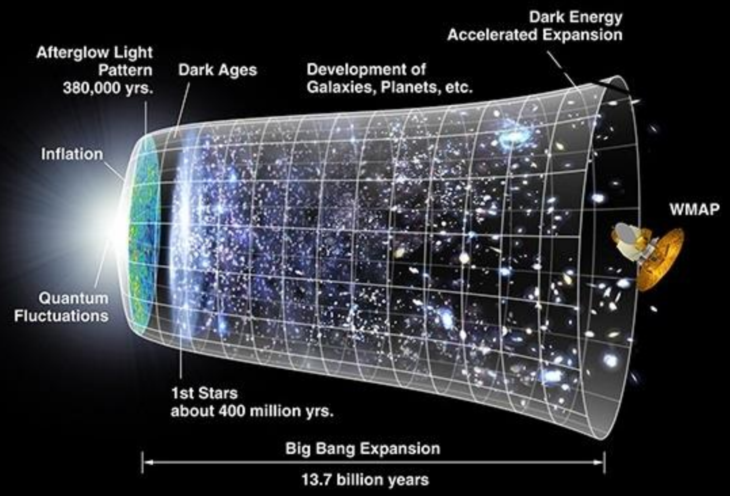

Running Non-Minimal Inflation with Stabilized Potential

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"Running Non-Minimal Inflation with Stabilized Potential,"

By *Nobuchika Okada* and *Digesh Raut*.

Arxiv: 1509.04439 (PRD Under Review)

Outline

- Outline of Single Field Inflationary Scenario

- Non-Minimal Inflation Scenario

- Inflaton in Particle Physics Context

- Motivation

  Inflationary Physics $\overset{Correlations}{\underset{?}{\Longleftrightarrow}}$ Low Energy Phenomenology

- Discuss general feature of RGE Improved Gauged Higgs type Inflation with Yukawa sector and analyze the issue of Potential Instability

Model

Non-Minimally Coupled B-L Higgs Inflation

Standard Big-Bang Cosmology and Inflation

- Flatness Problem (Fine-tuning Problem)
  - Our universe is very flat today. : $\Omega(t_0) = 1.02 \pm 0.02$.
  - Requires Extreme Fine-Tuning : 10^{-4} (recombination) and 10^{-16} (BBN)
- Horizon Problem
  - Angular size of causally connected patches at CMB $\simeq 1.6^{0}$

- Inflationary Solution
  - Phase of accelerated expansion before BBN era

Inflation: ⟶ $a(t_E) = a\,(t_I)\, e^{\sim 60}$

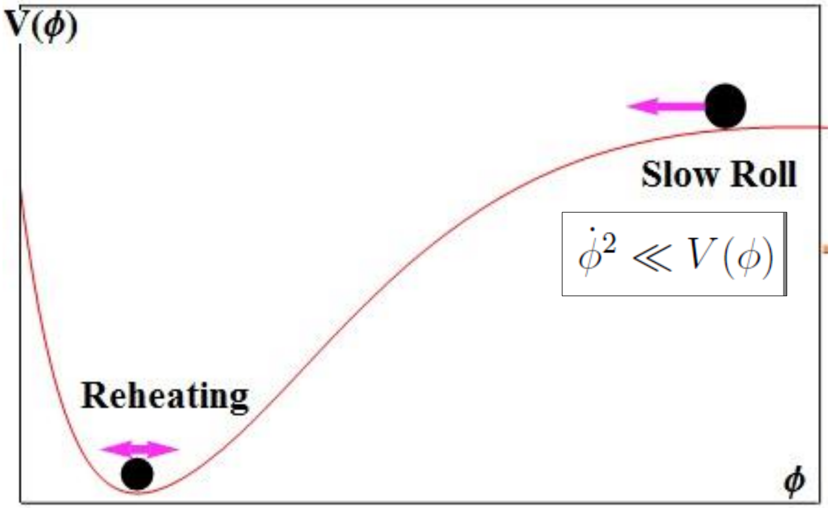
- Primordial Density Fluctuation
  - CMB Temperature Fluctuation ⟺ Primordial Density Fluctuation

$$\frac{\delta T}{T} \cong 10^{-5} \quad \text{(Planck +WMAP)}$$

  - Standard Big Bang Cosmology does not explain the origin of these fluctuations
  - Inflation scenario naturally generates such density fluctuation

Single Scalar Field: Slow Roll Inflation Scenario

- Slow Roll (Inflation Regime)



- Reheating (Decay Of Inflaton)

$$\ddot{\phi} + 3H\dot{\phi} + \Gamma\dot{\phi} + m_\phi^2\phi = 0\ .$$

- Decay width (Γ) : ϕ → SM particles
- Themalization of decay products recreates Standard Big Bang Scenario.

Non-Minimal $\lambda\phi^4$ Inflation Scenario

- Jordan Frame

$$\mathcal{S}_J = \int \mathrm{d}^4x \, \sqrt{-g} \left[-\frac{1}{2} f(\phi)\mathcal{R} - \frac{1}{2}(\nabla\phi)^2 - V_J(\phi) \right]$$

$$V_J = \frac{\lambda}{4}\phi^4$$

- Conformal Transformation :

$$g^E_{\mu\nu} \equiv f(\phi) \, g_{\mu\nu}$$

$$f(\phi) = 1 + \xi\phi^2$$

- Einstein Frame

$$\mathcal{S}_E = \int \mathrm{d}^4x \, \sqrt{-g_E} \left[-\frac{1}{2}\mathcal{R}_E - \frac{1}{2}[\nabla\sigma]^2 - \frac{V_E[\phi(\sigma)]}{f^2[\phi(\sigma)]} \right]$$

$$\sigma'(\phi)^2 = \frac{1 + (1 + 6\xi)\xi\phi^2}{(1 + \xi\phi^2)^2}$$

$$V_E = \frac{\lambda}{4} \frac{\phi^4}{(1 + \xi\phi^2)^2}$$

- Canonical Variables

Jordan Frame : ϕ

Einstein Frame : σ

Our Choice of Frame

Non-Minimal Inflation Scenario

- Slow-Roll Parameters and E-holding Number

$$\epsilon(\phi) = \frac{1}{2} m_P^2 \left(\frac{V_E'}{V_E \sigma'}\right)^2 ,$$

$$\eta(\phi) = m_P^2 \left[\frac{V_E''}{V_E (\sigma')^2} - \frac{V_E' \sigma''}{V_E (\sigma')^3}\right] ,$$

$$\zeta^2(\phi) = m_P^4 \left(\frac{V_E'}{V_E \sigma'}\right) \left(\frac{V_E'''}{V_E (\sigma')^3} - 3\frac{V_E'' \sigma''}{V_E (\sigma')^4} + 3\frac{V_E' (\sigma'')^2}{V_E (\sigma')^5} - \frac{V_E' \sigma'''}{V_E (\sigma')^4}\right)$$

Reduced Planck Mass (Mp = 1)

$$N = \frac{1}{\sqrt{2}} \int_{\phi_E}^{\phi_I} \frac{\mathrm{d}\phi}{\sqrt{\epsilon(\phi)}} \left(\frac{\mathrm{d}\sigma}{\mathrm{d}\phi}\right)$$

- Slow Roll Conditions

$$\{\epsilon,\ |\eta|,\ \zeta^2\} \ll 1$$

- Observables

$$n_s \simeq 1 - 6\,\epsilon + 2\,\eta,$$

$$r \simeq 16\,\epsilon,$$

$$\frac{dn_s}{d\ln k} \simeq 16\,\epsilon\,\eta - 24\,\epsilon^2 - 2\,\zeta^2.$$

$$n_s \simeq 0.9603 \pm 0.0073$$

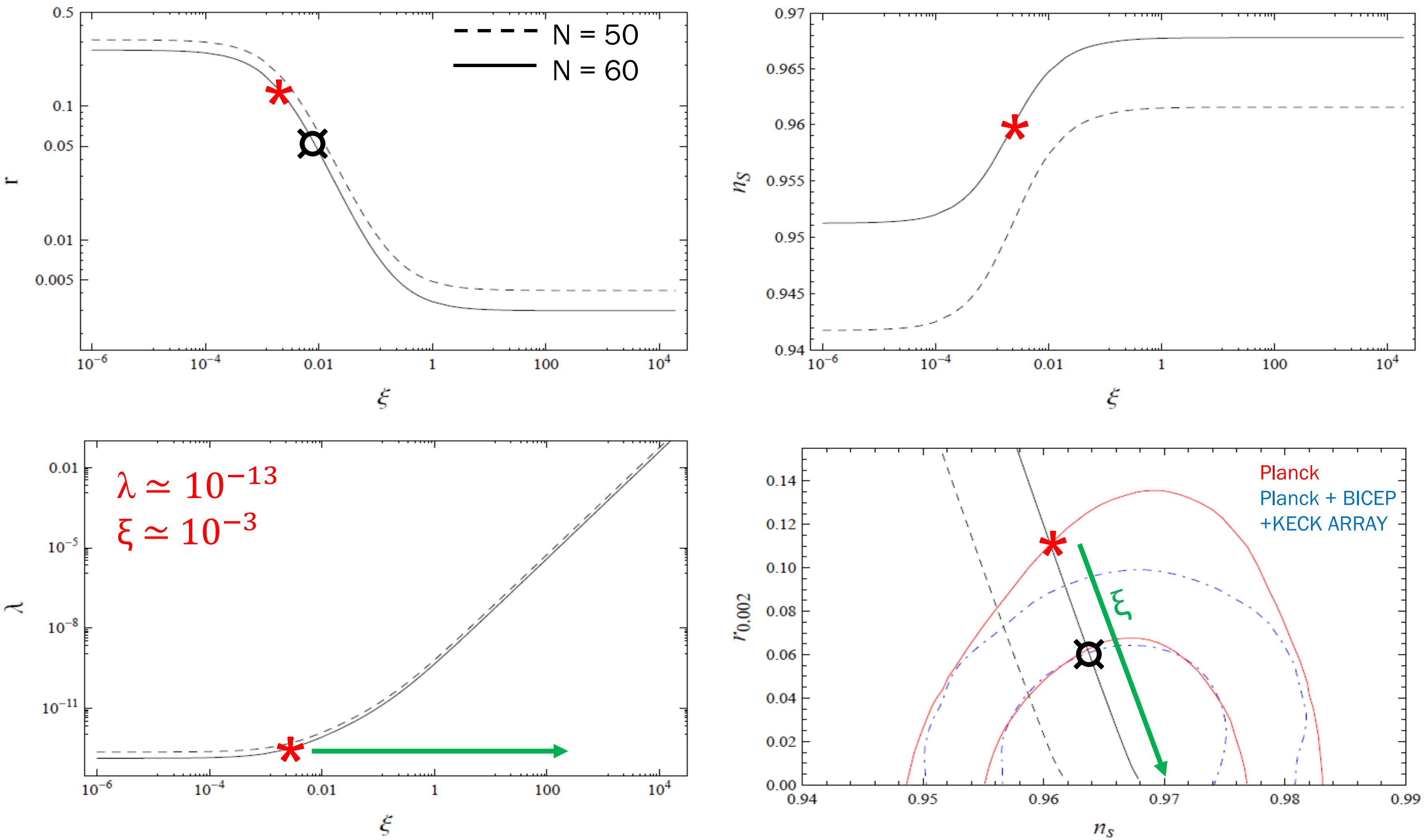
$$r \leq 0.11$$

$$\frac{dn_s}{d\ lnk} \simeq -0.0084 \pm 0.0082$$

Planck 2015 Measurements

Inflationary Predictions for Non-Minimal $\lambda\,\phi^4$ inflation,

Tree-level vs. Planck 2015



Non-Minimal B-L Inflation Scenario

- Minimal B-L(Baryon-Lepton) Extension of Standard Model
  - 3 generation of right handed Neutrinos (N_i) to make theory free of gauge anomaly.
  - B-L Higgs Field (φ) to break the B-L gauge symmetry.
  - B-L symmetry breaking generates Z' boson mass and Majorana mass for N_i.

$$\mathcal{L} \supset -\frac{1}{2}\sum_{i=1}^{3} Y \, \varphi \, \overline{N^c} N + \text{h.c.}$$

	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	$U(1)_{B-L}$
q_L^i	3	2	$+1/6$	$+1/3$
u_R^i	3	1	$+2/3$	$+1/3$
d_R^i	3	1	$-1/3$	$+1/3$
ℓ_L^i	1	2	$-1/2$	-1
NR^i	1	1	0	-1
e_R^i	1	1	-1	-1
H	1	2	$-1/2$	0
φ	1	1	0	$+2$

Model: NR with degenerate mass spectrum

  - See-Saw Mechanism
  - Mass Spectrum : $m_{NR} = \frac{1}{\sqrt{2}} Y_N v_{BL}, \;\; m_{Z'} = 2 g v_{BL}, \;\; m_\phi^2 = 2\lambda v_{BL}^2$

Non-Minimal B-L Inflation Scenario

- Relevant Tree Level Jordan Lagrangian and Masses

$$\mathcal{S}_J^{tree} = \int d^4x\sqrt{-g}\left[-\frac{1}{2}f(|\varphi|)\mathcal{R} + (D_\mu\varphi)^\dagger g^{\mu\nu}(D_\nu\varphi) - V(|\varphi|)\right]$$

$$D_\mu = \partial_\mu - i2gZ'_\mu$$

- B-L Higgs **(ϕ)** ➡ Inflaton

$$\varphi = (v_{BL} + \phi)/\sqrt{2}$$

- RGE Improved Non-minimal Potential

$$V_E(\phi) = \frac{1}{4}\lambda(\Phi)\ \Phi^4$$

$$\Phi \equiv \phi/\sqrt{1+\xi\phi^2}$$

- B-L RGE Running

$$16\pi^2\mu\frac{dg}{d\mu} = 12g^3,$$

$$16\pi^2\mu\frac{dY}{d\mu} = -6g^2Y + \frac{5}{2}Y^3,$$

$$16\pi^2\mu\frac{d\lambda}{d\mu} = 20\lambda^2 - (48g^2 - 6Y^2)\lambda + 96g^4 - 3Y^4.$$

Frame Independence

$\mu \Leftrightarrow \Phi$

RGE Improvement And Potential Instability

- RGE improved quartic coupling (λ)

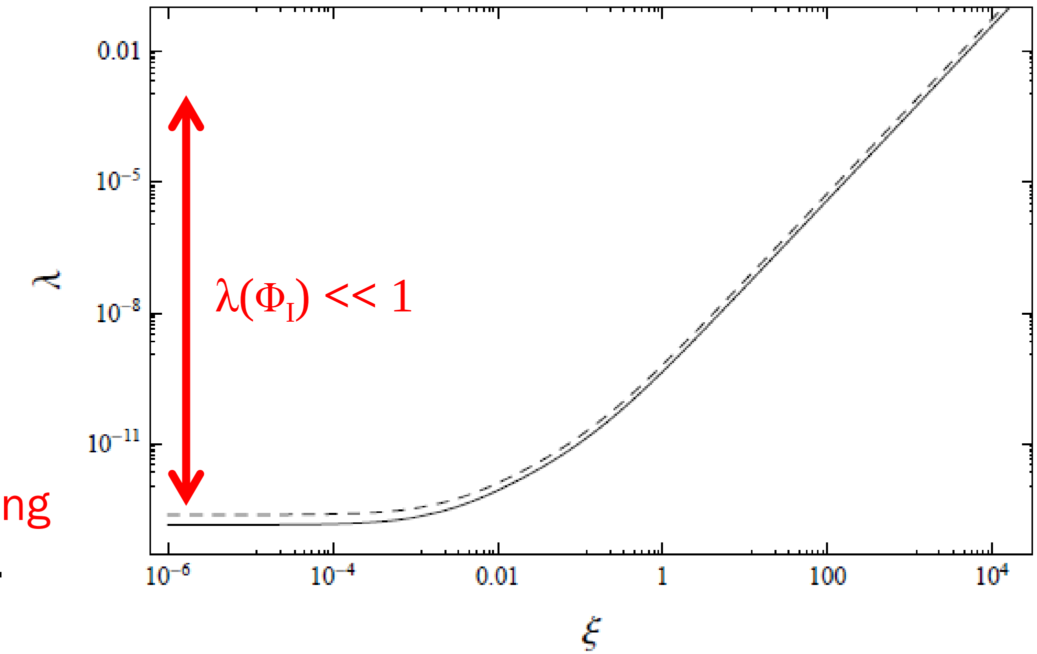
$$\beta_\lambda(\Phi_I) = \frac{1}{16\pi^2}\left[20\lambda^2 - 48g^2\lambda + 96g^4 + 6\lambda Y^2 - 3Y^4\right]$$

- Instability of RGE improved potential

  - Tree-level ⟶ $\lambda(\Phi_I)$
  - Sufficiently large g and Y

  $\lambda(\Phi) \propto \{g^4(\Phi_I), Y^4(\Phi_I)\}$

  - Potential Instability
    Possibility of Negative running of λ after RGE improvement.



RGE Improvement And Potential Instability

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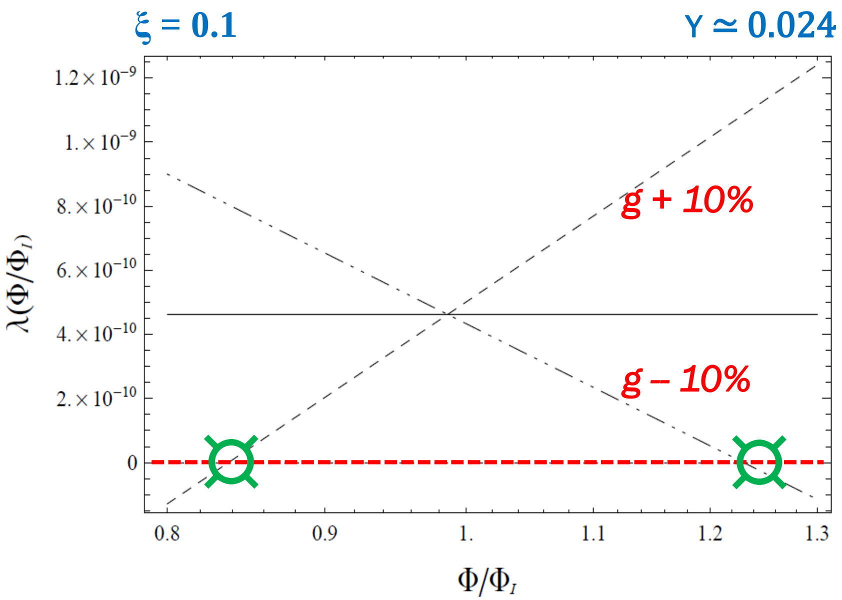
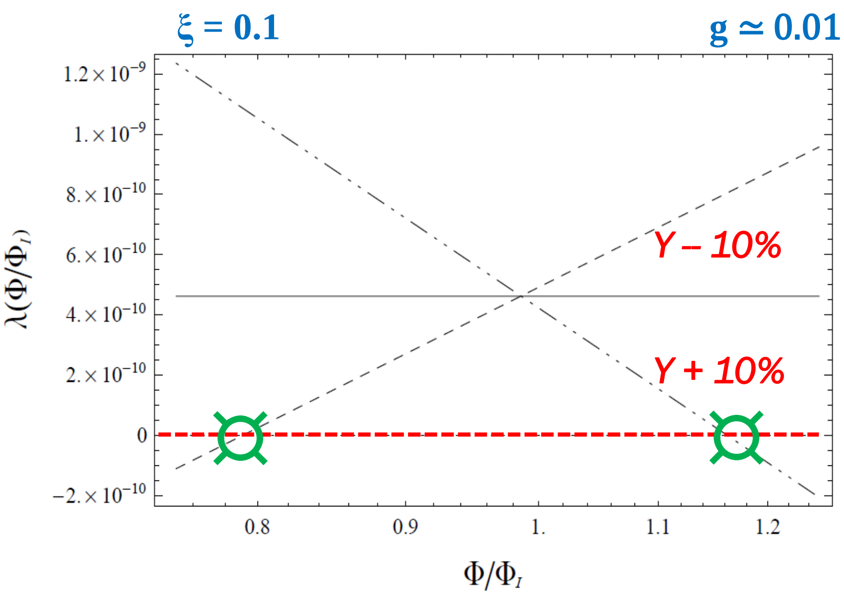




RGE Improvement And Potential Instability

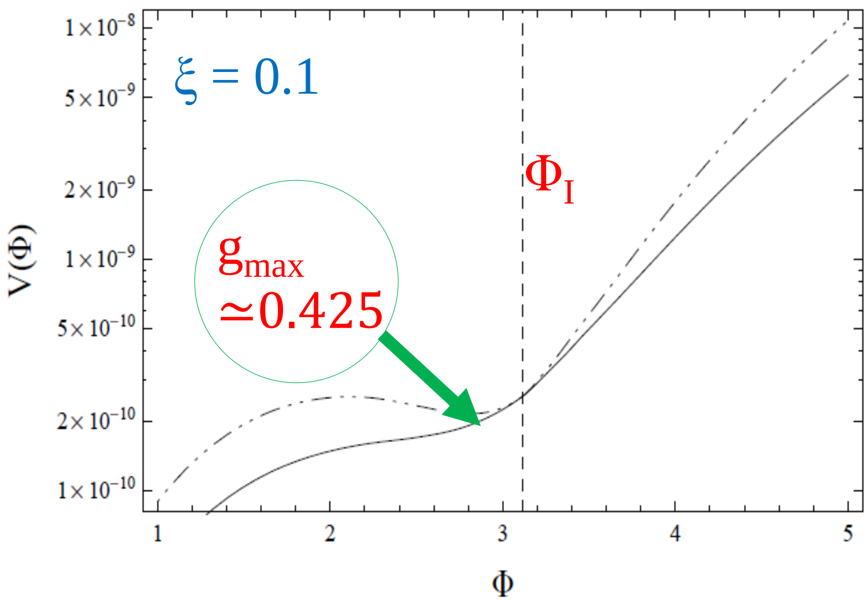
Generally ($g^2 , Y^2 \gtrsim \lambda^2$)

$\lambda(\Phi_I) << 1$

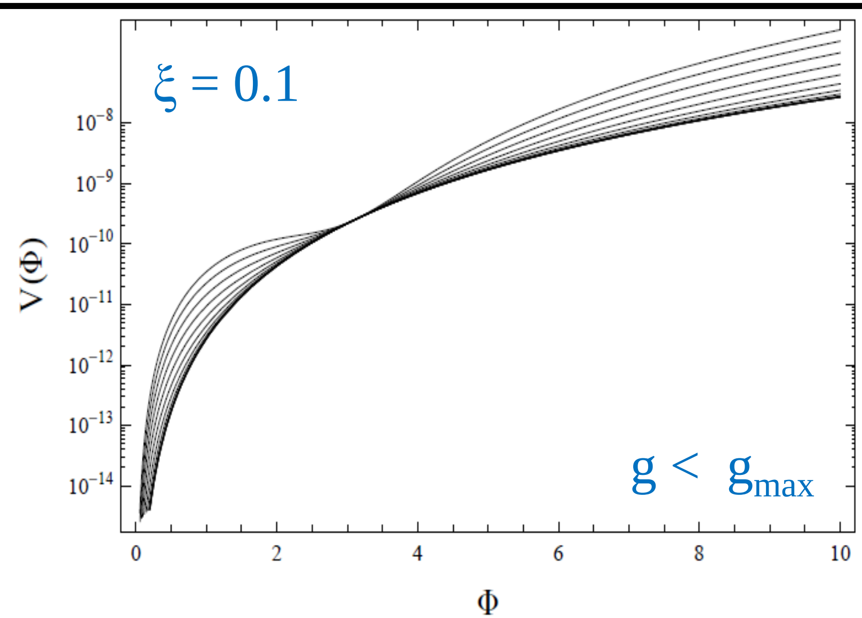
$$\beta_\lambda(\Phi_I) = \frac{1}{16\pi^2}\left[20\lambda^2 - 48g^2\lambda + 96g^4 + 6\lambda Y^2 - 3Y^4\right]$$

$\beta_\lambda (g=0, Y=0) \sim 0$

Stability Conditions → $\beta_\lambda(\Phi_I) = 0$, $\left.\frac{d\beta_\lambda(\Phi)}{d\Phi}\right|_{\Phi=\Phi_I} > 0$ → Unique Model parameters g and ξ



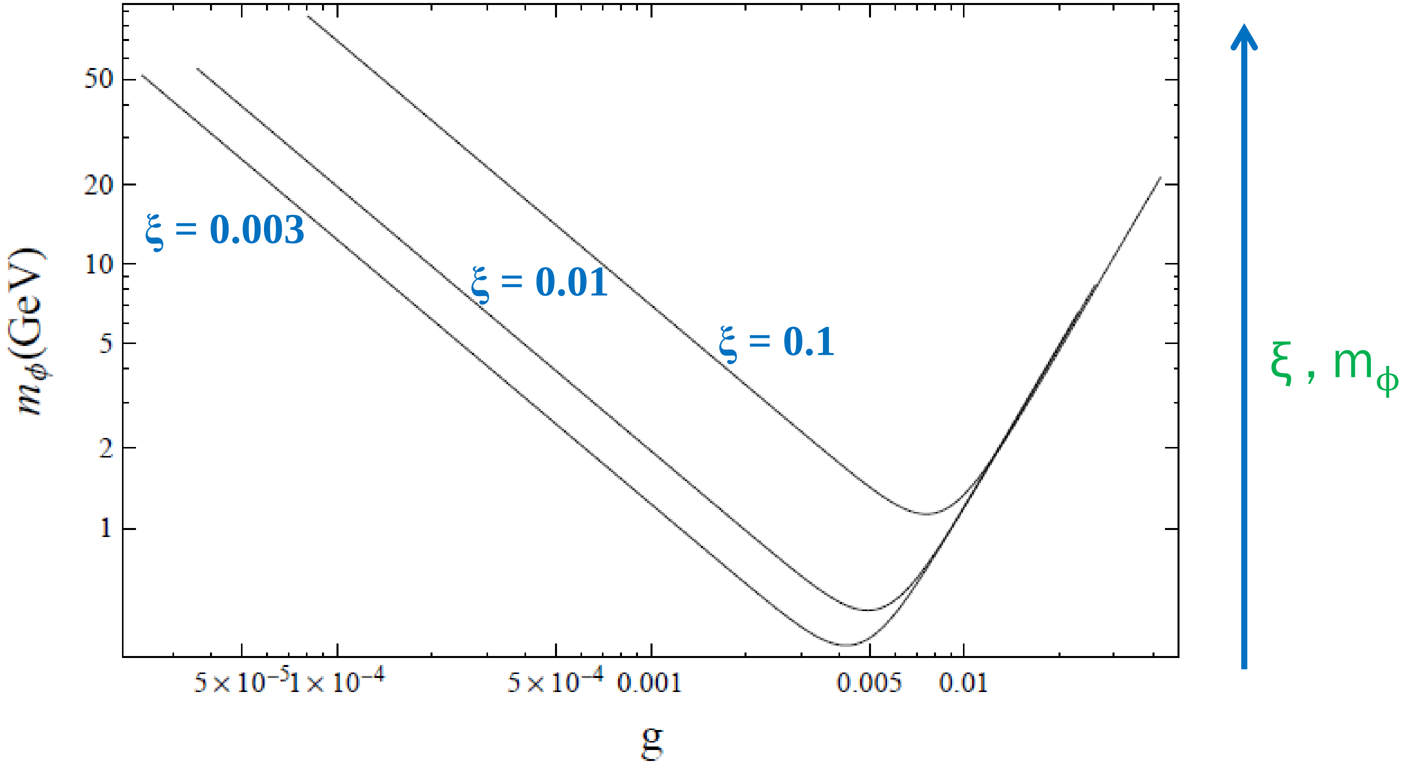
Second Minimum !!



Upper Bound (g_{max})

Low Energy Predictions

- $m_{z'}$ = 3 TeV (fixed) ⟶ VEV (ϕ)



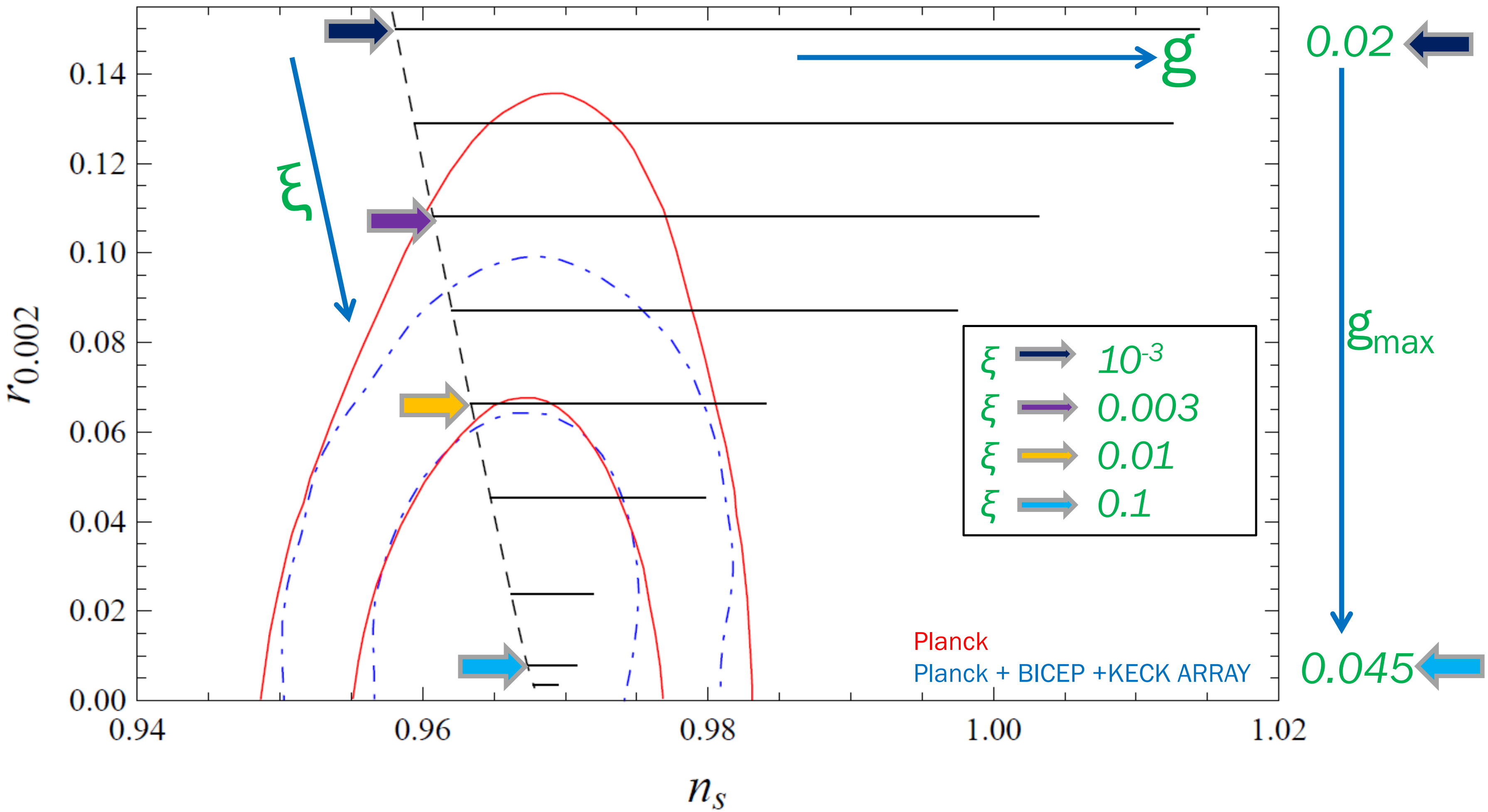
- ξ < 1

$m_{z'} / m_{NR} \simeq 0.84$

ξ >1
Inflationary Predictions same as tree level
Outside our interest

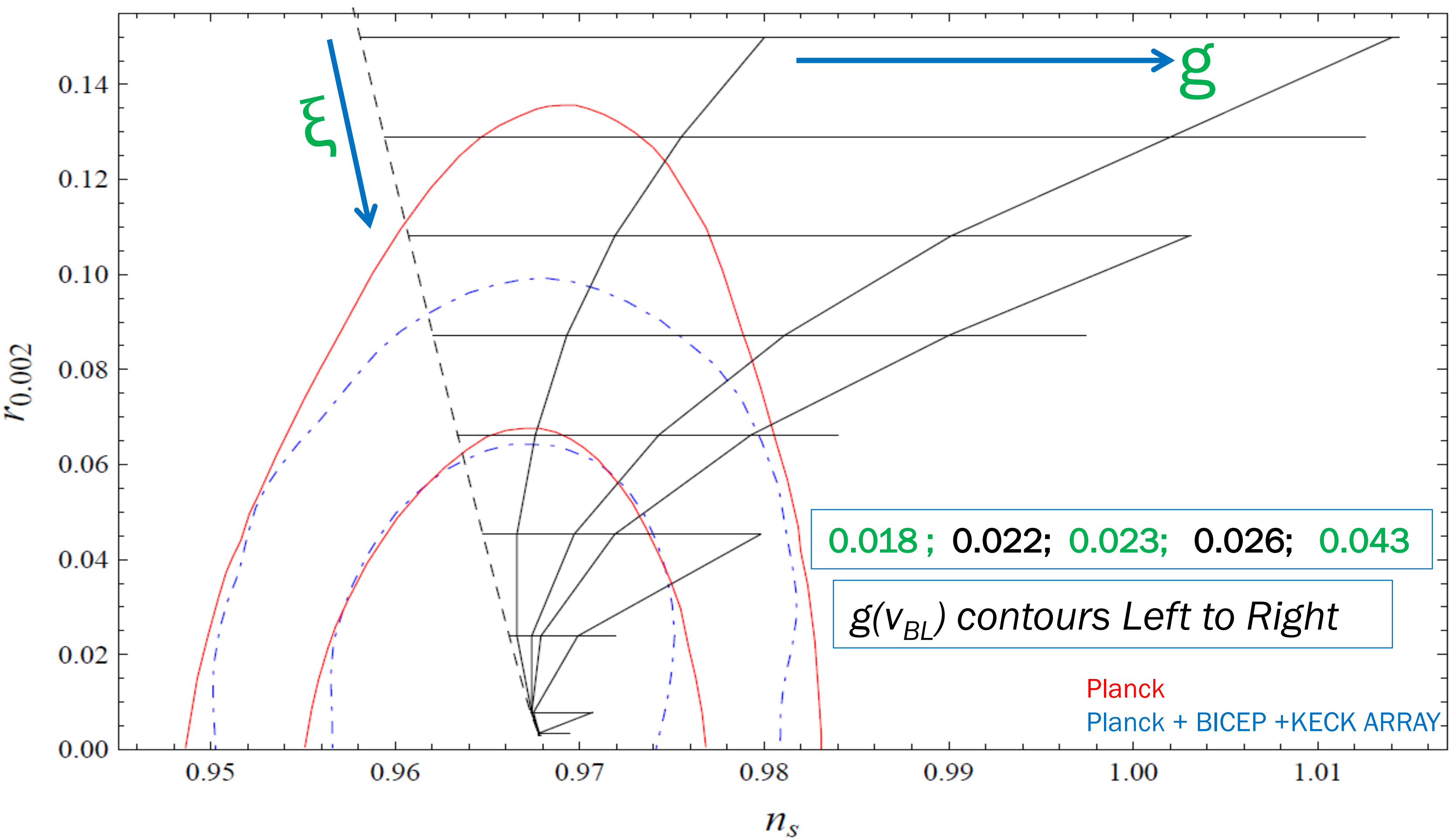
Inflationary Predictions & constraint on g

Stability Conditions → $n_s(g) \neq n_s^{\text{tree}}, \; r(g) = r^{\text{tree}}, \; \alpha(g) \neq \alpha^{\text{tree}}$



Inflationary Predictions & low energy correlations

ξ < 0.01 : Inflationary constraint on g_{max} more severe than theoretical constraint
ξ > 1 : Inflationary predictions same as tree-level



Conclusions

- We considered $\lambda\ \phi^4$ inflation with non-minimal gravitational coupling in context of B-L extension of SM with identification of B-L Higgs as Inflaton.

- The instability of the RGE improved potential due to small quartic coupling was solved by imposing the stability condition on the beta function of B-L Higgs quartic coupling.

- The RGE running gave correlations between the physics at very high energies(Inflation) and low energy physics observables.

- We analyzed the slow-roll inflation scenario, and showed that the spectral index (n_s) deviating from tree-level value.

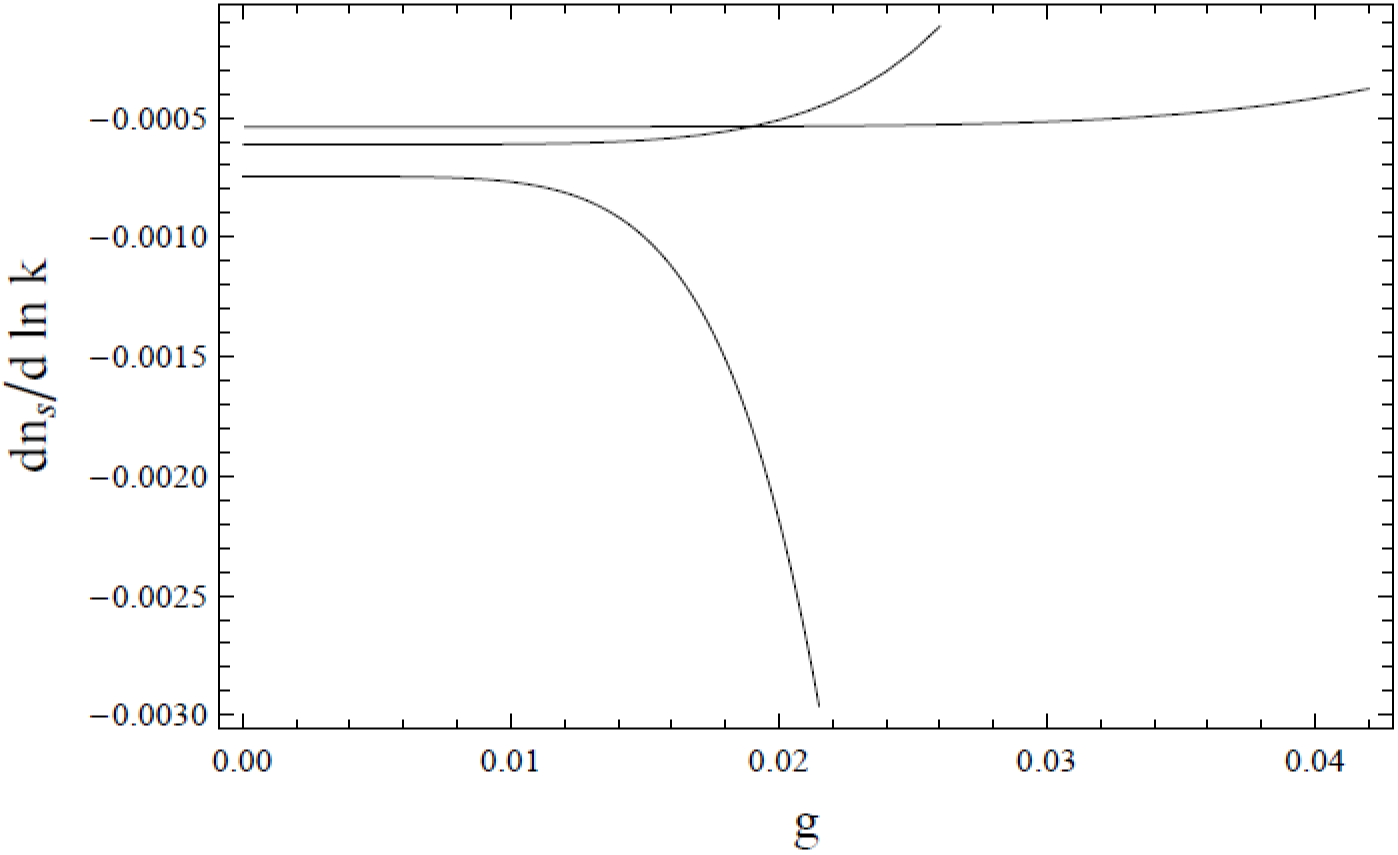
- Interestingly, Planck 2015 constraint for gauge coupling and hence the low energy observable is more severe than the theoretical constraint we imposed to avoid second minimum.

Thank You

arxiv: 1509.04439

Inflationary Prediction Running of Spectral Index

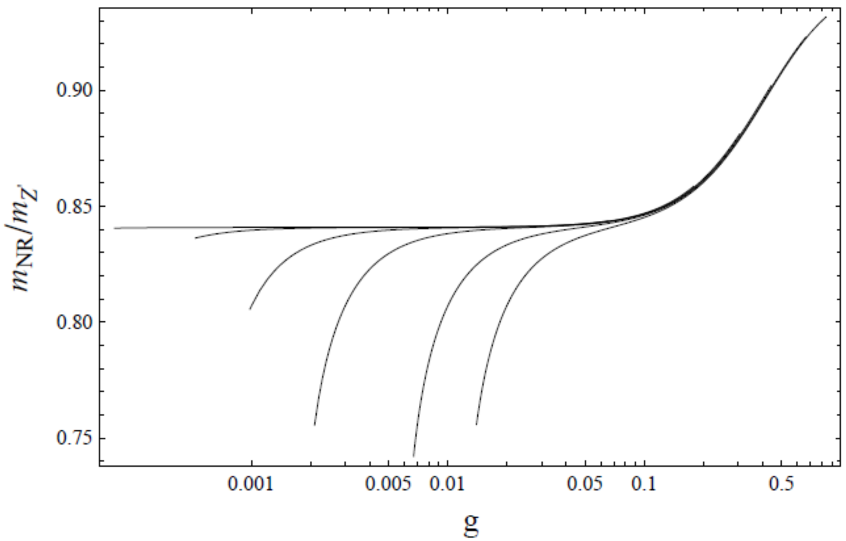
$$\frac{dn_s}{d\ lnk} \simeq -0.0084 \pm 0.0082$$

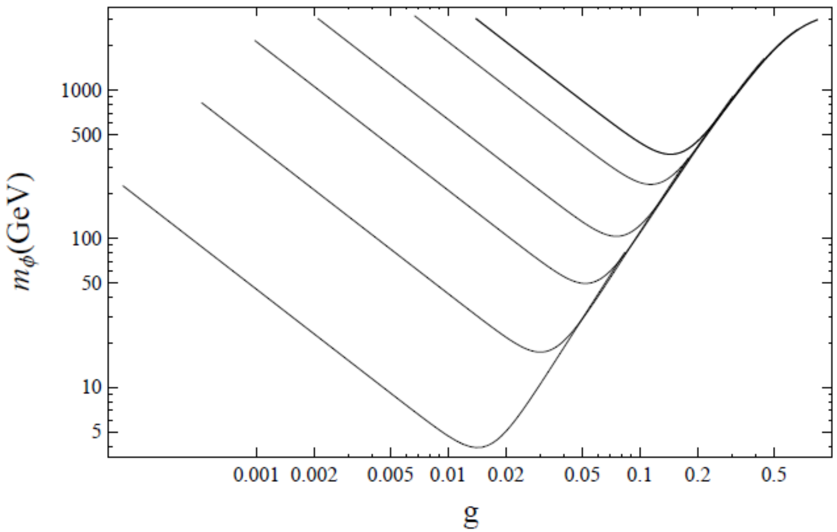


Low Energy Predictions ξ >1

- $m_{z'}$ = 3 TeV (fixed) ➡ VEV (ϕ)

Left to Right ξ = {1, 10, 50, 150, 500, 1000}





- Minimum m_ϕ increases with increased ξ

Reheating

- SM Higgs Mixing

$$\mathcal{L} \supset -\lambda' \left(H^\dagger H\right) \left(\varphi^\dagger \varphi\right)$$

- Diagonalization

$$\begin{bmatrix} h \\ \phi \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix}$$

$$2 v_{BL} v_{SM} \lambda' = (m_h^2 - m_\phi^2) \tan 2\theta$$

- B-L RGE Running and consistency condition

$$16\pi^2 \beta_\lambda \to 16\pi^2 \beta_\lambda + 2\lambda'^2$$

$$\frac{\lambda'^2}{48 g^4} \ll 1 \Rightarrow g \gg 6 \times 10^{-3}\, \theta$$

$\theta << 1$

$h \simeq \phi_1$ (SM Higgs)

$\phi \simeq \phi_2$ (B-L Higgs)

- SM Higgs Decay

$$\mathcal{L} \supset Y\, h\, \bar{\Psi}_L \Psi_R$$

$$h \simeq \phi_1 + \theta\, \phi_2$$

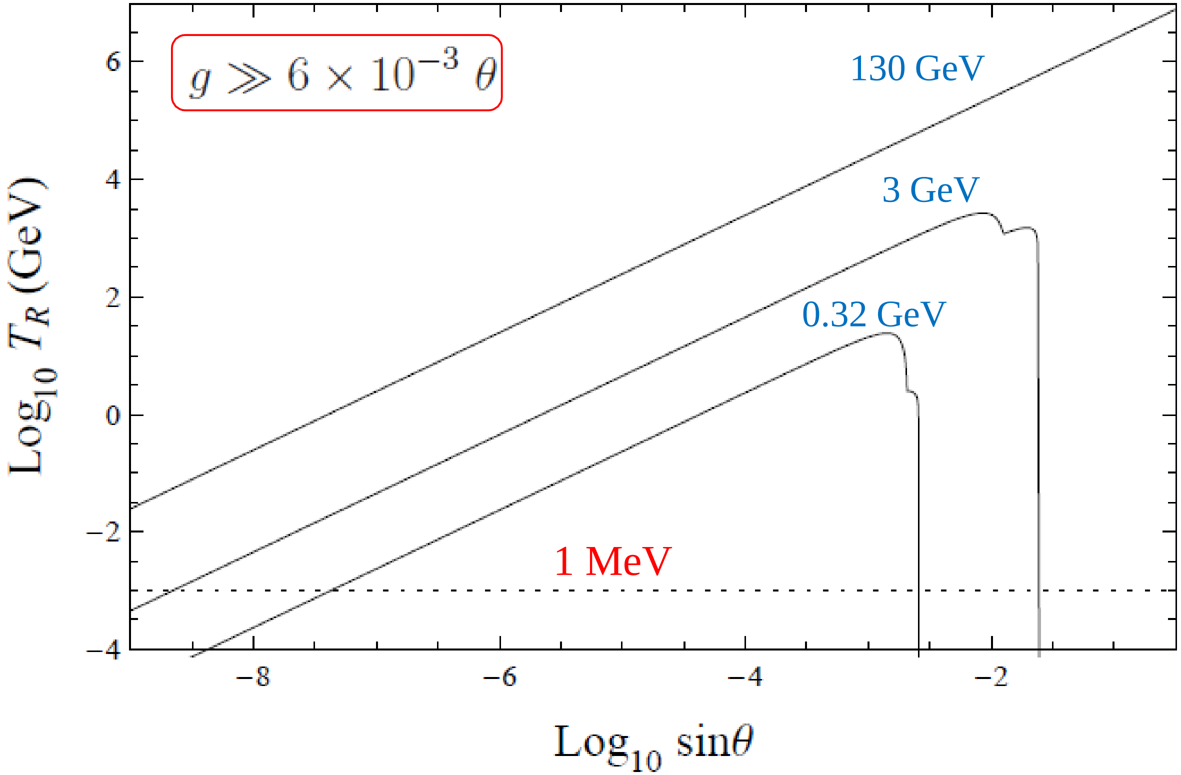
$$\Gamma_{\phi_2} = \sin^2\theta \times \Gamma_h(m_{\phi_2})$$

SM Higgs Decay Width with mass of SM Higgs Replaced by $m_{\phi 2}$

Reheating

- Reheating Temperature

$$T_R \simeq 0.2\ (100/g_*)^{1/4}\sqrt{\Gamma M_P}$$



BBN Constraint

$$T_R \gtrsim 1\ \text{MeV}$$