Gravity duals of boundary cones

1605.08588

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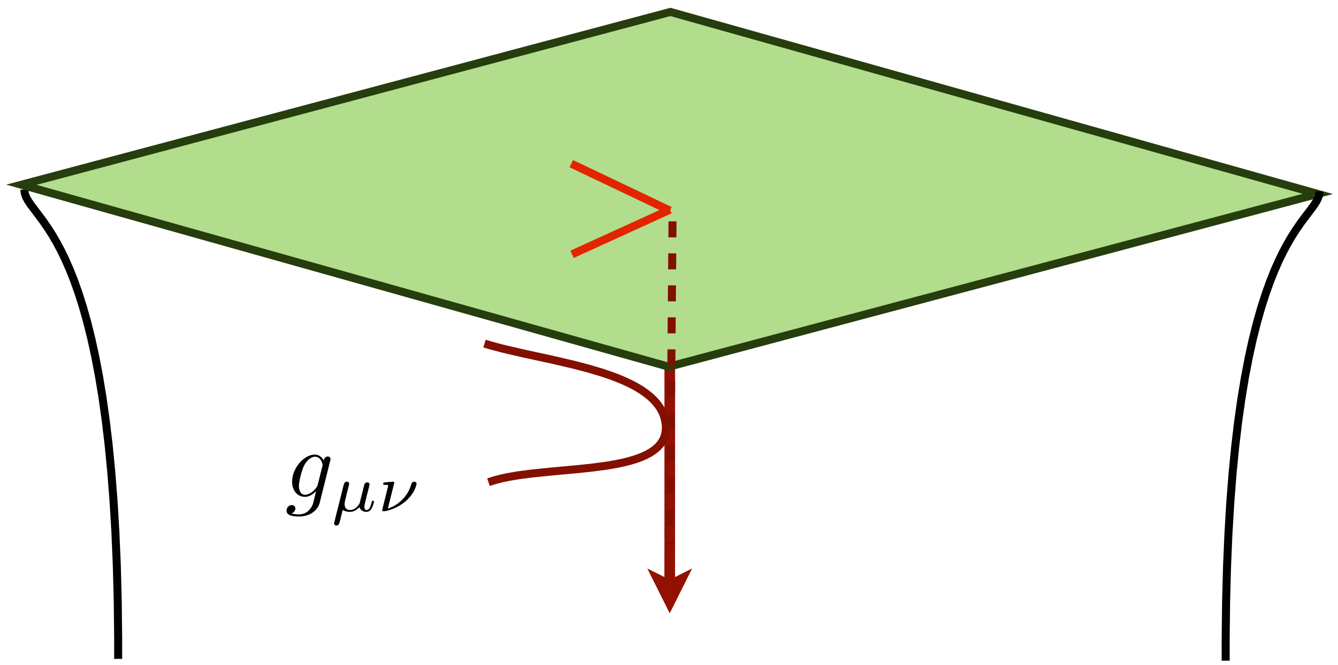
DAMTP

Purpose

- Discuss gravity duals to conical singularities

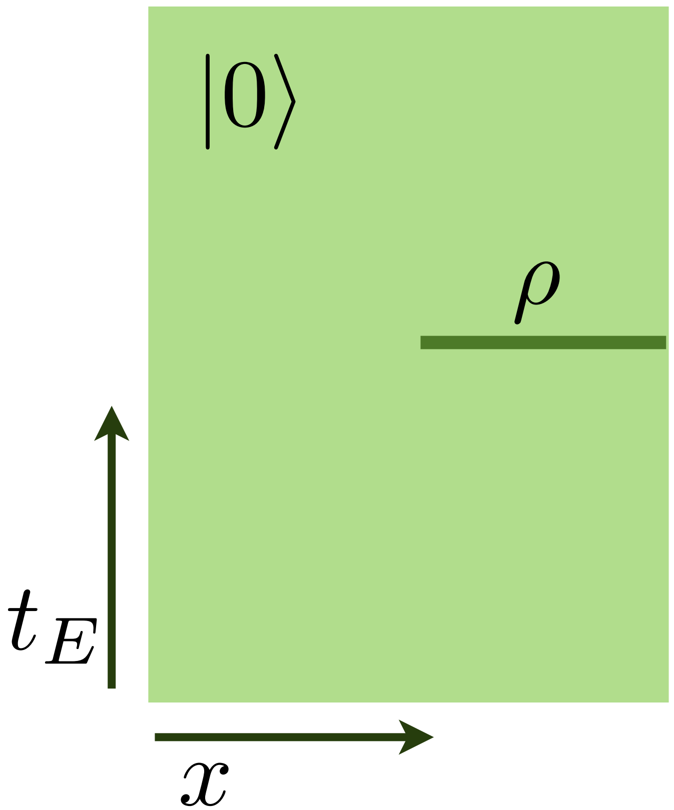


- Bulk setup: General Relativity, Euclidean, 5D

Motivation

- ‘Generalized Entropy’: Derivation of Ryu-Takayanagi by Lewkowycz and Maldacena
- Renyi entropy
- Open questions about generalization of RT to higher derivatives: ‘splitting problem’
- Singularity resolution

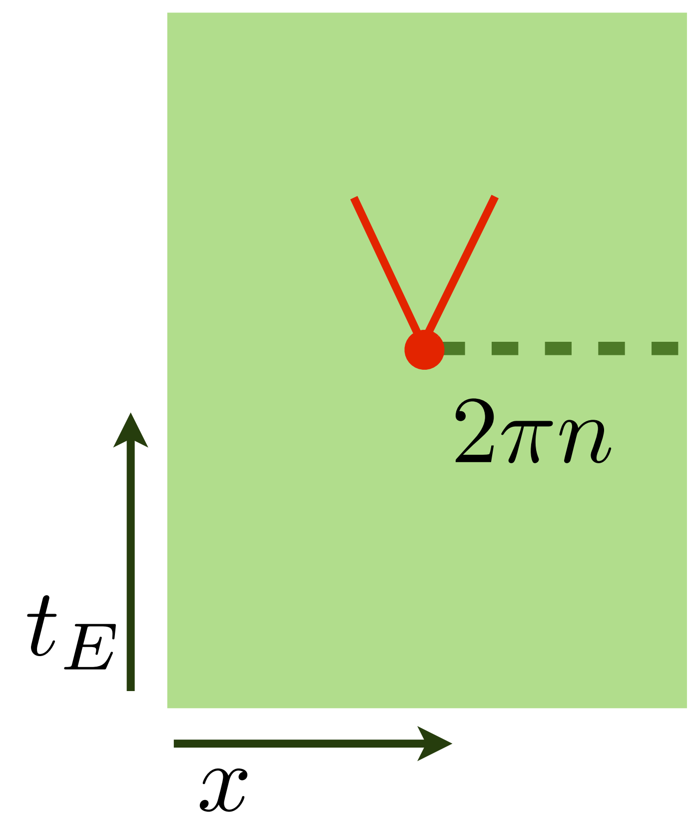
Replica trick:



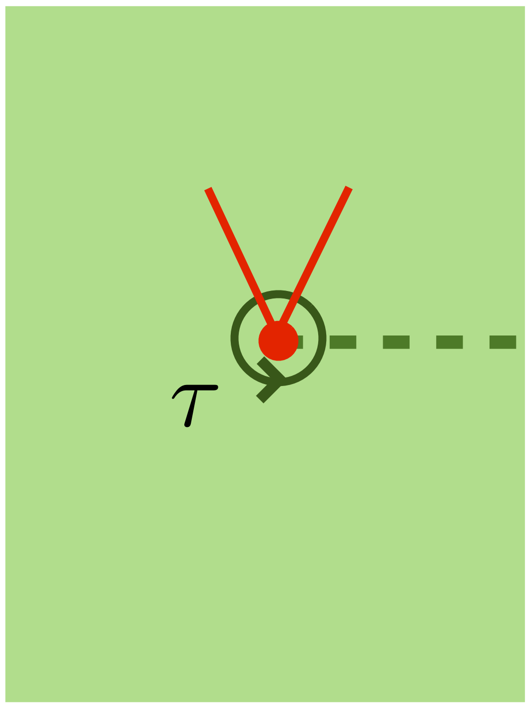
$$S = -\mathrm{tr}(\rho \log \rho)$$

$$S_n = -\frac{\log(\mathrm{tr}\rho^n)}{n-1}$$

$$S = \lim_{n \to 1} S_n$$



CHM derivation of RT



$$ds^2_{\mathrm{CFT}} = r^2 d\tau^2 + dr^2 + d\zeta\, d\bar{\zeta}$$

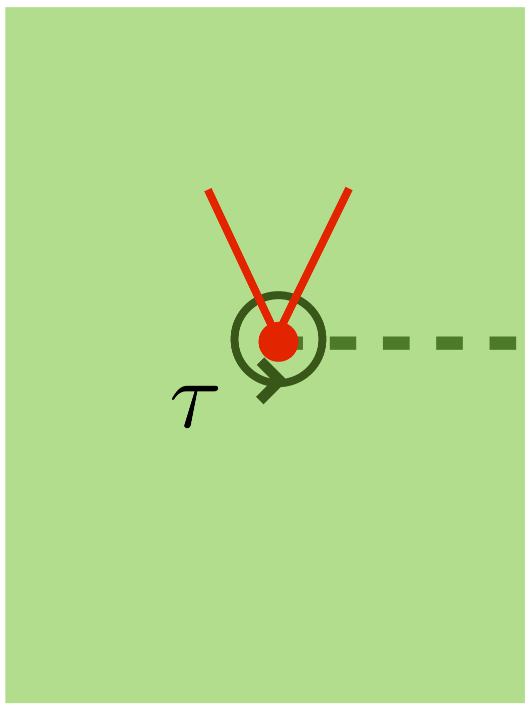
$$\tau \sim \tau + 2\pi n$$

$$ds^2_{\mathrm{CFT}} = d\tau^2 + \frac{dr^2 + d\zeta\, d\bar{\zeta}}{r^2}$$

Periodic, finite ‘time’ circle

Dual to Hyperbolic BH: Usual thermo, area

Cones in CCoords

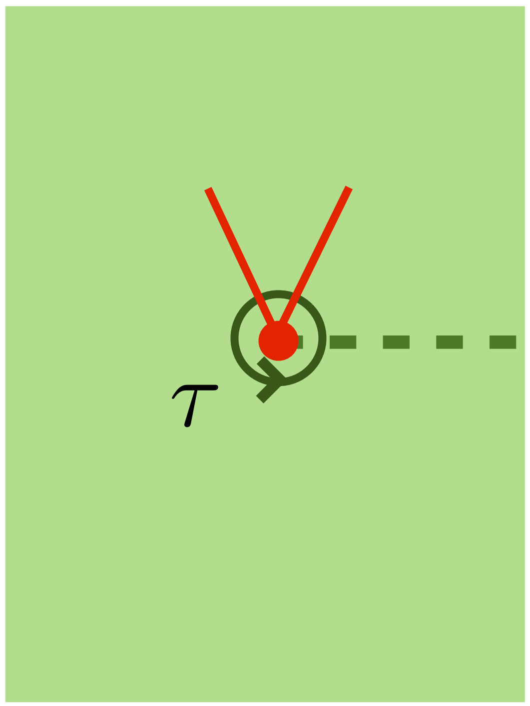


$$dr^2 + r^2 d\tau^2 = dz\, d\bar{z}$$

$$re^{i\tau} = z \to z^n$$

$$\boxed{n^2 (z\bar{z})^{n-1} dz\, d\bar{z}}$$

Cones in CCoords



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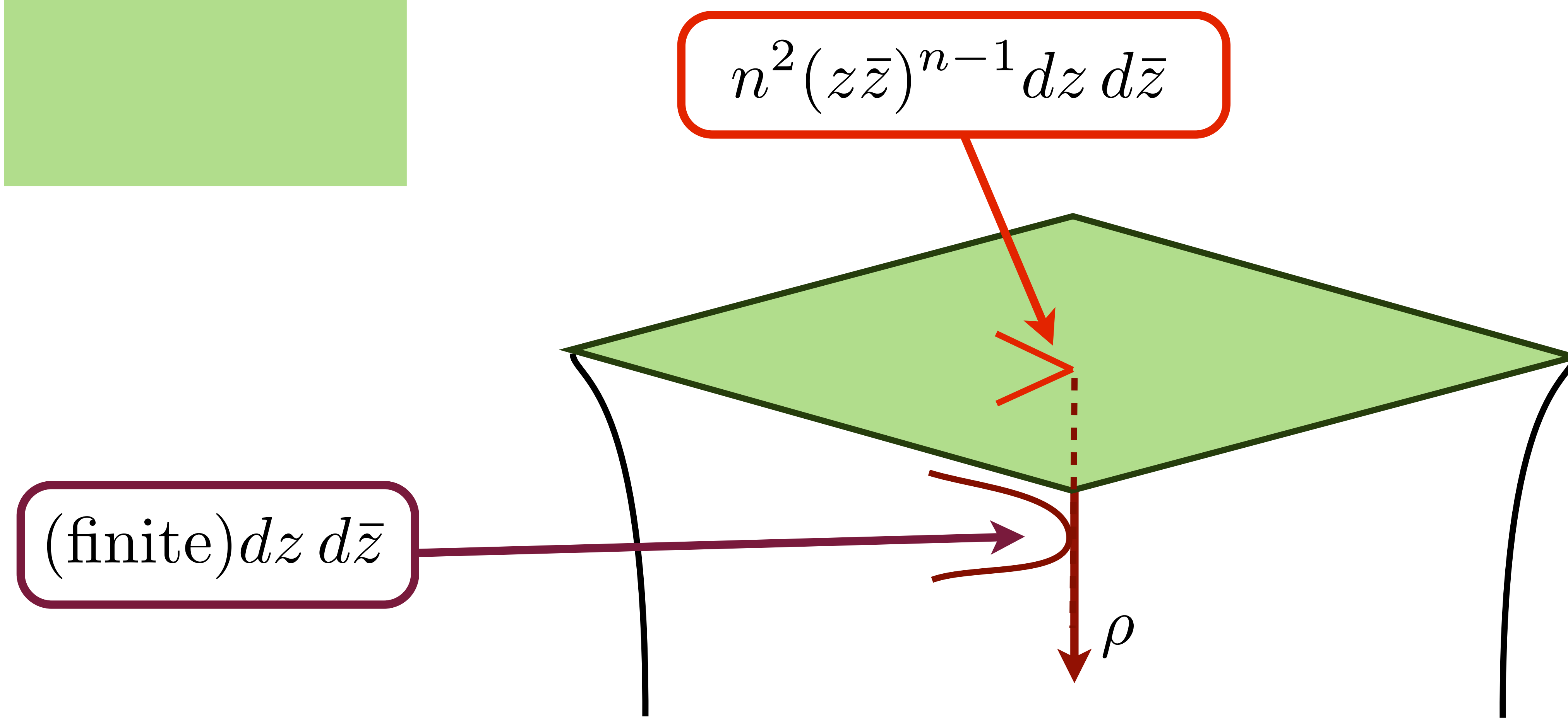
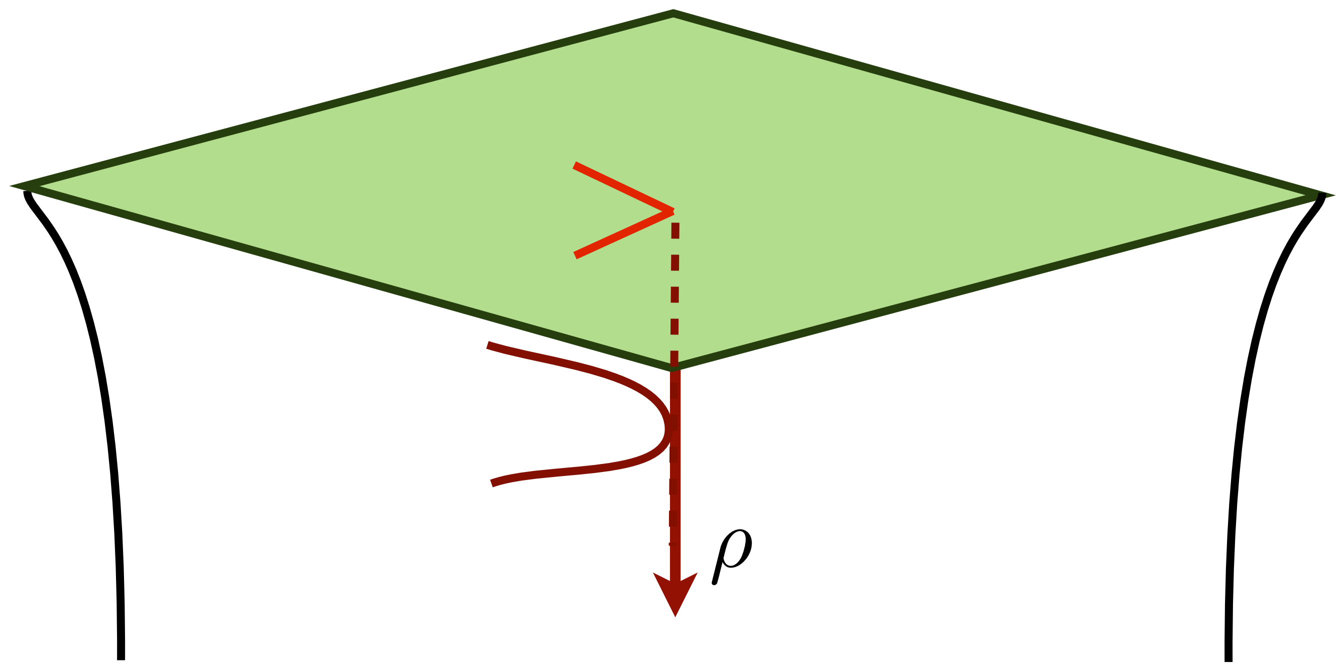


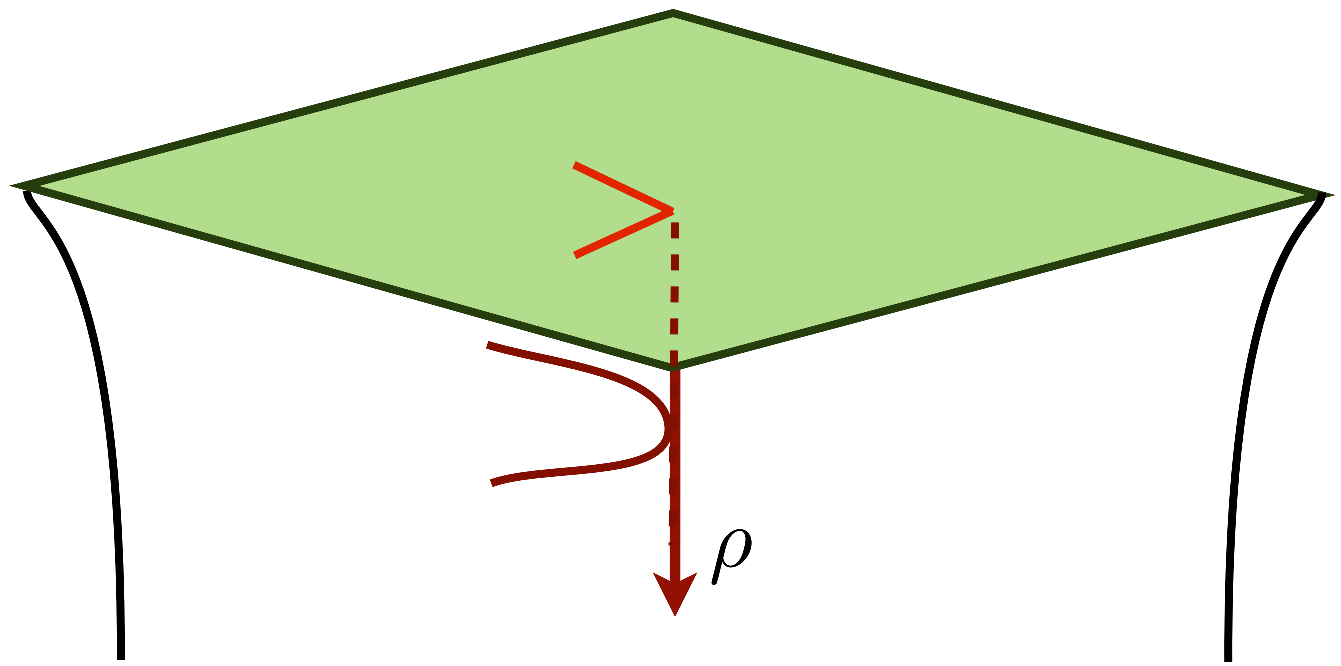
Gravity duals



$$\boxed{x = (z\bar{z})^n/\rho^2}$$

$$ds^2_{\mathrm{AdS}} = \frac{d\rho^2}{\rho^2} + \frac{d\zeta\, d\bar{\zeta}}{\rho^2} + \frac{(1+x)^{\frac{n-1}{n}}\, n^2 dz\, d\bar{z}}{\rho^{2/n}}$$

Gravity duals



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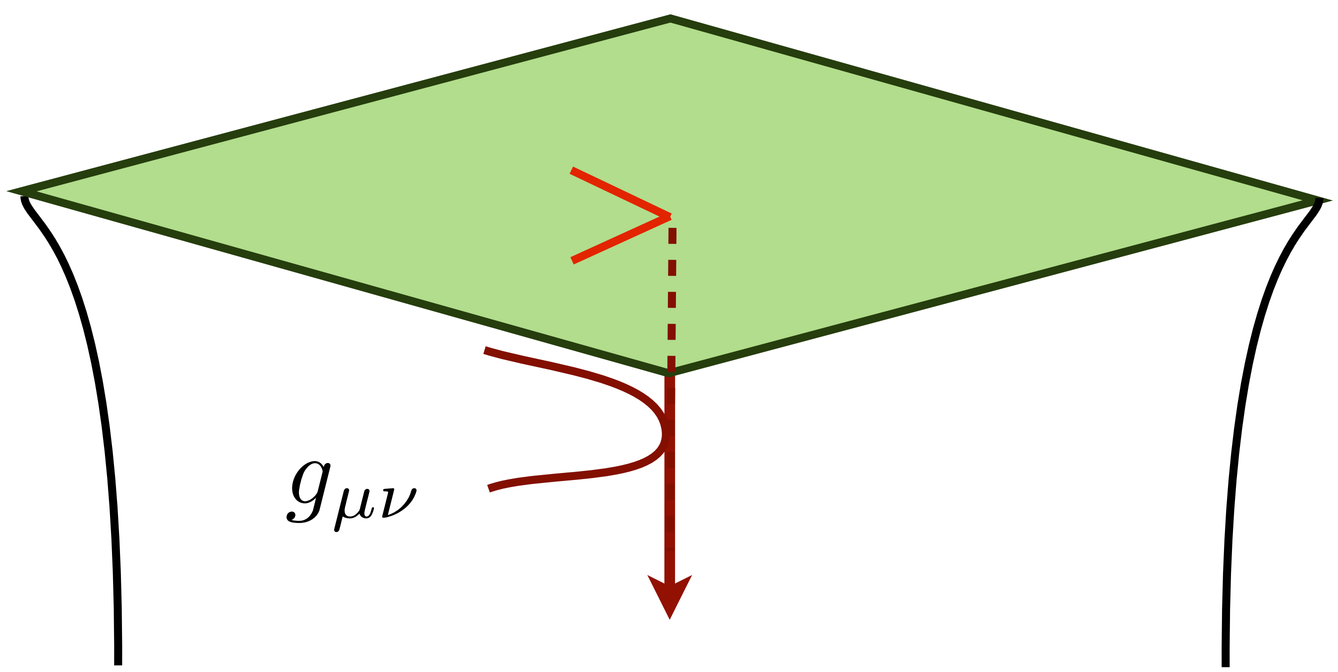
$$+ \frac{(n-1)}{1+x}\frac{-2d\rho^2 + d\zeta\, d\bar{\zeta} - 2dz\, d\bar{z}}{3\rho^2} + O\left((n-1)^2\right)$$

Summary so far

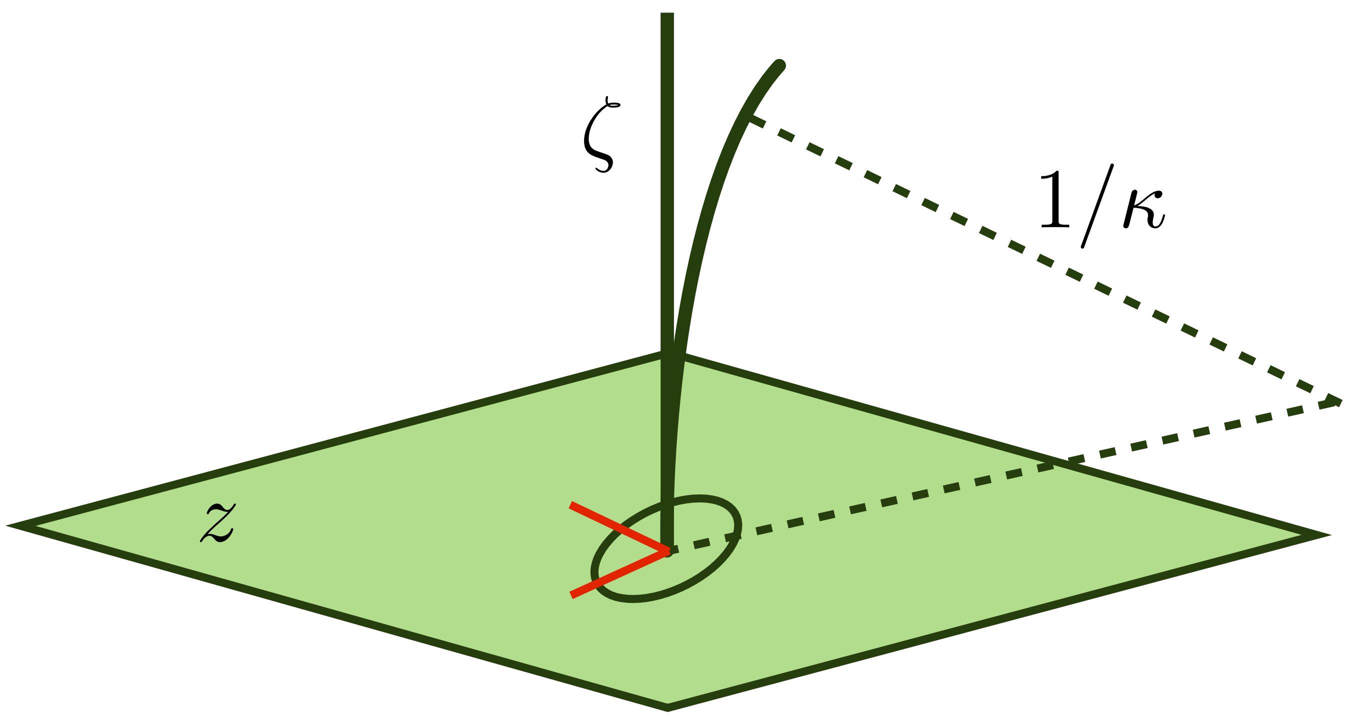
- A bulk diffeo makes the hyperbolic BH into



- This is naturally related to Renyis

Squashing the cone

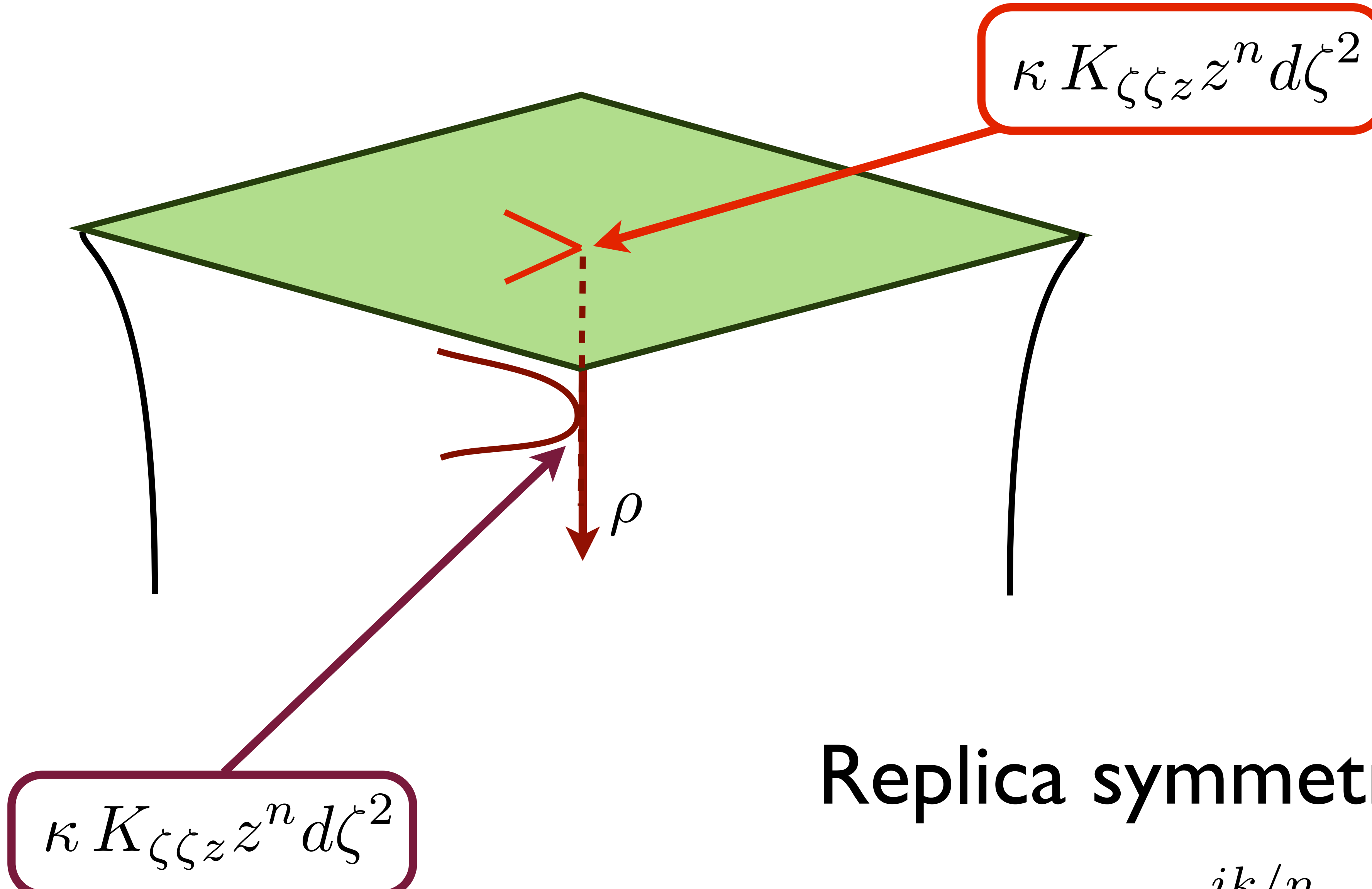
$$ds^2_{\text{CFT}} = d\zeta\, d\bar{\zeta} + dz\, d\bar{z} + 2\kappa\, K_{\zeta\zeta z} z\, d\zeta^2 + \text{c.c.} + O\left(\kappa^2\right)$$



$z \to z^n$

$$ds^2_{\text{CFT}} = d\zeta\, d\bar{\zeta} + n^2 (z\bar{z})^{n-1} dz\, d\bar{z} + 2\kappa\, K_{\zeta\zeta z} z^n\, d\zeta^2 + \text{c.c.} + O\left(\kappa^2\right)$$

Gravity duals of squashed cones



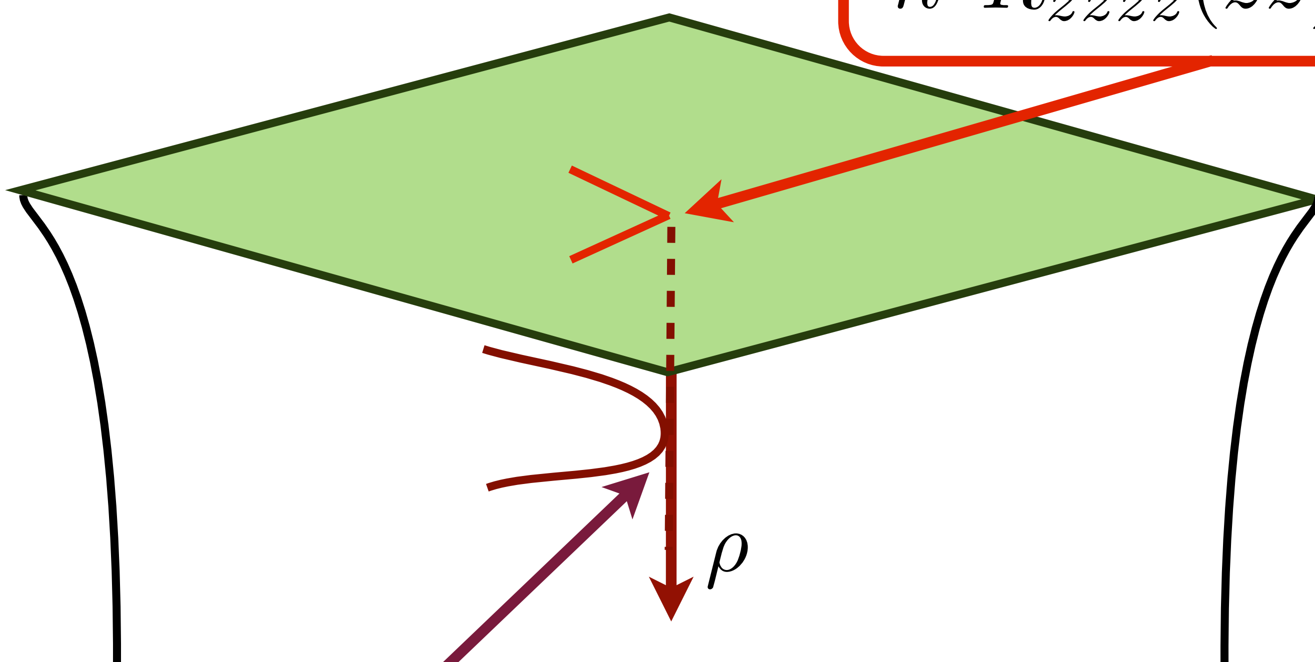
Replica symmetry

$$z \to z\, e^{ik/n}$$

Second order

Captures Riemann curvature

$$\kappa^2 R_{z\bar{z}z\bar{z}}(z\bar{z})^n n^2 (z\bar{z})^{n-1} dz\, d\bar{z}$$



$$\kappa^2 R_{z\bar{z}z\bar{z}} z\bar{z}\, dz\, d\bar{z}$$

Replica symmetry

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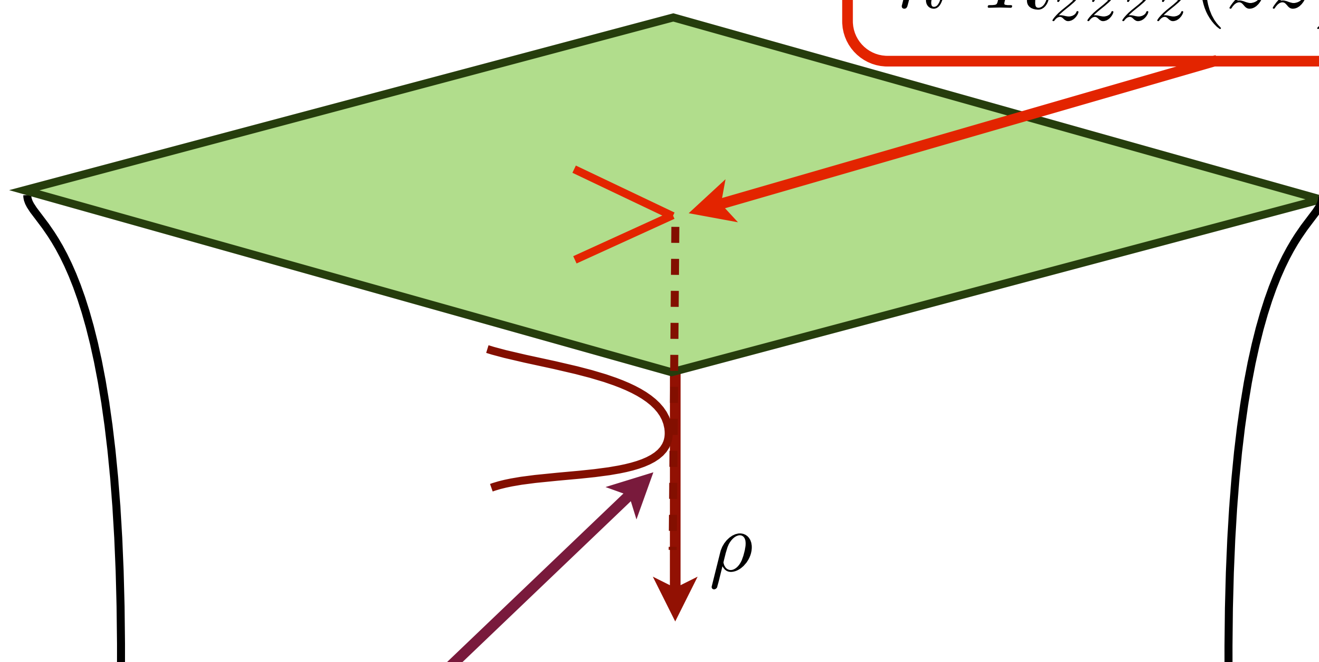

$$\kappa^2 \left(R_{z\bar{z}z\bar{z}} + 2K_{\zeta\zeta z}K_{\bar{\zeta}\bar{\zeta}\bar{z}}\right) z\bar{z}\, dz\, d\bar{z}$$
$$-2\kappa^2 K_{\zeta\zeta z}K_{\bar{\zeta}\bar{\zeta}\bar{z}}(z\bar{z})^n\, dz\, d\bar{z}$$

'Non-minimal' singularity resolution

Miao

Entropy for higher derivatives

- For $L(\mathrm{Riem})$ theories, generalized entropy:

$$S = \int \frac{\partial L}{\partial \mathrm{Riem}} + \int \sum_\alpha \left(\frac{\partial^2 L}{\partial \mathrm{Riem}^2} \right)_\alpha \frac{K^2}{1+q_\alpha}$$

Dong, JC

- In this expression, α labels terms in a certain decomposition of $\partial^2 L/\partial \mathrm{Riem}^2$, that depends on details of the cone resolution.

- Does not change Lovelock, $L(\mathrm{Riem}^2)$, $f(R)$

Conclusions

- How General Relativity in the bulk resolves boundary conical singularities
- Second order in κ. First order in $(n-1)$
- $\langle T_{\mu\nu} \rangle$. Traceless (up to contact terms)
- Non-minimal resolution impacts the entropy formula for Riem^k corrections.
- Logarithmic divergence of Renyi $f_b(n) \neq f_c(n)$

Lee, McGough, Safdi
Lewkowycz, Perlmutter

Dong

Thank you

Vacuum polarisation

For....

$$ds^2 = \left(\gamma_{ij} + \left[2\kappa\, K_{ijz} z + \kappa^2 Q_{ijzz} z^2 + \kappa^2 Q_{ijz\bar{z}} z\bar{z} + \text{c.c.}\right]\right) d\sigma^i d\sigma^j + 2\kappa\, A_{iz\bar{z}} (\bar{z}\, dz - z\, d\bar{z})\, d\sigma^i$$
$$- \frac{4}{3}\kappa^2 \left[R_{izz\bar{z}} z - \text{c.c.}\right] (\bar{z}\, dz - z\, d\bar{z}) d\sigma^i + \left(1 + 2\kappa^2 R_{z\bar{z}z\bar{z}} z\bar{z}\right) dz\, d\bar{z} + O(\kappa^3)$$

$$\gamma_{ij} d\sigma^i d\sigma^j = d\zeta\, d\bar{\zeta} - \frac{1}{3}\kappa^2 \mathcal{R}_{\zeta\bar{\zeta}\zeta\bar{\zeta}} \left(\bar{\zeta}\, d\zeta - \zeta\, d\bar{\zeta}\right)^2 + O(\kappa^3)\,,$$
$$2\kappa\, A_{iz\bar{z}} d\sigma^i = \kappa^2 F_{\zeta\bar{\zeta}z\bar{z}} (\bar{\zeta}\, d\zeta - \zeta\, d\bar{\zeta}) + O(\kappa^3)$$

We get....

$$
\begin{aligned}
\langle T\rangle = \frac{\ell^3(n-1)}{4\pi G(z\bar{z})^2}\Bigg\{ & \frac{-1}{6}\left(\gamma_{ij}d\sigma^i d\sigma^j - dz\,d\bar{z} + \frac{\bar{z}^2dz^2 + z^2d\bar{z}^2}{z\bar{z}}\right) \\
& - \kappa\left[\frac{K_{\zeta\zeta z}z\,d\zeta^2 + K_{\zeta\zeta\bar{z}}\bar{z}\,d\zeta^2}{6} + \text{c.c.}\right] \\
& + \kappa^2\frac{F_{\zeta\bar{\zeta}z\bar{z}}}{2}(z\,d\bar{z} - \bar{z}\,dz)(\zeta\,d\bar{\zeta} - \bar{\zeta}\,d\zeta) \\
& + \kappa^2\left[\frac{Q_{\zeta\bar{\zeta}zz}\,z^2}{9}\left(d\zeta\,d\bar{\zeta} - 4dz\,d\bar{z} + 2\frac{z^2d\bar{z}^2 + 3\bar{z}^2dz^2}{z\bar{z}}\right) + \text{c.c.}\right] \\
& + \kappa^2\left[\frac{K_{\zeta\zeta z}K_{\bar{\zeta}\bar{\zeta}z}z^2}{9}\left(-5d\zeta\,d\bar{\zeta} + 17dz\,d\bar{z} - \frac{7z^2d\bar{z}^2 + 30\bar{z}^2dz^2}{z\bar{z}}\right) + \text{c.c.}\right] \\
& - \kappa^2\left[\frac{Q_{\zeta\zeta z\bar{z}}z\bar{z}\,d\zeta^2}{3} + \text{c.c.}\right] \\
& - \kappa^2\left[\frac{K_{\zeta\zeta z,\bar{\zeta}}z\,d\zeta}{6}\left(\frac{3}{2}z\,d\bar{z} - 2\bar{z}\,dz\right) + \zeta\leftrightarrow\bar{\zeta} + \text{c.c.}\right] \\
& + \kappa^2\left[\frac{R_{\zeta zz\bar{z}}z\,d\zeta}{36}(8\bar{z}\,dz + 13z\,d\bar{z}) + \zeta\leftrightarrow\bar{\zeta} + \text{c.c.}\right] \\
& - \kappa^2\frac{5R_{\zeta\bar{\zeta}\zeta\bar{\zeta}}}{27}(z^2d\bar{z}^2 + \bar{z}^2dz^2) \\
& + \kappa^2\frac{R_{z\bar{z}z\bar{z}}z\bar{z}}{3}\left(2d\zeta\,d\bar{\zeta} - dz\,d\bar{z} + \frac{4}{9}\frac{\bar{z}^2dz^2 + z^2d\bar{z}^2}{z\bar{z}}\right) \\
& - \kappa^2\frac{2Q_{\zeta\bar{\zeta}z\bar{z}}z\bar{z}}{3}\left(d\zeta\,d\bar{\zeta} - \frac{5}{9}\frac{z^2d\bar{z}^2 + \bar{z}^2dz^2}{z\bar{z}}\right) \\
& - \kappa^2\left(K_{\zeta\zeta z}K_{\bar{\zeta}\bar{\zeta}\bar{z}} + \zeta\leftrightarrow\bar{\zeta}\right)z\bar{z}\left(-2d\zeta\,d\bar{\zeta} + \frac{2}{3}dz\,d\bar{z} + \frac{1}{6}\frac{\bar{z}^2dz^2 + z^2d\bar{z}^2}{z\bar{z}}\right) + O(\kappa^3)\Bigg\} \\
& + O\left((n-1)^2\right)
\end{aligned}
$$

Renyi

$$S_n = \cdots + \left(\frac{f_a(n)}{2\pi} \int \mathcal{R} \sqrt{\gamma}\, d^2\sigma + \frac{f_b(n)}{2\pi} \int K_{\{ij\}a} K^{\{ij\}a} \sqrt{\gamma}\, d^2\sigma \right.$$
$$\left. - \frac{f_c(n)}{2\pi} \int W_{ij}{}^{ij} \sqrt{\gamma}\, d^2\sigma \right) \log \epsilon + \ldots$$

- We find (GR)

$$f_a = a \left(1 - \frac{1}{2}(n-1) \right) \qquad f_b = c \left(1 - \frac{11}{12}(n-1) \right)$$

$$f_c = c \left(1 - \frac{17}{18}(n-1) \right)$$

agrees with Dong

Splitting problem

$$S=\int\frac{\partial L}{\partial \mathrm{Riem}}+\int\sum_{\alpha}\left(\frac{\partial^2 L}{\partial \mathrm{Riem}^2}\right)_{\alpha}\frac{K^2}{1+q_\alpha}$$

term	q_α
$\mathcal{R}_{ijkl}$	0
K_{ijz}	1/2
$\mathcal{Q}_{ijzz}$	1
$\mathcal{Q}_{ijz\bar{z}}-K_{ikz}\gamma^{kl}K_{jl\bar{z}}$	0
$R_{izz\bar{z}}$	1/2
$R_{z\bar{z}z\bar{z}}+\frac{1}{2}K_{ijz}K^{ij}{}_{\bar{z}}$	0

$$\begin{aligned} ds^2 &= \big(\gamma_{ij}+\big[2\kappa\,K_{ijz}z+\kappa^2\mathcal{Q}_{ijzz}z^2+\kappa^2\mathcal{Q}_{ijz\bar{z}}z\bar{z}+\mathrm{c.c.}\big]\big)\,d\sigma^i d\sigma^j+2\kappa\,A_{iz\bar{z}}(\bar{z}\,dz-z\,d\bar{z})\,d\sigma^i \\ &\quad-\frac{4}{3}\kappa^2\left[R_{izz\bar{z}}z-\mathrm{c.c.}\right](\bar{z}\,dz-z\,d\bar{z})d\sigma^i+\big(1+2\kappa^2R_{z\bar{z}z\bar{z}}z\bar{z}\big)\,dz\,d\bar{z}+O(\kappa^3)\end{aligned}$$