Scattering with partial information

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How can we understand (relativistic) scattering in the language of quantum measurement theory?

Measurement theory:

$$|0\rangle_A \otimes \sum_i c_i |i\rangle_S \xrightarrow{U} \sum_i c_i |i\rangle_A \otimes |i\rangle_S$$

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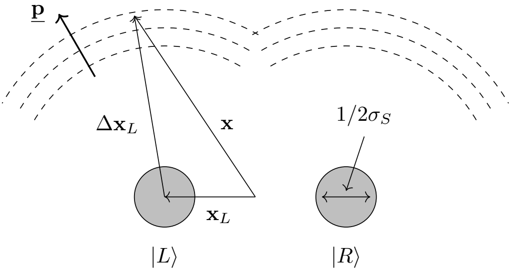
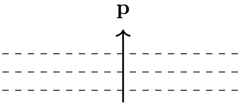
$\mathcal{H}^\pm = \mathcal{H}_A^\pm \otimes \mathcal{H}_S^\pm$

Questions

1. Given some property of S we want to measure, what do we observe with A?
2. How much $A - S$ entanglement does scattering generate?

1. Ex: verifying spatial superpositions





$$S = -\int d^4x \ \frac{1}{2}(\partial_\mu \phi_S)^2 + \frac{1}{2}(\partial_\mu \phi_A)^2 + \frac{1}{2}m_S^2\phi_S^2 + \frac{1}{2}m_A^2\phi_A^2 + \frac{\lambda}{4}\phi_S^2\phi_A^2$$

$$\rho = |\mathbf{p}^-\rangle \langle \mathbf{p}^-|_A \otimes \begin{pmatrix} \frac{1}{2} & \alpha \\ \alpha & \frac{1}{2} \end{pmatrix}_S$$

Convex family $0 \leq \alpha \leq 1$.

$\alpha \to 1$: coherent superposition of $|L\rangle$, $|R\rangle$
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Question: How do we measure α (in perturbation theory in λ?)

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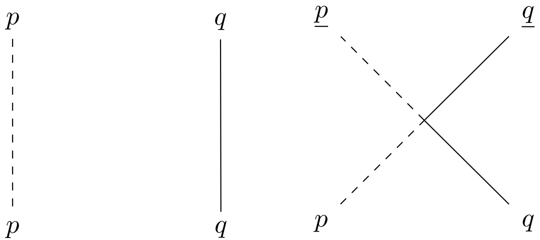
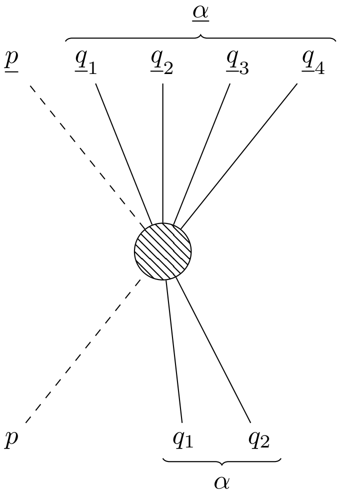


2. Entanglement entropy generated by scattering



$$|\psi\rangle = |\mathbf{p}\rangle_A^- \otimes |\alpha = \mathbf{q}_1, \mathbf{q}_2, \ldots\rangle_S^-$$

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$$\rho_A^+ = \begin{pmatrix} 1 + I_0 + F(\mathbf{p}) & & & \\ & F(\underline{\mathbf{p}}_1) & & \\ & & F(\underline{\mathbf{p}}_2) & \\ & & & \ddots \end{pmatrix}$$

$$I_0 = -2T\ \mathrm{Im} M_{\mathbf{p}\alpha\mathbf{p}\alpha}$$

$$F(\underline{\mathbf{p}}) = 2\pi T \sum_{\underline{\alpha}} \left| M_{\underline{\mathbf{p}}\underline{\alpha}\mathbf{p}\alpha} \right|^2 \delta_{\underline{\mathbf{p}}+\mathbf{p}_{\underline{\alpha}},\mathbf{p}+\mathbf{p}_\alpha} \delta(E^A_{\underline{\mathbf{p}}} + E^S_{\underline{\alpha}} - E^A_{\mathbf{p}} - E^S_\alpha)$$

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$F(\underline{\mathbf{p}})$: "conditional total cross-section"

So, entanglement entropy:

$$S_A = -(1 + I_0 + F(\mathbf{p}))\ln(1 + I_0 + F(\mathbf{p})) - \sum_{\underline{\mathbf{p}} \neq \mathbf{p}} F(\underline{\mathbf{p}}) \ln F(\underline{\mathbf{p}})$$

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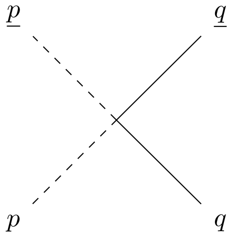
Assuming perturbation theory holds,

$$S_A \approx -\sum_{\underline{\mathbf{p}}} F(\underline{\mathbf{p}}) \ln F(\underline{\mathbf{p}})$$

Ex: $2 \to 2$ scattering in $\lambda \phi_A^2 \phi_S^2$ theory at lowest order.

(cf. Seki-Park-Sin 1412.7894, Peschanski-Seki 1602.00720)

$iM_{\underline{\mathbf{pq}}\mathbf{pq}} =$



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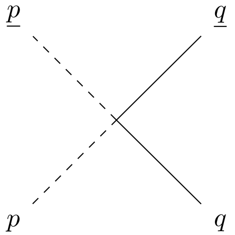

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$$S_A = -\frac{T}{V}\frac{\lambda^2}{16\pi}\frac{p_{cm}(E^A + E^S)}{(E^A E^S)^2}\ln\left[\frac{T^2}{V^2}\frac{\lambda^2}{16(E^A E^S)^2}\right]$$

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3. Black holes: entropy in soft modes? String scattering?