Causal Evolution of Bulk localized Excitations in AdS from CFT

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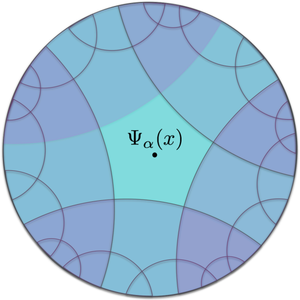
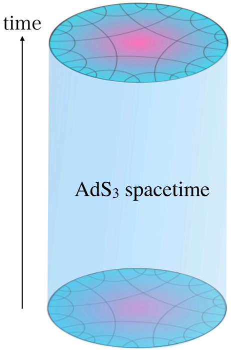
based on arXiv:1605.02835
KG and M.Miyaji and T.Takayanagi

Bulk locality in AdS/CFT



- Main interest is the bulk locality in AdS/CFT
- Important for understanding the structure of quantum gravity
  - link with quantum error correction?
    (*Harlow-Pastawski-Preskill-Yoshida*,$\cdots$)
  - state-dependent map between CFT and AdS?
    (*Papadodimas-Raju*,$\cdots$)
- How does the bulk locality/causality of AdS emerge from CFT?

Construction of bulk local states



- We construct CFT duals of bulk local states in AdS_3/CFT_2 in the large c limit.

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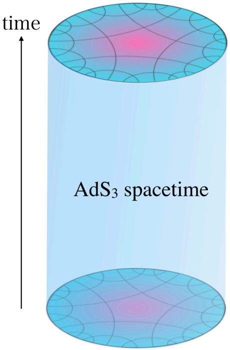

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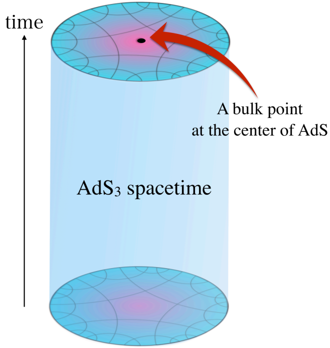

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- AdS_3 has a symmetry $SL(2,\mathbb{R}) \times SL(2,\mathbb{R})$; global part of Virasoro symmetry.
- Consider the bulk point at the center of AdS; $r=0$ on the time slice $t=0$.

- The bulk local state at $r = t = 0$ should be invariant under all $SL(2,\mathbb{R}) \times SL(2,\mathbb{R})$ transformations which keep the point invariant.

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- The solutions can be constructed from (analogue of) Ishibashi states for the $SL(2,\mathbb{R}) \times SL(2,\mathbb{R})$ symmetry

$$|\Psi_\alpha\rangle = \sqrt{\mathcal{N}} \sum_{k=0}^{\infty} (-1)^k e^{-\epsilon k} \frac{1}{N_k} (L_{-1})^k (\tilde{L}_{-1})^k |\alpha\rangle_L |\alpha\rangle_R$$

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- The averaged energy is

$$\Delta_{\Psi_\alpha} = \langle\Psi_\alpha| L_0 + \tilde{L}_0 |\Psi_\alpha\rangle \simeq \Delta_\alpha + \frac{1}{\epsilon} \qquad (\Delta_\alpha = h_\alpha + \bar{h}_\alpha)$$

- One can check the equivalence with the HKLL construction explicitly.

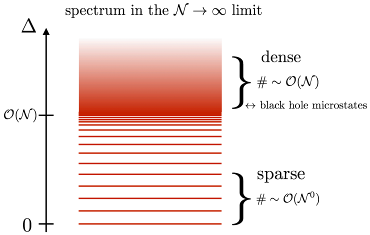
$$|\Psi_\alpha\rangle \Leftrightarrow \hat{\phi}_\alpha|0\rangle = \int dt' d\phi' K(t',\phi'|0,0,\phi) O_\alpha(t',\phi')|0\rangle$$

- We can take the point $r = 0$ to a point (r,ϕ,t) by using $SL(2,\mathbb{R})$ transformation $g(r,\phi,t)$.

$$|\Psi_\alpha(r,\phi,t)\rangle = g(r,\phi,t)|\Psi_\alpha\rangle$$

- These bulk local states satisfy the bulk equations of motion and two point functions reproduce the bulk to bulk propagators. (*Miyaji-Numasawa-Shiba-Takayanagi-Watanabe*)

Bulk locality from holographic CFTs



- The locality in AdS/CFT requires the large c limit with a large gap in the spectrum in a CFT, called holographic CFTs.
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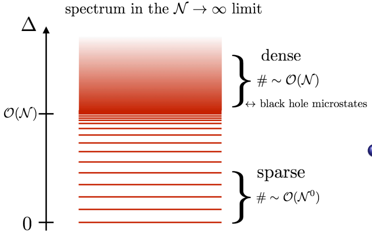

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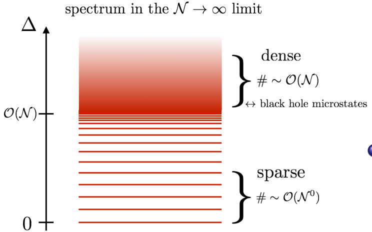

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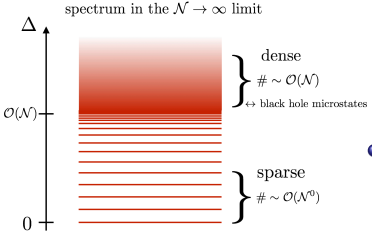

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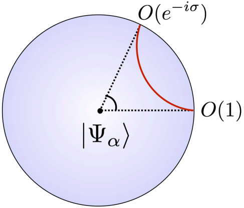
- However, the computation of the bulk two point function is universal in any CFTs and the result reproduces the expected gravity result in AdS even for non holographic CFTs such as free CFTs.
- Thus we study four point functions $\langle\Psi_\alpha|O(x)O(y)|\Psi_\alpha\rangle$.

Conformal block decomposition of the 4-pt. function



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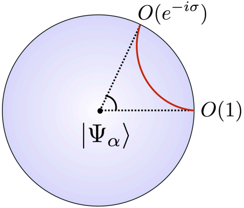

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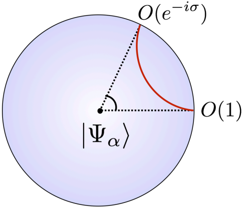

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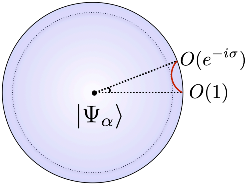
- Virasoro conformal block decomposition of $\langle O_\alpha O(1) O(e^{-i\sigma}) O_\alpha \rangle$
  $\Rightarrow$ In a holographic CFT case, it is dominated by the vacuum block.
  We mainly calculated the following case

$$h_\alpha = \mathcal{O}(1), \quad h_O = \mathcal{O}(c) \quad \text{in the large } c \text{ limit}$$

Short distance behavior and the first law of entanglement



- Behavior near $\sigma \sim 0$
  $\Rightarrow$ the two point function is expanded as

$$\langle\Psi_\alpha|O(1)O(e^{-i\sigma})|\Psi_\alpha\rangle\cdot|1-e^{-i\sigma}|^{4h_O} \simeq 1-\frac{\Delta_{\Psi_\alpha}\Delta_O}{c}\sigma^2$$

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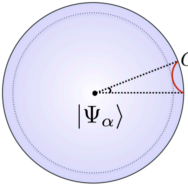

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- If we take the operator O to be a twist operator for the replica method with $h_O = c(n - 1/n)$, we can evaluate the increased amount of the entanglement entropy.

$$\Delta S_A \simeq \frac{\Delta_{\Psi_\alpha} \sigma^2}{6}$$

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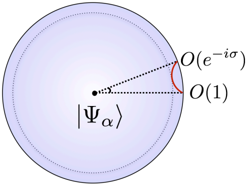

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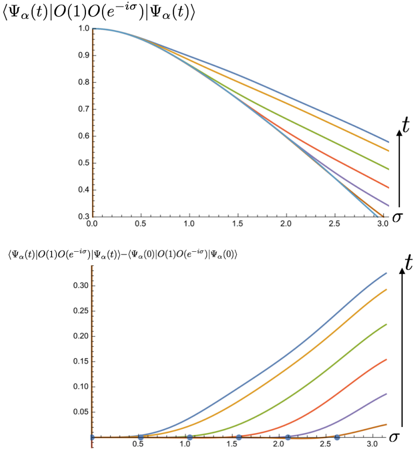
$$\Delta S_A \simeq \frac{\Delta_{\Psi_\alpha} \sigma^2}{6} = \Delta E_A / T_{ent}$$

$\Rightarrow$ The first law like relation of the entanglement entropy

- We also observed the time evolution of the four point functions.
- Especially when the mass of dual bulk scalar field is close to the BF bound,...



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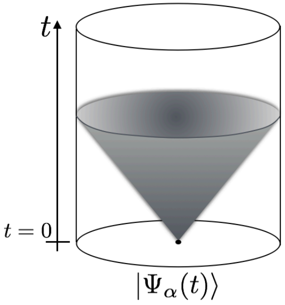
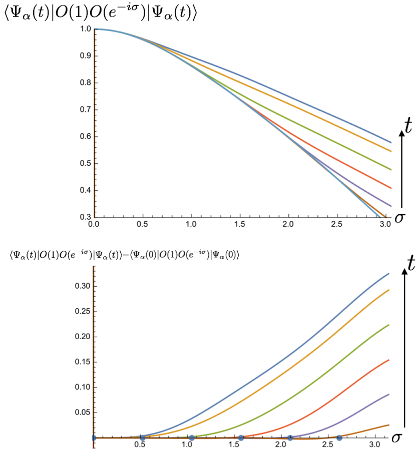
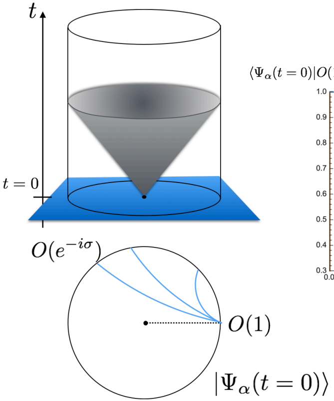
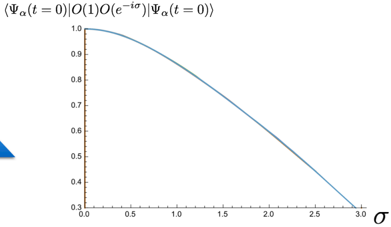




Causal evolution of bulk localized states

- The primary operators $O(1)$ and $O(e^{-i\sigma})$ can probe a light-like spread of the localized bulk excitation in AdS.





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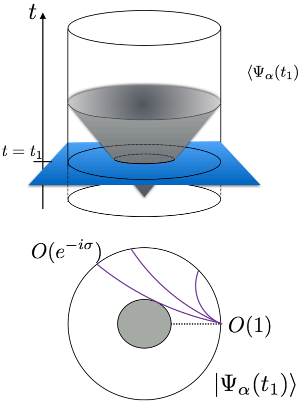

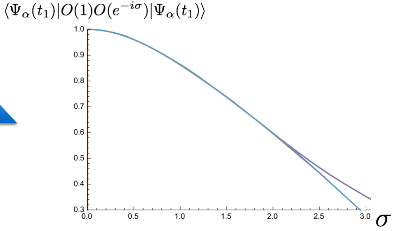

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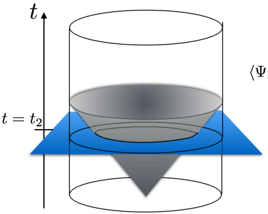
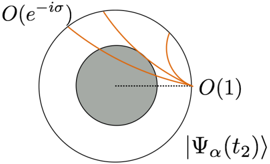
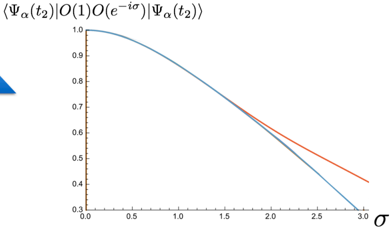
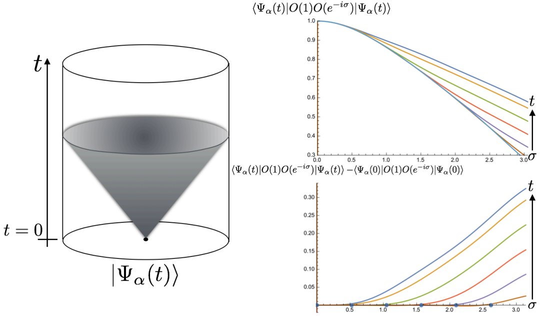






Causal evolution of bulk localized states

- When the mass of dual bulk scalar field is close to the BF bound, we observed the causal (light-like) propagation of the bulk excitations.

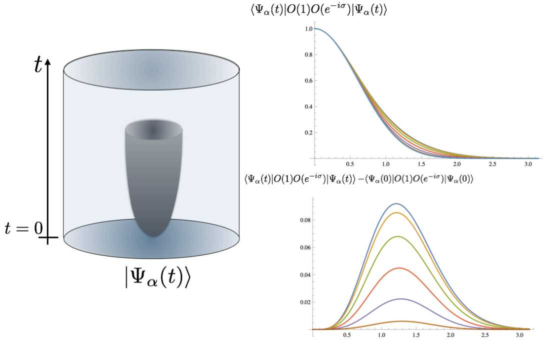


- Such behavior cannot be seen in the four point functions of free fermion CFTs; non-holographic CFTs

Summary

- We studied four point functions $\langle \Psi_\alpha | O(x) O(y) | \Psi_\alpha \rangle$.
- The short distance behavior is universal, and the increased amount of the entanglement obeys the first law like relation.
- In holographic CFTs we can see the light-like propagations of the bulk excitations, while we cannot see such behavior in non holographic CFTs.

- When the mass of dual bulk scalar field is much heavier than the BF bound, we cannot observe the light-like propagation of the bulk excitations. Also, each two point function is greatly suppressed.



- This is because the scalar field is very massive, the excitations does not reach the AdS boundary, and also the primary field in $|\Psi_\alpha\rangle$ gives a back reacted geometry (deficit angle spacetime).