Entanglement with Centers

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$$H \sim \bigoplus_k \left(H_V^k \otimes H_{\bar{V}}^k \right)$$

$$
\begin{aligned}
-\mathrm{Tr}\left(\rho_A \ln \rho_A\right) &= -\sum_k \mathrm{Tr}\left(p_k \rho_{A_k} \ln(p_k \rho_{A_k})\right) \\
&= {\color{red}-\sum_k p_k \ln p_k} - \sum_k \mathrm{Tr}\left(p_k \rho_{A_k} \ln \rho_{A_k}\right).
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-\sum_\phi \big(f(\phi)\Delta\big) \ln(f(\phi)\Delta) \longrightarrow -\ln(\Delta) - \int d\phi\, f(\phi) \ln f(\phi).
$$

Lagrangian Method

Non-trivial centers in a standard Hilbert space or standard quantum field theory (by removing operators) are equivalent to mentioning that we do not have quantum fluctuation on an entangling surface. We can consider classical solution in bulk and boundary, and quantum fluctuation in bulk to compute the entanglement entropy with centers.

The Result of Massless p-form Theory

The massless p-form theory in $(2p+2)$-dimensions has electric-magnetic duality. From the Hamitonian formulation, we find equivalent entanglement entropy between different centers from electric-magnetic-like duality.

$SU(N)$ Yang-Mills Gauge Theory

$$H_{LYMF} = \frac{g^2}{2} \sum_{l} E_l^a E_l^a - \frac{1}{g^2} \sum_{\Box} \left(\mathrm{Tr}\, U_\Box + \mathrm{Tr}\, U_\Box^\dagger \right) .$$

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$$\psi'(U_{l_1},\cdots,U_{l_{\partial A}},U_{l_{\partial\bar{A}}},\cdots,U_{l_k}) \equiv \psi(U_{l_1},\cdots,U_{l_{\partial A}}\cdot U_{l_{\partial\bar{A}}},\cdots,U_{l_k}),$$

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$$S_{EE} = n_A (D-2) \lambda^2 (- \ln \lambda^2 + 1 + 2 \ln N) + O(\lambda^3),$$

where n_A is the number of boundary links in the region A, and $\lambda \equiv \frac{2N}{g^4(N^2-1)}$.

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- The result of the massless p-form theory possibly implies global symmetry structure in centers.
- The properties of the entanglement entropy in strong coupling region are still unclear.