EPR Pairs, Local Projection and Quantum Teleportation in Holography

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with
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"Quantum Operations" in QFT ?
Its Holographic Duals ?

Entanglement Measures & Related Phenomena motivated by Quantum Information theory

EE, MI, Relative Entropy, Negativity, Complexity, Information metric

Scrambling, Quantum Error Correction, Distillation …

→ Hot topics in QFT & its Holographic Dual

Big Workshop !!

Big Collaboration !!

It from Qubit

Simons Collaboration on
Quantum Fields, Gravity and Information

How about Operational Aspects ?

Quantum Operations → Also important in many (quantum process or protocols) in QI

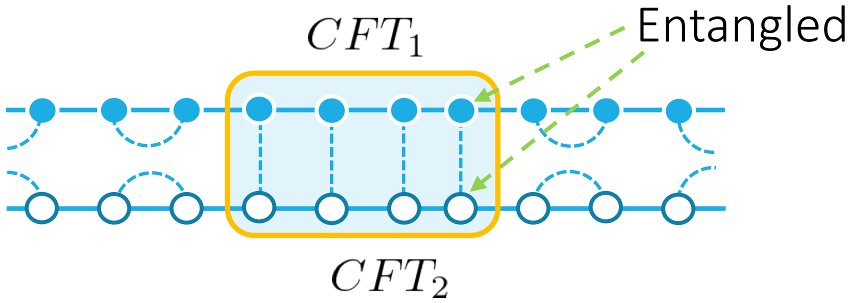
→ However, so far, NOT investigated well in QFT & Holography ….

Let's Try 3 Quantum Operations !!

$$|0\rangle_{CFT_1} \otimes |0\rangle_{CFT_2} \longrightarrow ??$$

Focus on
AdS_3/CFT_2

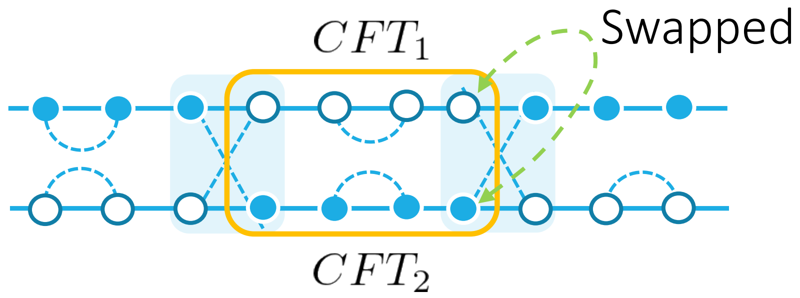
Partial Entangling



→ Partially prepare EPR pairs between two CFTs

$S_1 \propto$ Vol (operated region)

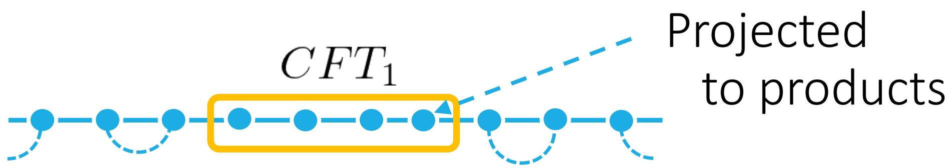
Partial Swapping



→ Partially exchange two CFTs (also EPR pairs)

$S_1 \propto$ (# of EPR pairs crossing the edges)

Local (Partial) Projection

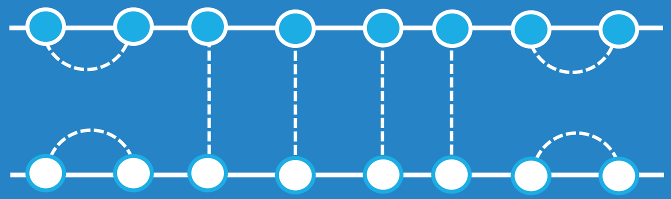


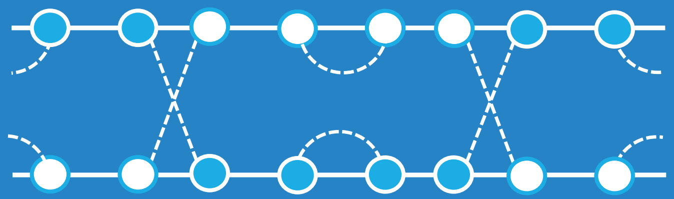
→ Partially project out a CFT

Reduce EE

QFT Analog & Holographic Dual of

"Quantum Teleportation"





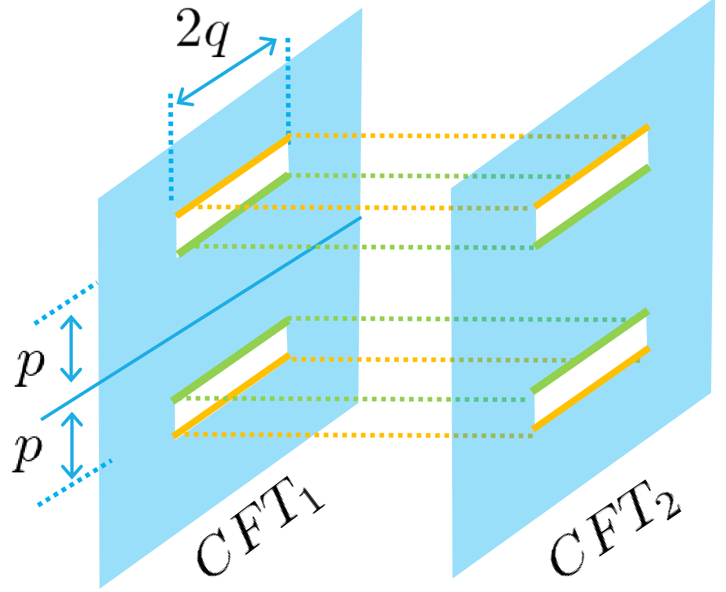
Entangling & Swapping between two CFTs

Path-Integral Pictures

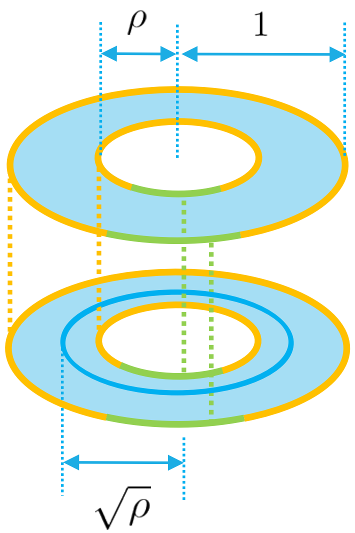
EEs after these operations

Path-Integral Pictures: Entangling

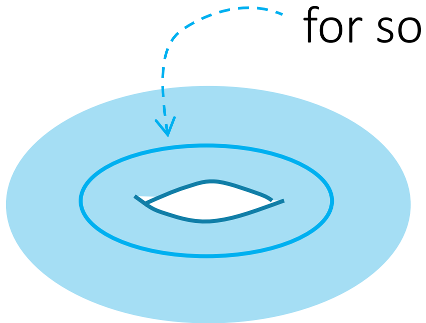
$$\prod_{x\in P}\left(\sum_{n_x}|n_x\rangle_1|n_x\rangle_2\right)\left(\sum_{m_x}\langle m_x|_1\langle m_x|_2\right)\otimes\prod_{x\in P^c}(I_x^1\otimes I_x^2)$$



Conformal Map



Non-contractive for solid torus



Torus with period $p/q \ll 1$

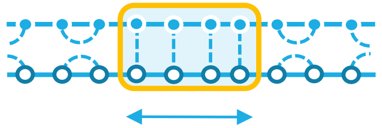
$$\tau = \tau_1 + i\tau_2 \simeq i\frac{q}{2p}$$

Dual ⟷ BTZ BH

EE between two CFTs

$$S_1 = \frac{\pi c}{3}\tau_2 \simeq \frac{\pi c}{6}\frac{q}{p} \quad \propto q$$

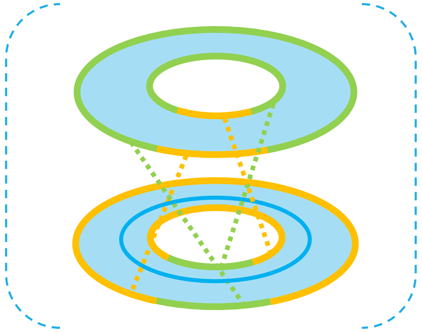
$p/q \ll 1$

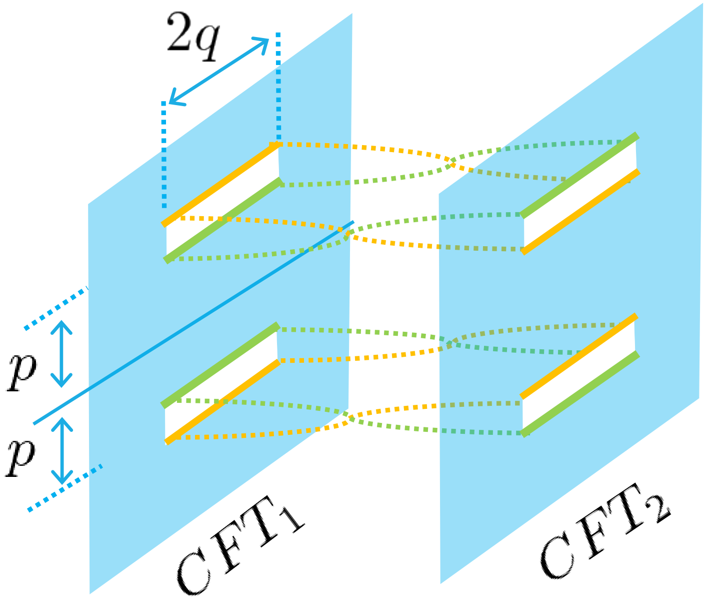


$\propto$ Volume of the region we operated

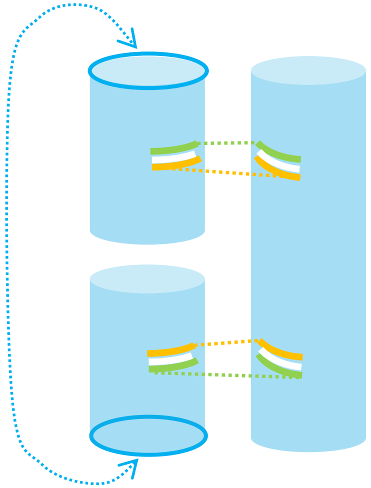
Path-Integral Pictures: Swapping

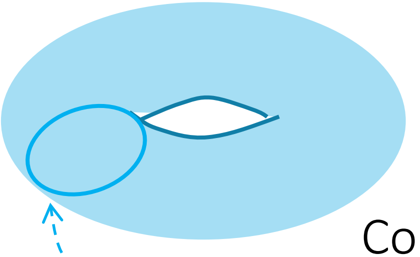
$$\prod_{x\in P}\left(\sum_{n_x,m_x}|n_x\rangle_1|m_x\rangle_2\langle m_x|_1\langle n_x|_2\right)\otimes\prod_{x\in P^c}(I_x^1\otimes I_x^2)$$





Conformal Map





Torus

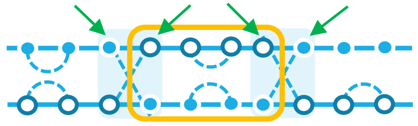
Contractive for solid torus

→ Different cycle as Entangling

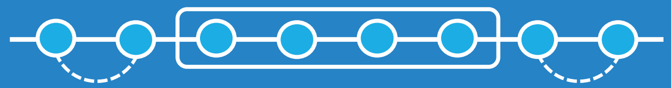
EE between two CFTs

$$S_1=\frac{\pi c}{3}\tau_2\simeq 2\cdot\frac{c}{3}\log\frac{q}{p}$$

$p/q\ll 1$ $\quad = 2\cdot S_{\mathrm{EE,1interval}}$



$\propto$ # of EPR pairs crossing the edges



Local Projection Measurement in a CFT

Local Projection as Boundary State

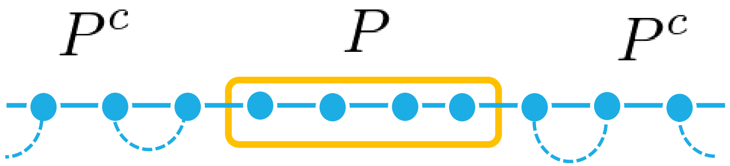
Path-Integral Pictures

EEs after the operation

Local Projection described by Boundary States

Local (Partial) Projection

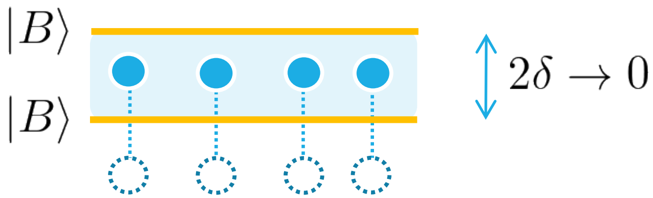
$$\hat{P} = \left(\prod_{x \in P} |\psi_x\rangle\langle\psi_x| \right) \otimes \left(\prod_{x \in P^c} I_x \right)$$



$\longrightarrow$ No real space entanglement at each point in P

✓ Factrization of n-pt function on Conformal Boundary states close each other

$$\frac{\langle B|e^{-\delta H}\mathcal{O}(x_1)\cdots\mathcal{O}(x_n)e^{-\delta H}|B\rangle}{\langle B|e^{-2\delta H}|B\rangle} \approx \prod_{i=1}^{n}\langle\mathcal{O}(x_i)\rangle$$



$\longrightarrow$ No real space entanglement [Miyaji-Ryu-Takayanagi-Wen '14]

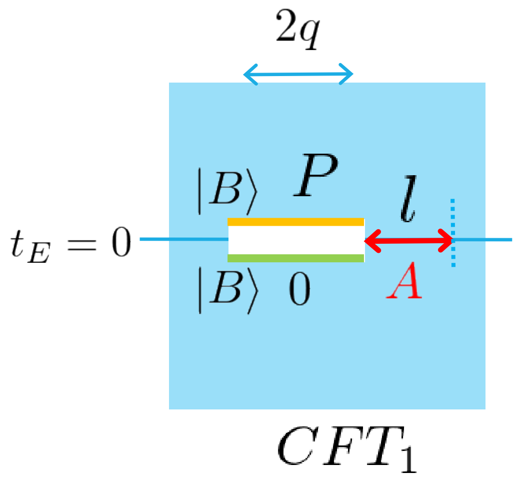
⇨ **Local Projection can be described by Boundary States $|B\rangle$** [Rajabpour'15]

More generally, $\prod_x U_x|B\rangle$

Path-Integral Pictures: Local Projection

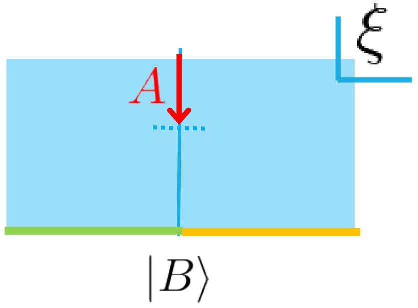
$$\left(\prod_{x\in P}|\psi_x\rangle\langle\psi_x|\right)\otimes\left(\prod_{x\in P^c}I_x\right)$$

1 interval jointed with 1-cut



$$w=\frac{x+iy}{\sqrt{2}}$$

$$\xi=\sqrt{\frac{\frac{q}{\sqrt{2}}+w}{\frac{q}{\sqrt{2}}-w}}$$



UHP with a twist operator

[Rajabpour'15]

$$\boxed{S_A=\frac{c}{6}\log\frac{2(l+2q)l}{aq}+\gamma_b}$$

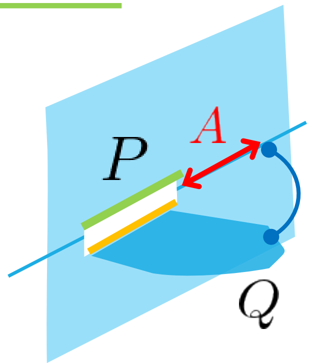
a : UV cut-off

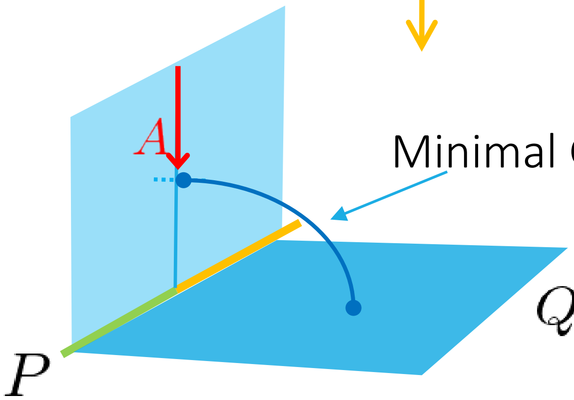
γ_b : Boundary entropy

(ignore in this talk)

Dual

Its Dual Picture





Q : Totally Geodesic Surface

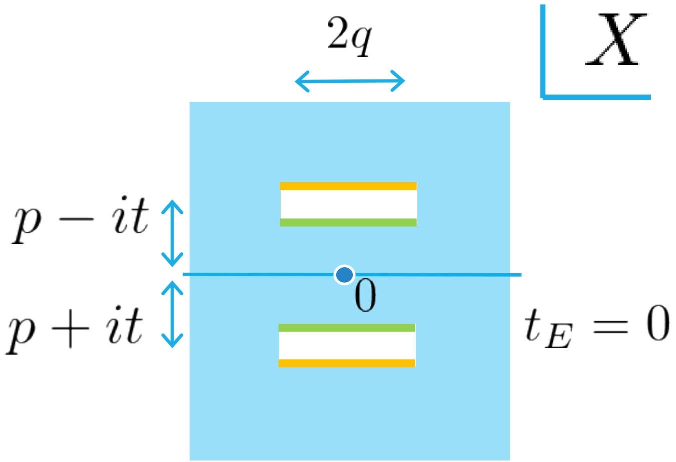
(ignore the tension in this talk)

AdS/BCFT :

[Takayanagi'11]

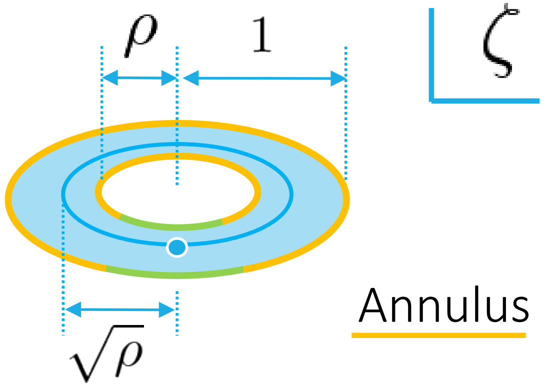
[Fujita-Tonii-Takayanagi'11]

2-cuts, 1-disjoint interval and Time-evolutions



$$X(\zeta)=2ip\left(K(\zeta/\sqrt{\rho})+K(\zeta\sqrt{\rho})-\frac{1}{2}\right)+t$$

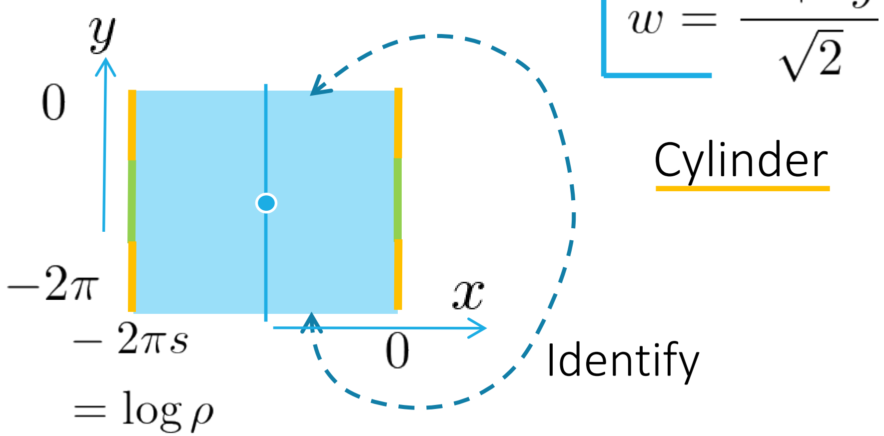
$$K(\zeta)=\frac{\zeta}{\zeta-1}+\sum_{k=1}^{\infty}\left(\frac{\rho^{2k}\zeta^{-1}}{1-\rho^{2k}\zeta^{-1}}-\frac{\rho^{2k}\zeta}{1-\rho^{2k}\zeta}\right)$$



$$\zeta=\rho\cdot e^{-\sqrt{2}w}$$

$$\frac{q}{p}\simeq\frac{1}{s}=\frac{-2\pi}{\log\rho}$$

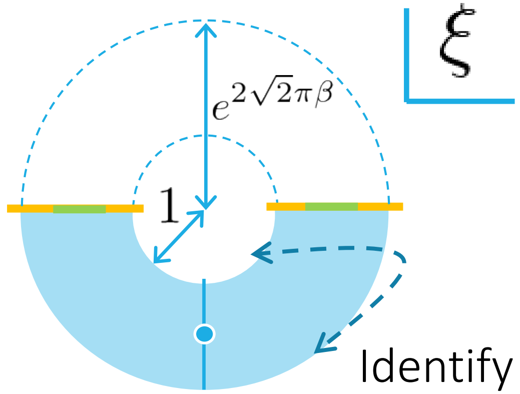
$$p/q\ll 1$$



$$\xi=e^{i\frac{w}{\sqrt{2}s}}$$

$$\rho=e^{-2\pi s}$$

$$\beta=\frac{1}{2\sqrt{2}s}$$



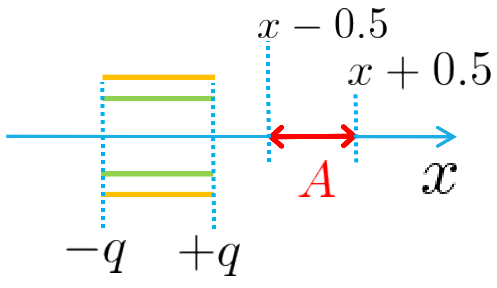
Half of Torus

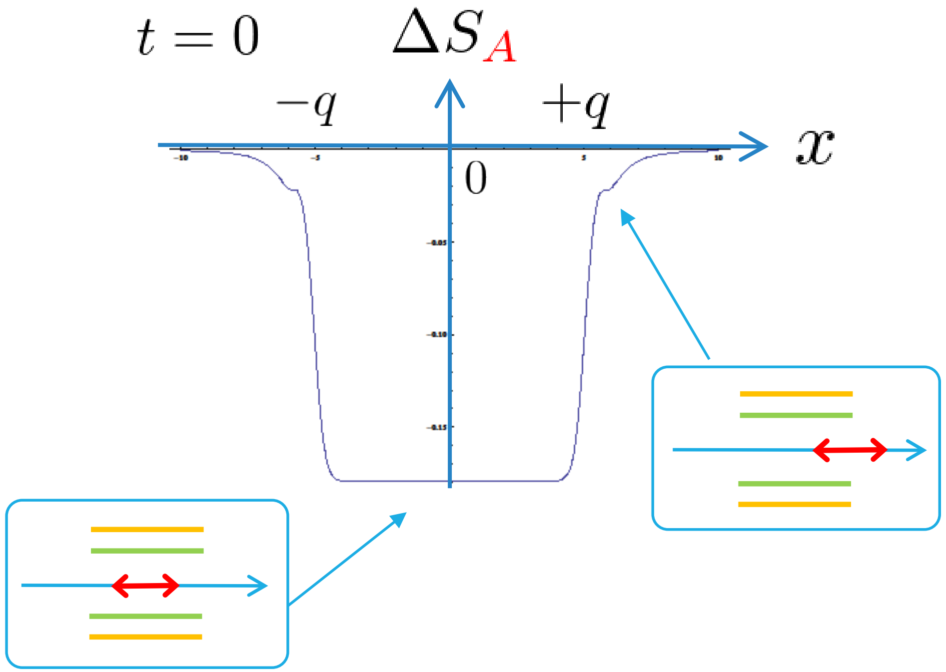
Dual

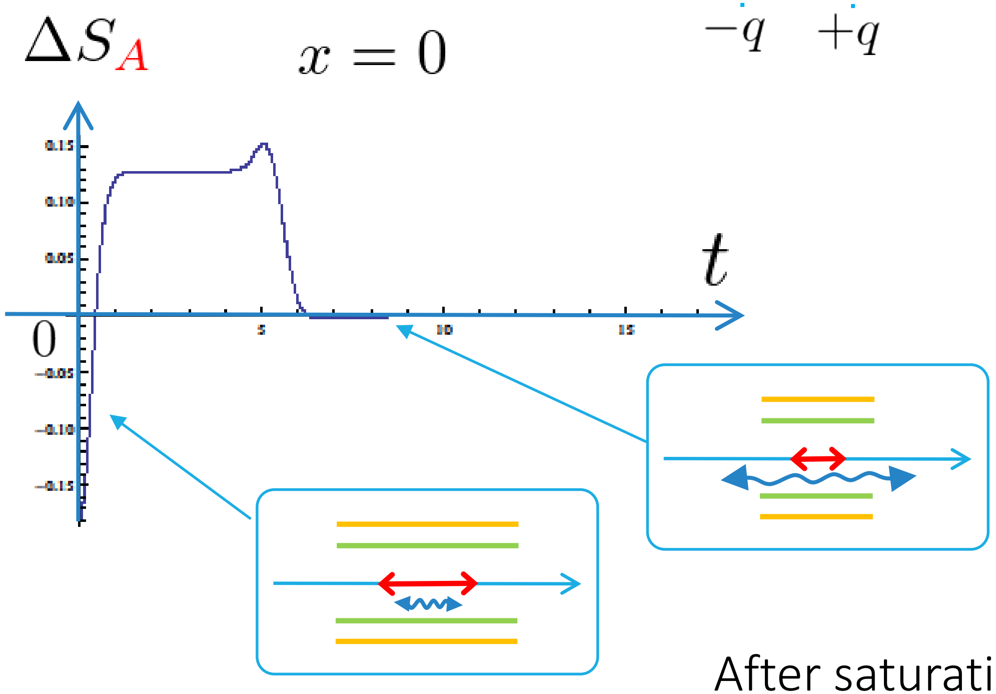
Half of BTZ BH

Example: EE in 2D Free Fermion CFT

$$\boxed{\Delta S_A = S_A - S_A^{G.S.}}$$







EE reduces by local projection at $t=0$

$$\Delta S_A \leq 0$$

ΔS_A grows linearly in time

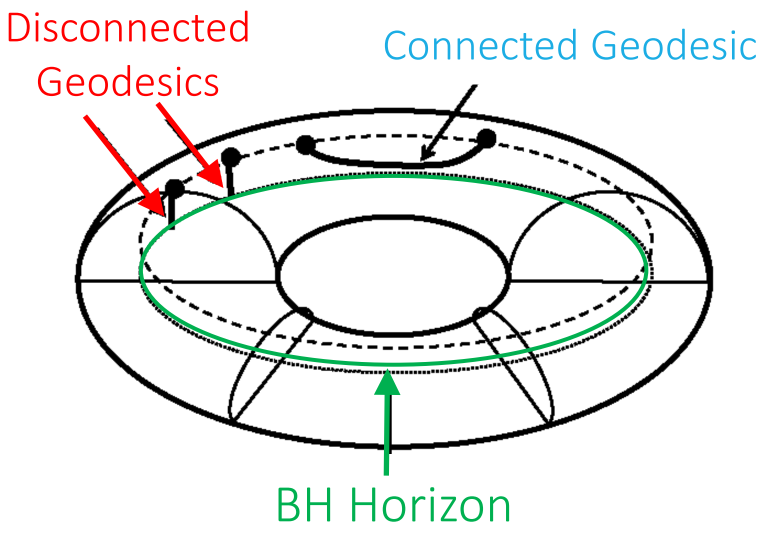
(like quantum quench)

After saturation,

ΔS_A goes to zero

(projection effect goes out)

Holographic EE after Projection or Entangling

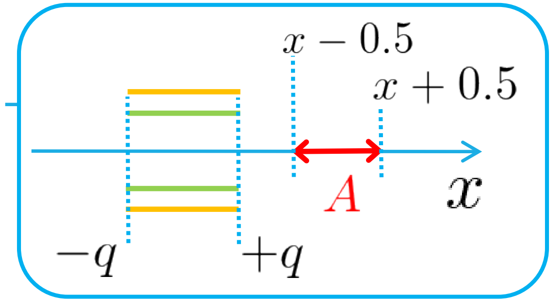


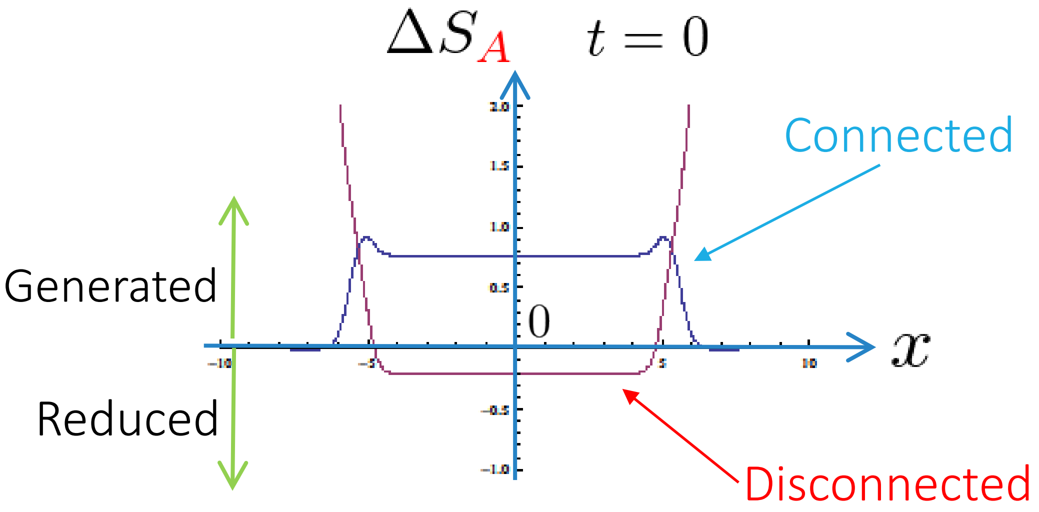
$$S_A = \frac{c}{6} \cdot \min_{\{\ \ \}} \text{(Geodesic Length)}$$

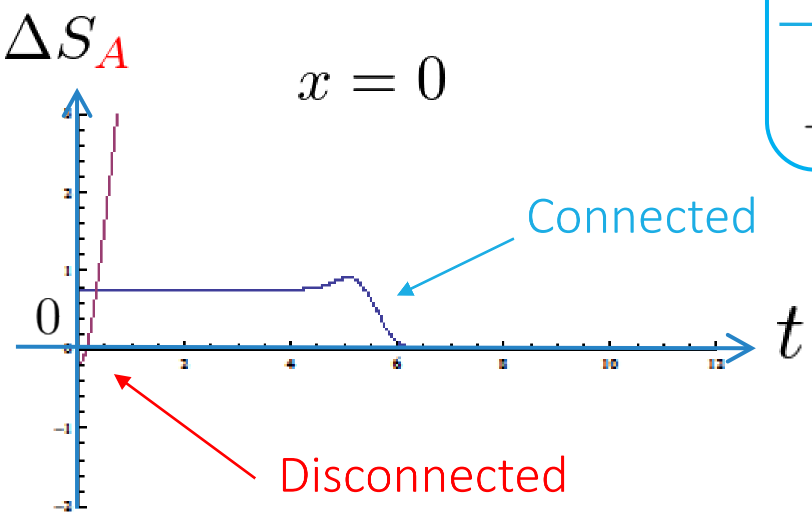
$$\beta = \frac{1}{2\sqrt{2}s} \simeq \frac{1}{2\sqrt{2}}\frac{q}{p}$$

Entangling (Whole Torus) → { Only Connected }

Projection (Half of Torus) → { Connected or Disconnected }





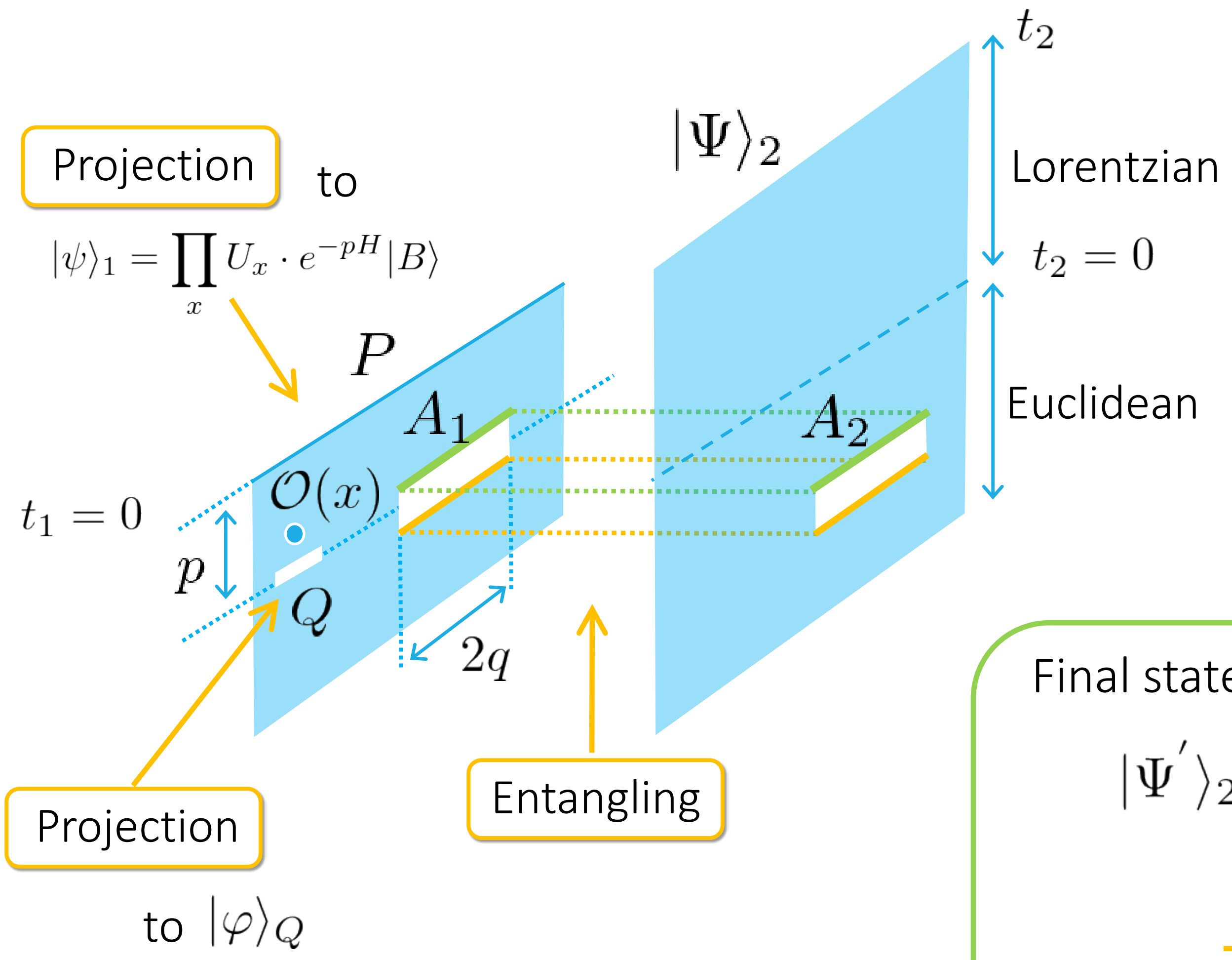


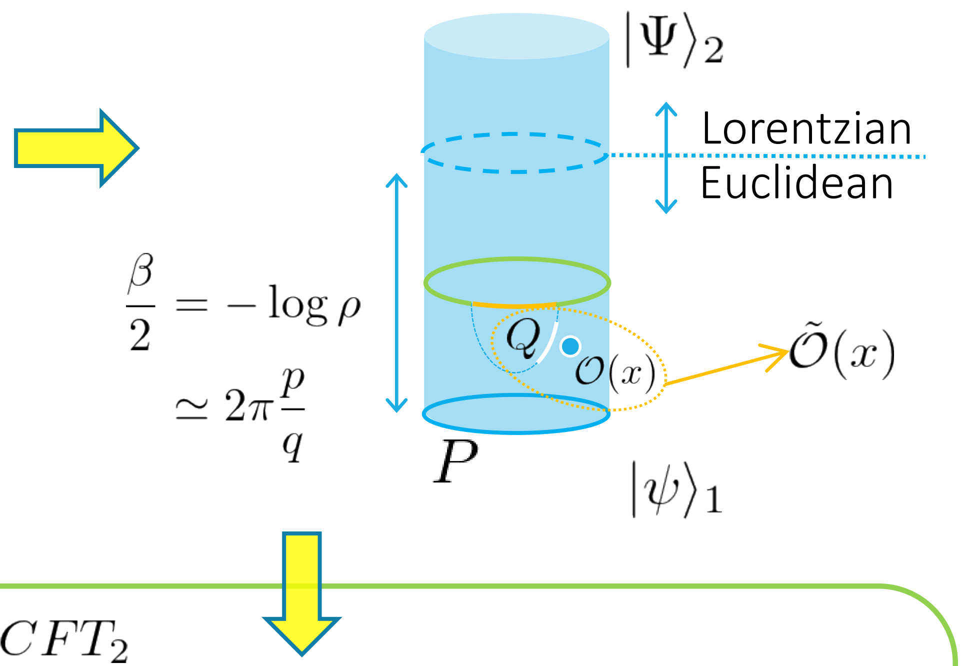
Analogue of Quantum Teleportation in CFT & Holography

“Quantum Teleportation” of Local Operator

Partial Entangling + Local Projection + Local Unitary Transformation

Path-Integral Pictures: Quantum Teleportation





Final state in CFT_2

$$|\Psi'\rangle_2 = \mathcal{N}'_{\mathcal{O}} \cdot e^{-\frac{\beta}{4}H} \cdot \tilde{\mathcal{O}}(x) \cdot e^{-\frac{\beta}{4}H} e^{it_2 H}|\psi\rangle_2$$

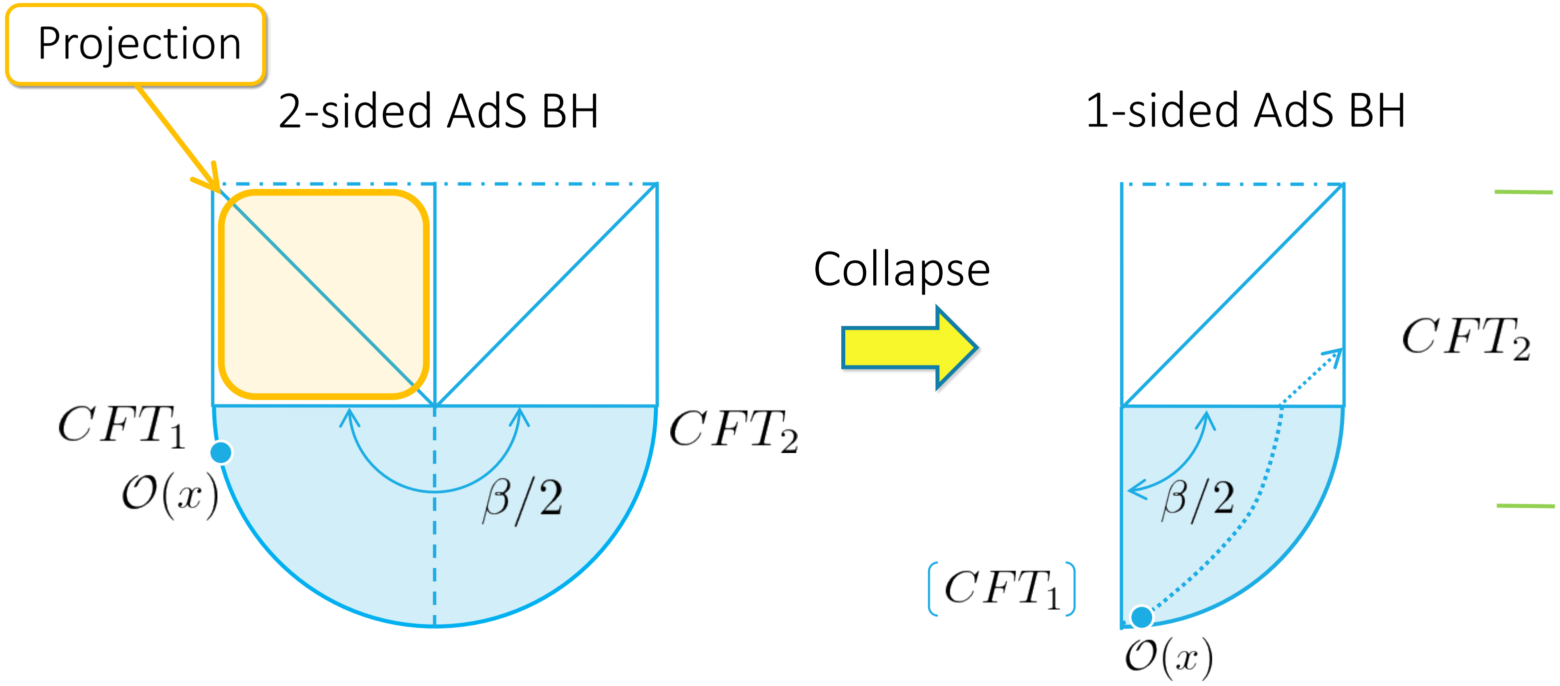
$$\xrightarrow{\exists U_\psi} \hat{\mathcal{N}}'_{\mathcal{O}} \cdot \underbrace{\mathcal{O}(x)|\varphi\rangle_Q}_{\tilde{\mathcal{O}}(x)} \otimes |\psi'(Q^c)\rangle$$

Teleported !

Holographic Model of Quantum Teleportation

Projecting out CFT_1 → Collapse of the holographic geometry

(The topology can change)



— Effective Temperature reduces by half

$$\beta \rightarrow \beta/2$$

— Teleportation to CFT_2

through the Euclidean BH

cf. [Susskind '14, '16]

→ Need more detail understandings ….

Traversable channel created by Swapping ??

↔ "Classical Communication" part ??

Summary

1604.01772 [hep-th]

3 Quantum Operations

$|0\rangle_{CFT_1} \otimes |0\rangle_{CFT_2} \longrightarrow$??

in CFTs & Holographic Duals

AdS_3/CFT_2

Partial Entangling

$$S_1 = \frac{\pi c}{3} \boxed{\tau_2} \simeq \frac{\pi c}{6} \frac{q}{p}$$

$\propto$ Vol(operated region)

Torus $\xleftrightarrow{\text{Dual}}$ BTZ BH

Time evolution

Partial Swapping

$$S_1 = \frac{\pi c}{3} \boxed{\tau_2} \simeq 2 \cdot \frac{c}{3} \log \frac{q}{p}$$

$\propto$(# of EPR pairs crossing the edges)

Torus $\longrightarrow$ Different Cycle as Entangling

Local (Partial) Projection

(Can) Reduce EE

Half of Torus $\xleftrightarrow{\text{Dual}}$ Half of BTZ BH

Time evolution

But, Generate EE sometime

(Growth as Quantum Quench)

CFT Analogue & Holographic Model of

"Quantum Teleportation" of Local Operators

Further Directions ...

— Higher dim. Generalizations

— More details on "Quantum Teleportation" in QFT & Holography

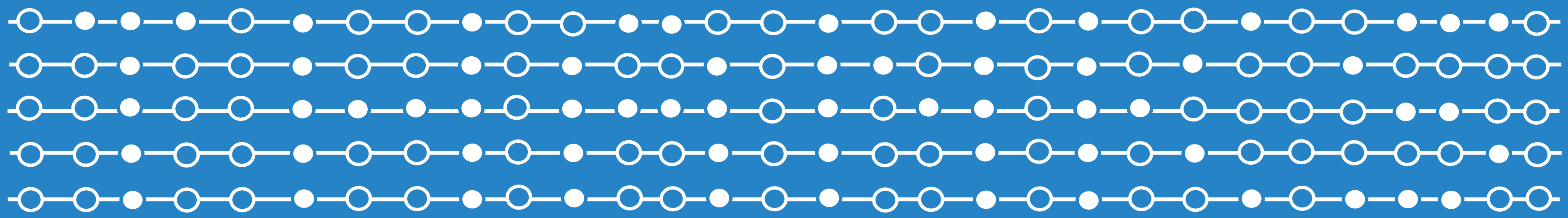
— Interpretation in Tensor Networks

— New Entanglement Measures in QFT & Holography

- for Multi-body Entanglement (GHZ, ...?)
- for Mixed States ... (Quantum Discord,...?)

from Local Operations ...etc

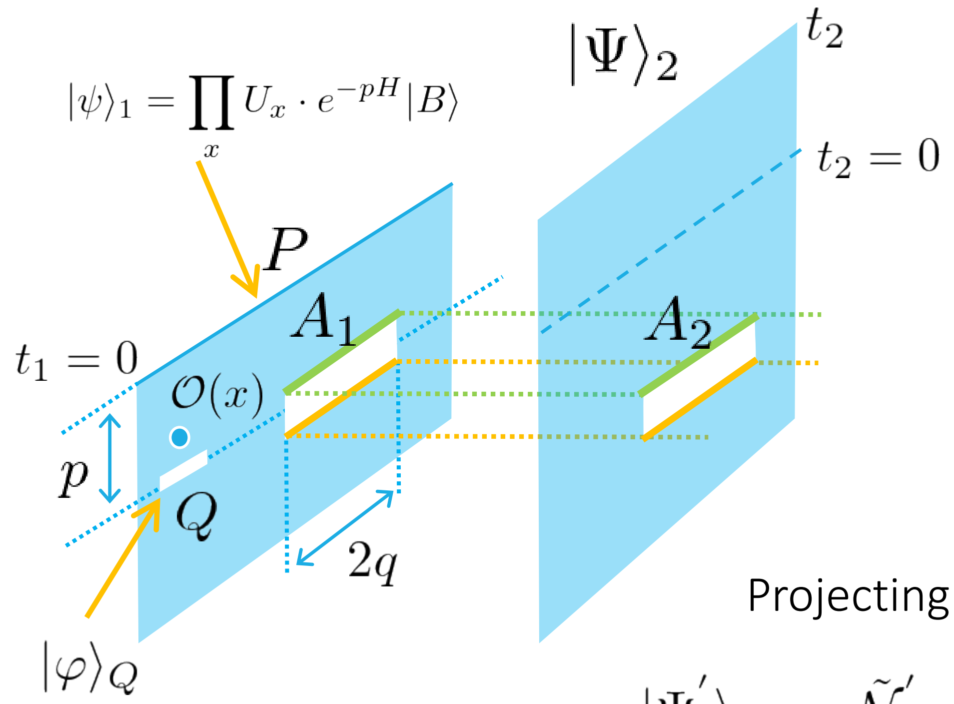
Play with Entanglement !!
Play with Operations !!



THANKS !!

Appendix

Some Details on QFT Analogue of “Quantum Teleportation”



The state after Entangling between A_1 and A_2

$$|\Psi\rangle_{12} = \prod_{x\in A_1}\left[\sum_{n_x}|n_x\rangle_{A_1}|n_x\rangle_{A_2}\right]\otimes|\Psi(A_1^c\cup A_2^c)\rangle_{12}$$

Act $\mathcal{O}(x)$ on the state

$\mathcal{O}(x) = \lambda_1\mathcal{O}_1(x) + \lambda\mathcal{O}_2(x)$

with some conditions for the linearity

$$|\tilde{\Psi}\rangle_{12} = \tilde{\mathcal{N}}_{\mathcal{O}}\cdot\mathcal{O}(x)|\Psi(A_1^c\cup A_2^c)\rangle_{12}$$

Projecting the state by $|\varphi\rangle_Q$ $\longrightarrow$ Projecting the state by

$$|\Psi^{'}\rangle_{12} = \tilde{\mathcal{N}}_{\mathcal{O}}^{'}\cdot\underbrace{\mathcal{O}(x)|\varphi\rangle_Q}_{\tilde{\mathcal{O}}(x)}\otimes\prod_{x\in A_1}\left[\sum_{n_x}|n_x\rangle_{A_1}|n_x\rangle_{A_2}\right]\otimes|\Psi((A_1^c-Q)\cup A_2^c)\rangle_{12}$$

Projecting the state by $|\psi\rangle_1$ $\longrightarrow$ Final state in CFT_2

$$|\Psi^{'}\rangle_2 = \mathcal{N}_{\mathcal{O}}^{'}\cdot e^{-\frac{\beta}{4}H}\cdot\tilde{\mathcal{O}}(x)\cdot e^{-\frac{\beta}{4}H}e^{it_2H}|\psi\rangle_2 \xrightarrow[\exists U_\psi]{} \hat{\mathcal{N}}_{\mathcal{O}}^{'}\cdot\underbrace{\mathcal{O}(x)|\varphi\rangle_Q}_{\tilde{\mathcal{O}}(x)}\otimes|\psi^{'}(Q^c)\rangle$$

Teleported !