Hyperbolicity, Boundary Causality and CHI in Gauss-Bonnet gravity

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Motivation/Summary

- Higher derivative theories source of interesting phenomena Brigante,Liu, Myers, Shenker, Yaida; de Boer, Kulaxizi, Parnachev; Ge, Sin; Cai, Nie, Sun; Cai, Nie,Ohta,Sun
- Can have superluminal modes
- Toy model: black holes in Gauss-Bonnet
- In large Gauss-Bonnet black holes boundary causality constrains allowed values of GB coupling Buchel, Escobedo, Myers, Paulos, Sinha, Smolkin; Hofman, Maldacena; Camanho, Edelstein
- Boundary causality in small black holes?
- Hyperbolicity
- Causal holographic information .

Hyperbolicity in Gauss-Bonnet

Izumi 2014; Reall, Tanahashi, Way 2014

In Lovelock theories the equations of motion $\mathcal{E}_I = 0$ depend linearly on $\partial_0^2 g_{\mu\nu}$

$$\frac{\partial \mathcal{E}_I}{\partial(\partial_0^2 g_J)} \partial_0^2 g_J + \cdots = 0$$

- Consider hypersurface Σ and coordinates (x_0, x_i) such that Σ has equation $x_0 = 0$
- Unique solutions only if

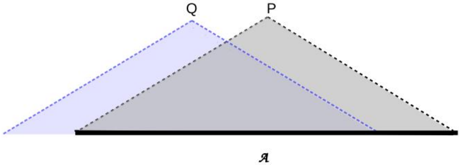
  $$P^I_J = \frac{\partial \mathcal{E}_I}{\partial(\partial_0^2 g_J)},$$

  is invertible $\rightarrow$ Σ is non-characteristic
- If P^I_J is not invertible, Σ is *characteristic*
- Characteristic hypersurface: hypersurface beyond which evolution is not unique.

- Given a metric

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2_{d-2}$$

- perturbations $A = \{S, V, T\}$
- the fastest mode propagates along a characteristic hypersurface.



- Characteristic hypersurfaces in Einstein gravity are always null
- Not in Gauss-Bonnet gravity H. Reall, N. Tanahashi, B.Way 1406.3379; K.Izumi 1406.0677

- Static, spherically symmetric backgrounds
- Can define an effective metric such that the characteristic hypersurface is null with respect to that metric.
- $A = \{S, V, T\}$

$$\left(-\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial r_*^2} - V_A^\ell(r)\right)\Psi_A^\ell(t,r) = 0$$

- Large ℓ

$$\left(-\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial r_*^2} - f(r)\frac{c_A(r)}{r^2}D^2\right)\Psi_A(t,r) = f(r)G_A^{\mu\nu}\partial_\mu\partial_\nu\Psi_A = 0$$

- characteristic hypersurface is null with respect to the corresponding effective metric,

$$G_{A\mu\nu}dx^{\mu}dx^{\nu} = -f(r)dt^2 + \frac{dr^2}{f(r)} + \frac{r^2}{c_A(r)}d\Omega^2$$

- In Lovelock theories the characteristic determinant factorizes

$$Q(x,\xi) = (G_S^{ab}(x)\xi_a\xi_b)^{n_S}(G_V^{ab}(x)\xi_a\xi_b)^{n_V}(G_T^{ab}(x)\xi_a\xi_b)^{n_T},$$

- If $c_A = 0$ theory is not hyperbolic

Gauss-Bonnet gravity

$$
S_{grav} = \frac{1}{16\pi G_N}\int d^5x\sqrt{-g}\left(R + \frac{12}{L^2} + \frac{\lambda L^2}{2}\mathcal{L}_{(2)}\right),
$$
$$
\mathcal{L}_{(2)} = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2
$$

Black hole solution,

$$
ds^2 = -\frac{f(r)}{f_\infty}dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_3^2,
$$

where

$$
f(r) = r^2\left[\frac{L^2}{r^2} + \frac{1}{2\lambda}\left(1 - \sqrt{1 - 4\lambda + 4\lambda\frac{\mu^4}{r^4}}\right)\right].
$$

and $f_\infty = \frac{1-\sqrt{1-4\lambda}}{2\lambda}$
Effective metrics from master equation Takahashi, Soda

$$G_{A\mu\nu} dx^\mu dx^\nu = -f(r)dt^2 + \frac{dr^2}{f(r)} + \frac{r^2}{c_A(r)} d\Omega^2$$

with,

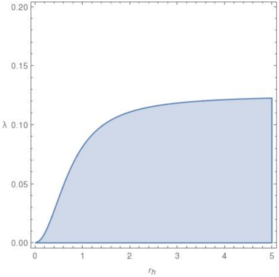
$$c_T = \frac{r^4(1-4\lambda) + 12\lambda(rh^2 + rh^4 + \lambda)}{r^4(1-4\lambda) + 4\lambda(rh^2 + rh^4 + \lambda)}$$

$$c_V = ...$$

$$c_S = \frac{4\lambda(rh^2 + rh^4 + \lambda) + r^4(-1+4\lambda)}{-4\lambda(rh^2 + rh^4 + \lambda) + r^4(-1+4\lambda)}$$

- $c_A \to 1$ as $r \to \infty$
- Only c_S can have a zero

Scalars control hyperbolicity



Tensors are the fastest, causal structure

Null Geodesics

Geodesic equations

$$\dot{t} = \frac{1}{f(r)}, \quad \dot{\phi} = \frac{\ell c_T(r)}{r^2}, \quad \dot{r} = \eta\sqrt{f_\infty - \ell^2 c_T(r) f(r)/r^2}$$

- Tensor effective metric
- Geodesics with endpoints at the boundary, $\ell^2 < 1$.
- Turning point: $f_\infty - \ell^2 c_T(r_m) f(r_m)/r_m^2$

Δt : time that a geodesic takes to travel from boundary to minimum radius back to boundary

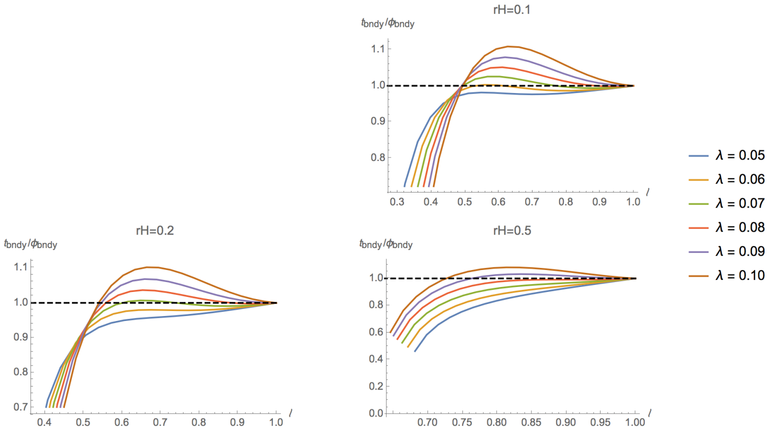
$$\Delta t = 2\int_{r_m}^{\infty} \frac{dr}{f(r)\sqrt{f_\infty - \ell^2 c_T(r) f(r)/r^2}}.$$

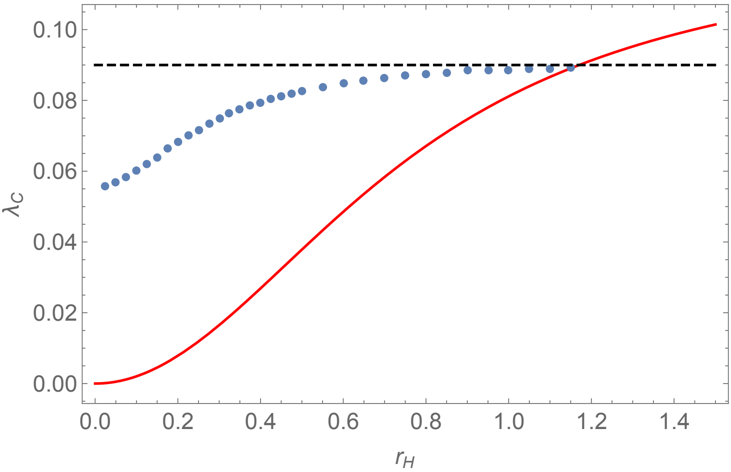
$\Delta\phi$: angle subtended –at the boundary– in Δt

$$\Delta\phi = 2\int_{r_m}^{\infty} \frac{\ell c_T(r) dr}{r^2\sqrt{f_\infty - \ell^2 c_T(r) f(r)/r^2}}.$$

Δt and $\Delta\phi$ numerically

Causality violation

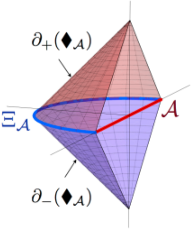



0.10
0.08
0.06
0.04
0.02
0.00
λ_C
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
r_H

Causal Holographic Information

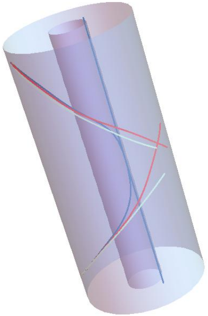
Hubeny, Rangaman 1204.1698; Hubeny, Rangamani, Tonni 1306.4324

- $\Xi_{\mathcal{A}} =$ rim of causal wedge
- $\chi = \frac{\Xi_{\mathcal{A}}}{4G_N}$

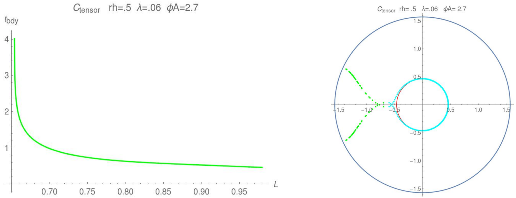


In Gauss-Bonnet causal structure is not generated by null geodesics

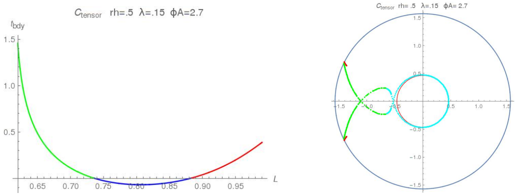
- Use previous framework, null geodesics in effective tensor metric
  - Shoot geodesics from tip of causal diamond ($p^\vee$ or $p^\wedge$)
  - Also geodesics without turning point



Causality OK



Causality violation



Conclusions

- For small black holes hyperbolicity is a more stringent requirement than boundary causality
- Non-monotonic behavior of t_{bdy} associated to bdy causality violation
- CHI.... to be continued

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- Negative λ
- Gao-Wald ?
- interpretation/dual of hyperbolicity ??
- entanglement wedge in higher derivative theories ?