Horizon as Critical Phenomenon

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Goal

A first principle derivation of AdS/CFT correspondence, which allows one to find holographic duals for general QFTs*

*For general QFTs, holographic duals can be non-classical / non-local. However, we would like to find a general prescription to construct them.

Other related approaches

General Connection between holography and RG

- E. T. Akhmedov, Phys. Lett. B 442 (1998) 152
- J. de Boer, E. Verlinde and H. Verlinde, J. High Energy Phys. 08, 003 (2000)
- S. R. Das and A. Jevicki, Phys. Rev. D 68 (2003) 044011.
- R. Gopakumar, Phys. Rev. D 70 (2004) 025009; ibid. 70 (2004) 025010.
- I. Heemskerk, J. Penedones, J. Polchinski and J. Sully, J. High Energy Phys. 10 (2009) 079.
- I. Heemskerk and J. Polchinski, arXiv:1010.1264
- T. Faulkner, H. Liu and M. Rangamani, arXiv:1010.4036.
- R. Koch, A. Jevicki, K. Jin and J. P. Rodrigues, arXiv:1008.0633.
- M. Douglas, L. Mazzucato, and S. Razamat, Phys. Rev. D 83 (2011) 071701.
- R. Leigh, O. Parrikar, A. Weiss, arXiv:1402.1430
- E. Mintun and J. Polchinski, arXiv:1411.3151

Plan

- RG flow as wavefunction collapse
- The collapse is described by holographic dual
- Horizon from dynamical critical point

From action to state

$$\big|S\big\rangle = \int D\phi \; e^{-S[\phi]} \big|\phi\big\rangle,$$

$$\big\langle \phi' \big| \phi \big\rangle = \prod_i \delta(\phi_i' - \phi_i)$$

- An action of QFT in D-dimensional space defines a D-dimensional quantum state
- The Boltzmann weight becomes wavefunction

Sources as variational parameters

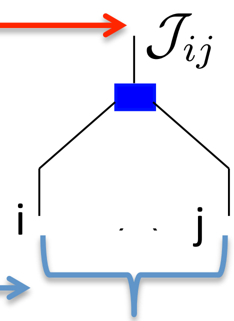
$$S = -\mathcal{J}^M \mathcal{O}_M$$

$$\big|\{\mathcal{J}\}\big\rangle = \int D\phi \, e^{\mathcal{J}^M \mathcal{O}_M} \big|\phi\big\rangle$$

- State can be labeled by the sources of operators

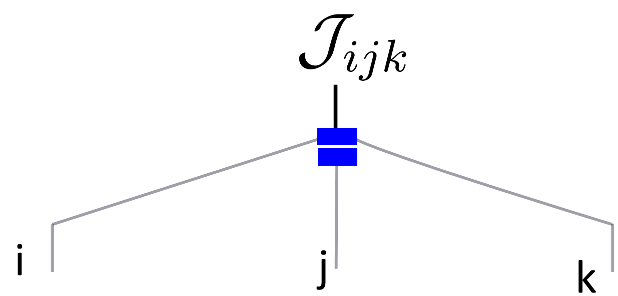
Tensor representation

$$e^{\boxed{\mathcal{J}_{ij}}\boxed{\phi_i\phi_j}}$$



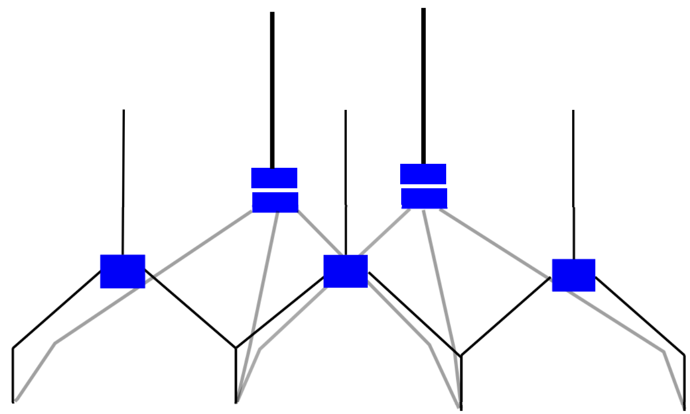
In general, O_M depends on multiple points in spacetime (e.g. bi-local operator in vector model, Wilson loop in gauge theory)

$$e^{\mathcal{J}_{ijk}(\phi_i\phi_j)(\phi_j\phi_k)}$$



O_M can be composite of multiple operators

Tensor representation



$$\big|\{\mathcal{J}\}\big\rangle \;=\; \int D\phi\; e^{\mathcal{J}^M\mathcal{O}_M}\big|\phi\big\rangle$$

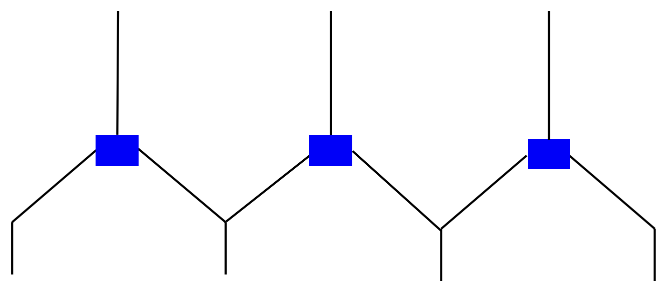
- Local action generates states given by a product of local tensors
- They are over-complete

Single-trace operator

$$\mathcal{O}_M = \sum c_M^{n_1,n_2,..} O_{n_1} O_{n_2} ..$$

- Minimal set of operators of which all singlet operators can be written as polynomial

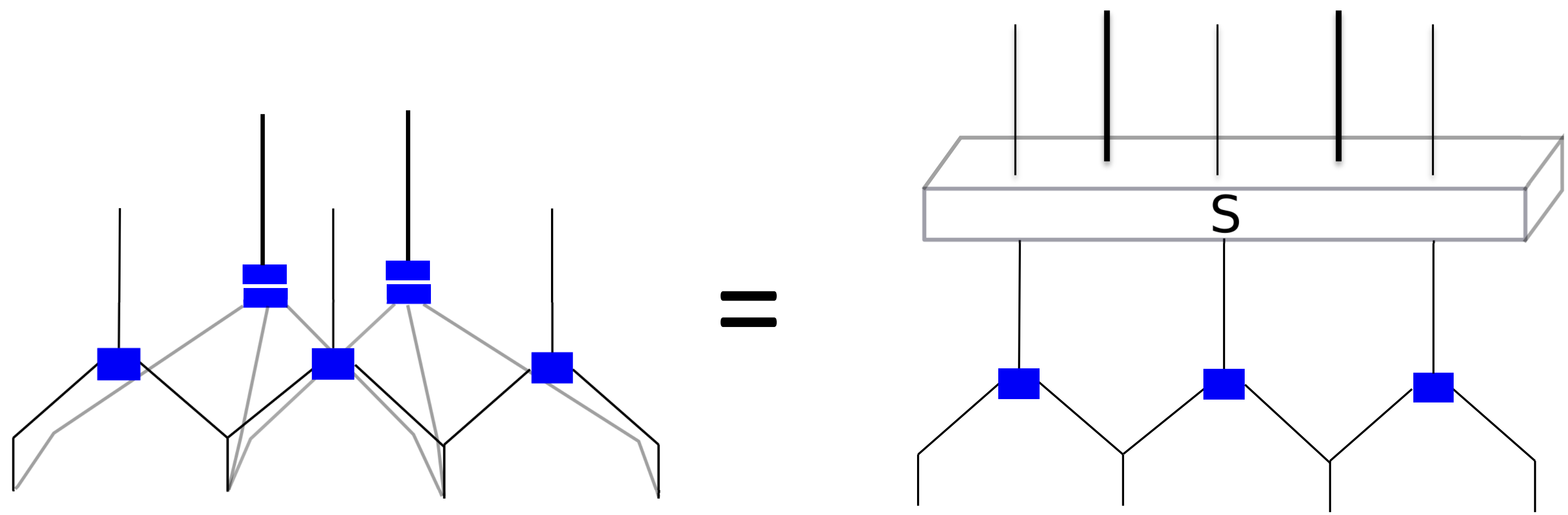
States generated from single-trace operators form a complete basis



$$|j\rangle = \int D\phi \; e^{j_n O_n} |\phi\rangle$$

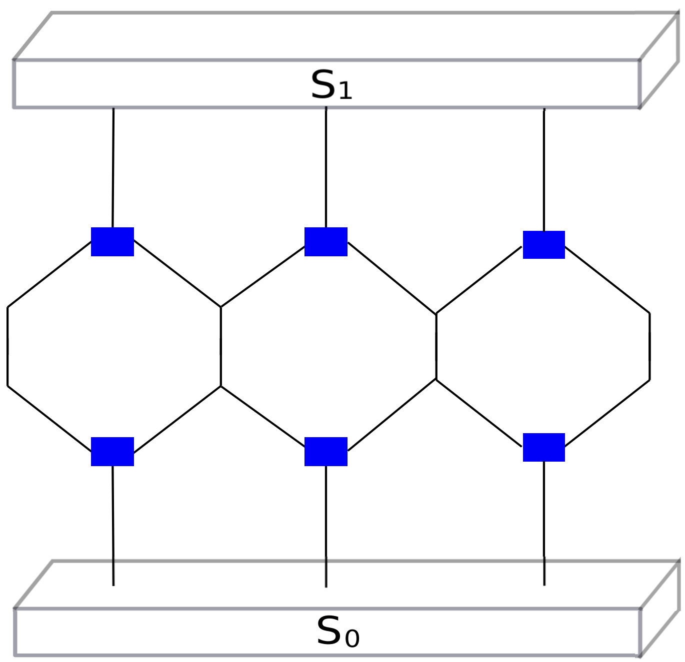
States generated from single-trace operators form a complete basis

$$\int D\phi \; e^{\sum_k \mathcal{J}^{n_1,n_2,..,n_k} O_{n_1} O_{n_2} .. O_{n_k}} \; |\phi\rangle = \int Dj \; \boxed{\Psi_S(\mathcal{J}, j)} \; |j\rangle$$



Partition function is an overlap between states

$$Z = \int D\phi \; e^{-(S_0+S_1)} = \langle S_0^* | S_1 \rangle$$



$$|S_0\rangle = \int D\phi \; e^{-S_0[\phi]} |\phi\rangle$$

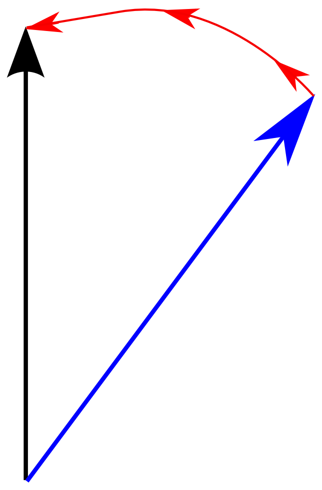
$$|S_1\rangle = \int D\phi \; e^{-S_1[\phi]} |\phi\rangle$$

RG flow as wave-function collapse

$$Z = \langle S_0 | S_1 \rangle = \langle S_0 | e^{-dz\hat{H}} | S_1 \rangle = \langle S_0 | S_1 + \delta S_1 \rangle$$

- $|S_0\rangle$ is the ground state of H^+ with zero energy
- H acting on $|S_1\rangle$ generates RG flow

$$Z = \langle S_0 | e^{-z\hat{H}} | S_1 \rangle$$



Example : Wilson-Polchinski RG equation

$$S_0 = \frac{1}{2}\int d^D k \; G_\Lambda^{-1}(k)\phi_k\phi_{-k}$$

$$S_1 = \text{ interactions}$$

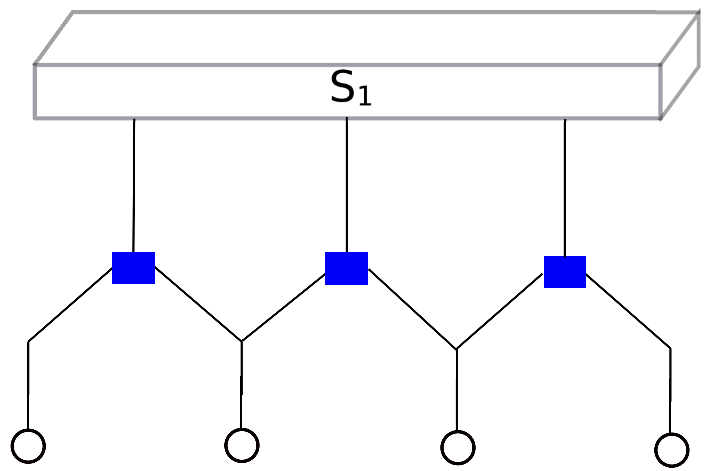
$$e^{-(S_1+\delta S_1)} = \left\langle \phi \middle| e^{-dz\hat{H}} \middle| S_1 \right\rangle$$

$$\hat{H} = \int dk \left[\frac{\tilde{G}(k)}{2}\hat{\pi}_k\hat{\pi}_{-k} - i\left(\frac{D+2}{2}\hat{\phi}_k + k\partial_k\hat{\phi}_k\right)\hat{\pi}_{-k}\right]$$

$$\tilde{G}(k) = \frac{\partial G_\Lambda(k)}{\partial \ln \Lambda}$$

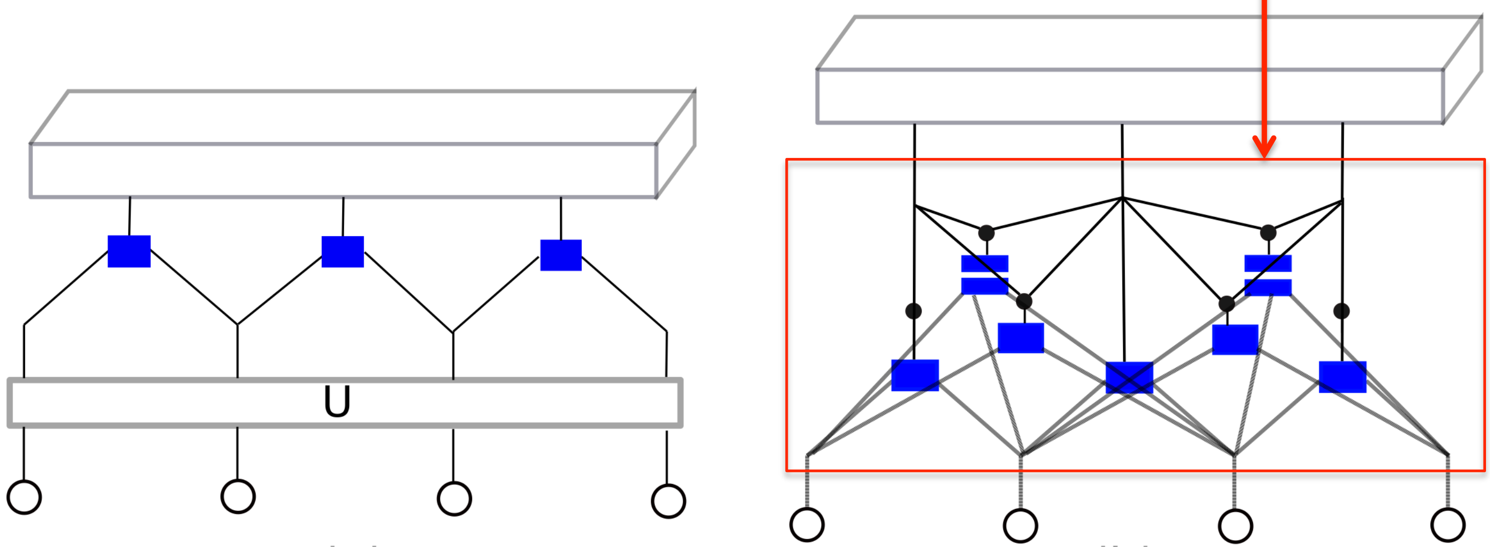
Direct product state for the reference state (tentative IR fixed point)

$$Z = \int Dj^{(0)} \ \left\langle S_0^* \middle| j^{(0)} \right\rangle \Psi(j^{(0)})$$



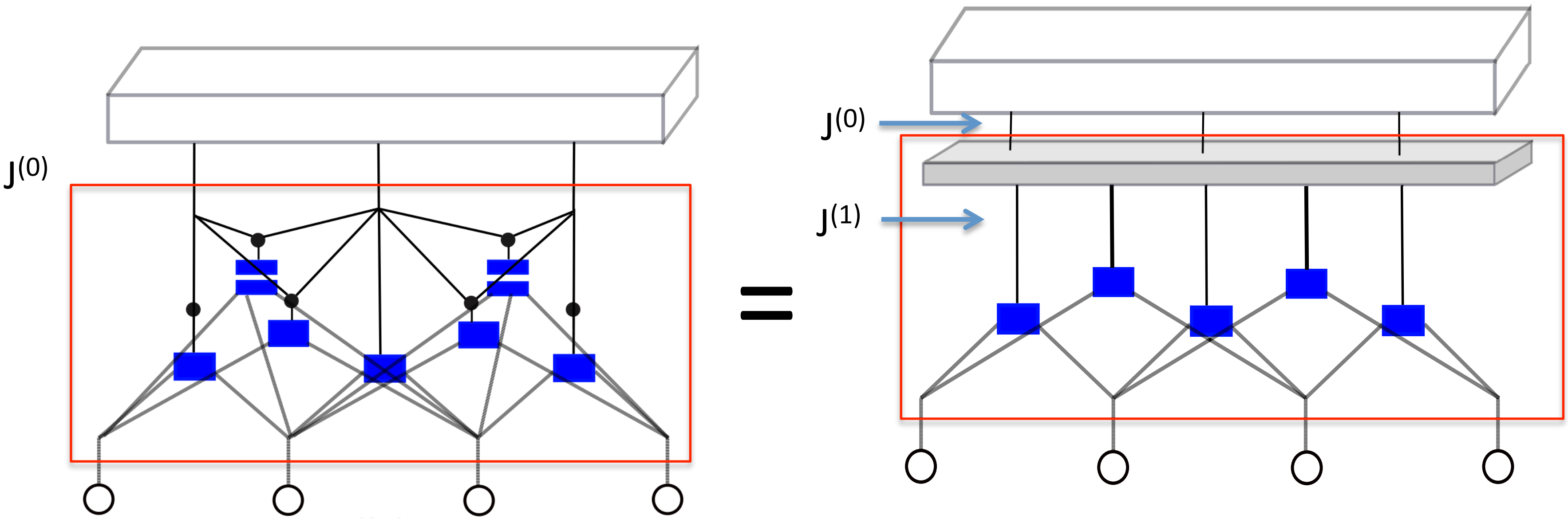
Coarse graining

$$Z = \int Dj^{(0)} \, \langle S_0^* | e^{-dz\hat{H}} | j^{(0)} \rangle \Psi(j^{(0)})$$

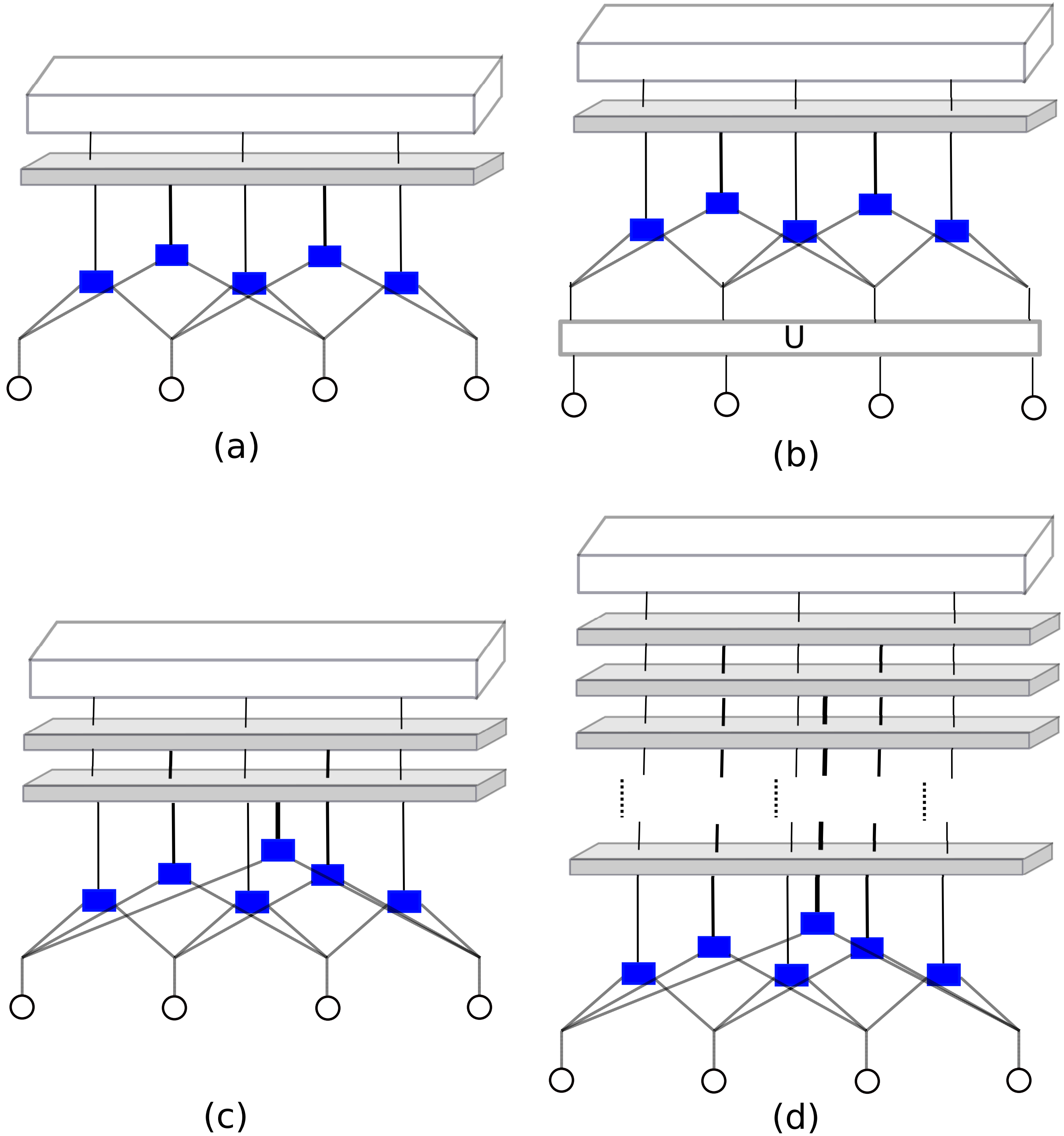


Quantum RG

$$e^{-dz\hat{H}}\left|j^{(0)}\right\rangle = \int Dj^{(1)}\; e^{-j_n^{*(1)}\left(j^{(1)n}-j^{(0)n}\right)-dz\mathcal{H}\left[j^{*(1)},j^{(0)}\right]}\left|j^{(1)}\right\rangle$$



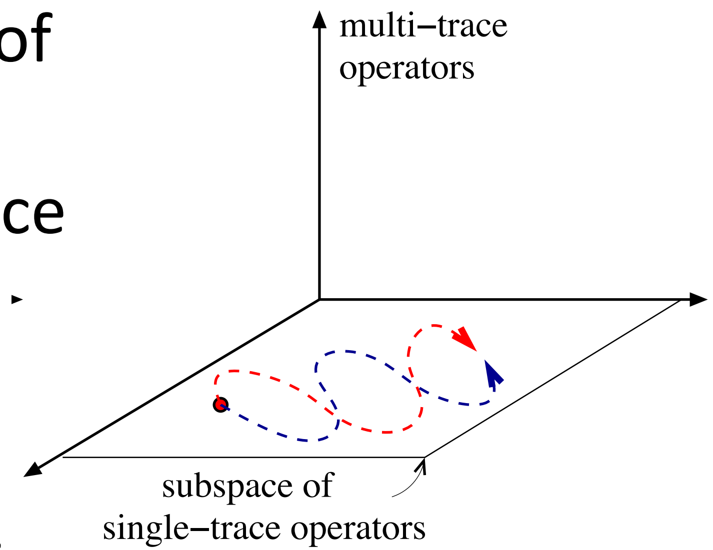
- State with multi-trace tensors can be written as a linear superposition of single-trace states
- Non-local single-trace tensors are generated


U
(a)
(b)
(c)
(d)

Quantum RG

$$Z = \int Dj^{'} Dj Dj(z) \;\; \Psi_0^*(j^{'}) \; e^{-\int dz (j^* \partial_z j + \mathcal{H}[j^*, j])} \Psi_1(j) \Big|_{j(0)=j, j(z)=j^{'}}$$

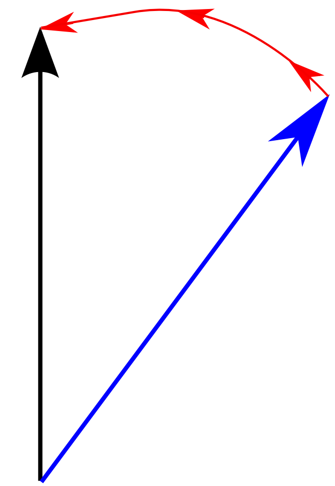
- The RG flow is confined to the space of single-trace sources
- Sum over all RG path in the single-trace space
- Single-trace sources are promoted to quantum operators $[j^n, j_m^\dagger] = \delta_m^n$
- Quantum RG to Wilsonian RG is what quantum computer is to classical computer



Further comments

- The bulk tensor network involves single-trace tensors of all sizes (no pre-assigned local structure) : kinematic non-locality is a necessary condition for diffeomorphism invariance in the bulk
- The bulk theory include dynamical gravity : the source for single-trace energy momentum tensor (metric) gets promoted to dynamical variables
- Regularization of quantum gravity boils down to regularization of QFT

Question



Is the projection always smooth ?

Answer : It depends on S_0 and S_1.

- If the full theory S_0+S_1 is in the same phase as $S_{0,}$ $|S_1>$ is smoothly projected.
- Otherwise, $e^{-z\,H}|S_1>$ undergoes a phase transition as a function of z

Example : Vector model

$$\mathcal{S} = \int d^D x \left[|\nabla \vec{\phi}|^2 + m^2 |\vec{\phi}|^2 + \frac{\lambda}{N} (|\vec{\phi}|^2)^2 \right]$$

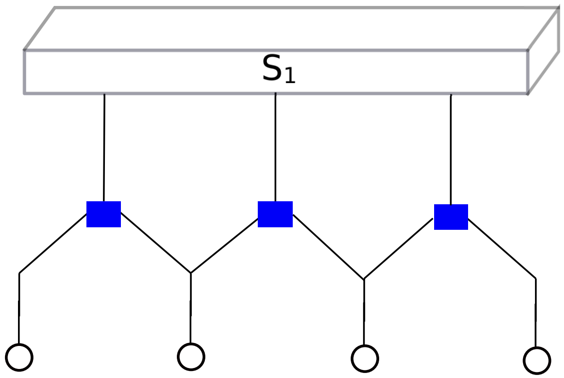
Lattice Regularization :

$$S_0 \;=\; m^2 \sum_i (\boldsymbol{\phi}_i^* \cdot \boldsymbol{\phi}_i)$$

$$S_1 \;=\; -\sum_{ij} t_{ij}^{(0)} \left(\boldsymbol{\phi}_i^* \cdot \boldsymbol{\phi}_j \right) + \frac{\lambda}{N} \sum_i (\boldsymbol{\phi}_i^* \cdot \boldsymbol{\phi}_i)^2$$

Example : Vector model

$$Z = \langle S_0 | t^{(0)} \rangle$$



Gapped phase (direct product state)

$$|S_0\rangle = \int D\boldsymbol{\phi}\ e^{-m^2 \sum_i \boldsymbol{\phi}_i^* \cdot \boldsymbol{\phi}_i} |\boldsymbol{\phi}\rangle,$$

Deformation to the gapped fixed point (entangled state)

$$|t^{(0)}\rangle = \int D\boldsymbol{\phi}\ e^{\sum_{ij} t_{ij}^{(0)} \boldsymbol{\phi}_i^* \cdot \boldsymbol{\phi}_j - \frac{\lambda}{N} \sum_i (\boldsymbol{\phi}_i^* \cdot \boldsymbol{\phi}_i)^2} |\boldsymbol{\phi}\rangle$$

Hamiltonian

$$\hat{H} = \sum_i \left[\frac{2}{m^2} \boldsymbol{\pi}_i \cdot \boldsymbol{\pi}_i^* + i(\boldsymbol{\phi}_i \cdot \boldsymbol{\pi}_i + \boldsymbol{\phi}_i^* \cdot \boldsymbol{\pi}_i^*) \right]$$

- H is not Hermitian, but has real eigenvalues (related to Hermitian through a similarity transformation)
- $|S_0\rangle$ is the ground state of H^+
- e^{-zH} gradually removes entanglement* in $|t^{(0)}\rangle$

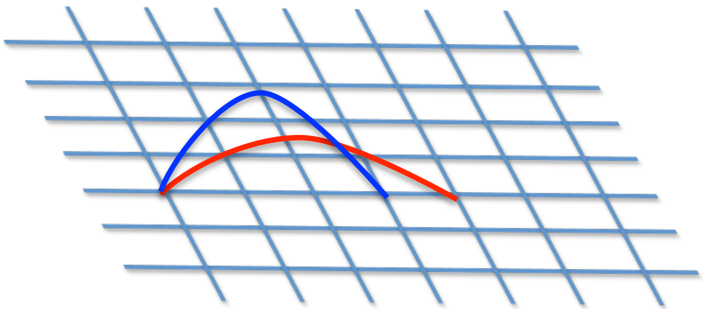
* Entanglement in spacetime

Bulk Hamiltonian (in a fixed gauge)

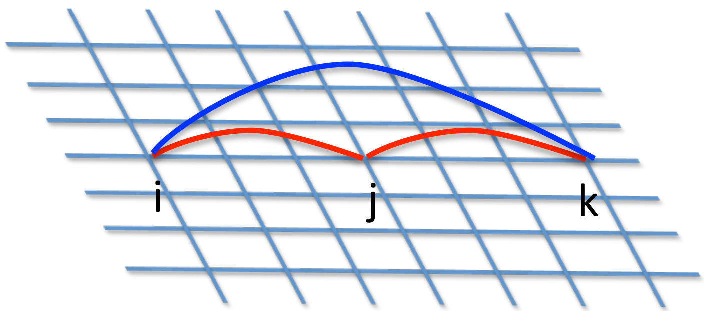
$$\hat{\mathcal{H}} = \sum_i \left[-\frac{2}{m^2} t_{ii} + \frac{4\lambda\left(1+\frac{1}{N}\right)}{m^2} t^\dagger_{ii} - 4\lambda\left(t^\dagger_{ii}\right)^2 - \frac{8\lambda^2}{m^2}\left(t^\dagger_{ii}\right)^3 \right] + \sum_{ij}\left[2 + \frac{4\lambda}{m^2}(t^\dagger_{ii} + t^\dagger_{jj})\right] t^\dagger_{ij}\; t_{ij} - \frac{2}{m^2}\sum_{ijk}\left[t^\dagger_{kj} t_{ki} t_{ij}\right]$$

- t^+_{ij} (t_{ij}) creates (annihilates) a quantum of connectivity
- The Hamiltonian describes evolution of quantum geometry in the bulk

Background independence



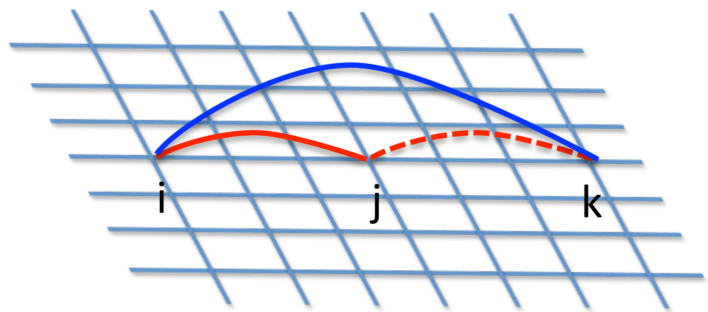
$$t^{\dagger}_{ik} t_{ij}$$



$$t^{\dagger}_{ik} t_{ij} t_{jk}$$

- There is no bare kinetic term for the bi-local object
- No pre-imposed background

Background independence



$$t_{ik}^{\dagger} t_{ij} t_{jk} \rightarrow t_{ik}^{\dagger} t_{ij} < t_{jk} >$$

- t_{ij} can move only in the presence of condensate
- The condensate, which is dynamical, determines the geometry on which t_{ij} propagates

Saddle point approximation

- In the large N limit, semi-classical RG path dominates the partition function
- At the saddle point, $t_{ij} \to \bar{t}_{ij}, \quad t^*_{ij} \to \bar{p}_{ij}$

$$\begin{aligned}
\partial_z \bar{t}_{ij} \quad &= -2\Big\{ \tfrac{2\lambda\,\delta_{ij}}{m^2} - \delta_{ij}\left[4\lambda + \tfrac{12\lambda^2}{m^2}\bar{p}_{ii}\right]\bar{p}_{ii} + \tfrac{2\lambda\,\delta_{ij}}{m^2}\textstyle\sum_k \left(\bar{t}_{ik}\bar{p}_{ik} + \bar{t}_{ki}\bar{p}_{ki}\right) \\
&\qquad + \left[1 + \tfrac{2\lambda}{m^2}\left(\bar{p}_{ii} + \bar{p}_{jj}\right)\right]\bar{t}_{ij} - \tfrac{1}{m^2}\textstyle\sum_k \bar{t}_{ik}\bar{t}_{kj} \Big\}, \\
\partial_z \bar{p}_{ij} \quad &= 2\left\{ -\tfrac{\delta_{ij}}{m^2} + \left[1 + \tfrac{2\lambda}{m^2}\left(\bar{p}_{ii} + \bar{p}_{jj}\right)\right]\bar{p}_{ij} - \tfrac{1}{m^2}\textstyle\sum_k \left(\bar{p}_{ik}\bar{t}_{jk} + \bar{t}_{ki}\bar{p}_{kj}\right) \right\}
\end{aligned}$$

Exact solution :

$$\begin{aligned}
\bar{T}_q(z) \;&=\; \frac{2\lambda}{m^2} + m^2 + \frac{2\lambda}{m^2}e^{-2z}(m^2\bar{p}_0(0) - 1) - m^2\frac{\delta^2 + q^2}{(1 - e^{-2z})(q^2 + \delta^2) + m^2 e^{-2z}}, \\
\bar{P}_q(z) \;&=\; \frac{e^{-2z}}{q^2 + \delta^2} + \frac{1 - e^{-2z}}{m^2}
\end{aligned}$$

Metric

- Fluctuations away from saddle point

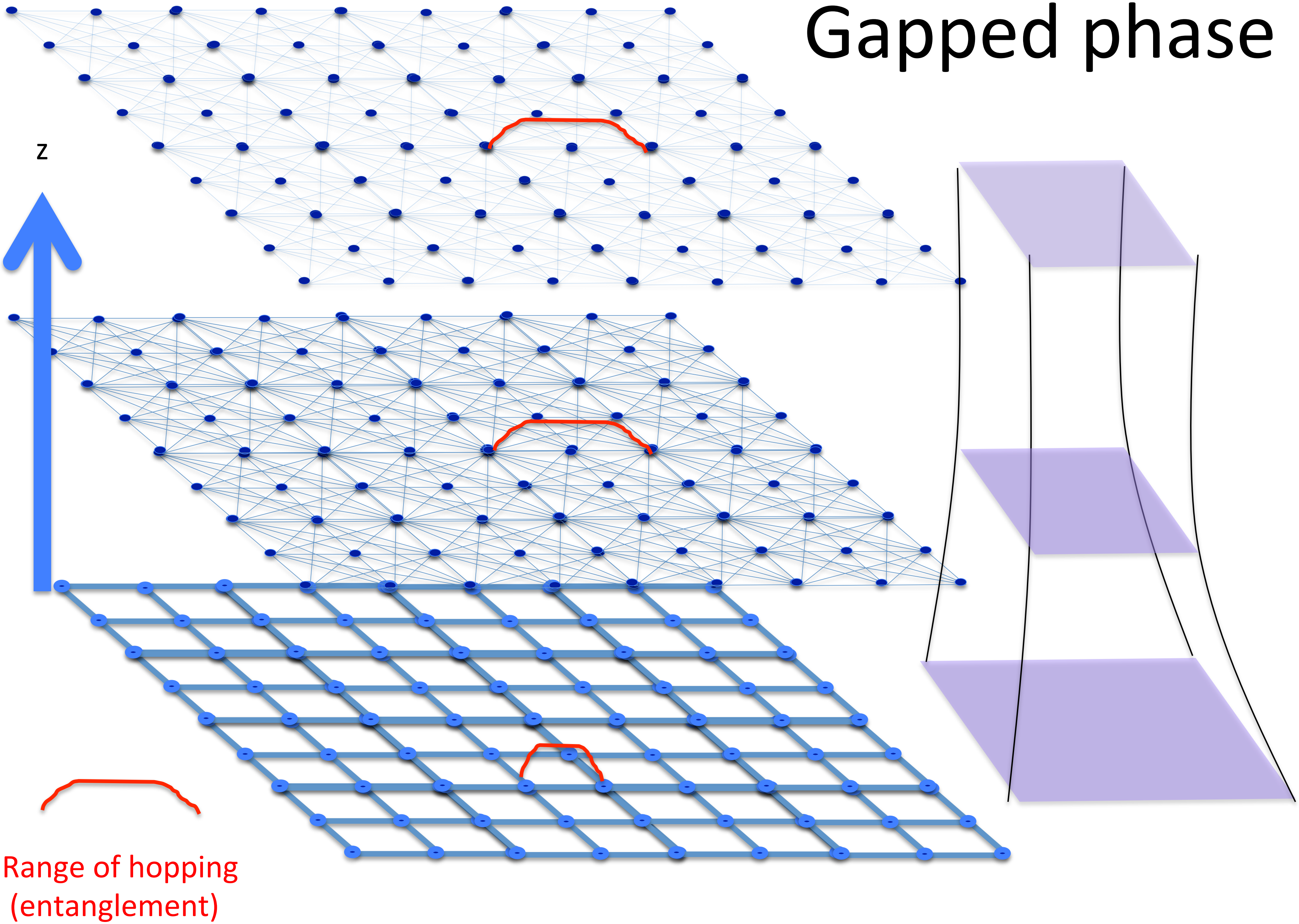
$$\tilde{t}_{ij} = t_{ij} - \bar{t}_{ij}$$

- Anti-symmetric component obeys a simple diffusive equation in the bulk

$$\tilde{t}^A_{ij} = \tilde{t}_{ij} - \tilde{t}_{ji}$$

$$\left(m\sqrt{g^{zz}}\partial_z - g^{\mu\nu}\partial_\mu\partial_\nu - g^{\mu\nu}\partial^{'}_\mu\partial^{'}_\nu + ...\right)\tilde{t}^A(x,x^{'},z) = 0$$

Gapped phase



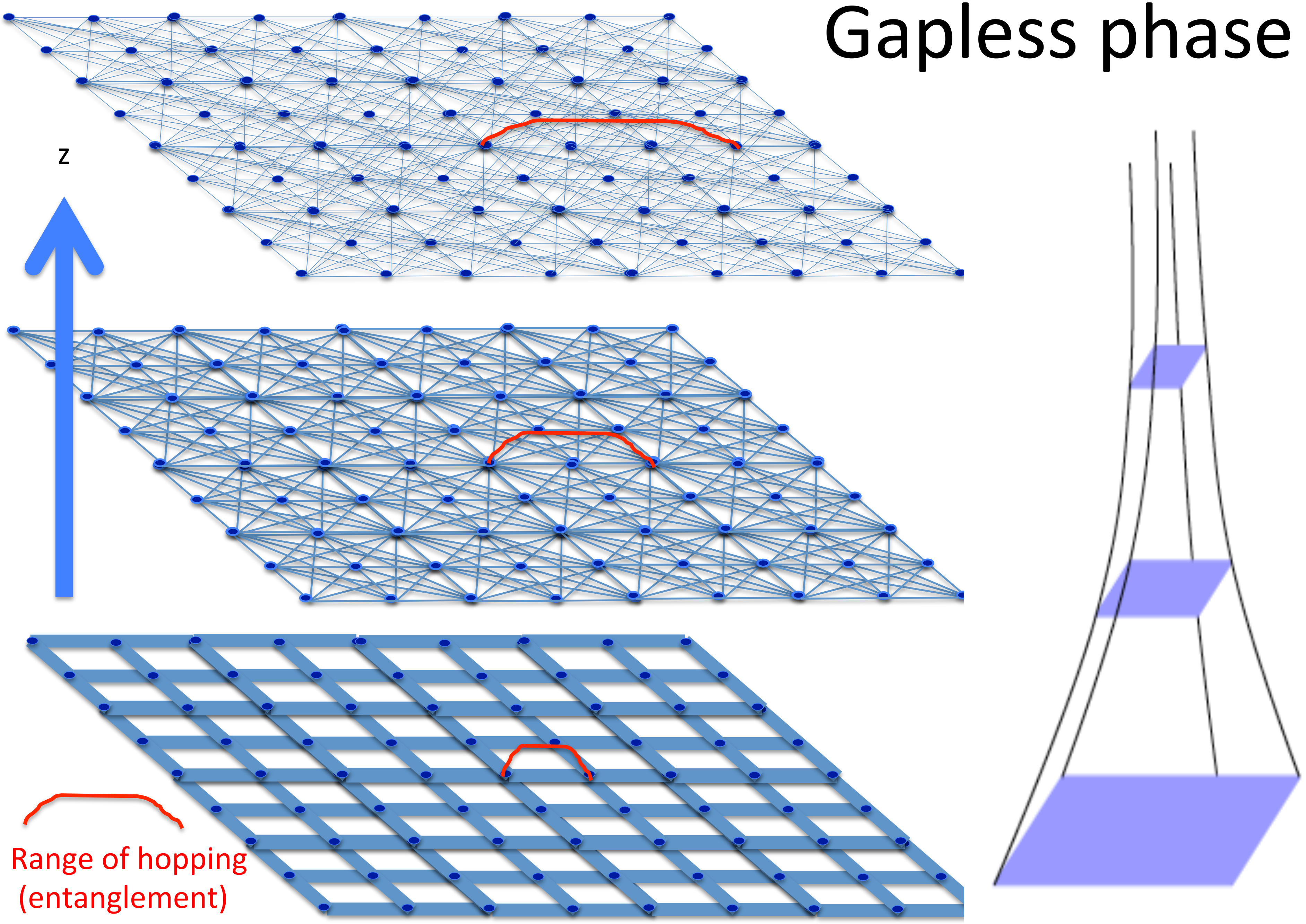
Gapped phase

- The range of entanglement (hopping) saturates in the large z limit
- The strength of hopping (entanglement) decays exponentially in z
- $e^{-z H}$ $|S_1>$ is smoothly projected to the direct product state in the large z limit
- The bulk terminates at a finite proper distance
- The proper distance measures the complexity : # of RG steps needed to remove all entanglement

[Susskind]

$$ds^2 = \left(\frac{1}{1 + \left(\frac{\delta}{m} e^z\right)^2} \right)^2 \frac{dz^2}{m^2} + \left(\left(\frac{\delta}{m}\right)^2 + e^{-2z} \right) \sum_{\mu=0}^{D-1} dx^\mu dx^\mu.$$

Gapless phase



Gapless phase

- The range of entanglement (hopping) keep increasing with increasing z
- $e^{-z\,H}\,|S_1\rangle$ can not be smoothly projected to the direct product state in the large z limit
- In the large z limit, the range of entanglement diverges : critical point -> Poincare horizon

$$ds^2 = \frac{dz^2}{m^2} + e^{-2z} \sum_{\mu=0}^{D-1} dx^\mu dx^\mu$$

- In metallic phase, horizon arises at finite z

[Q. Hu, SL, to appear]

Summary

- RG flow is a gradual wavefunction collapse
- The process of collapse is described by dual holographic theory via quantum RG
- Obstruction to smooth projection of one phase to another phase manifests itself as a horizon in the bulk

$$e^{-zH}\Big|S_1\Big\rangle$$

Dynamical quantum critical point
= Horizon