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Toward Quantum Energy Teleportation in (Holographic) CFTs

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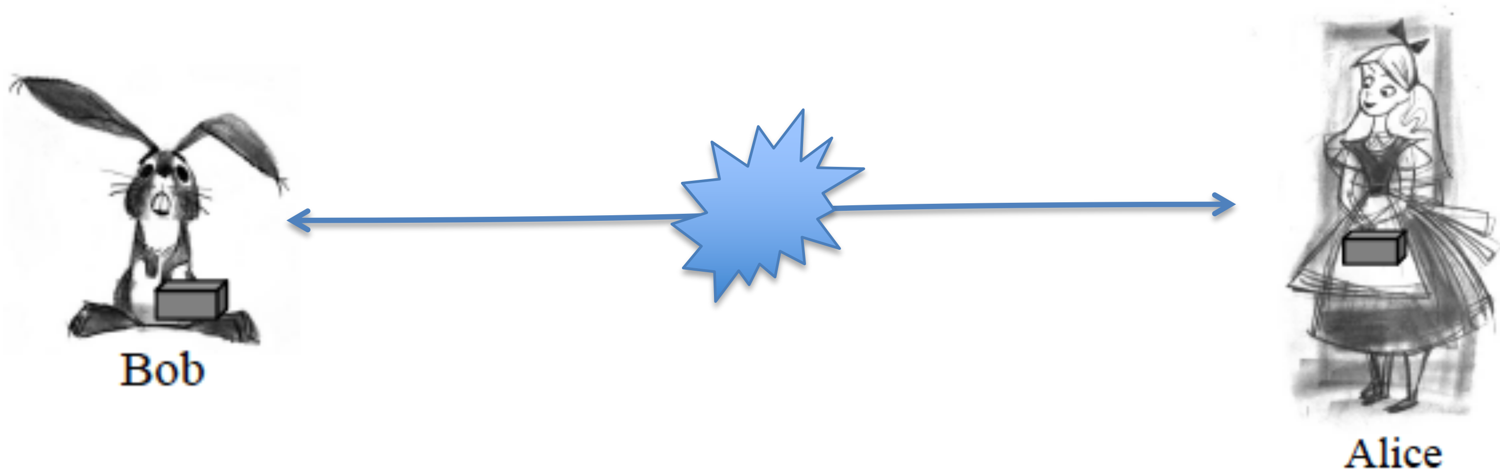
Motivation

- Motivated by the AdS/MERA and Kinematic space(KS)/MERA duality, we now have some way of understanding quantum information process (QIP) from the (AdS or KS) geometric point of view.
- Quantum state teleportation is the most well-known QIP.
- However, quantum energy teleportation (QET) is the simpler one in the QFT setup.
- We will study the QET for (holographic) CFTs.

Quantum Teleportation

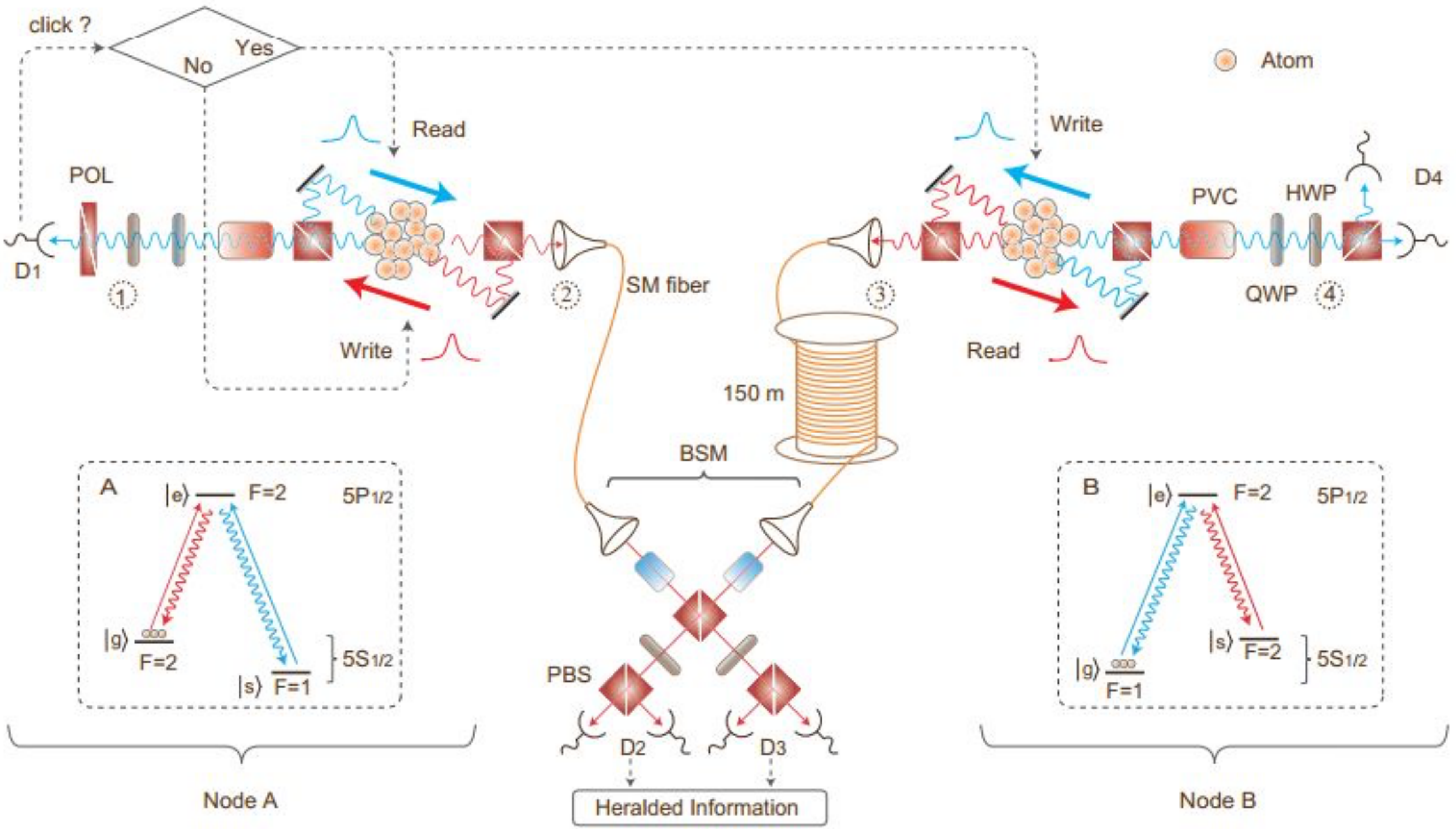


1. Alice and Bob share a pair of qubits in the Bell state. Alice want to send a qubit of unknown state to Bob.
2. Alice performs Bell measurement on her two qubits. This entangles Alice's 2 qubits and disentangle Bob's qubit from Alice's.
3. Alice sends Bob her measurement outcome by classical communication.
4. Bob performs proper local unitary operation (LU) on his qubit to recover Alice's unknown state.

LOCC

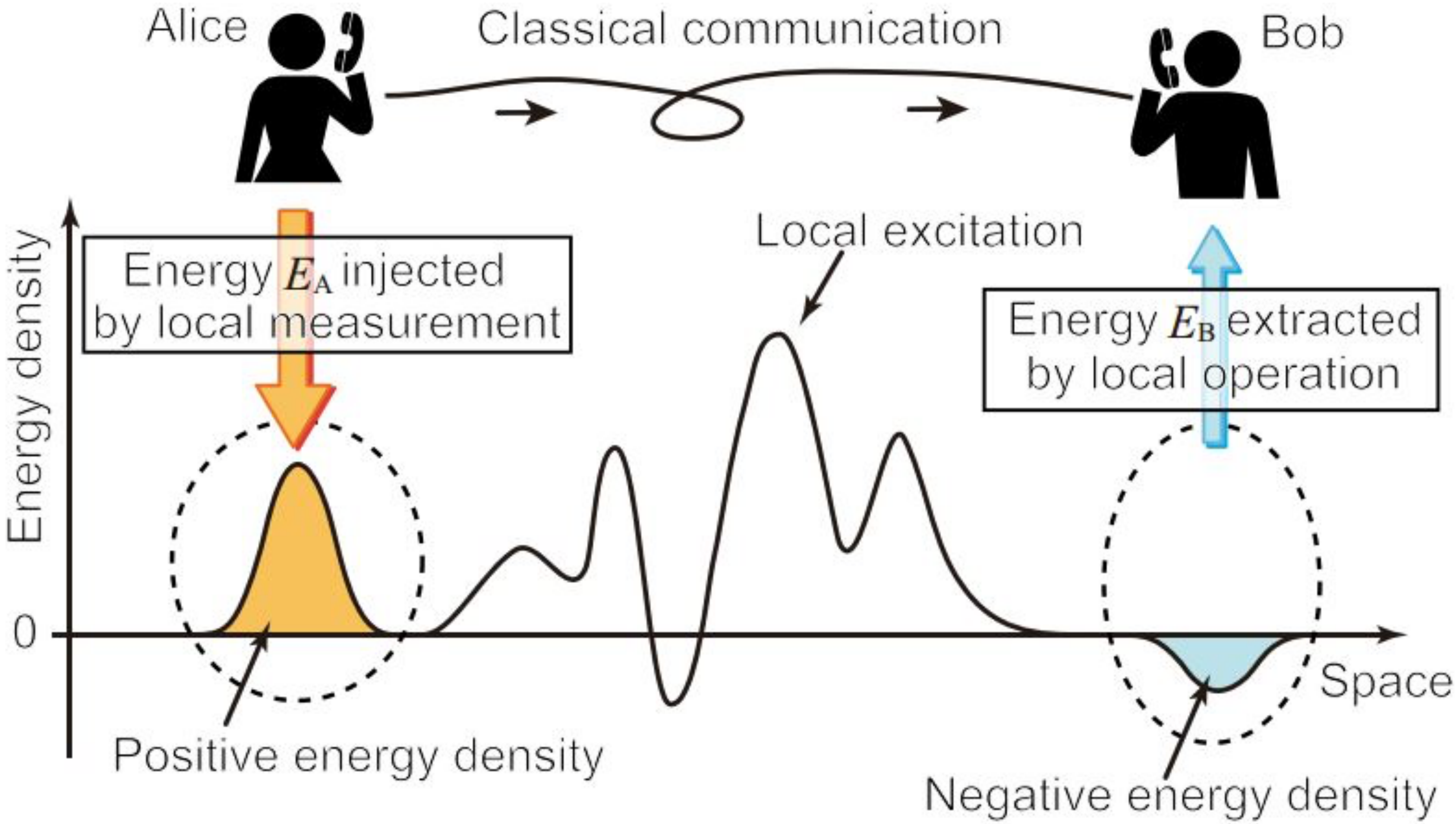
Phys. Rev. Lett. 70, 1895 (1993)

Experimental setup for quantum state teleportation



arXiv:1211.2892

QET (M. Hotta)



arXiv:1305.3955

Qubit model of QET

- Alice and Bob share an entangled ground state $|g\rangle$of following Hamiltonian: $$H_T = H_A + H_B + V$$

$$H_A := h\, \sigma_3 \otimes I_{2\times 2} + \frac{h^2}{\sqrt{h^2+k^2}}\, I_{4\times 4}, \qquad H_B := h\, I_{2\times 2} \otimes \sigma_3 + \frac{h^2}{\sqrt{h^2+k^2}}\, I_{4\times 4}$$

$$V := 2k\, \sigma_1 \otimes \sigma_1 + \frac{2k^2}{\sqrt{h^2+k^2}}\, I_{4\times 4}\ , \qquad \langle g|H_A|g\rangle = \langle g|H_B|g\rangle = \langle g|V|g\rangle = 0$$

1. Alice performs local projective operation (LPO)

$P_A[\alpha] := \frac{1}{2}(I_{4\times 4} + \alpha\, \sigma_1 \otimes \sigma_1)$ on $|g\rangle$, which inject the energy (passivity):

$$E_A[\alpha] = \langle g|P_A[\alpha] H_T P_A[\alpha]|g\rangle = \frac{h^2}{2\sqrt{h^2+k^2}} > 0\ .$$

The post-measurement state is $|M(\alpha)\rangle := \frac{1}{\sqrt{p_A[\alpha]}} P_A[\alpha]|g\rangle$ which is a product state and

$p_A[\alpha] := \langle g|P_A[\alpha]|g\rangle = 1/2$

The average injected energy is $\Delta E_A := \sum_\alpha p_A[\alpha] E_A[\alpha] = E_A[+1]$

Qubit model of QET (Cont’d)

2. Bob perform proper LU to extract energy. In this case, the LU is $U[\theta;\beta] := e^{-i\beta\theta\ I_{2\times 2}\otimes\sigma_2}$

   Note this LU acts only on qubit B.

3. Bob can extract the energy (for single LU)

$$E_B[\alpha,\beta] := E_A[\alpha] - \langle g|U^\dagger[\theta;\beta]P_A[\alpha]H_T P_A[\alpha]U[\theta;\beta]|g\rangle = \frac{\sin\theta[\alpha\beta hk\cos\theta - (h^2+2k^2)\sin\theta]}{\sqrt{h^2+k^2}}\ .$$

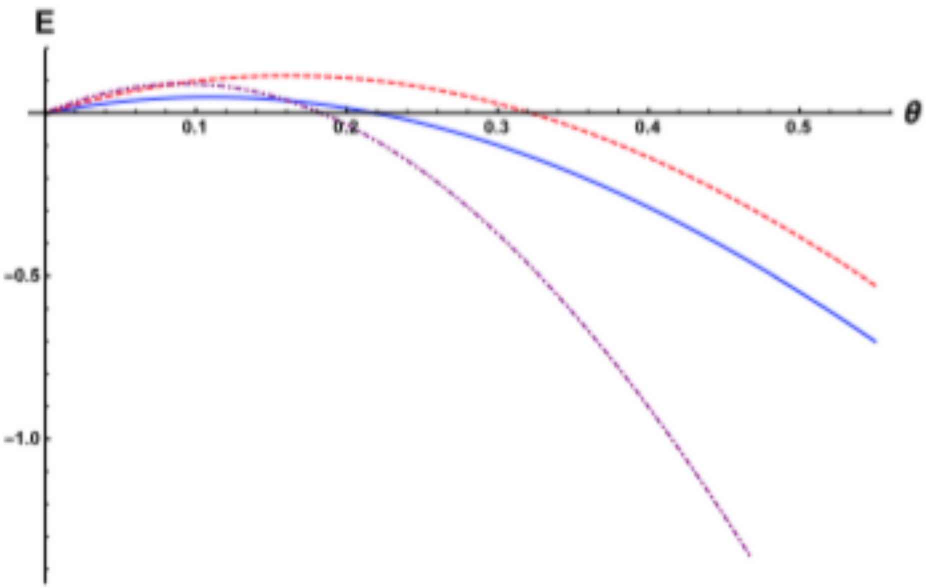
   A. If Alice’s measurement outcome does not feedback to Bob’s LU, then the average extraction energy is negative, i.e.,

$$\Delta E_B|_{\beta=1} := p_A[+1]E_B[+1,+1] + p_A[-1]E_B[-1,+1] = -\frac{(h^2+2k^2)\sin^2\theta}{\sqrt{h^2+k^2}} \leq 0$$

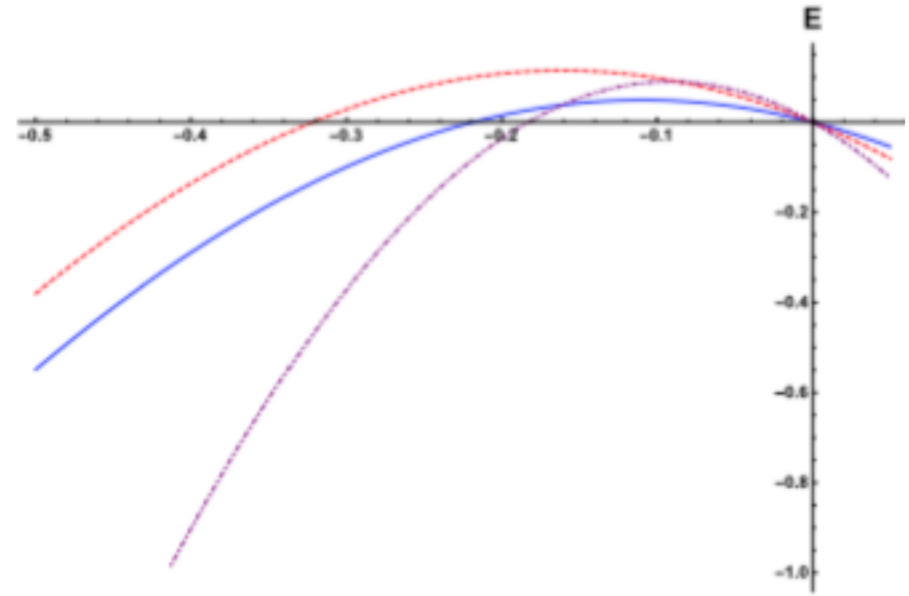
   B. Otherwise, there is a window for the extraction energy to be positive, i.e.,

$$\Delta E_B|_{\alpha\beta=1} := p_A[+1]E_B[+1,+1] + p_A[-1]E_B[-1,-1] = E_B[+1,+1]$$

$$\Delta E_B|_{\alpha\beta=-1} := p_A[+1]E_B[+1,-1] + p_A[-1]E_B[-1,+1] = E_B[+1,-1]$$



$$\Delta E_B|_{\alpha\beta=1} = E_B[\alpha = +1, \beta = +1]$$



$$\Delta E_B|_{\alpha\beta=-1} = E_B[\alpha = +1, \beta = -1]$$

It suggests that one could extract energy even without feedback via LOCC.

Holographic QET --- Scheme

- To realize the QET for (holographic) CFTs, we need to find the corresponding operations for LPO and LU.
- Once these operations are constructed, we just evaluate the corresponding energy density at each step.

- When acting on the ground state with LPO, it injects energy so that the resultant state is an excited state.
- In CFT2 once we know resultant stress tensor after LPO, we can obtain the corresponding Banados' geometry for further holographic QIP such as holographic LU.

$$ds^2 = R^2\Big\{\frac{dz^2}{z^2} + L(w)dw^2 + \overline{L}(\overline{w})d\overline{w}^2 + (\frac{1}{z^2} + z^2 L(w)\overline{L}(\overline{w}))dwd\overline{w}\Big\}$$

$$L(w) := \frac{6}{c}T(w), \qquad \overline{L}(\overline{w}) := \frac{6}{c}\overline{T}(\overline{w}) \qquad c := \frac{3R}{2G_N}$$

arXiv:1604.01772
arXiv:1501.07831

LPO in CFTs

- LPO in QFT is defined as follows: project a local region of a ground state into some particular states, i.e.,

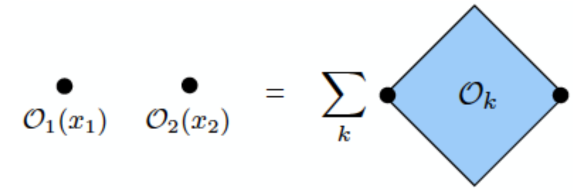
$$\mathcal{P} = \prod_{x \in \mathcal{A}} O(x)|0_x\rangle\langle 0_x|O^\dagger(x) \otimes \prod_{x \in \mathcal{A}^c} I_x$$

- LPO is non-local (but act on finite region) and should also obey the projector conditions, i.e., $\mathcal{P}_i \mathcal{P}_j = \delta_{ij} \mathcal{P}_i$
- One LPO proposal in CFT2 is realized by a conformal map $\xi(w)$ from a slit −q<x<q on C to UHP. This projects the interval into product state of BCFT.
- The Banados' geometry is characterized by

$$T(w) = \frac{c}{12}\{\xi, w\}_S = \frac{c}{8}\frac{q^2}{(w^2 - q^2)^2} > 0$$ (passivity of vacuum!)

arXiv:1604.03110

OPE blocks



- The OPE blocks are kinematic object in CFTs, i.e.,

$$\mathcal{O}_i(x_1)\mathcal{O}_j(x_2) = |x_1 - x_2|^{-\Delta_i - \Delta_j} \sum_{k \in primaries} C_{ijk}\mathcal{B}_k^{ij}(x_1, x_2)$$

- OPE blocks are conformal scalar for $\Delta_i = \Delta_j$.
- In CFT2, it takes the form:

$$\mathcal{B}_k\left(x_1, x_2\right) = \frac{\Gamma\left(2h_k\right)\Gamma(2\bar{h}_k)}{\Gamma(h_k)^2\,\Gamma(\bar{h}_k)^2}\int_{\diamond_{12}} dw\, d\bar{w} \left(\frac{(w - z_1)(z_2 - w)}{z_2 - z_1}\right)^{h_k - 1}\left(\frac{(\bar{w} - \bar{z}_1)(\bar{z}_2 - \bar{w})}{\bar{z}_2 - \bar{z}_1}\right)^{\bar{h}_k - 1}\mathcal{O}_k\left(w, \bar{w}\right)$$

Or, in the formalism of shadow operator:

$$\mathcal{B}_k^{ij}\left(x_1, x_2\right) \propto \int d^d z\, |x_1 - x_2|^{\Delta_i + \Delta_j} \left\langle \mathcal{O}_i\left(x_1\right)\mathcal{O}_j\left(x_2\right)\tilde{\mathcal{O}}_{k\,\mu\nu\ldots}\left(z\right)\right\rangle \mathcal{O}_k^{\mu\nu\ldots}\left(z\right)$$

- The OPE blacks for primaries form a complete set for positive-operator-valued measure (POVM) of quantum measurement theory.

- The stress tensor after LPO is $T(w) = \langle 0|\mathcal{B}_k^\dagger(x_1, x_2)\ \mathcal{T}(w)\ \mathcal{B}_k(x_1, x_2)|0\rangle$

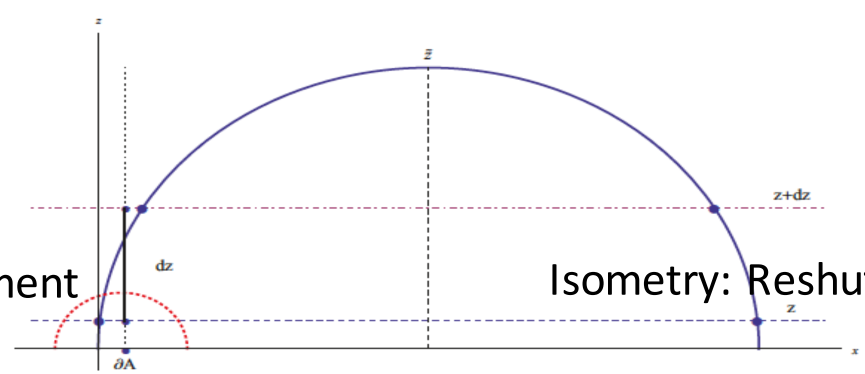
Thanks Bartek Cezch for sharing his insight on this.

LU in (holographic) CFT

- In general, LU should also be realized by a conformal map as for LPO. This is especially the case in realizing the MERA of CFT (arXiv:1510.07637).
- For convenience at this stage, we adopt the Surface/State (SS) duality (arXiv:1506.01353): Different bulk surfaces are related by LU (or conformal map in MERA).
- Motivated by in KS/MERA (arXiv:1512.01548), we can understand the entanglement renormalization in SS duality:

(arXiv:1507.04633)

disentanglement



Isometry: Reshuffle entanglement

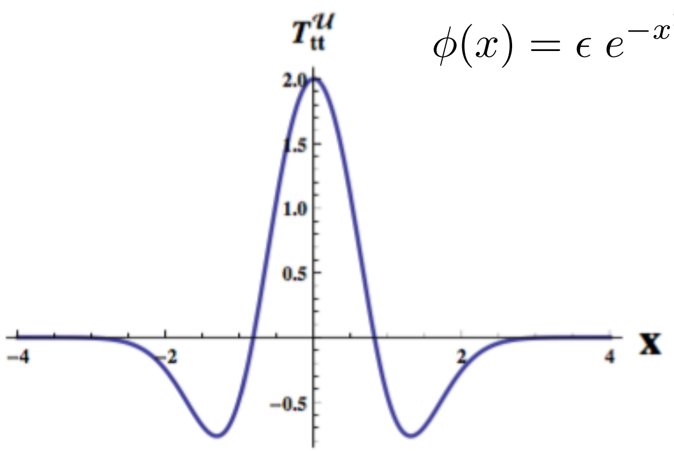
Holographic LU (cont'd)

- At this stage LU is considered to be dual to a local bumpy deformation on the UV cutoff surface:

- We then evaluate the holographic stress tensor on the deformed surface. Unlike the stress tensor for nice UV slice, it diverges even w/o LPO.
- In contrast to LPO, it is not positive-definite. Moreover, this UV piece will be used as the counter term for stress tensor with both LPO & LU.

$$T_{tt}^{\mathcal{U}} = \lim_{\epsilon \to 0} \frac{c}{12} \left[\left(\frac{\phi'}{\epsilon} \right)^2 - 2 \frac{\phi''}{\epsilon} \right]$$

$$\phi(x) = \epsilon \, e^{-x^2}$$



QET for holographic CFT2

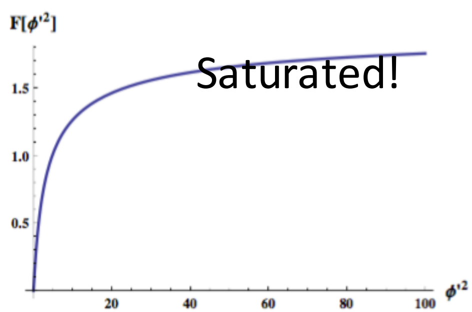
- Let's assume the Banados' geometry after LPO, and then evaluate the stress tensor in this background for the bumpy UV slice.
- Then, subtracting the UV counter term for LU, we obtain the regularized extraction energy density:

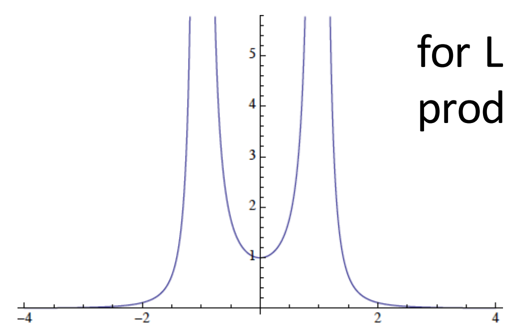
$$\Delta\rho_{\mathcal{B}}^{(reg)} = F\left[\phi'^2\right] T_{tt}^{\mathcal{P}}$$

- This is a relation of linear response, and is positive definite.

$$F\left[\phi'^2\right] := \frac{4\left(-1+\sqrt{1+\phi'^2}\right)+\phi'^2\left(-5+4\sqrt{1+\phi'^2}\right)}{2\left(1+\phi'^2\right)^{3/2}}$$

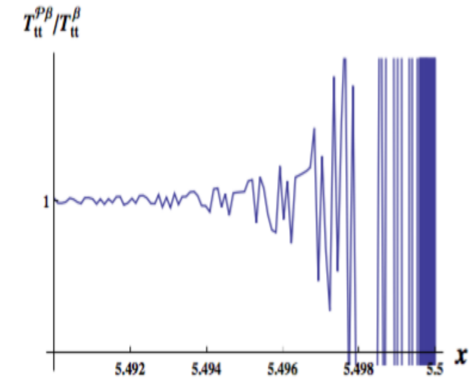
$$T_{tt}^{\mathcal{P}} = \frac{c}{8}\frac{q^2}{(x^2-q^2)^2}$$





for LPO to BCFT product state

Finite T hQET



- Consider hQET in planar BTZ background:

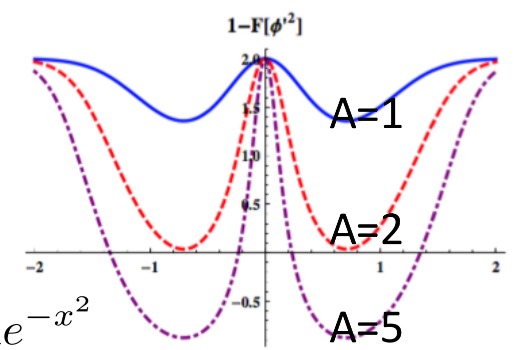
$$ds^2 = \frac{R^2}{z^2}\Big\{ -(1-\frac{\pi^2 z^2}{\beta^2})^2 dt^2 + (1+\frac{\pi^2 z^2}{\beta^2})^2 dx^2 + dz^2 \Big\} \qquad T^{\beta}_{tt} = \frac{c\pi^2}{3\beta^2}.$$

- The stress tensor $T^{\mathcal{P}\beta}_{tt}$ due to LPO is positive & highly fluctuating:
- However, the injected energy due to LPO is smoothly oscillating and positive-definite:

$$T^{Inj,\beta}_{tt} = \frac{3\sin^2\frac{2\pi q}{\beta}}{4\left(\cos\frac{2\pi q}{\beta} - \cos\frac{2\pi x}{\beta}\right)^2} T^{\beta}_{tt}$$

- These feature reflects the underlying state is thermally excited.
- Due to the same cause, Large LU can extract energy.
- As the evaluation of extraction energy density is universal once the LPO geometry is given. Thus, we have

$$\Delta\rho^{(reg)\beta}_{\mathcal{B}} = F\left[\phi'^2\right] T^{\mathcal{P}\beta}_{tt}.$$



$$\phi(x) = Ae^{-x^2}$$

$$T^{(reg)\mathcal{U}\beta}_{tt} := T^{\mathcal{U}\beta}_{tt} - T^{\mathcal{U}}_{tt} = (1 - F\left[\phi'^2\right])T^{\beta}_{tt}$$

Conclusions

1. Unlike the usual QET, in (holographic) CFTs we can extract energy without the need of feedback via CC.

2. The peculiar features of injected energies due to LPO & LU reflect thermal fluctuations. However, the positivity of QET remains.

3. The positivity of extraction energy density could be related to some quantum energy condition.

4. In higher D, the OPE blocks as POVM and LU as conformal map may induce more general QET results.