Holographic quantum error-correcting code - exactly solvable toy models for the AdS/CFT correspondence

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joint work with

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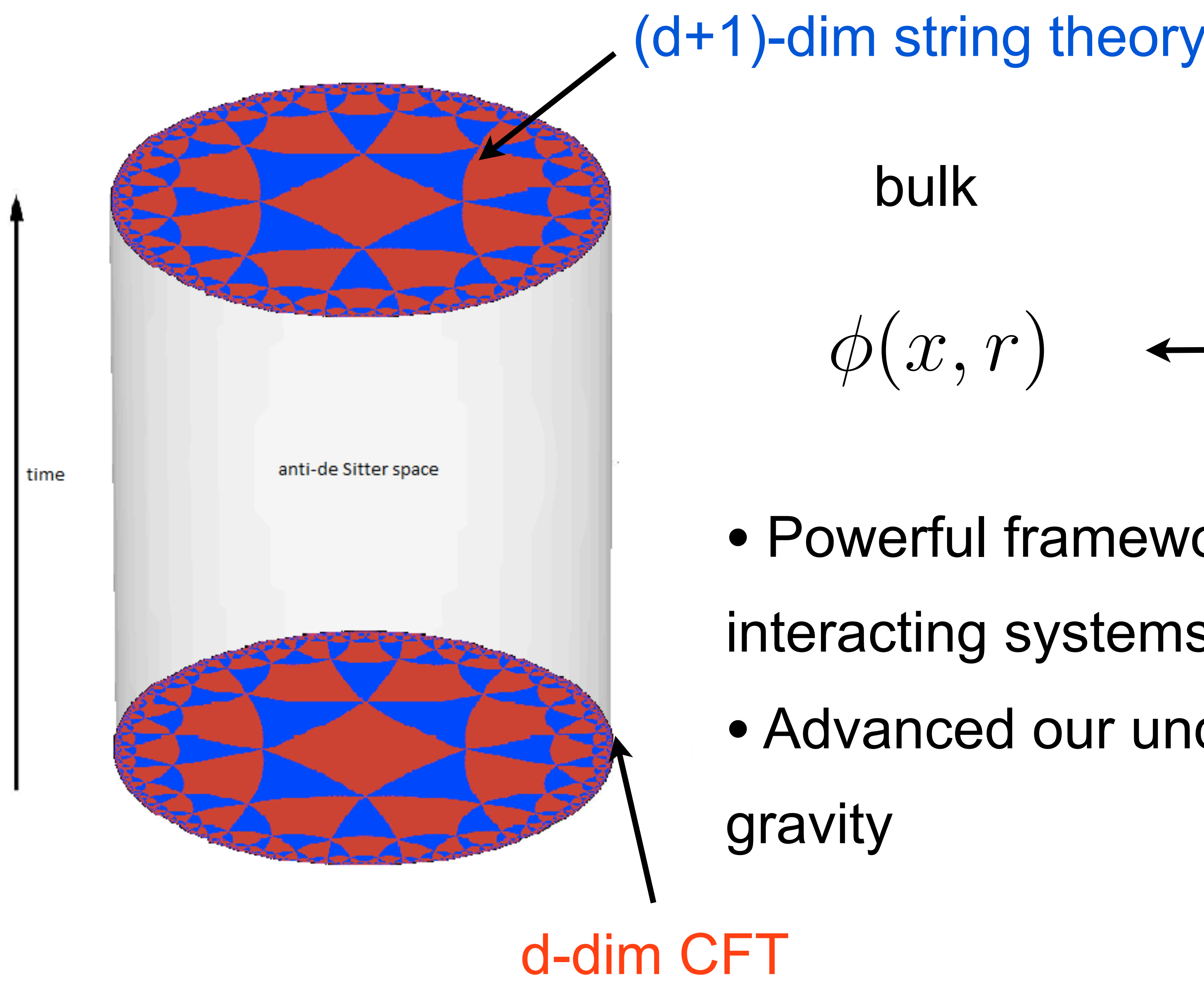
Introduction

AdS/CFT correspondence (Maldacena 98)



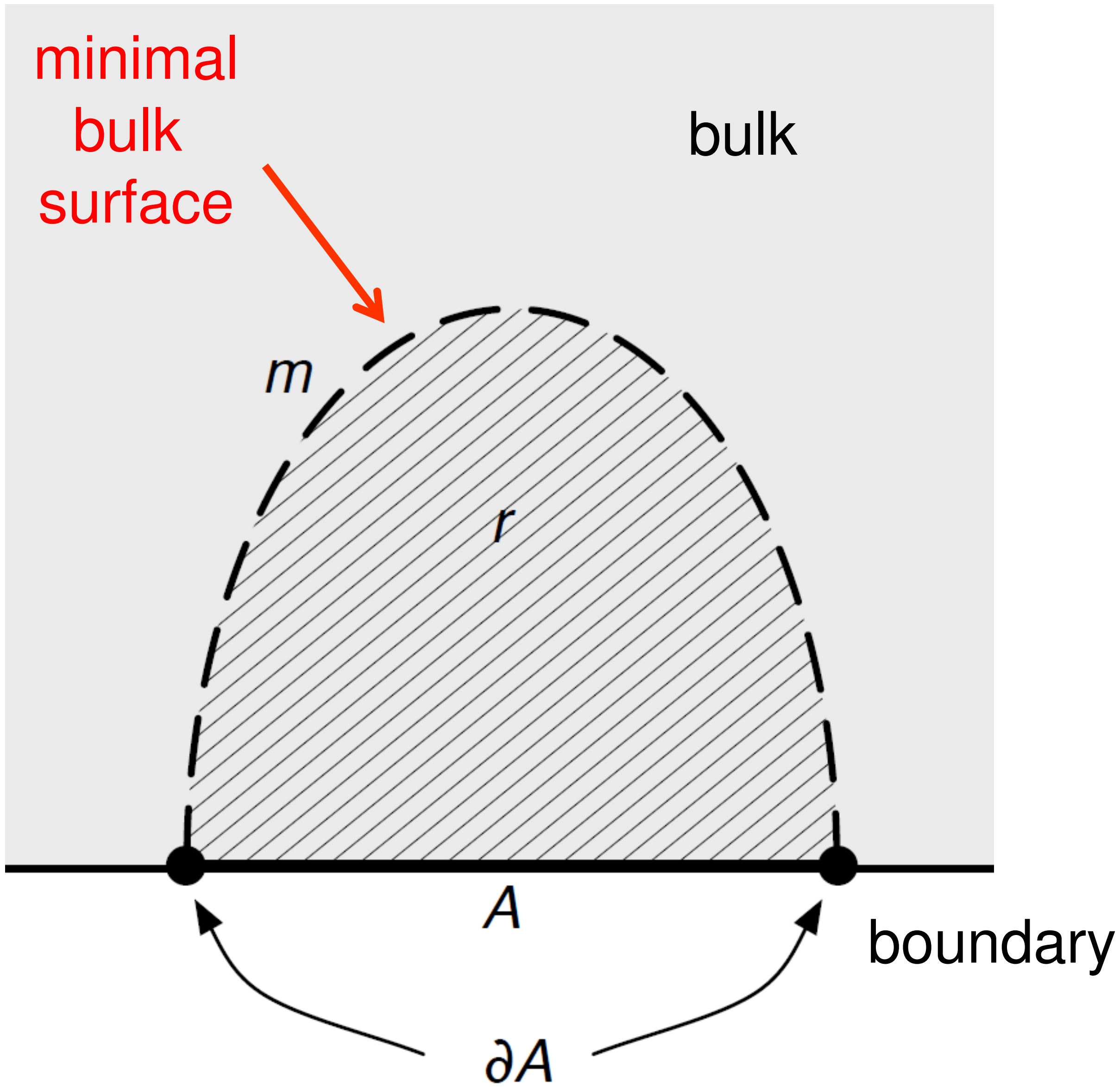
bulk — boundary

$$\phi(x,r) \longleftrightarrow \mathcal{O}(x)$$

- Powerful framework to study strongly-interacting systems
- Advanced our understanding of quantum gravity

Ryu-Takayanagi formula (06)

- Bulk/Boundary duality to Geometry/Entanglement duality



$$S(A) = \frac{1}{4G_N} \min_{\partial m = \partial A} \mathrm{area}(m)$$

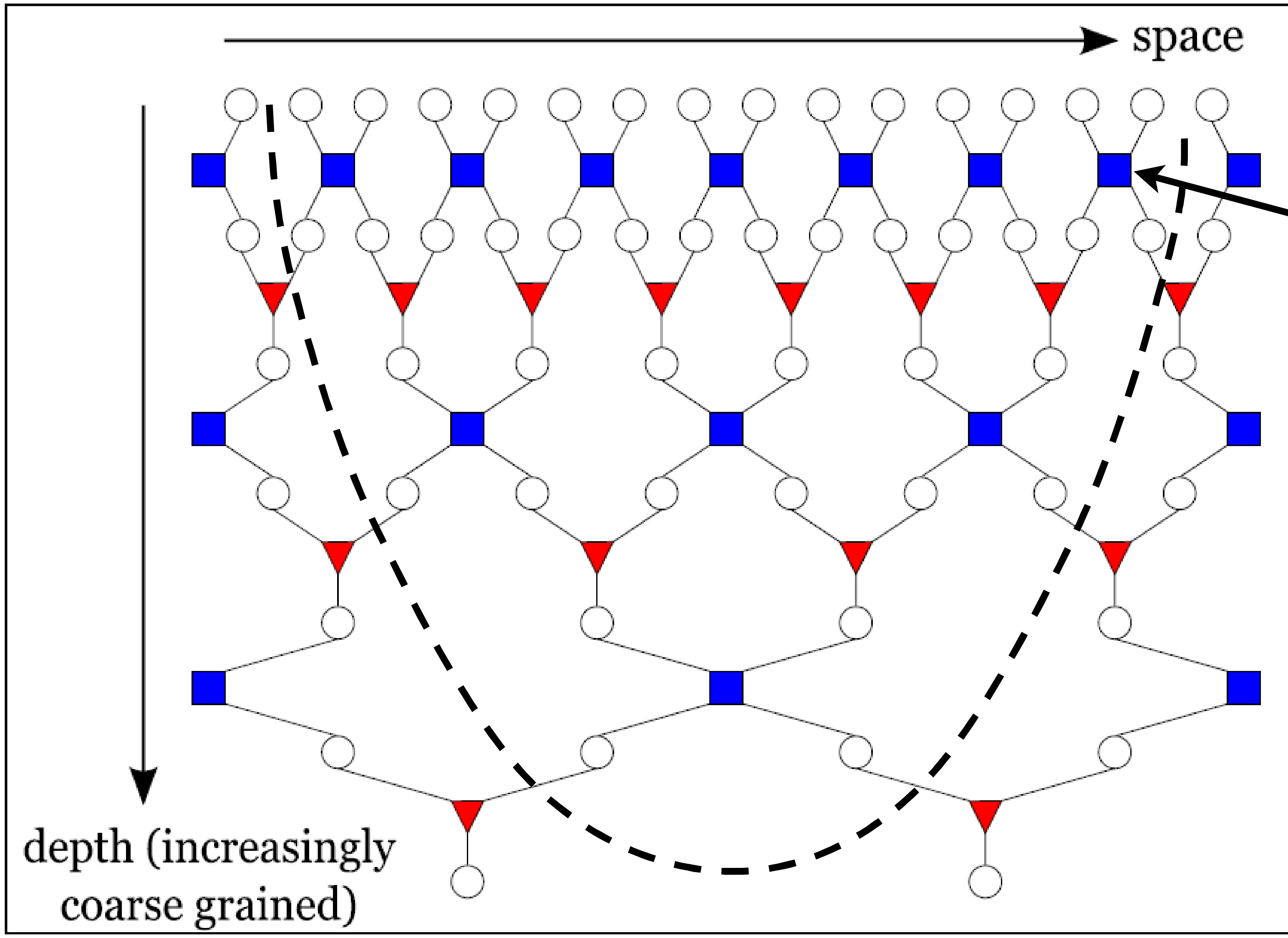
Entanglement

↕

Geometry (Space-time)

MERA (Vidal 07)

- Powerful numerical method to study strongly-correlated systems.

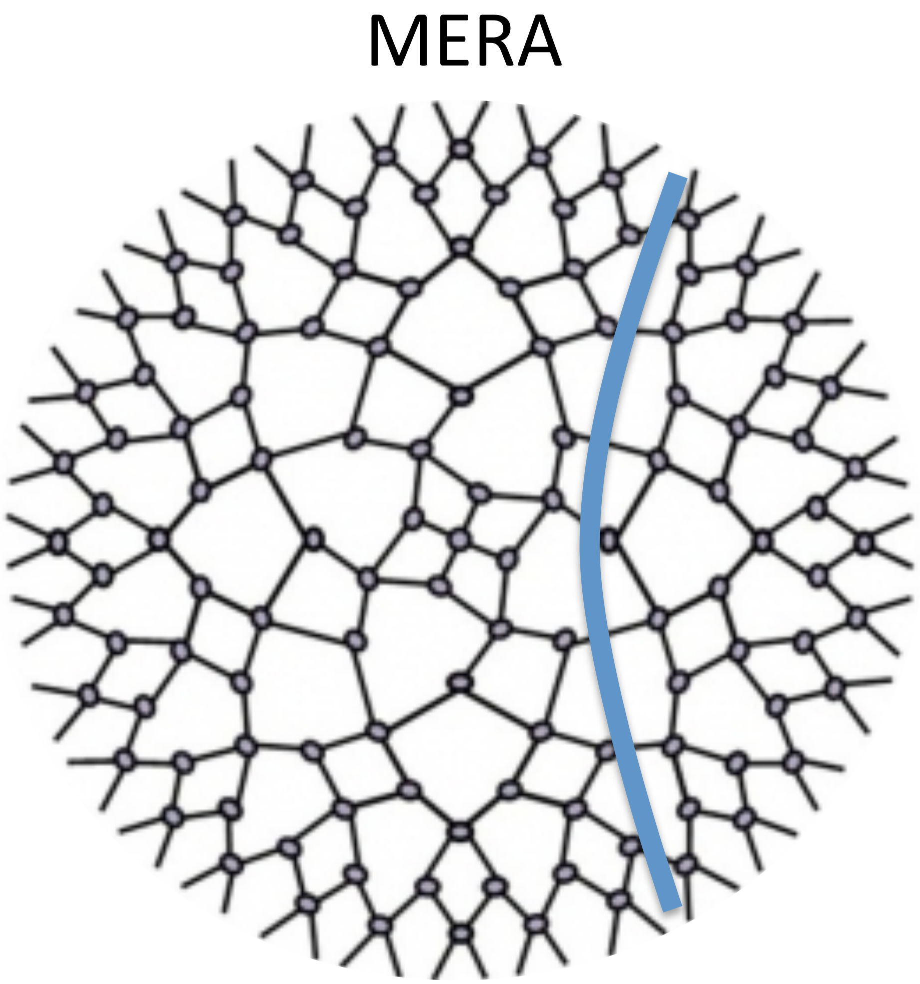


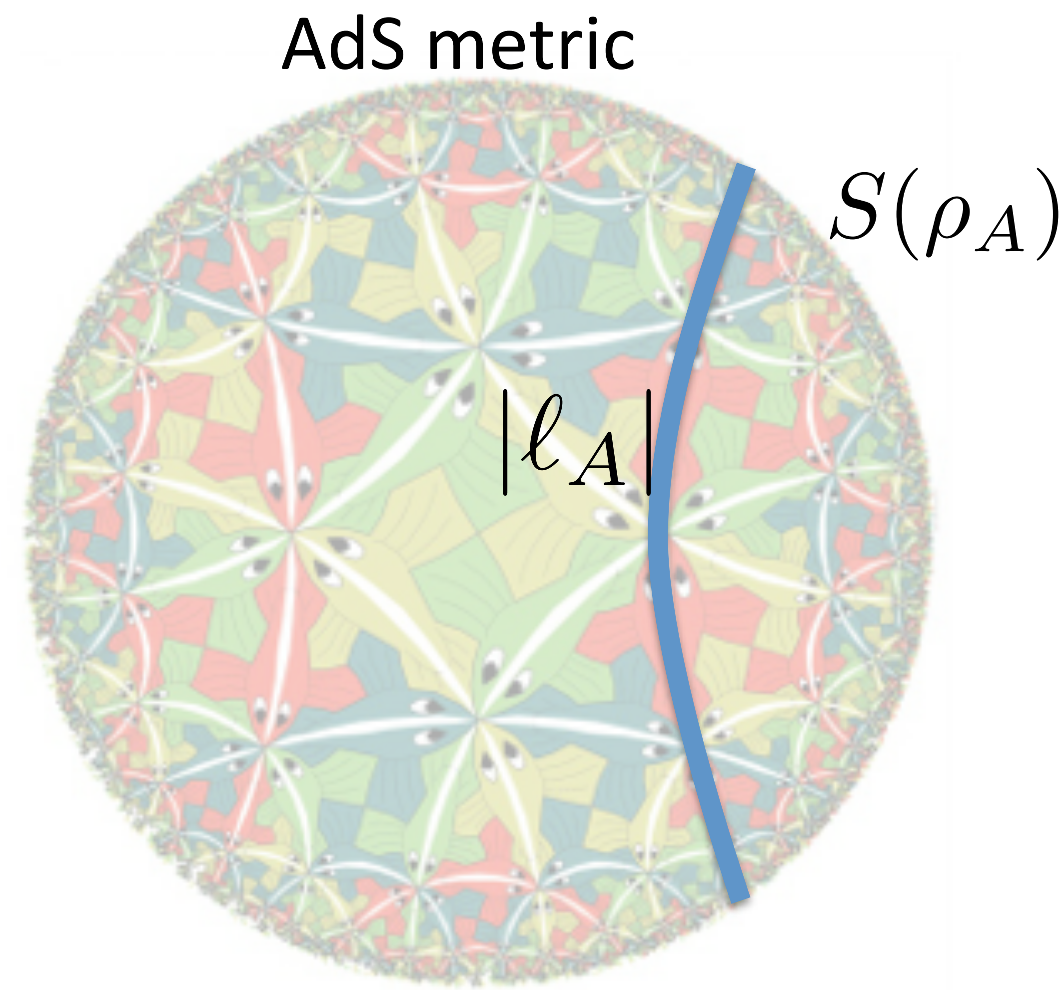
remove short-range entanglement

MERA = Multiscale entanglement renormalization ansatz

AdS/CFT as a tensor network (Swingle 09)

AdS/CFT correspondence can be explained by a tensor network ?

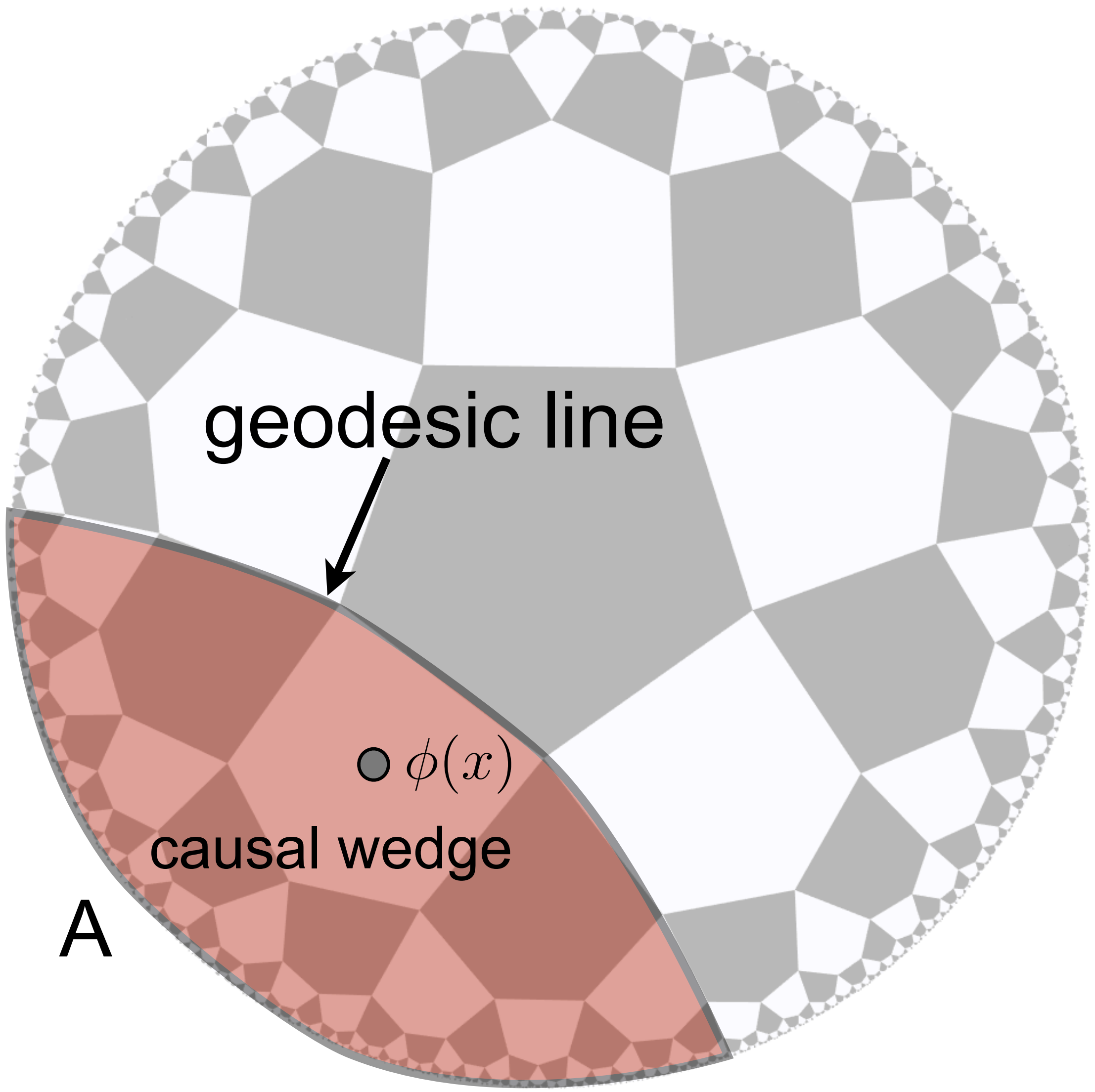




The bulk-locality paradox

Rindler-wedge reconstruction

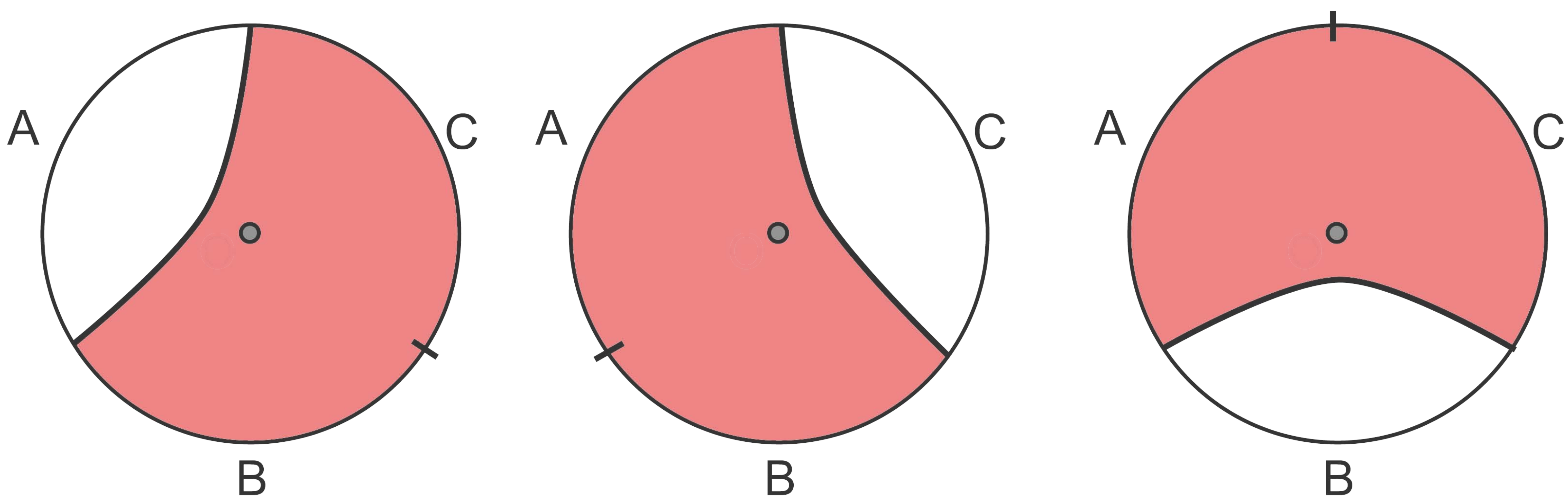
A bulk operator ϕ can be represented by some integral of local boundary operators supported on A if and only if ϕ is contained inside the causal wedge of A.



$$\phi(x) = \int_{\mathbb{S}^{d-1}\times\mathbb{R}} dY\, K(x;Y)\mathcal{O}(Y),$$

Bulk locality paradox

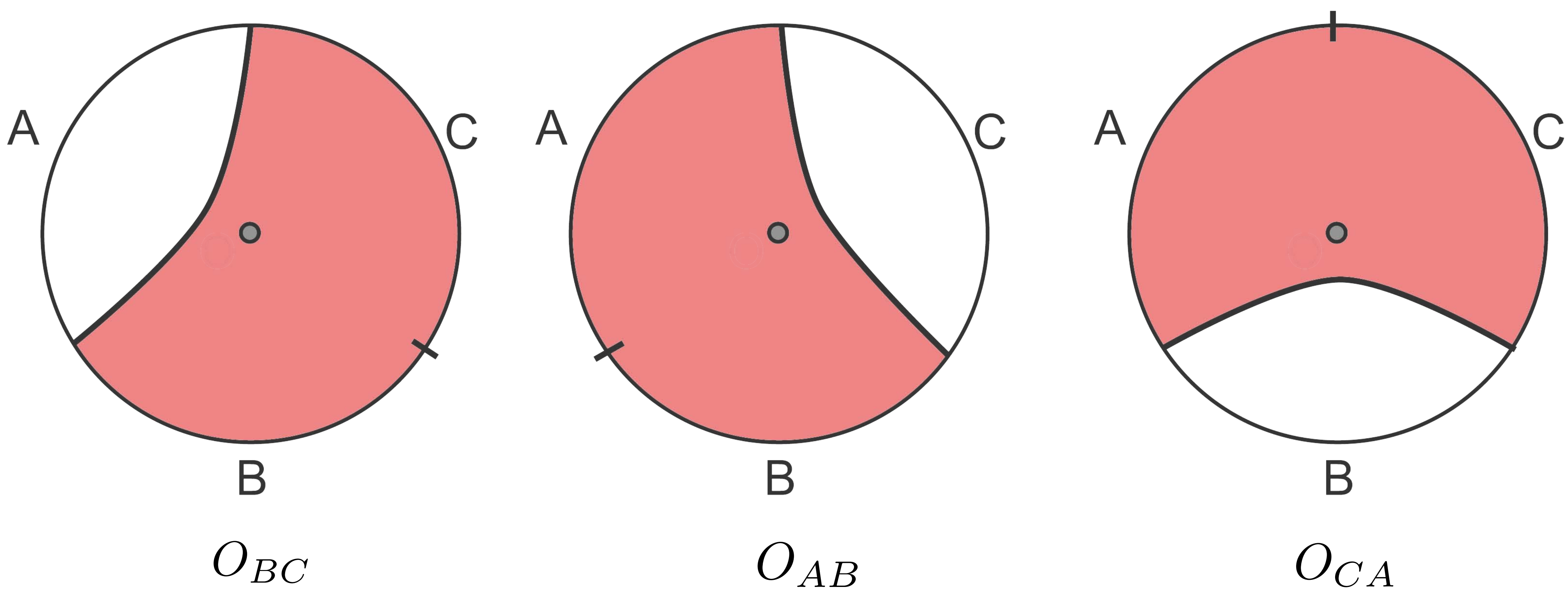
All the bulk operators must correspond to identity operators on the boundary.



If so, the AdS/CFT correspondence seems boring ...

AdS/CFT is a quantum code ?

Solution: T*he AdS/CFT correspondence can be viewed as a quantum error-correcting code*.

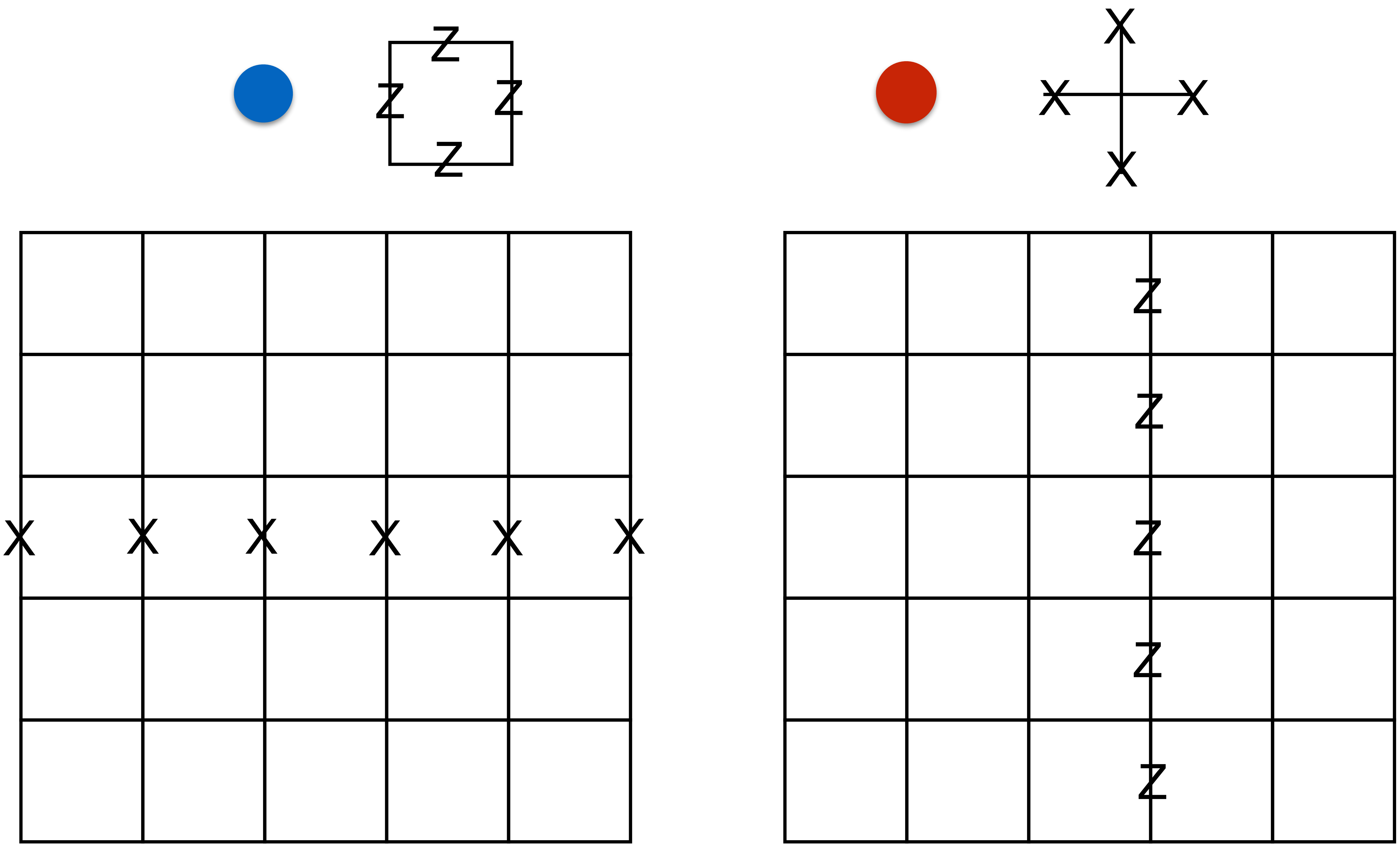


They are different operators, but act in the same manner in a low energy subspace.

cf. Quantum secret-sharing code

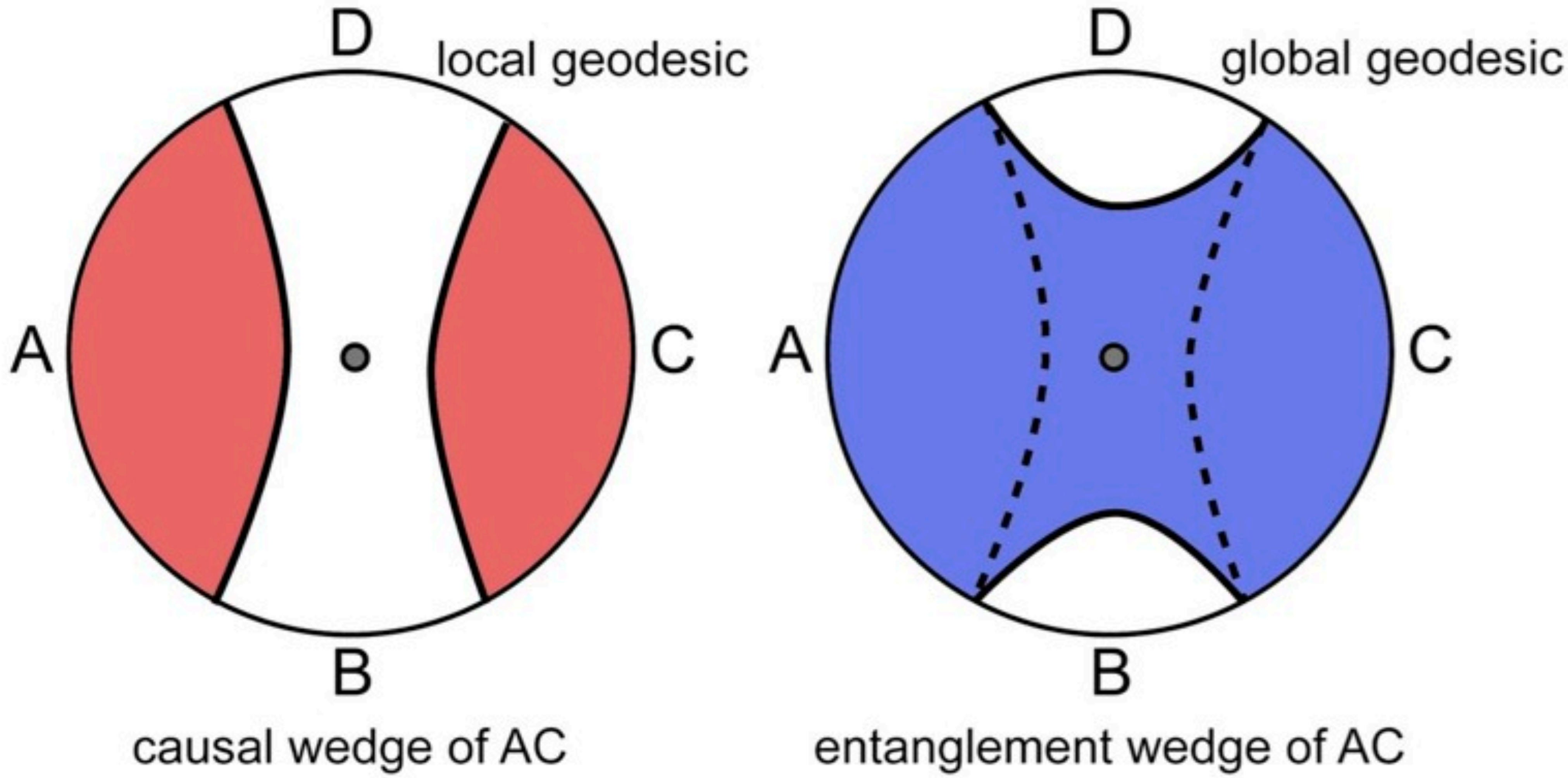
Logical operators in the Toric code

- String-like Pauli X and Pauli Z logical operators



Entanglement wedge reconstruction

Operators in the entanglement wedge can be reconstructed (?)



(entanglement wedge may extend over the singularity).

* Whether this is possible or not remains open.

Holographic code

Minimal model

Let us construct the simplest toy model !

- 5 qubits on the boundary
- 1 qubit on the bulk

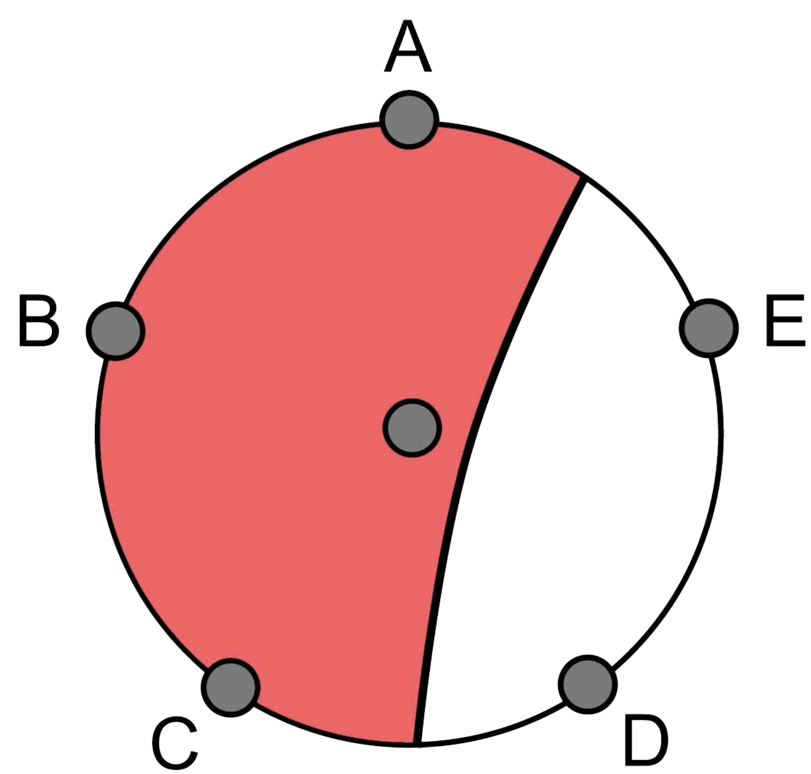
Minimal model

Let us construct the simplest toy model !

- 5 qubits on the boundary
- 1 qubit on the bulk

Causal wedge reconstruction implies

A bulk operator must have representations on ABC, BCD, ...



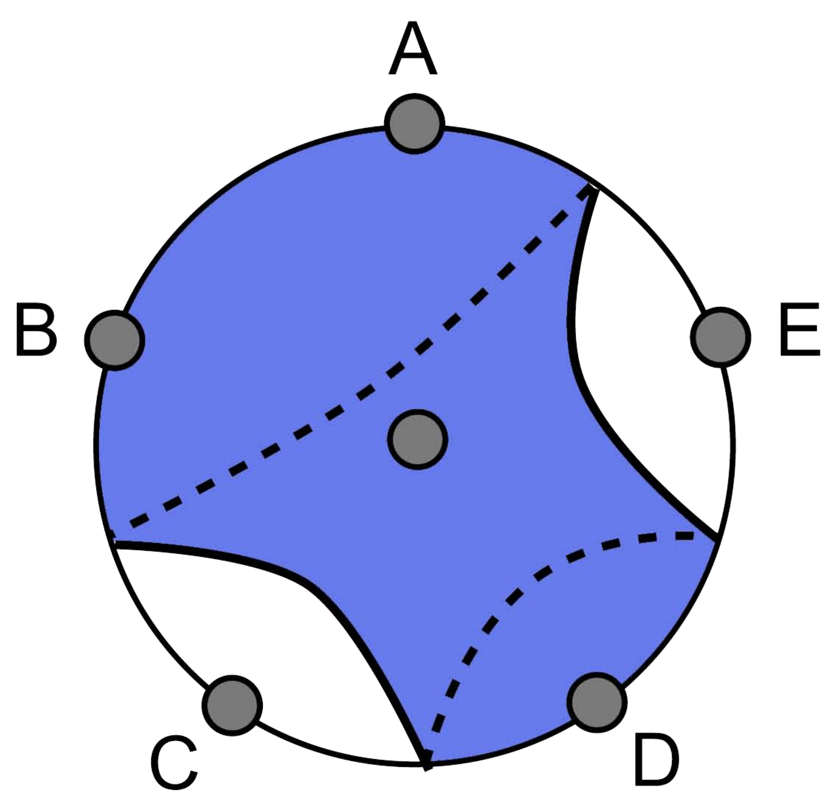
Minimal model

Let us construct the simplest toy model !

- 5 qubits on the boundary
- 1 qubit on the bulk

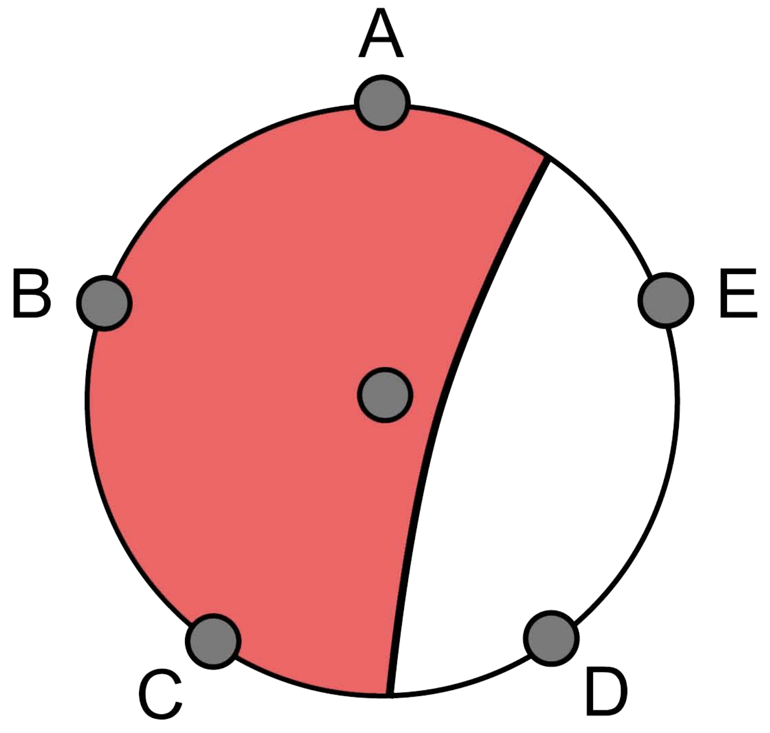
Entanglement wedge reconstruction implies

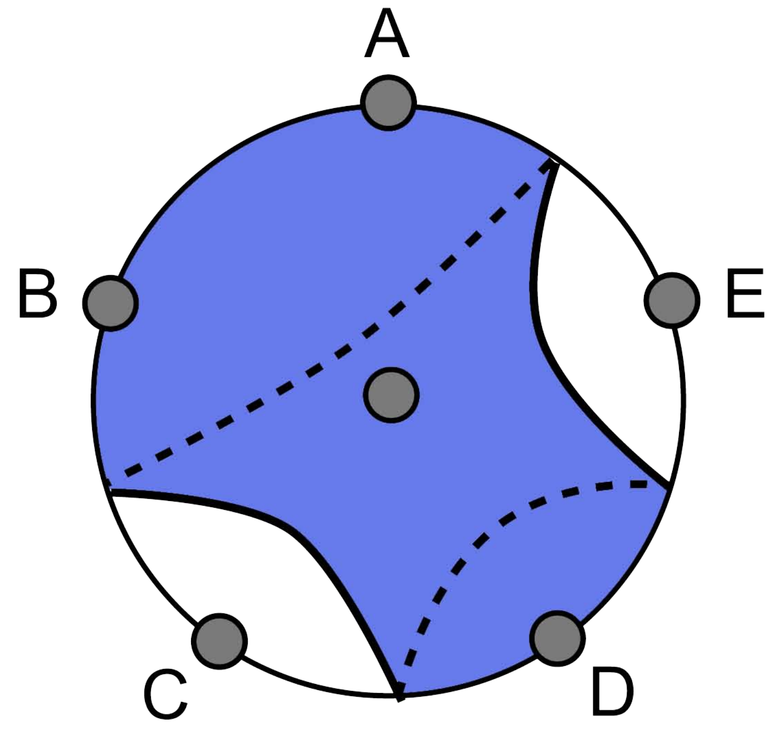
A bulk operator must have representations on ABD, BCE, ...



Desired properties

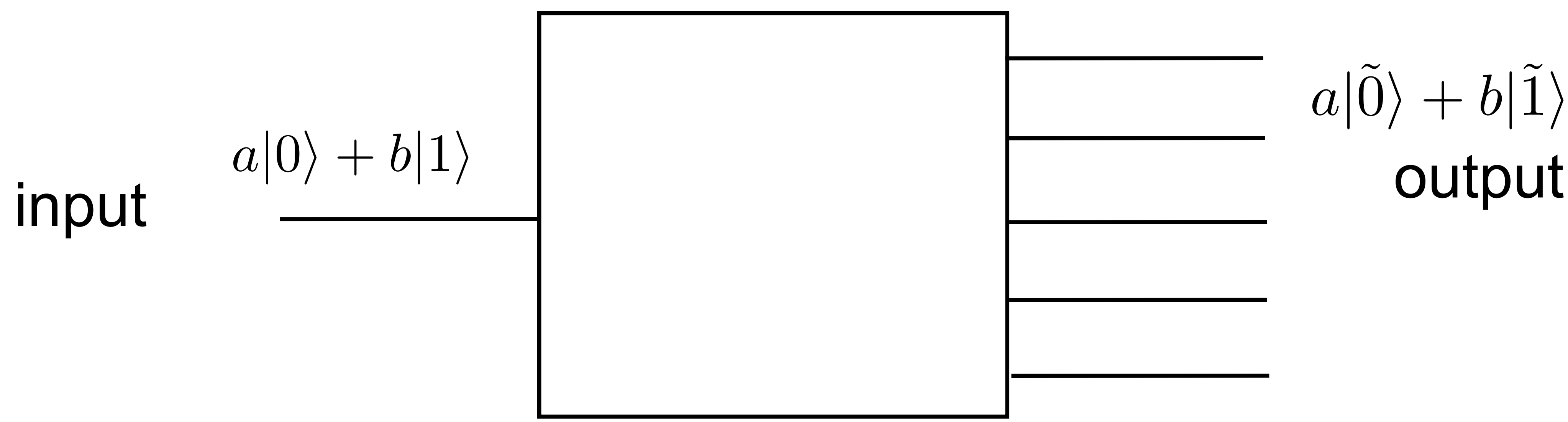
A bulk operator must have representations on any region with three qubits.





Five qubit code

Encode one logical qubit into five physical qubits.



Pauli X, Z $\longrightarrow$ logical Pauli X, Z

Logical Pauli X,Z have representations on any region with three qubits.

Five-minute introduction to five-qubit code (1)

- For a system of 5 qubits, consider the following stabilizer operators

$$S_1 = X \otimes Z \otimes Z \otimes X \otimes I$$

$$S_2 = I \otimes X \otimes Z \otimes Z \otimes X$$

$$S_3 = X \otimes I \otimes X \otimes Z \otimes Z$$

$$S_4 = Z \otimes X \otimes I \otimes X \otimes Z$$

They pairwise commute.

Eigenvalues +1, -1

- The codeword space is specified by $\mathcal{C} = \{|\psi\rangle : \underline{S_j|\psi\rangle = |\psi\rangle} \ \forall j\}$

  4 constraints for 5 qubits, so there are two states $|\psi_0\rangle$ and $|\psi_1\rangle$ in C.

  $|\psi\rangle = \alpha|\psi_0\rangle + \beta|\psi_1\rangle$ encodes one logical qubit in an entangled state.

- Logical operators are given by $\overline{X} = X \otimes X \otimes X \otimes X \otimes X$

$$\overline{Z} = Z \otimes Z \otimes Z \otimes Z \otimes Z$$

  Logical operators commute with stabilizer operators, but act non-trivially inside C

$$[\overline{X}, S_j] = [\overline{Z}, S_j] = 0$$

  Logical operators are like Pauli X and Z operators for the logical qubit

Five-minute introduction to five-qubit code (2)

- There are many logical operators which are equivalent inside C

$$S_1 = X \otimes Z \otimes Z \otimes X \otimes I$$
$$S_2 = I \otimes X \otimes Z \otimes Z \otimes X$$
$$S_3 = X \otimes I \otimes X \otimes Z \otimes Z$$
$$S_4 = Z \otimes X \otimes I \otimes X \otimes Z$$

stabilizer operators

$$\overline{X} = X \otimes X \otimes X \otimes X \otimes X$$
$$\overline{Z} = Z \otimes Z \otimes Z \otimes Z \otimes Z$$

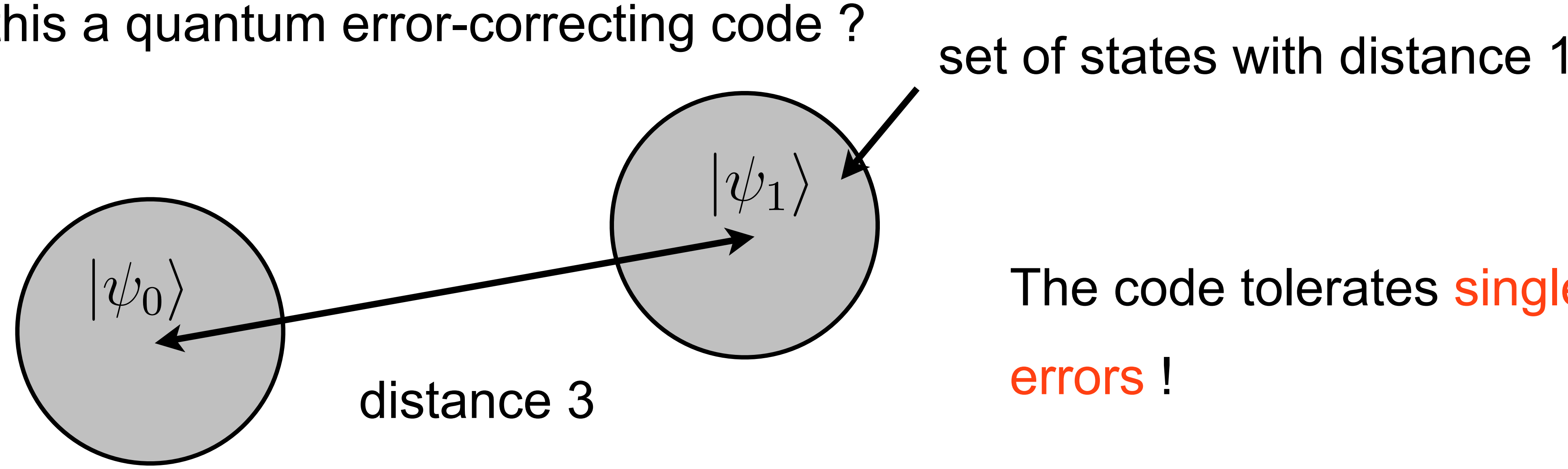
logical operators

eg) $\overline{X} \sim \overline{X} S_1 = I \otimes Y \otimes Y \otimes I \otimes X$

- Five-qubit code has code distance 3

Logical operators must have supports on at least three qubits.

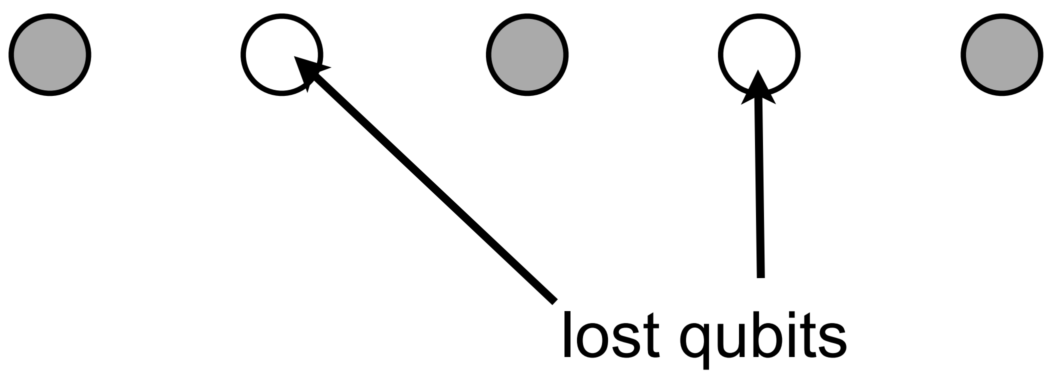
- Why is this a quantum error-correcting code ?



The code tolerates single-qubit errors !

Five-minute introduction to five-qubit code (3)

For any subset of three qubits, logical X and logical Z operators can be found.



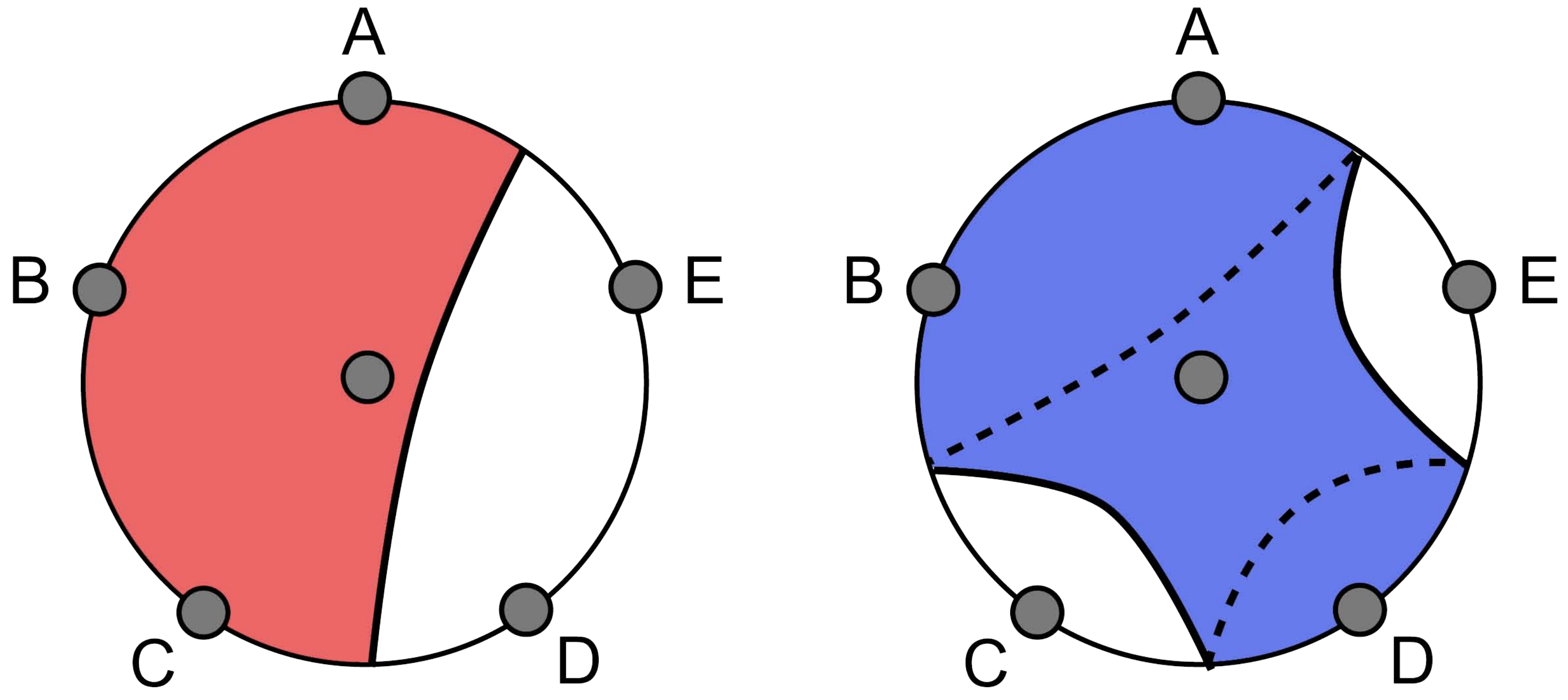
Both Pauli X and Pauli Z logical operators can be supported on shaded qubits.

The code can tolerate loss of 2 qubits.

Logical Pauli X,Z have representations on any region with three qubits.

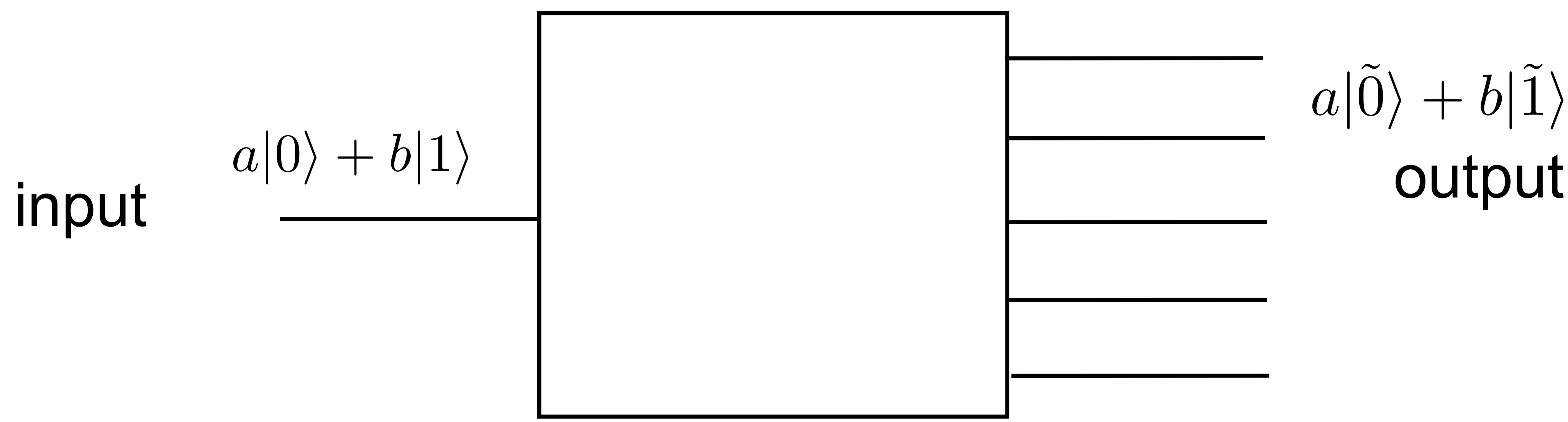
Desired properties

A bulk operator must have representations on any region with three qubits.



Five qubit code

Encode one logical qubit into five physical qubits.

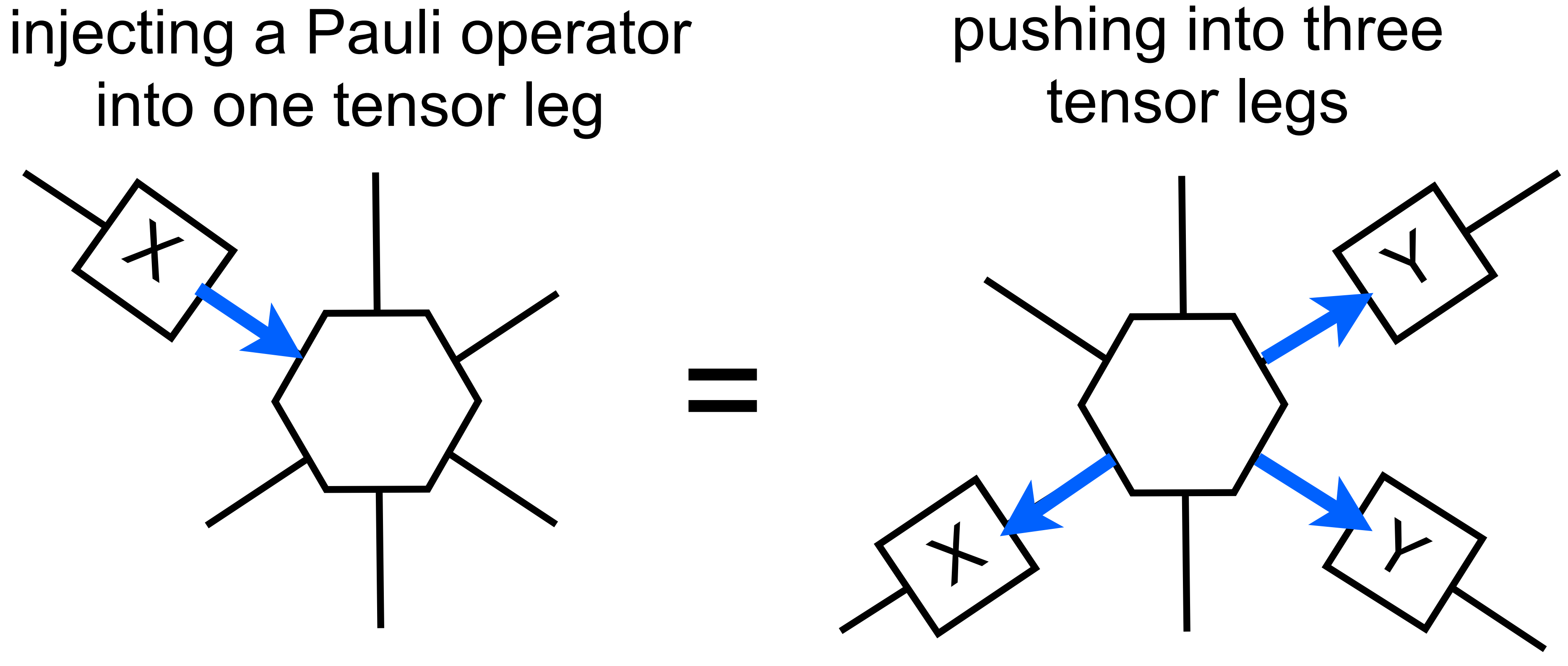


Pauli X, Z $\longrightarrow$ logical Pauli X, Z

Logical Pauli X,Z have representations on any region with three qubits.

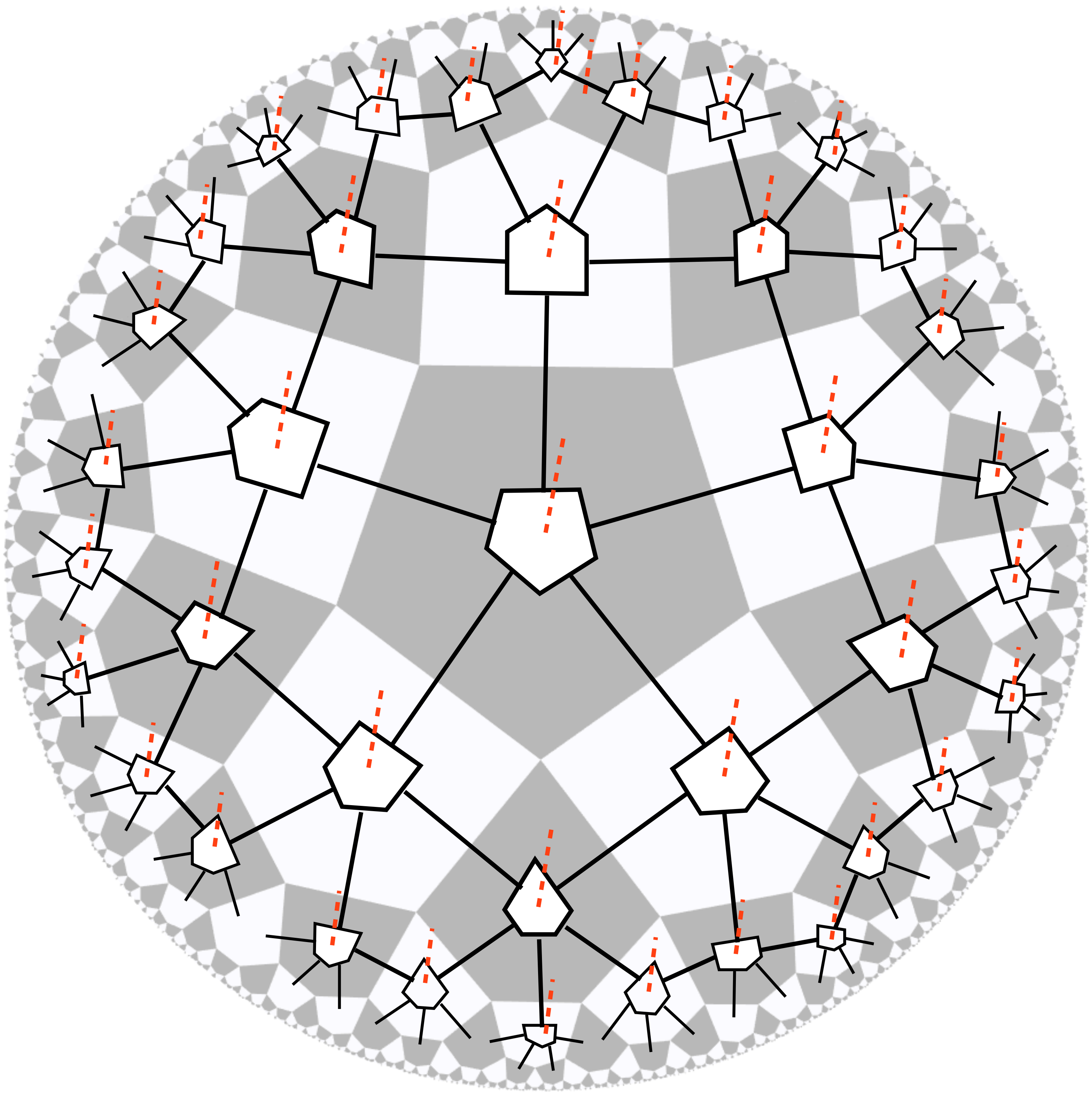
Six-qubit tensor

Tensor pushing

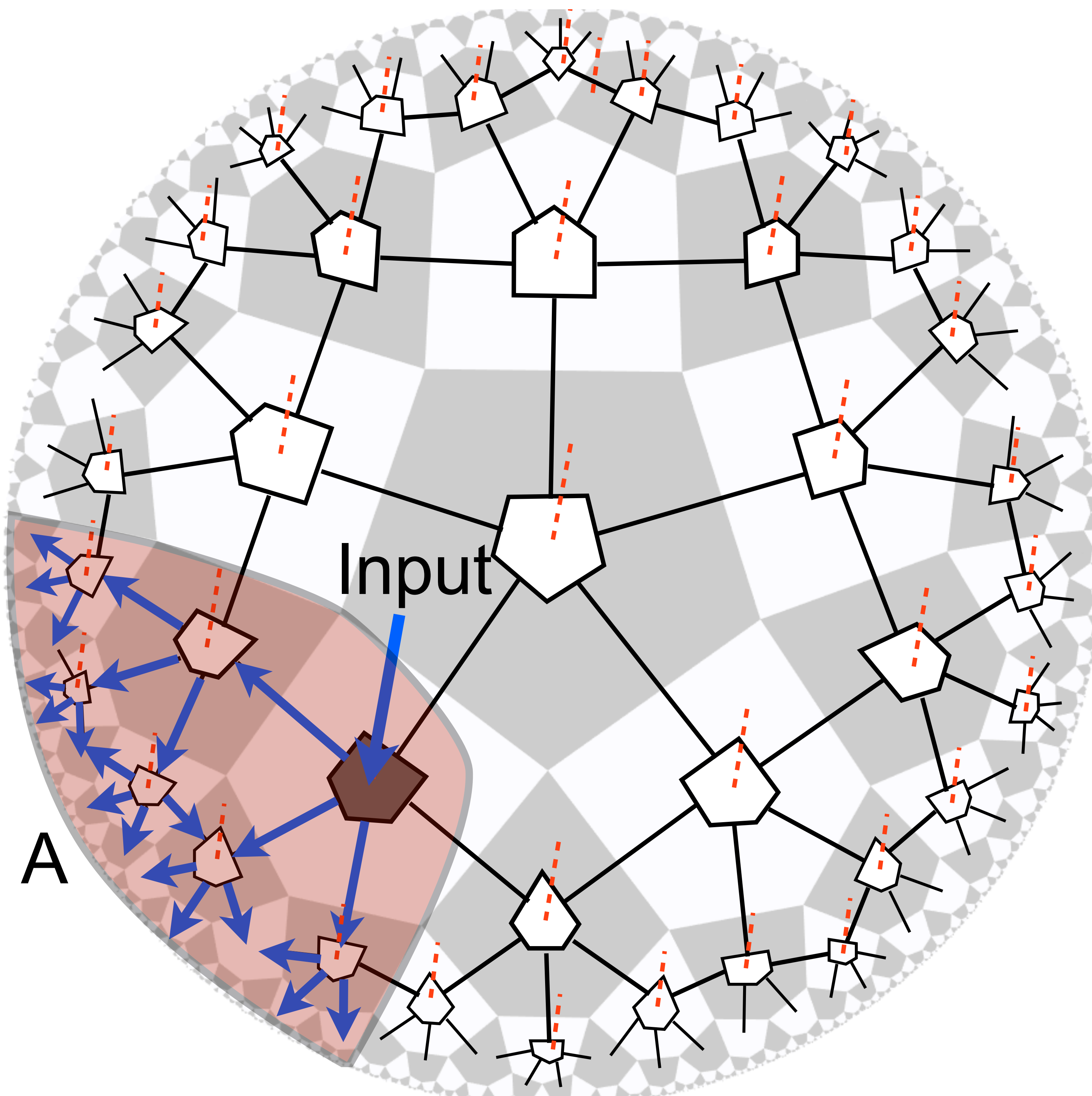


Any leg can be used as an input leg !!!

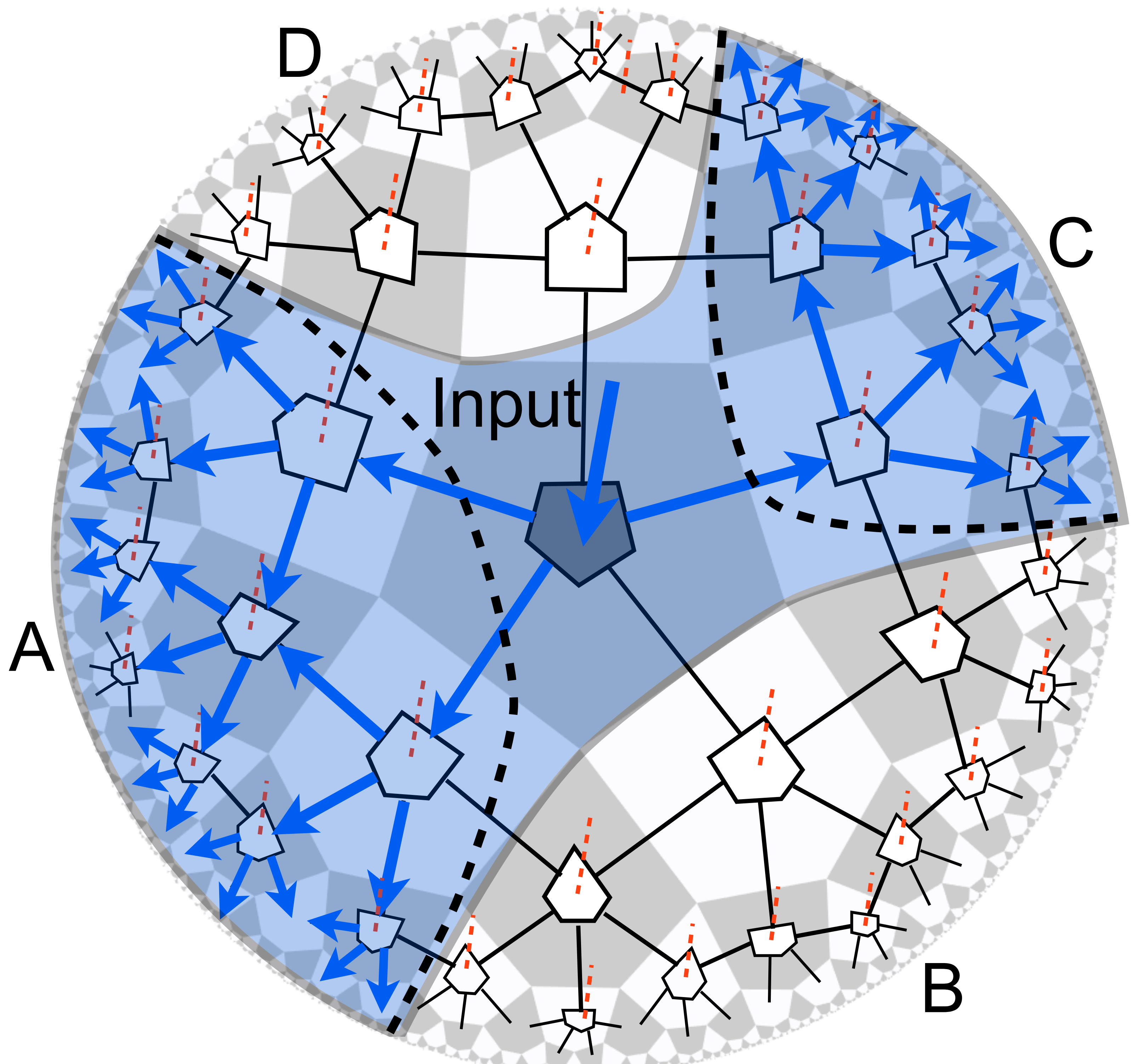
Holographic Code



Causal wedge reconstruction



Entanglement wedge reconstruction

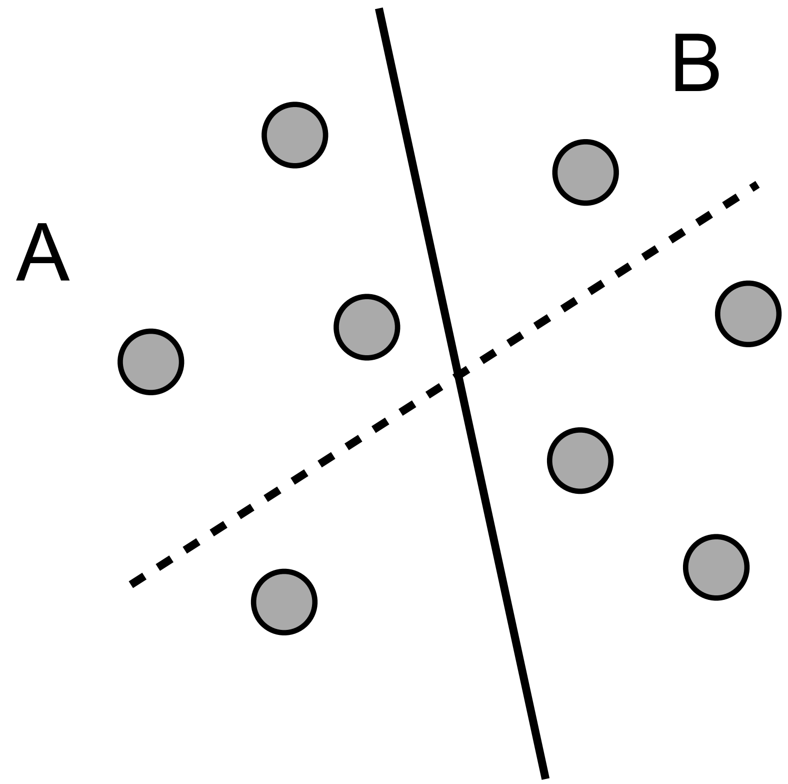


Generic properties : Perfect tensors

Perfect state / tensor

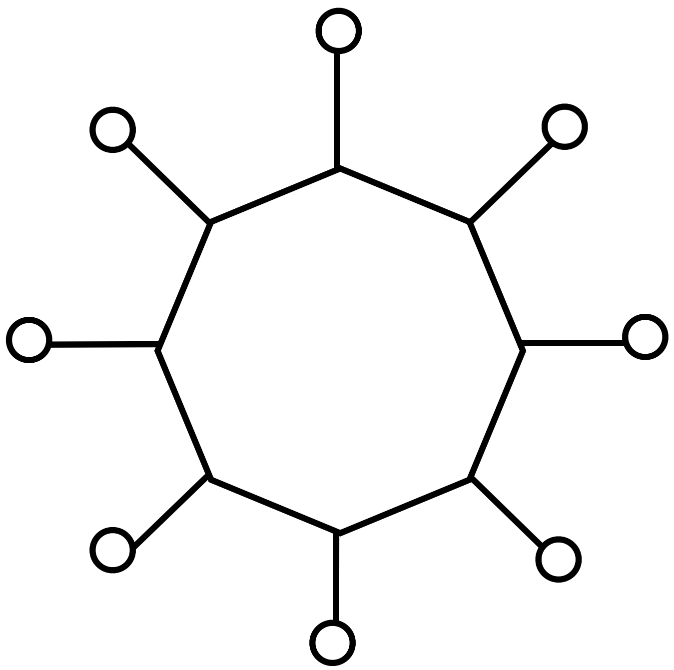
- A pure state with maximal entanglement in any bipartition

Perfect state (2n spins)



$$\rho_A \propto I_A \qquad \text{for all} \quad |A| \leq n$$

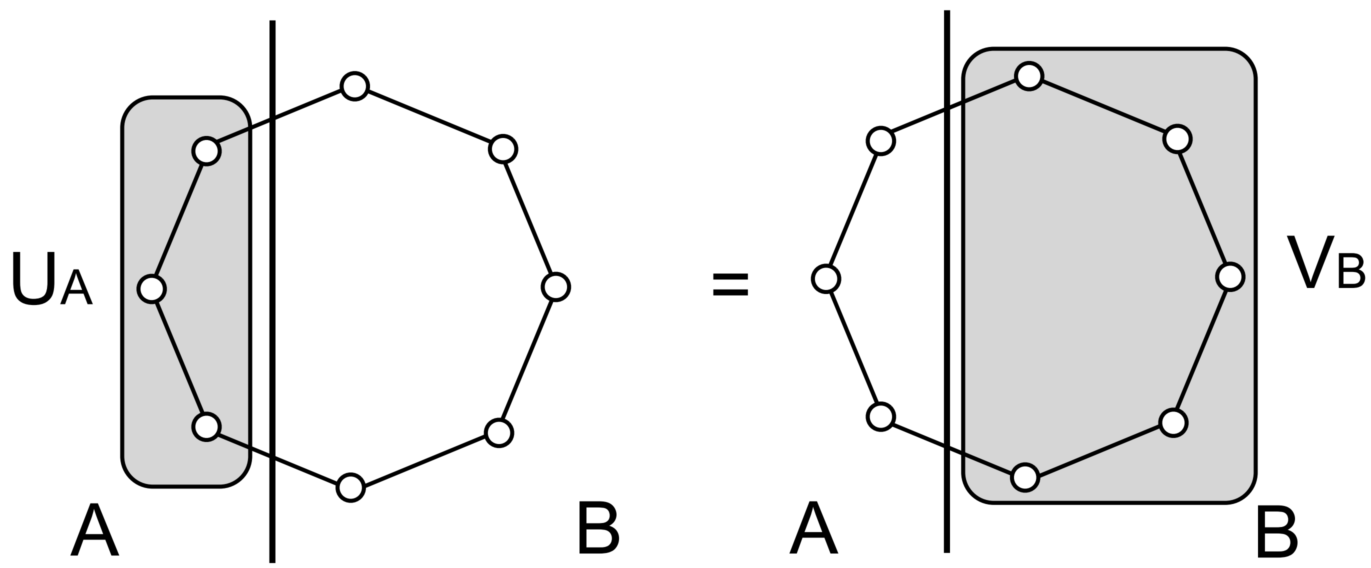
Perfect tensor (2n legs)



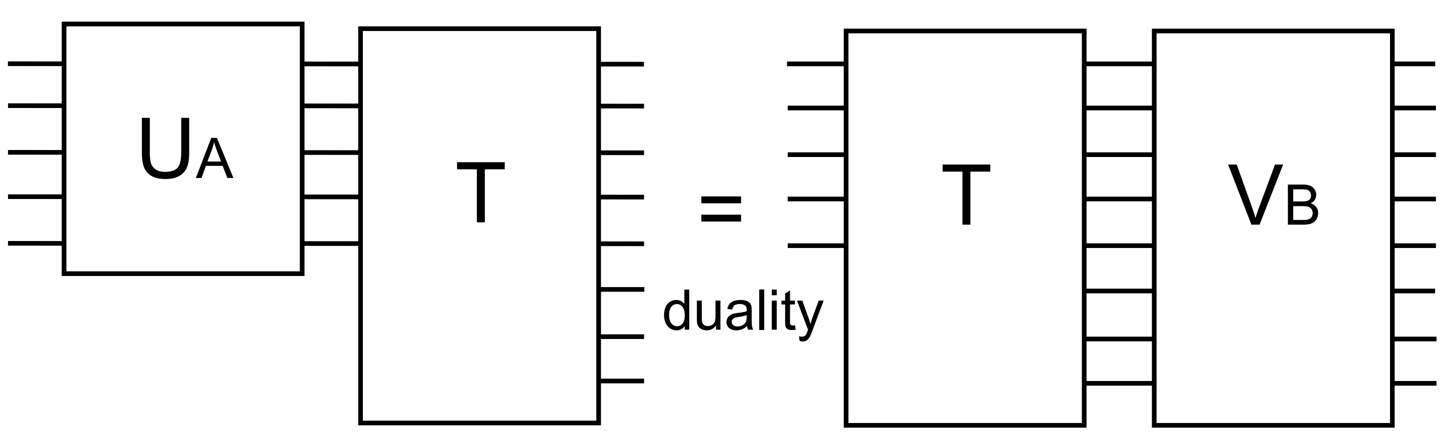
$$|\psi\rangle = \sum_{i_1=0}^{v-1} \sum_{i_2=0}^{v-1} \cdots \sum_{i_n=0}^{v-1} T_{i_1 i_2 \dots i_n} |i_1 i_2 \dots i_n\rangle$$

Duality of unitary operators

- Given U_A, there always exists V_B such that $U_A \otimes I|\psi\rangle = I \otimes V_B|\psi\rangle$



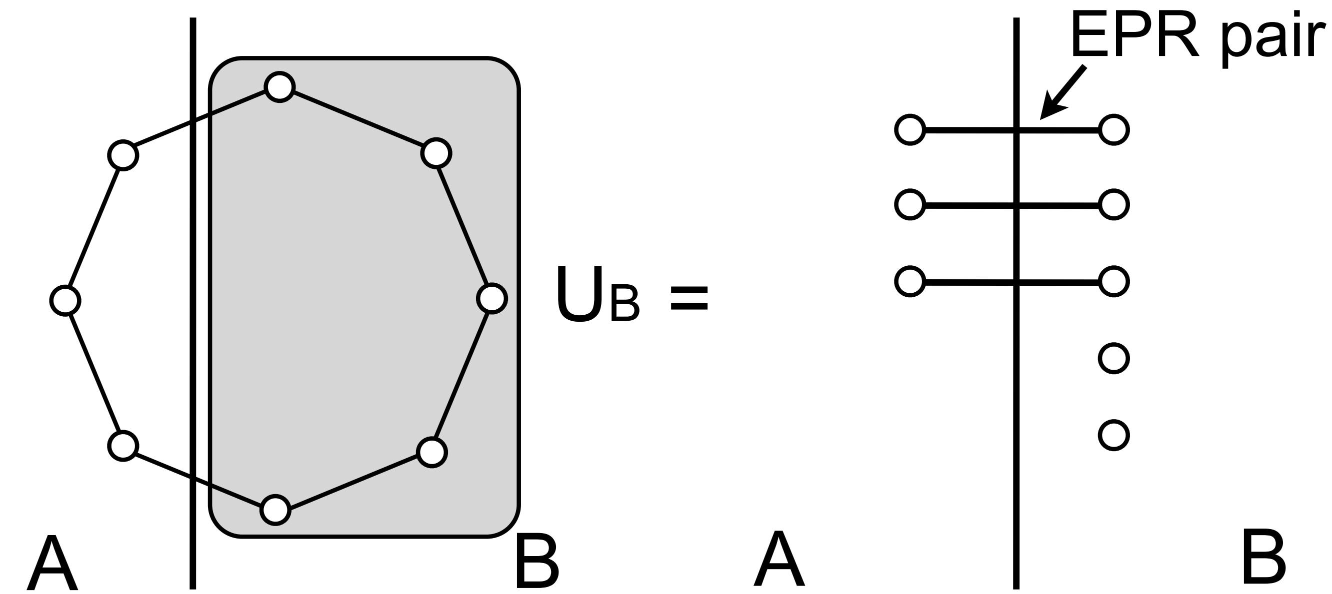
"gate teleportation"



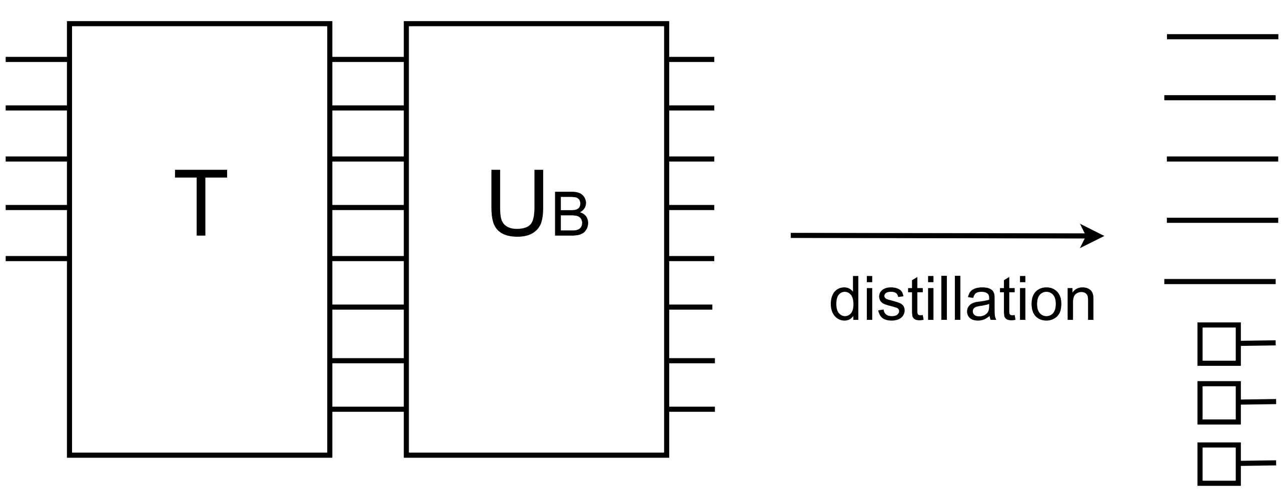
will lead to bulk/ boundary duality...

Distillation of EPR pairs

- By applying a unitary only on B, EPR pairs can be distilled

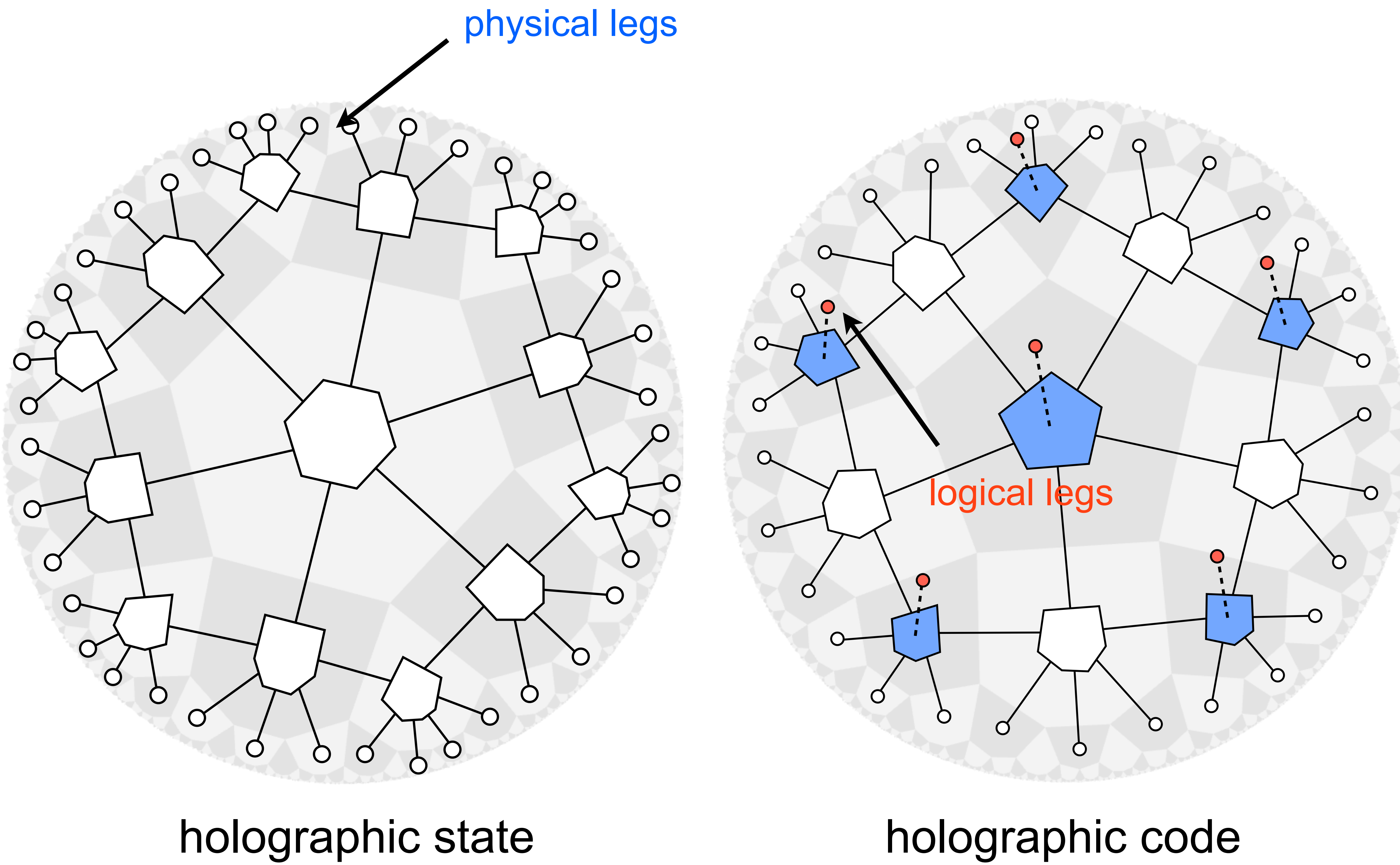


only on B !



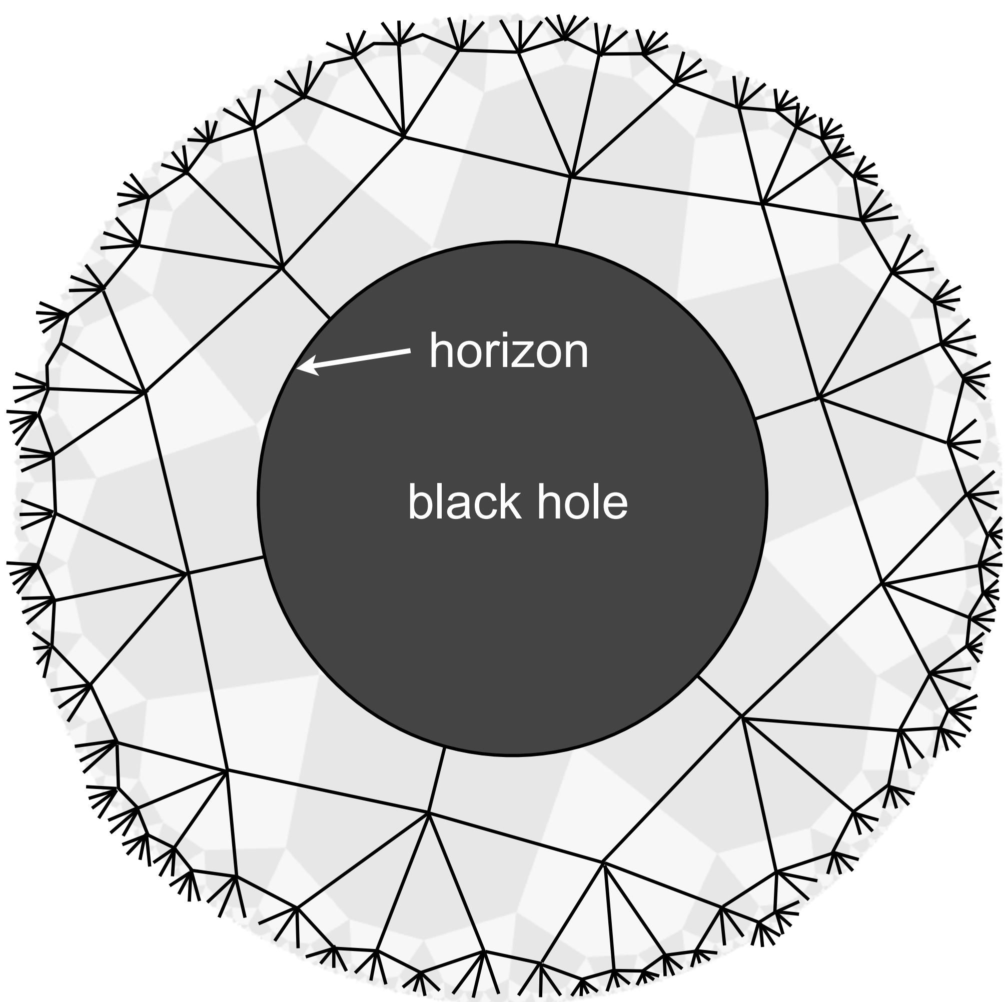
will lead to the RT formula...

Holographic quantum state / code



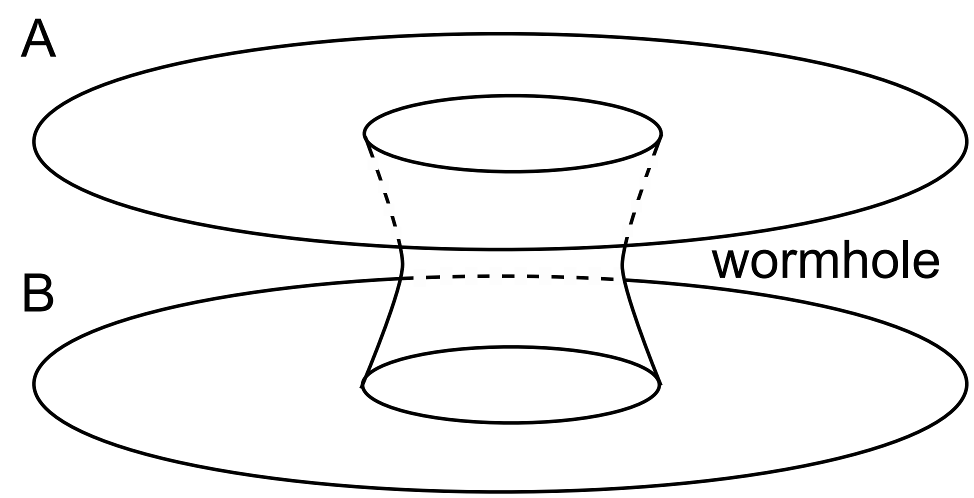
A black hole and wormhole

As a mixed state ρ



inject maximally mixed state

As a purified state $|\phi\rangle$



$$Tr_B(|\phi\rangle\langle\phi|) = \rho$$

Yes, perfect states exist

- 2n : total number of spins
  v : spin dimension

- For v=2, perfect states with n=1,3 exist

  -- EPR pair
  -- 6-qubit state (5-qubit code)

- For large n, perfect states with $v \sim O(n^{1/2})$ exist

- Pick a Haar random pure state, then it is a nearly perfect state (canonical typicality).

Holographic state (bipartition)

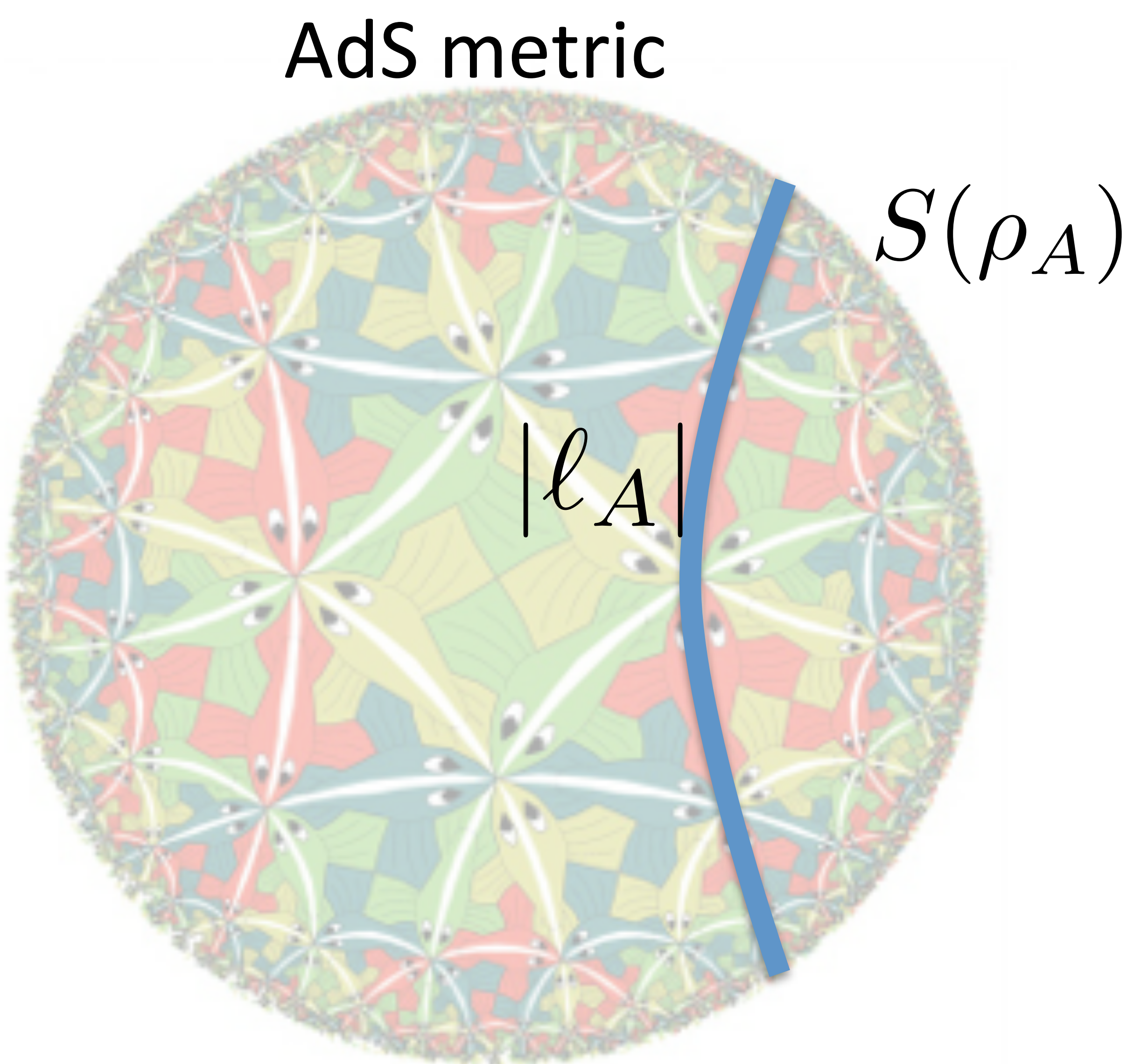
Ryu-Takayanagi formula, it's exact !

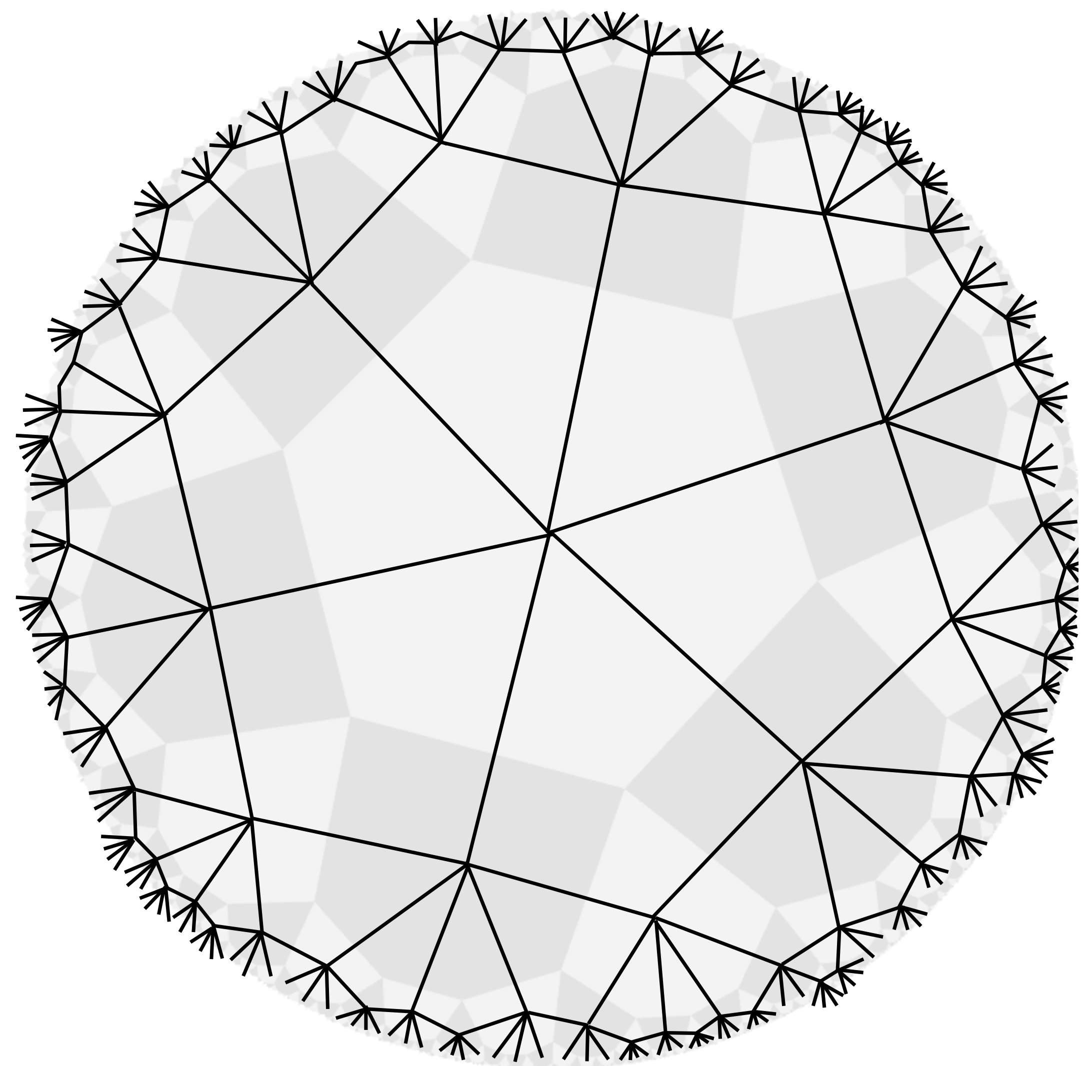
[Claim] Entanglement entropy for A (connected region) is equal to the geodesic length.

$$S_A = \log_2 \boxed{v} \cdot \boxed{\gamma_A}$$

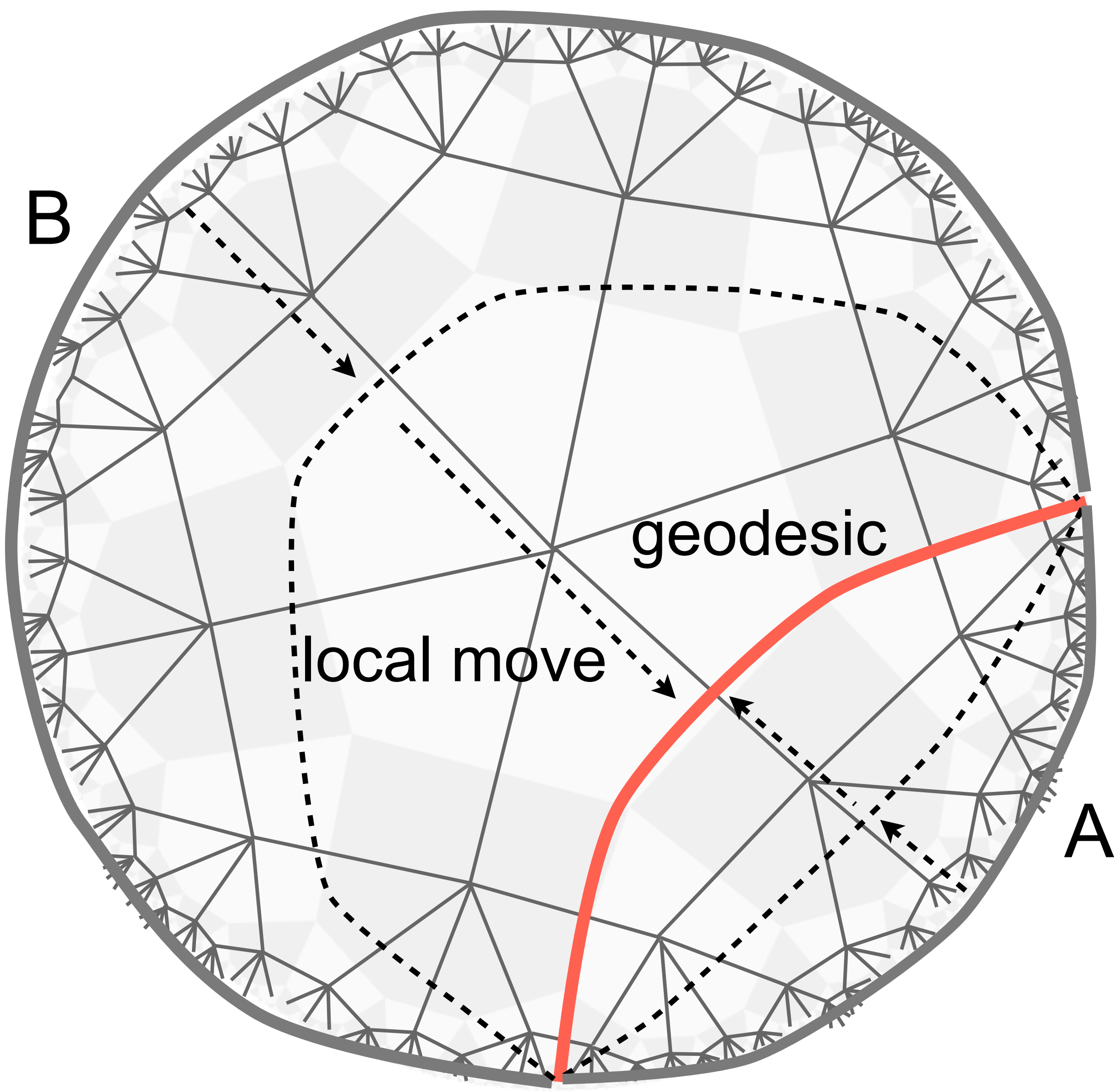
spin dim

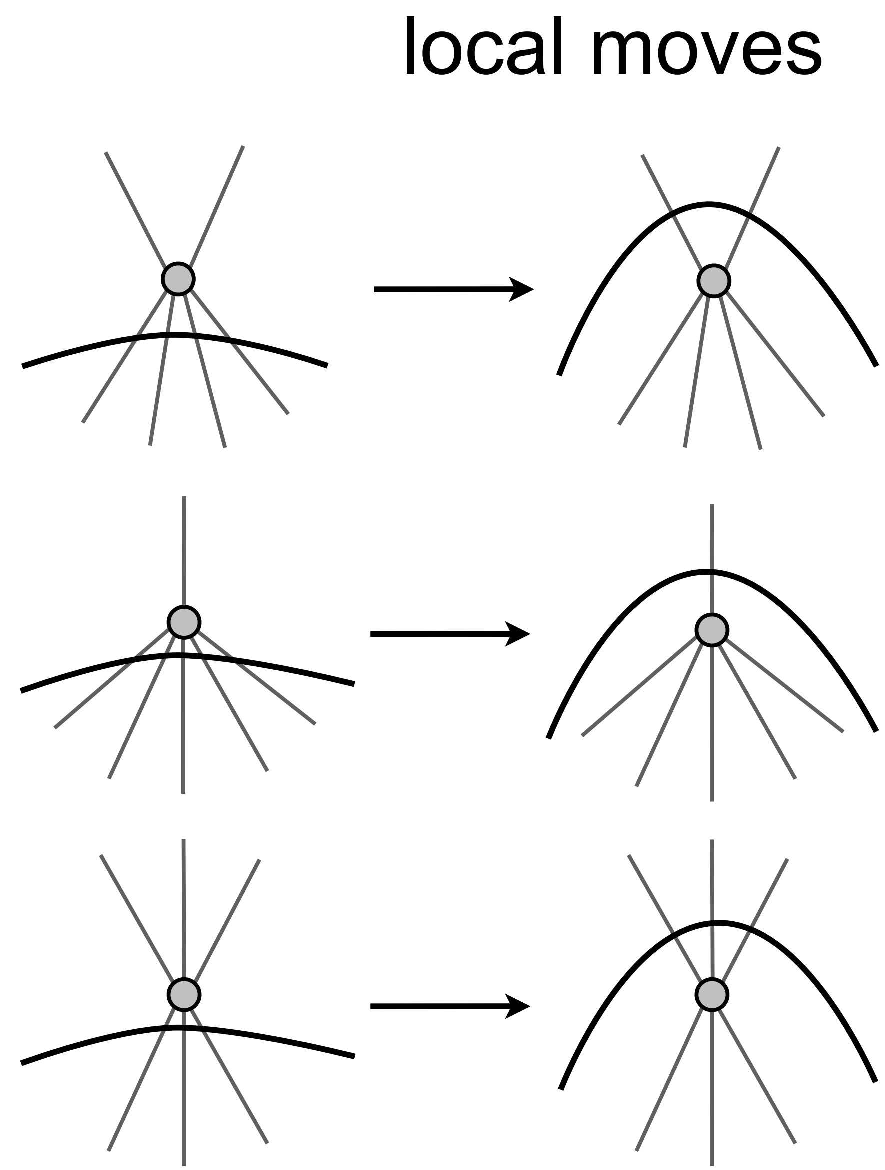
length





Geodesic line from local moves

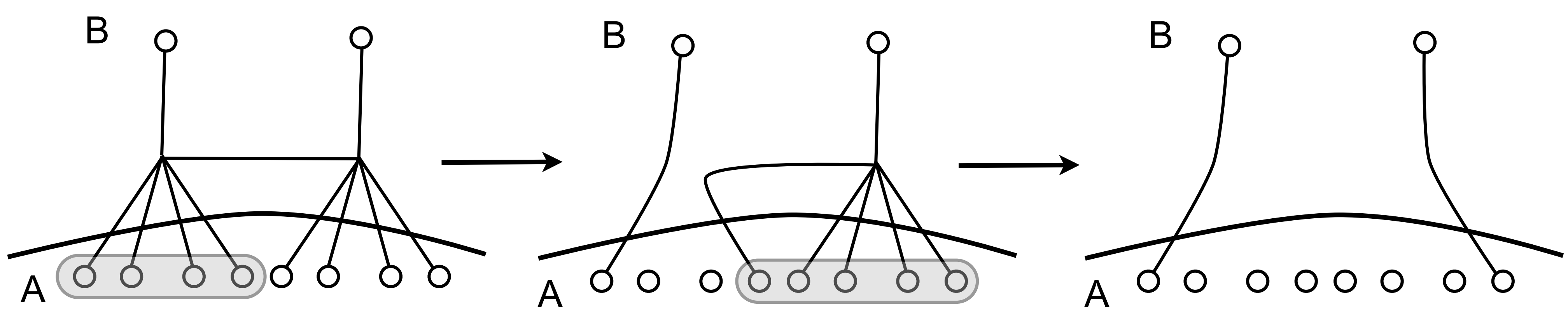




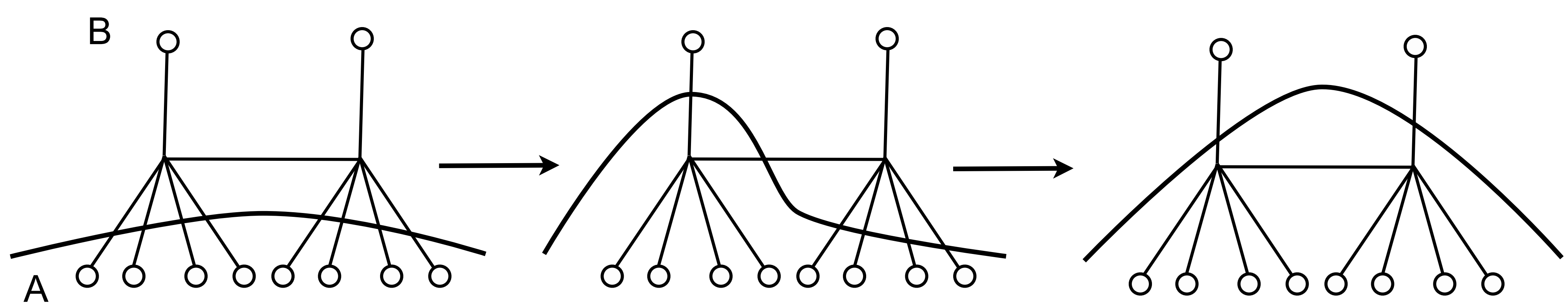
Local move = distillation of EPR pairs

- Local moves distill EPR pairs and decouple "junks".

distillation of EPR pairs



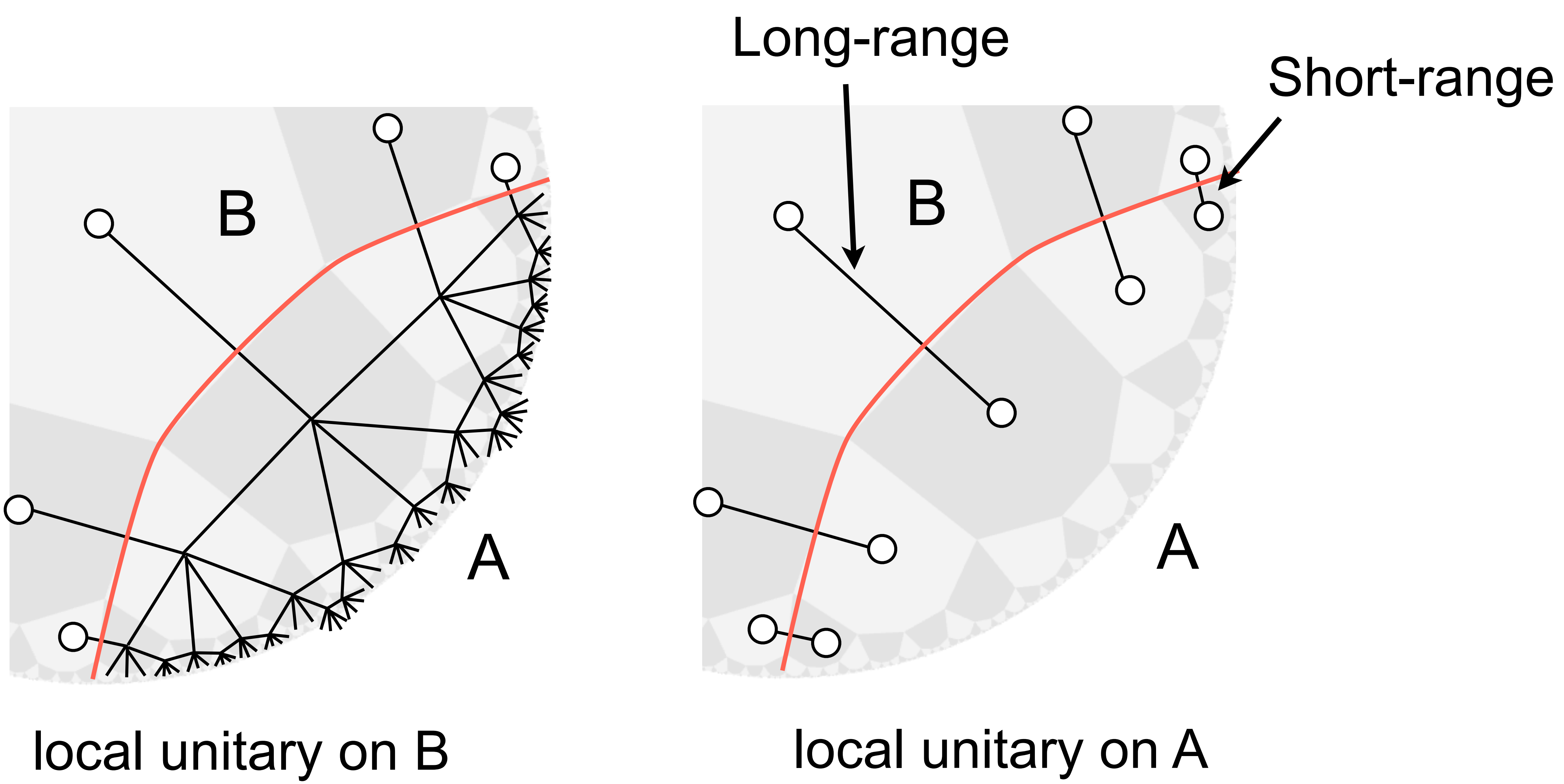
local move



Geometric map of entanglement

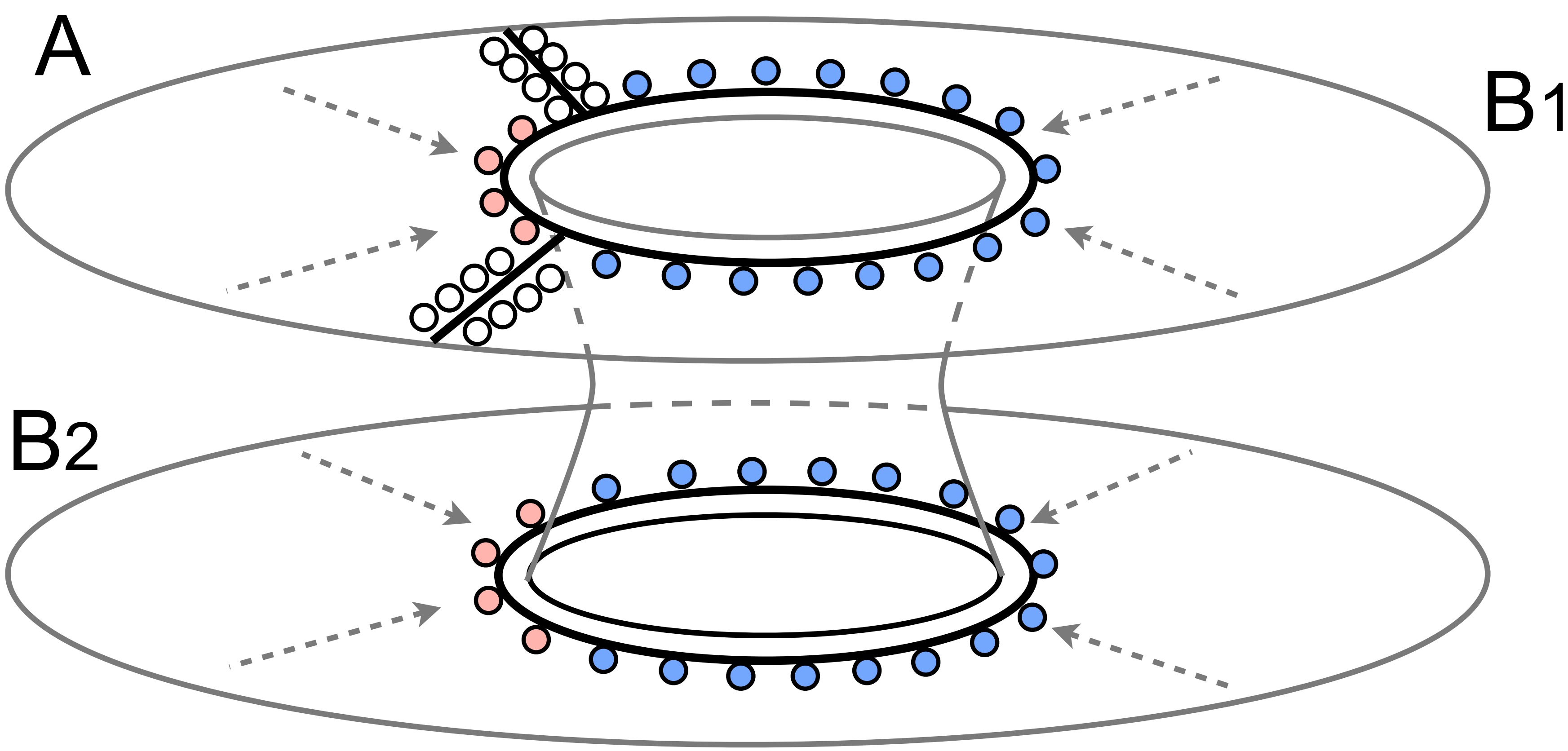
- Graphical representation of entanglement in the AdS/CFT correspondence.

Geodesic line = EPR pairs



Entanglement in a black hole

- EPR pairs along the wormhole (ER=EPR ?)
- RT formula with a black hole

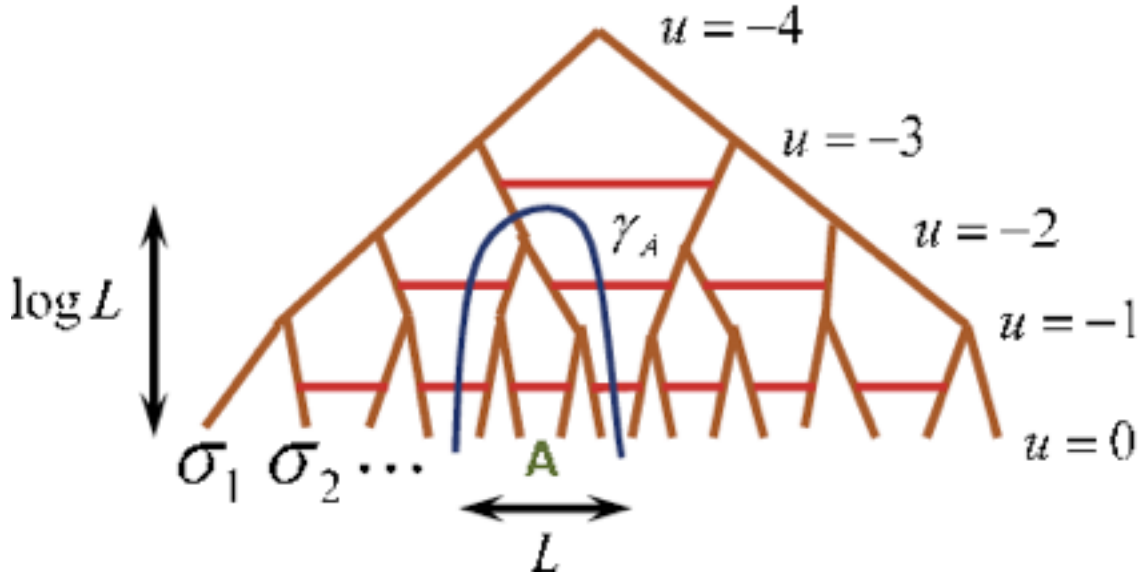


Holographic state
(multi-partition)

Multi-partite entanglement

- It is not difficult to create a wavefunction with $S_A \propto \log(L)$, but...

MERA



eg) distribute EPR pairs in a tensor tree

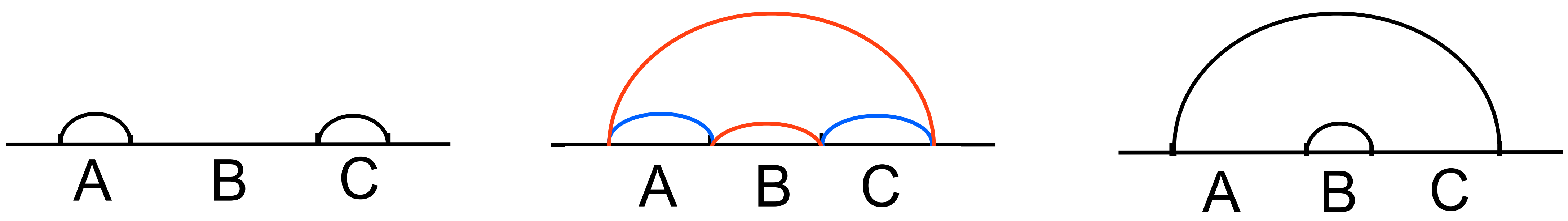
- Negativity of tripartite entanglement entropy (any "holographic state")

$$I(A,B,C) = S_A + S_B + S_C - S_{AB} - S_{AC} - S_{BC} + S_{ABC}$$

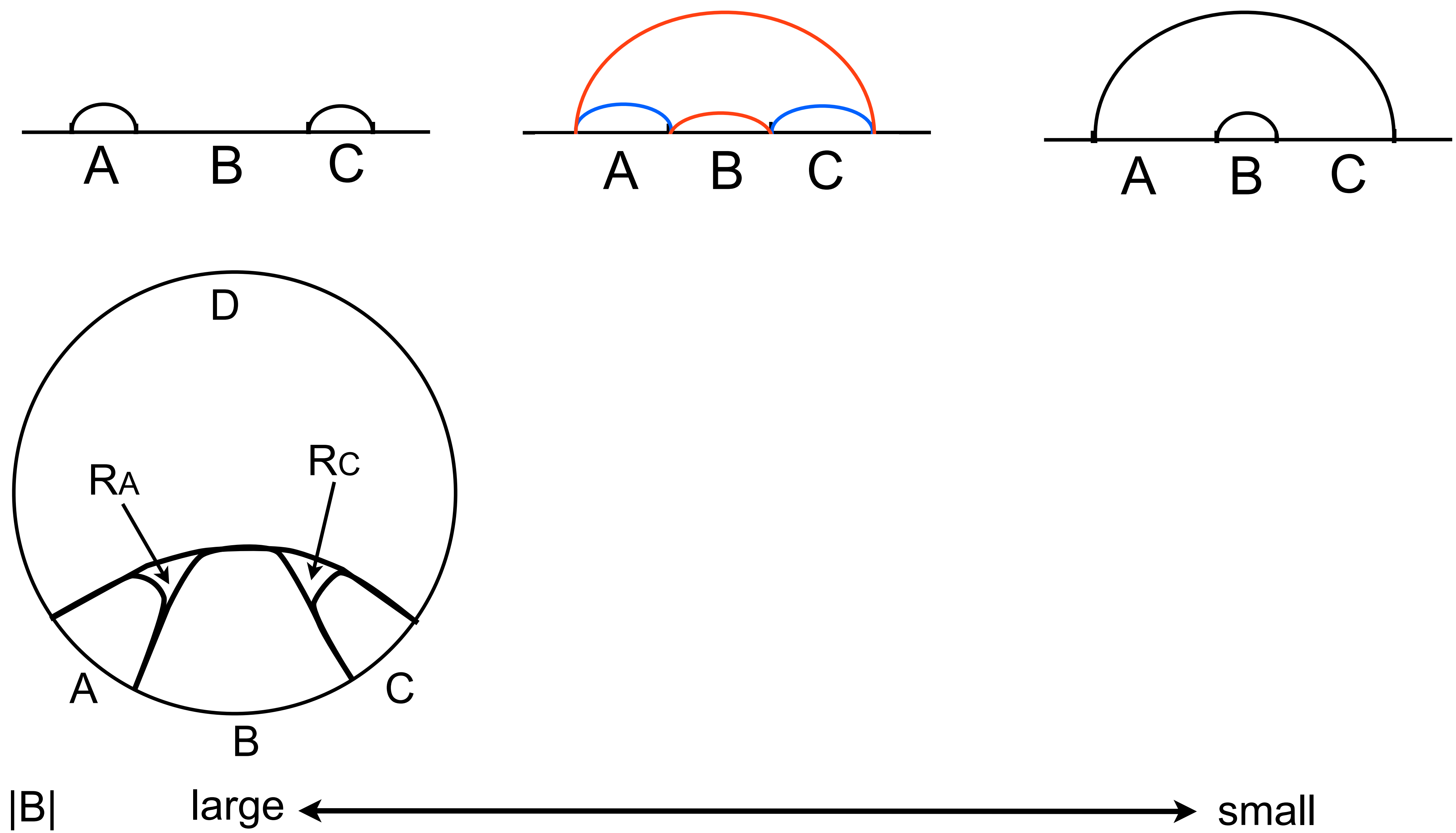
-- EPR pair, then I(A,B,C)=0.

-- GHZ state, then I(A,B,C)>0.

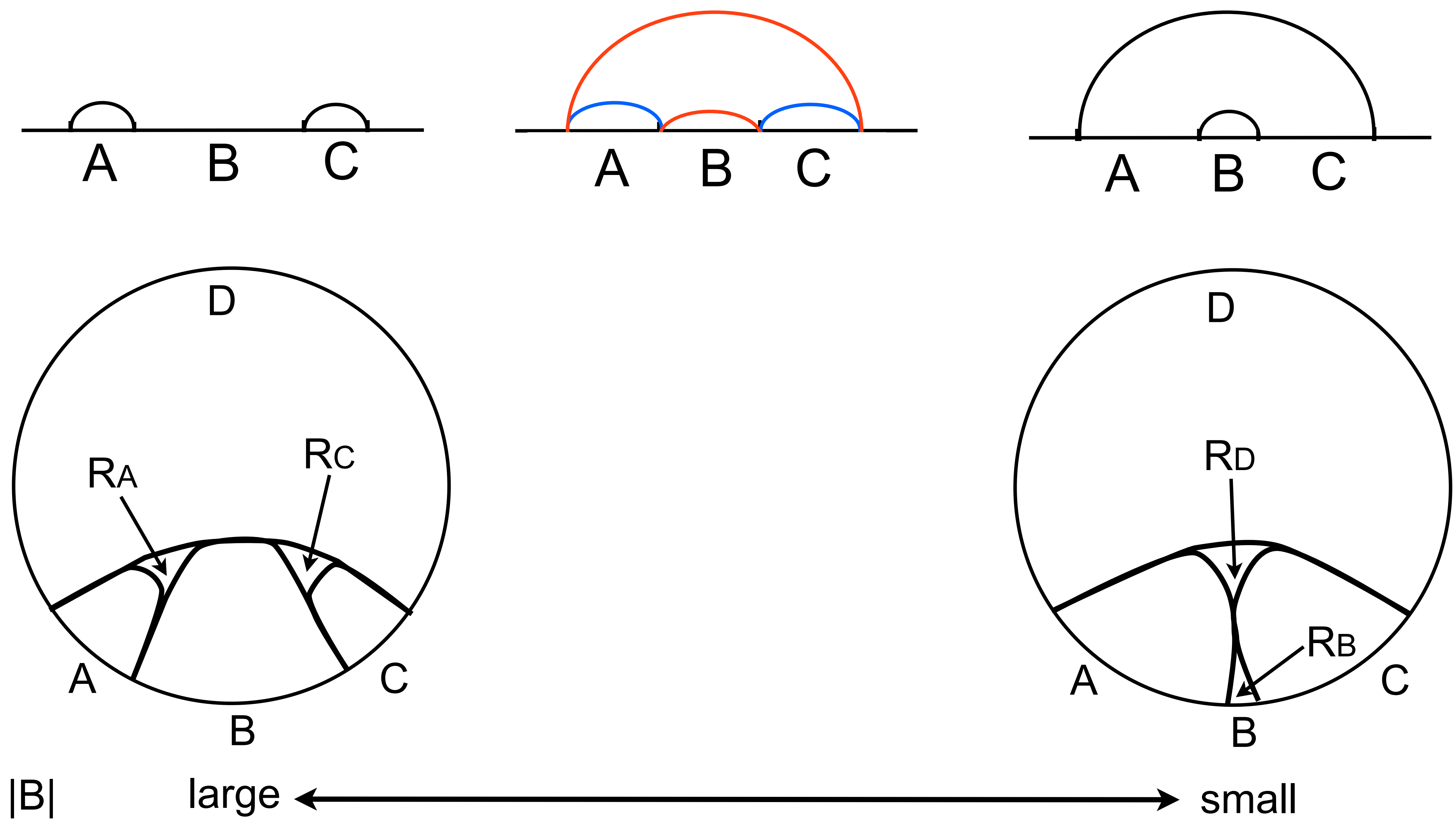
Multi-partition and residual regions



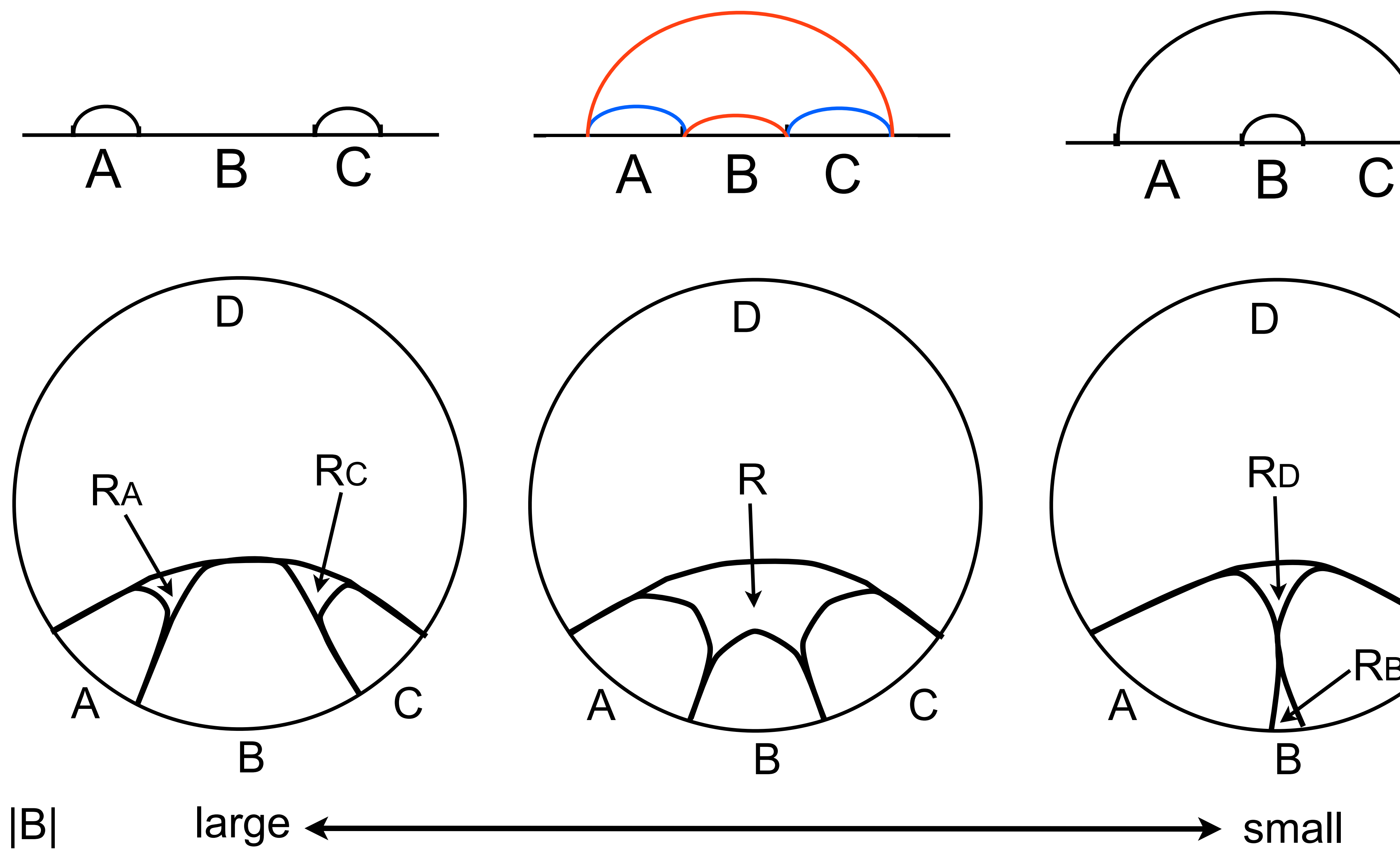
Multi-partition and residual regions



Multi-partition and residual regions

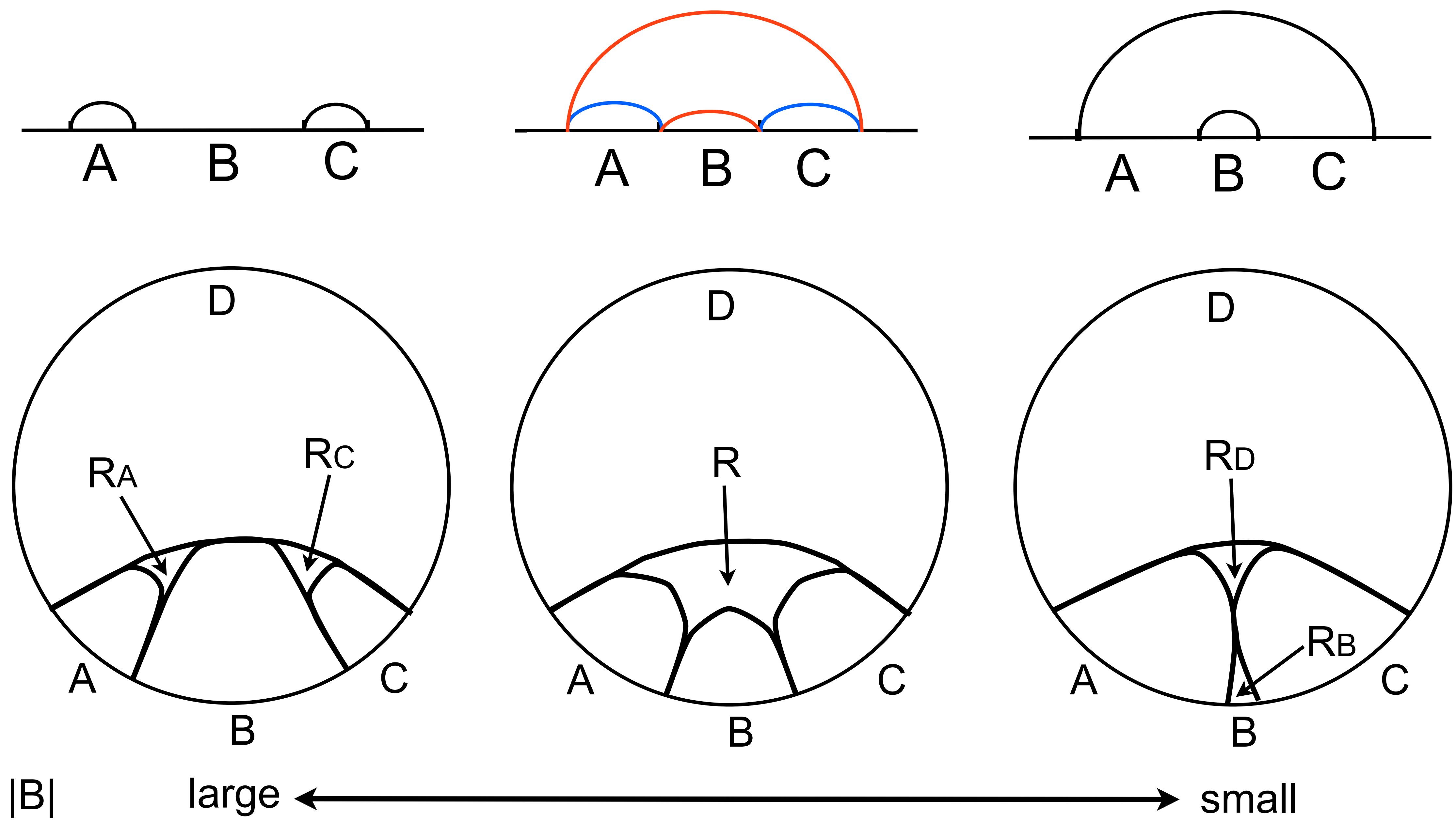


Multi-partition and residual regions

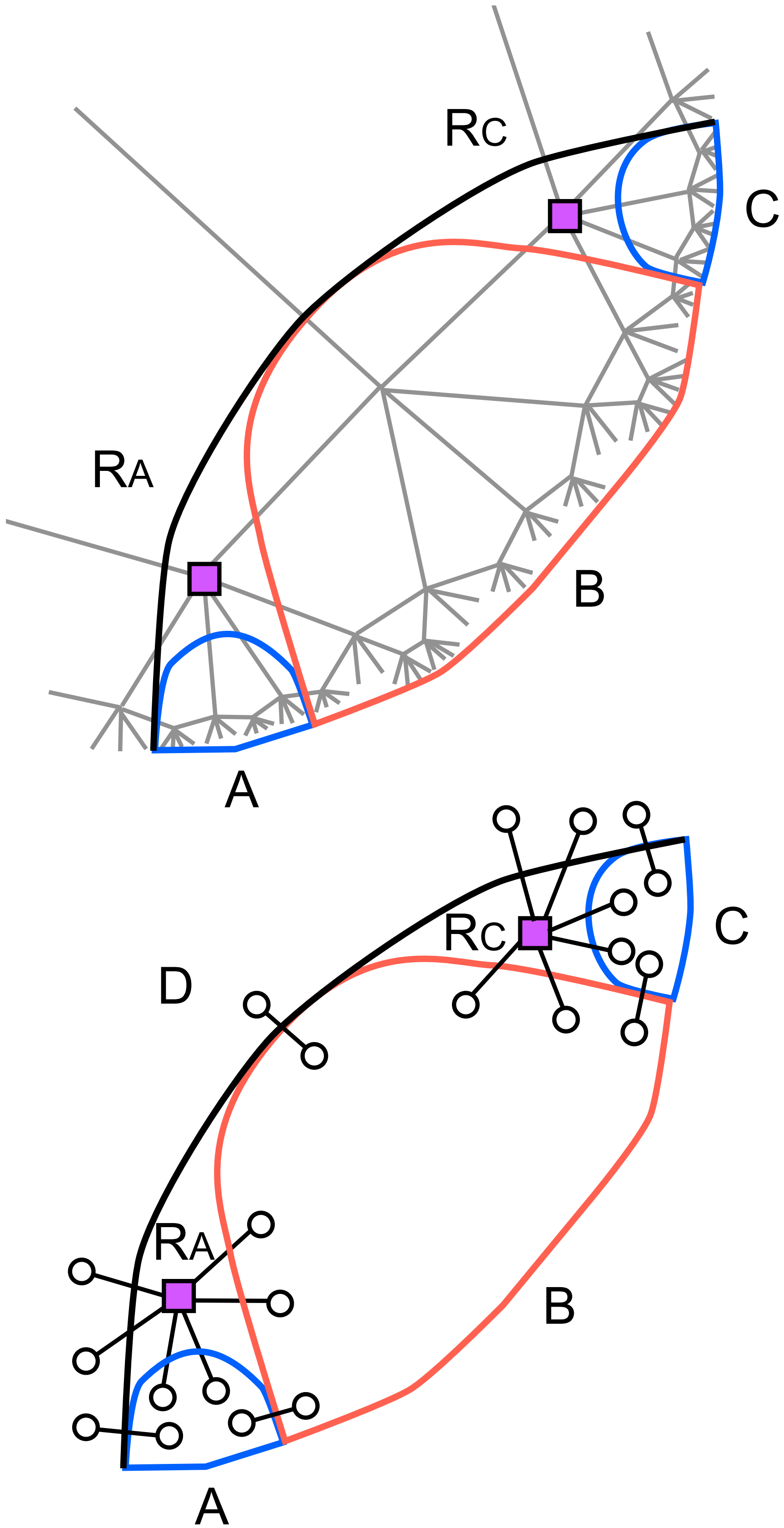


Multi-partition and residual regions

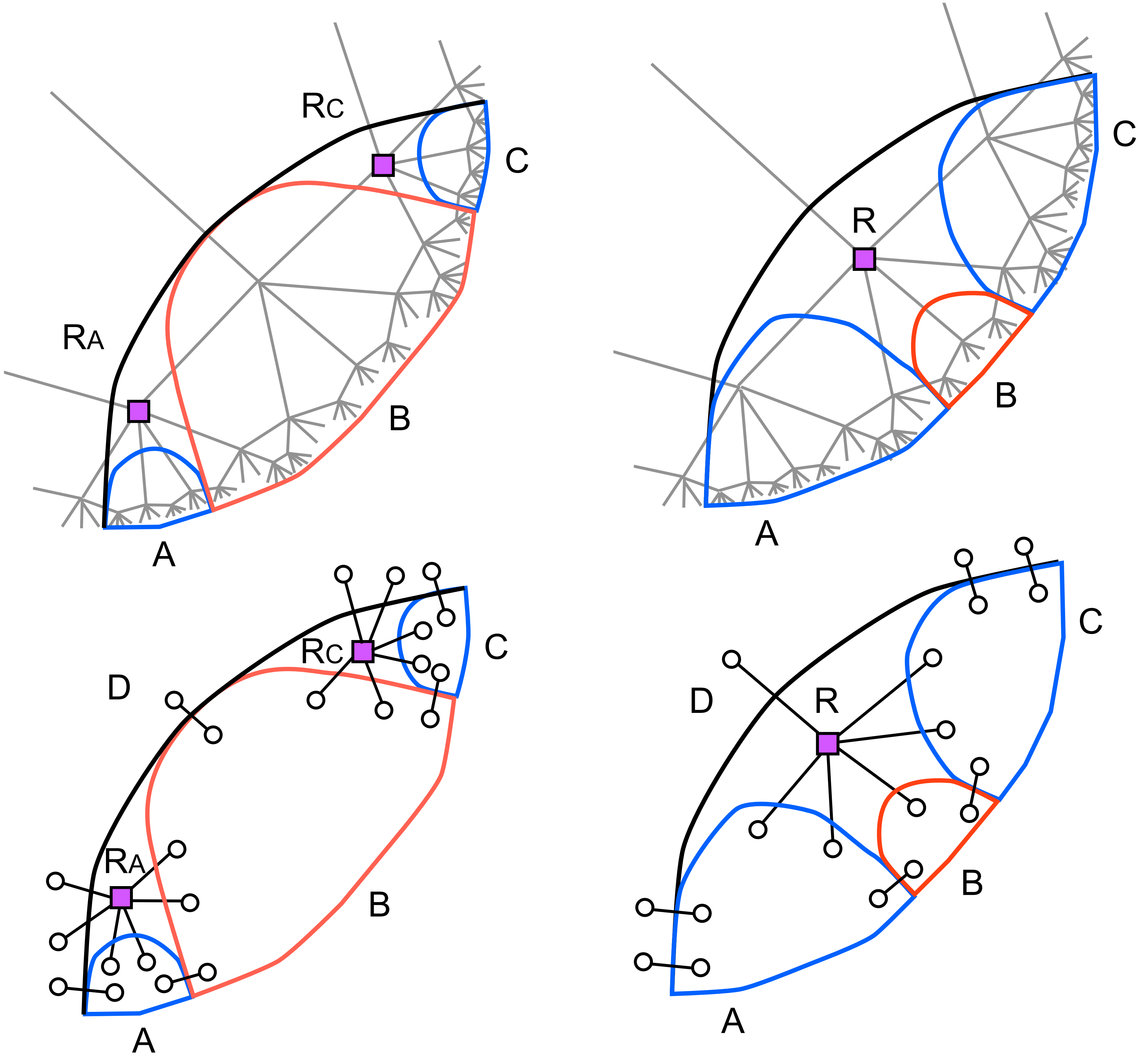
- Identified segment of geodesic lines = EPR pairs
- Residual regions = Multipartite entanglement ?



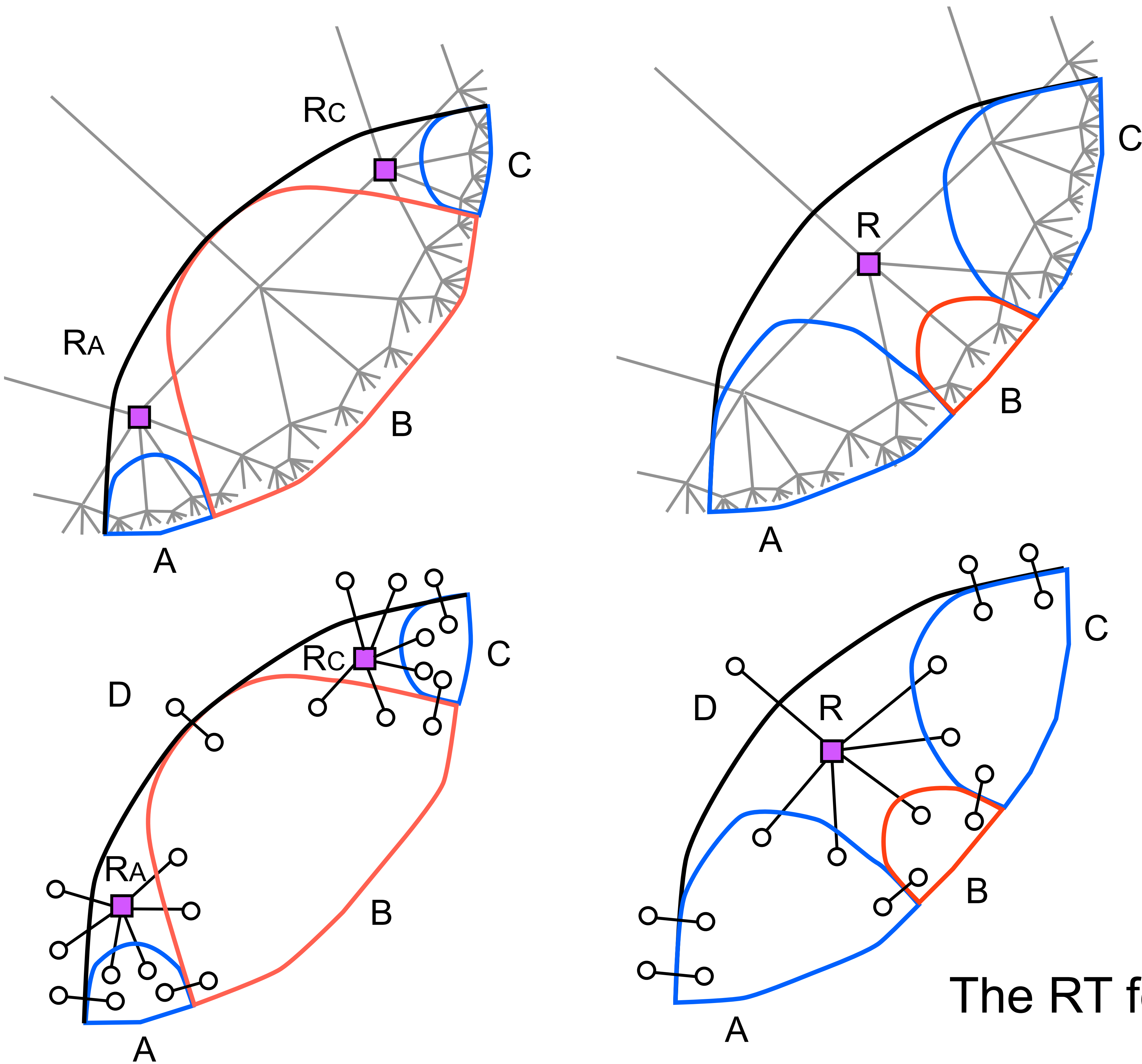
Residual regions in holographic state



Residual regions in holographic state



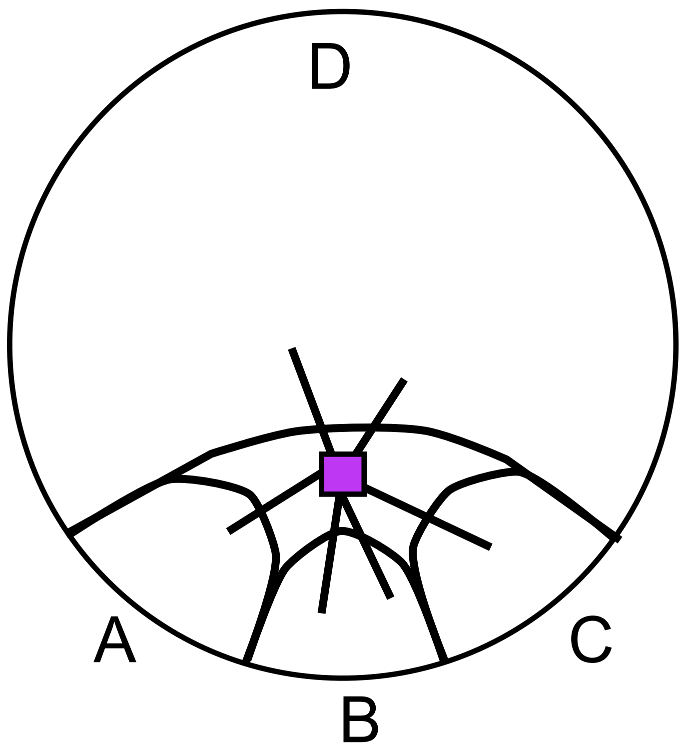
Residual regions in holographic state



The RT formula still holds

Negativity of tripartite entanglement

• Perfect tensor (state) is the key for negative tripartite entanglement !



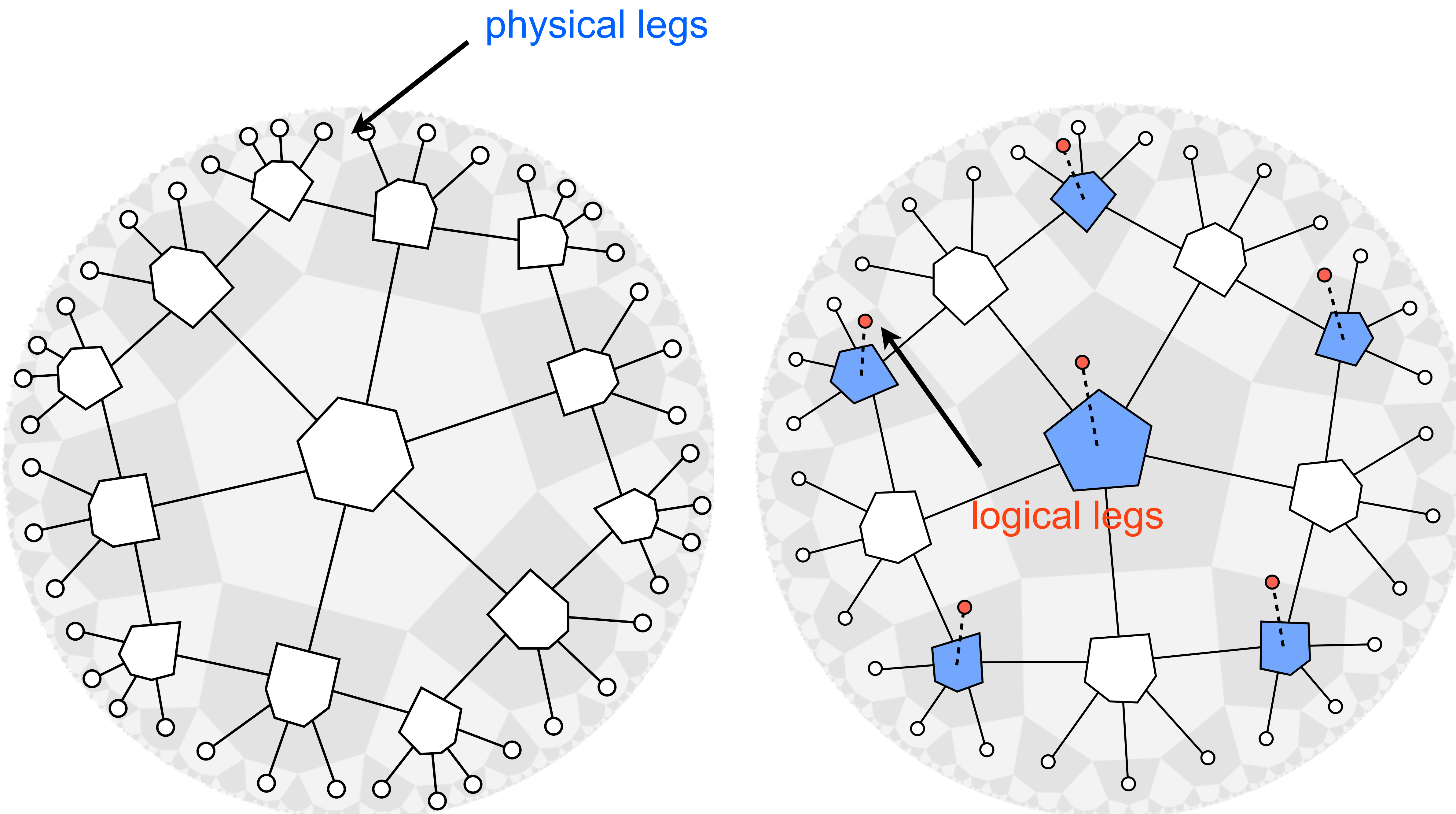
-- Split 2n-perfect state into four subsets A, B, C, D.

-- Assume 0< |A|, |B|, |C|, |D| < n+1

Then the tripartite entanglement is always negative !

$$I(A,B,C) = S_A + S_B + S_C - S_{AB} - S_{AC} - S_{BC} + S_{ABC}$$

Holographic quantum state / code



holographic state

holographic code

Future works

Fast scrambler ?   Computational complexity and Einstein-Rosen bridge ?