Mazur-Suzuki bounds in holography

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TCM

Drude weight

Absence of interactions, impurities, lattice defects $\longrightarrow$ Ballistic transport
Infinite dc-conductivity

$$K\delta(\omega)$$

Ideal transport not expected in interacting systems; exceptions:

1-dim systems:

Spin ½-XXZ chain Zotos (99), Fujimoto *et al.* (03), Sirker *et al.* (11), Karrasch *et al.* (12)

Repulsive Hubbard Fujimoto *et al.* (98)

Existence of $\mathbf{K}$ is guaranteed by the Mazur-Suzuki bounds

Mazur (69), Suzuki (71)

In this talk

Mazur (classical) and Suzuki (quantum) bounds

How to compute MS-bound in holographic theories

MS bounds in translational invariant holographic theories

Saturation
Non-saturation

Application of MS bounds in theories with weak breaking of translational invariance

Mazur *classical* – Suzuki *quantum*

Mazur

Exact conservation laws $\Longrightarrow$ Bound on $\bar{A}^2$

$\frac{dQ_k}{dt} = 0$ Orthogonal Q_i

$$\bar{A} \equiv \lim_{t\to\infty} \frac{1}{t} \int_0^\infty dt' A(t')$$

$$\boxed{\langle \bar{A}^2 \rangle_\beta = \lim_{t\to\infty} \frac{1}{t} \int_0^t \langle A(0) A(t') \rangle_\beta \geq \sum_i \frac{\langle A Q_i \rangle_\beta^2}{\langle Q_i Q_i \rangle_\beta}}$$

Mazur 69

Mazur *classical* – Suzuki *quantum*

Mazur

Exact conservation laws

$$\frac{dQ_k}{dt} = 0 \quad \text{Orthogonal } Q_i$$

$\Longrightarrow$

Bound on $\bar{A}^2$

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Mazur (69)

Suzuki

$$\boxed{\lim_{t\to\infty} \frac{1}{t} \int_0^t \langle A(0)B(t') \rangle_\beta = \sum_{i=1}^{\infty} \frac{\langle AQ_i \rangle_\beta \langle BQ_i \rangle_\beta}{\langle Q_i^2 \rangle_\beta}}$$

Suzuki (71)

If nonzero then A and B are nonergodic

Re-derivation: 1-d quasi-local cons. quantities.
Infinite chain. Lieb-Robinson bounds.

Illievski *et al.* (13)

Mazur-Suzuki bound in holography

$$K = \frac{\beta}{V} \lim_{t\to\infty} \langle J(t) J(0) \rangle \geq K_{\text{MS}} \equiv \frac{\beta}{V} \sum_{i}^{N} \frac{\langle J Q_i \rangle^2}{\langle Q_i Q_i \rangle} \, , N < \infty$$

In holographic theories with translational invariance:

$$J = J_x \qquad\qquad Q_i = \Pi_x$$

$$\langle J_x \Pi_x \rangle = \frac{1}{\beta} \lim_{\omega\to 0, q\to 0} \left[G_{\Pi_x J_x}(\omega, q) - G_{\Pi_x J_x}(0, q) \right]$$

$$\langle \Pi_x \Pi_x \rangle = \frac{1}{\beta} \lim_{\omega\to 0, q\to 0} \left[G_{\Pi_x \Pi_x}(\omega, q) - G_{\Pi_x \Pi_x}(0, q) \right]$$

Universality in holographic models

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$$\sigma_{\mathrm{dc}} = \sigma_{\mathrm{Q}} + K\delta(\omega)$$

- Einstein-Maxwell (EM)
- EM-dilaton (EMd)
- Single R-charge background (d=5 *N*=2 $U(1)^3$ gauged SUGRA)

'Universal' Drude weight:

T>0, boundary is AdS,
massless gauge field

$$K_{\mathrm{U}} = \frac{\rho^2}{\epsilon + P}$$

Davison *et al.* (15)

- Multiple R-charges (STU) black holes

$$\boxed{K_{ij} = (-1)^{i+j}\,\frac{\rho_i \rho_j}{\epsilon + P}}$$

Saturation of MS bound

Saturation of MS bound

Einstein Maxwell (EM): d+1 dim. Reissner-Nördstrom

$$\langle \Pi_x \Pi_x \rangle \propto \chi_0 = \epsilon + P \qquad \langle J_x \Pi_x \rangle \propto \rho$$

$$K_{\mathrm{MS}} = \frac{\beta}{V_{d-1}} \frac{\langle J_x \Pi_x \rangle^2}{\langle \Pi_x \Pi_x \rangle} = \frac{\rho^2}{\epsilon + P} \quad (= K = K_{\mathrm{U}})$$

1 R-charged black hole:

$$\sigma \sim i \frac{Q^2}{2\kappa^2 \omega L} + \mathcal{O}(\omega')$$

DeWolfe *et al.* (11)

$$G_{\Pi_x, \Pi_x}(\omega, q) = \ldots \qquad G_{\Pi_x J_x}(\omega, q) \ldots$$

Son *et al.* (06)

$$K_{\mathrm{MS}} = \frac{Q^2}{2\kappa^2 L} \quad (= K = K_{\mathrm{U}})$$

EM-dilaton: $K_{\mathrm{MS}} = K = K_{\mathrm{U}}$

$$K = K_{\mathrm{U}} \underset{?}{\Longrightarrow} \text{saturation}$$

Deviation from universality:

Deviation from universality: U(1) symmetry breaking

$$K = K_U + K_{\text{Superfluid}}$$

EM+complex scalar

$$S = S_{EM} - \frac{1}{2\kappa^2}\int d^{d+1}x|\partial\Psi + iA\Psi|^2 + m^2|\Psi|^2$$

Gubser (08)
Hartnoll *et al.* (08)

$\langle J_x \Pi_x \rangle \quad \langle \Pi_x \Pi_x \rangle$ same as normal phase (RN) $\quad \Psi = 0$

$$K_{\text{MS}} = \frac{\rho^2}{\epsilon + P} < K$$

EMd+A² Franco *et al.* (09), Goutéraux *et al.* (13), Ren *et al.* (15)

$$S_{\text{EMdW}} = \frac{1}{2\kappa^2}\int d^{p+1}x\sqrt{-g}\left[R - \frac{1}{2}(\partial\phi)^2 + V(\phi) - \frac{Z(\phi)}{4}F^2 - \frac{W(\phi)}{2}A_\mu A^\mu\right]$$

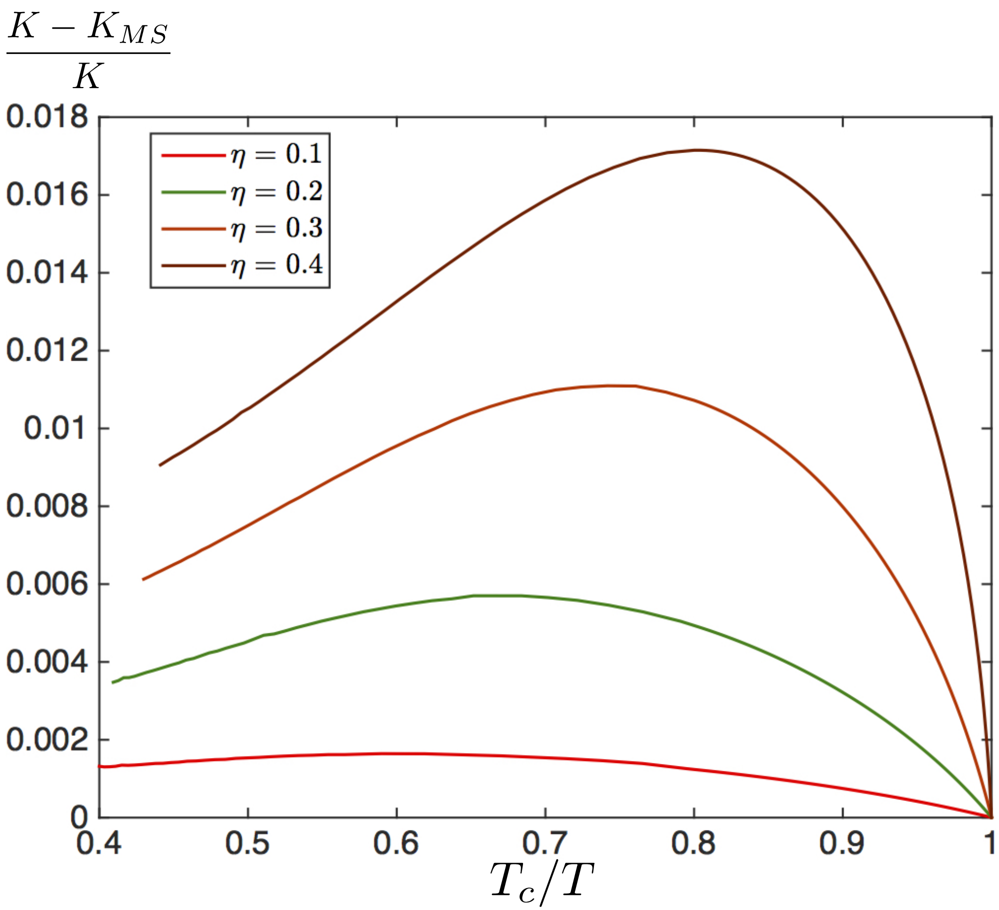
2nd order PT: $Z'(\phi = 0) = 0$

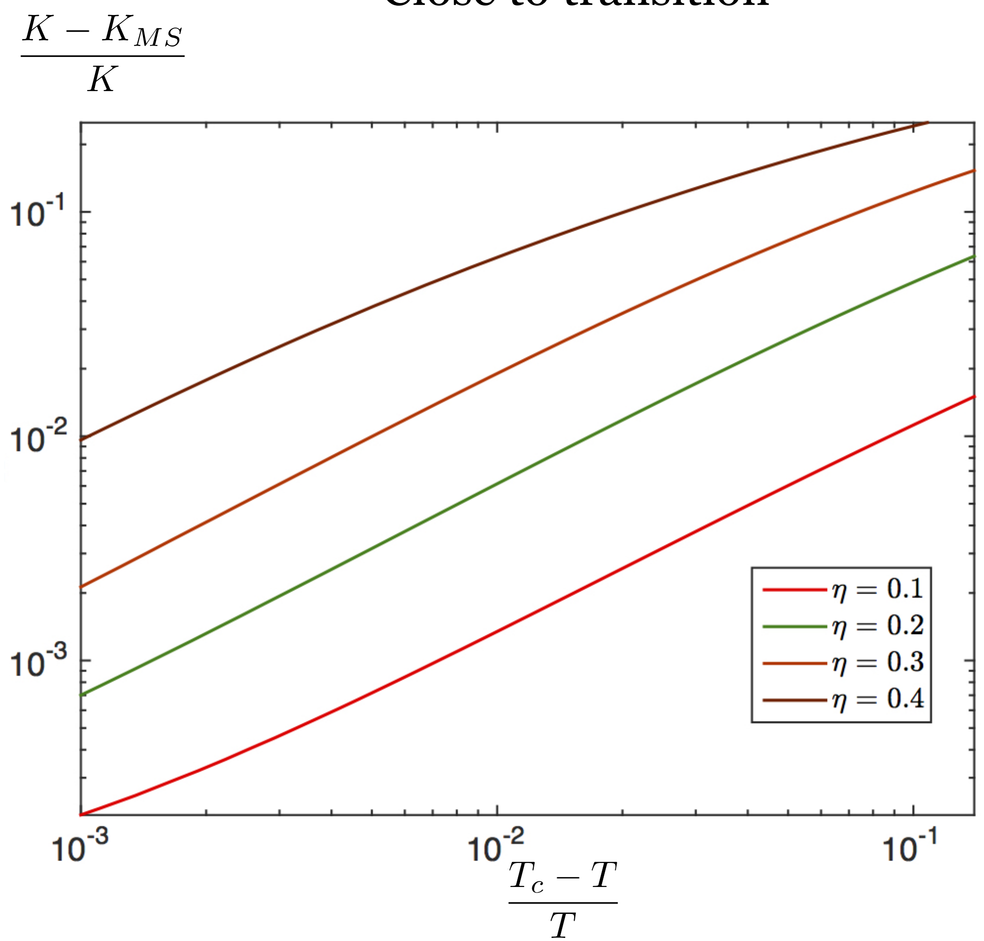
Deviation from universality: U(1) symmetry breaking

$$K > K_{\rm MS} = \frac{\rho^2}{\epsilon + P}$$

$$K_{\rm Superfluid} \propto \langle \mathcal{O} \rangle^2 \propto \frac{T_c - T}{T_c}$$

Close to transition





Parameters: $Z(\phi) = \cosh(\gamma\phi)$, $V(\phi) = -2\Lambda - \frac{2m^2}{\delta^2}\sinh^2(\delta\phi)$, $W(\phi) = W_0\left[-1 + \cosh^2(\eta\phi/2)\right]$

$\delta\gamma = 1$, $\lambda = 1/2$, $W_0 = 1$

Deviation from universality: Lifshitz

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Asymptotically Lifshitz Einstein-Procca

$$S = \frac{1}{16\pi G_{d+2}} \int d^{d+1}x\sqrt{-g}\left(R - 2\Lambda - \frac{1}{4}F^2 - \frac{1}{2}m^2A^2\right)$$

Korovin *et al.* (13)

$$m^2 = d - 1 + (d-2)\eta^2$$

$$ds^2 = -c(r)b(r)^2dt^2 + \frac{dr^2}{c(r)} + r^2(dx^2 + dy^2),$$

$$c = \underbrace{\boxed{c_0(r)}}_{\downarrow} + \eta^2\mu^2c_1(r) \qquad b = 1 + \eta^2\mu^2b_1(r)$$

AdS Schwarzschild
black brane

$$\Updownarrow$$

$$z = 1 + \eta^2$$

Deviation from universality: Lifshitz

$$\delta g_{xt} = \eta^2 g_1 e^{-i\omega t}, \quad \delta A_x = \eta a_x e^{-i\omega t}$$

$$a_x \simeq \left(1 - \frac{r_0^3}{r^3}\right)^{-i\frac{\omega}{3r_0}} \left(\underbrace{\boxed{a_x^{(0)}(r)}}_{\downarrow\, K} + \underbrace{\omega a_x^{(1)}(r)}_{\downarrow\, \sigma_Q} + \dots\right)$$

Drude weight: $K = \frac{\alpha}{16\pi G} \frac{\eta^2}{36 r_0}$ $\quad \alpha > 0$

'Universal' prediction: $K_{\mathrm{U}} \propto \mu^2 \eta^2$

MS-bound: $K_{\mathrm{MS}} = 0 + \mathcal{O}(\eta^3)$

Weak breaking of translational invariance

Weak breaking of translational invariance

EMd+axions

EMd+A²+axions
(U(1) sym. breaking)

$$\psi_I = \alpha x_I$$

$$S_{\text{axion}} = -\frac{1}{2\kappa^2}\int \sqrt{-g}\frac{Y(\phi)}{2}\sum_I^{d-1}(\partial\psi_I)^2 d^{d+1}x$$

Weak breaking: $\alpha \ll T \quad \tau \gg T$

$\tau?$ Dominant quasi-normal mode

$$\begin{array}{c}\text{Translationally} \\ \text{invariant theory} \\ K \geq K_{\text{MS}}\end{array} \xrightarrow[\alpha \ll T]{} \begin{array}{c}\text{Perturbed theory} \\ \sigma_{\text{dc}}?\end{array}$$

Bound on (part of) conductivity?

EMd

$$K_{\mathrm{MS}}$$

MS-bound of translationally invariant theory

EMd+axions

$$\sigma_{\mathrm{dc}} = Z_H C_H^{\frac{d-3}{2}} + \frac{\rho^2}{\alpha^2 C_H^{\frac{p-1}{2}} Y_H}$$

Goutéraux (14)

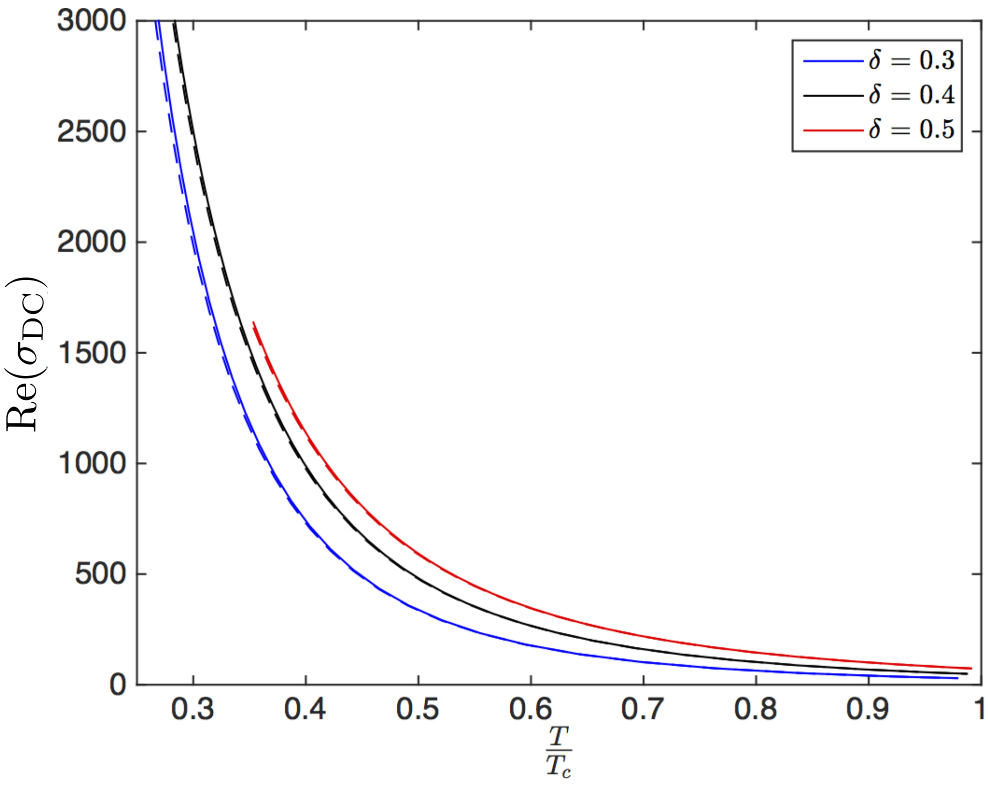
Bound on K $\longrightarrow$ bound on $\sigma_{\mathrm{dc}}^{\mathrm{reg}}$ for a given τ?

$$\boxed{\sigma_{\mathrm{dc}}^{\mathrm{reg}} = Z_H C_H^{\frac{d-3}{2}} + K_{\mathrm{MS}}\tau}$$

Bound on conductivity?

$$\sigma_{\text{dc}}^{\text{reg}} = Z_H C_H^{\frac{d-3}{2}} + K_{\text{MS}}\tau \qquad \text{(dashed lines)}$$

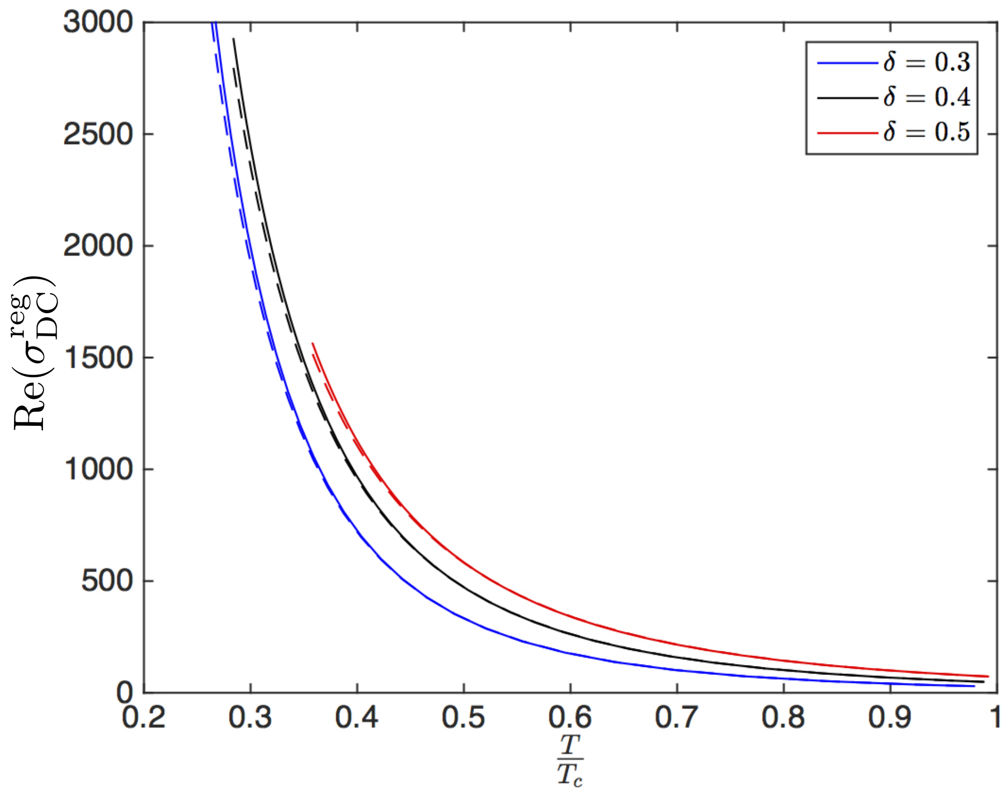
EMd+axions



$$\delta\gamma = 1 \;, \lambda = \frac{1}{2} \;, \frac{\alpha}{r_0} = \frac{1}{2}$$

EMd+A²+axions

$K_{Superfluid}\delta(\omega)$



$$\delta\gamma = 1 \;, \lambda = \frac{1}{2} \;, \frac{\alpha}{r_0} = \frac{1}{2} \qquad W_0 = 1 \;, \frac{\eta}{r_0} = \frac{1}{10}$$

Summary

$$K = \frac{\beta}{V} \lim_{t\to\infty} \langle J(t)J(0)\rangle \geq K_{\rm MS} \equiv \frac{\beta}{V} \sum_i^N \frac{\langle JQ_i\rangle^2}{\langle Q_iQ_i\rangle}$$

- MS bound saturated if $K = K_{\rm U} = \dfrac{\rho^2}{\epsilon + P}$
- MS bound is not saturated in:

  Asymptotically Lifshitz $K_{\rm MS} = 0$

  Spontaneous U(1) symmetry breaking

- $K_{\rm MS}$ (transl. inv. theory) controls the *coherent* part of $\sigma_{\rm dc}^{\rm reg}$ (weak breaking transl. inv.)

Curiosities:

- Saturation of MS-bound: Generalized Gibbs Ensemble? Mierzejewski (14)
- Existence of Drude weight: nonergodicity. *Quantum integrability?*

  Zotos *et al.* (97) Castella (95)

Deviation from universality: Lifshitz 2

Asymptotically Lifshitz EMd $F = dA\,, G = dB$

$$S = \frac{1}{16\pi G_4}\int d^4x\sqrt{-g}\left(R - 2\Lambda - \frac{1}{2}\partial_\mu\phi\partial^\nu\phi \;-\frac{e^{\lambda_1\phi}}{4}F^2 - \frac{e^{\lambda_2\phi}}{4}G^2\right)$$

$B_t(r\to\infty)\to\mu$ Tarrío *et al.*, 11

$A_t(r\to\infty)\to\infty$

$$z = 2$$

$$\delta g_{xt} = \tilde{g}_{xt}e^{-i\omega t}, \quad \delta B_x = \tilde{B}_x e^{-i\omega t}$$

MS-bound:

$$K_{\mathrm{MS}} = 0 < K$$