On interaction-free measurements

NII
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PHYSICAL REVIEW LETTERS

VOLUME 74 | 12 JUNE 1995 | NUMBER 24

Interaction-Free Measurement

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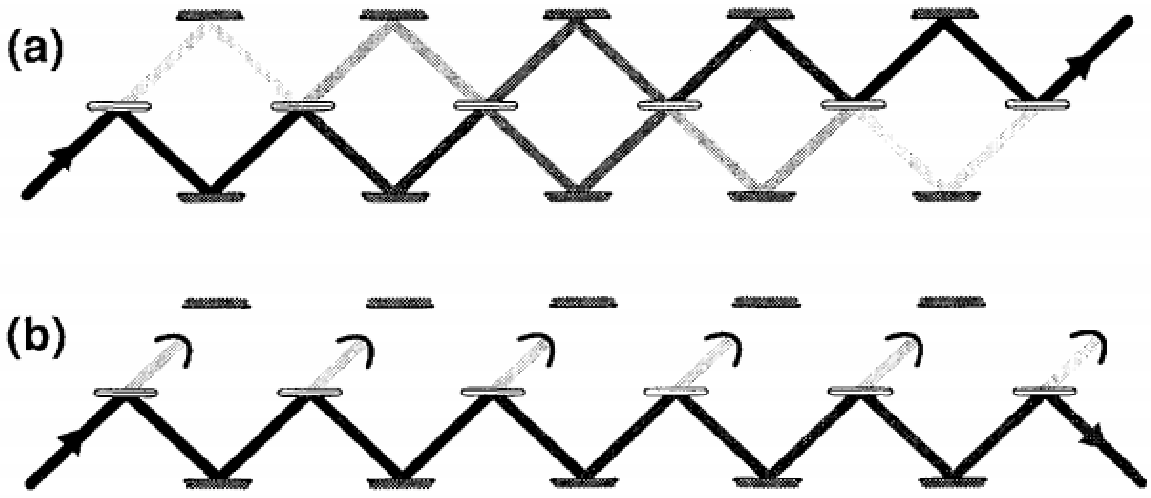
(Received 19 September 1994)

We show that one can ascertain the presence of an object in some sense without interacting with it. One repeatedly, but weakly, tests for the presence of the object, which would inhibit an otherwise coherent evolution of the interrogating photon. The fraction of "interaction-free" measurements can be arbitrarily close to 1. Using single photons in a Michelson interferometer, we have performed a preliminary demonstration of some of these ideas.

PACS numbers: 03.65.Bz, 42.50.Dv

Detection of the presence of something interacting with the incoming particle, with high probability, but with almost no interaction at all

KWHZK protocol



(a) nothing is in the blackbox

(b) something is present, the amplitude into the backbox is negligible -> almost no interaction !!

The state of the object inside is almost unchanged

$$D_{box} = O(T^{-1})$$

D_{box}: distortion of the state in the box
T: times of the interaction

In this talk ….

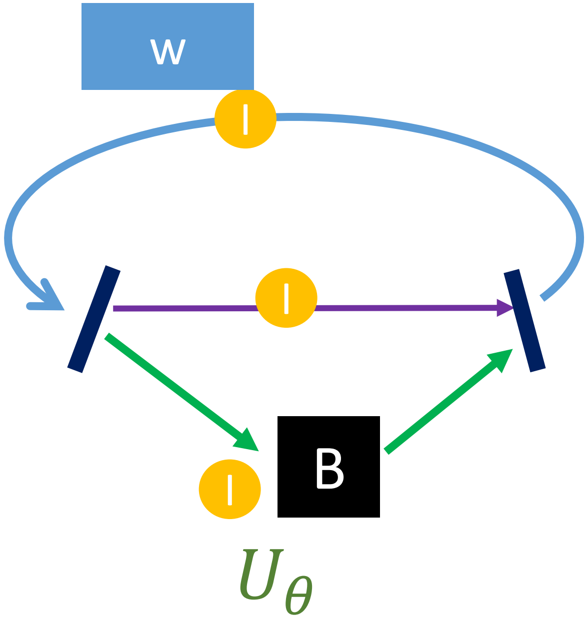
1. Optimality of KWHZK protocol in a certain setting
   (adversary method,
   classical technique of query complexity)

$$D_{box} \geq O(T^{-1})$$

2. Detection of unitary operation with almost certainty,
   and no distortion at all to the input at all to the input particle.

$$D_{input} = 0$$

KWHZK is optimal : setting



The blackbox operation is given as c-U

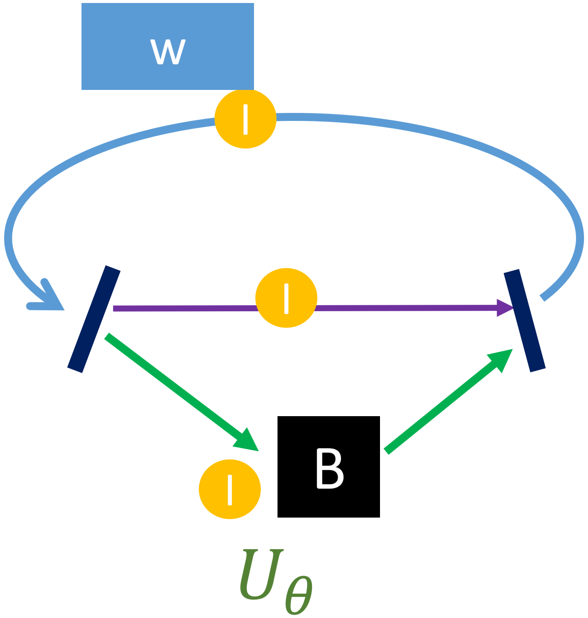
$$\begin{bmatrix} 1 & 0 \\ 0 & U_\theta \end{bmatrix}$$

$$U_0 = 1, U_1 \neq 1$$

$c - U_\theta$: unitary on $H_C \otimes H_I \otimes H_B$

which path = $H_C = \mathbf{C}^2$

KWHZK is optimal : setting



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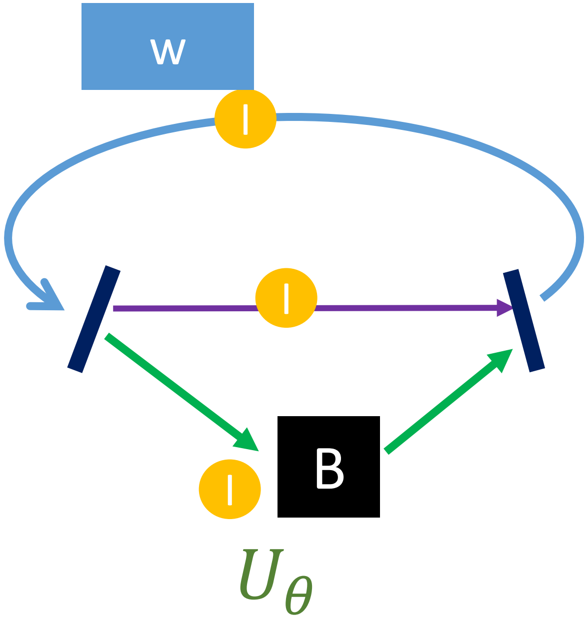
$H_B = H_{B'}^{\otimes n}$: initial state $|f>^{\otimes n}$

at each interaction, U acts one of $H_{B'}$ ‘s ,

(the part interacting with the input is always |f>)

KWHZK is optimal : setting



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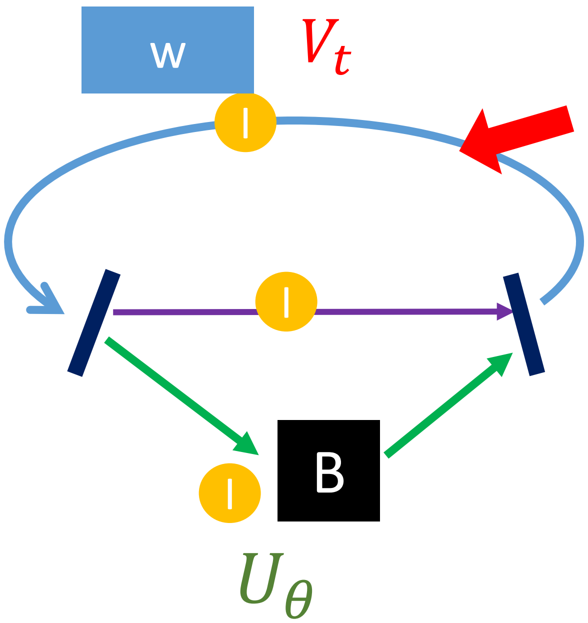


Do some information processing on the particle,

V_t: Unitary on $H_W \otimes H_C \otimes H_{\mathrm{I}}$
t : t-th loop, Hw : working space

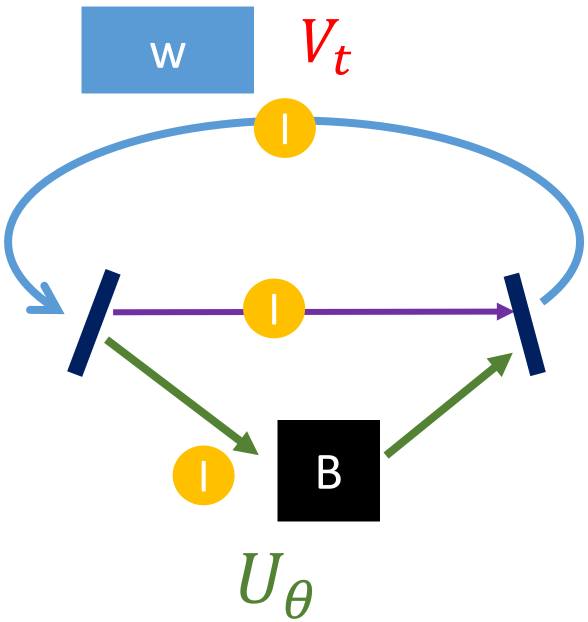
$$\begin{bmatrix} 1 & 0 \\ 0 & U_\theta \end{bmatrix} \qquad U_0 = 1, U_1 \neq 1$$

$c - U_\theta$: unitary on $H_C \otimes H_I \otimes H_{B'}$

which path = $H_C = \mathbf{C}^2$

$H_B = H_{B'}^{\otimes n}$: initial state $|f>^{\otimes n}$

KWHZK is optimal : setting



I: the inner degree of the input particle

B: the box , = n copies of B'

C: which path=$\mathbf{C}^2$

W : working space

$c - U_\theta$: on $H_C \otimes H_I \otimes H_{B'}$,

$$\begin{bmatrix} 1 & 0 \\ 0 & U_\theta \end{bmatrix} \quad U_0 = 1, U_1 \neq U_I \otimes U_B$$

V_t: Unitary on $H_W \otimes H_C \otimes H_I$

t : t-th loop, W: working space

$H_B = H_{B'}^{\otimes n}$: initial state $|f>^{\otimes n}$

After T times loop,

We measure other than B

KWHZK is optimal : statement

$|\Phi_{\theta,t}\rangle$ The state of the whole system after the t-th loop when U_θ is true blackbox op

$$D_t := \left\| \left| \Phi_{\theta,t} \right\rangle \left\langle \Phi_{\theta,t} \right| |_{\mathcal{H}_B} - \left(|f\rangle \langle f| \right)^{\otimes t} \right\|_1$$

The distortion of the blackbox

$$\left\| \mathrm{tr}_{\mathcal{H}_B} \left| \Phi_{0,T} \right\rangle \left\langle \Phi_{0,T} \right| - \mathrm{tr}_{\mathcal{H}_B} \left| \Phi_{1,T} \right\rangle \left\langle \Phi_{1,T} \right| \right\|_1 \geq 1 - \varepsilon$$

Can distinguish U_0=1 and U_1 with prob 1-ε after T-th step

$$\boxed{D_T \geq (C+1)^{-2} (1-\varepsilon)^2 \frac{1}{T} = O\left(\frac{1}{T}\right)}$$

KWHZK is optimal : proof

$|\Phi_{\theta,t}\rangle$ is hard to handle.

$$|\psi_t\rangle := \langle f^{\otimes t}|\Phi_{\theta,t}\rangle \qquad |\varphi_t\rangle := \langle f^{\otimes t}|\Phi_{0,t}\rangle$$

on $H_W \otimes H_C \otimes H_I$

Still can bound key quantities from safer side

$$D_t \geq 1 - \| \psi_t \|^2$$

RHS:Entanglement between H_B and the rest

$$\| \mathrm{tr}_{\mathcal{H}_B} |\Phi_{0,t}\rangle\langle\Phi_{0,t}| - \mathrm{tr}_{\mathcal{H}_B} |\Phi_{1,t}\rangle\langle\Phi_{1,t}| \|_1$$

$$\leq \| |\varphi_t\rangle\langle\varphi_t| - |\psi_t\rangle\langle\psi_t| \|_1 + 1 - \|\psi_t\|^2$$

KWHZK is optimal :proof

Lemma

$$\left\| |\varphi_t\rangle\langle\varphi_t| - |\psi_t\rangle\langle\psi_t| \right\|_1 - \left\| |\varphi_{t-1}\rangle\langle\varphi_{t-1}| - |\psi_{t-1}\rangle\langle\psi_{t-1}| \right\|_1$$

$$\leq C\sqrt{\|\psi_{t-1}\|^2 - \|\psi_t\|^2} \qquad C := 6(1 - \|\langle f|U_1|f\rangle\|^2)^{-1/2}$$

KWHZK is optimal :proof

By this lemma,

$$\left\| |\varphi_T\rangle\langle\varphi_T| - |\psi_T\rangle\langle\psi_T| \right\|_1 \leq C \sum_{t=1}^{T} \sqrt{\|\psi_{t-1}\|^2 - \|\psi_t\|^2}$$

$$\leq C \sqrt{T \sum_{t=1}^{T} \left(\|\psi_{t-1}\|^2 - \|\psi_t\|^2 \right)} = C \sqrt{\left(1 - \|\psi_T\|^2\right) T}$$

Thus by

$$D_t \geq 1 - \|\psi_t\|^2$$

$$\left\| \mathrm{tr}_{\mathcal{H}_B} |\Phi_{0,t}\rangle\langle\Phi_{0,t}| - \mathrm{tr}_{\mathcal{H}_B} |\Phi_{1,t}\rangle\langle\Phi_{1,t}| \right\|_1 \leq \left\| |\varphi_t\rangle\langle\varphi_t| - |\psi_t\rangle\langle\psi_t| \right\|_1 + 1 - \|\psi_t\|^2$$

We have the assertion

Conjecture 1

By KWHZK-like protocol,

Cannot distinguish two different operations that are not identity operation

(can distinguish nothing or something,

but cannot detect what it is)

Complete distortion-free detection of unitary

Given one out of {c-U_θ}, $\theta \in \{1,..,k\}$

U_θ : acting on H_I only (not on $H_I \otimes H_B$)

can be any non-commutative unitaries

Detect θ

with arbitrarily high success probability $1-\varepsilon$

without any distortion at all to the input state !

Complete distortion-free detection
commutative case

Given one out of $\{c\text{-}U_\theta\}$, $\theta \in \{1,..,k\}$

U_θ : acting on H_I only (not on $H_I \otimes H_B$)

commute with each other

Detect θ

with arbitrarily high success probability 1-ε

without any distortion at all to the input state !

Use phase estimation

H_I is set to common eigenvectors

which never be disturbed.

can make the error probability arbitrarily small

Complete distortion-free detection of unitary

Main idea : Reduce the problem to commutative case by preprocessing

Here we treat the following special case only:
general case is similar

$$U_1 = \begin{bmatrix} e^{\sqrt{-1}a} & 0 \\ 0 & e^{\sqrt{-1}b} \end{bmatrix}$$

U_2: arbitrary 2x2

Observation: let X be one of Pauli matrices

$$U_1 X U_1 X = \begin{bmatrix} e^{\sqrt{-1}a} & 0 \\ 0 & e^{\sqrt{-1}b} \end{bmatrix} \begin{bmatrix} e^{\sqrt{-1}b} & 0 \\ 0 & e^{\sqrt{-1}a} \end{bmatrix} = e^{\sqrt{-1}(a+b)} \mathbf{1}$$

Complete distortion-free detection of unitary

$$U_1 = \begin{bmatrix} e^{\sqrt{-1}a} & 0 \\ 0 & e^{\sqrt{-1}b} \end{bmatrix}$$ U_2: arbitrary 2x2

Observation: let X be one of Pauli matrices

$$U_1XU_1X = \begin{bmatrix} e^{\sqrt{-1}a} & 0 \\ 0 & e^{\sqrt{-1}b} \end{bmatrix} \begin{bmatrix} e^{\sqrt{-1}b} & 0 \\ 0 & e^{\sqrt{-1}a} \end{bmatrix} = e^{\sqrt{-1}(a+b)}\mathbf{1}$$

So if $U_2XU_2X \neq U_1XU_1X$, the problem is reduced to The commutative case.

But what if $U_2XU_2X = U_1XU_1X$, ?

Complete distortion-free detection of unitary

$$U_1 = \begin{bmatrix} e^{\sqrt{-1}a} & 0 \\ 0 & e^{\sqrt{-1}b} \end{bmatrix}$$

U_2: arbitrary 2x2

$$X_{\vec{\lambda}} := \begin{bmatrix} 0 & e^{\sqrt{-1}\lambda_1} \\ e^{\sqrt{-1}\lambda_2} & 0 \end{bmatrix}$$

$$U_1 X_{\vec{\lambda}} U_1 X_{\vec{\lambda}} = e^{\sqrt{-1}(a+b)}\mathbf{1}$$

Lemma : if U_1 and U_2 does not commute, there is at least one $\vec{\lambda}$ such that $U_2 X U_2 X \neq c1$

Since $U_1 X_{\vec{\lambda}} U_1 X_{\vec{\lambda}} \neq U_2 X_{\vec{\lambda}} U_2 X_{\vec{\lambda}}$, we are done.

Complete distortion-free detection of unitary

If dim ≧3....

$$X^k_{U,\vec{\lambda}} := \sum_{i=0}^{d-1} e^{\sqrt{-1}\lambda_i} |e_{i+k}\rangle \langle e_i|$$

$$F_{U,\Lambda}(U') := \left(X^{d-1}_{U,\vec{\lambda}_{d-1}}\right)^\dagger U' X^{d-1}_{U,\vec{\lambda}_{d-1}} \cdot \ldots \cdot \left(X^{1}_{U,\vec{\lambda}_{1}}\right)^\dagger U' X^{1}_{U,\vec{\lambda}_{1}} \cdot \left(X^{0}_{U,\vec{\lambda}_{0}}\right)^\dagger U' X^{0}_{U,\vec{\lambda}_{0}}$$

Lemma 2 *There is* Λ *with* $F_{U,\Lambda}(U') \neq c\mathbf{1}$ *for all* U' *with* $[U,U'] \neq 0$.

If more than 2 unitaries....

1. Repeat 1 vs the rest many times
2. Since the state keeps to be a pure state all the process,
   one can make one side of the error zero
   thus can avoid the accumulation of the error probability

Yet to find out more …

1. For KWHZK protocol, optimization of the constant
2. For complete distortion-free unitary detection, trade-off between # of using the backbox and the error probability

Thanks for attentions, if you are still awake

1. Optimality of KWHZK protocol in a certain setting

$$D_{box} \geq O(T^{-1})$$

2. Dtection of unitary operation with almost certainty,

 and no distortion to the input at all to the input particle.

$$D_{input} = 0$$

Conclusion