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Interaction Effect on entanglement propagation in 2d RCFT

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Based on
arXiv:1606.xxxxx[hep-th]

Motivation

To understand how entanglement spreading depends on systems?

This leads to understand

- Scrambling in Black hole

  [Hayden-Preskill 07, Sekino-Susskind 08 etc...]

- Quantum Chaos in Many body system

  [Stanford-Shenker 13, Stanford-Shenker-Maldacena 15 etc...]

- Contrast to holographic CFTs, rational CFTs(integrable) are expected to behave oppositely

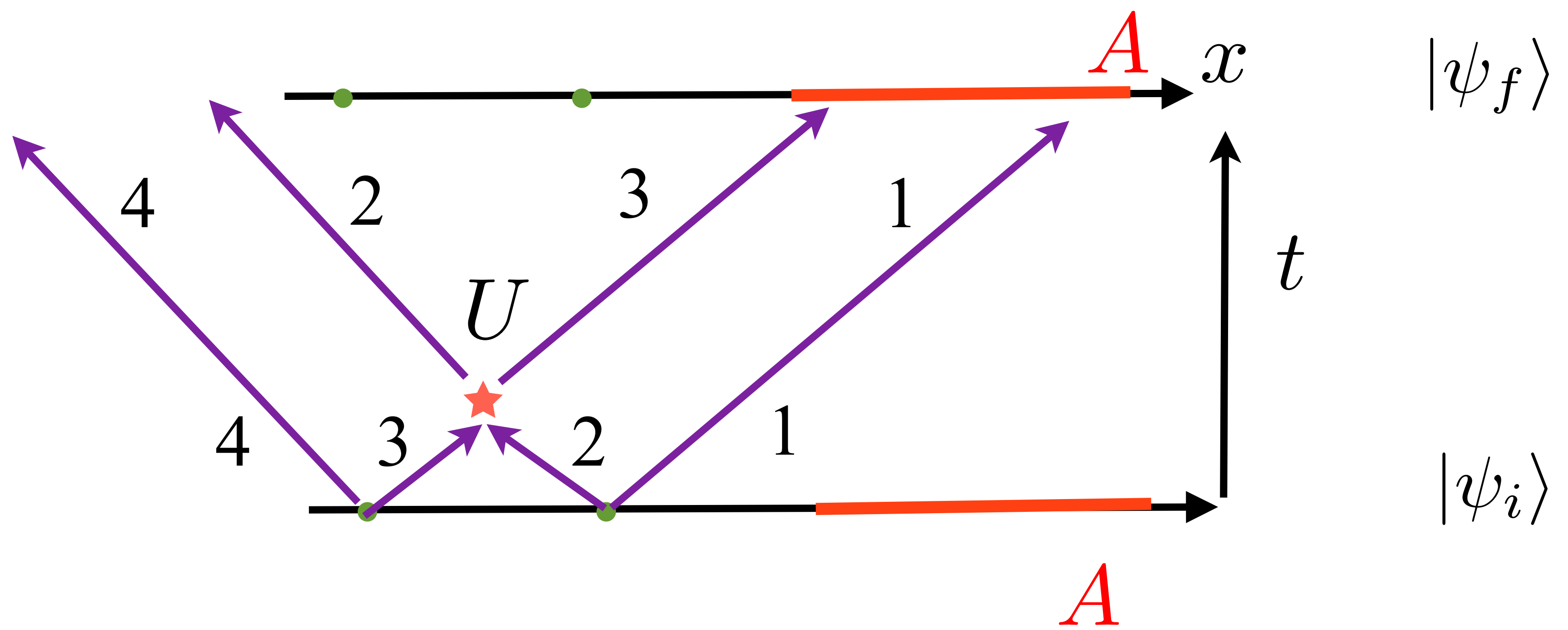
To understand these problem, we study the scattering effect on entanglement propagation in RCFTs !

Results

Entanglement is “Conserved” in Ising model.

・ scattering effect on EPR pairs

[Casini-Liu-Mezei,15]



$$|\psi_i\rangle = \frac{1}{1+|\alpha|^2}\left(|00\rangle_{12} + \alpha\,|11\rangle_{12}\right) \otimes \left(|00\rangle_{34} + \alpha\,|11\rangle_{34}\right)$$

$$|\psi_f\rangle = (1 \otimes U \otimes 1)\,|\psi_i\rangle$$

Effect on EE

$$S^{(i)}_{13} = 2(-p_\alpha \log p_\alpha - (1-p_\alpha)\log(1-p_\alpha)) \qquad p_\alpha = \frac{1}{1+|\alpha|^2}$$

$$S^{(f)}_{13} = -p_1 \log p_1 - (p_\alpha - p_1)\log(p_\alpha - p_1) - p_2 \log p_2 - (1-p_\alpha-p_2)\log(1-p_\alpha-p_2)$$

Entanglement Scrambling

[Asplund-Bernamonti-Galli-Hartman15]

Scrambling of entanglement for excited state

Global Quench: homogeneous, global excitation

[Calabrese-Cardy 05]

change the theory (Hamiltonian) at $t = 0$

$$H(\lambda_0) \to H(\lambda)$$

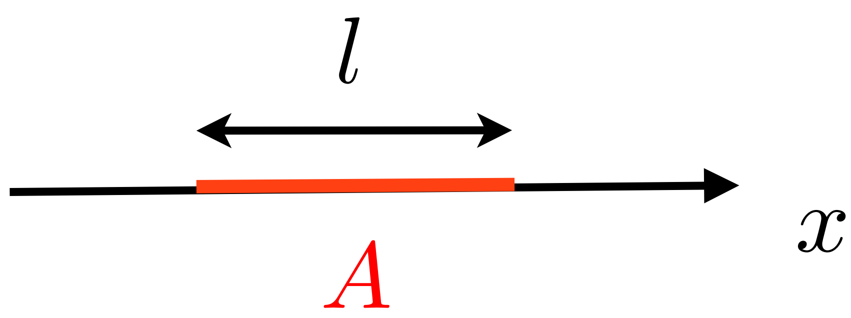
$H(\lambda_0)$:mass scale $1/\beta$ $\quad$ $H(\lambda)$: CFT

(ground state: $|\psi_0\rangle$ $\quad$ $|\psi\rangle$)

excited state for $H(\lambda)$

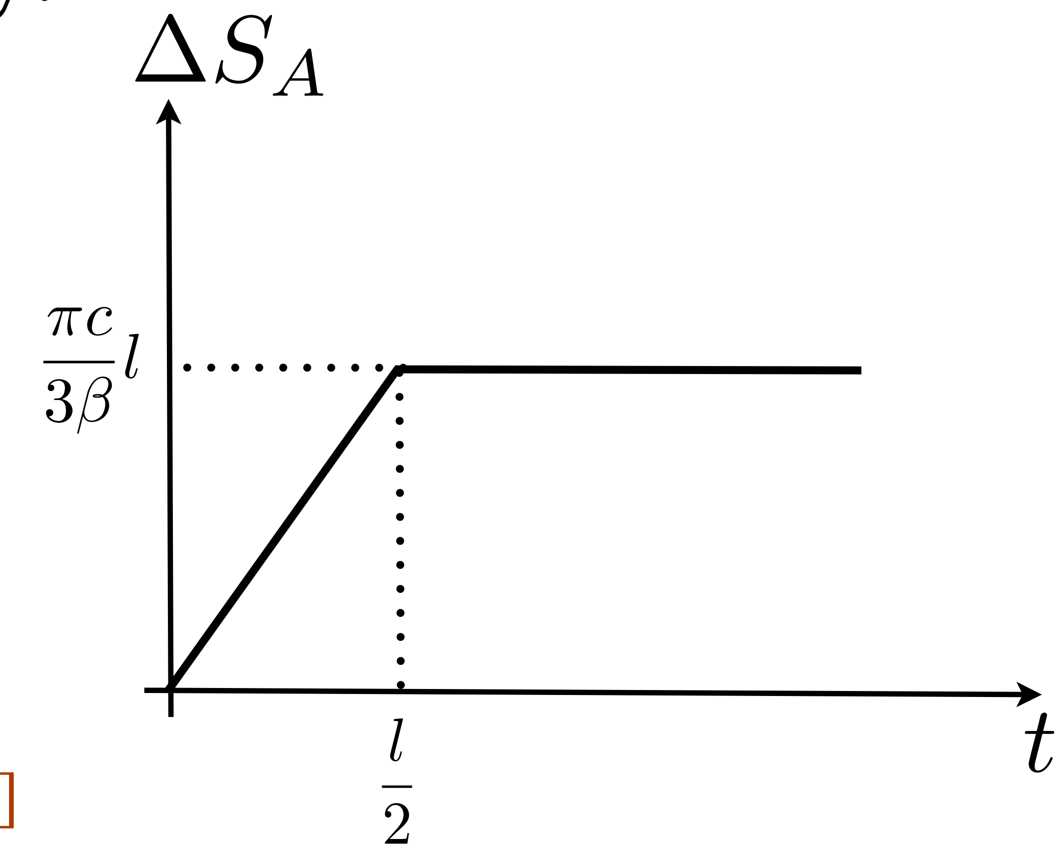
Consider the time evolution of $|\psi(t)\rangle = e^{-iH(\lambda)t}|\psi_0\rangle$

Time evolution of entanglement entropy:

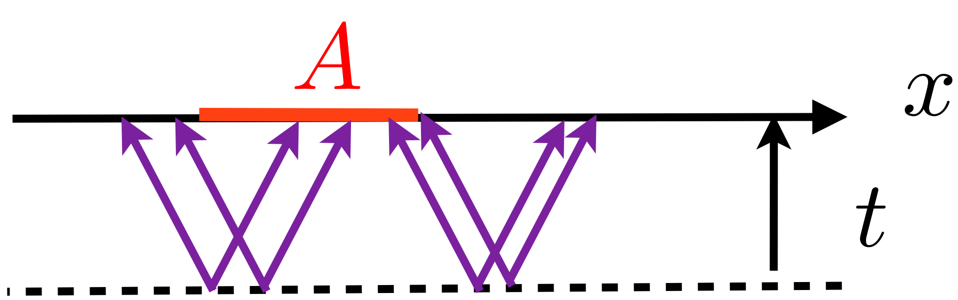


$$\Delta S_A(t) = \begin{cases} \frac{2\pi c}{3\beta} t & (t < l/2), \\ \frac{\pi c}{3\beta} l & (t > l/2) \end{cases}$$

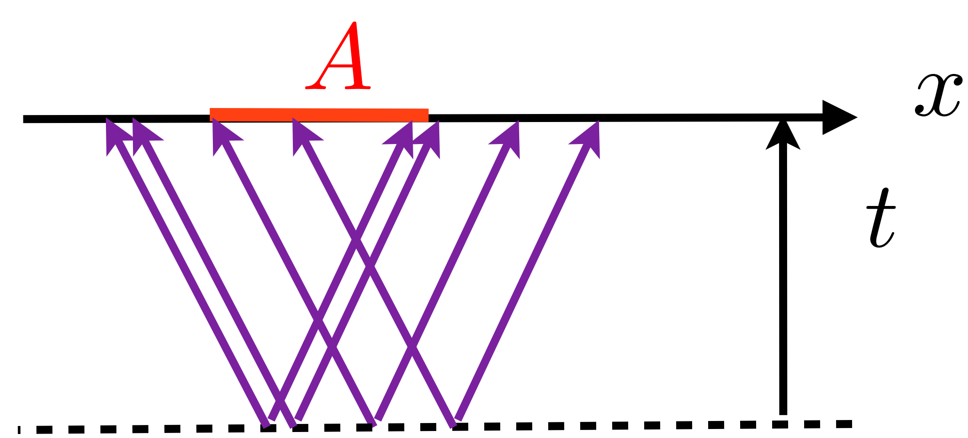
[Calabrese-Cardy 05]



Original Physical interpretation



$t < l/2$



$t > l/2$

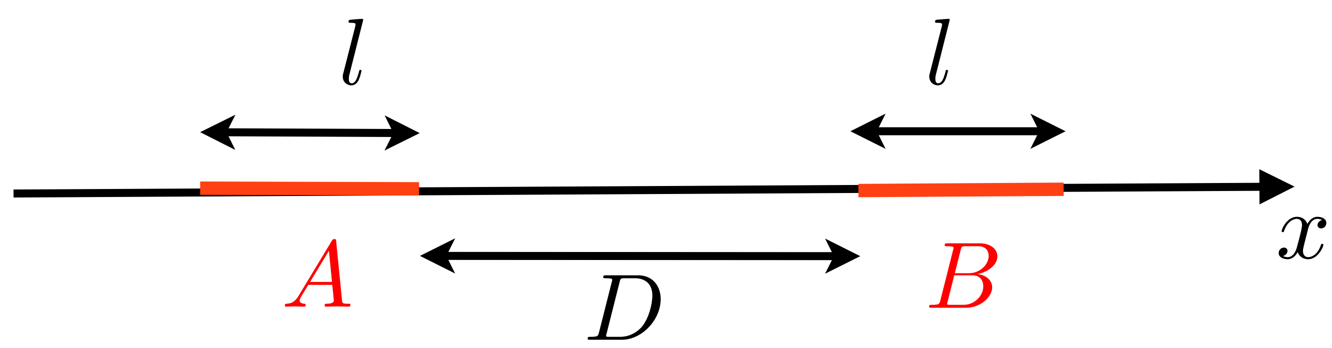
Propagation of entangling particle

At $t = l/2$, EE saturates.

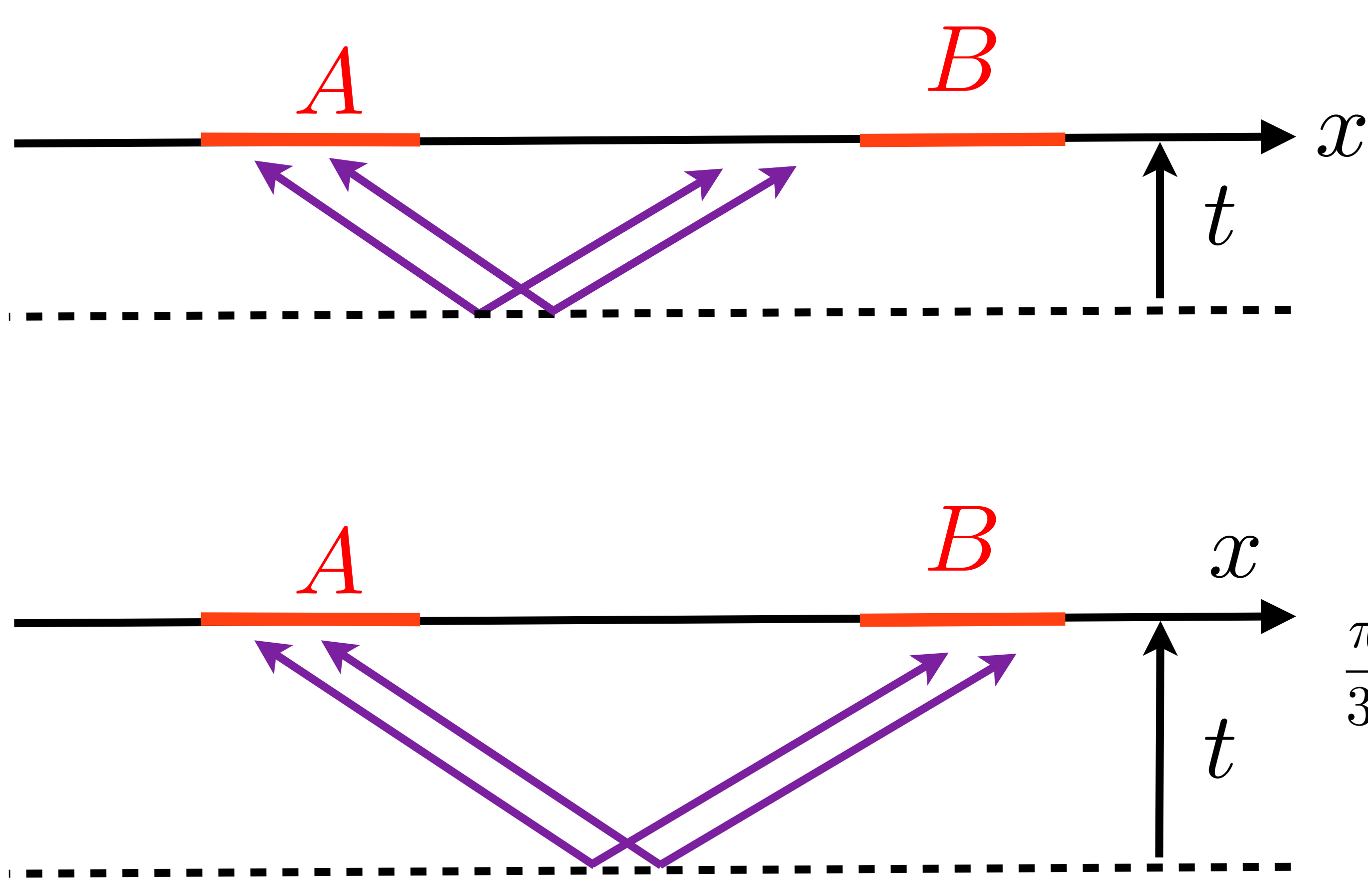
Two interval case

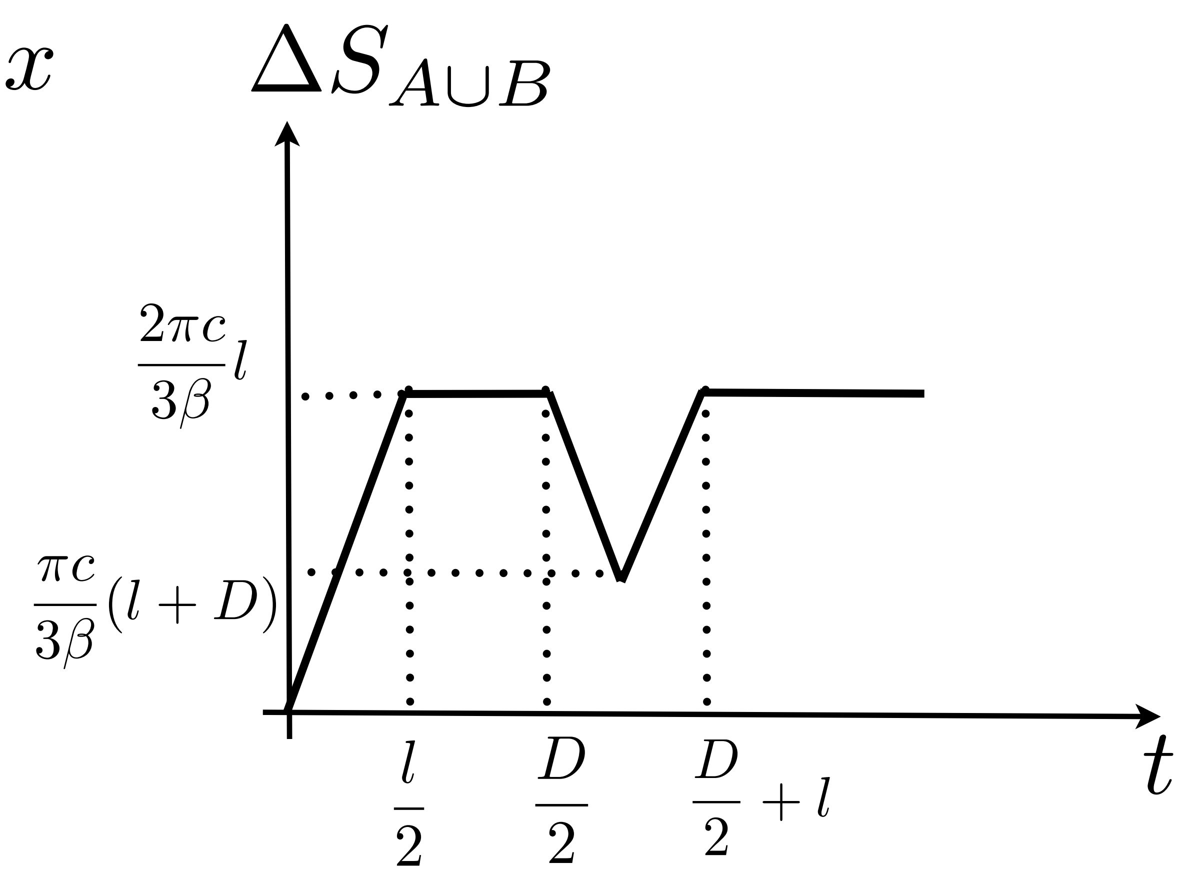
[Asplund-Bernamonti-Galli-Hartman,15]

Behavior is different in free theory and holographic CFT!

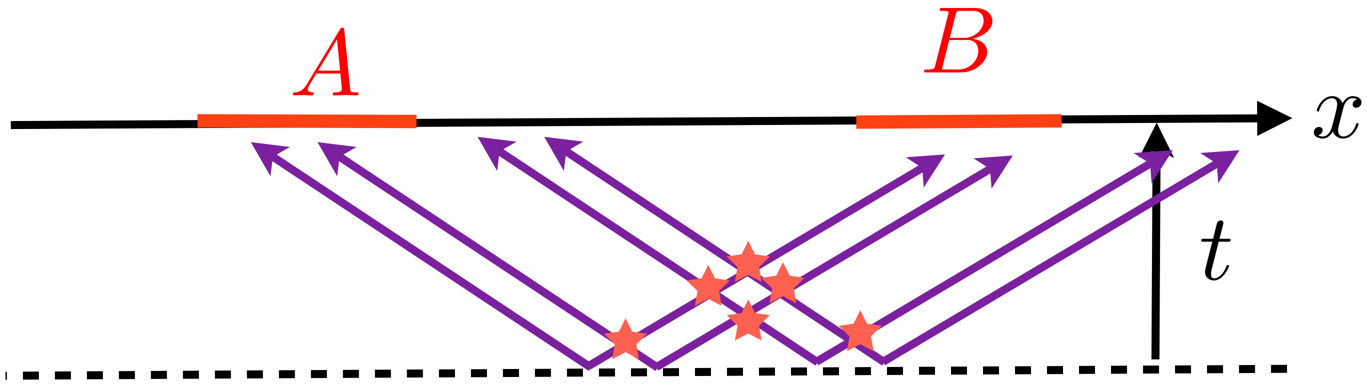


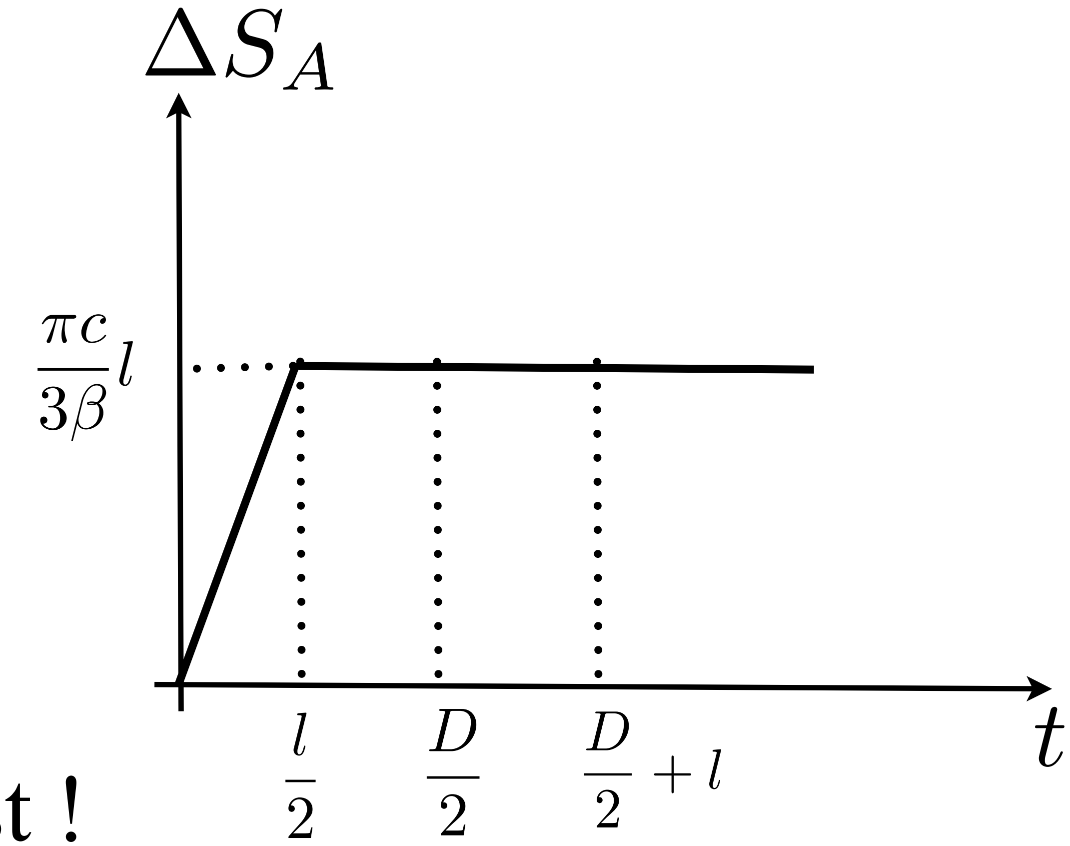
・Free theory





・Holographic CFT





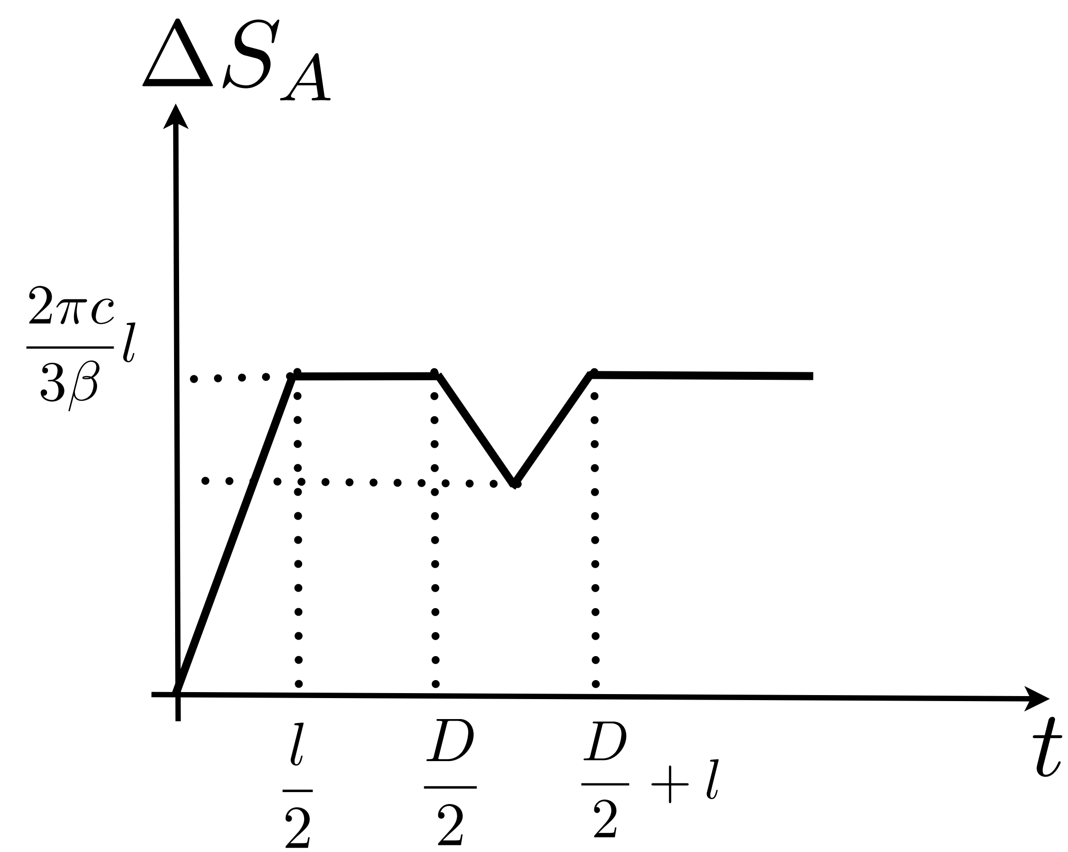
Many scattering occurs

⇒quasi-particle dip doesn’t exist !

Generically:

Dip is smaller than free theory

How about RCFT case (integrable interaction)?



In RCFT, quasiparticle dip exists

[Asplund-Bernamonti-Galli-Hartman 15]

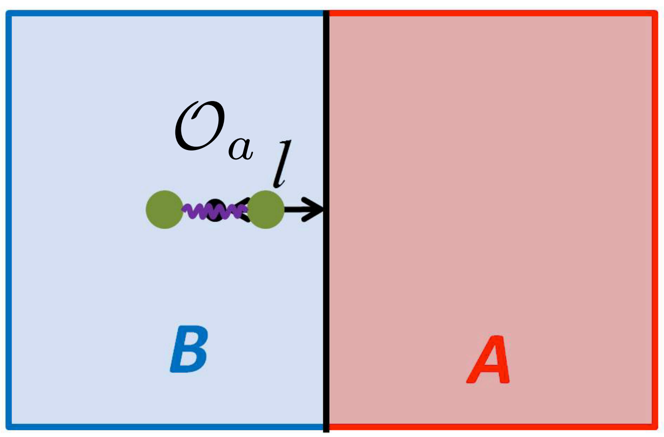
Interaction effect in RCFTs

To study the scattering effect, **let's consider the local excitation**

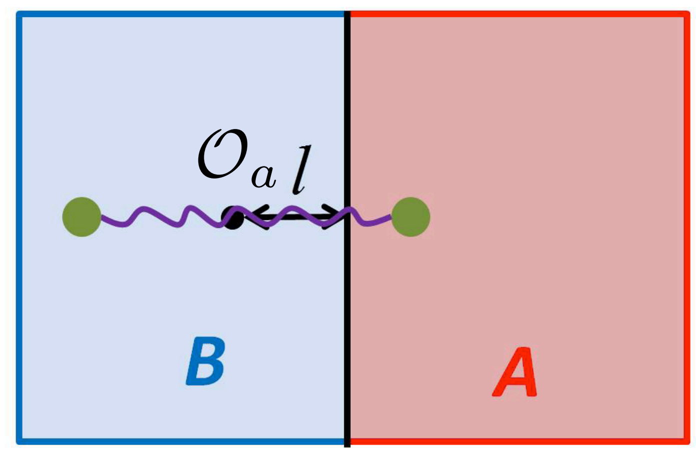
- How to prepare the (entangled) particle?

  $\Rightarrow$ Insertion of local operator

$$|\Psi\rangle \propto \mathcal{O}_a(-l)\,|0\rangle$$



$t < l$

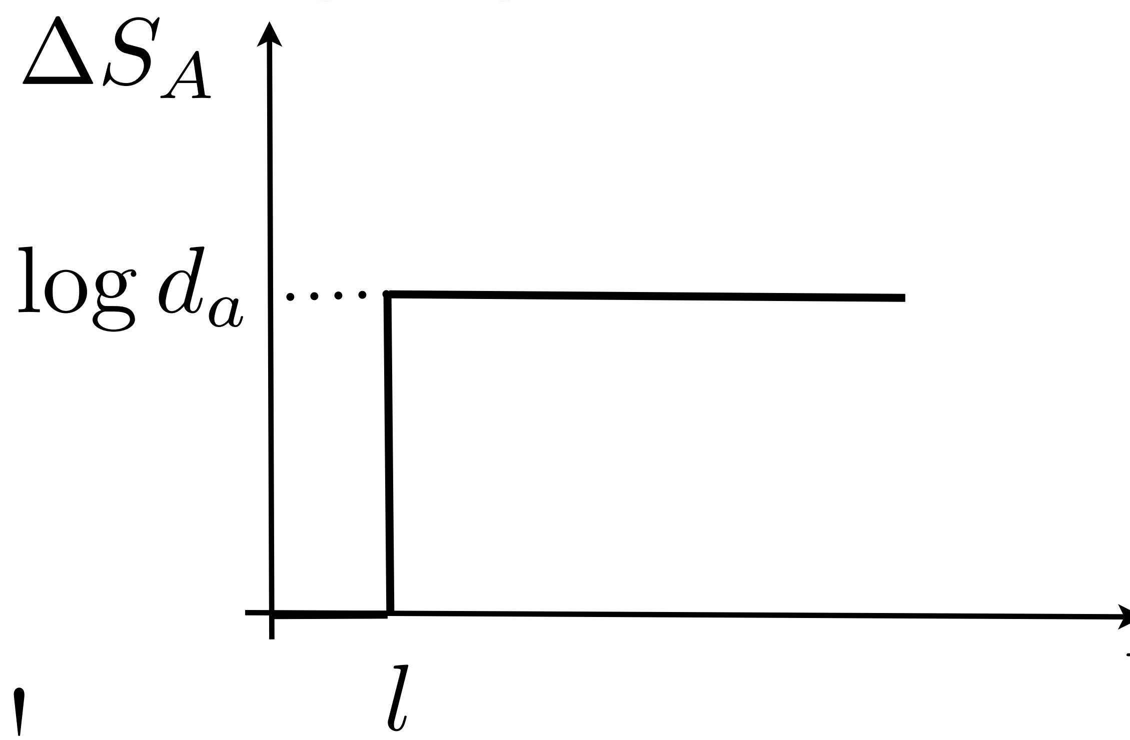


$t > l$

In rational CFT,

$$\Delta S_A^{(n)}(t) = \begin{cases} 0 & (t < l), \\ \log d_a & (t > l) \end{cases}$$

[He-TN-Takayanagi-Watanabe 14]



$\Rightarrow$created particle is entangled !

・Ising Model case

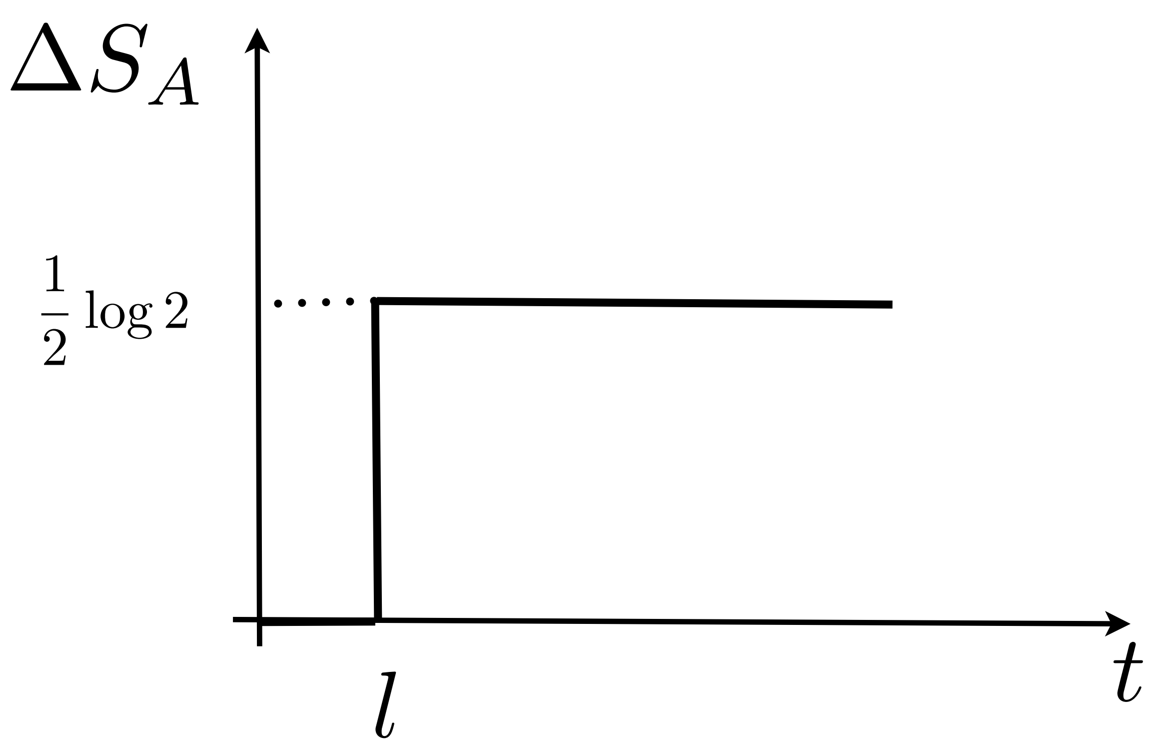
There are 3 primary fields: $\mathbb{I}$, σ , ϵ

(Identity, Spin op, Energy op)

$$\log d_{\mathbb{I}} = 0$$
$$\log d_{\sigma} = \log \sqrt{2}$$
$$\log d_{\epsilon} = 0 \quad \text{(no entanglement)}$$

⇒Time evolution of (Renyi) EE for spin op. is given by

$$\Delta S_A^{(n)}(t) = \begin{cases} 0 & (t < l), \\ \frac{1}{2}\log 2 & (t > l) \end{cases}$$



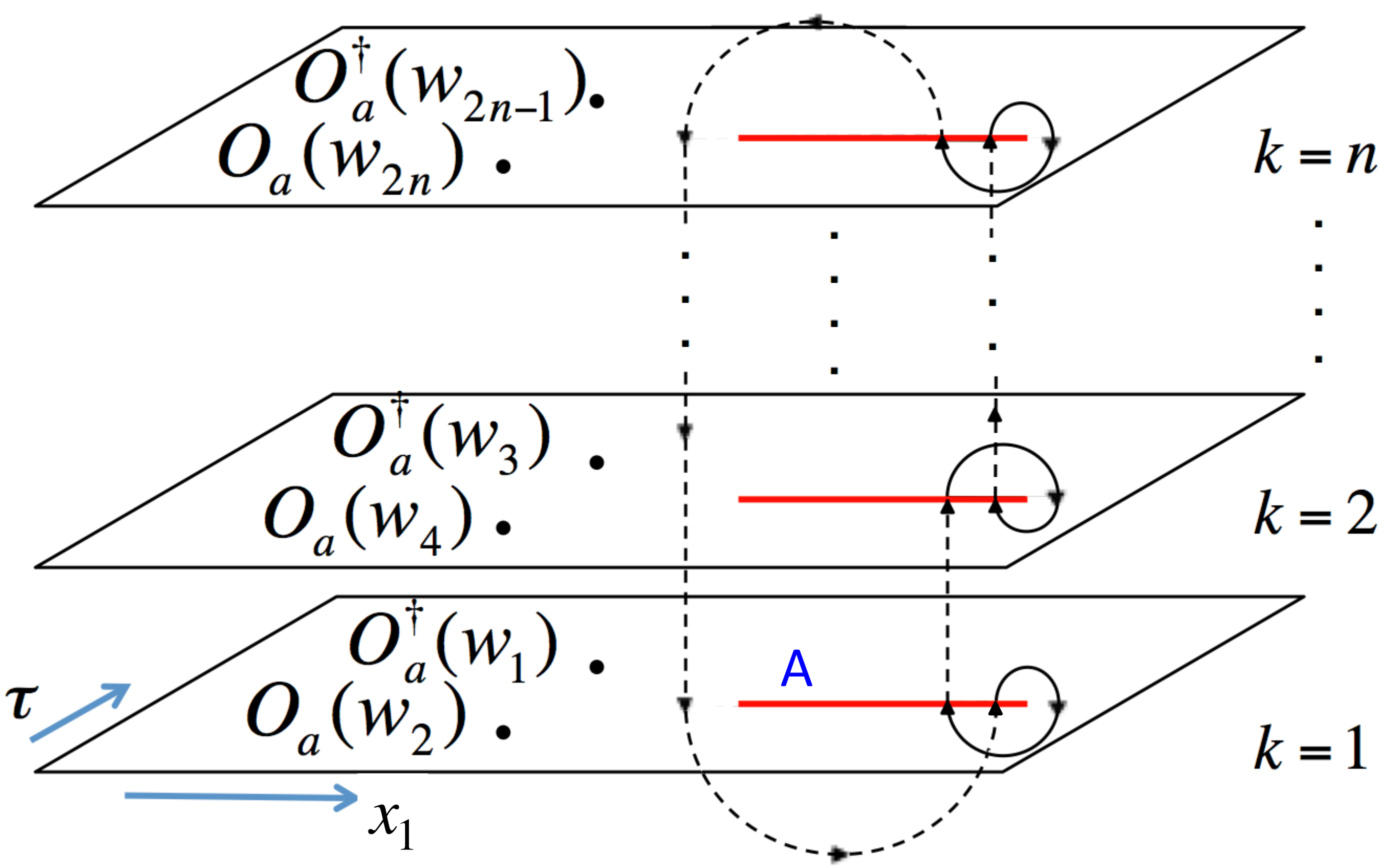
- Calculation of (Renyi) EE

⇒Use ***Replica Method***

We can express the $\mathrm{Tr}_A \rho_A^n$ in terms of 2n-pt correlation function on Σ_n :

$$
\begin{aligned}
\Delta S_A^{(n)} &= \frac{1}{1-n}[\log \mathrm{Tr}_A(\rho_A^{(ex)})^n - \log \mathrm{Tr}_A(\rho_A^{gr})^n] \\
&= \frac{1}{1-n}[\log \langle \mathcal{O}^\dagger(w_1)\mathcal{O}(w_2)\cdots\mathcal{O}^\dagger(w_{2n-1})\mathcal{O}(w_{2n})\rangle_{\Sigma_n} - \log \langle \mathcal{O}^\dagger(w_1)\mathcal{O}(w_2)\rangle_{\Sigma_1}]
\end{aligned}
$$

[Nozaki-TN-Takayanagi 14]

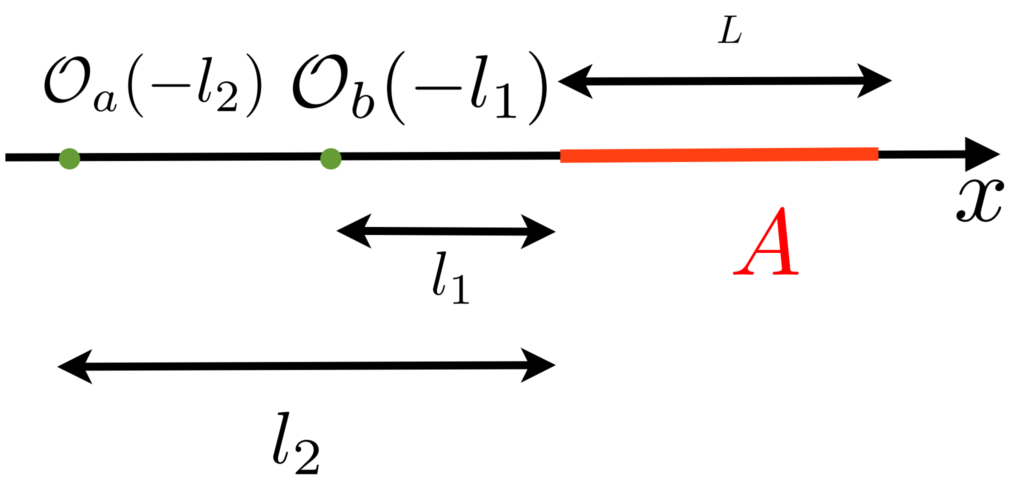


w_{2k-1}, w_{2k} :the coodinate of the inserted local operator on the k-th sheet

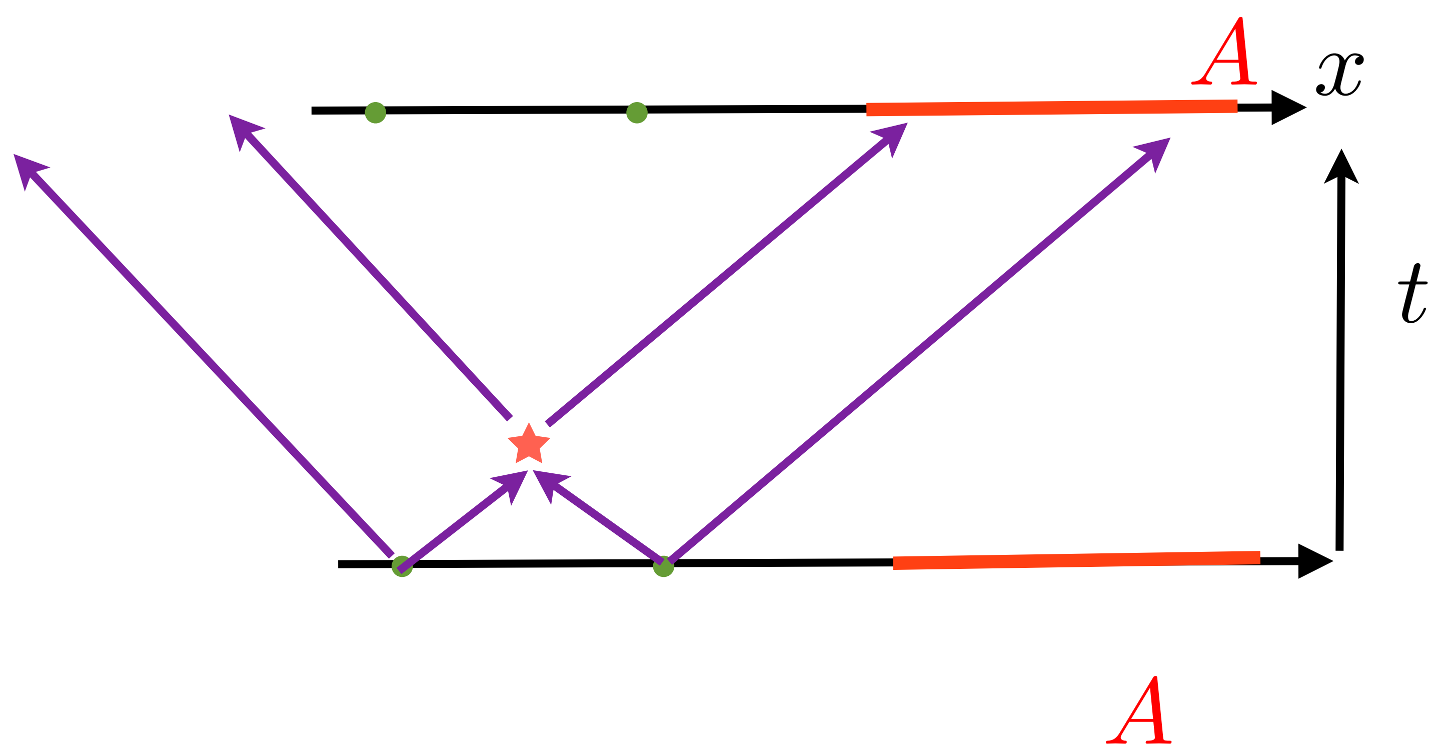
・two excitation

Insertion of two local operator

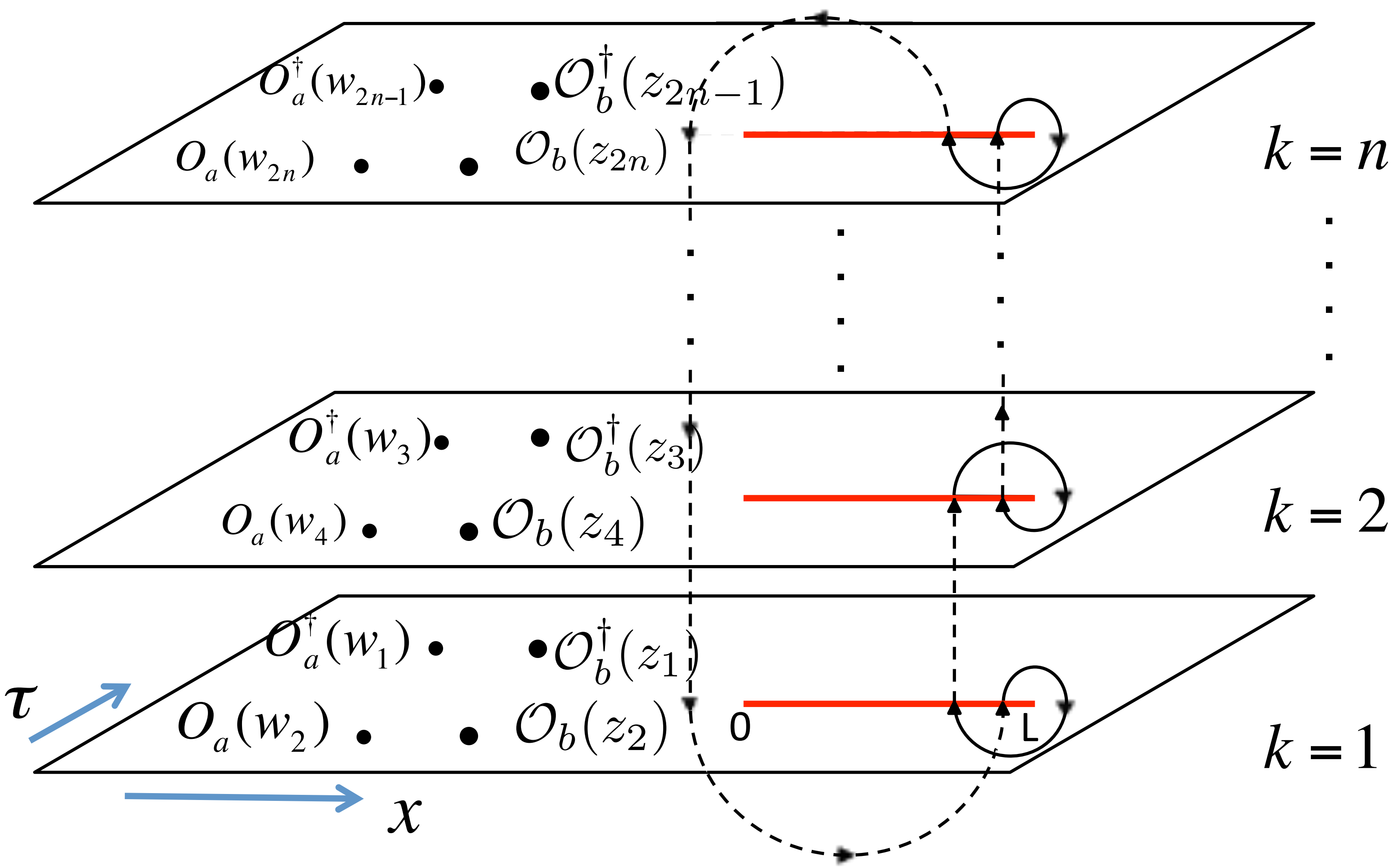
$$|\Psi\rangle \propto \mathcal{O}_a(-l_2)\mathcal{O}_b(-l_1)\,|0\rangle$$



scattering at $t = \dfrac{l_2 - l_1}{2}$



- Calculation of (Renyi) EE



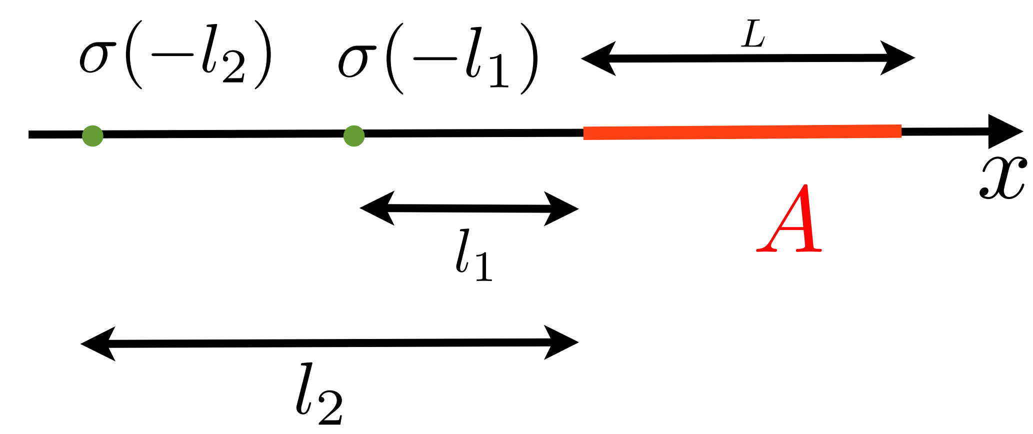
For example, we need 8pt function to calculate 2nd Renyi

⇒Consider Ising CFT (all correlation function are known)

・Result [TN in preparation]

Insertion of two local operator

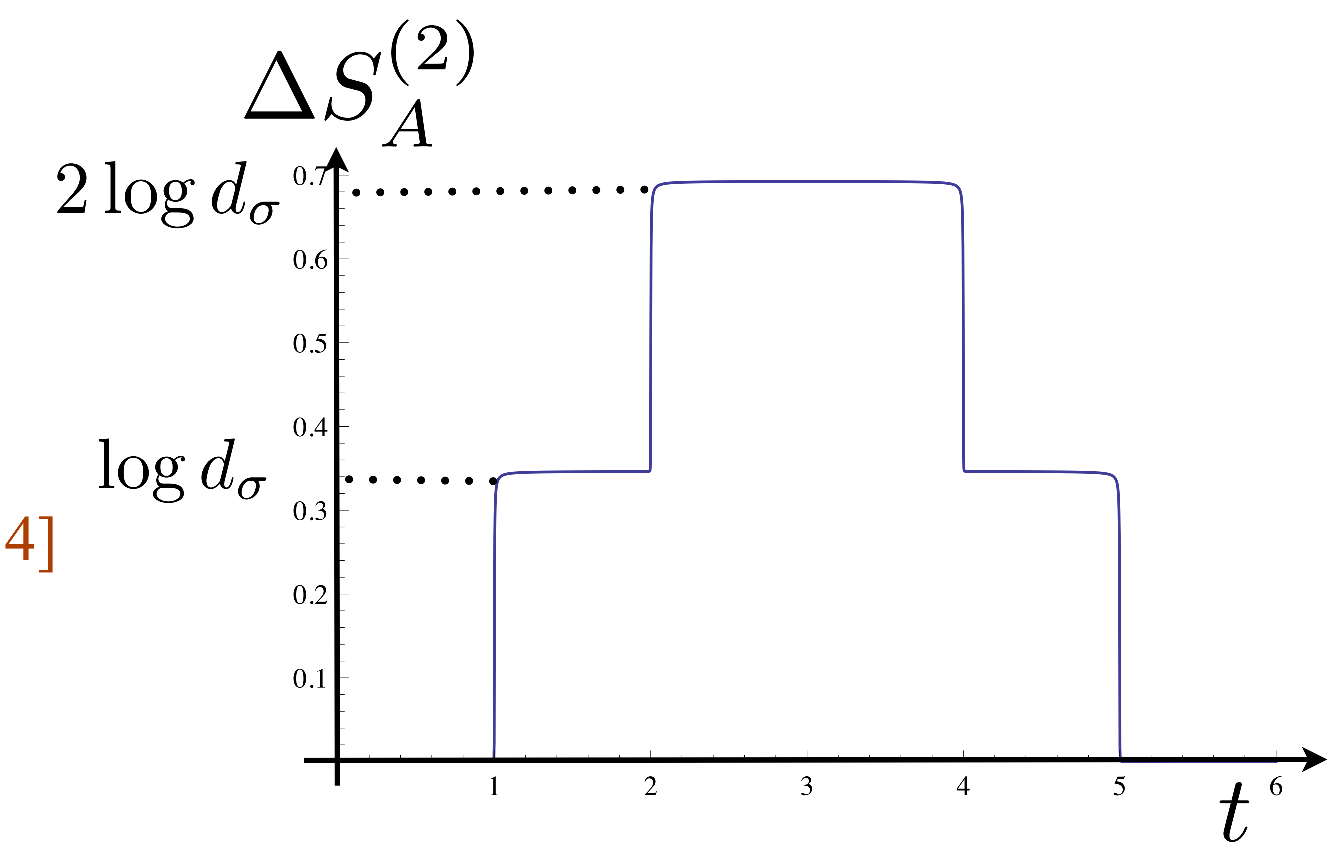
$$|\Psi\rangle \propto \sigma(-l_2)\sigma(-l_1)\,|0\rangle$$



(Renyi)Entanglement Entropy

=Sum of two entanglement

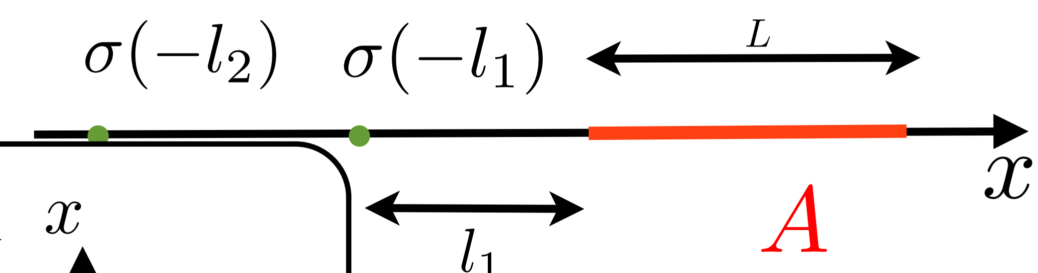
[cf: Sum rule of free theory, Nozaki 14]



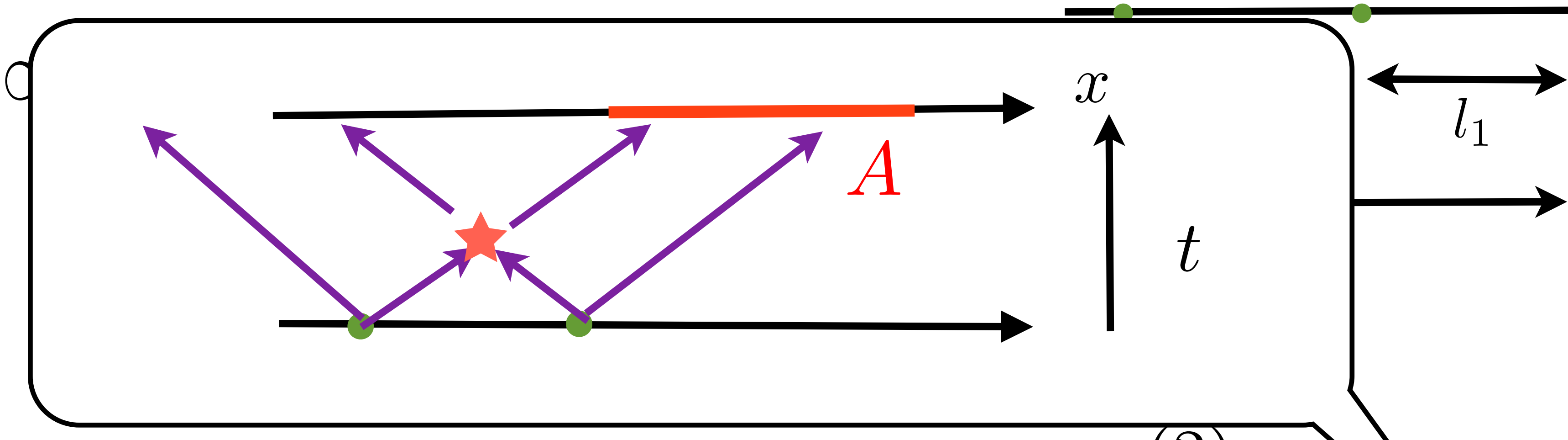
⇒Entanglement is not changed (or conserved) !

・Result [TN in preparation]

Insertion of two local operator



$|\Psi\rangle$



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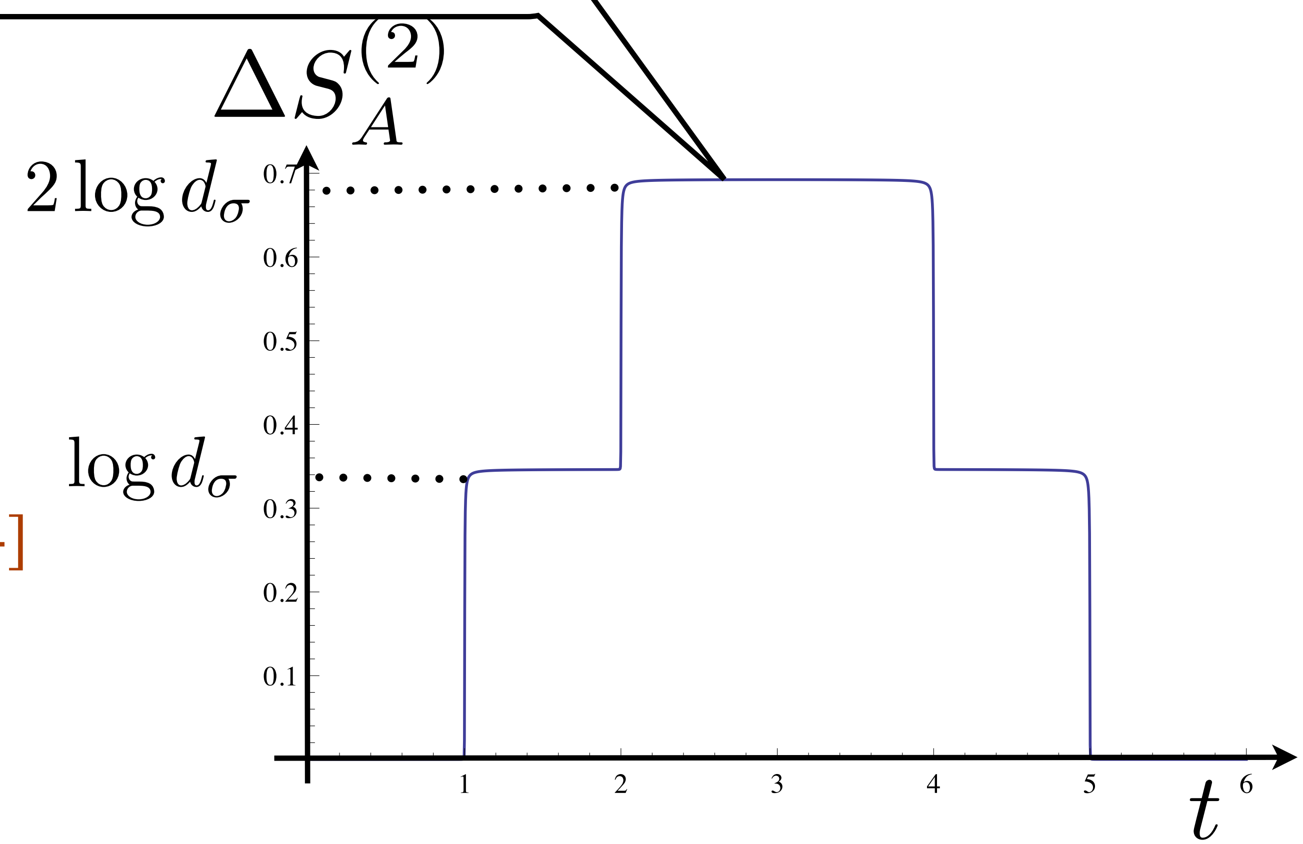

[cf: Sum rule of free theory, Nozaki 14]

⇒Entanglement is not changed (or conserved) !

Summary

- We consider the scattering effect to entanglement propagation in Ising CFT.
- Entanglement is not changed under the scattering of integrable interaction.
- The result is consistent with the formar results of entanglement scrambling in global quenches(dip exists).

Future work

- Confirm in general RCFT
- Confirm for general Renyi EE
- How about the case of Holographic CFTs?

(Maybe difficult to prepare “quasi-particle”...)