Real time confinement in a quantum quench

Pasquale Calabrese

SISSA-Trieste

Kyoto, June 15th 2016

Quantum matter, spacetime and information

Based on arXiv:1604.03571

with Mario Collura, Marton Kormos, Gabor Takacs

Quantum quench dynamics

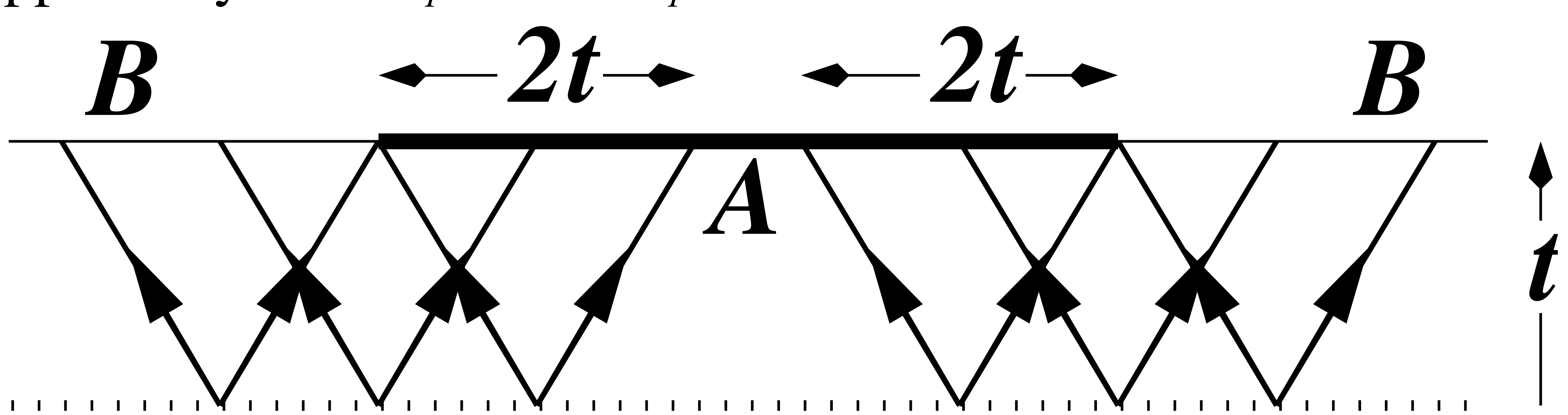
- A many-body quantum system is prepared in the ground-state of H_0, *i.e.* $|\Psi_0\rangle$
- At $t=0$, $H_0 \Rightarrow H$, *i.e.* a Hamiltonian parameter is quenched
- For $t>0$, it evolves unitarily: $|\Psi(t)\rangle = e^{-iHt}\,|\Psi_0\rangle$
- No contact with "external" world
- What are the main features of the dynamics?
- What about a "stationary state"?

The study of quench dynamics has been boosted by cold-atom experiments in the last decade or so

Light-cone spreading of entanglement entropy

PC, J Cardy 2005

- After a global quench, the initial state $|\psi_0\rangle$ has an extensive excess of energy
- It acts as a source of quasi-particles at t=0. A particle of momentum p has energy E_p and velocity $v_p = dE_p/dp$
- For $t > 0$ the particles moves semiclassically with velocity v_p
- particles emitted from regions of size of the initial correlation length are entangled, particles from far points are incoherent
- The point $x \in A$ is entangled with a point $x' \in B$ if a left (right) moving particle arriving at x is entangled with a right (left) moving particle arriving at x'. This can happen only if $x \pm v_p t \sim x' \mp v_p t$



Light-cone spreading of entanglement entropy

PC, J Cardy 2005

- The entanglement entropy of an interval A of length ℓ is proportional to the total number of pairs of particles emitted from arbitrary points such that at time *t*, $x \in A$ and $x' \in B$

- Denoting with *f(p)* the rate of production of pairs of momenta $\pm p$ and their contribution to the entanglement entropy, this implies

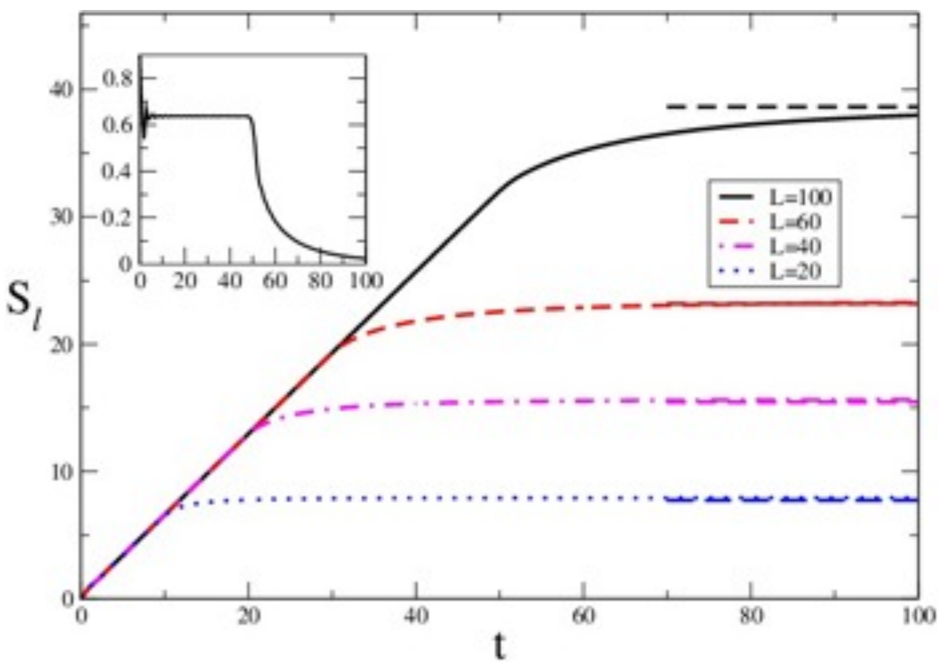
$$
\begin{aligned}
S_A(t) &\approx \int_{x'\in A} dx' \int_{x''\in B} dx'' \int_{-\infty}^{\infty} dx \int f(p)dp\delta\big(x'-x-v_pt\big)\delta\big(x''-x+v_pt\big) \\
&\propto t\int_0^\infty dp f(p) 2v_p \theta(\ell-2v_pt) + \ell \int_0^\infty dp f(p)\theta(2v_pt-\ell)
\end{aligned}
$$

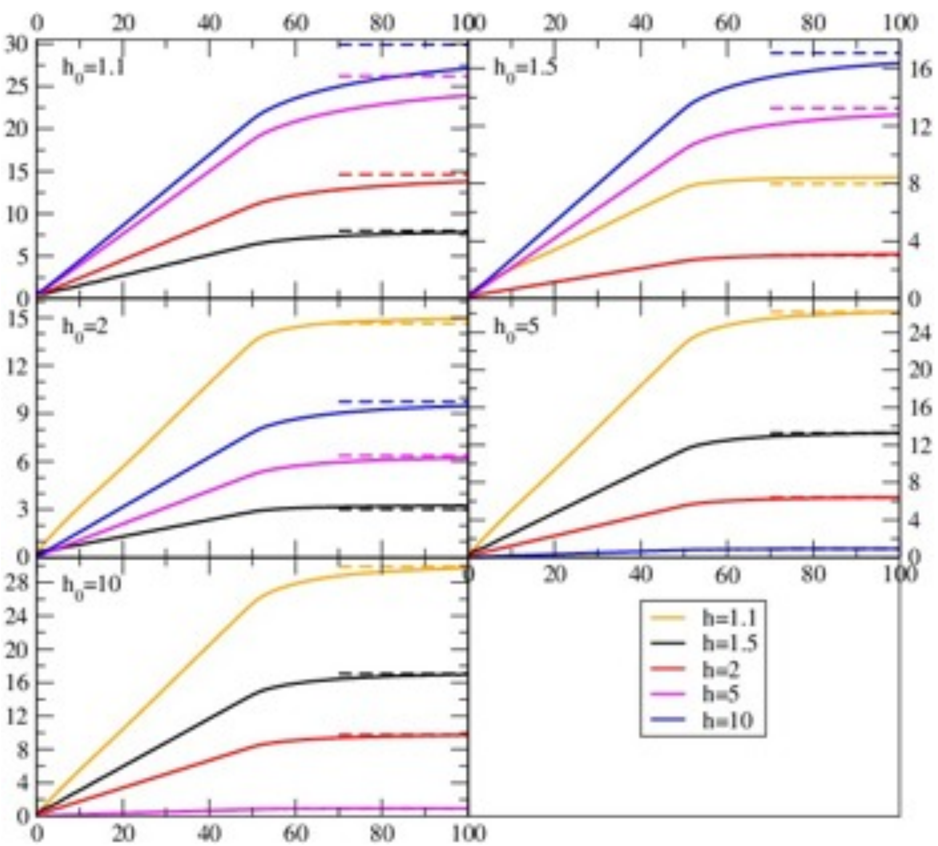
- When v_p is bounded (e.g. Lieb-Robinson bounds) $|v_p|<v_{\max}$, the second term is vanishing for $2\,v_{\max}\,t<\ell$ and the entanglement entropy grows linearly with time up to a value linear in ℓ

One example

Transverse field Ising chain

PC, J Cardy 2005





Analytically for t, $\ell \gg 1$ with t/ℓ constant

M Fagotti, PC 2008

$$S(t) = t \int_{2|\epsilon'|t<\ell} \frac{d\varphi}{2\pi} 2|\epsilon'| H(\cos\Delta_\varphi) + \ell \int_{2|\epsilon'|t>\ell} \frac{d\varphi}{2\pi} H(\cos\Delta_\varphi)$$

$$\cos\Delta_\varphi = \frac{1-\cos\varphi(h+h_0)+hh_0}{\epsilon_\varphi \epsilon^0_\varphi} \qquad H(x) = -\frac{1+x}{2}\log\frac{1+x}{2} - \frac{1-x}{2}\log\frac{1-x}{2}$$

Light-cone spreading of correlations

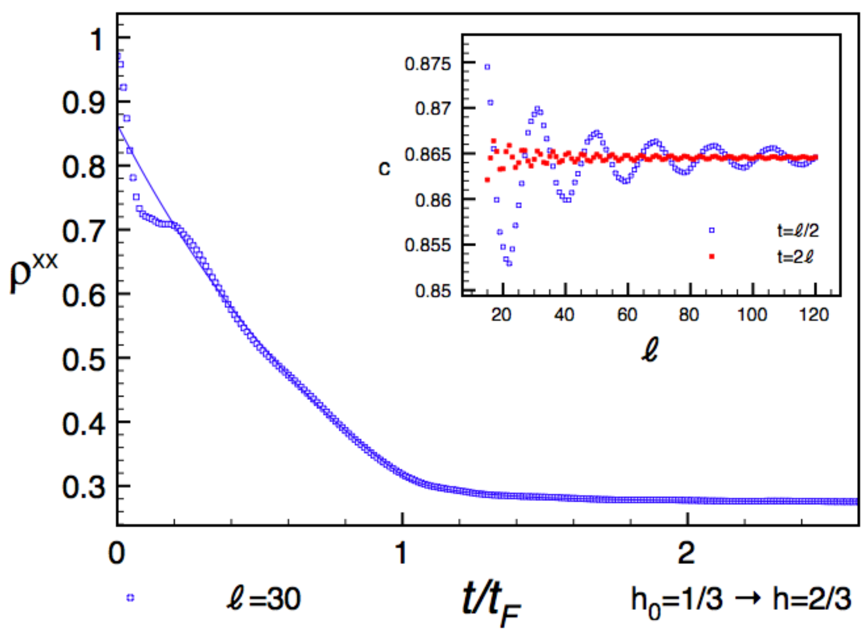
The same scenario is valid for correlations:

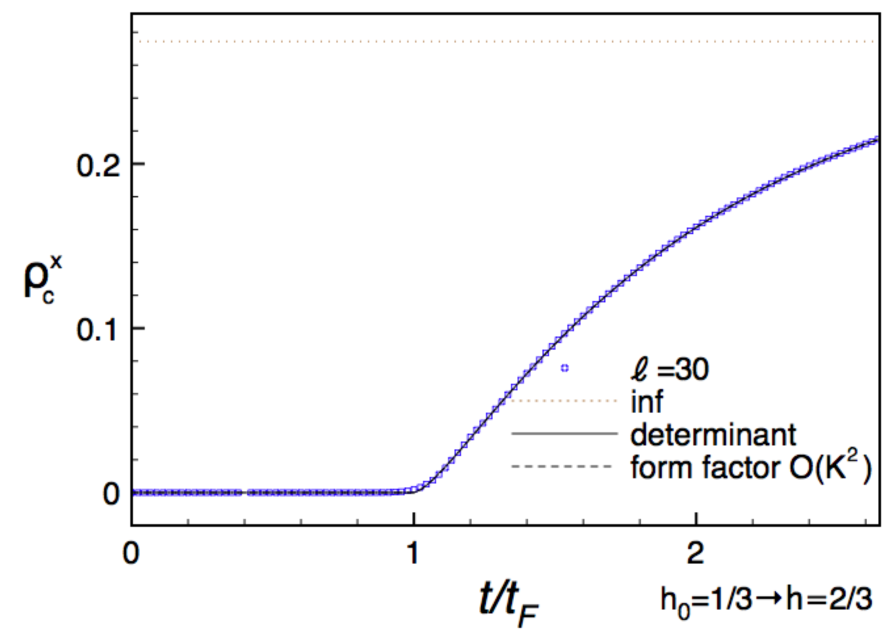
PC, J Cardy 2006/07

- **Horizon**: points at separation r become correlated when left- and right-moving particles originating from the same point first reach them
- If $|v_p|<v_{\max}$, connected correlations are then frozen for $t < r/2v_{\max}$

Example: Ising model within ferromagnetic phase

PC, F Essler, M Fagotti 2011/12

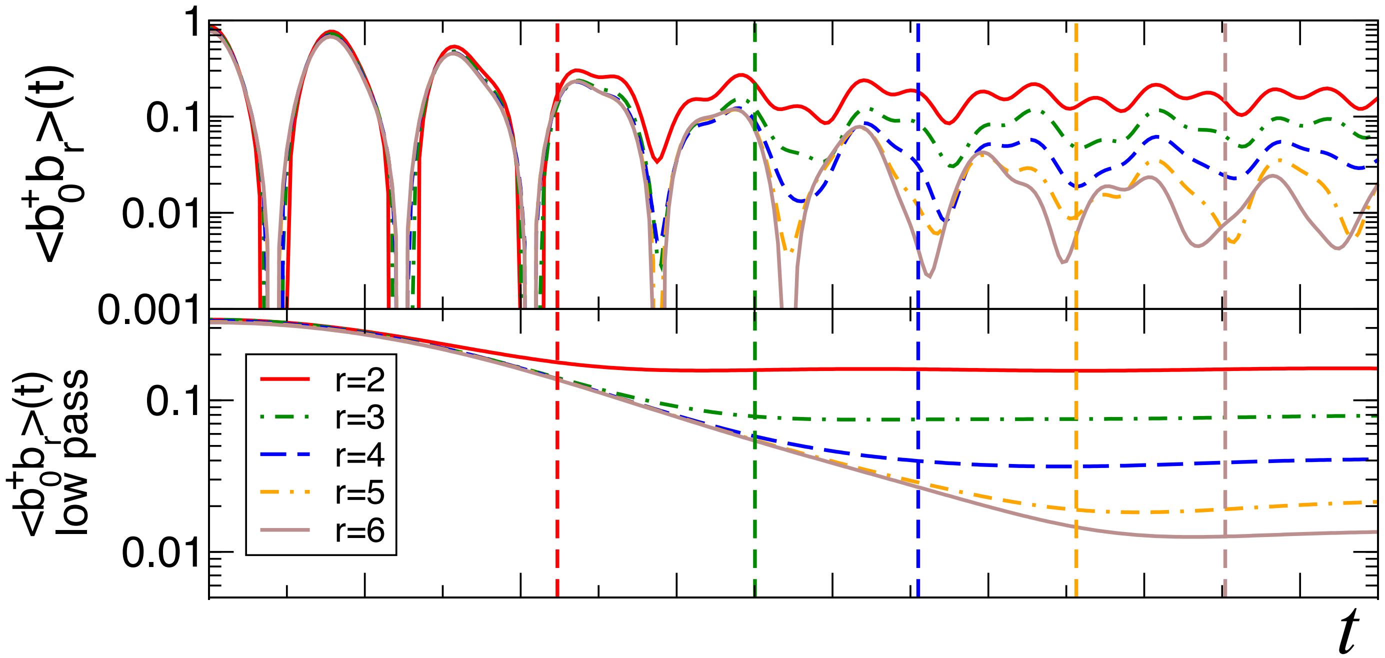




Light cone in interacting models

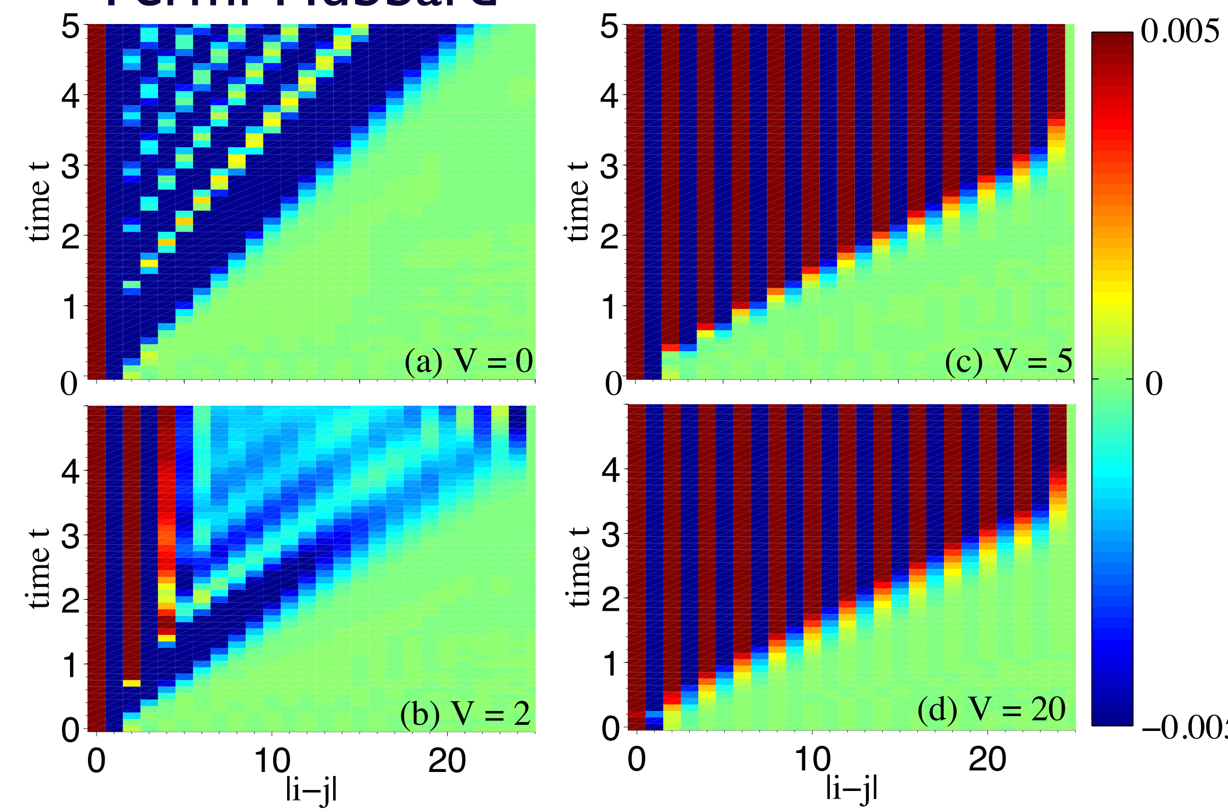
Kollath-Lauechli ’08

Bose-Hubbard



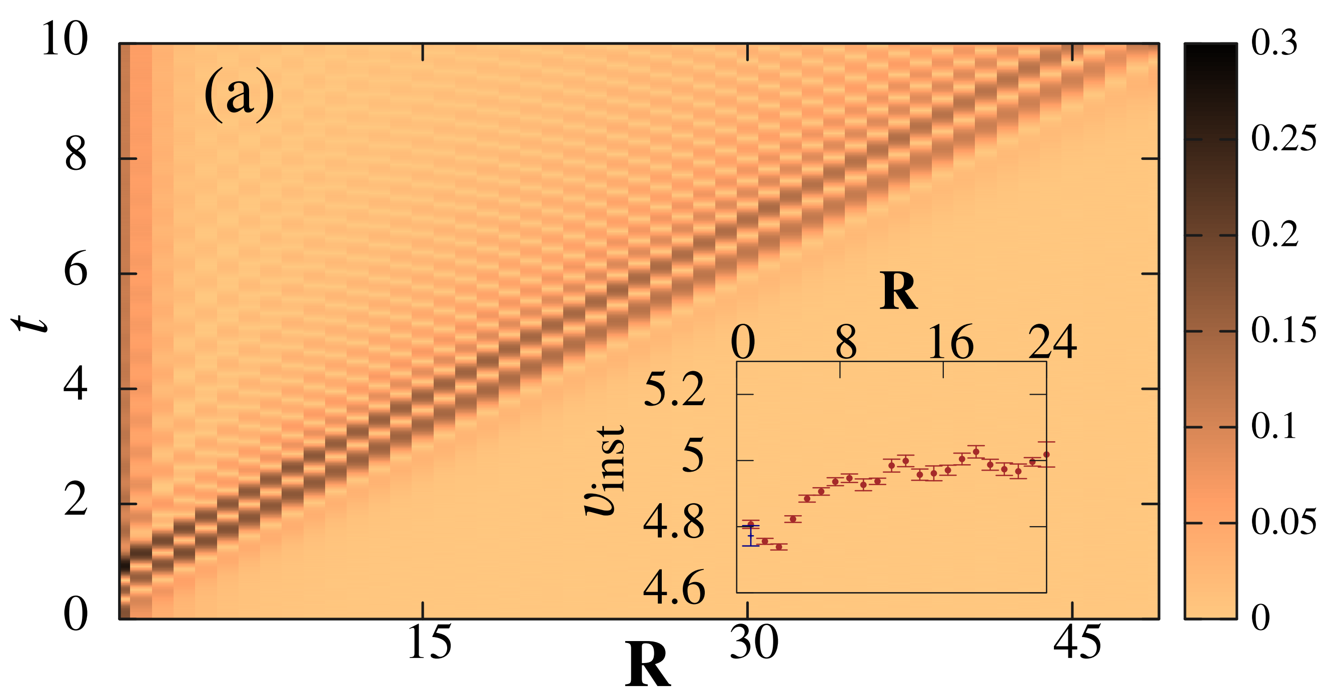
Manmana et al ’08

Fermi-Hubbard



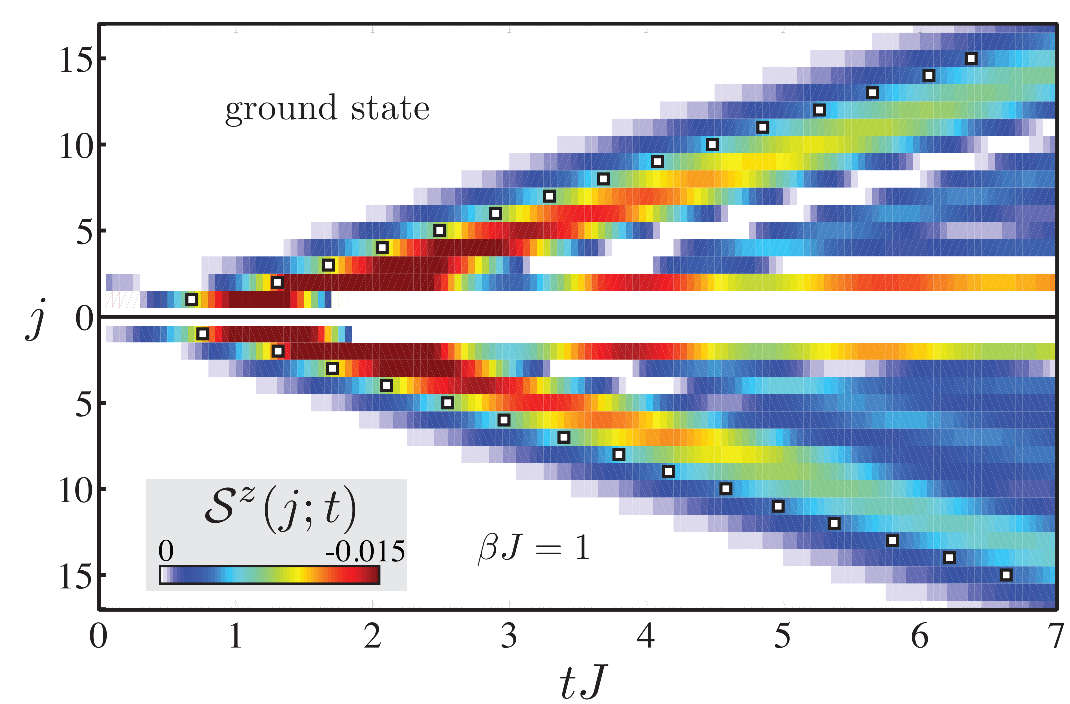
Carleo et al, ’14

Bose-Hubbard



Bonnes, Essler, Lauchli ’14

XXZ spin chain



Light cone in experiment

M. Cheneau et al, Nature 481, 484 (2012)

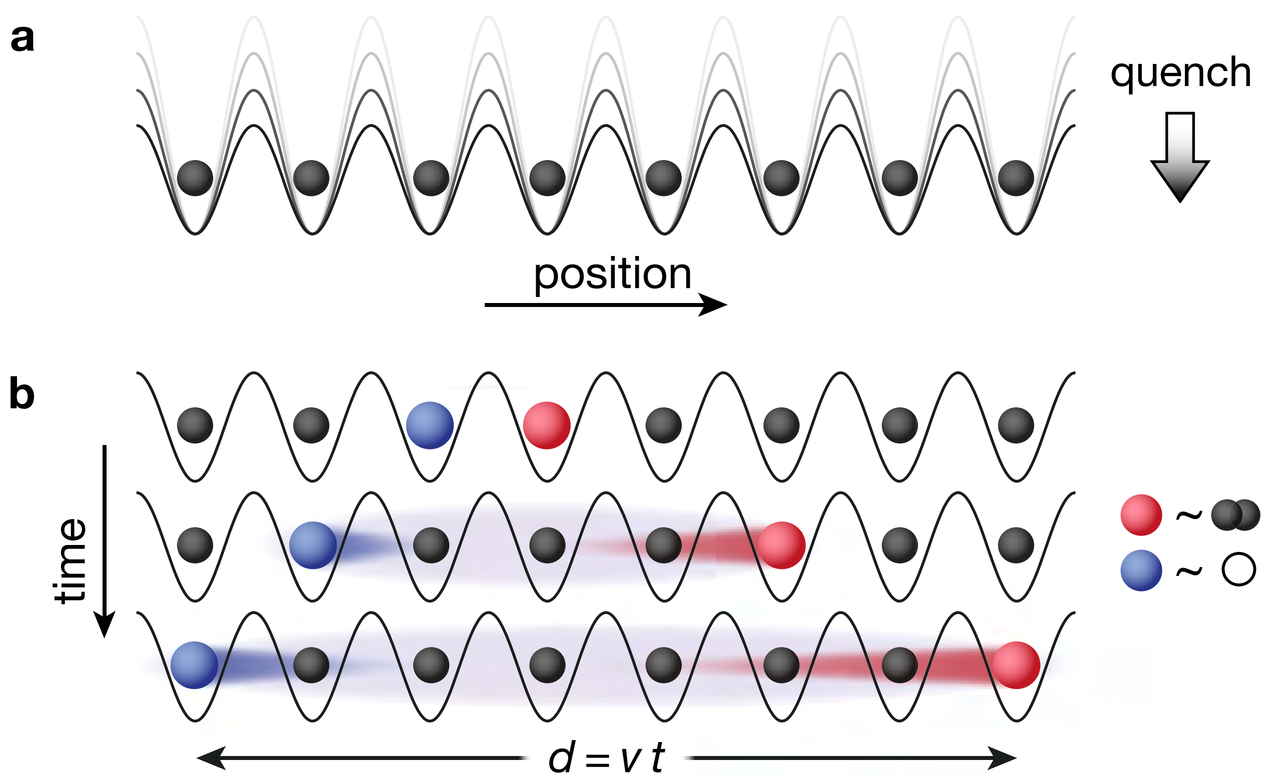
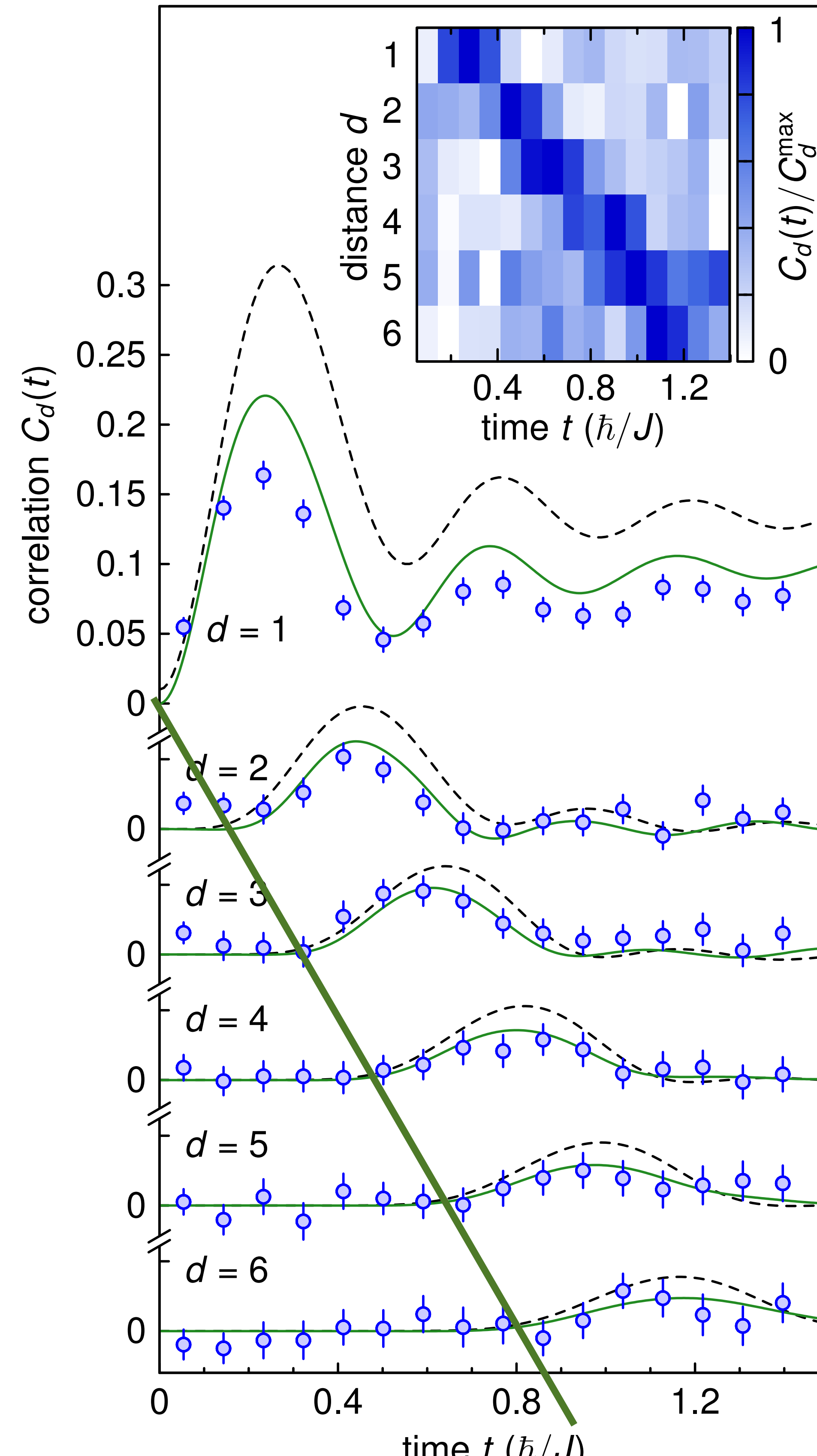


FIG. 1. **Spreading of correlations in a quenched atomic Mott insulator.** **a**, A 1D ultracold gas of bosonic atoms (black balls) in an optical lattice is initially prepared deep in the Mott-insulating phase with unity filling. The lattice depth is then abruptly lowered, bringing the system out of equilibrium. **b**, Following the quench, entangled quasiparticle pairs emerge at all sites. Each of these pairs consists of a doublon (red ball) and a holon (blue ball) on top of the unity-filling background, which propagate ballistically in opposite directions. It follows that a correlation in the parity of the site occupancy builds up at time t between any pair of sites separated by a distance $d = vt$, where v is the relative velocity of the doublons and holons.



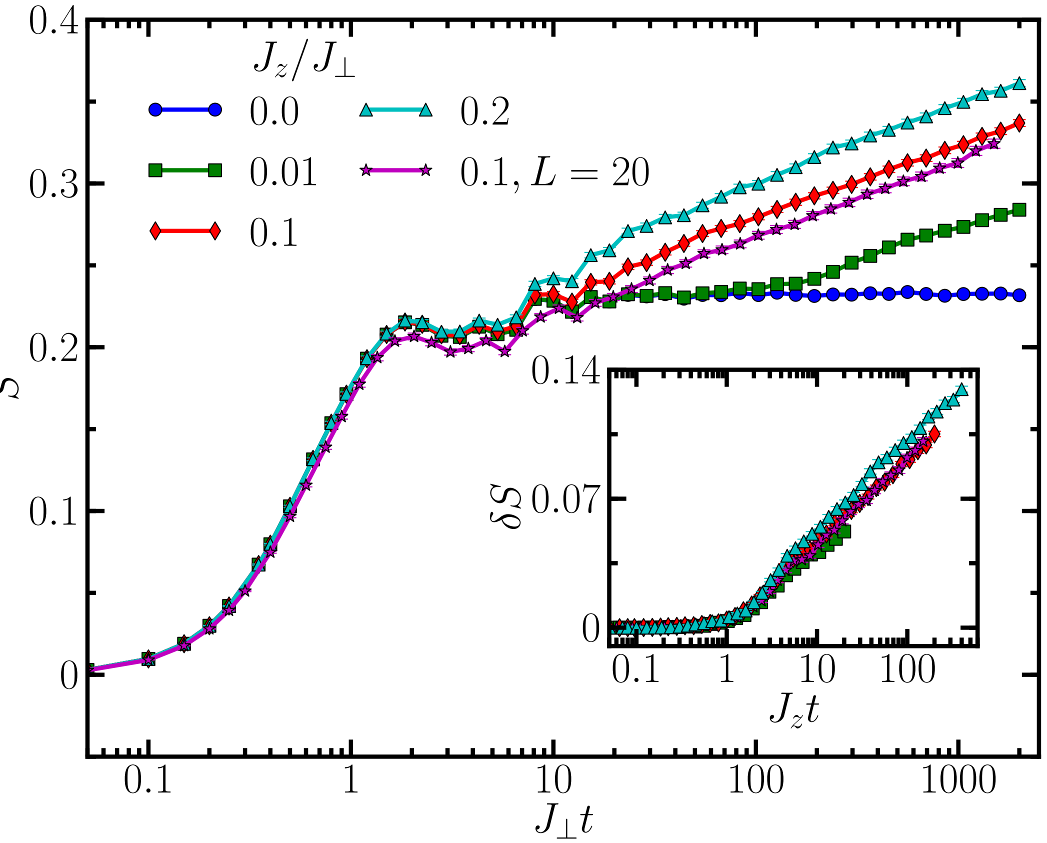
Some no light-cone spreadings

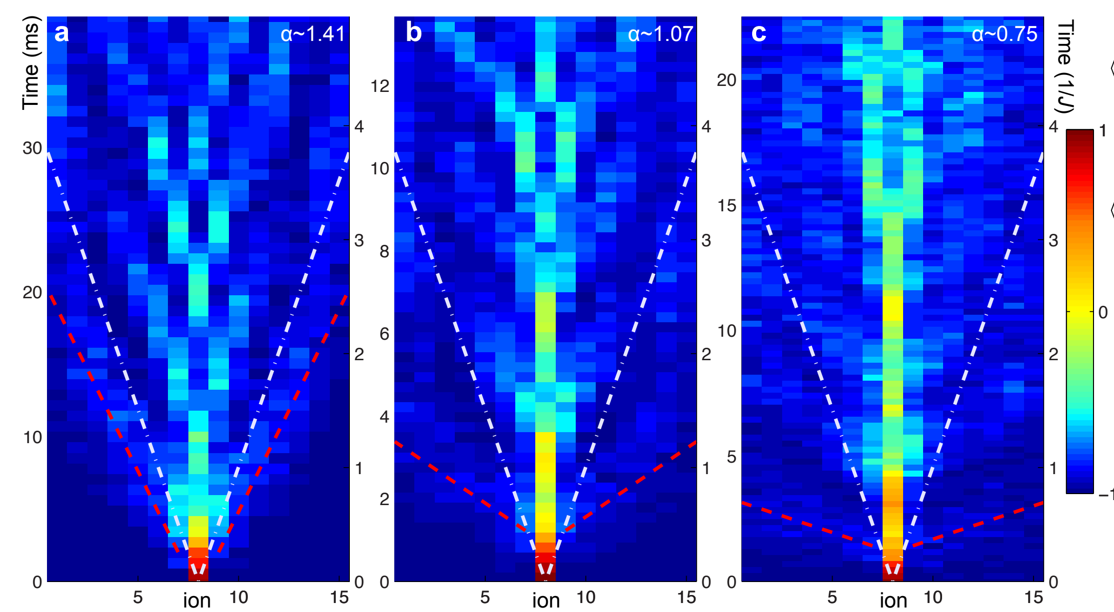
MBL, logarithmic growth of entanglement:

Long-range interaction:

Bardason, Pollmann, Moore, '12

Jurcevic et al Nature 511, 202 (2014)





When the range of interaction is long enough there is no light cone

see also: De Chiara et al '05
Burrell & Osborne '07
Vosk and Altman '13

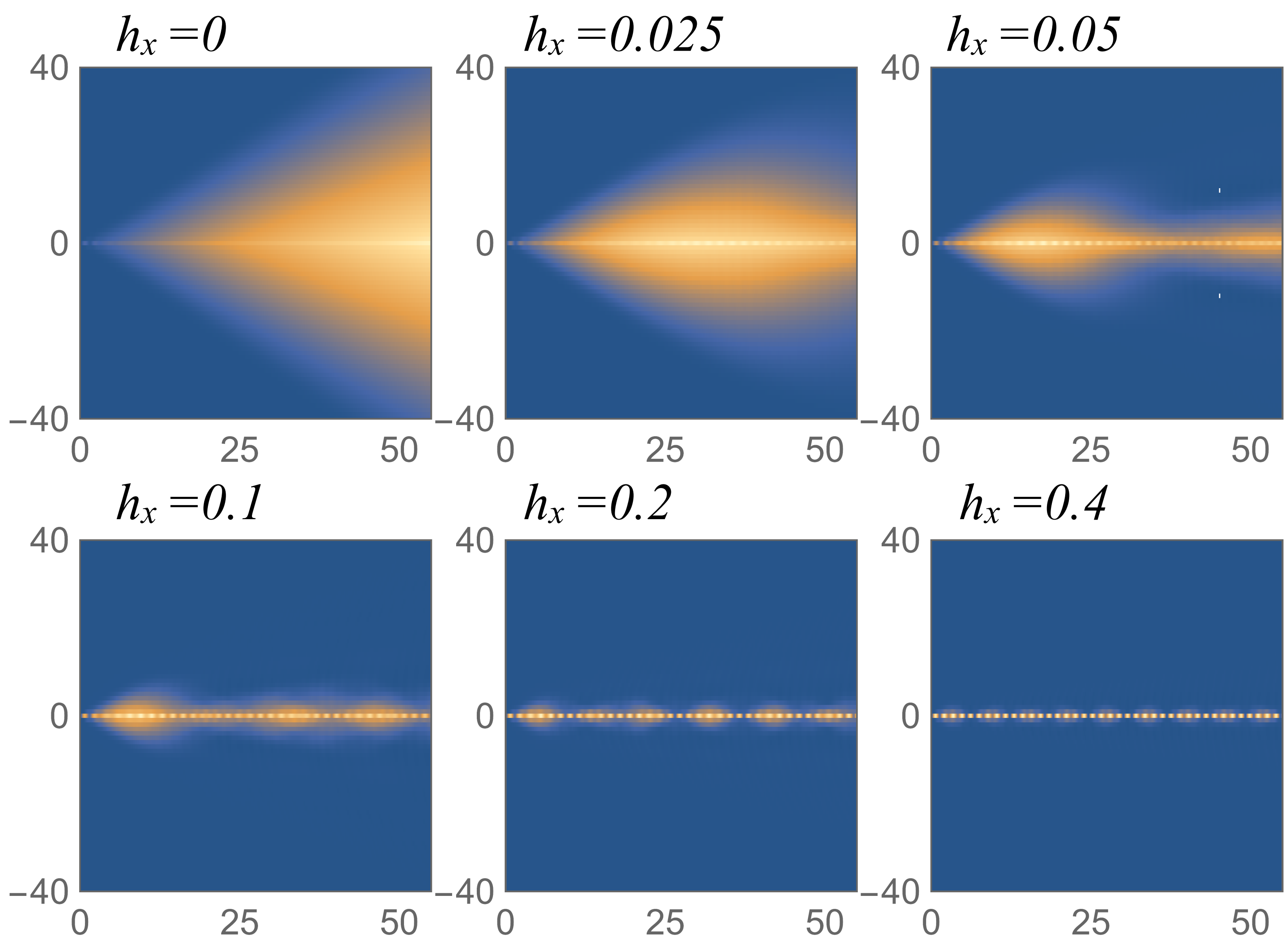
see also: Hauke & Tagliacozzo, '13
Schachenmayer et al '13
Richerme et al '14

Disappearance of the light cone

Starting from the ferromagnetic state (all spins up) and evolving with

$$H = -J \sum_{j=1}^{L} \left[\sigma_j^x \sigma_{j+1}^x + h_z \sigma_j^z + h_x \sigma_j^x \right]$$

with h_z= 0.25. Connected longitudinal correlation $\langle \sigma_1^x \sigma_{m+1}^x \rangle_c$



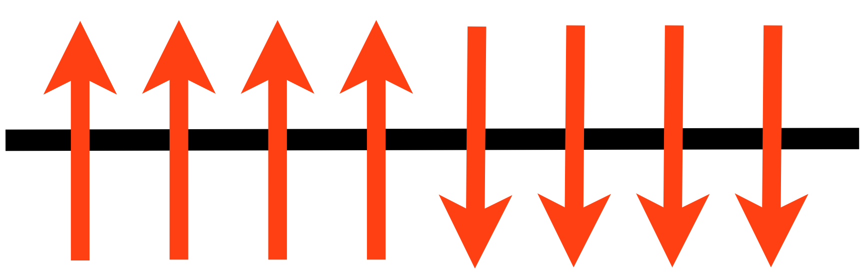
Why??

Confinement in the Ising model

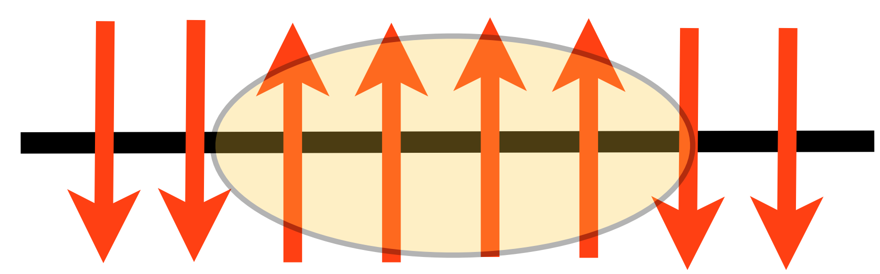
McCoy & Wu ’78
Bhaseen, Tsvelik ’04
+ many more, sorry

$$H = -J \sum_{j=1}^{L} \left[\sigma_j^x \sigma_{j+1}^x + h_z \sigma_j^z + h_x \sigma_j^x \right]$$

- For $h_x = 0$, free fermions with dispersion $\varepsilon(k) = 2J\sqrt{1 - 2h^z \cos k + {h^z}^2}$
- $h_z = 1$ separates two massive phases
- For $h_z < 1$ (ferro phase), the massive fermions can be seen as domain walls separating domains of magnetization $\sigma = (1 - h_z^2)^{1/8}$
- h_x induces an attractive interaction between DW that can be approximated as a linear potential $V(x) = \chi\, |x|$, with $\chi = 2Jh_x\sigma$
- DW do not propagate freely and they get confined into **mesons**



Free DW



Bound state = meson

A simple approximation for the meson spectrum

Rutkevich '08

Consider two fermions in 1D with Hamiltonian

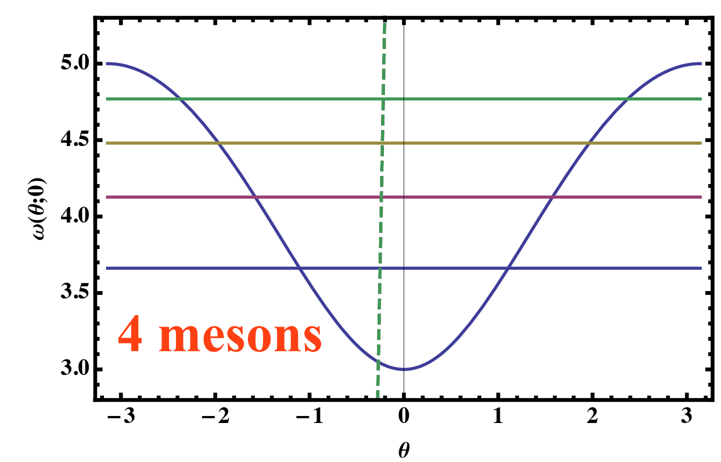
$$\mathcal{H} = \varepsilon(\theta_1) + \varepsilon(\theta_2) + \chi|x_2 - x_1| = \omega(\theta;\Theta) + \chi|x|$$

$$\omega(\theta;\Theta) = \varepsilon(\theta + \Theta/2) + \varepsilon(\theta - \Theta/2)$$

This can be quantized semiclassically a'la Bohr-Sommerfeld

- The number and the energies of mesons depend on h_x, h_z, Θ

$h_x = 0.1$, $h_z = 0.25$, $\Theta = 0$



- When ω has a single minimum one obtains

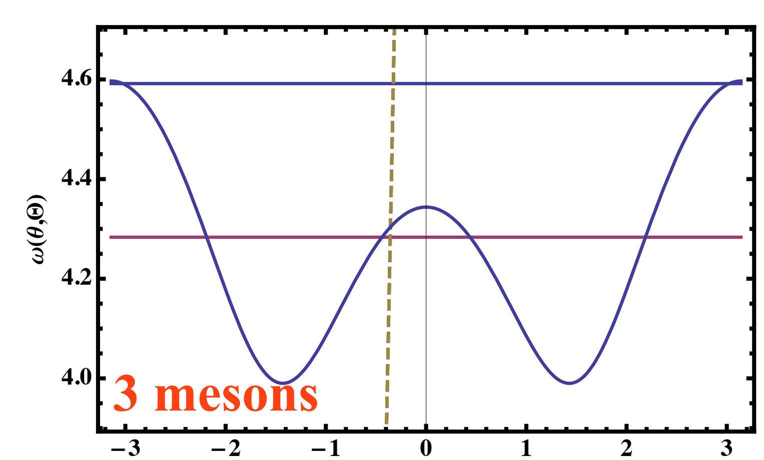
$$2E_n(\Theta)\theta_a - \int_{-\theta_a}^{\theta_a} \mathrm{d}\theta\, \omega(\theta;\Theta) = 2\pi\chi(n - 1/4)\,, \qquad n = 1, 2, \ldots$$

where θ_a is the solution of $\omega(\theta_a(n;\Theta);\Theta) = E_n(\Theta)$

$h_x = 0.1$, $h_z = 0.5$, $\Theta = 3$

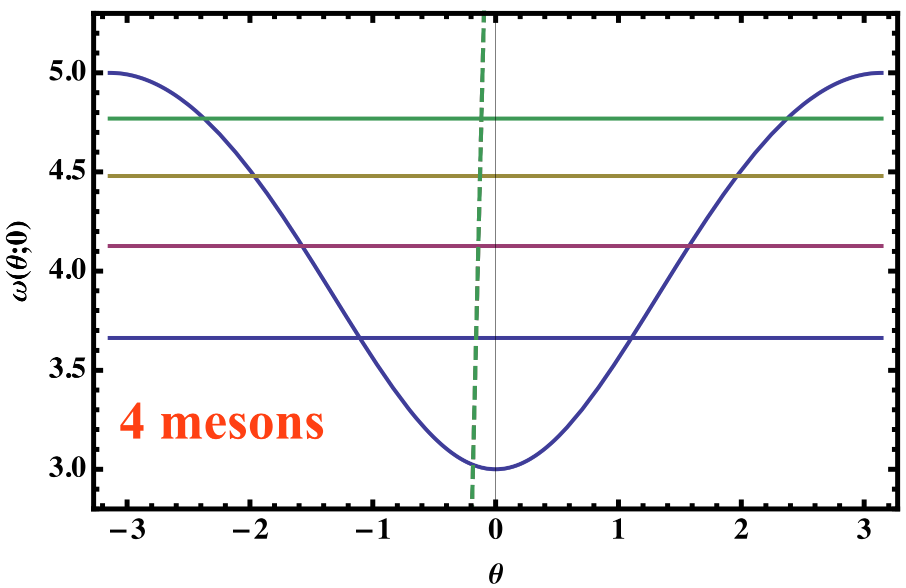
- For two minima

$$E_n(\Theta)(\theta_a - \theta_b) - \int_{-\theta_b}^{\theta_a} \mathrm{d}\theta\, \omega(\theta;\Theta) = \pi\chi(n - 1/2)\,, \qquad n = 1, 2, \ldots$$



A simple approximation for the meson spectrum

h_x =0.1, h_z= 0.25, Θ=0



The four masses are

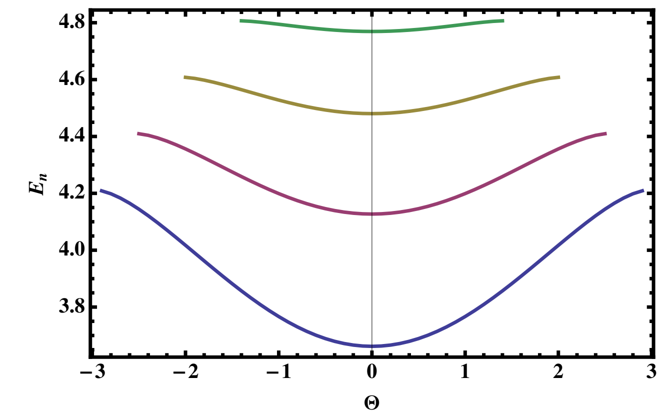
m_1=3.662 m_2=4.127 m_3=4.48 m_4=4.77

$E_n(\theta)$ is the dispersion relation of the mesons

$$v_n(\Theta) = \frac{\mathrm{d}E_n(\Theta)}{\mathrm{d}\Theta}$$

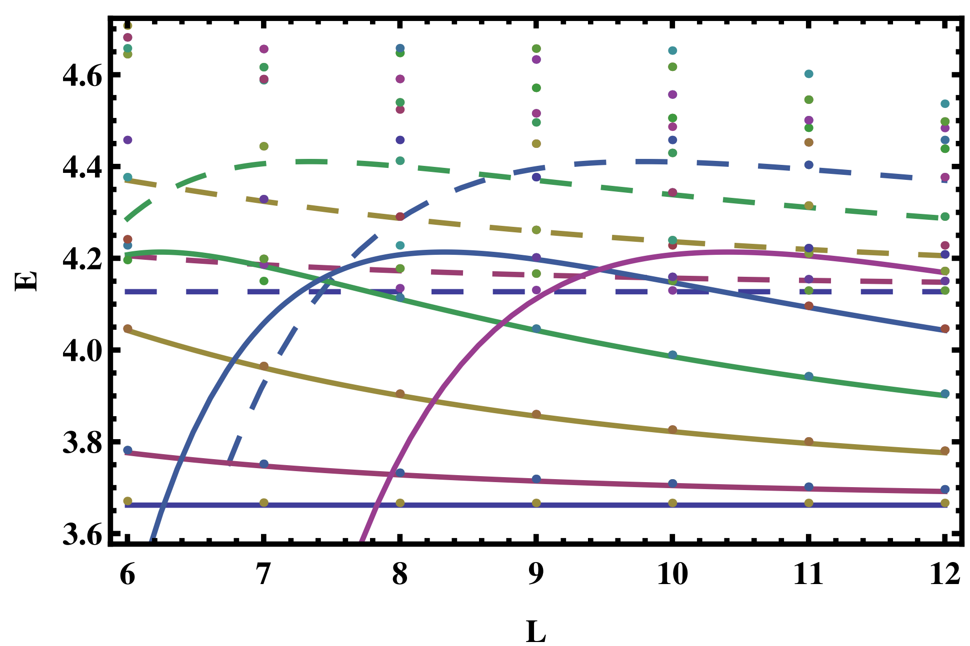
$v_{\max}$ = 0.274, 0.166,0.094, 0.004

$v_{\max}$ of DW = 0.5



◉ Comparison with exact diagonalization:

h_x =0.1, h_z= 0.5



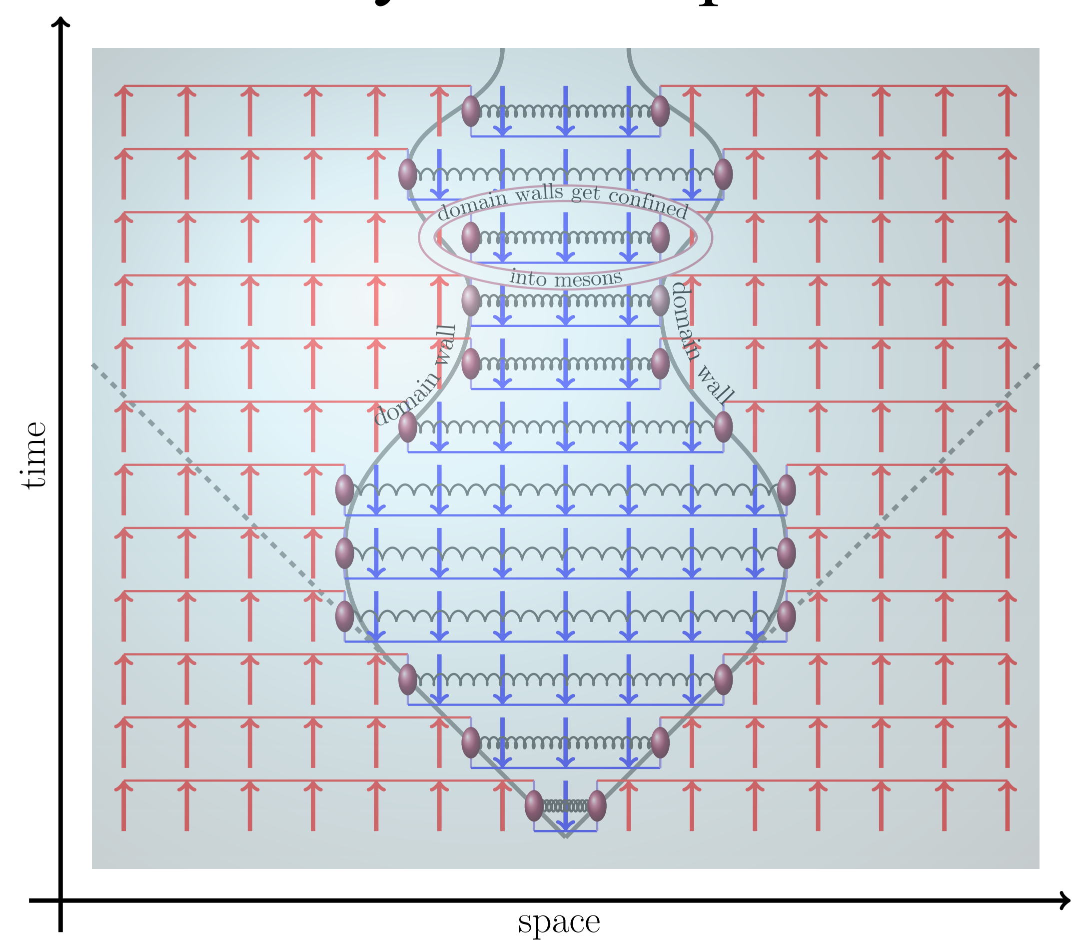
2nd meson 1pt states

1st meson 1pt states

Back to quenches

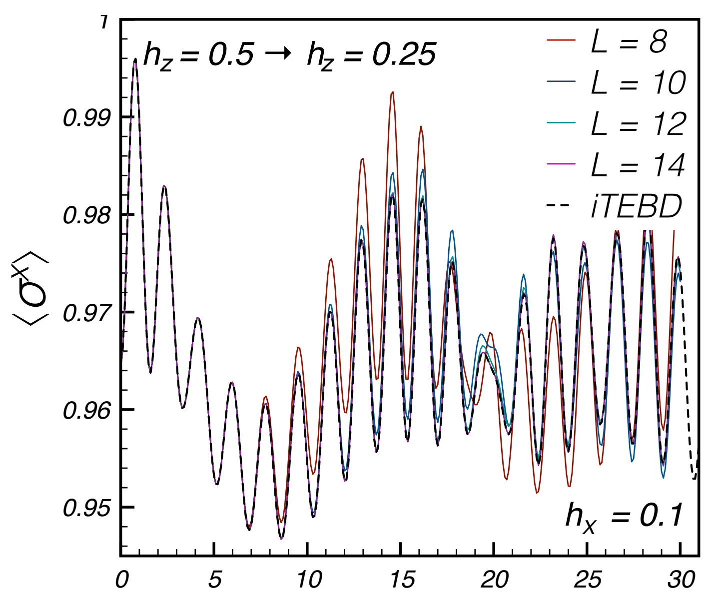
What happens if there are mesons in the spectrum of the postquench Hamiltonian in the quasi-particle picture?

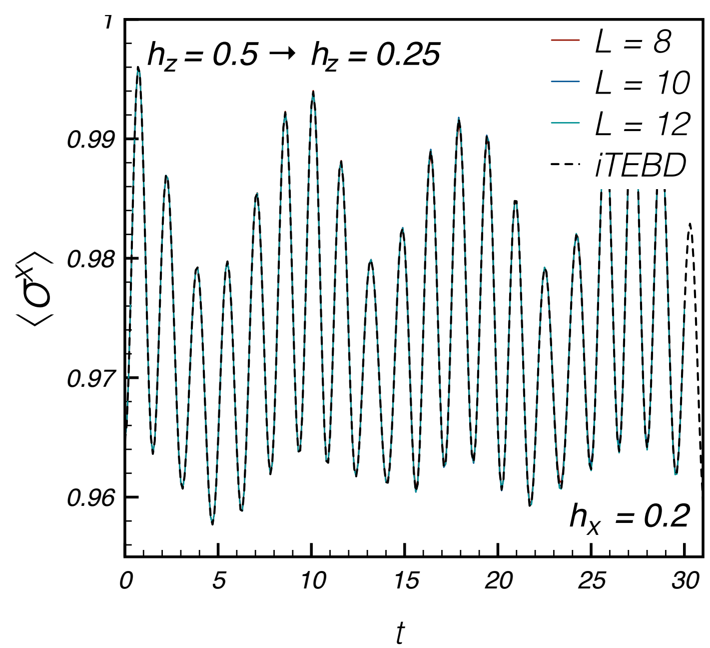
- $|\psi_0\rangle$ acts as a source of quasi-particles at t=0
- pairs of quasi-particles move in opposite directions with velocity v_p
- moving away the quasi-particle feel the attractive interaction
- Interaction will eventually turn the particle and start oscillations

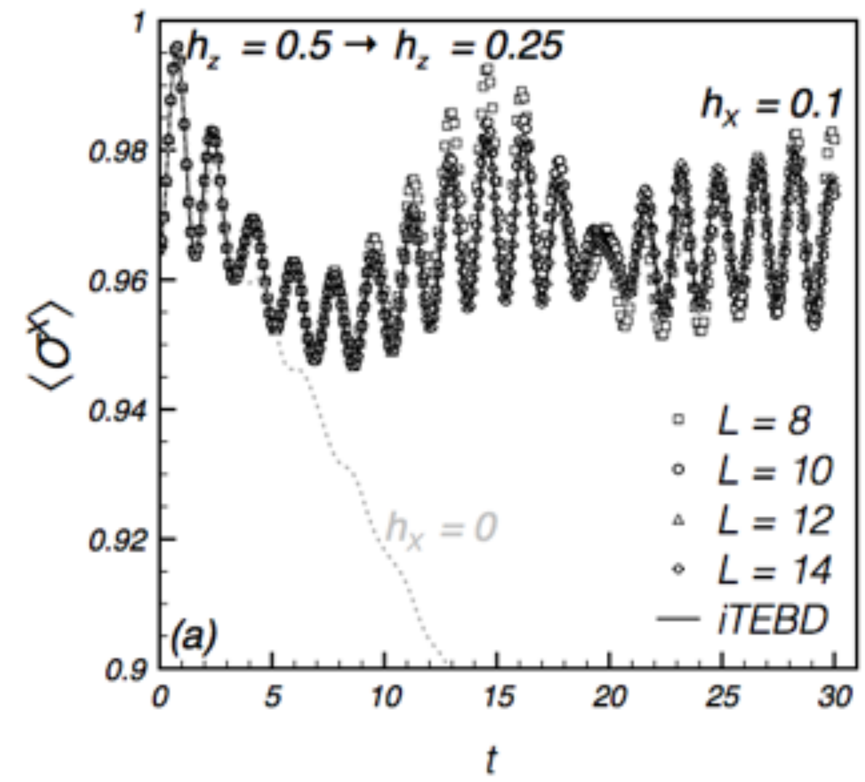


1-pt function $\langle \sigma^x \rangle$

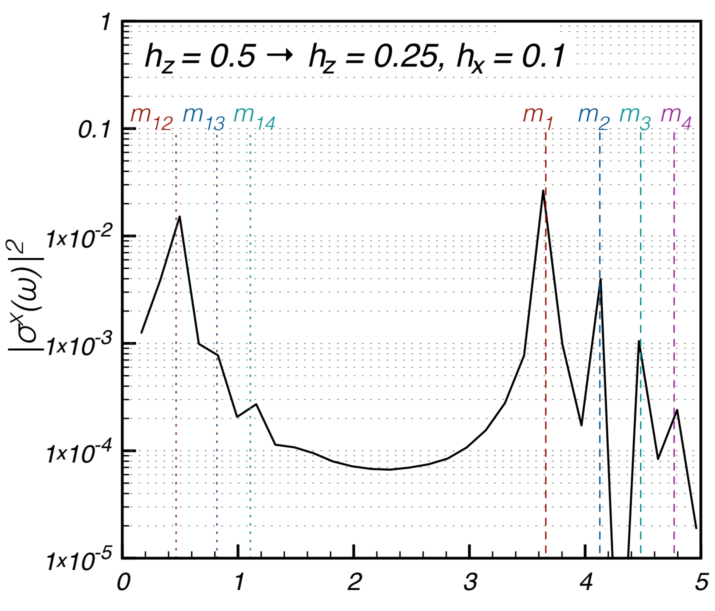
Quenches ferro to ferro

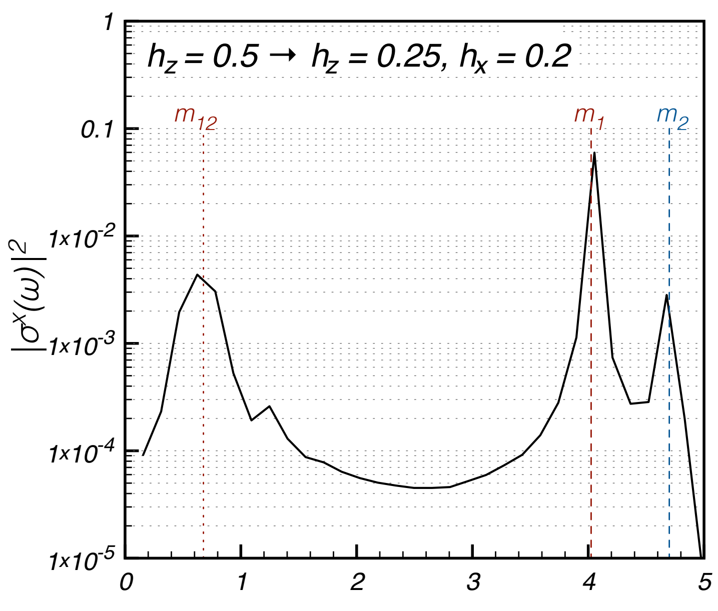






Power spectrum of $\langle \sigma^x \rangle$





$m_2 - m_1 = 0.46,\ m_1 = 3.7,$

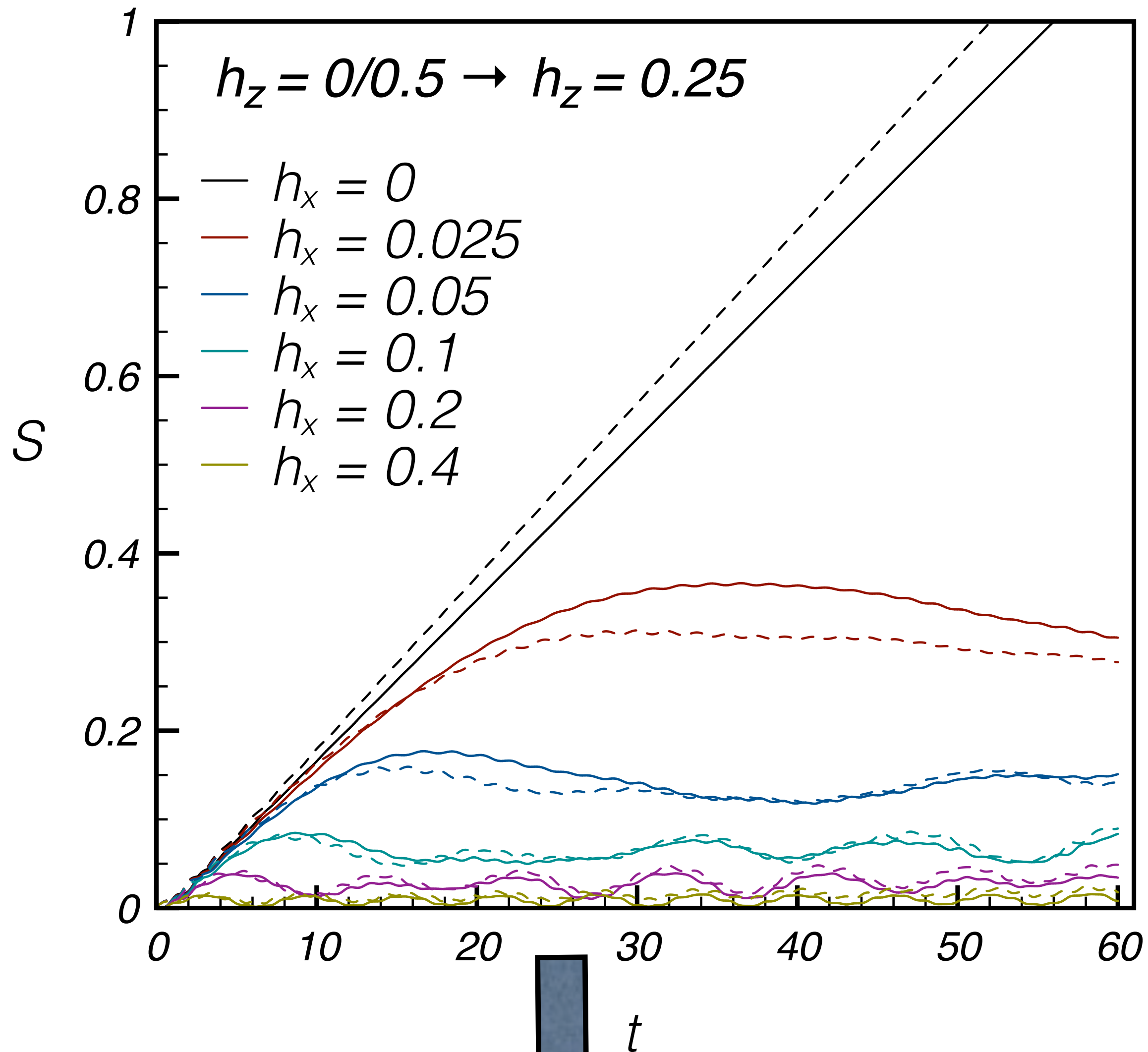
$m_2 = 4.1,\ m_3 = 4.5$

$m_2 - m_1 = 0.68,\ m_1 = 4.0,$

$m_2 = 4.7$

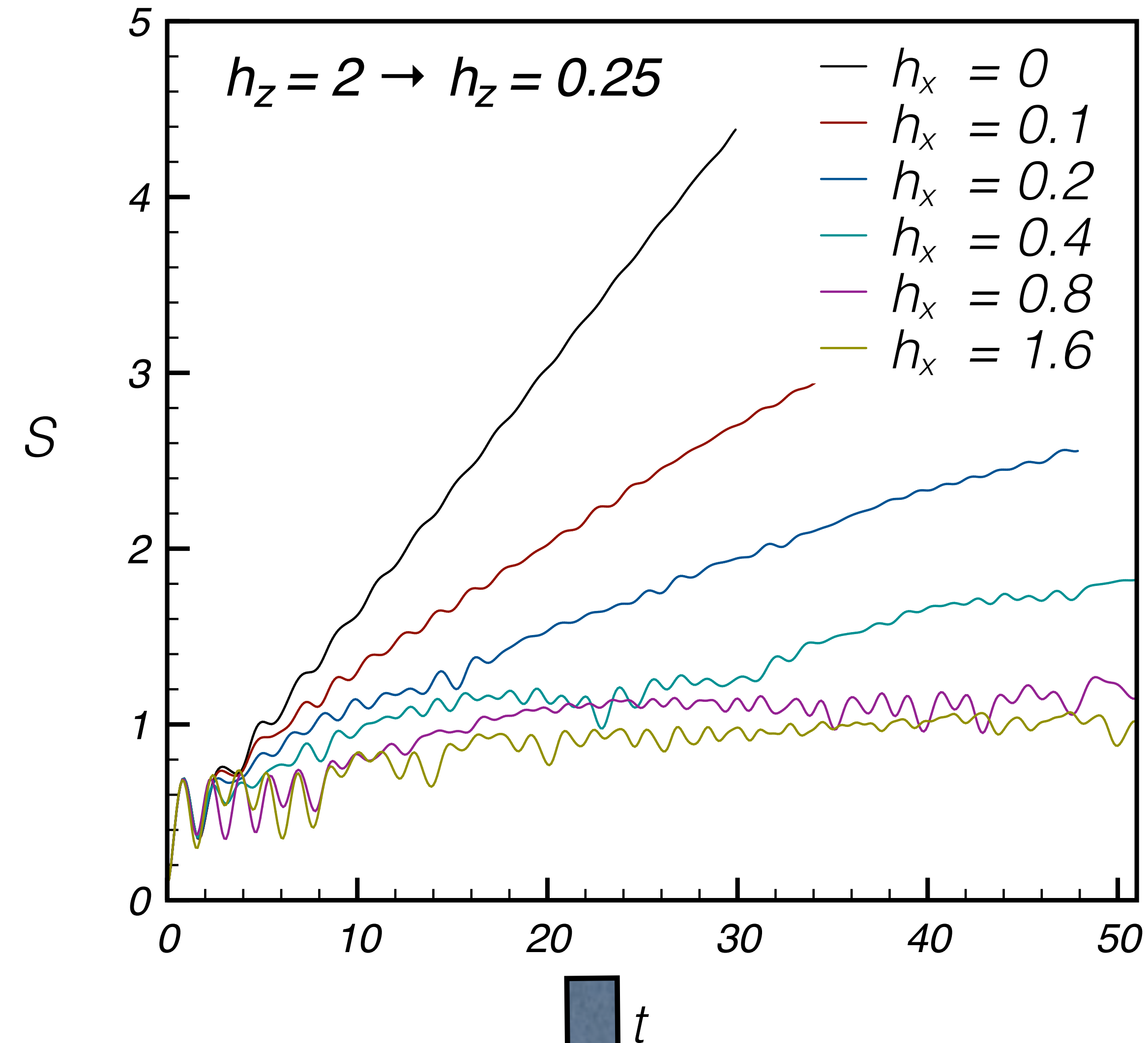
Half-chain entanglement entropy

Ferro-to-Ferro



The entanglement entropy does not **seem to grow** indefinitely but oscillates around a finite value

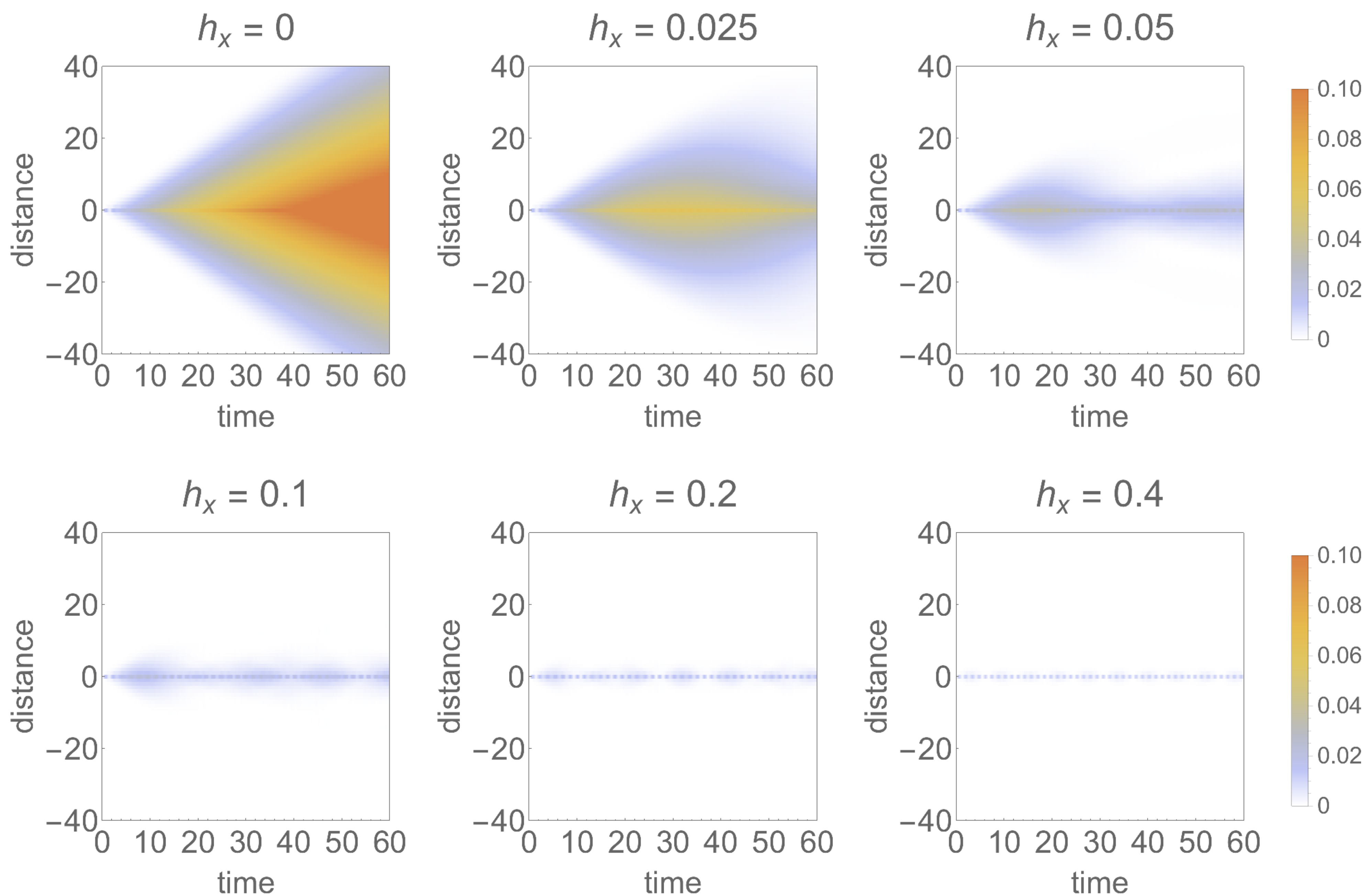
Para-to-Ferro



The entanglement entropy grows but much slower than in the integrable case

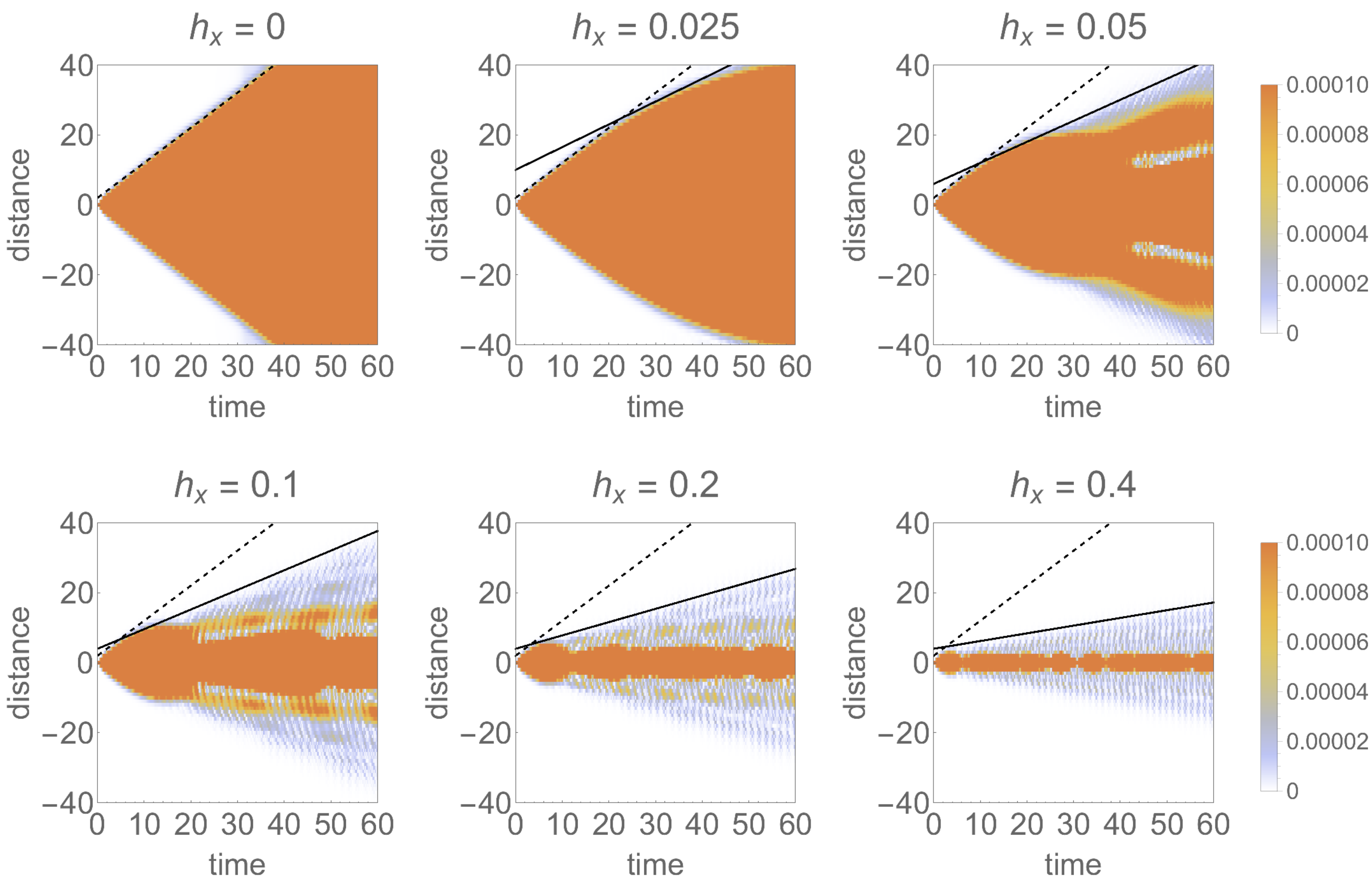
Connected correlation functions

$$h_z^0 = 0,\ h_x^0 = 0,\ h_z = 0.25$$



Connected correlation functions

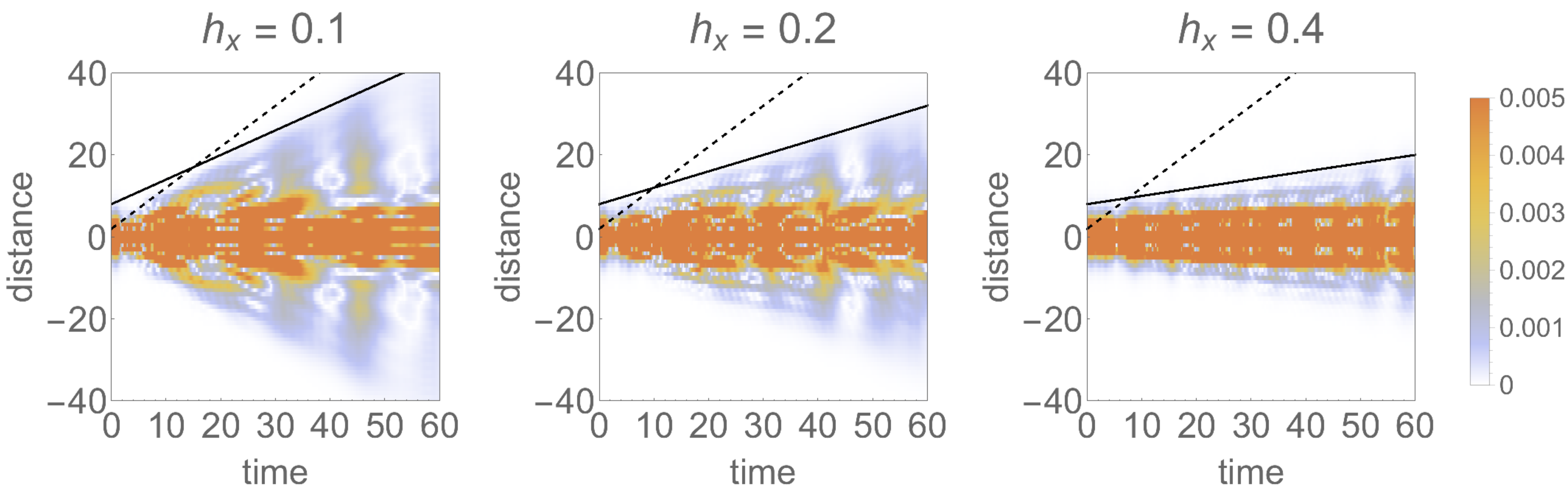
Let's zoom:



Conclusions: there is a feeble light-cone (a factor 10^{-3}) having the **mesons velocity**!

Quench para→ferro

$h_z^0=2,\ h_x^0=0,\ h_z=0.25$



Conclusions: The light-cone is visible, but it has the mesons velocity!

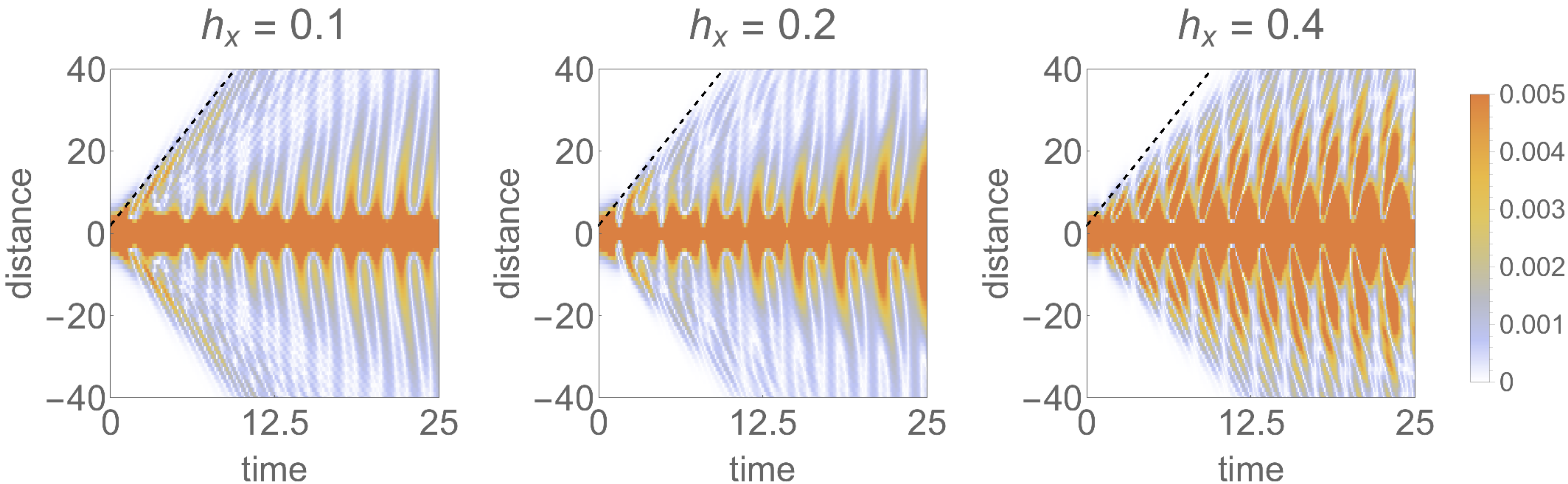
Physical interpretation

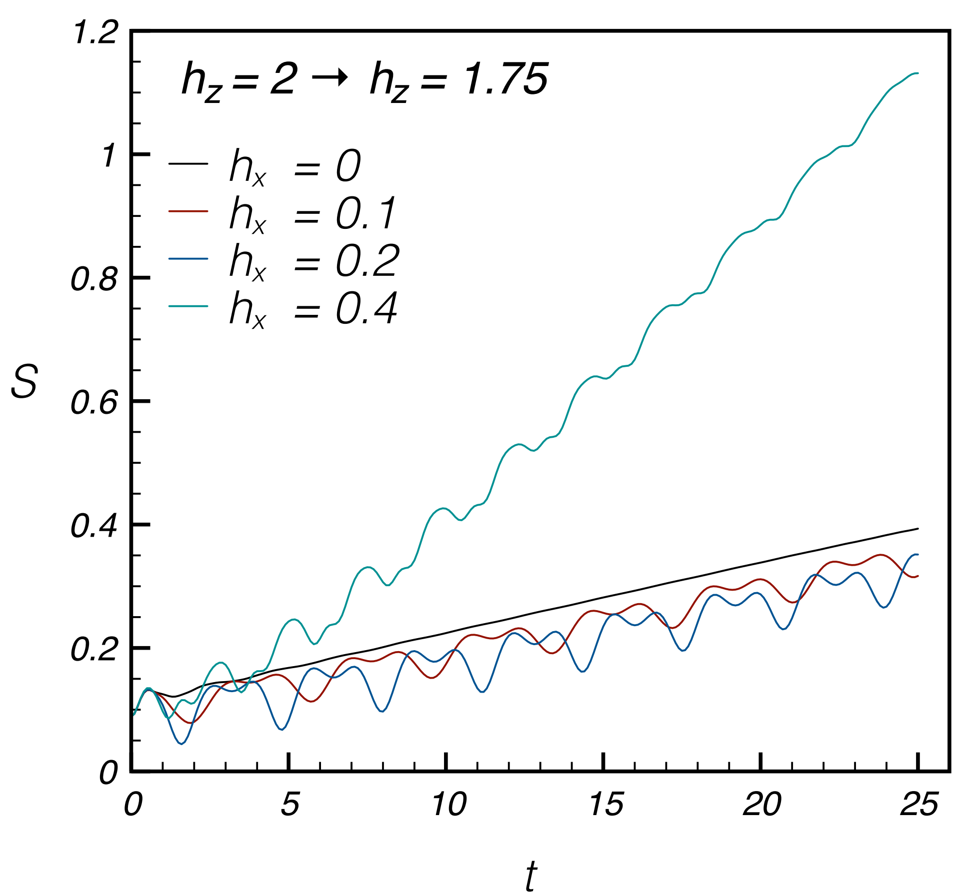
For h_x=0, the initial state can be written in the postquench basis as

$$|\psi_0\rangle = \prod_{k>0}(1 + iK(k)a_k^\dagger a_{-k}^\dagger)|0\rangle$$

- h_x confines the domain walls into mesons.
- When *K(k)* is small, the state is dominated by the linear terms which only contain *(k, -k)* pairs that get confined into mesons at rest.
- Quadratic terms (and higher) lead to propagating mesons, but can be seen only when *K(k)* is large enough.
- Mesons have velocities that are very different from the domain walls,

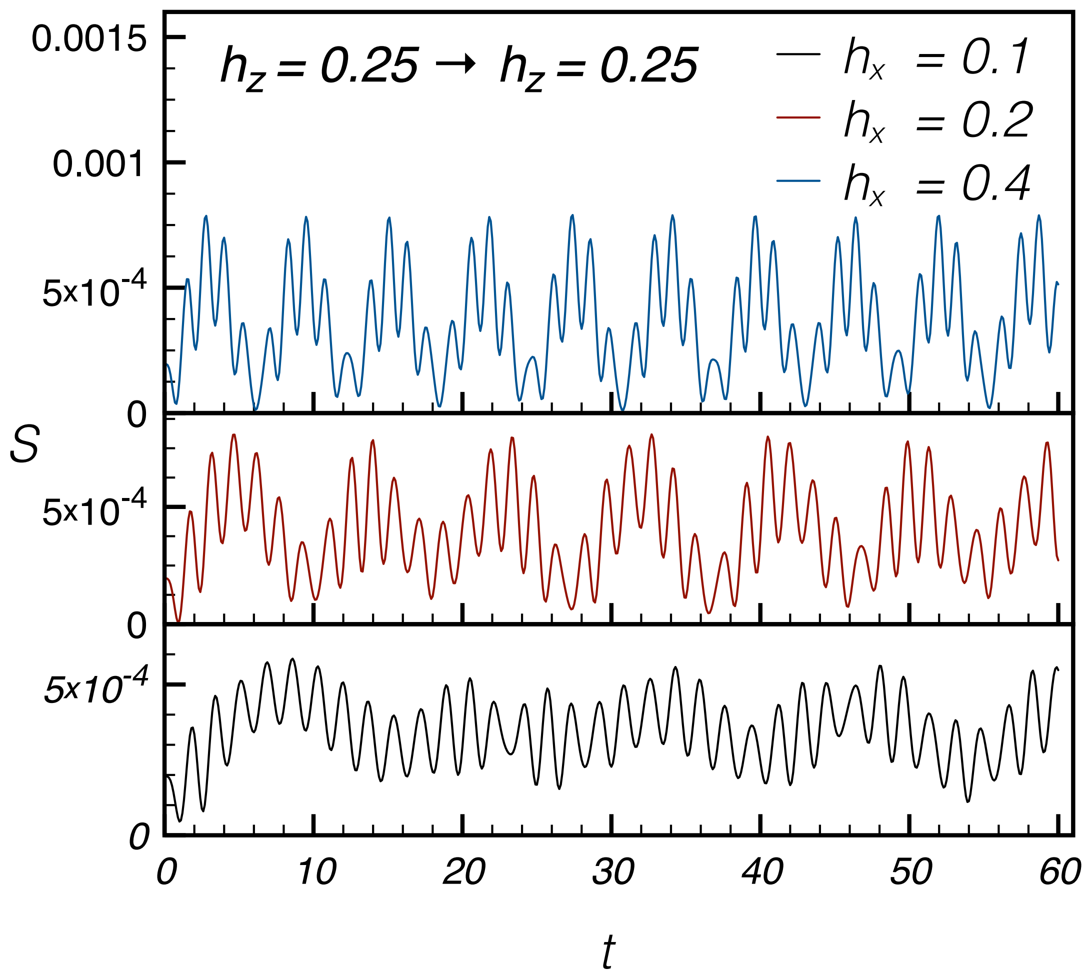
Quench para→para

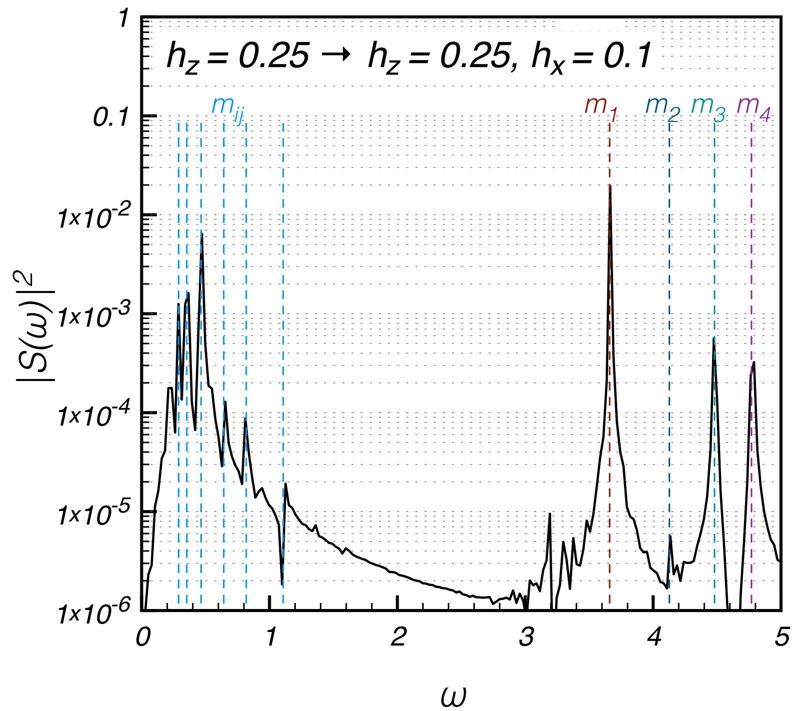


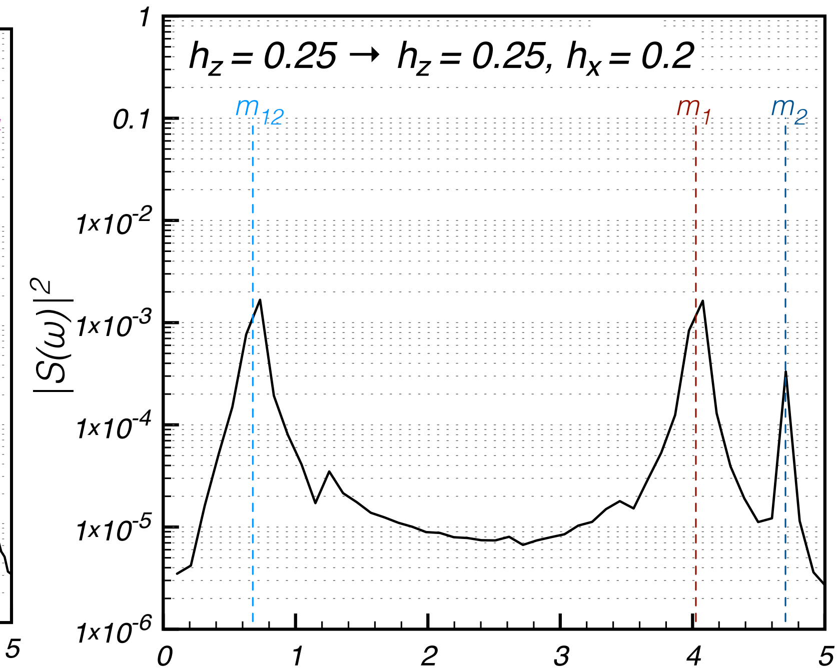


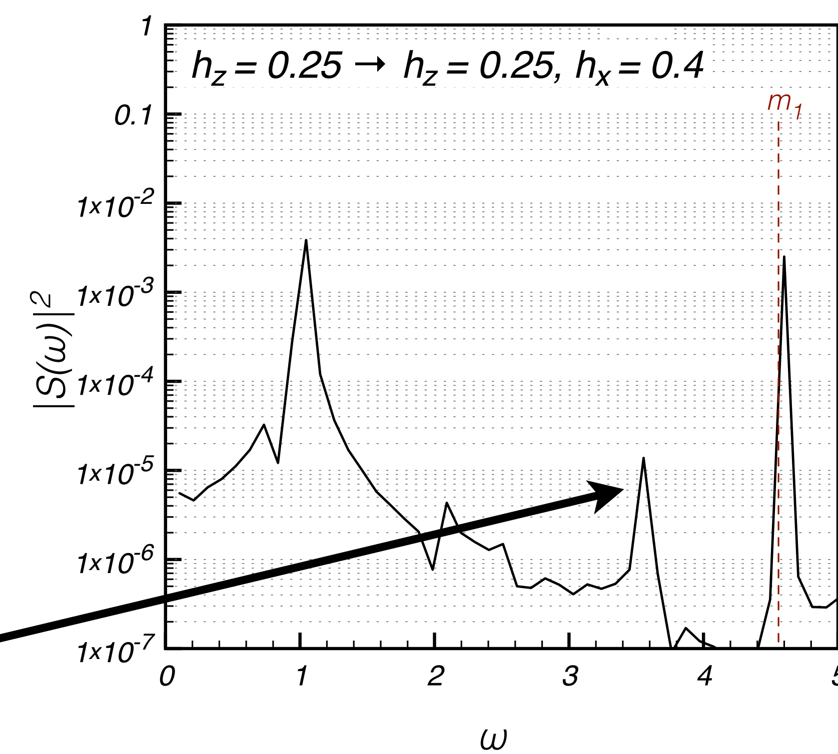
No dramatic qualitative difference with increasing h_x, reflecting the absence of mesons

No transverse (mass) quench









The masses of the mesons are seen even more neatly due to the absence of transient and drift

$\Rightarrow$ **entanglement quench spectroscopy**

Conclusions

In the Ising chain, mesons freeze the light cone spreading of correlations and entanglement

Questions:

- Is it a general property of the many models displaying confinement? Presumably yes, possible to check numerically
- Can it prevent thermalization? as in Banuls et al 2011?
- What about prethermalization?
- Is it true in higher dimensions? e.g. in QCD?

maybe holographically one can have some hints