Hydrodynamic theory of quantum fluctuating superconductivity

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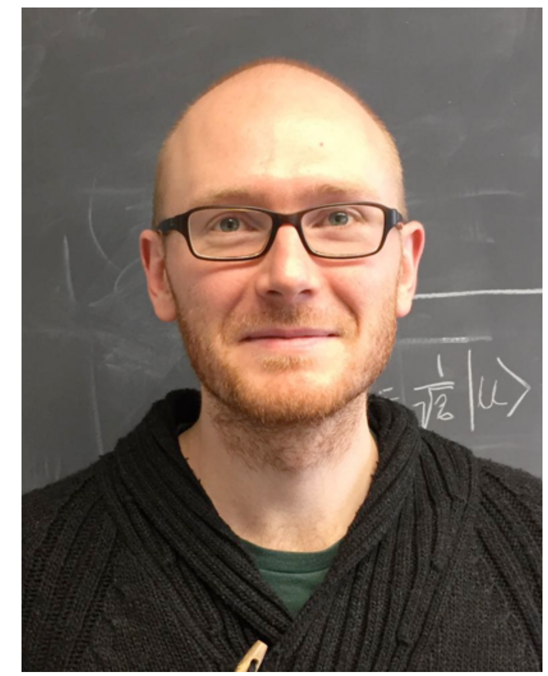

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Hydrodynamic description of conventional metals

- Hydrodynamics:
  → Universal low energy, long wavelength physics.
  → Conserved charges, their currents, Goldstone bosons.

- Conservation law:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot j = 0$$

- Constitutive relation (derivative expansion):

$$j = -D \nabla \rho + \cdots$$

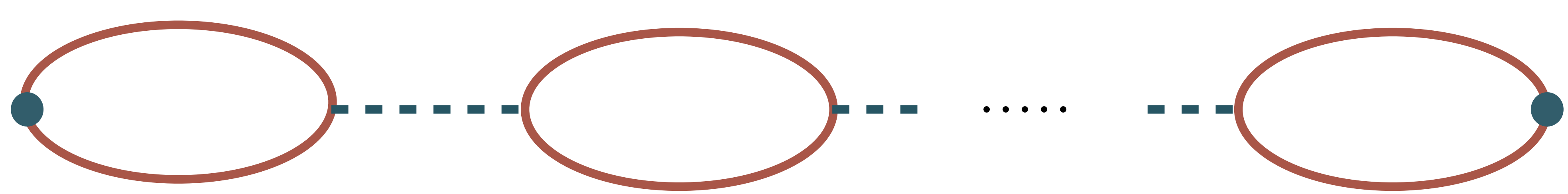
- Conductivity (Einstein relation):

$$j = -D \nabla \rho = -D \frac{\partial \rho}{\partial \mu} \nabla \mu = D \chi \, E = \sigma E$$

Comment on screening by Maxwell fields

- Charge in a metal does not diffuse, it decays exponentially.
- This comes from solving Maxwell's equations + Ohm's law.
- The Einstein relation for the conductivity still holds.
- σ measured with respect to total, not external, electric field:

$$j = \sigma E_{\rm tot} = \sigma \frac{E_{\rm ext}}{\epsilon(\omega,k)} = \frac{\sigma E_{\rm ext}}{1-\frac{1}{k^2}\chi(\omega,k)} = \frac{-i\omega D\chi}{i\omega - D(k^2+\chi)} E_{\rm ext}$$



Superfluid hydrodynamics

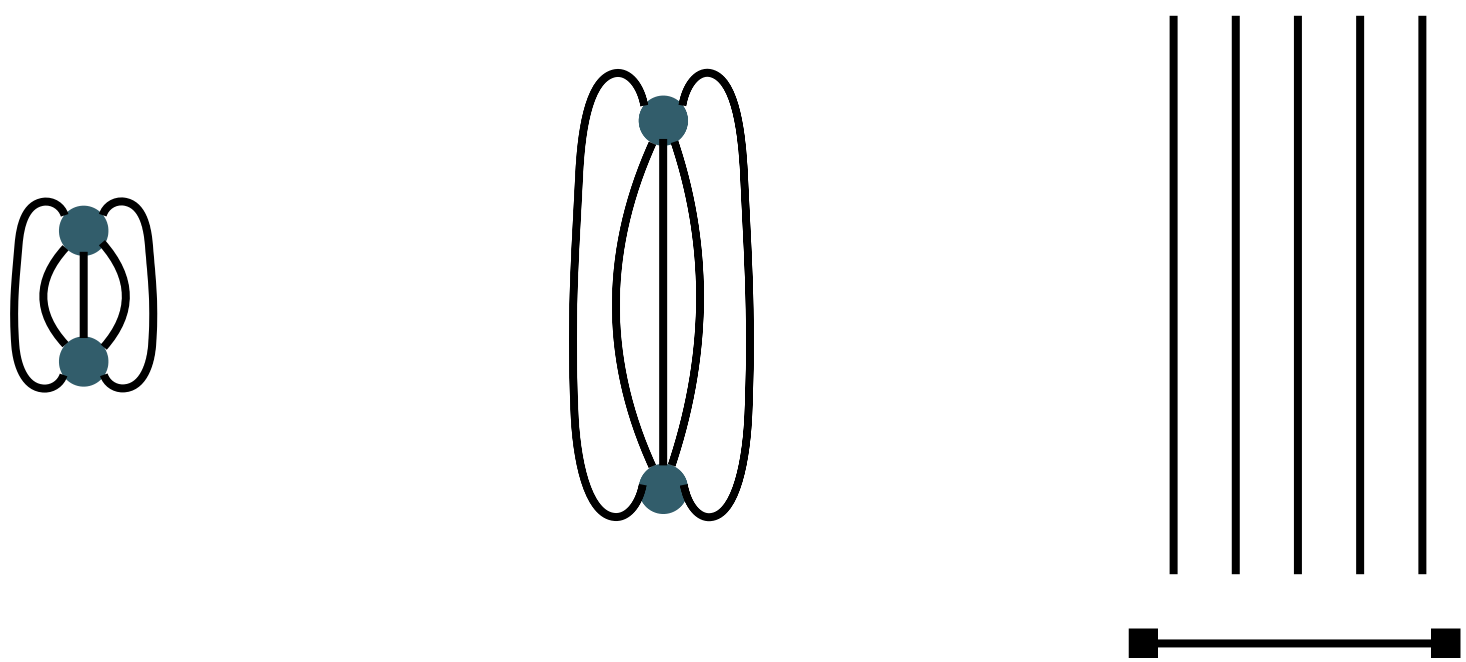
- Phase ϕ of the order parameter appears in hydrodynamics.
- $u_\phi = \dfrac{1}{m}\nabla\phi$ is the superfluid velocity.
- 'Josephson relation':
$$m\frac{\partial u_\phi}{\partial t} = \nabla\frac{\partial\phi}{\partial t} = -\nabla\mu + \cdots$$
- Constitutive relation:
$$j = \frac{\rho_s}{m}u_\phi - D\nabla\rho + \cdots$$
- (super-)Conductivity:
$$j = -\left(\frac{\rho_s}{m^2}\frac{i}{\omega} + D\chi\right)\nabla\mu = \left(\frac{\rho_s}{m^2}\frac{i}{\omega} + D\chi\right)E = \sigma(\omega)E$$

Superconductivity

- ∞ conductivity because: diffusion → second sound mode.
- In a superconductor, the U(1) symmetry is gauged, i.e. coupled to electromagnetism.
- This gaps out the Goldstone/sound mode in the same way the diffusive mode was previously gapped.
- However, the conductivity is, as before, measured with respect to the total electric field. So the unscreened (superfluid) hydrodynamics determines the conductivities.

Vortices and supercurrent relaxation

- In two space dimensions, above picture incomplete.
- Motion of vortices can wind and unwind the supercurrent.



$\Delta\phi = 2\pi$

- Expect supercurrent relaxation rate Ω:

$$\sigma(\omega) = \frac{\rho_s}{m^2} \frac{1}{-i\omega + \Omega}$$

Vortices and supercurrent relaxation

- This problem is well understood in some regimes:
  → Thermal BKT proliferation of vortices above T_{BKT}.

- Classical picture: vortices pushed across sample by 'superfluid Magnus force'
  → The core of the vortices is in the normal state.
  → Therefore, motion of vortices creates dissipation.
  → Get

$$\Omega \sim \frac{n_f A_v}{\sigma_{\mathrm{n}}}$$

[Bardeen-Stephen '65]

- Much controversy, however, about whether (quantum) phase-disordered superconductors exist at T = 0.

[review: Phillips-Dalidovich '03]

In the remainder

- Lightening overview of some experiments.
- Develop a fully quantum effective field theoretic formalism for the conductivity of phase-disordered superconductors.
- Illustrate formalism with two examples:

  (i) ‘Check’: Elegant (re)derivation of Bardeen-Stephen result.

  (ii) Phase disordering by a Chern-Simons interaction ['topologically ordered superfluid vortex liquid'].

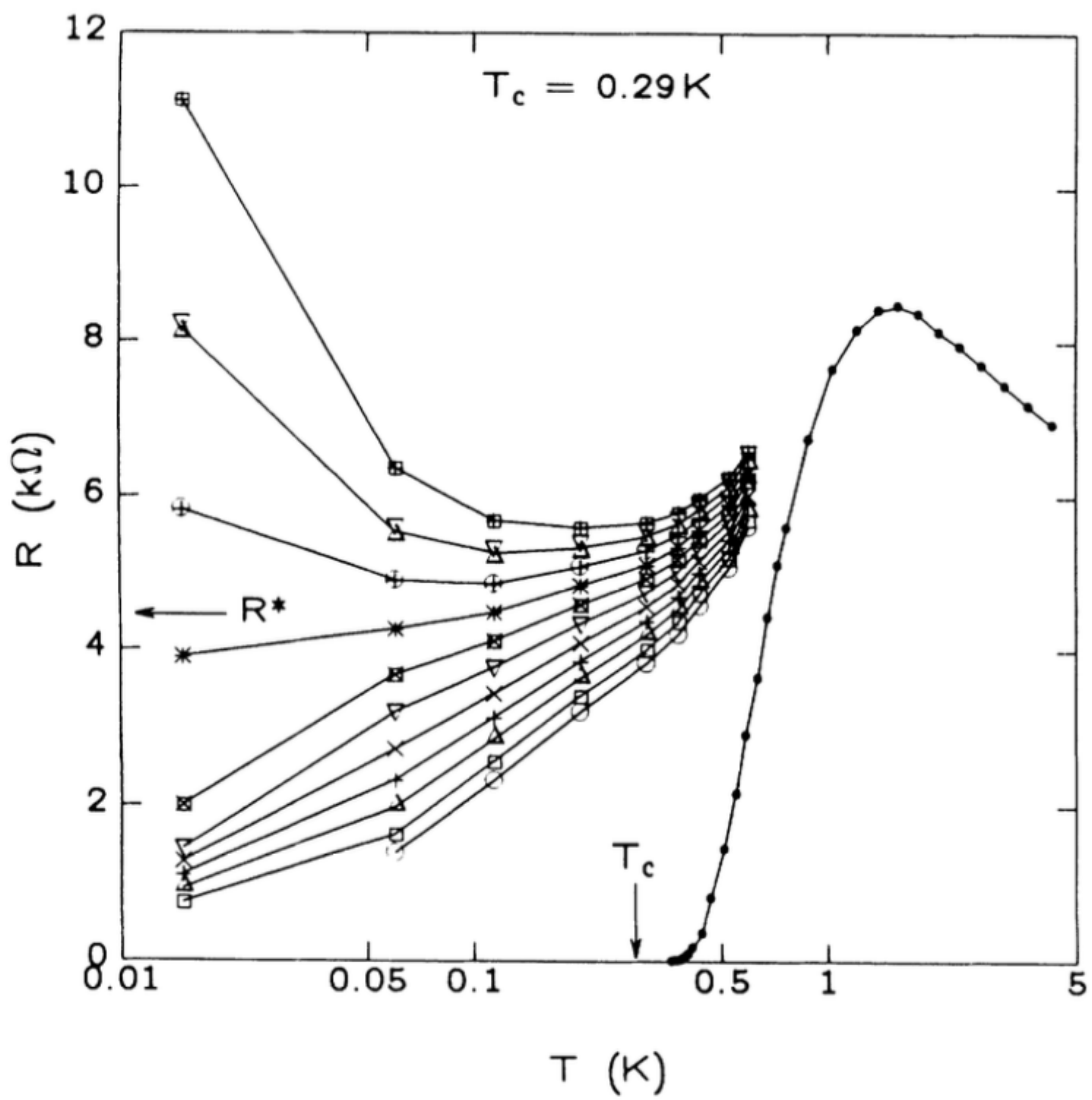
Superfluid-insulator transitions

- In two (spatial) dimensions, conventional theory suggests that as $T \to 0$ electrons will either localize or pair up.
- That is, the phase of matter one expects to find is either an insulator or a superconductor.
- Indeed, early experiments suggested that disordered thin films undergo superconductor-insulator transitions as a function of magnetic field or thickness ($\approx$ 1/disorder).

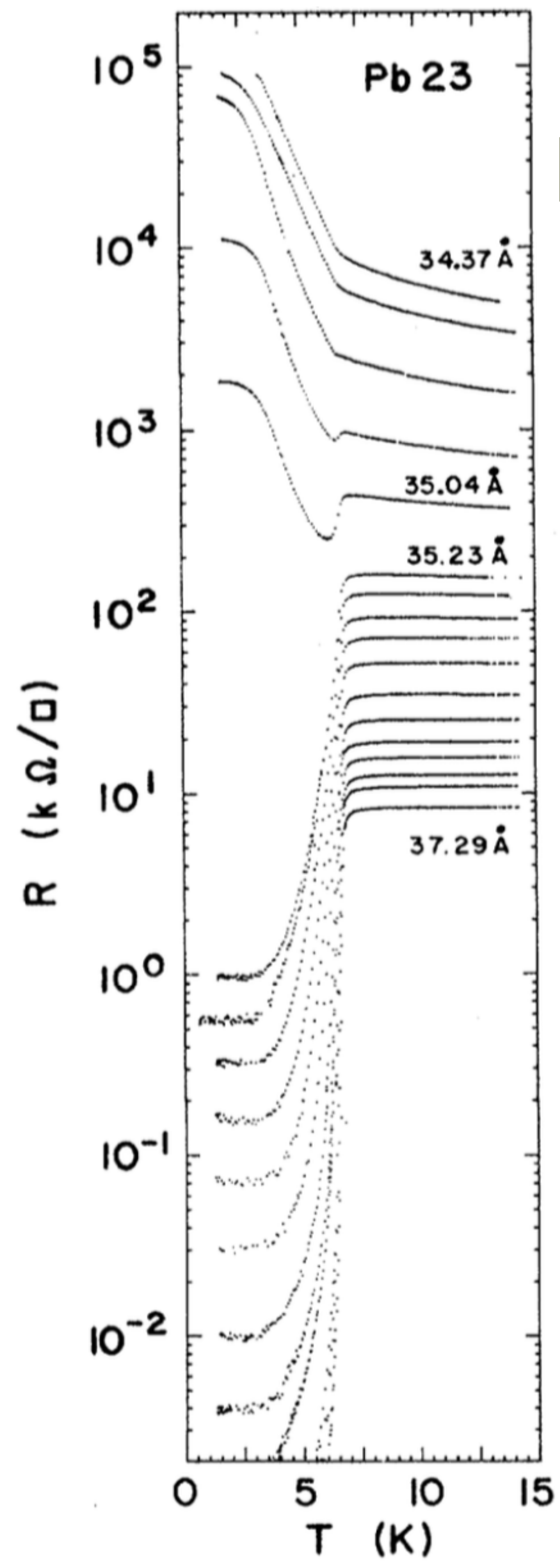
Destroys superconductivity

Favors localization

Superfluid-insulator transitions



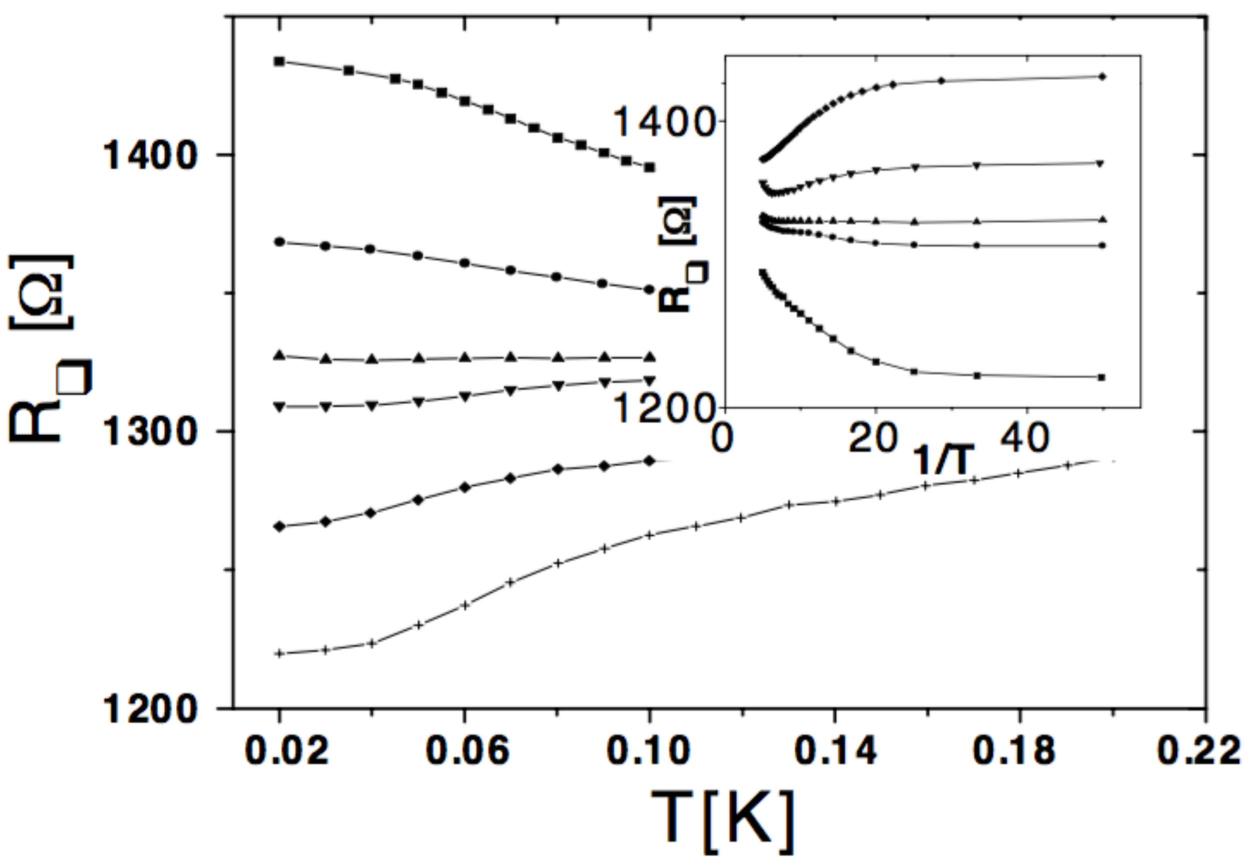
[Hebard and Paalanen ’90, α-InO_x]

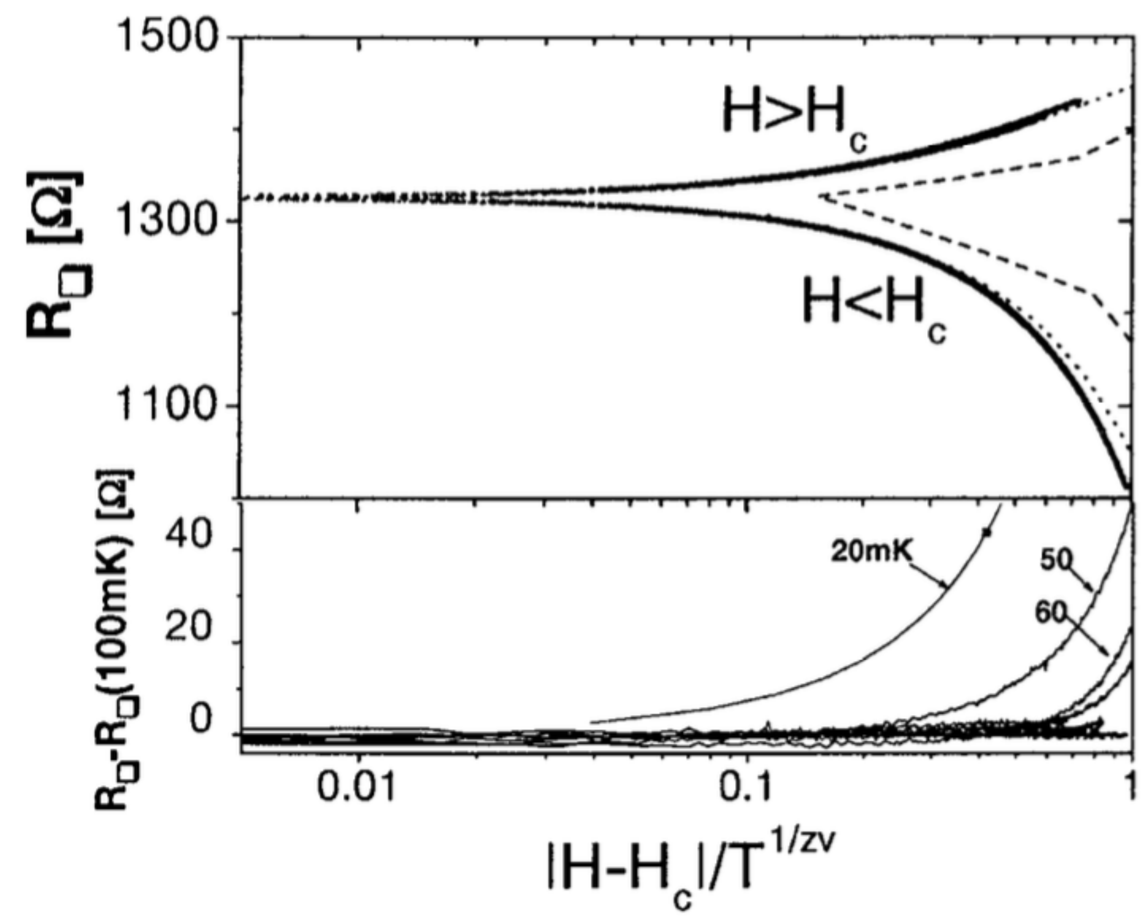


[Jaeger et al. ’89, Pb]

Metallic phases in two dimensions

- Problematically for ‘conventional’ understanding, in weakly disordered films a metallic phase intervenes (at T = 0!) between the superconductor and insulator.





[Mason, Kapitulnik ’99, α-MoGe]

Metallic phases in two dimensions

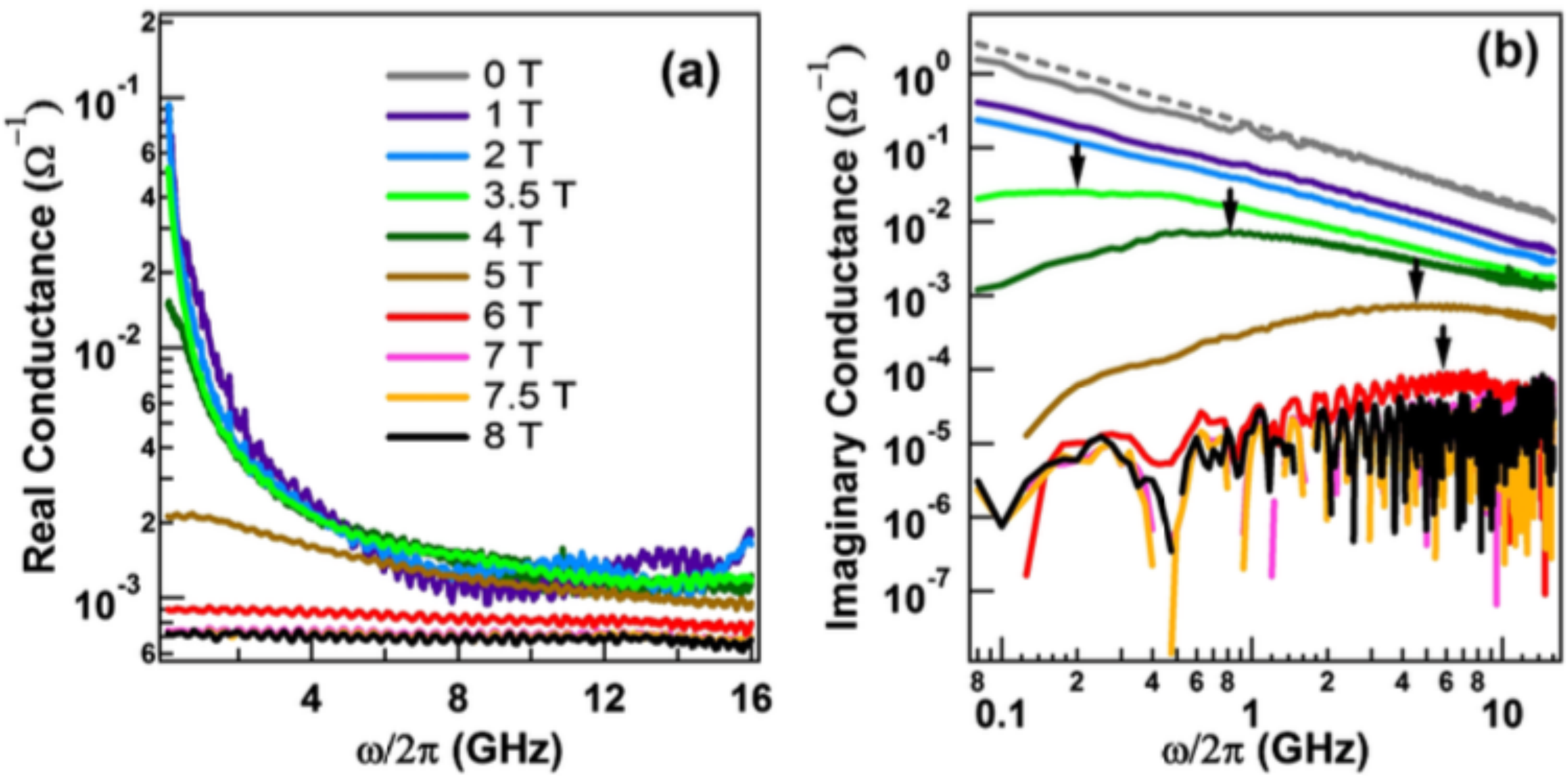
- Often, the residual resistivity of the metallic phase is much smaller than the “normal state” resistivity of the material at temperatures above the “mean field” superconducting temperature.

- Suggests the low energy degrees of freedom of the metallic phases are not the normal state quasiparticles.

- Natural to think of as “failed superconductors” where (quantum!) phase fluctuations have destroyed phase coherence.

Metallic phases in two dimensions

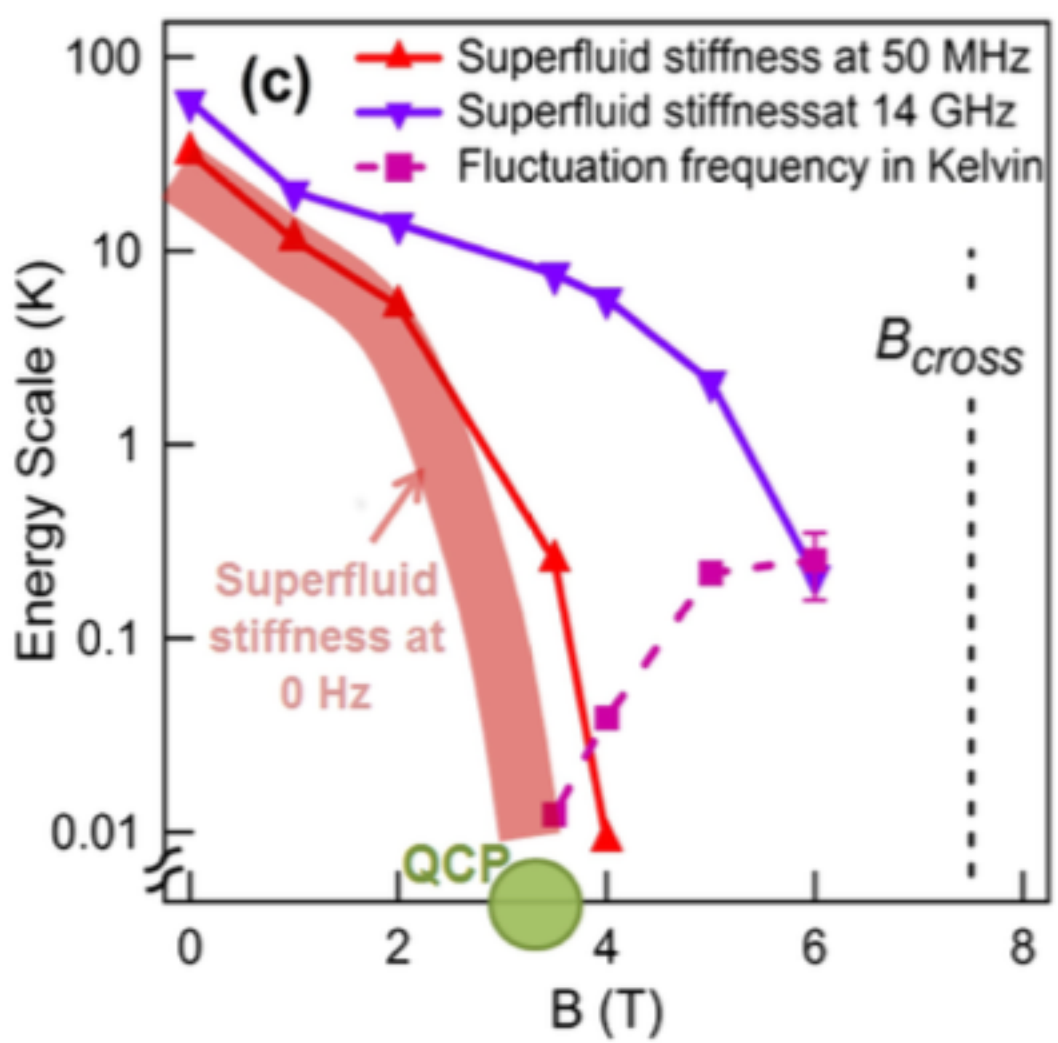
- Direct motivation for our work: observation of a Drude-like peak in the metallic phase of InO_x.



[Liu, Pan, Wen, Kim, Sambandamurthy, Armitage '13]

Metallic phases in two dimensions

- The width of the Drude-like peak goes to zero at the same magnetic field where superconductivity appears.



[Liu, Pan, Wen, Kim, Sambandamurthy, Armitage ’13]

Memory matrix formalism

- Most discussions of this physics have involved semi-microscopic models with uncontrolled approximations.
- Instead: work in a limit where a hierarchy of timescales allows an effective field theoretic approach.
- Small parameter will be the supercurrent relaxation rate. I.e. want $\Omega \ll T$, etc.
- (Approach inspired by studies in holographic systems over past few years, where slow mode was momentum.)

Memory matrix formalism

[Logic goes back to: Götze and Wölfle '72, Forster '75, …]

- Suppose that $H = H_0 + \varepsilon\, \Delta H$, with $[\Delta H, J_\phi] \neq 0$.

- Then the decay of J_ϕ is slow and dominates σ:

$$\sigma(\omega) = \frac{\chi^2_{J J_\phi}}{\chi_{J_\phi J_\phi}} \frac{1}{-i\omega + \Omega} + \cdots$$

- But now we have a formula for Ω! :

$$\Omega = \epsilon^2 \, \frac{1}{\chi_{J_\phi J_\phi}} \lim_{\omega \to 0} \left. \frac{\operatorname{Im} G^R_{i[\Delta H, J_\phi]\, i[\Delta H, J_\phi]}(\omega)}{\omega} \right|_{\epsilon = 0} .$$

Spectral density of states into which J_ϕ can decay. Cf. Fermi Golden rule.

Supercurrent relaxation

- Recap: if an 'almost conserved' operator carries current, rate of the decay determines the conductivity.
- In our case of interest today: $J_\phi = \frac{1}{m} \int d^2x \nabla\phi$
- Need an interaction that doesn't commute with J_Φ.
- Natural building block:

$$\pi_\phi = \frac{\partial f}{\partial \dot{\phi}} = -\frac{\partial f}{\partial \mu} = \rho \,.$$

i.e. charge density is canonically conjugate to the phase:

$$[\phi(x), \rho(y)] = i\delta(x-y) \,.$$

Supercurrent relaxation

- Thus a simple, generic perturbation of the superfluid state is the short range Coulombic interaction:

$$\Delta H = \frac{\lambda}{2} \int d^2x \, \rho(x)^2 \, .$$

- At first glance looks like commutator is trivial total derivative:

$$i[\Delta H, J_\phi] = -\frac{\lambda}{m} \int d^2x \nabla \rho(x)$$

- However, the phase appearing in J_ϕ is only defined outside of vortex cores! Above integral is then also only over the outside of vortex cores. Integral over all space vanishes:

  → integral over vortex cores.

Supercurrent relaxation

- The memory matrix formula for Ω becomes an integral of the two point function of ρ over the vortex core.

- Using the diffusive behavior of ρ in normal state, the Bardeen-Stephen formula drops out exactly.

$$\Omega \sim \frac{n_f A_v}{\sigma_{\mathrm{n}}}$$

- So we discover the quantum origin of this formula. Charge interactions enhance phase fluctuations:

$$\Delta\rho \, \Delta\phi \gtrsim \hbar$$

Beyond Bardeen-Stephen

- Real life vortices are not infinitely large. The diffusive form of the charge density correlator is therefore not exact. For small vortices, it will not even be approximately correct.

- Work in progress: generalize Bardeen-Stephen formula allowing for non-diffusive dynamics of the charge density.

- Part of the controversy around T=0 metallic phases is ‘where does the dissipation occur’? From our approach it is manifest that if the phase-relaxing interacting is local, dissipation must be due to vortex cores.

Supercurrent relaxation without parity

- With parity and time-reversal broken, a second very natural ΔH exists.

- Suppose the low energy effective theory is coupled to an emergent Chern-Simons gauge field:

$$\mathcal{L} = \mathcal{L}_{\mathrm{matter}} + j_\mu \left(A^\mu + a^\mu \right) - \frac{1}{2\lambda'} \epsilon^{\mu\nu\rho} a_\mu \partial_\nu a_\rho$$

- Integrating out the gauge field generates

$$\mathcal{L}' = \frac{\lambda'}{2} j_\mu \frac{\epsilon^{\mu\nu\rho}\partial_\rho}{\partial_\sigma \partial^\sigma} j_\nu \quad \Rightarrow \quad \Delta H = \frac{\lambda'}{2} \int \frac{d^2 k}{(2\pi)^2} \frac{\rho_{-k} (\nabla \times j)^z_k}{k^2} + \mathrm{h.c.}$$

Supercurrent relaxation without parity

- Non-locality of induced interaction leads to a nonzero time dependence of J_ϕ everywhere. In fact:

$$i[\Delta H, J^i_\phi] = -\frac{\lambda'}{m}\epsilon^{ij} J^j \, .$$

- Rough physical picture:
  Current = Flow of charge
  → Flow of emergent magnetic flux (CS term)
  → Flow of vortices
  → Relaxation of supercurrent in transverse direction!

- Ω depends on charge flow in normal component.

Supercurrent relaxation without parity

- Result for conductivities:

$$\sigma_{xx} = -\frac{m^2}{\lambda'^2 \rho_s} \frac{\omega(\omega\Omega + i(\Omega^2 + \Omega_H^2))}{(-i\omega + \Omega)^2 + \Omega_H^2} \, ,$$

$$\sigma_{xy} = -\frac{1}{\lambda'} - \frac{m^2}{\lambda'^2 \rho_s} \frac{\omega^2 \Omega_H}{(-i\omega + \Omega)^2 + \Omega_H^2} \, ,$$

- Feature: ‘supercyclotron resonance’ at

$$\omega_\star = \pm\Omega^H - i\Omega = \frac{\lambda' \rho_s}{m^2} \frac{1}{\pm 1 - \lambda'(\pm\sigma_0^H - i\sigma_0)} \, .$$

Conductivities of the normal component of superfluid.

Chern-Simons superfluid hydrodynamics

- The expressions for the conductivities can be alternatively derived directly from superfluid hydrodynamics coupled to a Chern-Simons gauge field.

- Dissipation in this case is not due to vortex cores, but to the normal component of the superfluid.

- If the normal component only has a Hall conductivity (e.g. a superfluid coupled to a quantum Hall state), obtain nontrivial dissipationless frequency dependent dynamics.

Recap [see arXiv/1602.08171]

- Superfluid relaxation occurs if perturbations of effective Hamiltonian do not commute with the supercurrent.
- Starting with perturbations of superfluid hydrodynamics gives controlled entry point. This works even if the underlying microscopic dynamics is strongly correlated.
- Gave two examples, with and without parity:
  (1) With parity: recovered Bardeen-Stephen.
  (2) Without parity: ‘supercyclotron resonance’ determined by conductivities of normal component.