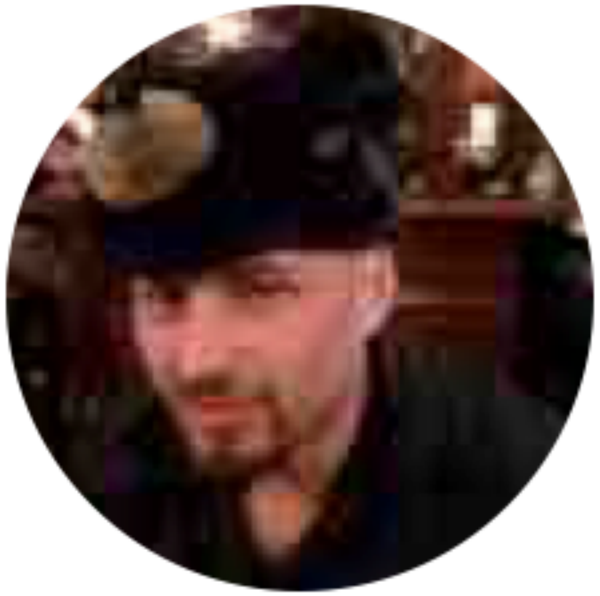
Branes in the AdS/CFT Correspondence

thanks to

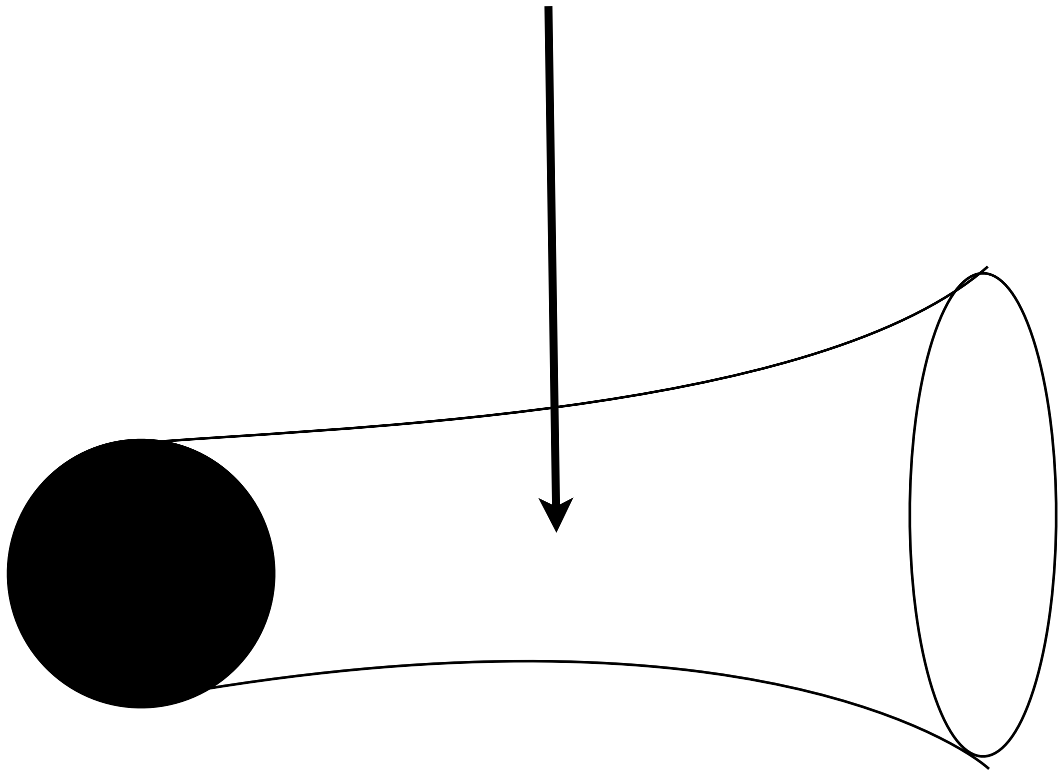
Gabriele La Nave

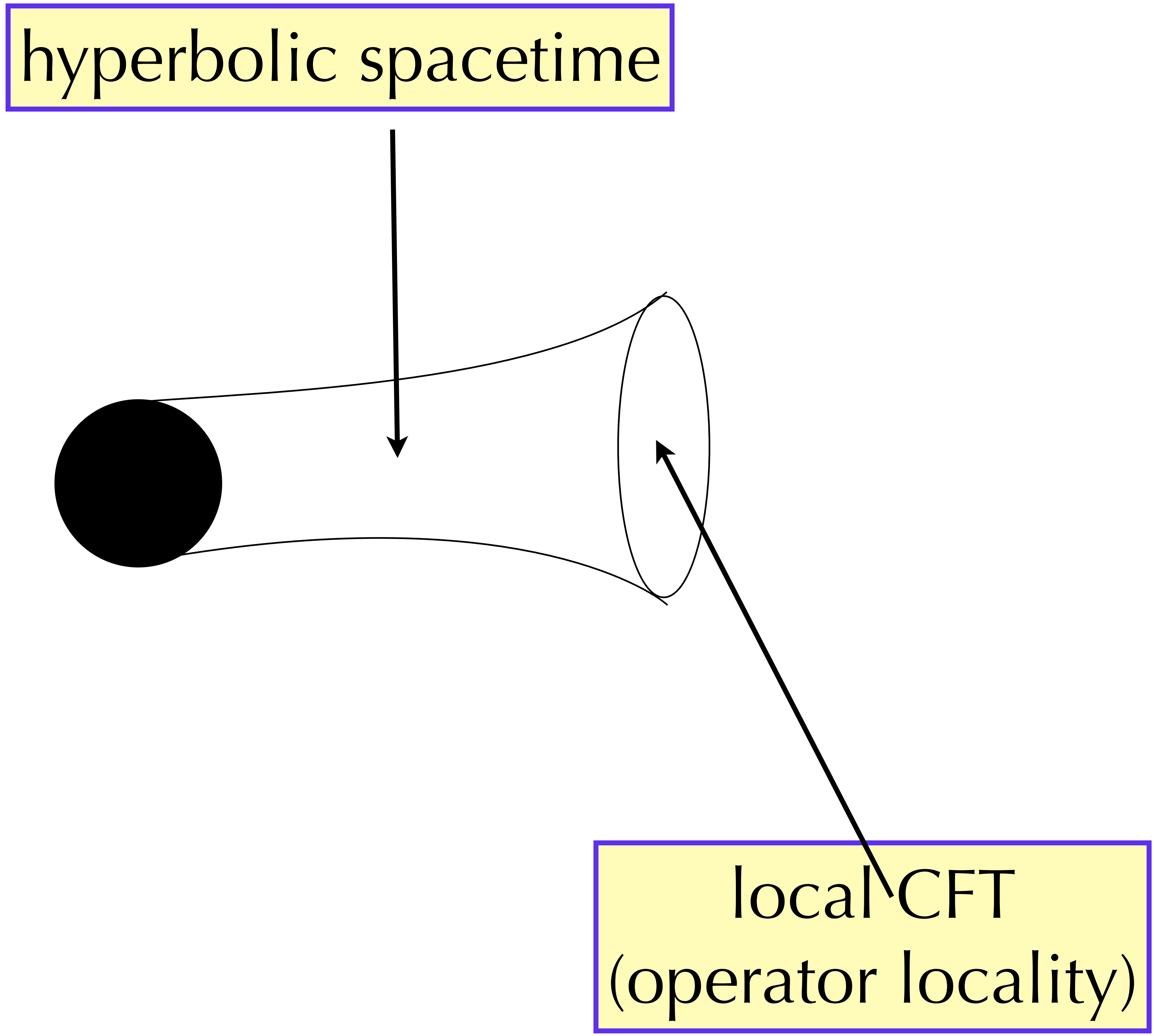


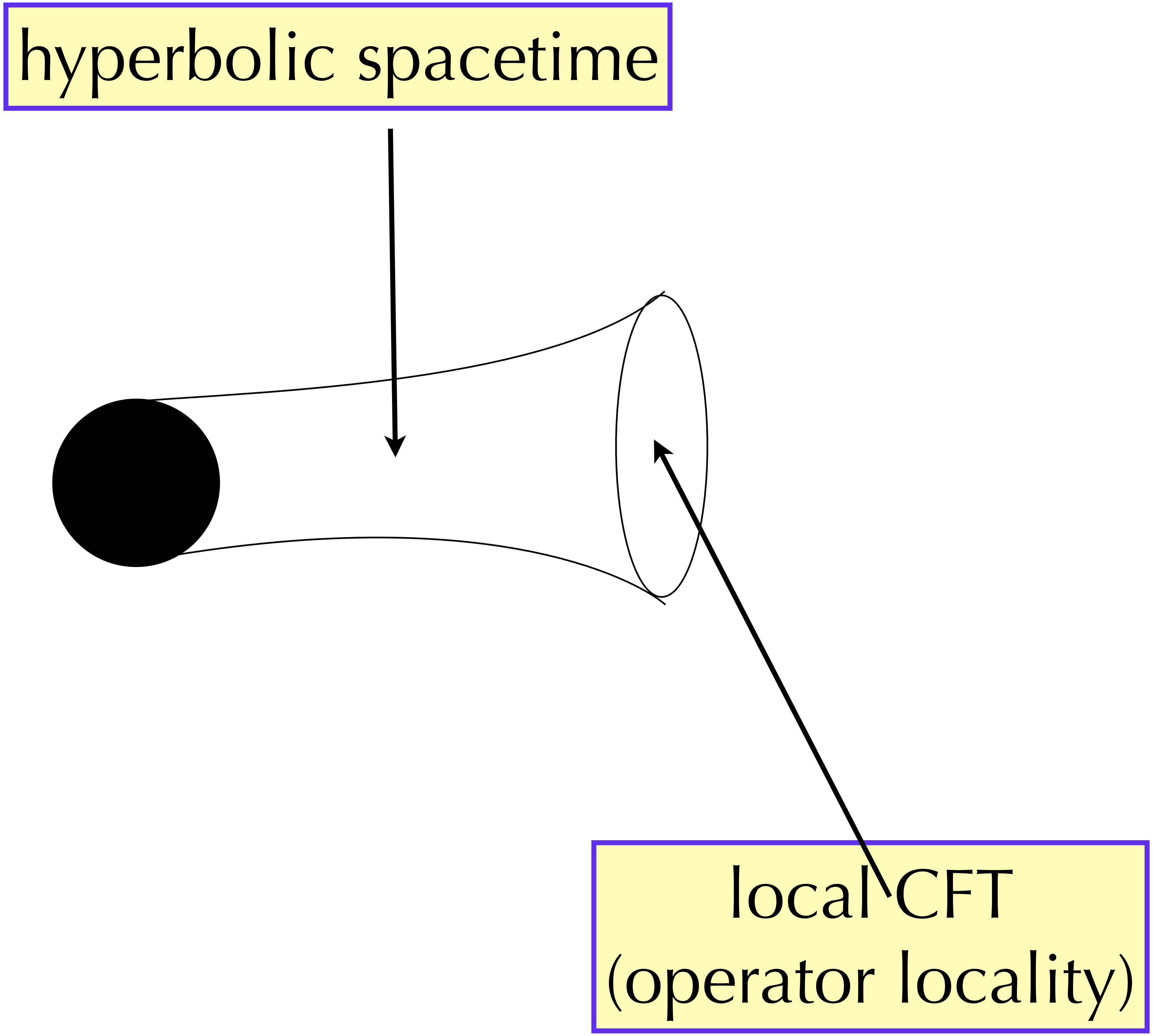
NSF

arxiv:1605.07525

hyperbolic spacetime

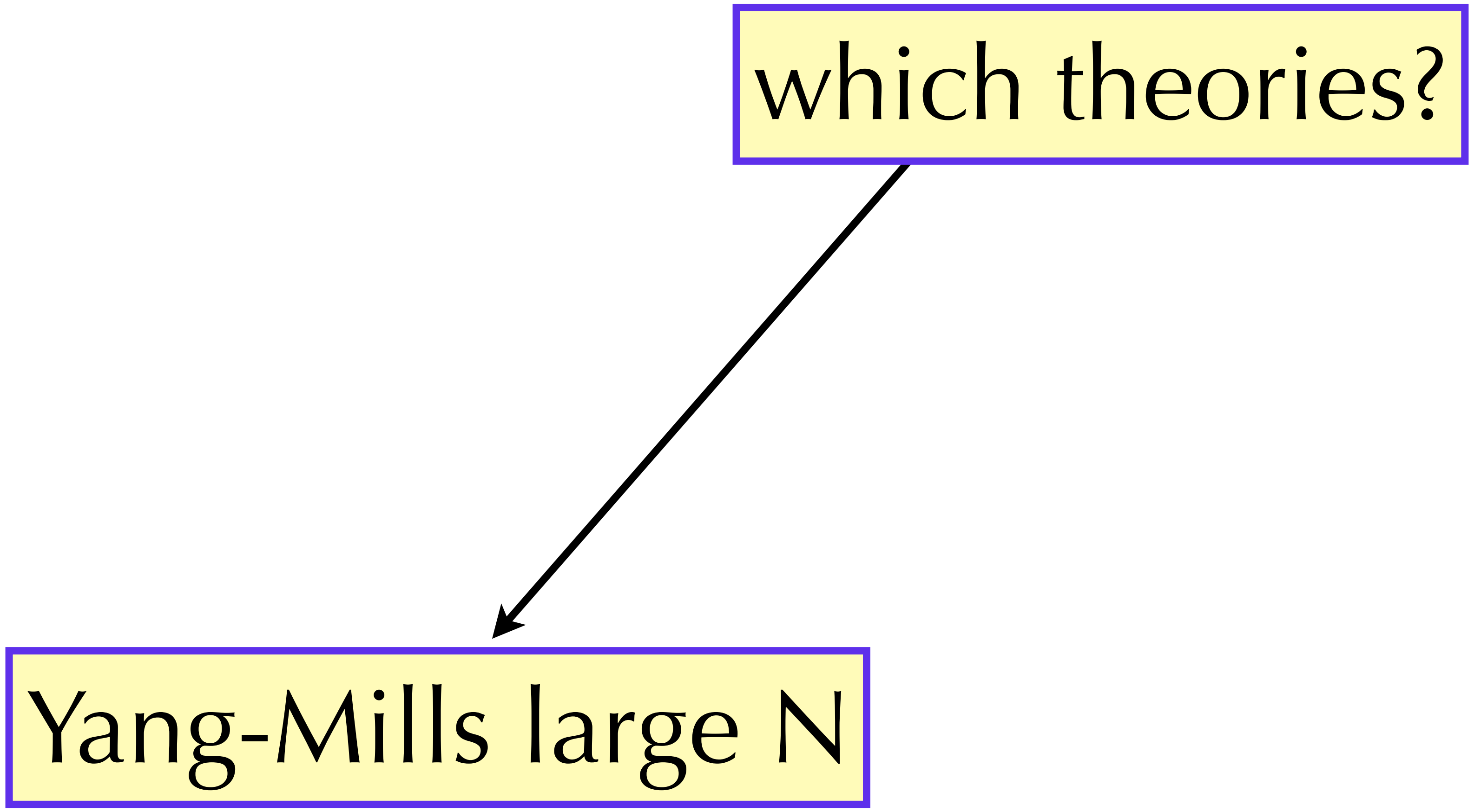


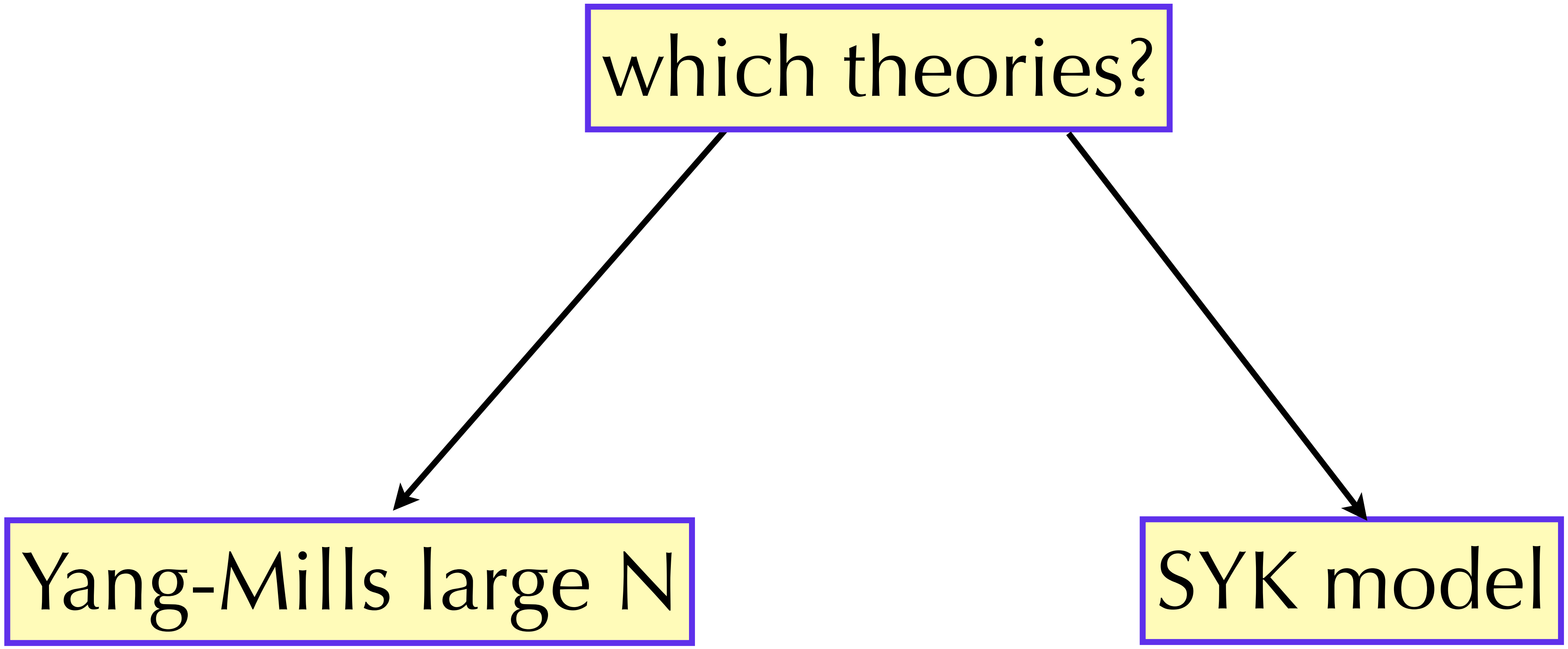

hyperbolic spacetime
local CFT
(operator locality)



1-1 state correspondence

which theories?


which theories?
Yang-Mills large N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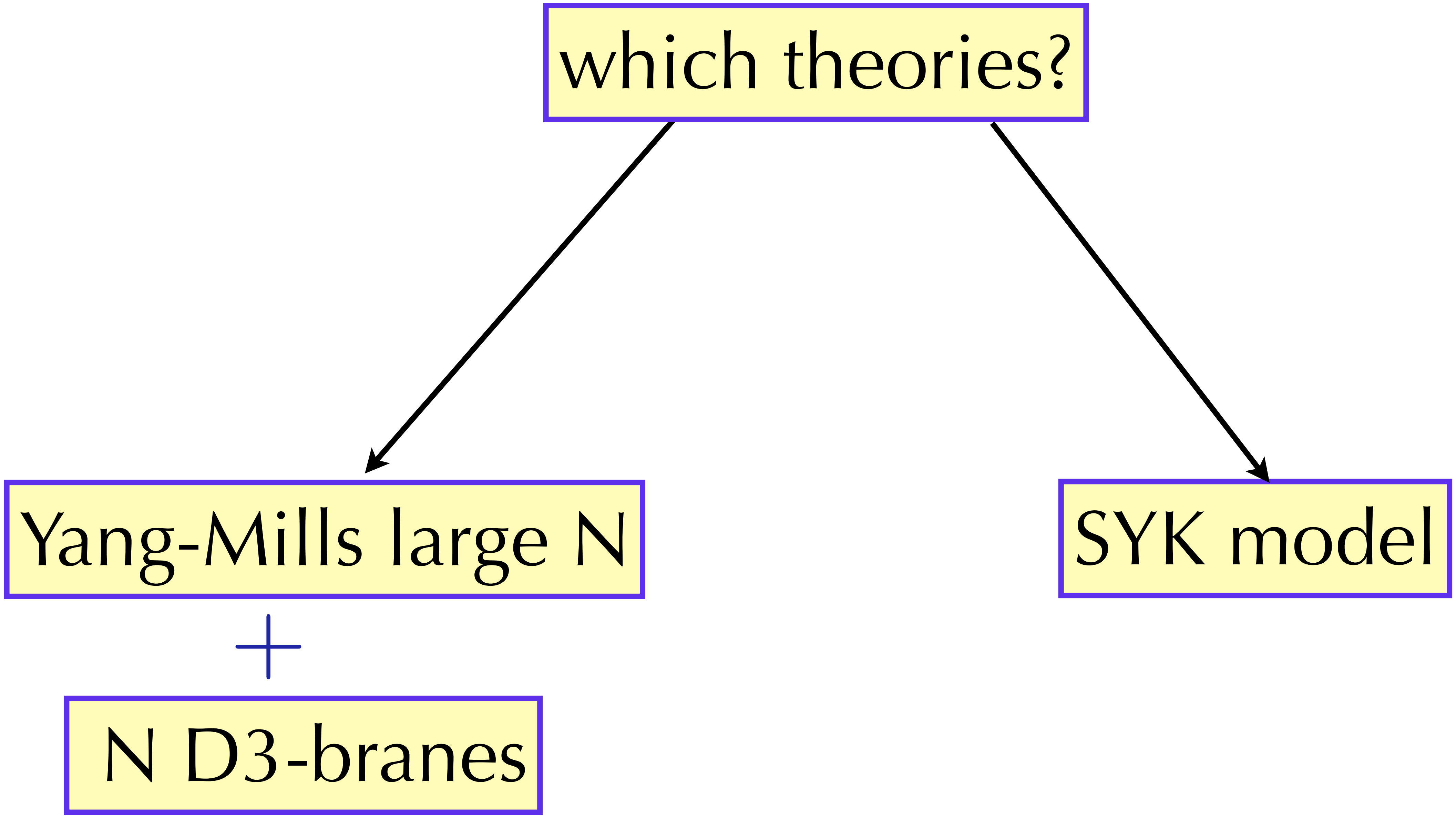
SYK model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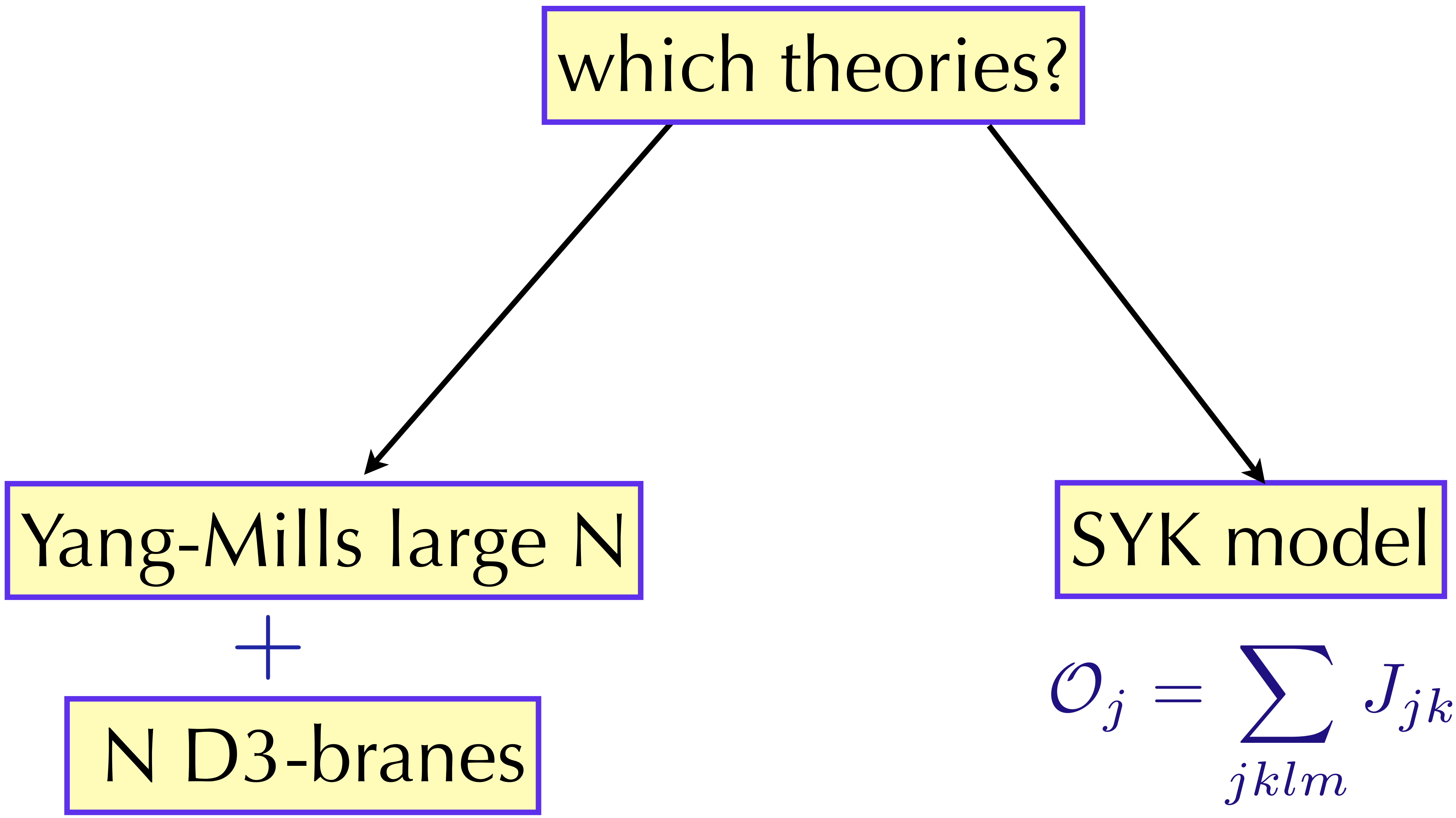
+
N D3-branes
SYK model



$$\mathcal{O}_j = \sum_{jklm} J_{jklm}\chi_k\chi_l\chi_m$$

which theories?

Yang-Mills large N

+

N D3-branes

SYK model

$$\mathcal{O}_j = \sum_{jklm} J_{jklm} \chi_k \chi_l \chi_m$$

completely different limits

which theories?

Yang-Mills large N

$+$

N D3-branes

SYK model

$$\mathcal{O}_j = \sum_{jklm} J_{jklm} \chi_k \chi_l \chi_m$$

completely different limits

geodesic completeness

$$ds^2_{\mathbb{H}} = \frac{1}{y^2}(dx^2 + dy^2) \qquad \mathbb{H}^2$$

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non-zero Christoffel symbols

$$\begin{cases} \Gamma^x_{xy} = \Gamma^x_{yx} = -\dfrac{1}{y} \\ \Gamma^y_{xx} = -\Gamma^y_{yy} = \dfrac{1}{y} \end{cases}$$

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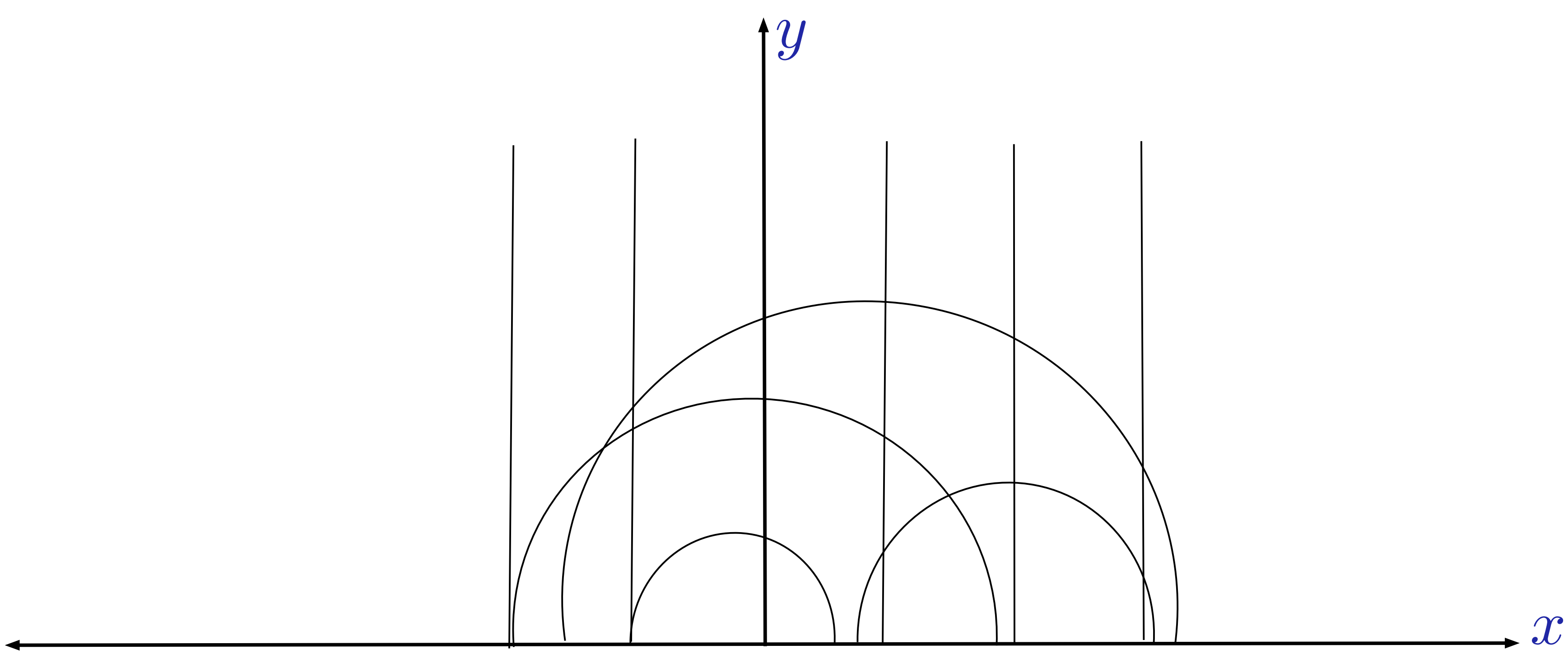

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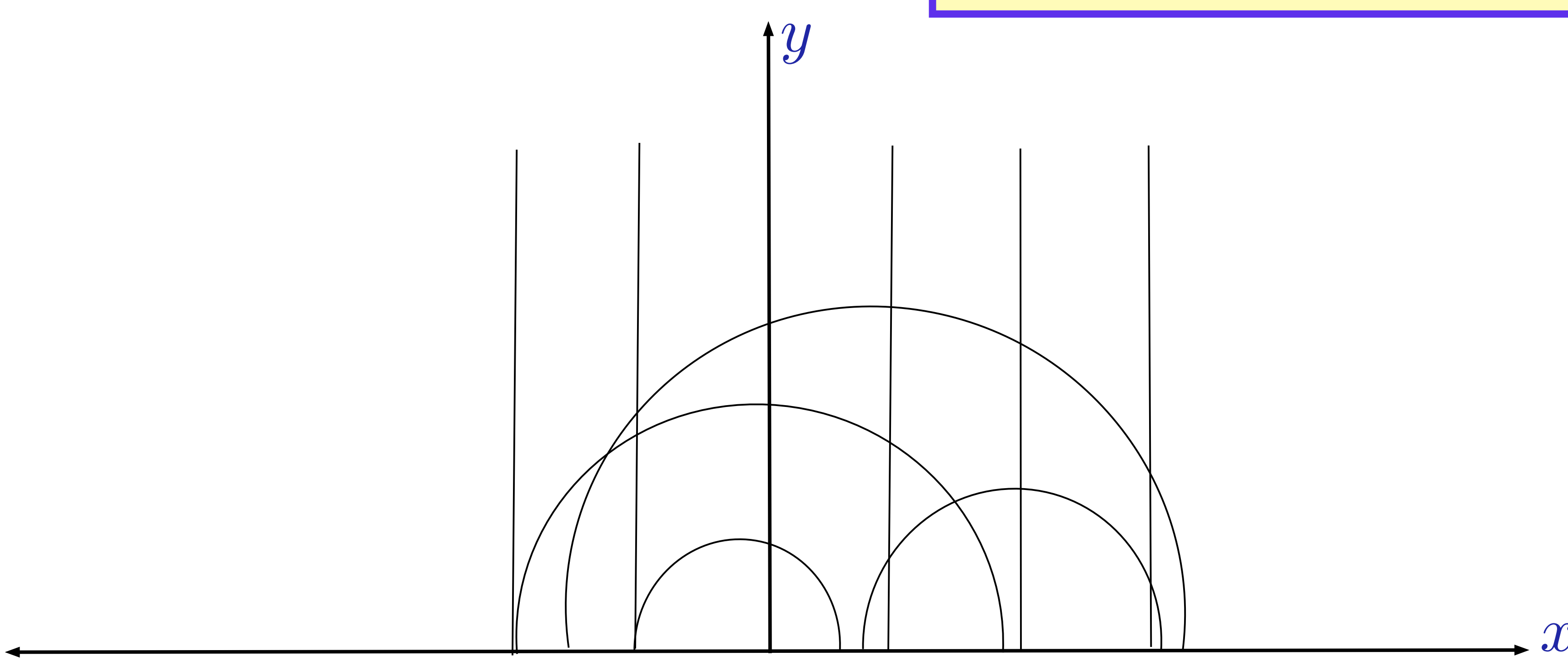
geodesics

cover all spacetime



what if?

$$ds^2 = -e^{-2|y|/L} g_{\mu\nu} dx^\mu dx^\nu + dy^2$$

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boundary locality

Type IIB String theory

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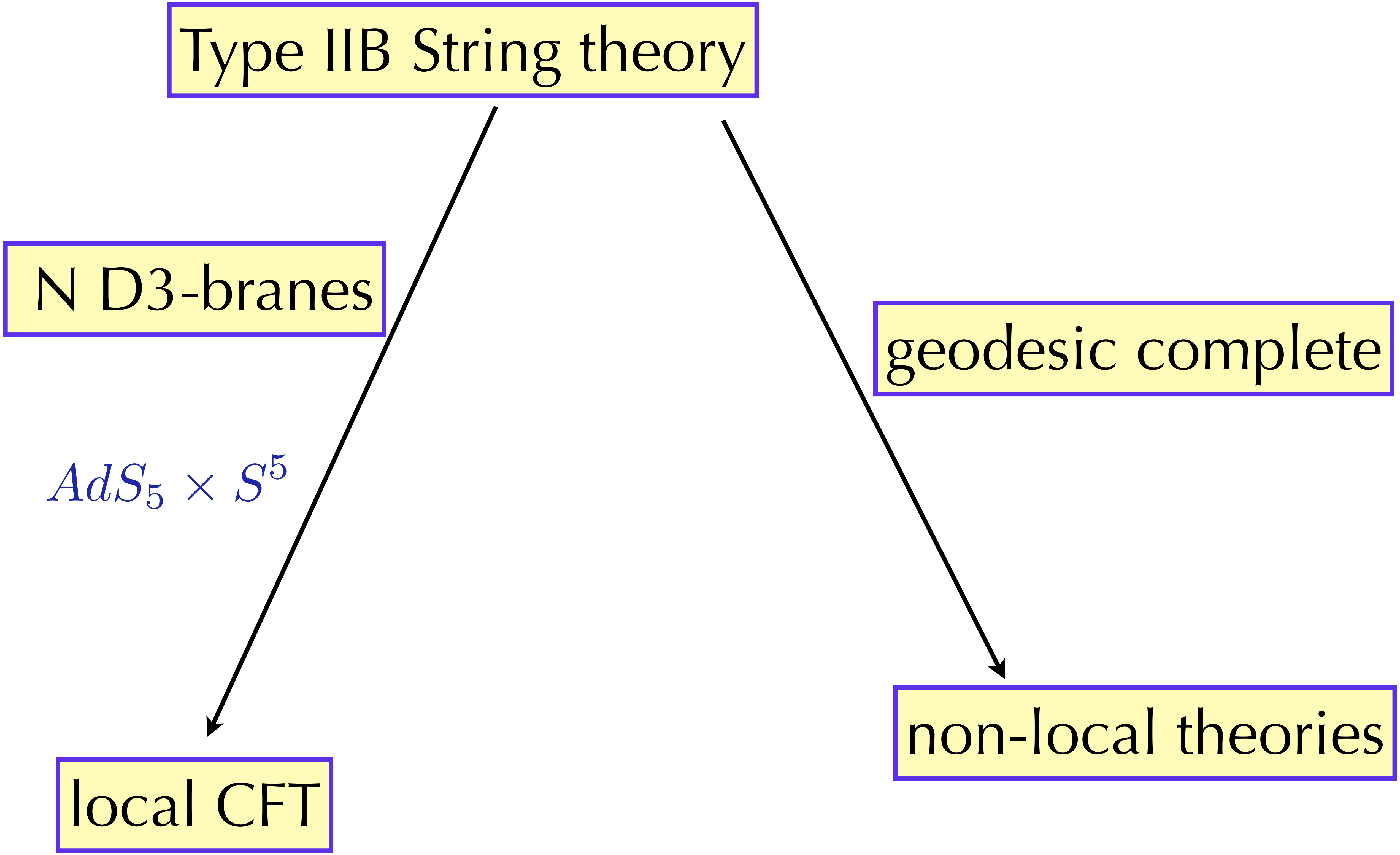
N D3-branes

$AdS_5 \times S^5$

local CFT


Type IIB String theory
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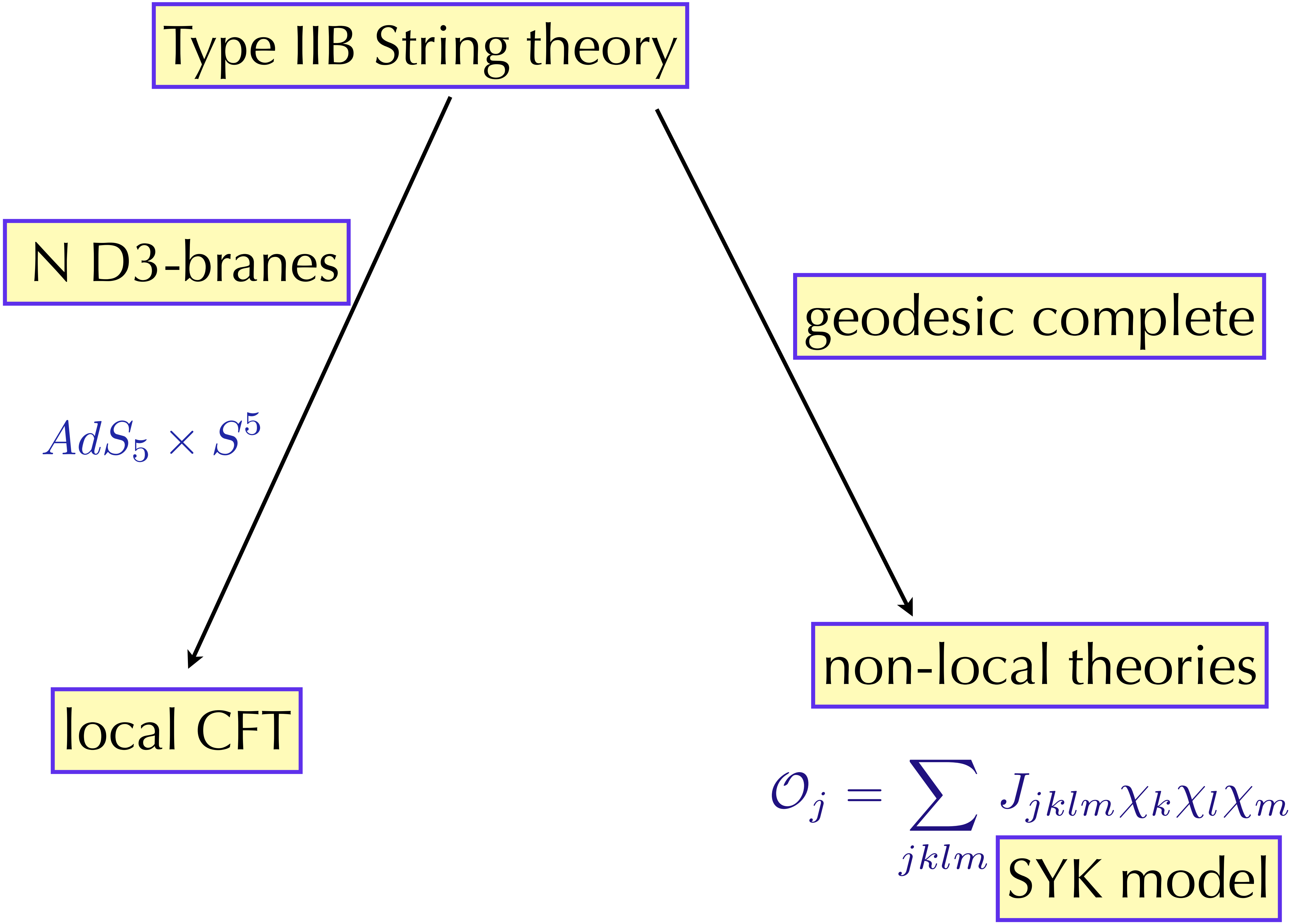
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$\mathcal{O}_j = \sum_{jklm} J_{jklm} \chi_k \chi_l \chi_m$
SYK model

does he feel his weight?

No

GR

No

Equivalence principle

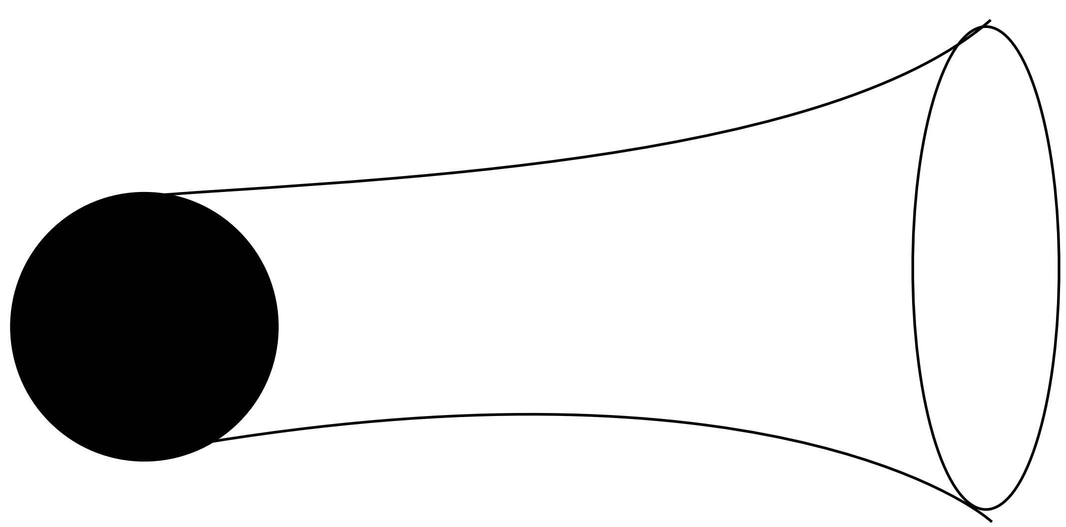
GR

No

Equivalence principle

no local measurement can ever tell you about a uniform gravitational field

any theory with gravity
has less observables
than a theory without it!



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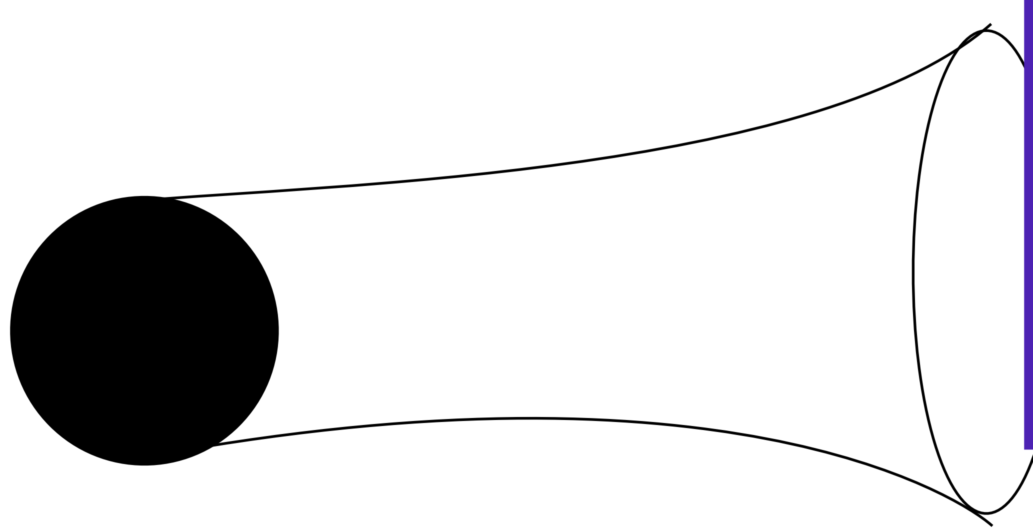

how can a
local CFT
emerge at the
boundary?

any theory with gravity
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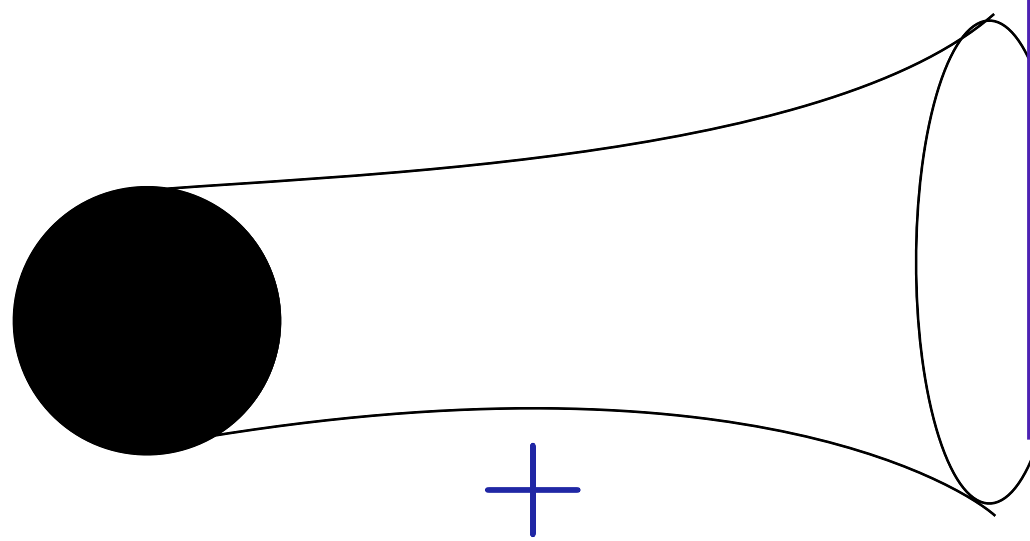

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N D3-branes

standard holography

$$S = S(g_{\mu\nu}, A_\mu, \phi, \cdots)$$

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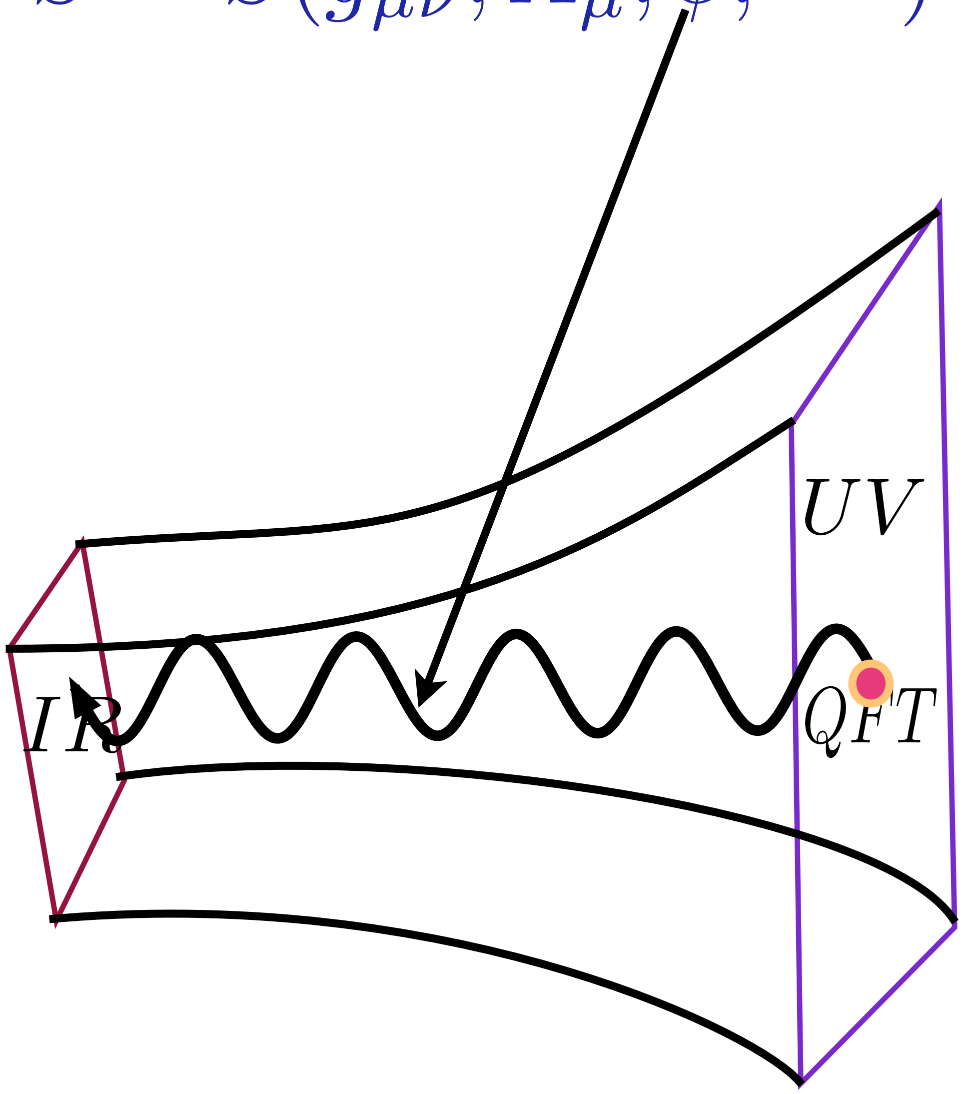

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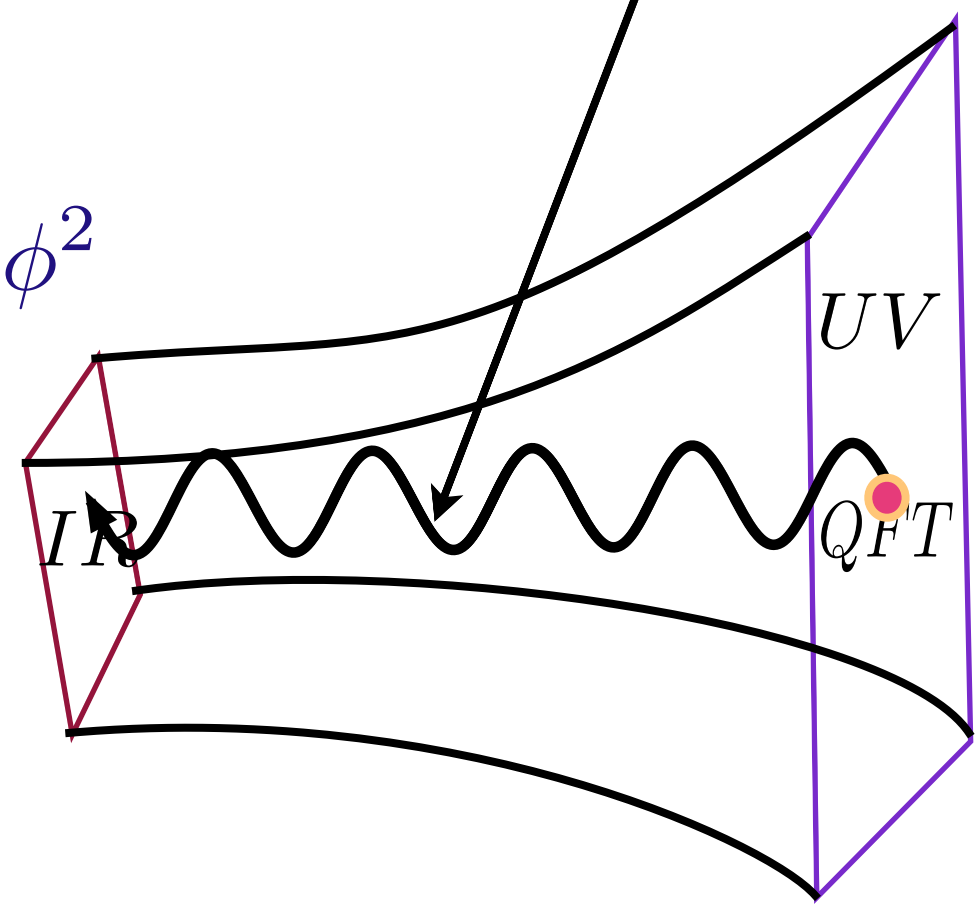
$$S = S(g_{\mu\nu}, A_\mu, \phi, \cdots)$$

$$(\partial_\mu \phi)^2 + m^2\phi^2$$



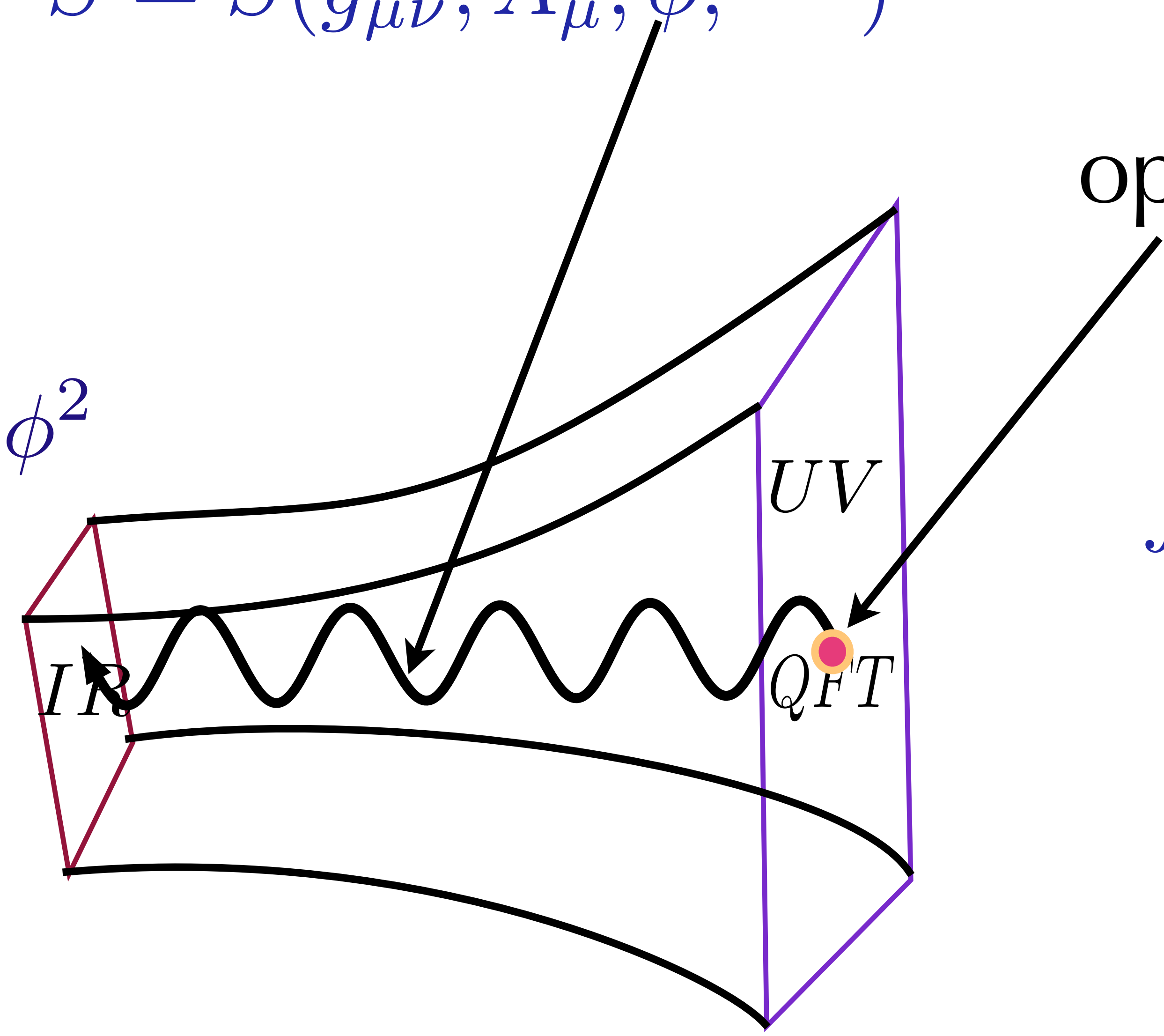
standard holography

$$S = S(g_{\mu\nu}, A_\mu, \phi, \cdots)$$

operators $\mathcal{O}$

$$(\partial_\mu \phi)^2 + m^2\phi^2$$

$$\int d^4x \phi_0 \mathcal{O}$$



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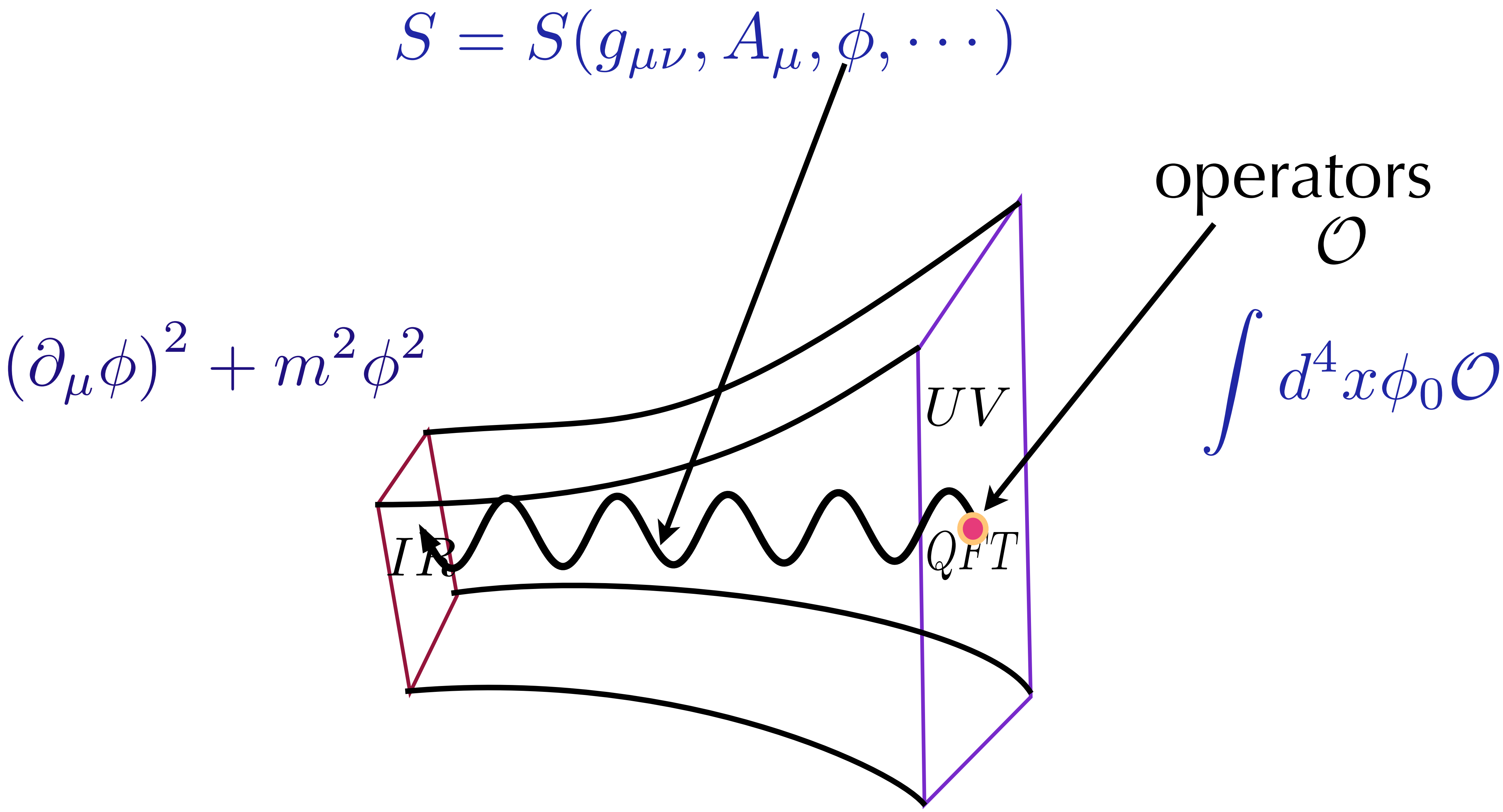

AdS=CFT claim:

$$\langle e^{\int_{S^d} \phi_0 \mathcal{O}} \rangle_{\text{CFT}} = Z_S(\phi_0)$$

$$Z_S(\phi_0)$$

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super-gravity partition function
averaged over all double-pole
metrics
that impose boundary
conformality

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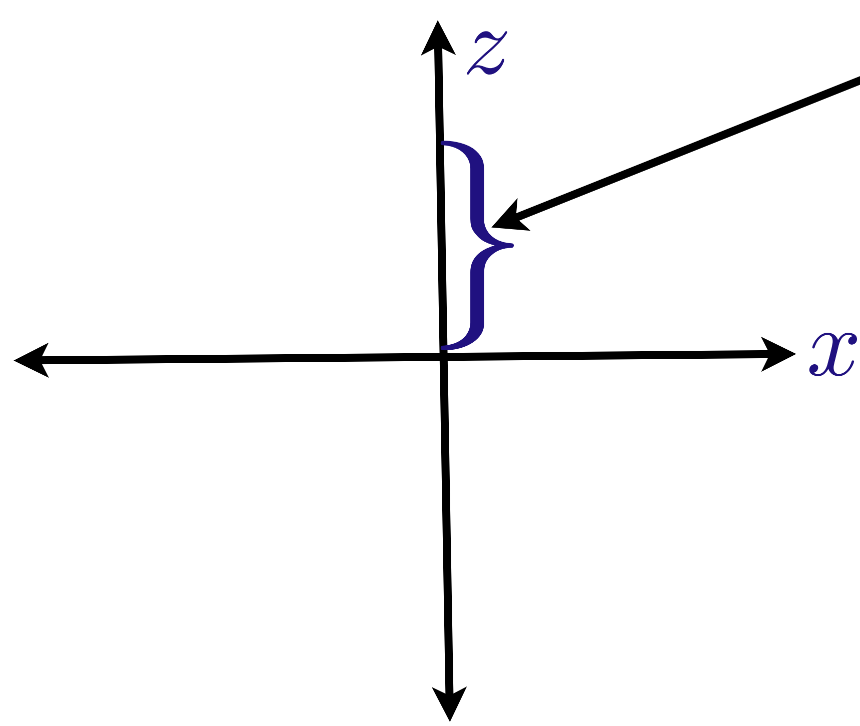
Why should the boundary
be conformal?

AdS metric: Euclidean signature

$$ds^2 = \frac{dz^2 + \sum_i dx_i^2}{z^2}$$

what is the length of this segment?



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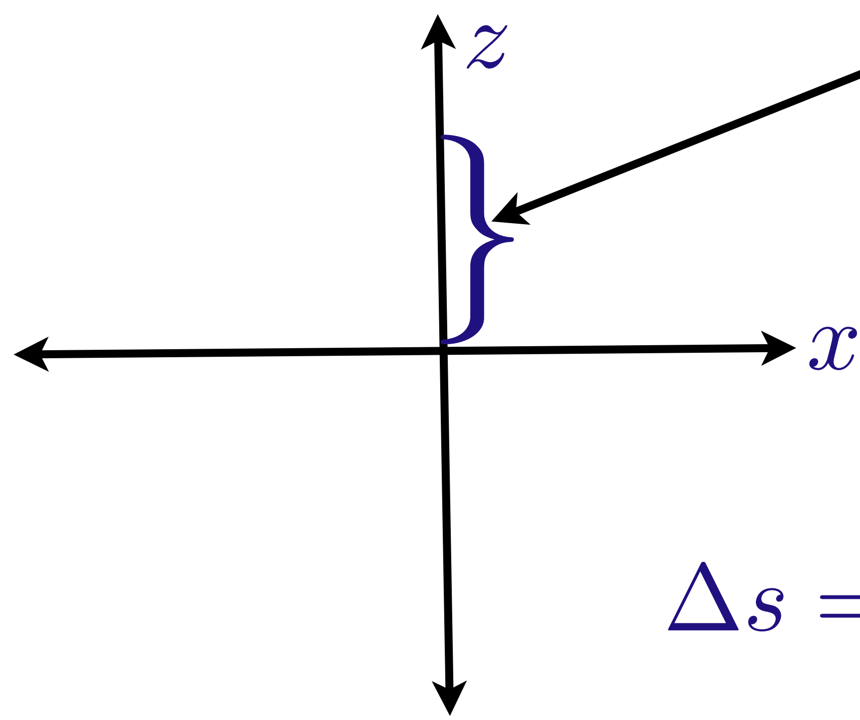

$$\Delta s = \int_0^{z_0} \frac{dz}{z} = \ln(z_0/0) = \infty$$

metric at boundary is not well defined

$$z^2 ds^2 = dz^2 + \sum_i dx_i^2$$

solves problem

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works for any real w

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boundary can only be specified conformally

requires boundary conformality

$$\langle e^{\int_{S^d} \phi_0 \mathcal{O}} \rangle_{\rm CFT} = Z_S(\phi_0)$$

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composite operator in interacting theory

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what is $\mathcal{O}$?

composite operator in interacting theory

$$\mathcal{O} = C_{\mathcal{O}} \lim_{z \to 0} z^{-\Delta} \phi(x, z)$$

Polcinski: 1010.6134

can $\mathcal{O}$ be determined exactly in some cases?

redo Witten's massive scalar field calculation explicitly

$$S_\phi = \frac{1}{2}\int \underbrace{d^{d+1}u\sqrt{g}}_{dV_g}\left(|\nabla\phi|^2 + m^2\phi^2\right)$$

to establish correspondence

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↓

$$(-\nabla)^\gamma\phi_0$$ Reisz fractional Laplacian

$$(-\Delta)^\gamma f(x) = C_{d,s} \int_{\mathbf{R^d}} \frac{f(x) - f(\xi)}{|x - \xi|^{d+2\gamma}} d\xi$$

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equations of motion

$$-\Delta\phi - s(d-s)\phi = 0 \qquad -\Delta\phi = \nabla_i\nabla^i\phi$$

$$m^2 = -s(d-s) \qquad s = \frac{d}{2} + \frac{1}{2}\sqrt{d^2+4m^2}$$

bound

$$m^2 \geq -d^2/4$$

BF bound

solutions

$$\phi = F z^{d-s} + G z^{s}, \quad F, G \in \mathcal{C}^{\infty}(\mathbb{H}),$$

$$F = \phi_0 + O(z^2), \quad G = g_0 + O(z^2)$$

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$$\phi_0 = \lim_{z \to 0} \phi$$

boundary of AdS_{d+1}

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$$\int_{z>\epsilon} dV_g \phi\partial_\mu\phi$$

restriction $\text{pf} \int_{z>\epsilon} \left(|\partial\phi|^2 - s(d-s)\phi^2\right) dV_g = -d \int_{z=0} \phi_0 \, g_0$

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finite part from integration by parts

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finite part from integration by parts

use Caffarelli-Silvestre extension theorem (2006)

$$g(x,0) = f(x)$$

$$\Delta_x g + \frac{a}{z}\partial_z g + \partial_z^2 g = 0$$

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$$\gamma = \frac{1-a}{2}$$

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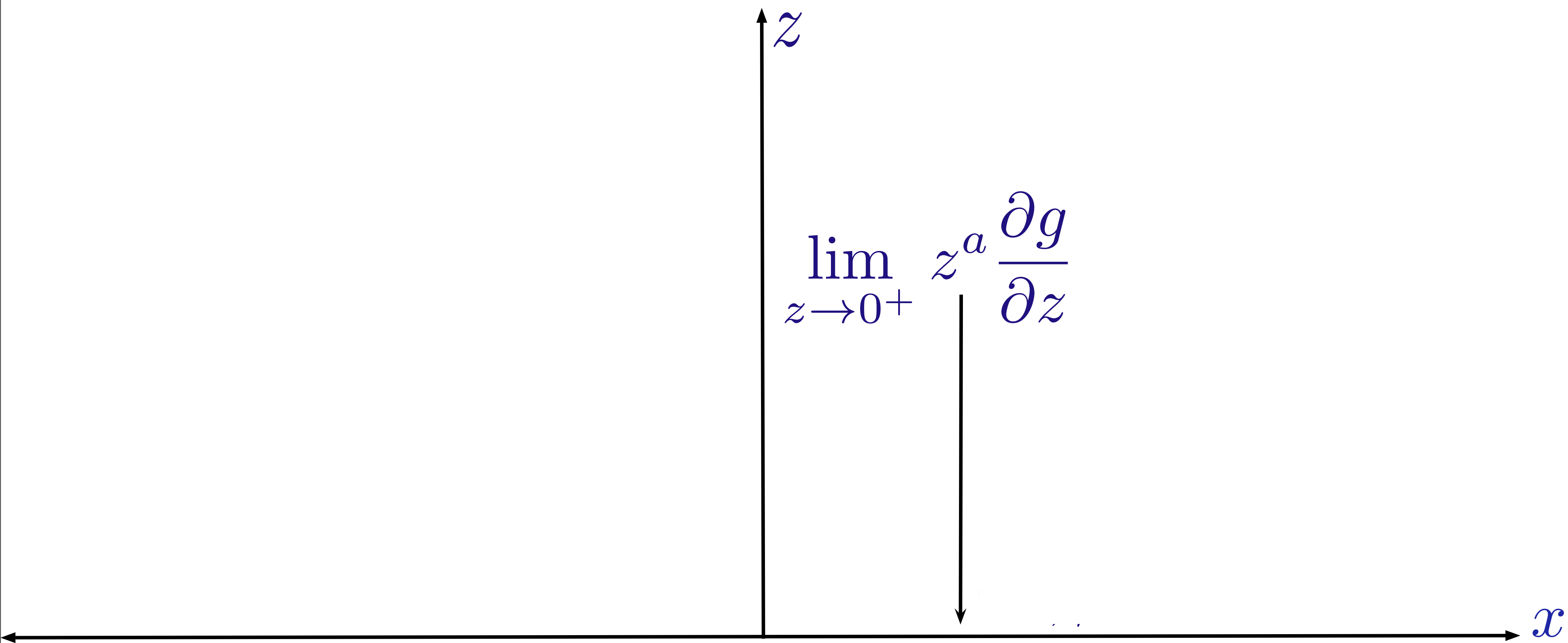
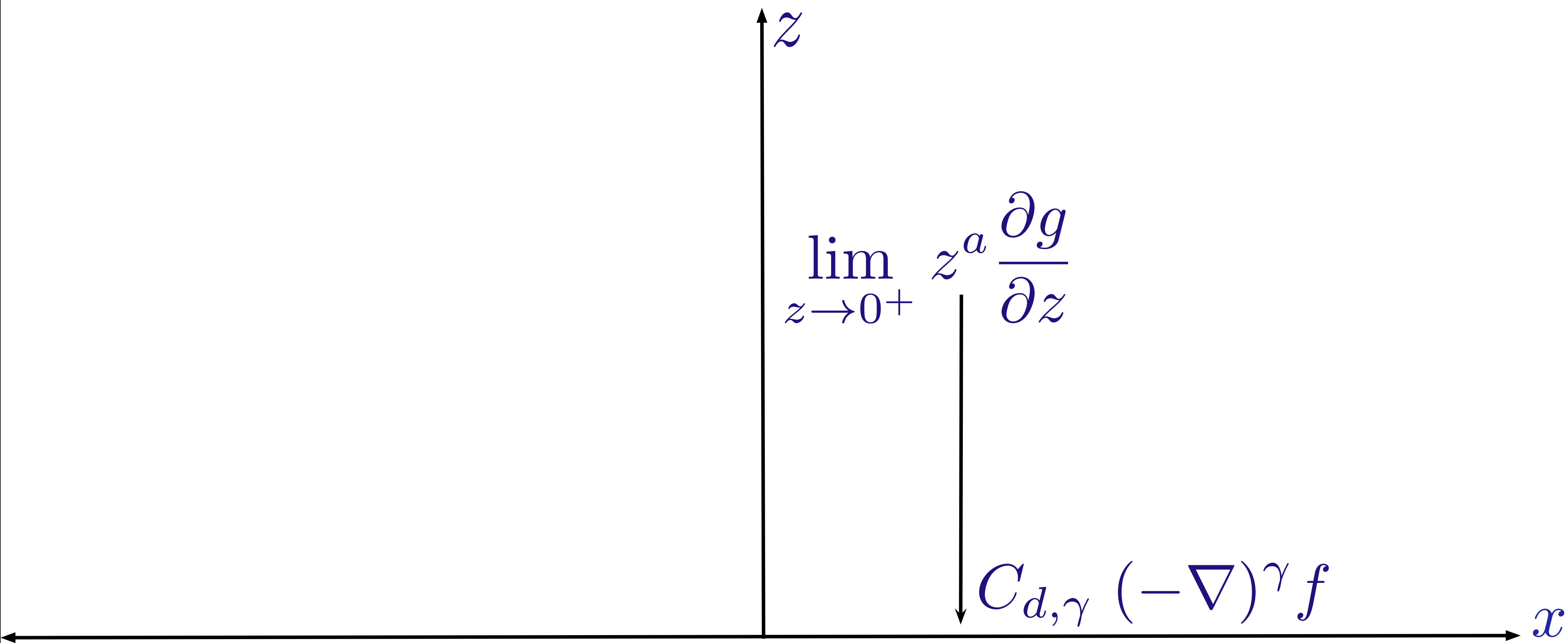
non-local


z
$\lim_{z \to 0^+} z^a \frac{\partial g}{\partial z}$
x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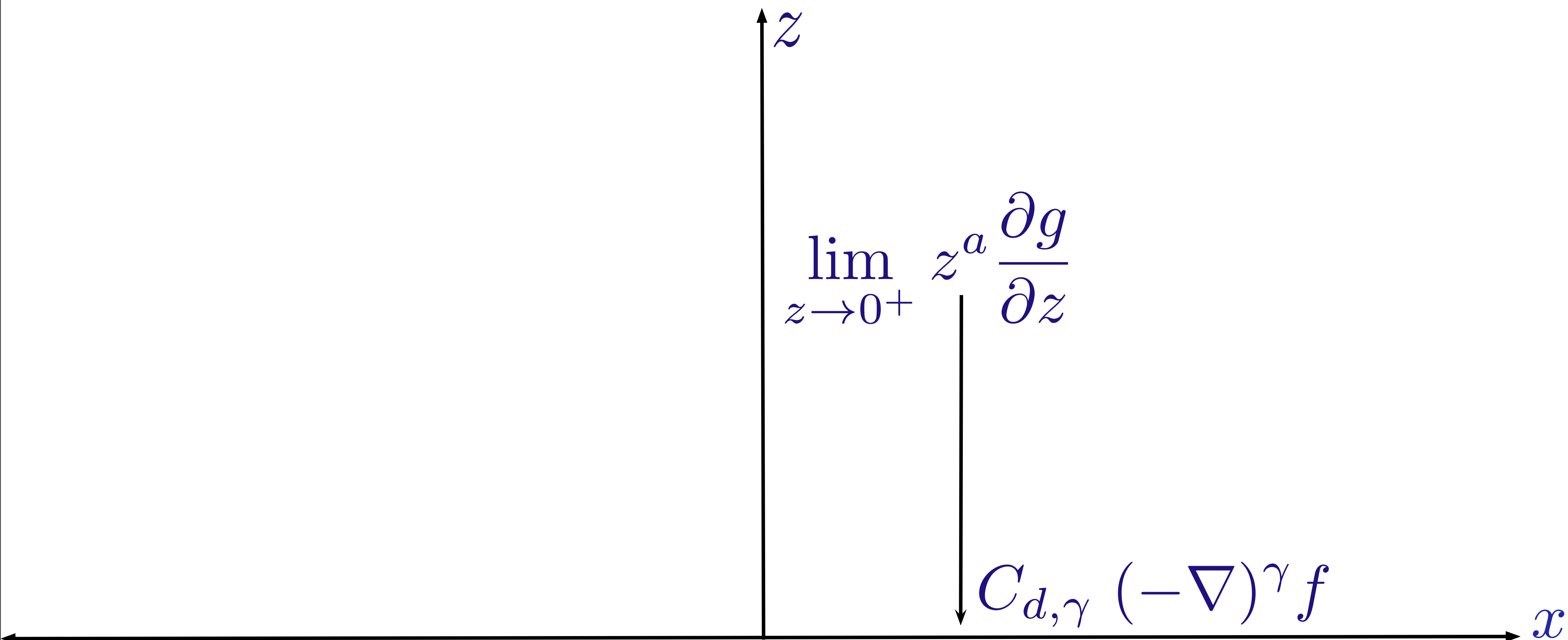

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ϕ

solves massive scalar problem

$$\phi$$

solves massive scalar problem

$$g = z^{\gamma - d/2}\phi$$

solves CS extension problem

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↓

$$\mathcal{O} = (-\Delta)^{\gamma}\phi_0$$

the $\mathcal{O}$ for massive scalar field consistency with Polcinski

$$\mathcal{O} = C_{\mathcal{O}} \lim_{z \to 0} z^{-\Delta} \phi(x, z)$$

use Caffarelli/ Silvestre

consistency with Polcinski

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$$\mathcal{O} = (-\Delta)^{\gamma}\phi_0 \quad \Longrightarrow \quad |x - x'|^{-d-2\gamma}$$

2-point correlator

$$\mathcal{O} = (-\Delta)^{\gamma}\phi_0 \longrightarrow |x - x'|^{-d-2\gamma}$$

2-point correlator

$$\langle e^{\int_{S^d} \phi_0 \mathcal{O}} \rangle_{\text{CFT}} = Z_S(\phi_0)$$

AdS-CFT correspondence but operators are non-local !!

simpler proof:

Reisz fractional Laplacian

$$(-\Delta)^{\gamma} f(x) = C_{d,s} \int_{\mathbf{R^d}} \frac{f(x) - f(\xi)}{|x-\xi|^{d+2\gamma}} d\xi$$

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pseudo-differential operator

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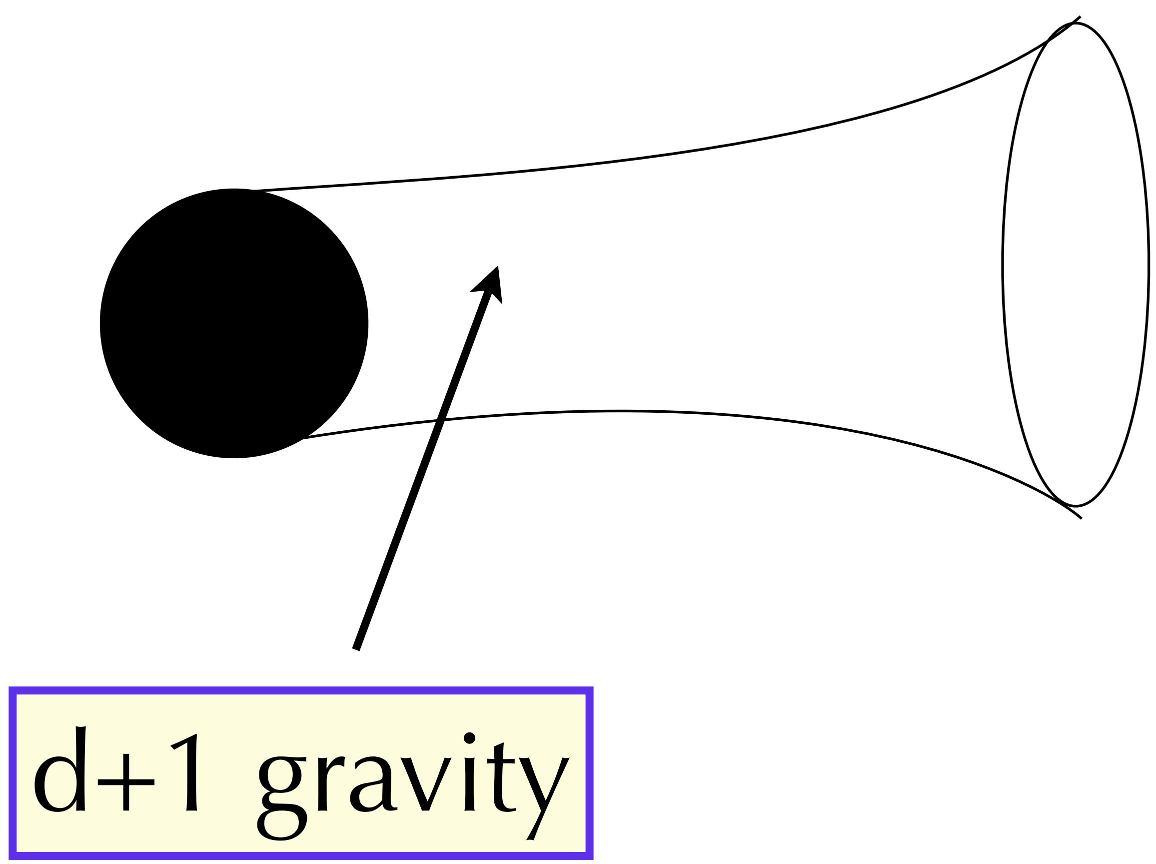
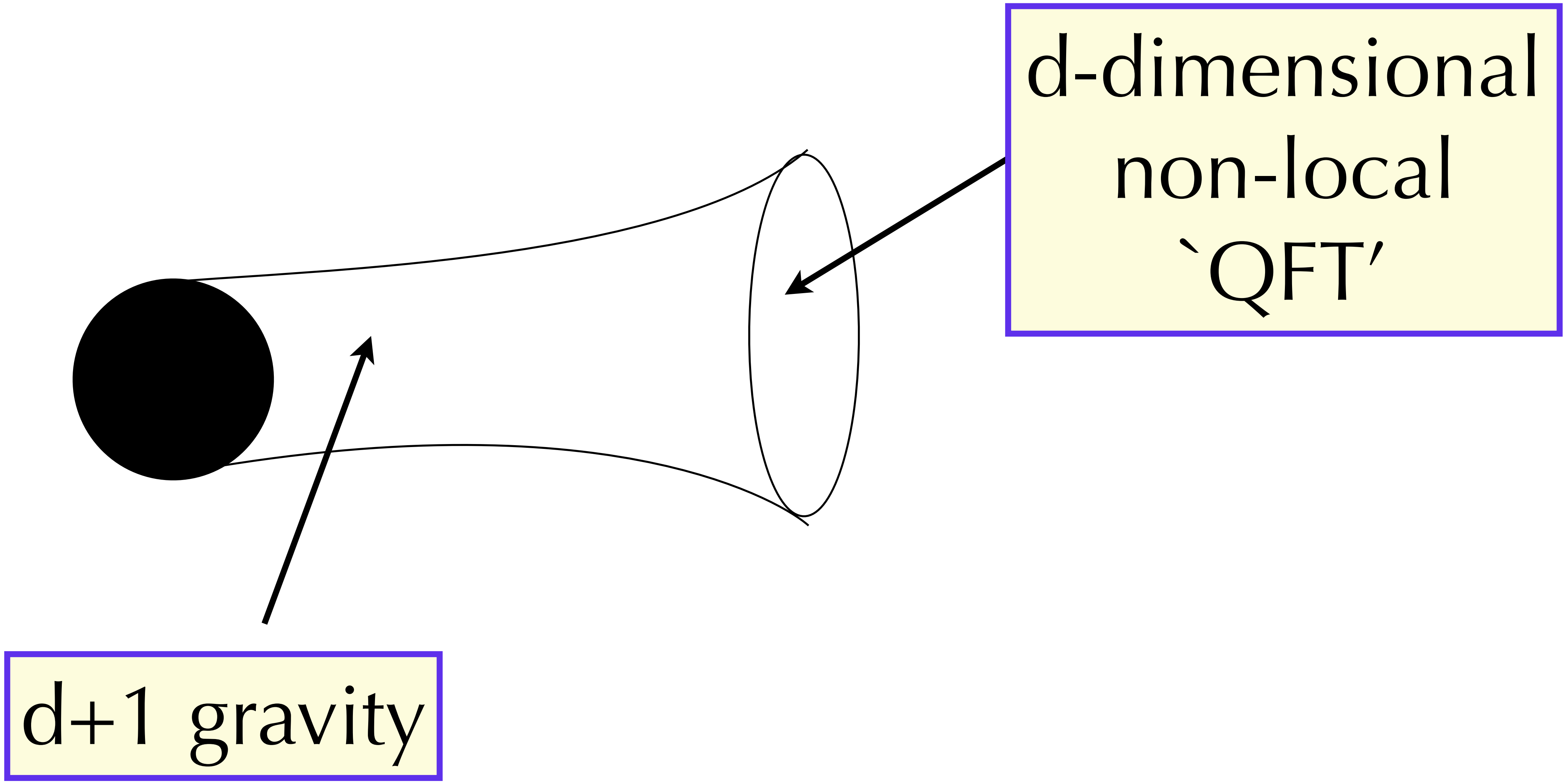
undo convolution

$$I(\phi) \propto \int d\mathbf{x}d\mathbf{x}' \frac{\phi_{\mathbf{0}}(\mathbf{x})\phi_{\mathbf{0}}(\mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|^{\mathbf{2}(\lambda+\mathbf{d})}}$$


d+1 gravity


d-dimensional
non-local
`QFT'
d+1 gravity

bulk conformality

$$S = S_{\rm gr}[g] + S_{\rm matter}(\phi)$$

$$S_{\rm matter} = \int_M d^{d+1}x \sqrt{g}\mathcal{L}_m$$

conformal sector

bulk conformality

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conformal sector

$$\mathcal{L}_m := |\partial\phi|^2 + \left(m^2 + \frac{d-1}{4d}R(g)\right)\phi^2$$

scalar curvature

bulk conformality

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$$S_{\rm matter} = \int_M d^{d+1}x\sqrt{g}\mathcal{L}_m$$

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\`conformal mass'

scalar curvature

on Riemannian (M,g) manifold of dimension N=d+1

conformal Laplacian

$$L_g = -\Delta_g + \frac{N-2}{4(N-1)} R_g = -\Delta_g + \frac{d-1}{4d} R_g$$

on Riemannian (M,g) manifold of dimension N=d+1

conformal Laplacian

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conformal change

$$A_w(\varphi) = e^{-bw} A(e^{aw}\varphi)$$

$$\hat{g} = y^2\, g$$

on Riemannian (M,g) manifold of dimension N=d+1

conformal Laplacian

$$L_g = -\Delta_g + \frac{N-2}{4(N-1)} R_g = -\Delta_g + \frac{d-1}{4d} R_g$$

conformal change

$$A_w(\varphi) = e^{-bw} A(e^{aw} \varphi)$$

$$\hat{g} = y^2 g$$

$$L_g(\varphi) = y^{\frac{d+3}{2}} L_{\hat{g}} \left(y^{-\frac{d-1}{2}} \varphi \right)$$

hyperbolic metric

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$$m^2 - \frac{d^2-1}{4} = -s(d-s)$$

$$s = \frac{d}{2} + \frac{\sqrt{4m^2+1}}{2} \longrightarrow m^2 > -1/4$$

stability independent of dimensionality

construct $\mathcal{O}$

eom

$$-\Delta_g \phi + \frac{d-1}{4d} R_g \phi = m^2 \phi$$

$$-\Delta \phi + \left(m^2 - \tfrac{d^2-1}{4} \right) \phi = 0$$

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solutions

$$\gamma = \sqrt{4m^2+1}$$

$$\phi = F y^{\frac{d}{2}-\gamma} + G y^{\frac{d}{2}+\gamma}, \quad F, G \in \mathcal{C}^\infty(\mathbb{H}), \quad F = \phi_0 + O(y^2), \quad G = g_0 + O(y^2)$$

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redefinition

$$g = y^{\gamma - \frac{d}{2}}\,\phi, \;\longrightarrow\; \lim_{y\to 0} y^{1-2\gamma} \frac{\partial g}{\partial y} = 2\gamma g_0$$

CS extension problem

scattering matrix

$$S(s)F = G|_M$$

source

response

conformal Laplacian

$$P_\gamma[d\tau^2, h] := d_\gamma\, S\left(\frac{d+1}{2} + \gamma\right), \quad d_\gamma = 2^{2\gamma}\frac{\Gamma(\gamma)}{\Gamma(-\gamma)}$$

bulk

boundary

Chang/Gonzalez
1003.0398

$$P_\gamma \in (-\Delta_{\hat{g}})^\gamma + \Psi_{\gamma-1}$$

pseudo-differential operator

Chang/Gonzalez
1003.0398

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in general

$$P_k = (-\Delta)^k + \text{lower order terms}$$

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1003.0398

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in general

$$P_k = (-\Delta)^k + \text{lower order terms}$$

$$P_1 = -\Delta + \frac{d-1}{4(d-1)} R_g$$

scattering problem

$$P_\gamma f = d_\gamma S\left(\tfrac{d}{2} + \gamma\right) = d_\gamma \, h$$

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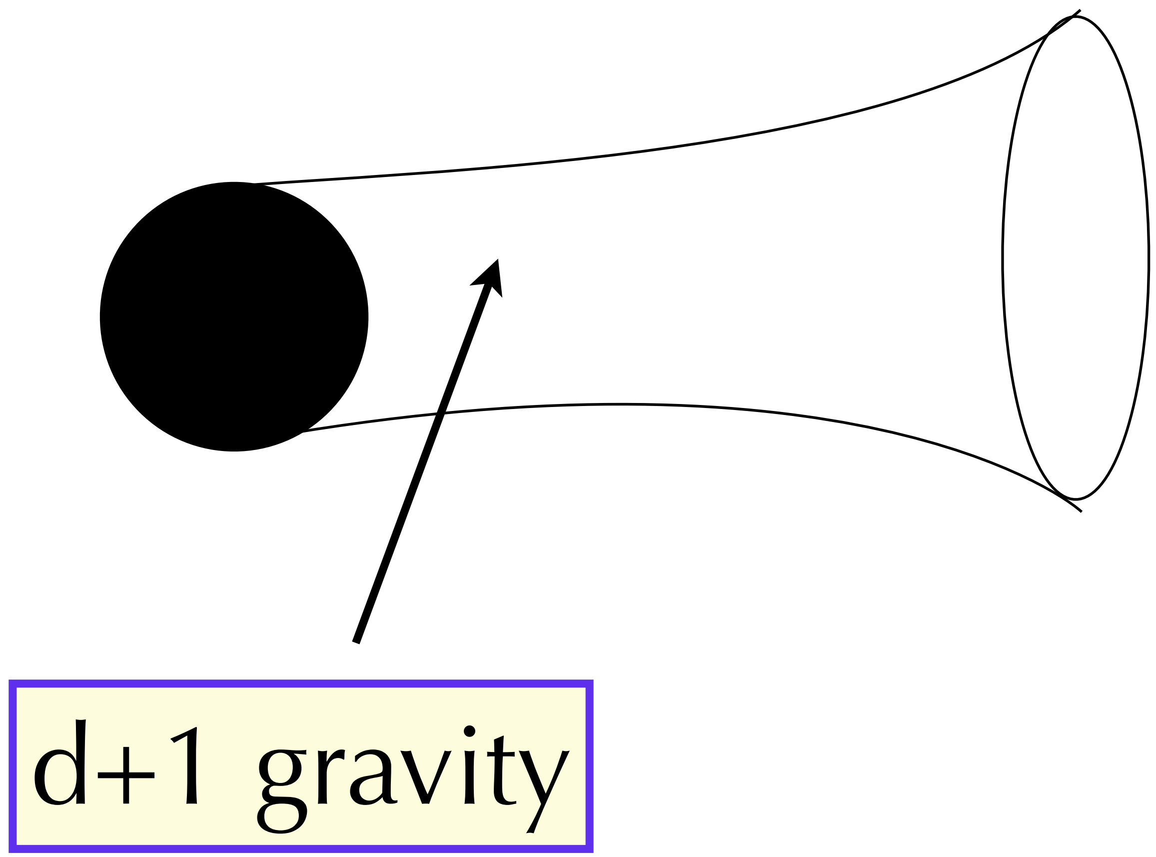
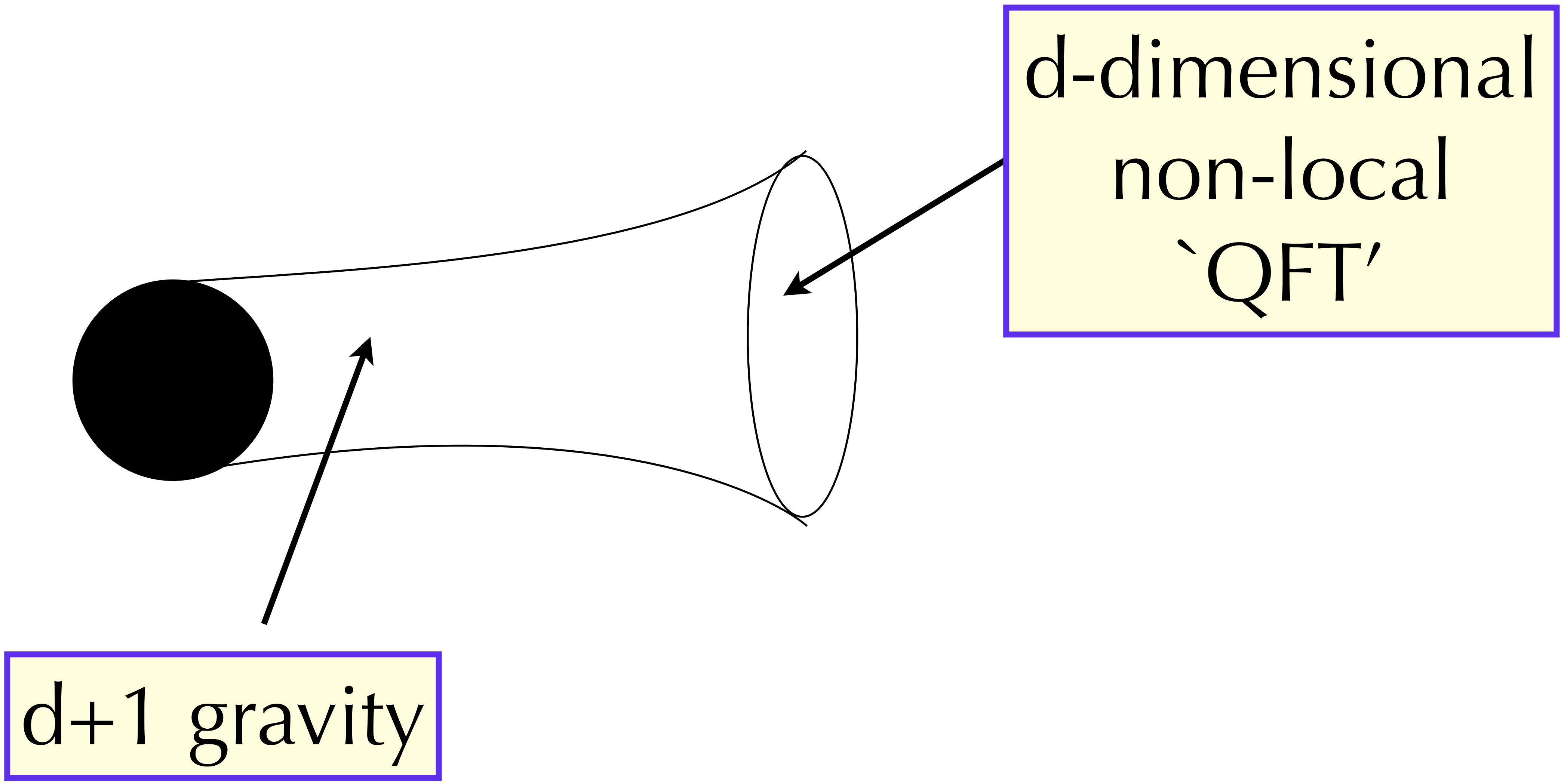
fractional conformal Laplacian


d+1 gravity


d-dimensional
non-local
`QFT'
d+1 gravity

What about Maldacena conjecture?

What about Maldacena conjecture?

Type IIB String `action'

$$S = \int d^{10}x\sqrt{-g}\left(e^{-2\phi}(R + 4|\nabla\phi|^2) - \frac{2e^{2\alpha\phi}}{(D-2)}F^2\right)$$

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$D = 7$ $\Big\downarrow$ **extremal solution**

$$ds_L^2 = H^{-1/2}(r)\,\eta_{\mu\nu}dx^\mu dx^\nu + H^{1/2}(r)\,\delta_{mn}dx^m dx^n$$

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D3-branes

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horizon at r=0

D3-branes

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rescale AdS metric

$$ds^2 \rightarrow ds_L^2$$

rescale AdS metric

$$ds^2 \to ds_L^2$$

$$\Box_{ds^2}^{conf}(\phi) + m^2\phi = L^2 \left(\Box_{ds_L^2}^{conf} + \frac{m^2}{L^2} \right) \phi$$

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$$\left(\Box_{ds_L^2}^{conf} + \frac{m^2}{L^2}\right)\phi = 0$$

what determines the exponent?

$$(-\Delta)^{\gamma} \longrightarrow \gamma = \frac{\sqrt{4\frac{m^2}{L^2} + 1}}{2}$$

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$$(-\Delta)^{\gamma} \rightarrow \gamma = \frac{\sqrt{4\frac{m^2}{L^2}+1}}{2}$$

$$\lim_{L\to+\infty (N\to\infty)} \gamma = \frac{1}{2}$$

$\Downarrow$

non-locality vanishes

more generally

$$\mathbb{R}^{3,1} \times K_6$$

$$ds^2 = f^{-1/2}\eta_{\mu\nu}dx^\mu dx^\nu + f^{1/2}\delta_{mn}dx^m dx^n$$

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more generally

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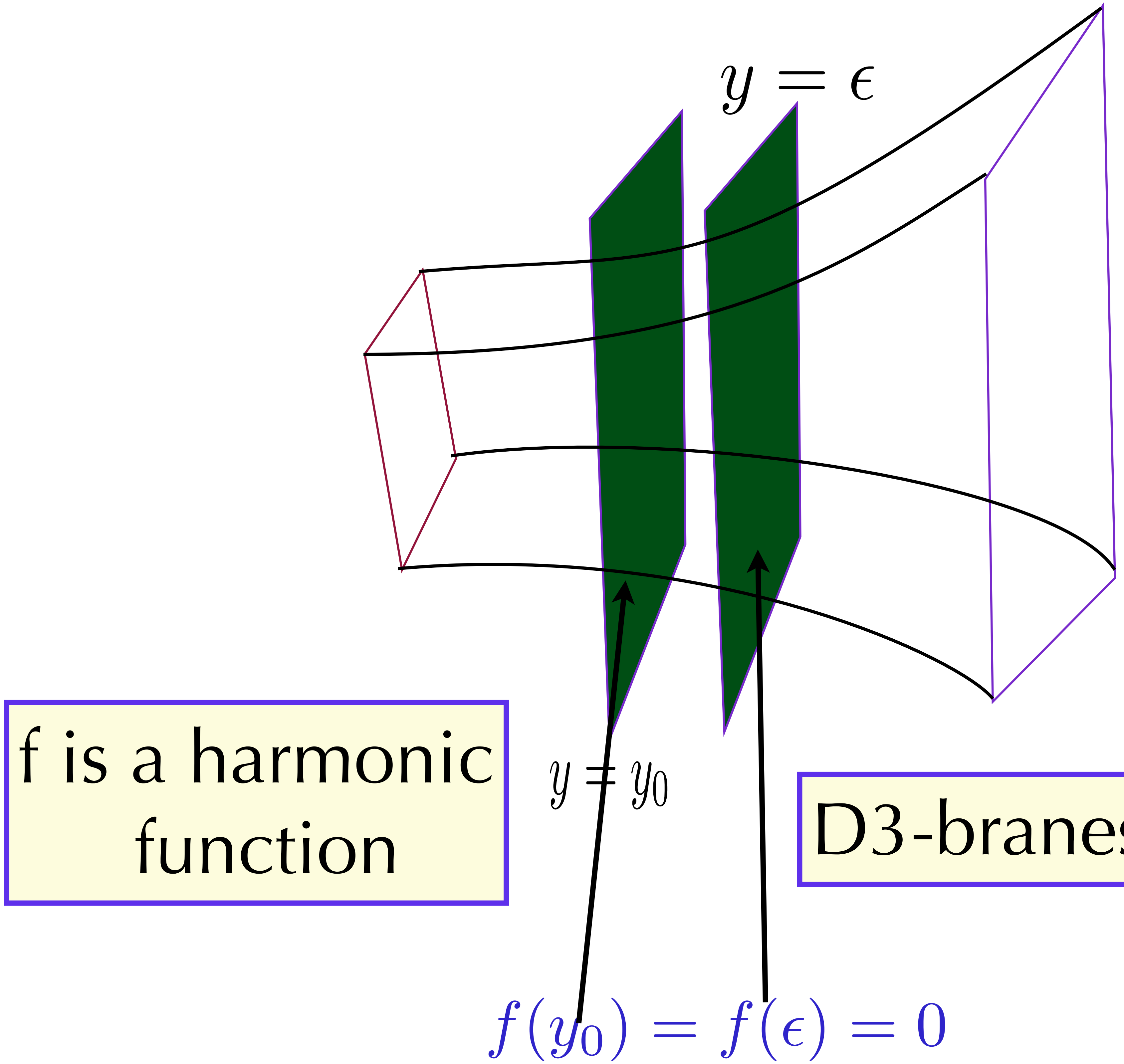
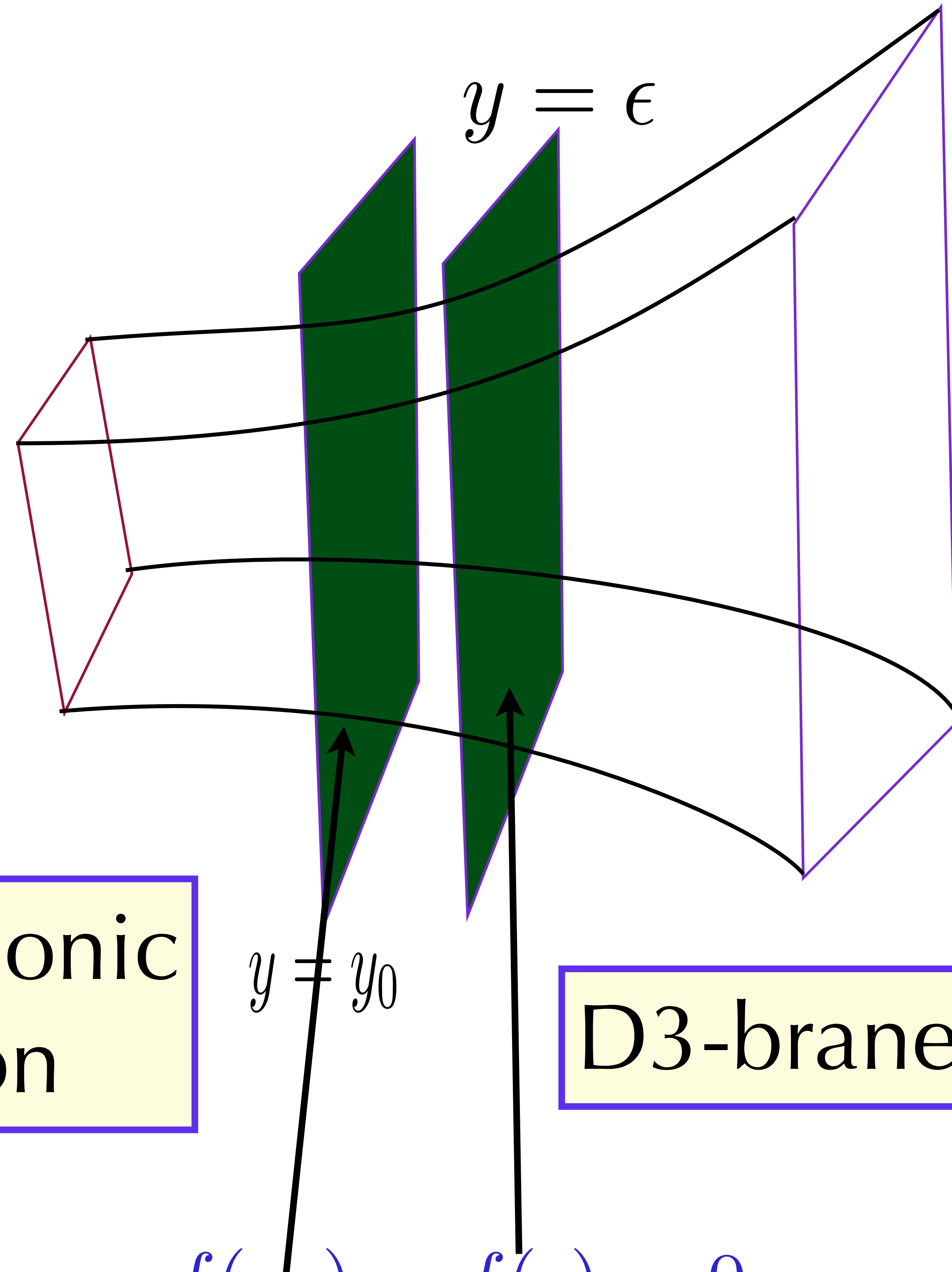
$N\delta(r)$

density of D3-branes



$y = \epsilon$
f is a harmonic function
$y = y_0$
D3-branes
$f(y_0) = f(\epsilon) = 0$



f is a harmonic function

D3-branes

$f(y_0) = f(\epsilon) = 0$

requires absolute-value singularity

|y| singular metrics (GI)

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Randall-Sundrum

$$ds^2 = -e^{-2|y|/L} g_{\mu\nu} dx^\mu dx^\nu + dy^2$$

$$y \in [-\pi R, \pi R]$$

|y| singular metrics (GI)

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massive-particle action at Brane at πR

$$\int d^4x \sqrt{-g} \left(g^{\mu\nu} \partial_\mu \hat{\phi} \partial_\nu \hat{\phi} + m^2 e^{-2\pi R/L} \hat{\phi}^2 \right),$$

$$\hat{\phi} = e^{-\pi R/L} \phi$$

$$\lim_{R/L \to \infty} m^2 e^{-2\pi R/L} \to 0$$

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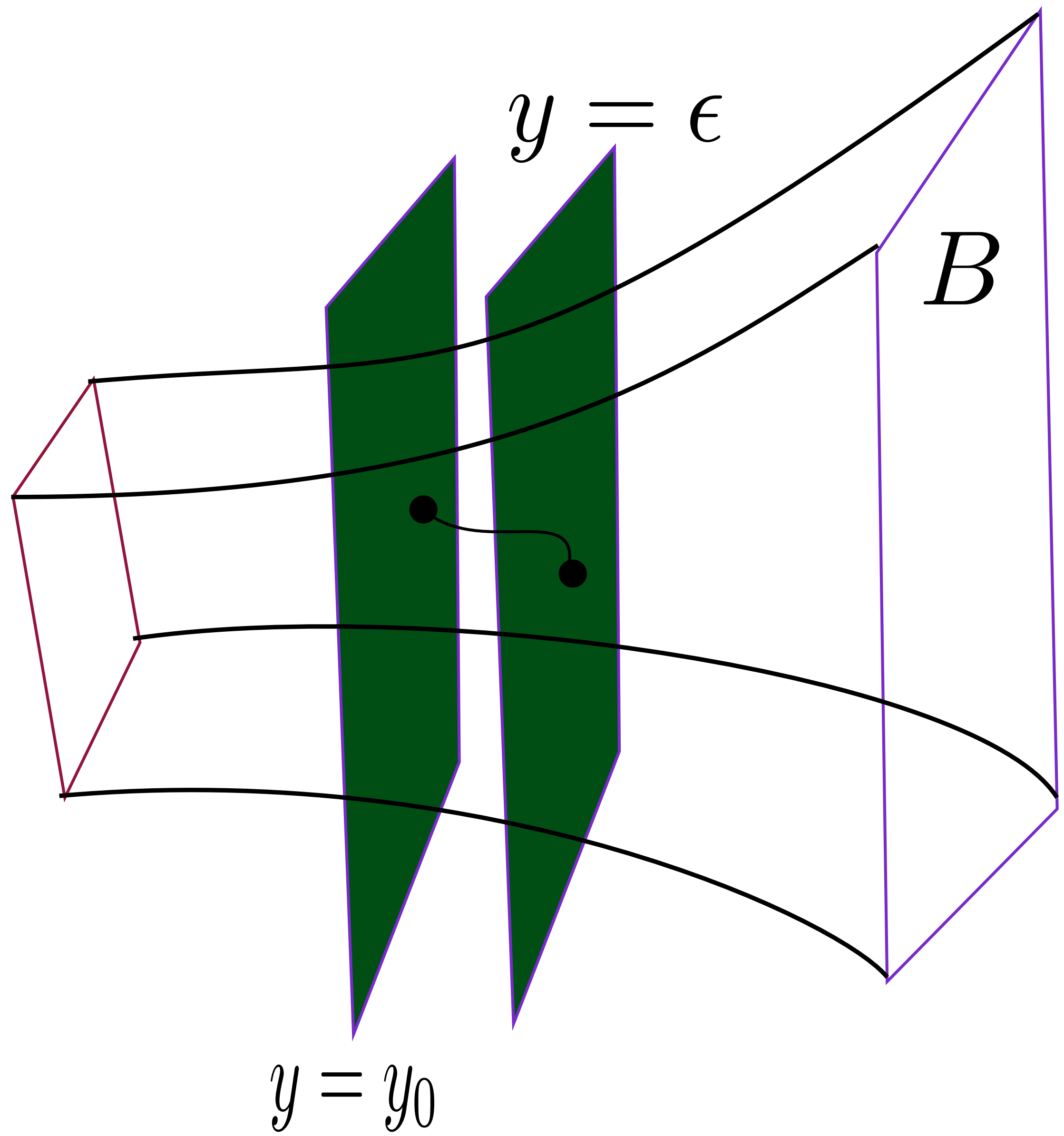
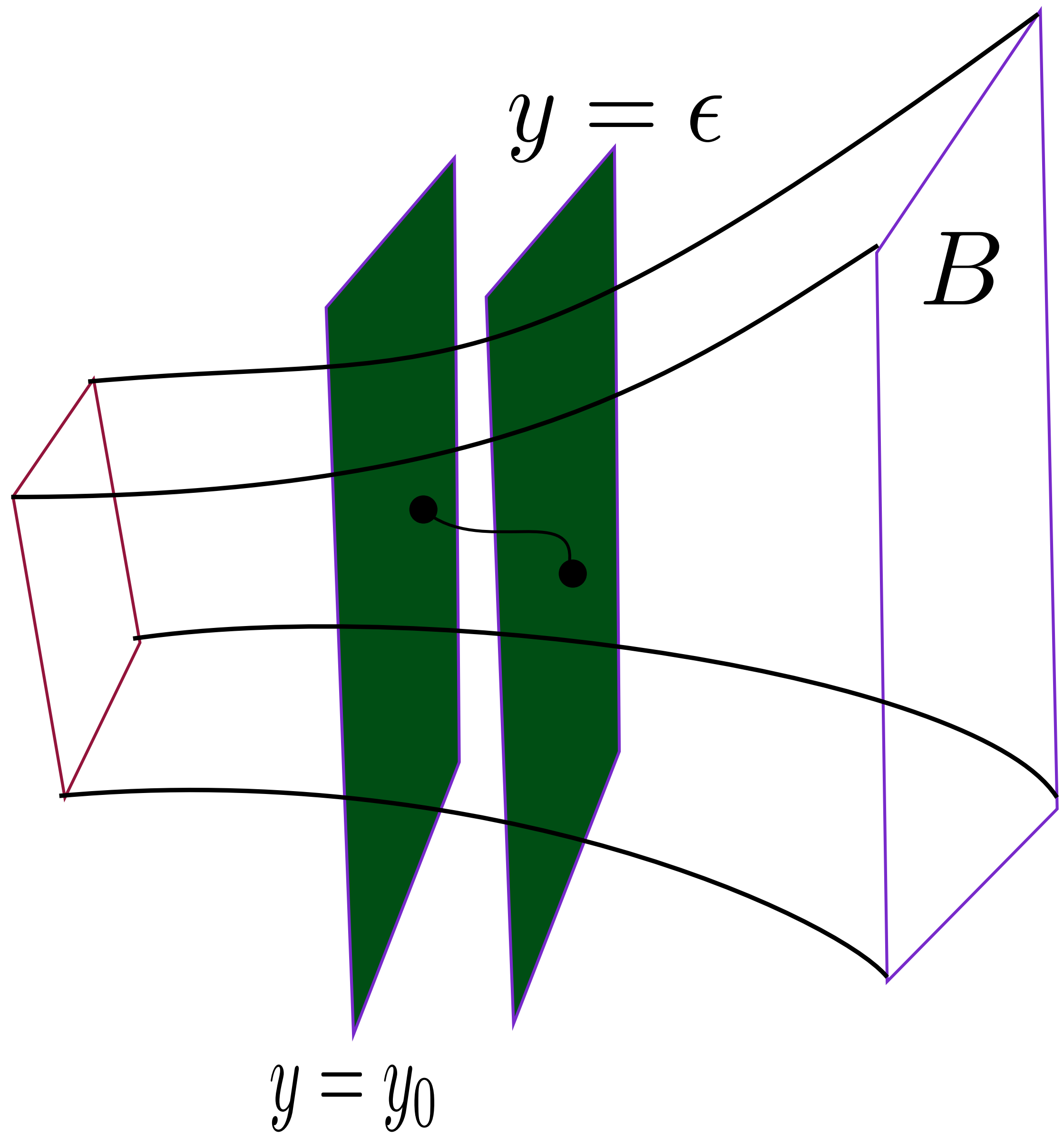
$$\Big\downarrow \quad \gamma = \frac{1}{2}$$

non-locality vanishes



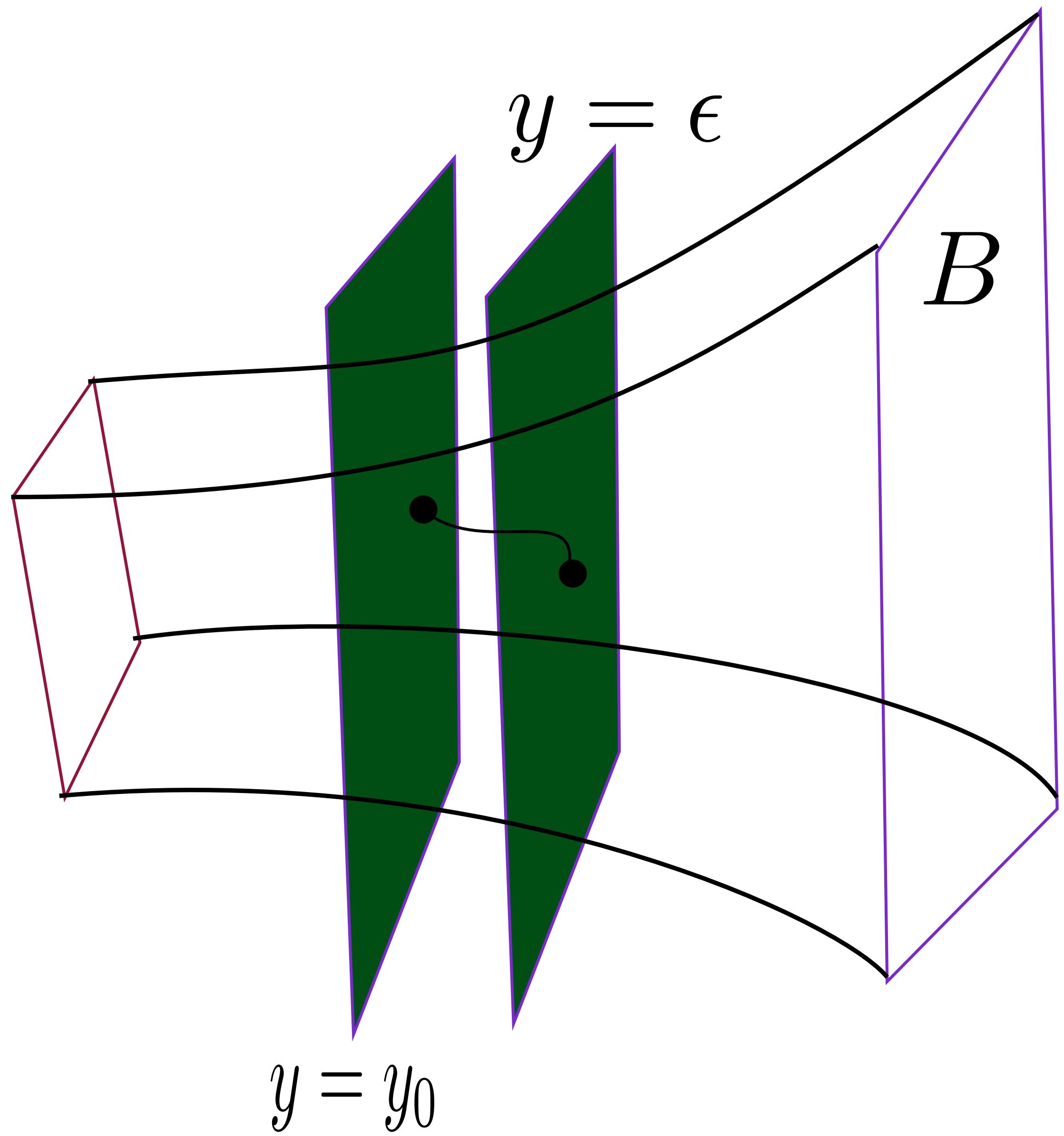
$$m^2 = -\frac{1}{\alpha'} + (\ln\epsilon)^2/(2\pi\alpha')^2$$



$$m^2 = -\frac{1}{\alpha'} + (\ln \epsilon)^2/(2\pi\alpha')^2$$

positive mass

$$|\ln \epsilon| > 2\pi\sqrt{\alpha'}$$



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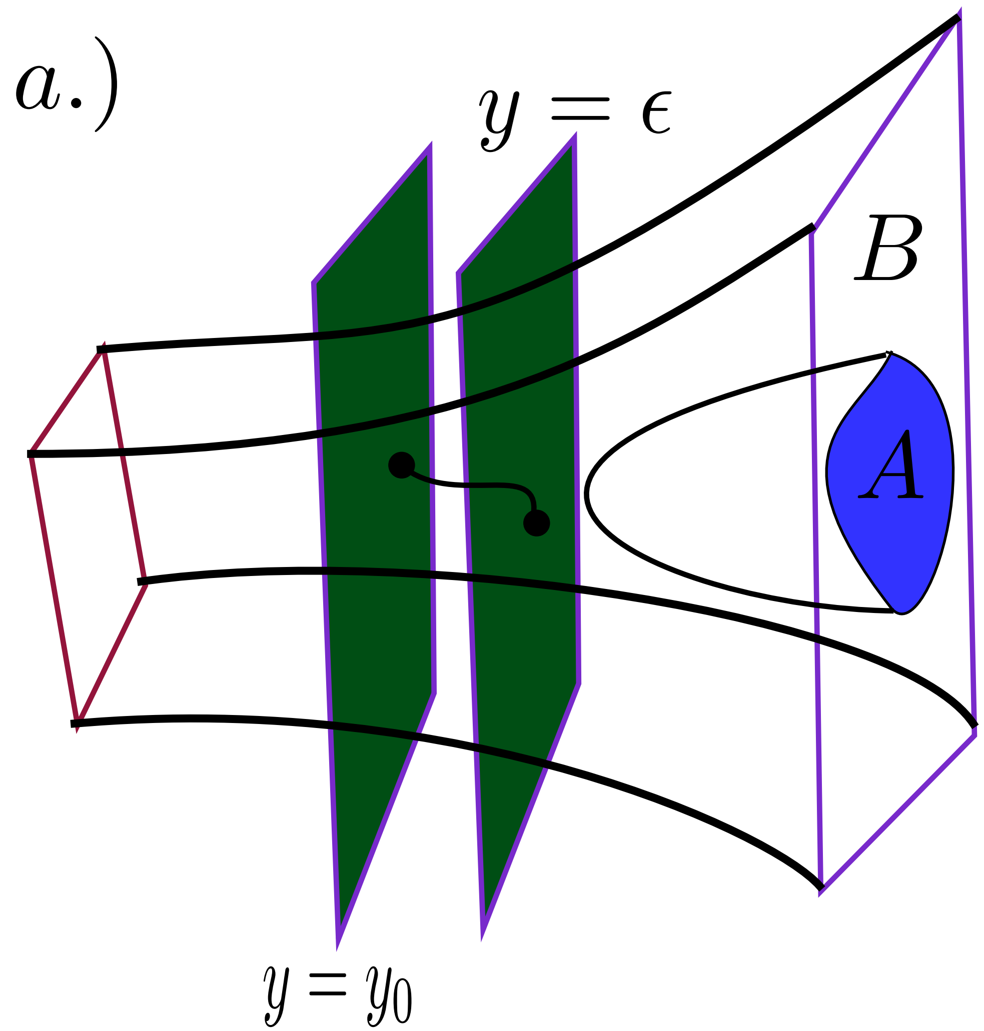
non-locality vanishes

Branes in Type IIB
string theory
eliminate non-local boundary
interactions

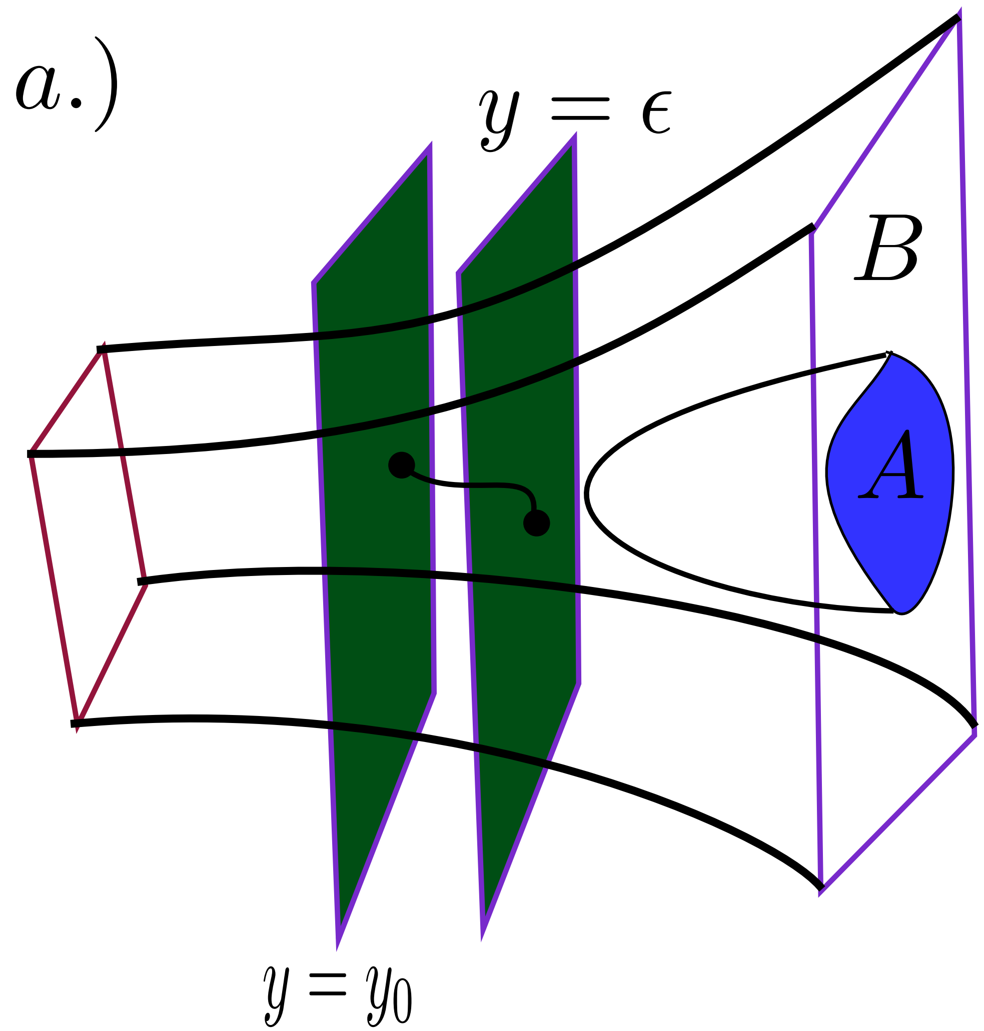
are there any consequences
for the enganglement entropy?

yes



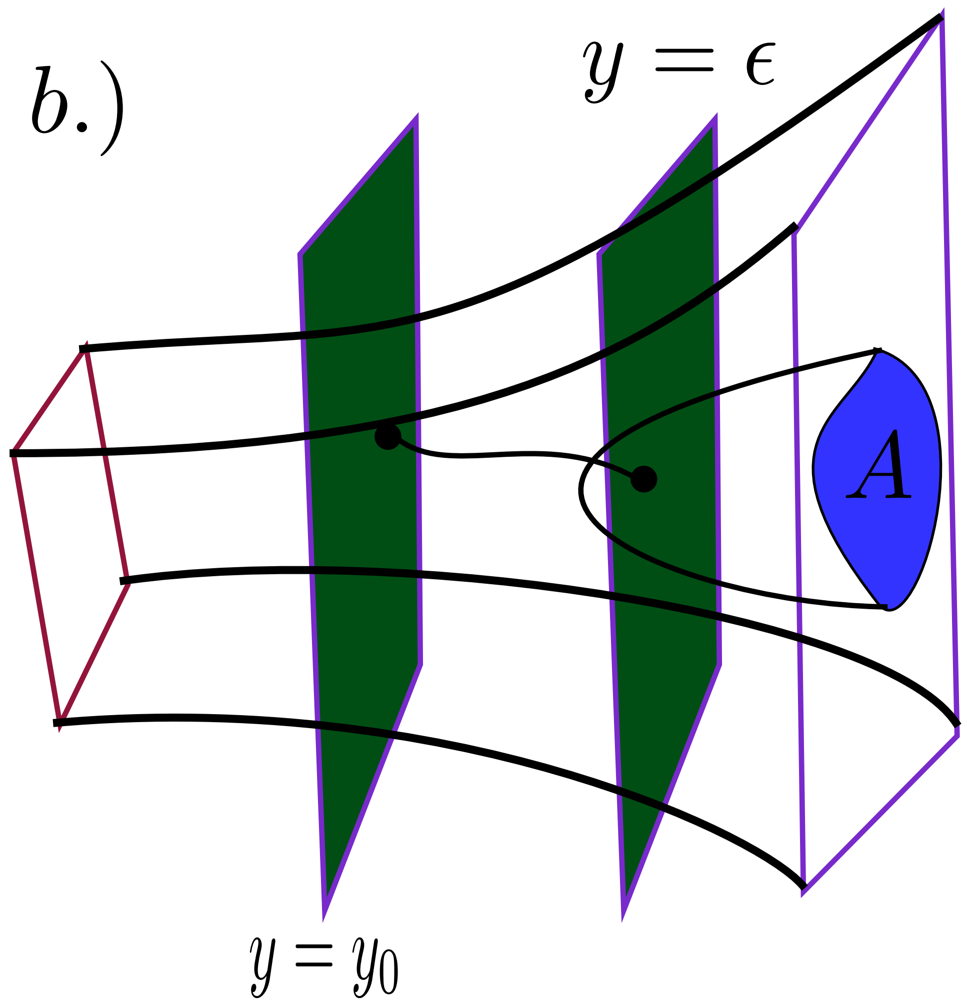
minimal surface avoids the D3-brane

ok



minimal surface avoids the D3-brane

what happens as brane approaches boundary?



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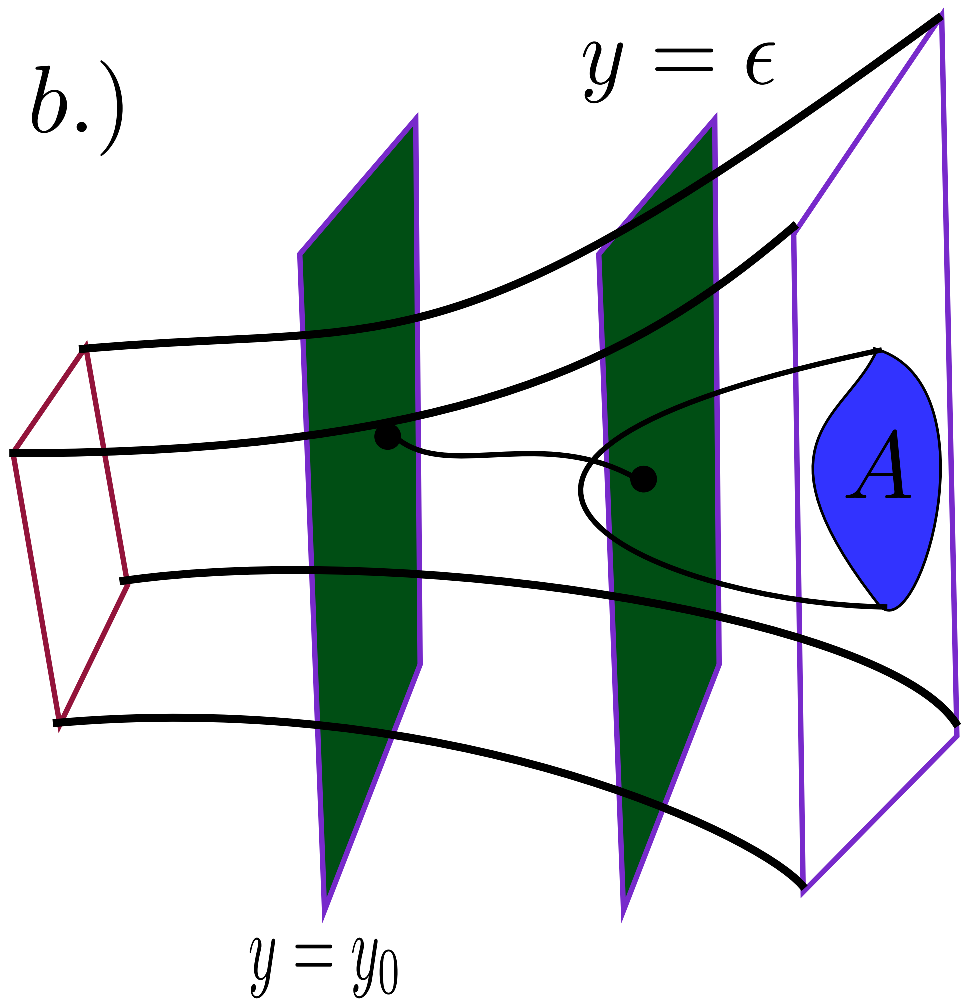

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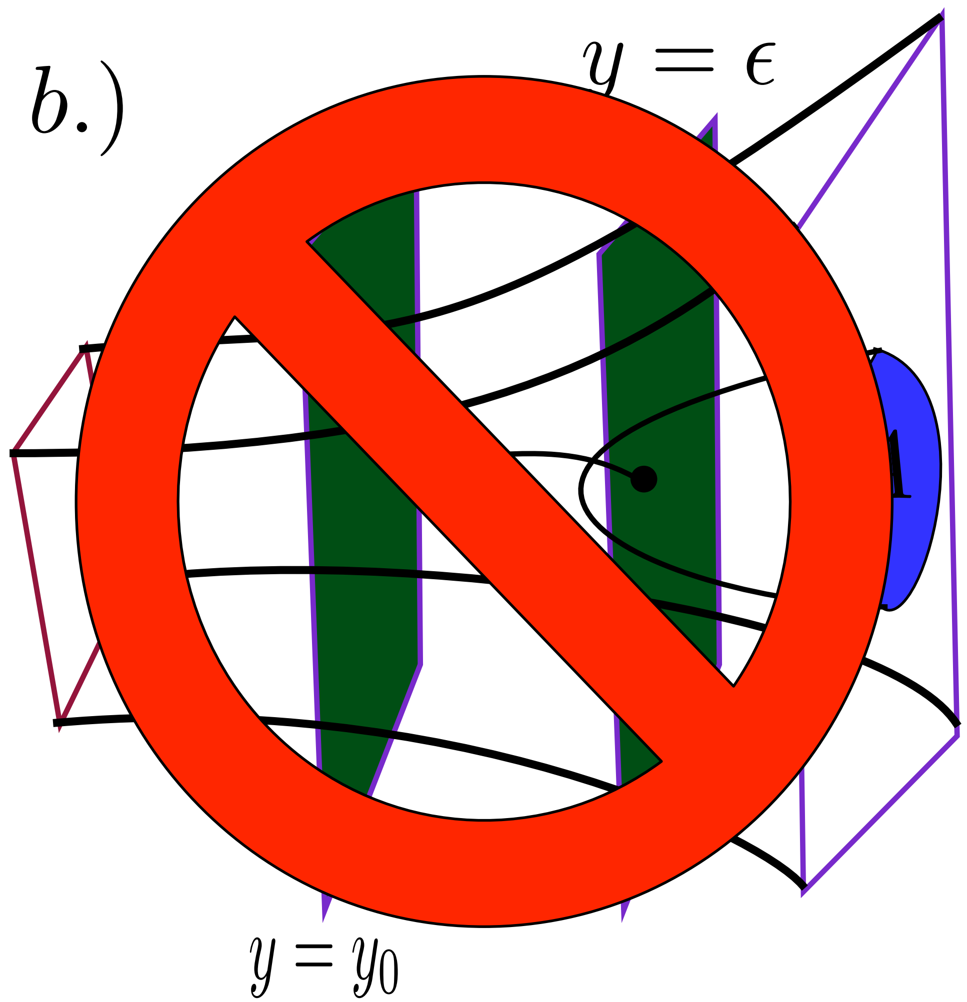
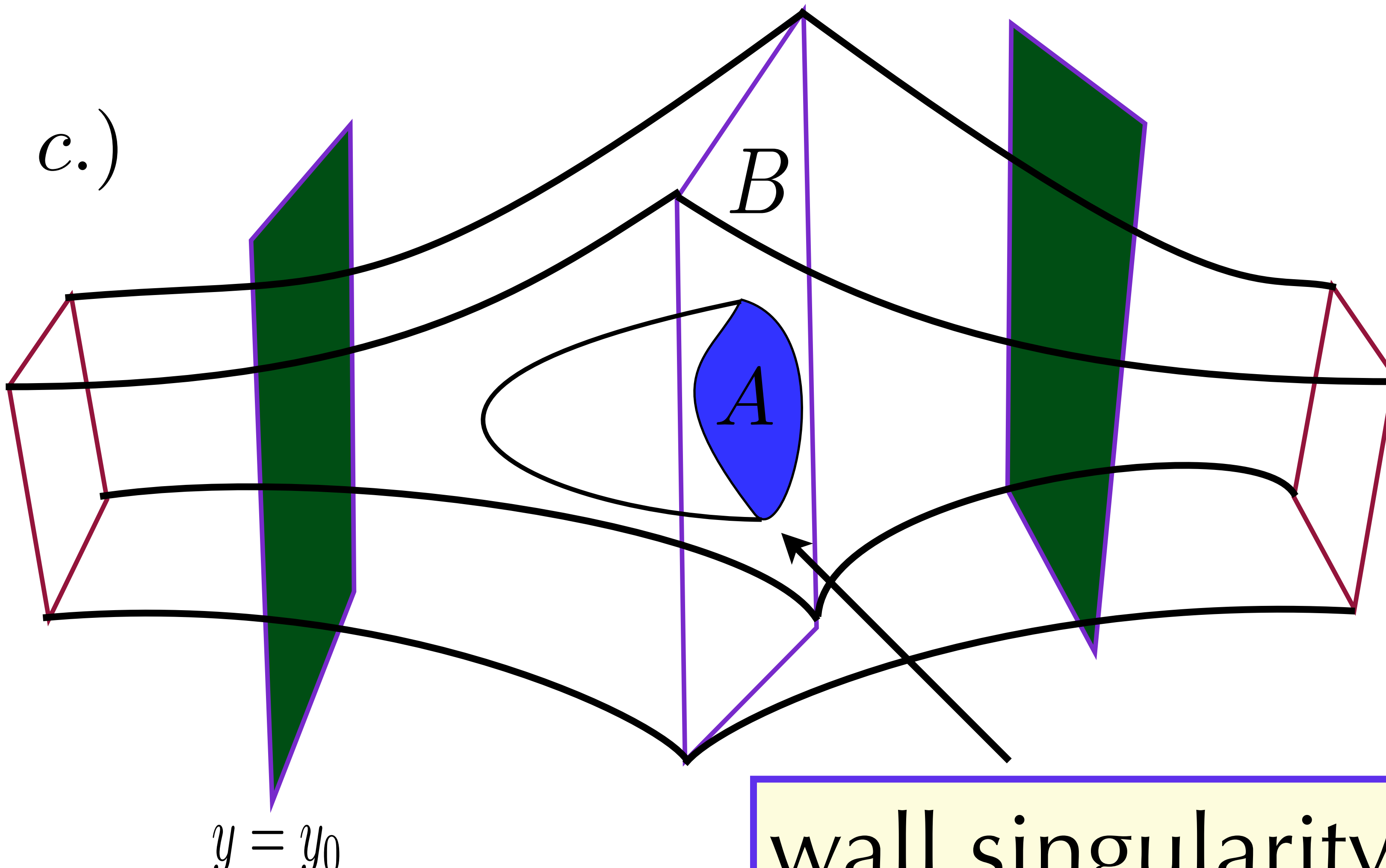


minimal surface must avoid brane

$\epsilon = 0$

metric doubles $S^1/\mathbb{Z}_2$



wall singularity

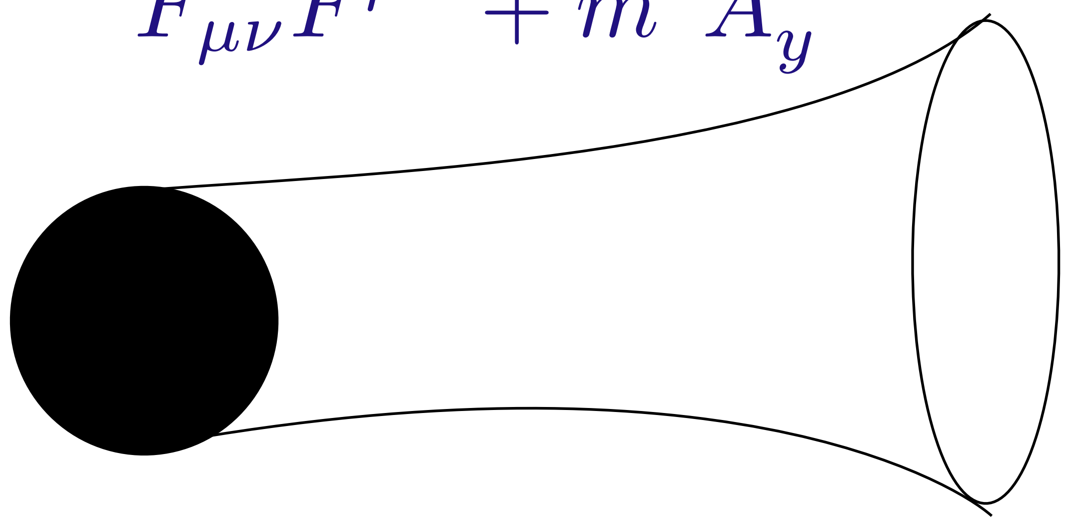
entropy vanishes $R/L = \infty$

higher-dimensional minimal surfaces can avoid singularities higher-dimensional minimal surfaces can avoid singularities

is this how the entanglement entropy should be formulated??

application: gauge fields with anomalous dimensions

$$F_{\mu\nu}F^{\mu\nu} + m^2 A_y^2$$



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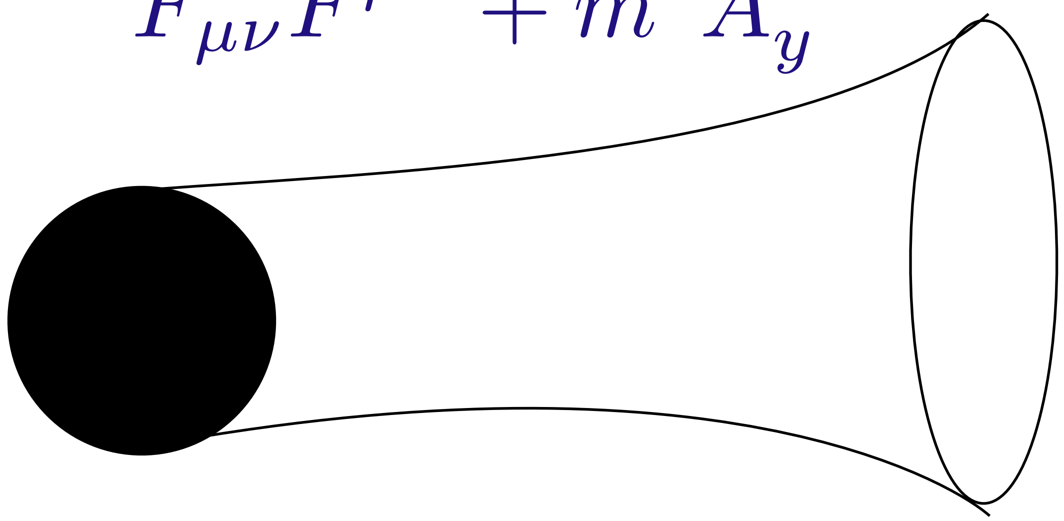

$$A_\mu^\perp \partial_\mu^\gamma A^{\perp\mu}$$

$$\gamma = \sqrt{d^2 + m^2 - 1}/2$$

application: gauge fields with anomalous dimensions

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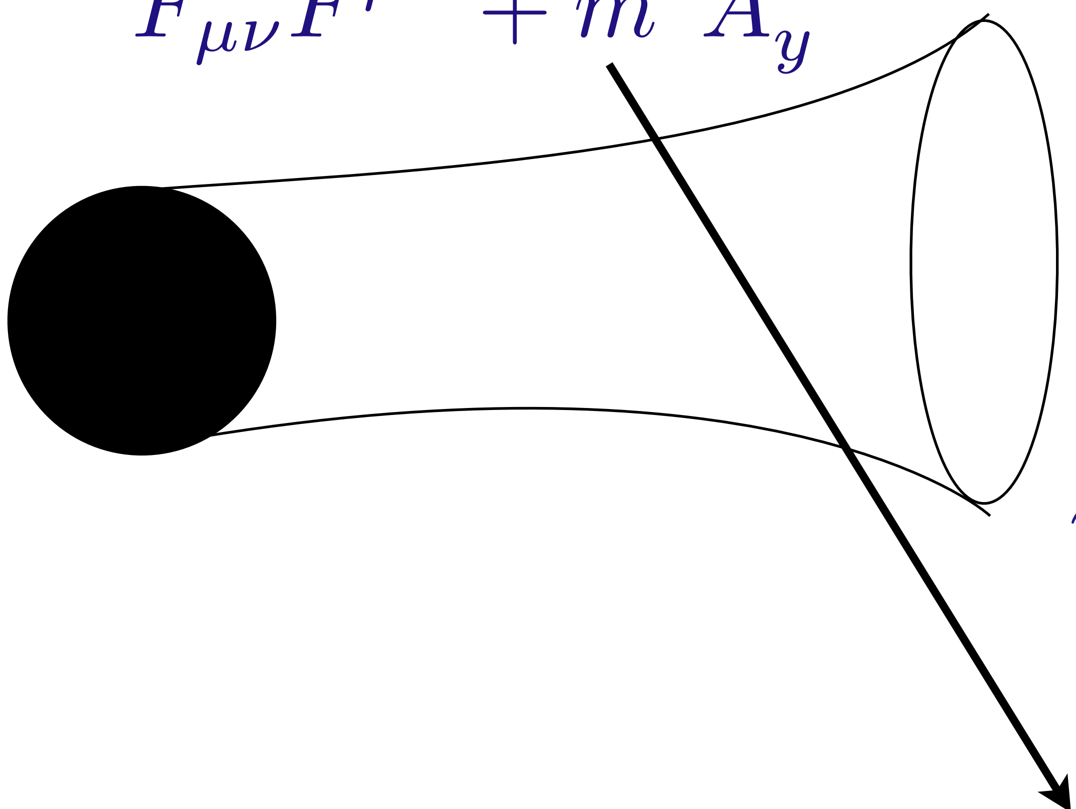
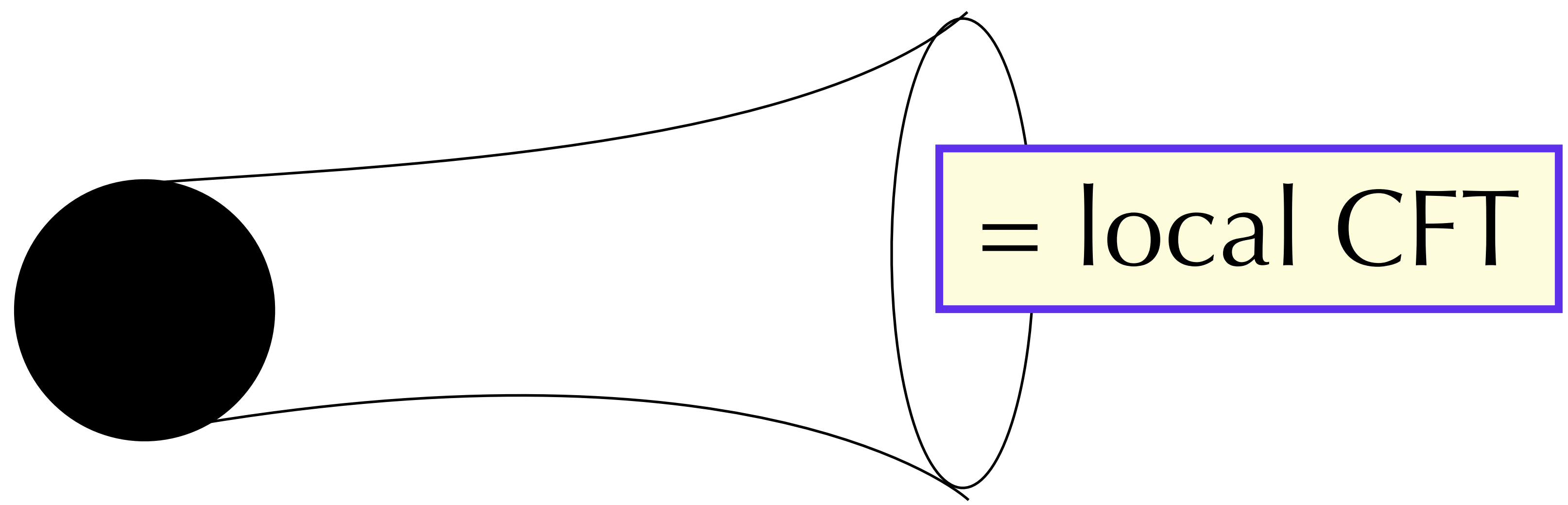


$$A^{\perp}_{\mu}\partial^{\gamma}_{\mu}A^{\perp\mu}$$

$$\gamma = \sqrt{d^2 + m^2 - 1}/2$$

dynamical `Higgs' mode

gauge-gravity correspondence



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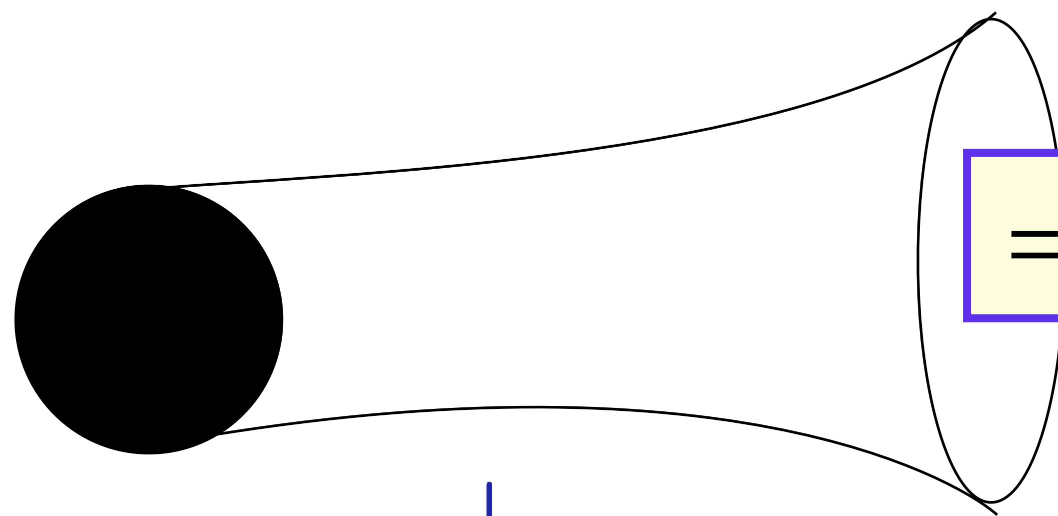
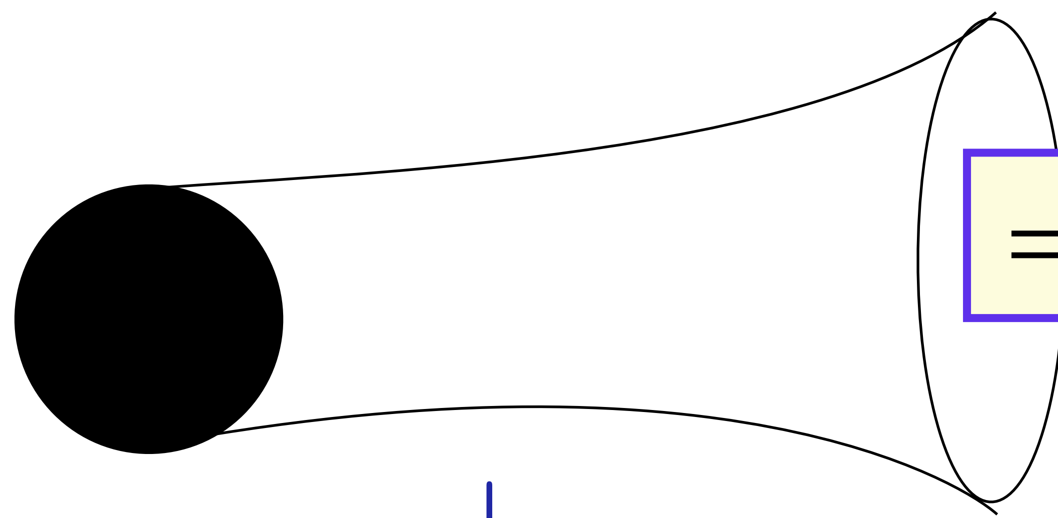


= local CFT

+

N D3-branes

gauge-gravity correspondence



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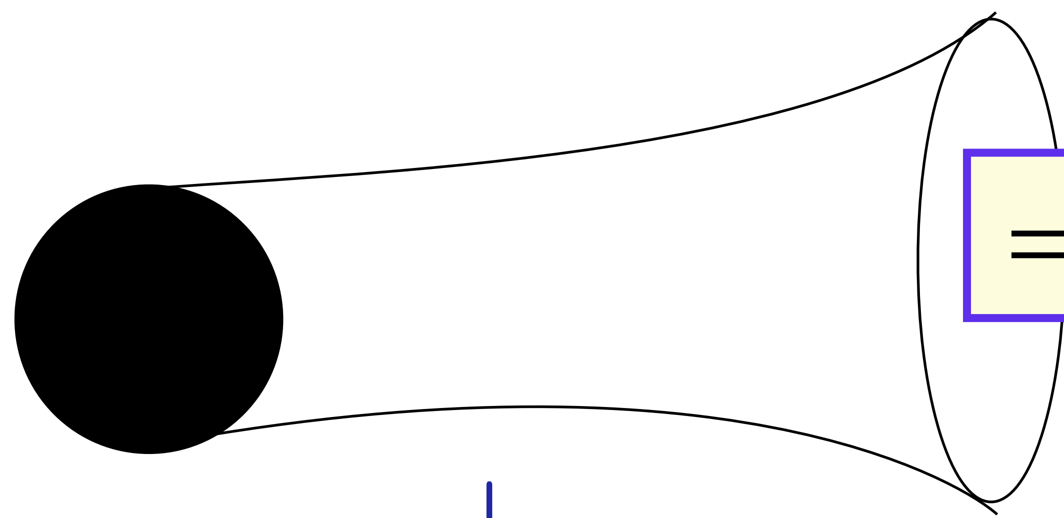
N D3-branes

entanglement entropy?

gauge-gravity correspondence



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SYK model is different