Flat Space Holography and Entanglement Entropy in $2+1$ Dimensions

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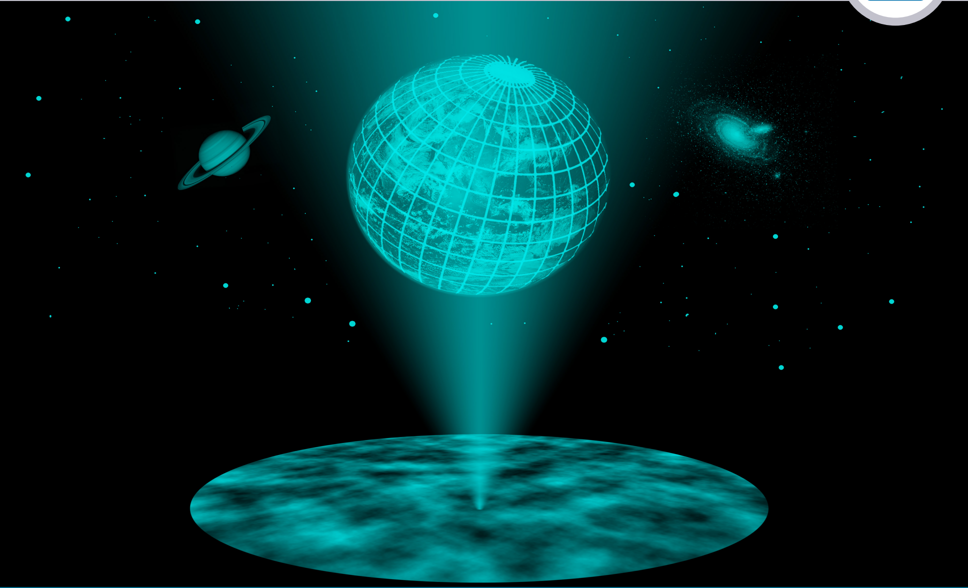
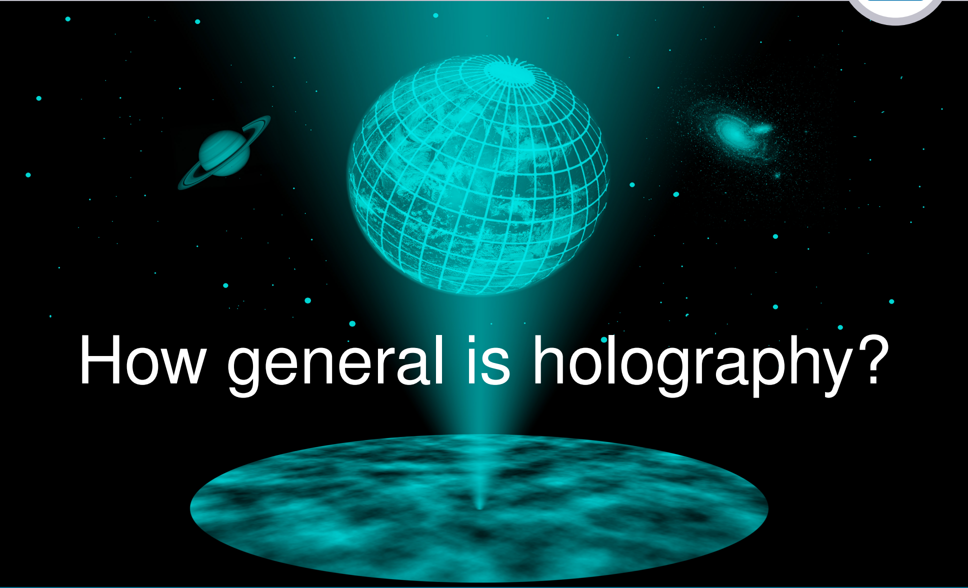
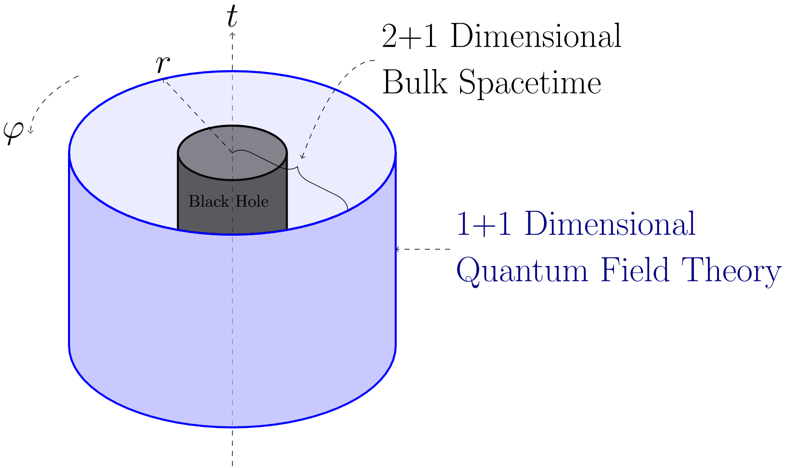
Doktoratskolleg
Particles and Interactions





How general is holography?



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2+1 Dimensional
Bulk Spacetime
Black Hole
1+1 Dimensional
Quantum Field Theory

$$S_{CS}[\mathcal{A}] = \frac{k}{4\pi}\int_{\mathcal{M}}\left\langle \mathcal{A}\wedge d\mathcal{A} + \frac{2}{3}\mathcal{A}\wedge\mathcal{A}\wedge\mathcal{A}\right\rangle$$

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- AdS_3: $\mathcal{A}\in\mathfrak{so}(2,2)\sim\mathfrak{sl}(2,\mathbb{R})\oplus\mathfrak{sl}(2,\mathbb{R})$,
- dS_3: $\mathcal{A}\in\mathfrak{so}(3,1)$,
- Flat Space: $\mathcal{A}\in\mathfrak{isl}(2,1)\sim\mathfrak{sl}(2,\mathbb{R})\in_{\mathrm{ad}}(\mathfrak{sl}(2,\mathbb{R}))_{\mathrm{Ab}}$.

$$S_{CS}[\mathcal{A}] = \frac{k}{4\pi} \int_{\mathcal{M}} \left\langle \mathcal{A} \wedge \mathrm{d}\mathcal{A} + \frac{2}{3} \mathcal{A} \wedge \mathcal{A} \wedge \mathcal{A} \right\rangle$$

- AdS$_3$: $\mathcal{A} \in \mathfrak{so}(N, 2) \sim \mathfrak{sl}(N, \mathbb{R}) \oplus \mathfrak{sl}(N, \mathbb{R})$,
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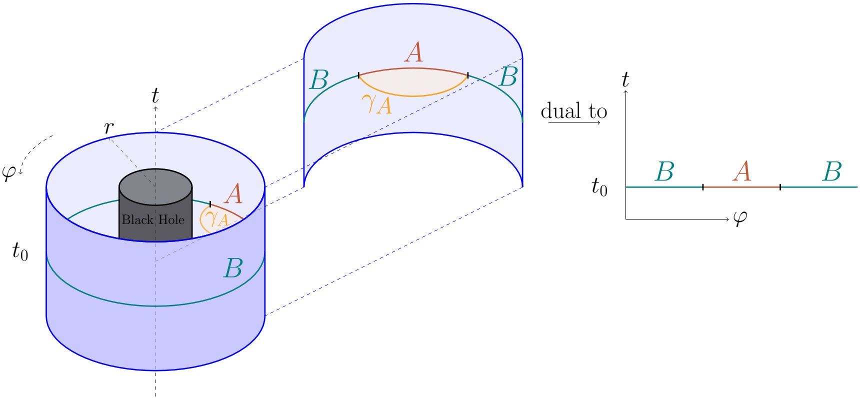
Holgraphic Entanglement Entropy

Geodesics as a Gravity Dual



$$W_{\mathcal{R}}(C) = \mathrm{Tr}_{\mathcal{R}}\left[\mathcal{P}\exp\left(\int_C \mathcal{A}\right)\right] = \int \mathcal{D}U \exp\left(-S(U;\mathcal{A})_C\right),$$

$$S_{\mathrm{EE}} = -\log\left[W_{\mathcal{R}}(C)\right].$$

Ammon, M., Castro, A., and Iqbal, N. (2013).
Wilson Lines and Entanglement Entropy in Higher Spin Gravity.
JHEP, 1310:110.

de Boer, J. and Jottar, J. I. (2014).
Entanglement Entropy and Higher Spin Holography in AdS_3.
JHEP, 04:089.

$\mathfrak{sl}(2,\mathbb{R})$ and $\mathfrak{vir}$

$$[L_n, L_m] = (n-m)L_{n+m} + \frac{c}{12}n(n^2-1)\delta_{n+m},$$
$$[\bar{L}_n, \bar{L}_m] = (n-m)\bar{L}_{n+m} + \frac{\bar{c}}{12}n(n^2-1)\delta_{n+m},$$
$$[L_n, \bar{L}_m] = 0.$$

AdS_3

Flat Space

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$$\mathfrak{so}(2,2) \sim \mathfrak{sl}(2,\mathbb{R}) \oplus \mathfrak{sl}(2,\mathbb{R})$$
$$\Downarrow$$
$$W_{\mathcal{R}}^{c}(C) \times W_{\mathcal{R}}^{\bar{c}}(C)$$
$$\Downarrow$$
$$S_{\text{EE}} = -\log\left[W_{\mathcal{R}}^{c}(C)\right] - \log\left[W_{\mathcal{R}}^{\bar{c}}(C)\right]$$

AdS$_3$

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AdS$_3$

$\mathfrak{isl}(2,\mathbb{R})$ and $\mathfrak{bms}_3$

$$[L_n, L_m] = (n-m)L_{n+m} + \frac{c_L}{12}n(n^2-1)\delta_{n+m},$$
$$[L_n, M_m] = (n-m)M_{n+m} + \frac{c_M}{12}n(n^2-1)\delta_{n+m},$$
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Flat Space

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$$?\Downarrow?$$
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Flat Space

Assumption

$$U = U_L U_M \quad \text{for} \quad U \in ISL(2, \mathbb{R}),$$

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$$\Downarrow \checkmark$$

$$W_{\mathcal{R}}^{c_L}(C) \times W_{\mathcal{R}}^{c_M}(C),$$

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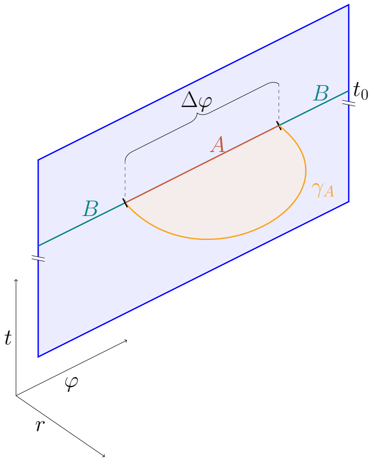
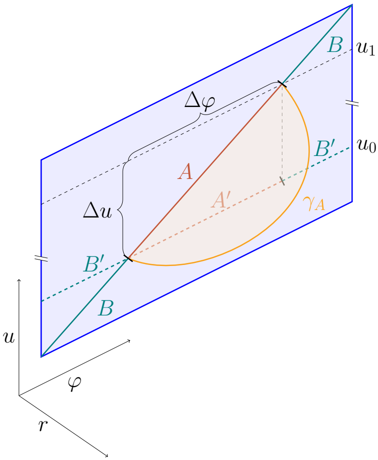
Flat Space Holography
Entangling Intervals



AdS_3



Flat Space

$$\mathrm{d}s^2 = \mathcal{M}\,\mathrm{d}u^2 - 2\,\mathrm{d}u\,\mathrm{d}r + 2\mathcal{N}\,\mathrm{d}u\,\mathrm{d}\varphi + r^2\,\mathrm{d}\varphi^2,$$

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$\mathfrak{isl}(2,\mathbb{R})$ Chern-Simons Gauge Field

$$\mathcal{A} = b^{-1}(\mathrm{d}+a)b \quad \text{with} \quad b = e^{\frac{r}{2}M_{-1}},$$

$$a = \left(M_1 - \frac{\mathcal{M}}{4}M_{-1}\right)\mathrm{d}u + \left(L_1 - \frac{\mathcal{M}}{4}L_{-1} - \frac{\mathcal{N}}{2}M_{-1}\right)\mathrm{d}\varphi.$$

$$ds^2 = \mathcal{M}\,\mathrm{d}u^2 - 2\,\mathrm{d}u\,\mathrm{d}r + 2\mathcal{N}\,\mathrm{d}u\,\mathrm{d}\varphi + r^2\,\mathrm{d}\varphi^2,$$

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- $\mathcal{M} = \mathcal{N} = 0$: Null-Orbifold (NO),

$$ds^2 = \mathcal{M}\,du^2 - 2\,du\,dr + 2\mathcal{N}\,du\,d\varphi + r^2\,d\varphi^2,$$

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- $\mathcal{M} = \mathcal{N} = 0$: Null-Orbifold (NO),
- $\mathcal{M} = -1$, $\mathcal{N} = 0$: Global Flat Space (GFS),

Flat Space Holography

Metric and Connection

$$ds^2 = \mathcal{M}\,du^2 - 2\,du\,dr + 2\mathcal{N}\,du\,d\varphi + r^2\,d\varphi^2,$$

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- $\mathcal{M} = \mathcal{N} = 0$: Null-Orbifold (NO),
- $\mathcal{M} = -1$, $\mathcal{N} = 0$: Global Flat Space (GFS),
- $\mathcal{M} \neq 0$, $\mathcal{N} \neq 0$: Flat Space Cosmologies (FSC).

Flat Space Holographic Entanglement Entropy

$$\text{Null-Orbifold}: S_{EE} = \frac{c_L}{6} \ln\left[\frac{r_0 \Delta\phi}{2}\right] + \frac{c_M}{6}\frac{\Delta u}{\Delta\phi},$$

$$\text{(Global) FS}: S_{EE} = \frac{c_L}{6} \ln\left[r_0 \sin\left(\frac{\Delta\phi}{2}\right)\right] + \frac{c_M}{12} \cot\left(\frac{\Delta\phi}{2}\right)\Delta u.$$

Bagchi, A., Basu, R., Grumiller, D., and Riegler, M. (2015).
Entanglement entropy in Galilean conformal field theories and flat holography.
Phys. Rev. Lett., 114(11):111602.

Wilson line $\Rightarrow$ Wilson loop

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Flat Space Holographic Thermal Entropy

$$S_{\text{Th}} = \frac{\pi}{6}\left(c_L\sqrt{\mathcal{M}} + c_M\frac{\mathcal{N}}{\sqrt{\mathcal{M}}}\right).$$

Basu, R. and Riegler, M. (2016).
Wilson Lines and Holographic Entanglement Entropy in Galilean Conformal Field Theories.
Phys. Rev., D93(4):045003.

Riegler, M. (2015).
Flat space limit of higher-spin Cardy formula.
Phys. Rev., D91(2):024044.

- Extend GCFT calculations in order to verify higher-spin results.

Conclusion
Extensions and Outlook

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Conclusion
Extensions and Outlook

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- Geometric bulk derivation using geodesics or covariant HEE proposal.
- Testing various information theoretic properties like SSA, mutual information etc.
- Holographic reconstruction of the bulk theory using EE.

Thank you for your attention!