“Aharonov-Bohm effect” and entanglement entropy in conformal field theory

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Based on
N.Shiba, arXiv:1606.xxxxx

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1.Introduction

Entanglement entropy (EE) is the quantity which measures the degree of entanglement.

EE plays important roles in various fields of quantum physics including quantum information theory, string theory, condensed matter physics, and the physics of the black hole.

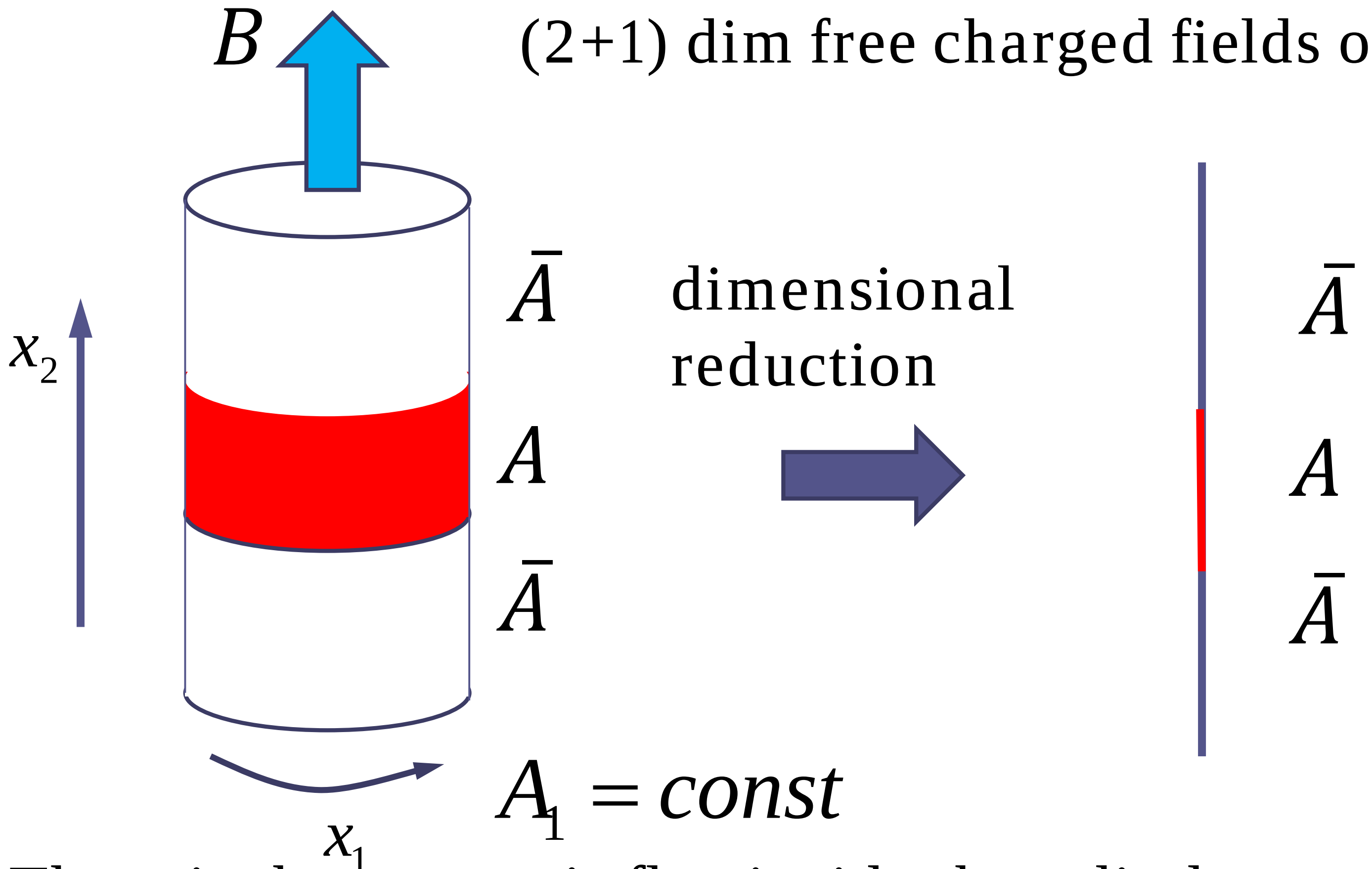
The Aharonov Bohm (AB) effect is a fundamental quantum phenomenon in which an electrically charged particle is affected by an electromagnetic potential A_μ , despite being confined to a region in which both the magnetic field B and electric field E are zero.

The dependence of EE with an AB phase, $\Phi = \oint_C dx^\mu A_\mu$, in QFT was studied by Arias, Blanco, Casini 2014 .

In our work, we consider a different setup and obtain some exact results.

Arias, Blanco, Casini 2014

(2+1) dim free charged fields on the cylinder

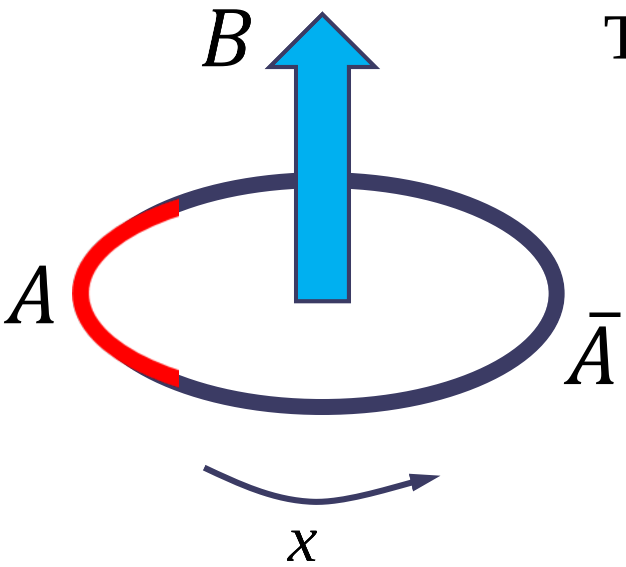


There is the magnetic flux inside the cylinder.

They considered EE of an annular strip A.

In this case, the EE becomes the EE in (1+1) dimension for an infinite tower of massive fields by the dimensional reduction.

2.Our Setup



(1+1) dim charged fields on the ring

There is the magnetic flux inside the ring.

$$x \sim x + L$$

$$A_x = const$$

$$\Phi = \oint_C dx^\mu A_\mu$$

We consider EE of one interval A for the charged field on the ring.

We assume that the charged field on the ring is (1+1) dim CFT.

In this case, we cannot use the dimensional reduction.
Instead, we can use 2d CFT technique and obtain
some exact results.

3.EE in 2d CFT with the AB phase

$w = x + i\tau \qquad x \sim x + L$

$A_x = const \qquad A : w_1 \le w \le w_2 \qquad |w_2 - w_1| = l$

First, we eliminate A_x by a gauge transformation

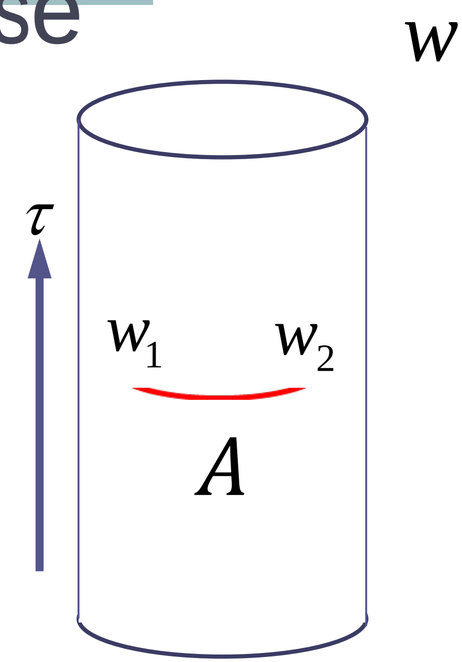
$$\phi'(x) = e^{-iqxA_x}\phi(x)$$

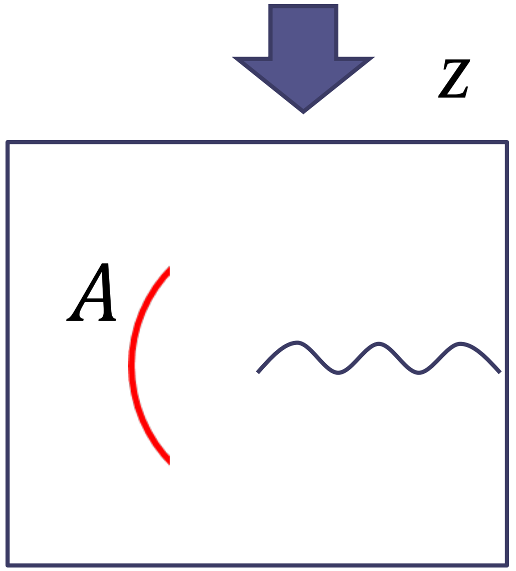
The charged field $\phi'(x)$ has the following boundary condition

$$\phi'(x+L) = e^{-i2\pi\nu}\phi'(x) \qquad \nu = \frac{qLA_x}{2\pi} = \frac{q\Phi}{2\pi}$$

The cylinder is mapped to the complex plane by a conformal map

$$z = \exp[-i2\pi w / L]$$





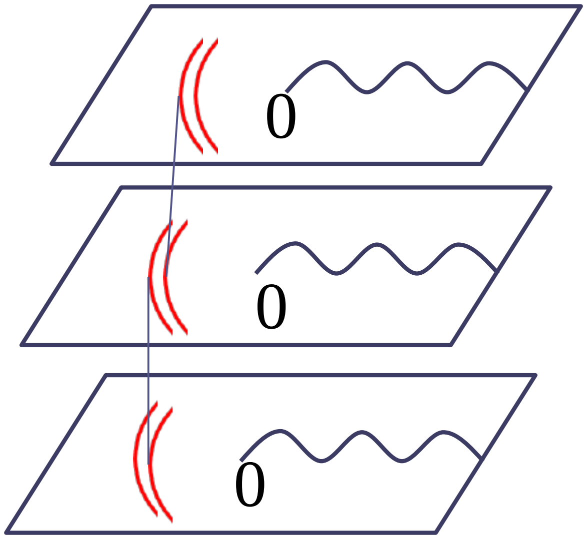
The twist boundary condition is represented as the insertion of twist operators at zero and infinity.

So, by using the replica method, we can express $Tr\rho_A^n$ as a four point function of the twist operators

$$Tr\rho_A^n = \frac{Z_n}{(Z_1)^n} = \frac{\langle T_n(z_1)\tilde{T}_n(z_2)\sigma_\nu(0)\tilde{\sigma}_\nu(\infty)\rangle}{\langle \sigma_\nu(0)\tilde{\sigma}_\nu(\infty)\rangle^n}$$

$$T_n : \phi_j \to \phi_{j+1} \qquad \sigma_\nu : \phi_j \to e^{i2\pi\nu}\phi_j$$

$$\tilde{T}_n : \phi_{j+1} \to \phi_j \qquad \tilde{\sigma}_\nu : \phi_j \to e^{-i2\pi\nu}\phi_j$$



A representation of the Riemann surface for n=3

$$S_A^{(n)} = \frac{1}{1-n}\ln tr\rho_A^n$$

Two different twist operators T_n and σ_ν appear.

3.EE in non compact charged free complex scalar field

For free fields, we can diagonalize T_n and σ_ν simultaneously by the discrete Fourier transformation

$$\tilde{\varphi}_k = \sum_{j=0}^{n-1} e^{i2\pi jk/n} \varphi_j$$

$$T_n : \tilde{\varphi}_k \to e^{i2\pi k/n} \tilde{\varphi}_k \qquad \sigma_\nu : \tilde{\varphi}_k \to e^{i2\pi\nu} \tilde{\varphi}_k$$

$$\tilde{T}_n : \tilde{\varphi}_k \to e^{-i2\pi k/n} \tilde{\varphi}_k \qquad \tilde{\sigma}_\nu : \tilde{\varphi}_k \to e^{-i2\pi\nu} \tilde{\varphi}_k$$

For free fields, the various k-modes decouple and the partition function is given by

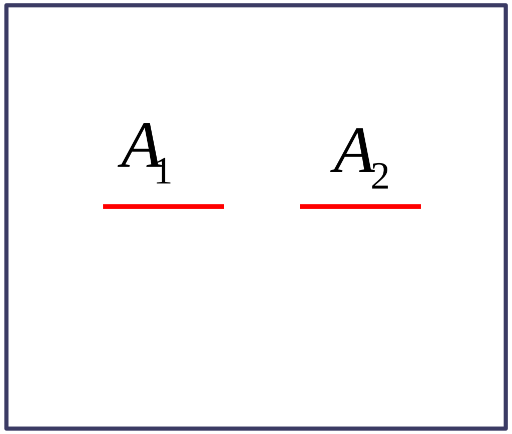
$$Tr\rho_A^n = \frac{Z_n}{(Z_1)^n} = \prod_{k=1}^{n-1} \frac{Z_{k,n}}{Z_1} = \prod_{k=1}^{n-1} \frac{\left\langle \sigma_{k/n}(z_1)\tilde{\sigma}_{k/n}(z_2)\sigma_\nu(0)\tilde{\sigma}_\nu(\infty) \right\rangle}{\left\langle \sigma_\nu(0)\tilde{\sigma}_\nu(\infty) \right\rangle}$$

◎Comment on EE of two intervals without the AB phase

EE of two intervals without the AB phase can be obtained by the following four point functions of twist operators.

Calabrese,Cardy,Tonni 2009

$$Tr\rho^n_{A_1\cup A_2}=\prod_{k=1}^{n-1}\left\langle \sigma_{k/n}(z_1)\tilde{\sigma}_{k/n}(z_2)\sigma_{k/n}(z_3)\tilde{\sigma}_{k/n}(z_4)\right\rangle$$



one interval with the AB phase

$$Tr\rho^n_A=\prod_{k=1}^{n-1}\frac{\left\langle \sigma_{k/n}(z_1)\tilde{\sigma}_{k/n}(z_2)\sigma_{\nu}(0)\tilde{\sigma}_{\nu}(\infty)\right\rangle}{\left\langle \sigma_{\nu}(0)\tilde{\sigma}_{\nu}(\infty)\right\rangle}$$

Because $\nu\neq k/n$, we need more general four point functions of twist operators.

$Z_{k,n}=\left\langle \sigma_{k/n}(z_1)\tilde{\sigma}_{k/n}(z_2)\sigma_{\nu}(0)\tilde{\sigma}_{\nu}(\infty)\right\rangle$ can be obtained by the method developed in CFT on orbifolds.

The results

$$Z_{k,n} = \kappa^2 |x|^{-2k/n(1-k/n)} \pi(I_{k/n}(x,\bar{x}))^{-1}$$

$$S_A^{(n)} = \frac{1}{1-n}\sum_{k=1}^{n-1} \ln Z_{k,n}$$

where κ^2 is a constant and

$$I_{k/n}(x,\bar{x})/\pi$$

$$= \frac{\Gamma(k/n)\Gamma(1-\nu)}{\Gamma(k/n-\nu+1)}\left[F(\bar{x})G(1-x)+F(x)G(1-\bar{x})\right] + |F(x)|^2 \pi\left(\cot\pi\nu - \cot(\pi k/n)\right)$$

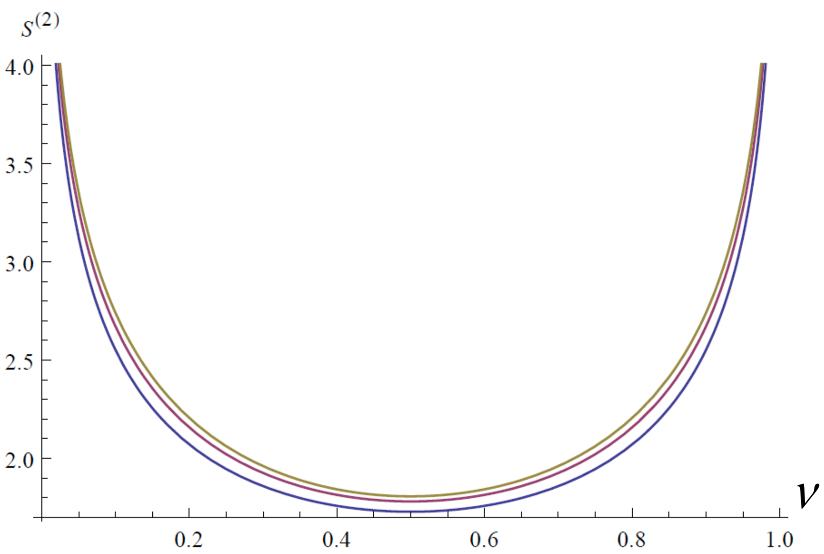
$$F(x) \equiv {}_2F_1(1-\nu, k/n, 1; x) \qquad G(1-x) \equiv {}_2F_1(1-\nu, k/n, 1+k/n-\nu; 1-x)$$

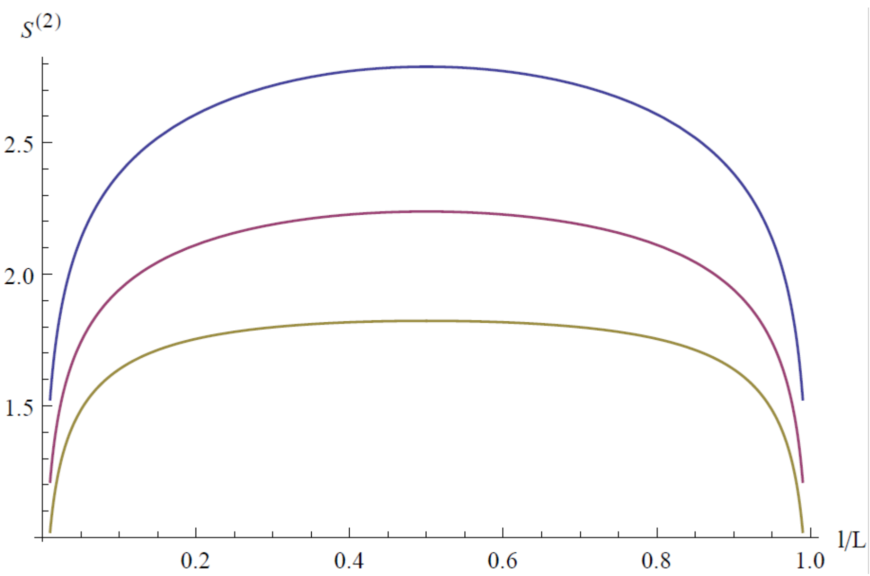
where x is the cross ratio

$$x = \frac{z_1 - z_2}{z_1} = 1 - e^{-i2\pi l/L} \qquad |x| = 2\sin(\pi l/L)$$

It is difficult to obtain $S_A = \lim_{n\to 1}\left[-\frac{\partial}{\partial n}\ln tr\rho_A^n\right]$

We show $S_A^{(2)} = -\ln tr\rho_A^2$





From top to bottom:

$l/L = 1/6,\ 1/4,\ 1/3$

From top to bottom:

$\nu = 1/10,\ 1/5,\ 1/2$

$S_A^{(2)}$ become the minimum value for $\nu = 1/2$

Some properties of the (Renyi) EE

$$S_A^{(n)} = \frac{1}{1-n} \ln tr \rho_A^n \qquad \nu = \frac{qLA_x}{2\pi} = \frac{q\Phi}{2\pi}$$

small AB phase limit $\lim_{\nu \to 0} S_A^{(n)} = \ln(1/\nu) + const$

In $\nu \to 0$ limit, there is a $\ln(1/\nu)$ divergence.
This divergence comes from the homogeneous component of the field. There is a similar divergence in EE in the massless limit of a massive scalar field in 2 dimension.

Short interval and small AB phase limit

$$\sin(\pi l / L) << 1 \qquad \nu << 1 \qquad \nu \ln \sin(\pi l / L) << 1$$

$$S_A^{(n)} = \ln(1/\nu) + \frac{1}{3}(1 + 1/n - 6\nu) \ln \sin(\pi l / L) + const$$

$$S_A = \ln(1/\nu) + (2/3 - 2\nu) \ln \sin(\pi l / L) + const$$

4.Conclusion

We considered the dependence of EE with an AB phase, $\Phi = \oint_C dx^\mu A_\mu$, in CFT on the ring.

The Renyi EE is expressed by the four point function of twist operators.

$$S_A^{(n)} = \frac{1}{1-n} \ln tr\rho_A^n$$

$$Tr\rho_A^n = \frac{Z_n}{(Z_1)^n} = \frac{\left\langle T_n(z_1)\tilde{T}_n(z_2)\sigma_\nu(0)\tilde{\sigma}_\nu(\infty)\right\rangle}{\left\langle \sigma_\nu(0)\tilde{\sigma}_\nu(\infty)\right\rangle^n}$$

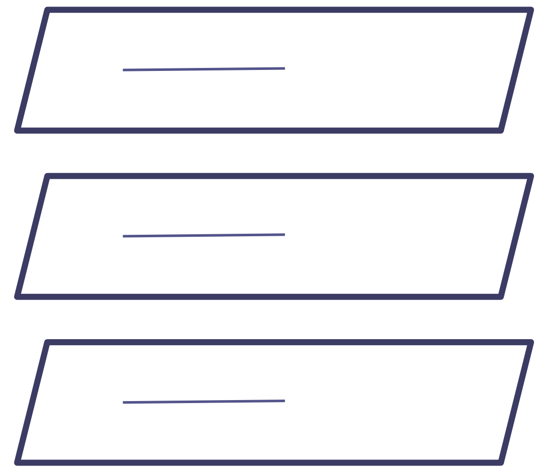
We obtained some results for EE in non compact charged free complex scalar field.

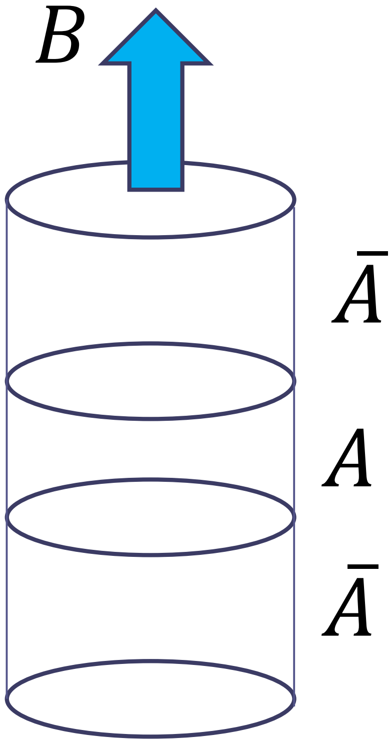
$$S_A^{(n)} = \frac{1}{1-n}\sum_{k=1}^{n-1} \ln Z_{k,n} \qquad Z_{k,n} = \kappa^2 |x|^{-2k/n(1-k/n)} \pi(I_{k/n}(x,\bar{x}))^{-1}$$

Future work:

finite temperature, time dependence, etc

$$Tr\rho_A^n = \frac{Z_n}{(Z_1)^n}$$

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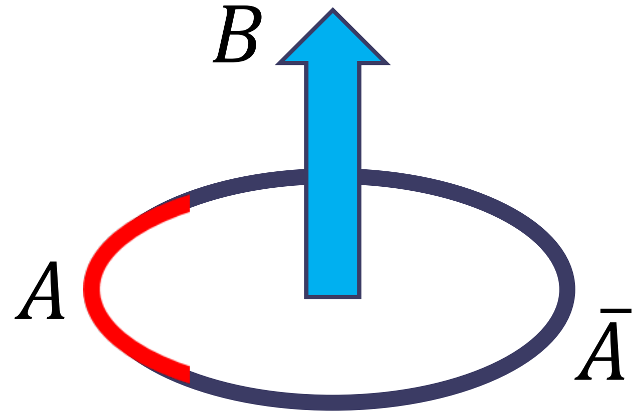
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