Remarks on holographic complexity & singularities

J.L.F. Barbon
Work with E. Rabinovici 150909291

+

Work with J. Martin-Garcia 151000349

+

Work in progress …
On general grounds, there are conflicting expectations regarding the “complexity” of space-time singularities.
On general grounds, there are conflicting expectations regarding the “complexity” of space-time singularities.

Bang singularities are usually regarded as “simple” ...

whereas Crunch singularities could be expected to be quite complex.

This simply reflects standard implicit assumptions regarding the thermodynamic arrow of time.
The Bang/Crunch asymmetry and its relation to the arrow of time has been forcefully emphasised by Penrose, among others.
Even BKL universality shows classical chaos

\[ ds^2_{K_i} \approx -dt^2 + \sum_j t^2 p_j \, d\vec{x}_j^2 \]

The succession of \( p_j \) is determined by a hyperbolic billiard game
In this talk I will explore some examples accessible to AdS/CFT constructions. 

“Complexity” will be defined through the so-called Volume/Complexity (V/C) duality.

Brown, Hartman, Maldacena, Roberts, Susskind, Stanford, Swingle, Zhao, …
VOLUME / COMPLEXITY duality

Extremal codimension-1 surface $\Sigma$

$$C \sim \frac{\text{Vol}(\Sigma)}{GR_{\text{AdS}}}$$

See also Miyaji, Numasawa, Shiba, Takayanagi, Watanabe
Swingle

MERA-like
TCN for \( l_{\text{vac}} > C_{\text{FT}} \)

“Complexity of entanglement”, as measured by the size of some “optimal” tensor network

Size of TN ↔ Volume of spatial slice
Black hole singularities are “repulsive” w.r.t. C/V duality

\[ C(t \to \infty) \sim S T t \]
Black hole singularities are “repulsive” w.r.t. C/V duality

\[ C \sim \text{Action}_{\text{wedge}} \]

\[ C(t \to \infty) \sim STt \]
Despite the codimension-1 surface not "probing" directly the singularity, it assigns linearly growing complexity to a tensor network state in its "causal domain".

However, if we could somehow take $\Sigma$ to the singularity, the CV ansatz would assign small complexity ...
In this talk I will discuss some attempts at “forcing” the maximal codim-1 surface to “directly probe” the singularity.

If the maximal surface does not go to the singularity, we try to bring the singularity to the maximal surface.

Since $\Sigma$ is defined by its UV boundary condition, it should be enough to engineer singularities which are visible in the UV of the CFT, i.e. AdS “cosmological” singularities.
This is usually done by “driving” the CFT to the singularity

\[ H_{\text{CFT}} \rightarrow H_{\text{CFT}} + J(t) \mathcal{O} \]

\[ J(t) \] singular as \[ t \rightarrow t_* \]

with \( \mathcal{O} \) marginal or relevant, and chosen so that we can approximate the bulk dynamics.
Simple case: a component of the e.m. tensor (marginal)

namely, the CFT is put on a singular frame

A case with easy holographic dual is Kasner

\[ ds^2_K = -dt^2 + \sum_j t^{2p_j} dx_j^2 \quad \Rightarrow \quad ds^2_{\text{bulk}} = \frac{dr^2}{r^2} + r^2 \, ds^2_K \]

Ricci-flat bdry

Awad, Das, Michelson, Nampuri, Narayan, Trivedi …

Engelhardt, Hertog, Horowitz
Extremal cod-1 surfaces are dominated by UV

\[ C \sim N^2 V_{CFT}(t) \Lambda^{d-1} \]

\[ V_{CFT}(t) \sim t \sum_j p_j = t \]
Another set of examples use an accelerated brane in finite-time collision course with the AdS boundary.

\[ t_\ast = \frac{\pi}{2} \]

\[ \mu(t) \sim \frac{1}{\cos(t)} \]

Hertog, Horowitz, Bernamonti, Craps, Maldacena, Harlow, Susskind, Barbon, Rabinovici...

RG domain wall $\mu(t)$
\[
\begin{align*}
\text{AdS} + & \quad \text{AdS} - \\
\text{Nothing} & \\
\eta = \eta_* \approx \pi \\
\rho = \infty & \\
\chi = 0 & \\
\tau = 0 & \\
\eta = \eta_* & \\
\rho = \infty & \\
\chi = 0 & \\
\tau = 0 & \\
\eta = \eta_* & \\
\rho = \infty & \\
\chi = 0 & \\
\tau = 0 &
\end{align*}
\]
\[
\frac{dC_{UV}}{dt} + \frac{dC_{IR}}{dt} < 0
\]

\(\Sigma_{IR}\) grows at the expense of \(\Sigma_{UV}\), but its induced metric has smaller radius of curvature.
This time, the TN picture is that of nested HTNs …

with time-dependent interface

Define the appropriate tensors at the interface
What about the inverted case?

\[
E \sim N V_{\text{CFT}} \mu_{\text{max}}^d
\]
Complexity of antibrane imploding state

\[ C(t) \]

black hole formation

black-hole complexity growth
CONCLUSIONS

- Complexity of singularities is an important future question
- Some singularities engineered in AdS get a tentative complexity classification via C/V duality
- Deep IR singularities get growing complexity
- UV singularities get shrinking complexity
- Simple models for TN real-time evolution with holographic interpretation