Remarks on holographic complexity & singularities

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Work in progress ···

On general grounds, there are conflicting expectations regarding the "complexity" of space-time singularities

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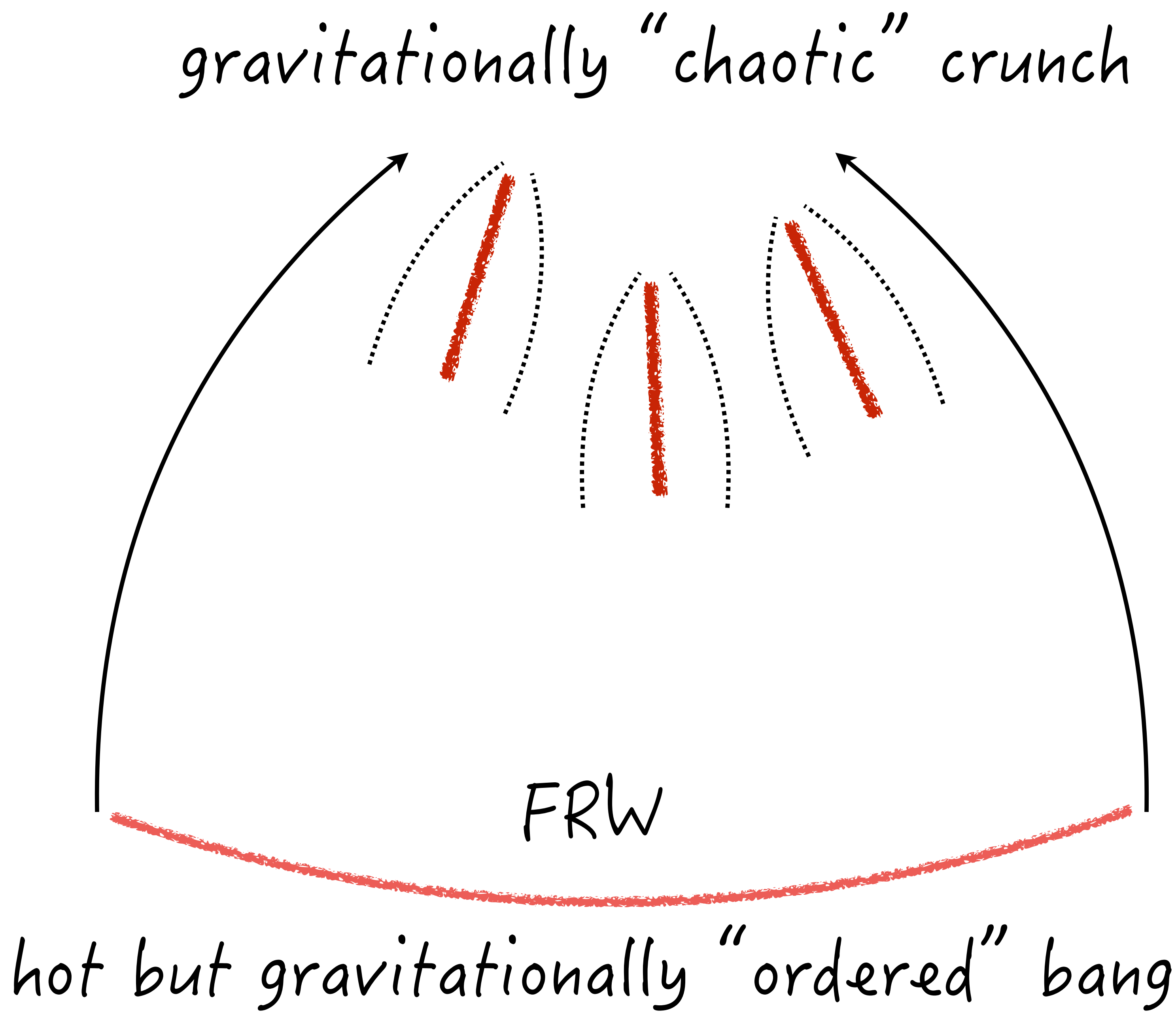
Bang singularities are usually regarded as "simple"

...

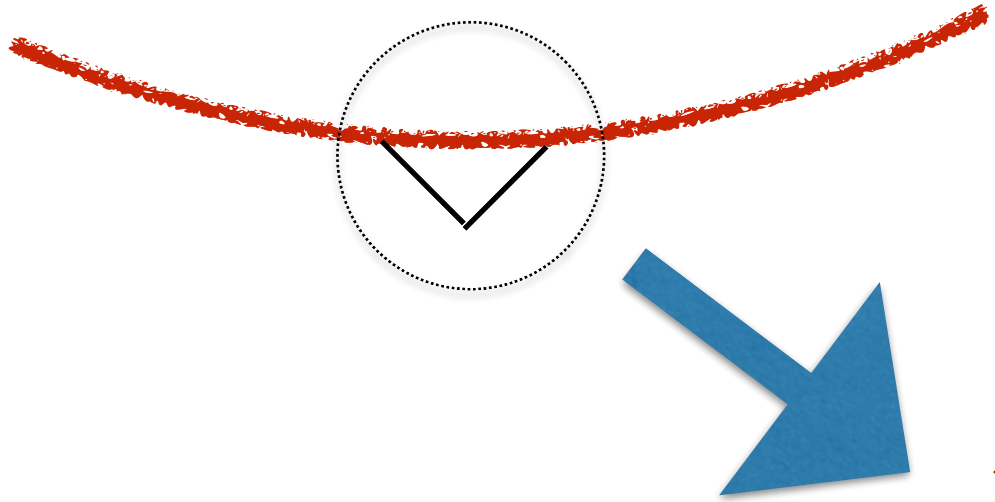
whereas Crunch singularities could be expected to be quite complex

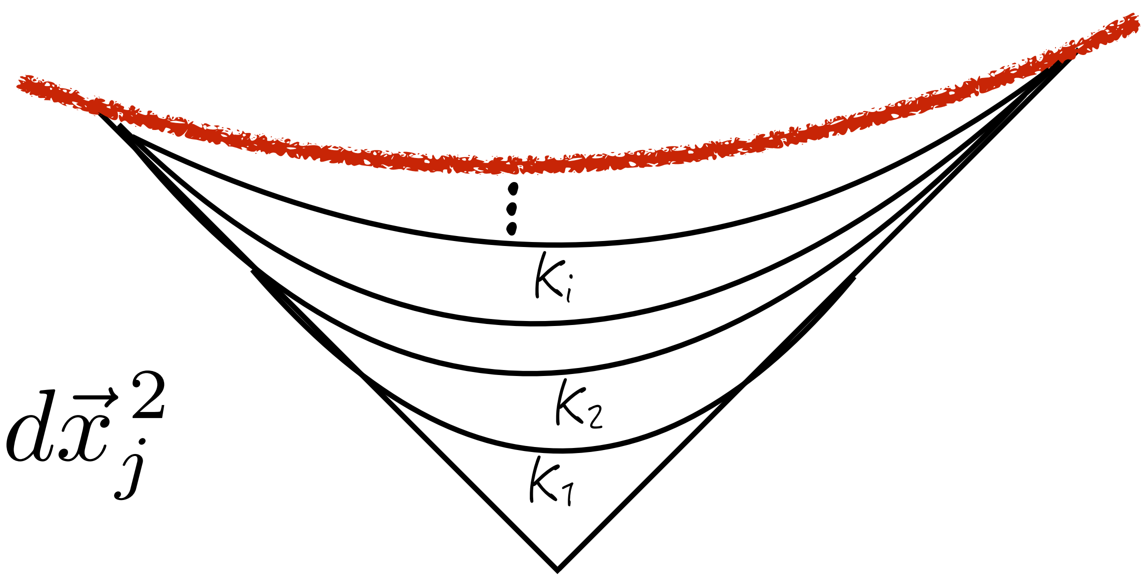
This simply reflects standard implicit assumptions regarding the thermodynamic arrow of time



The Bang/Crunch asymmetry and its relation to the arrow of time has been forcefully emphasised by Penrose, among others

Even BKL universality shows classical chaos





$$ds^2_{K_i} \approx -dt^2 + \sum_j t^{2p_j}\, d\vec{x}_j^{\,2}$$

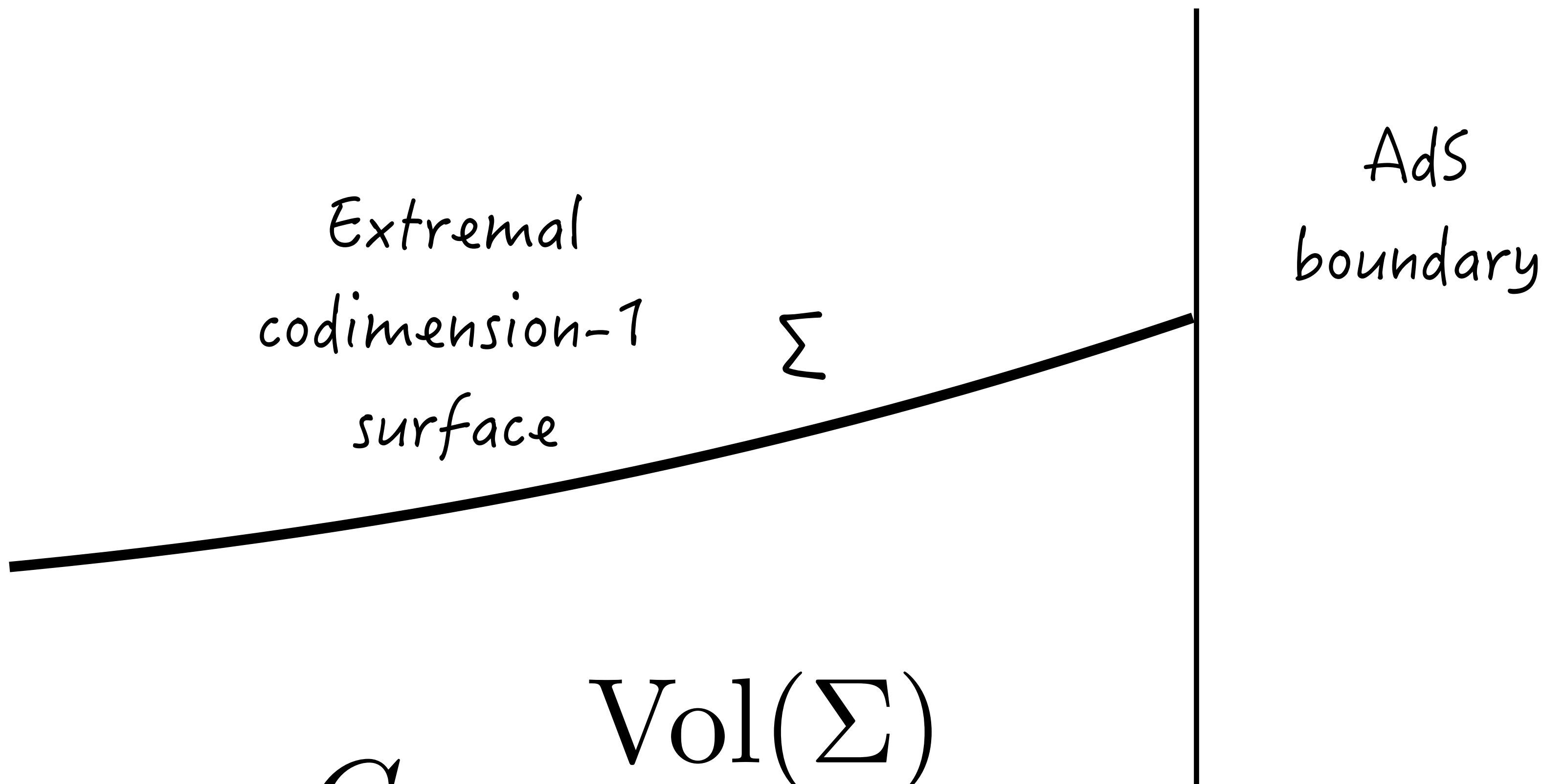
The succession of p_j is determined by a hyperbolic billiard game

In this talk I will explore some examples accesible to AdS/CFT constructions

"Complexity" will be defined through the so-called Volume/Complexity (V/C) duality

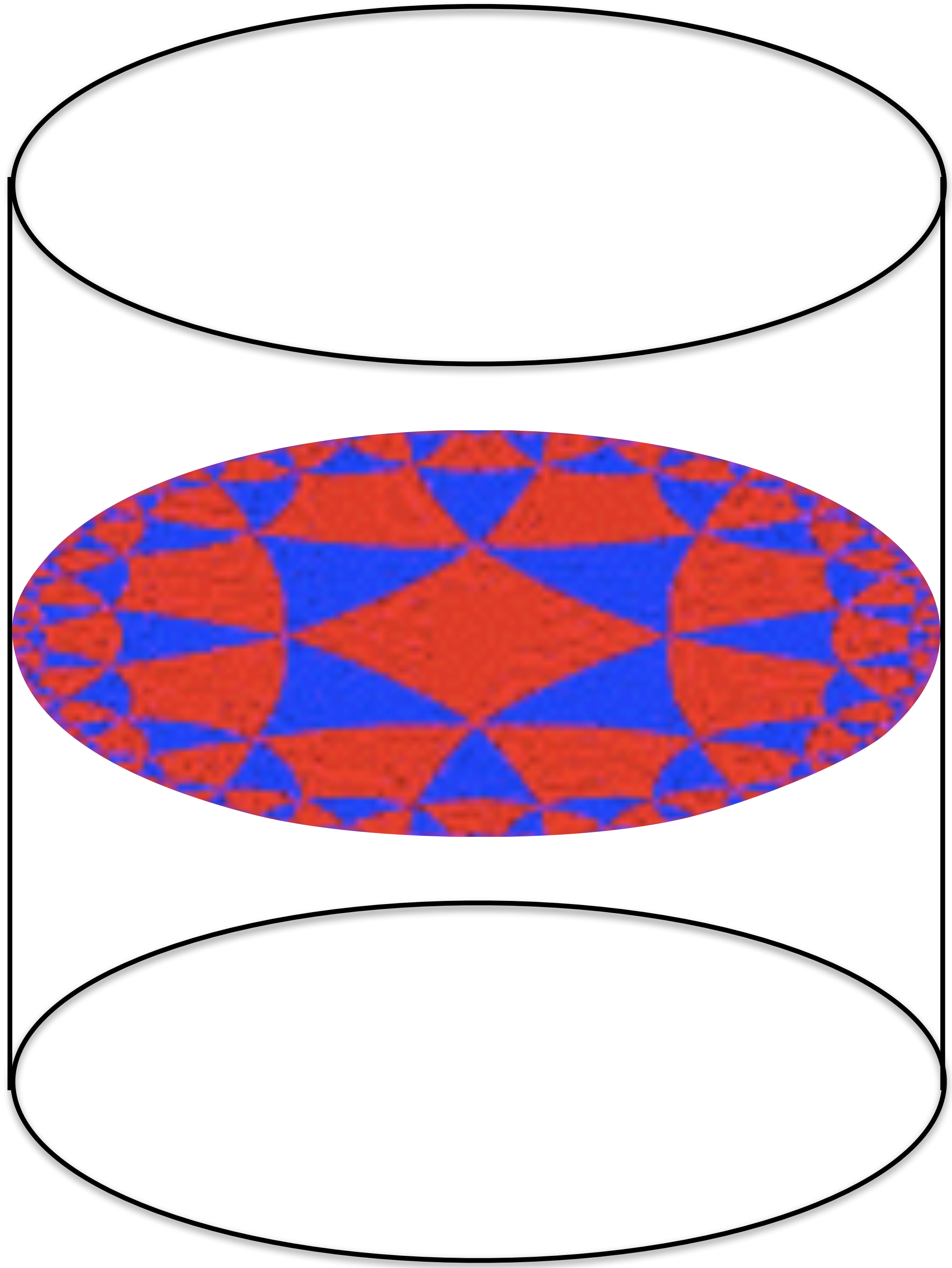
Brown, Hartman, Maldacena, Roberts, Susskind, Stanford, Swingle, Zhao, ⋯

VOLUME / COMPLEXITY duality



$$C \sim \frac{\mathrm{Vol}(\Sigma)}{G R_{\mathrm{AdS}}}$$

See also Miyaji, Numasawa, Shiba, Takayanagi, Watanabe



Swingle

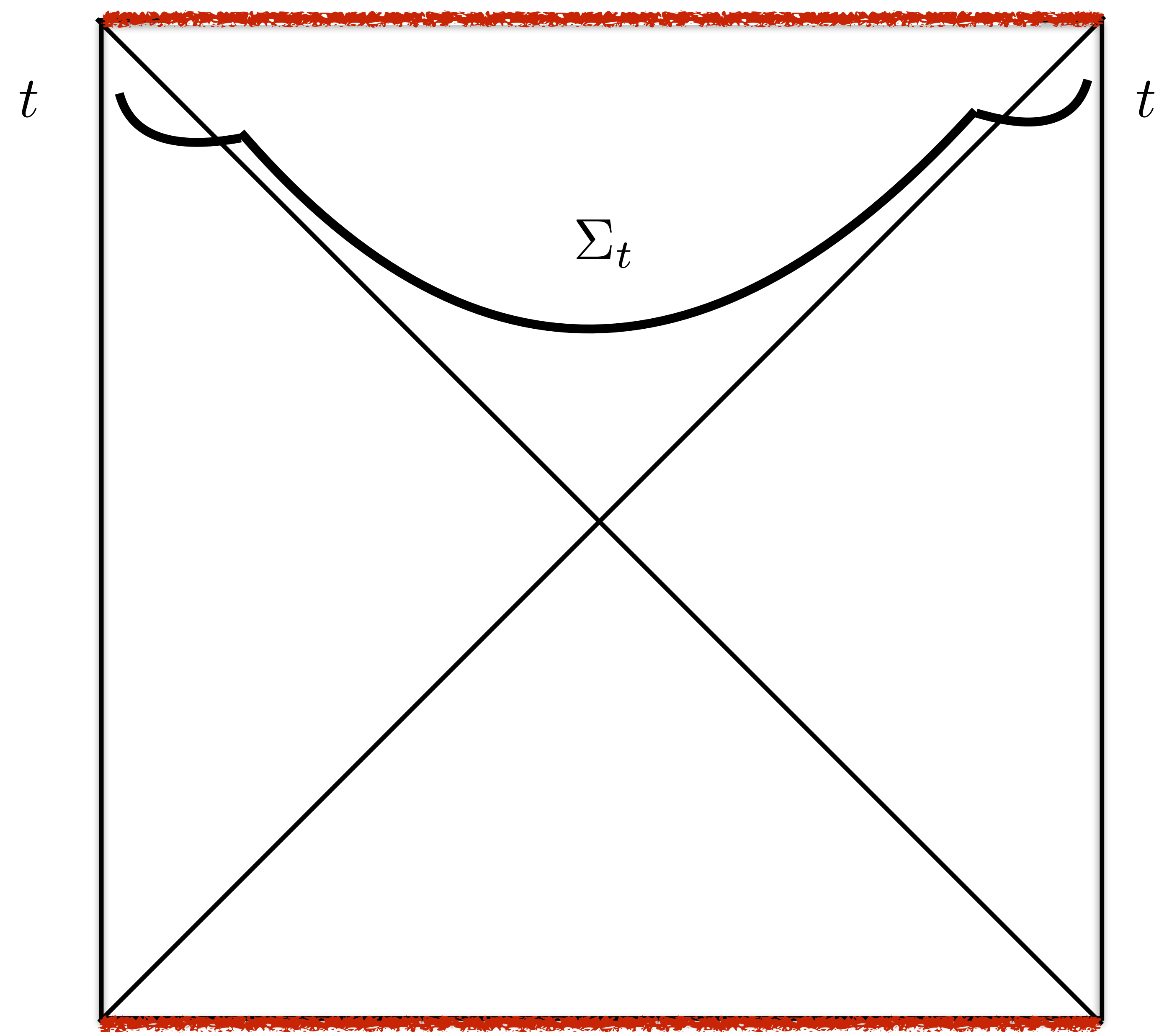
MERA-like

TN for $|vac\rangle_{CFT}$

"Complexity of entanglement", as measured by the size of some "optimal" tensor network

Size of TN ⟷ Volume of spatial slice

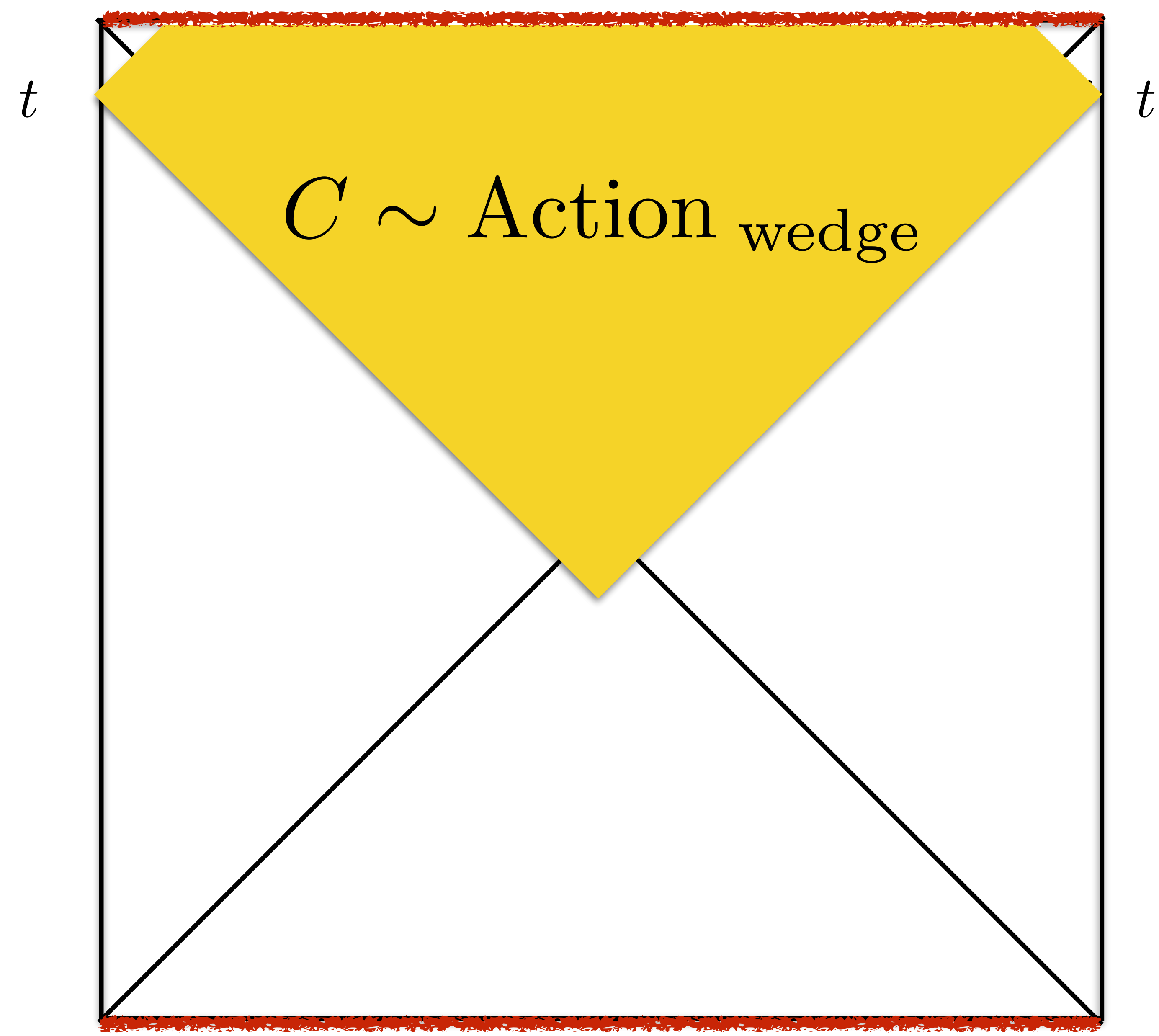
Black hole singularities are "repulsive" w.r.t. C/V duality



$$C(t \to \infty) \sim S\,T\,t$$

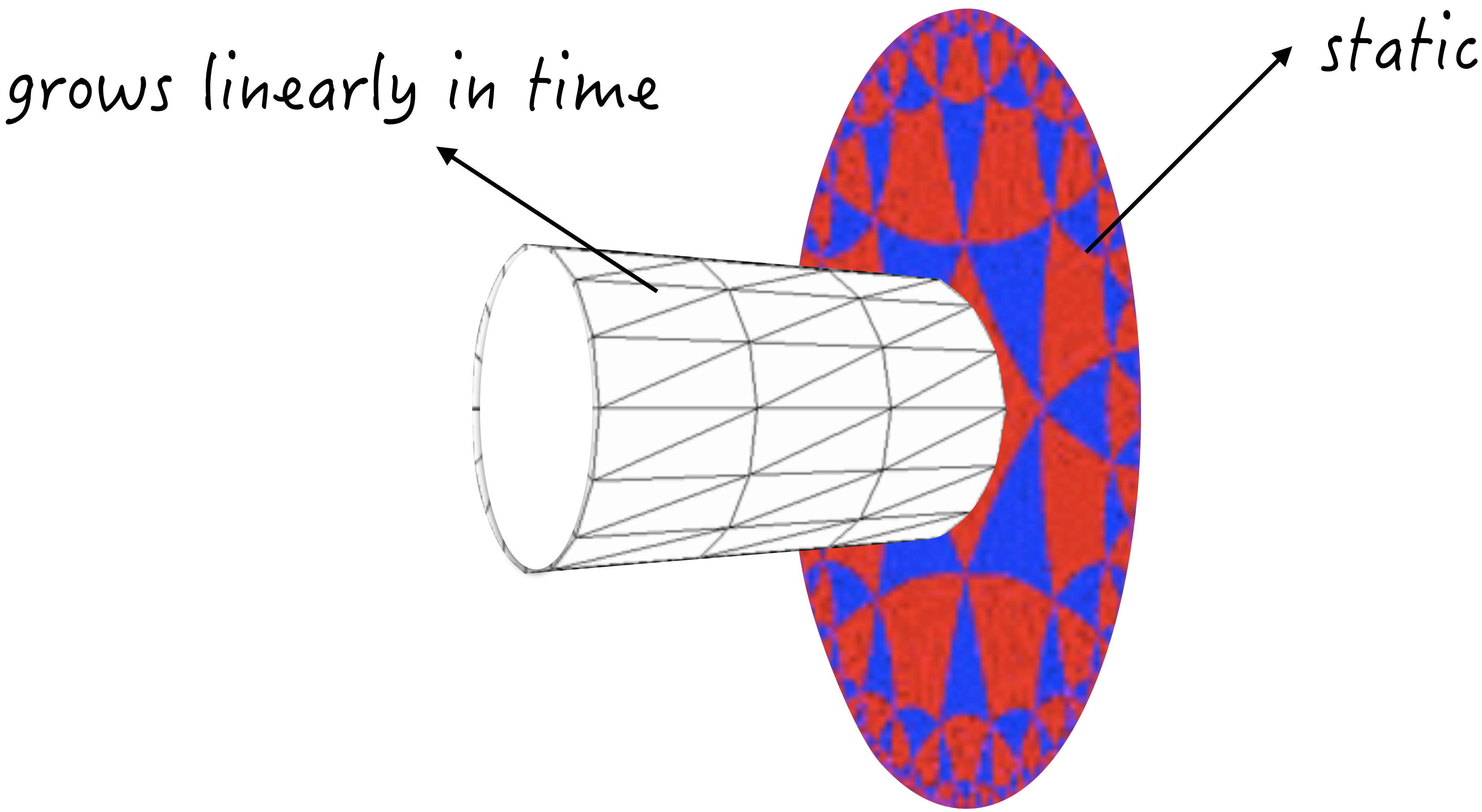
Black hole singularities are "repulsive" w.r.t. C/V duality

Brown, Roberts,
Susskind, Swingle
& Zhao



$$C(t \to \infty) \sim S\,T\,t$$

Despite the codimension-1 surface not "probing" directly the singularity, it assigns linearly growing complexity to a tensor network state in its "causal domain"



However, if we could somehow take Σ to the singularity, the CV ansatz would assign small complexity ...

In this talk I will discuss some attempts at "forcing" the maximal codim-1 surface to "directly probe" the singularity

If the maximal surface does not go to the singularity, we try to bring the singularity to the maximal surface

Since Σ is defined by its UV boundary condition, it should be enough to engineer singularities which are visible in the UV of the CFT, i.e. AdS "cosmological" singularities

This is usually done by "driving" the CFT to the singularity

$$H_{\rm CFT} \to H_{\rm CFT} + J(t)\,\mathcal{O}$$

$J(t)$ singular as $t \to t_\star$

with $\mathcal{O}$ marginal or relevant, and chosen so that we can approximate the bulk dynamics

Simple case: a component of the e.m. tensor (marginal)

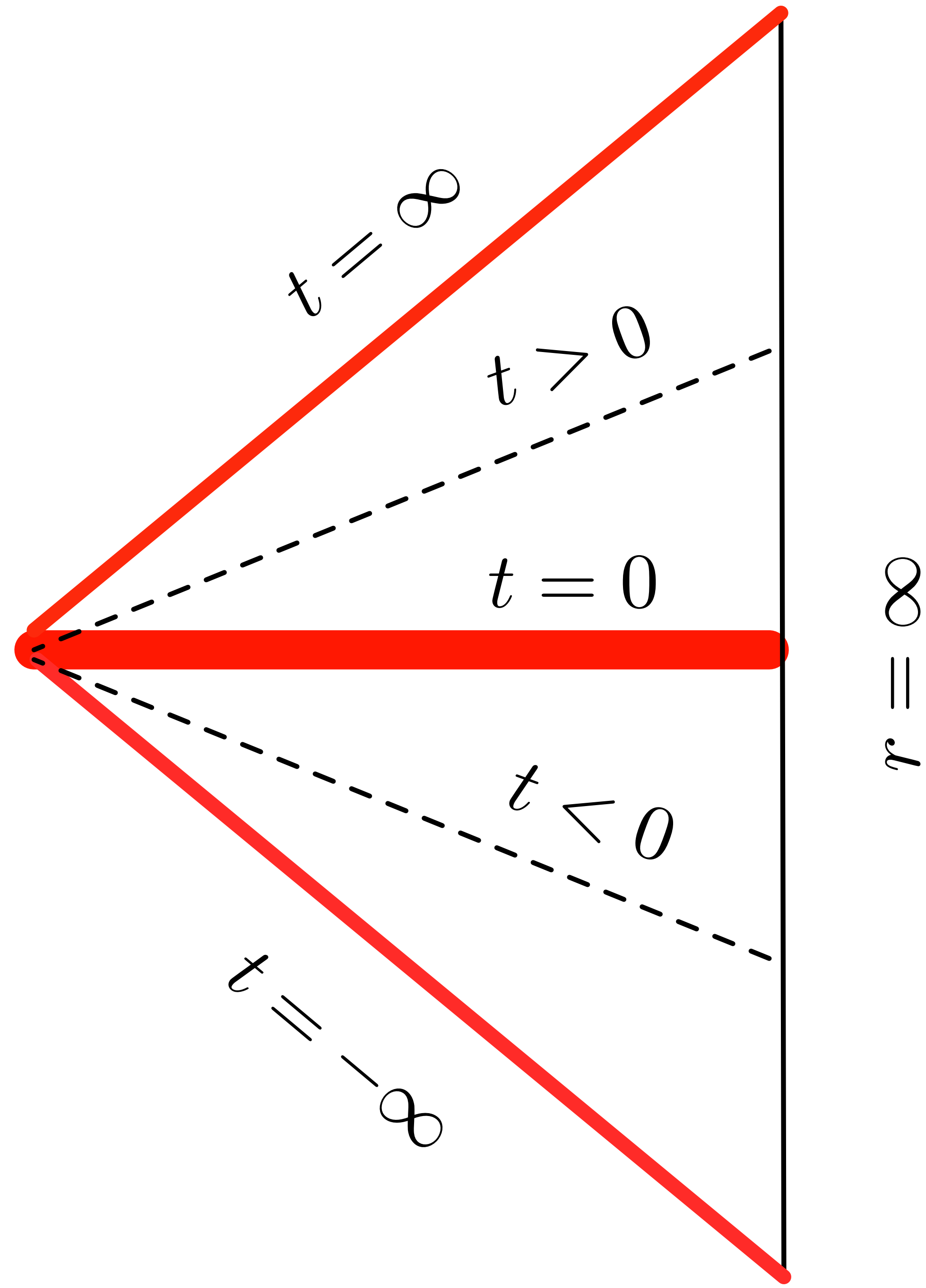
namely, the CFT is put on a singular frame

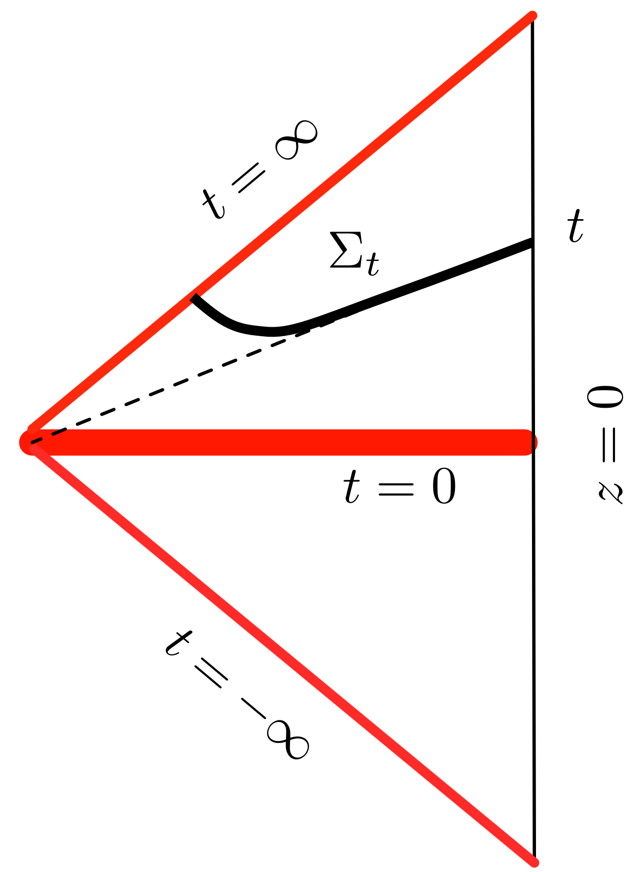
A case with easy holographic dual is Kasner

$$ds_K^2 = -dt^2 + \sum_j t^{2p_j} dx_j^2 \quad \xrightarrow{\text{Ricci-flat bdry}} \quad ds^2_{\rm bulk} = \frac{dr^2}{r^2} + r^2\, ds_K^2$$

Awad, Das, Michelson, Nampuri, Narayan, Trivedi ···

Engelhardt, Hertog, Horowitz


$t = \infty$
$t > 0$
$t = 0$
$t < 0$
$t = -\infty$
$r = \infty$

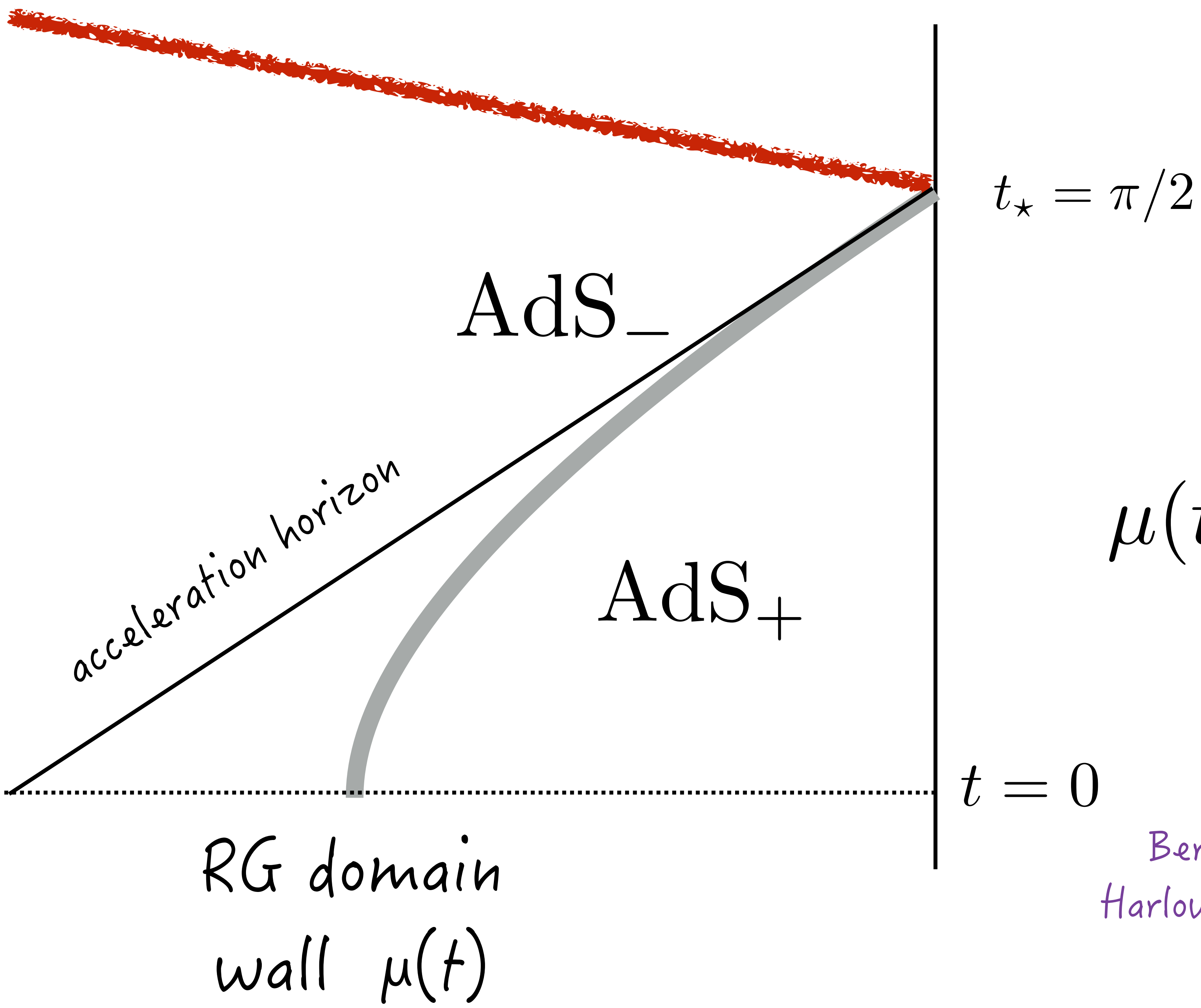


Extremal cod-1 surfaces are dominated by UV

$$C \sim N^2 \, V_{\rm CFT}(t) \, \Lambda^{d-1}$$

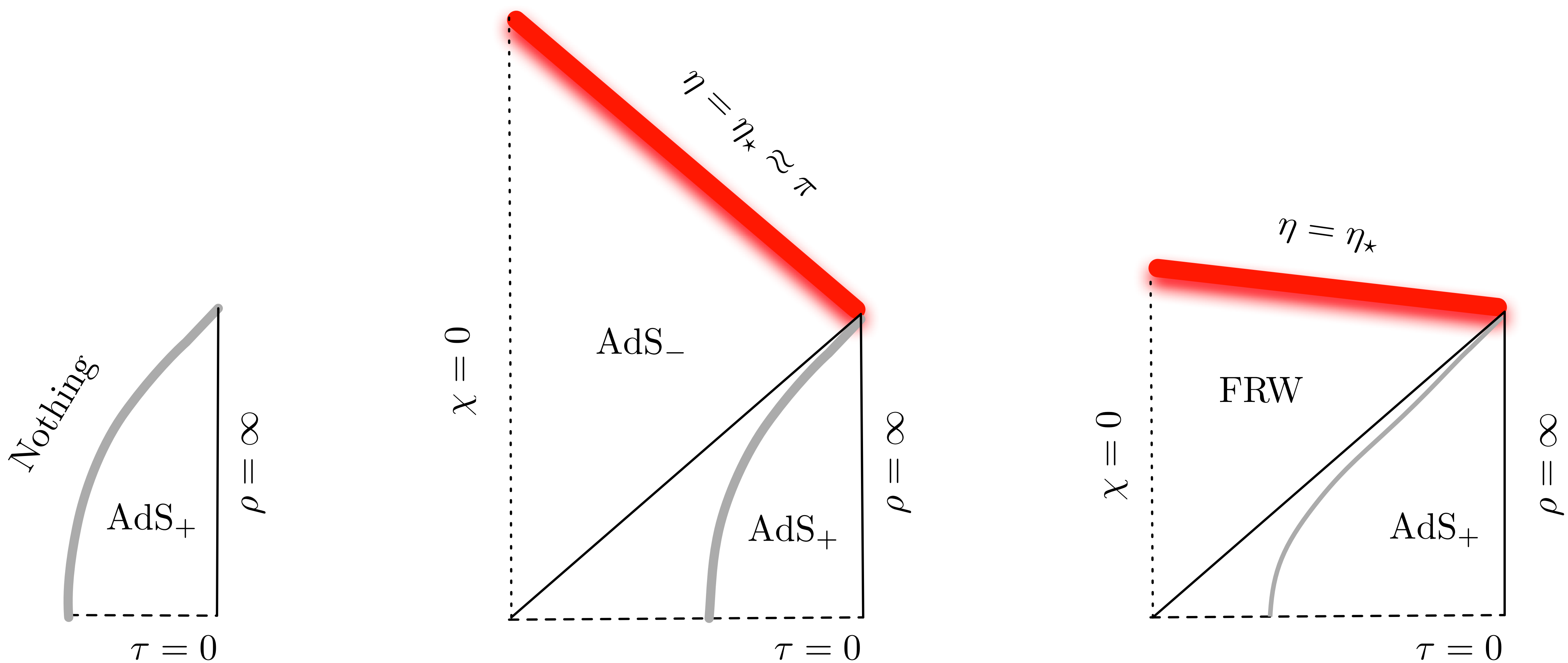
$$V_{CFT}(t) \sim t^{\sum_j p_j} = t$$

Another set of examples use an accelerated brane in finite-time collision course with the AdS boundary

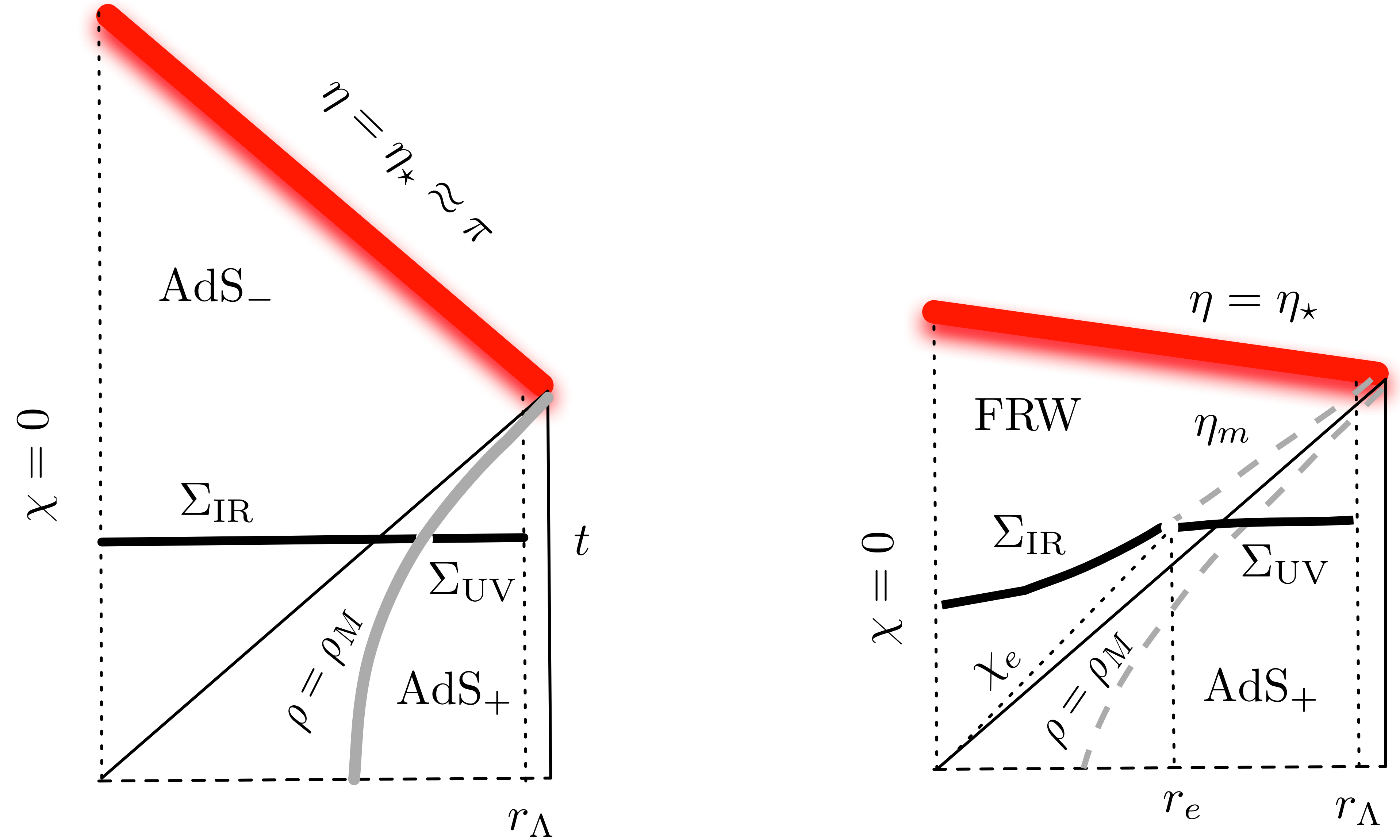


$$\mu(t) \sim \frac{1}{\cos(t)}$$

Hertog, Horowitz,
Bernamonti, Craps, Maldacena,
Harlow, Susskind, Barbon, Rabinovici
...



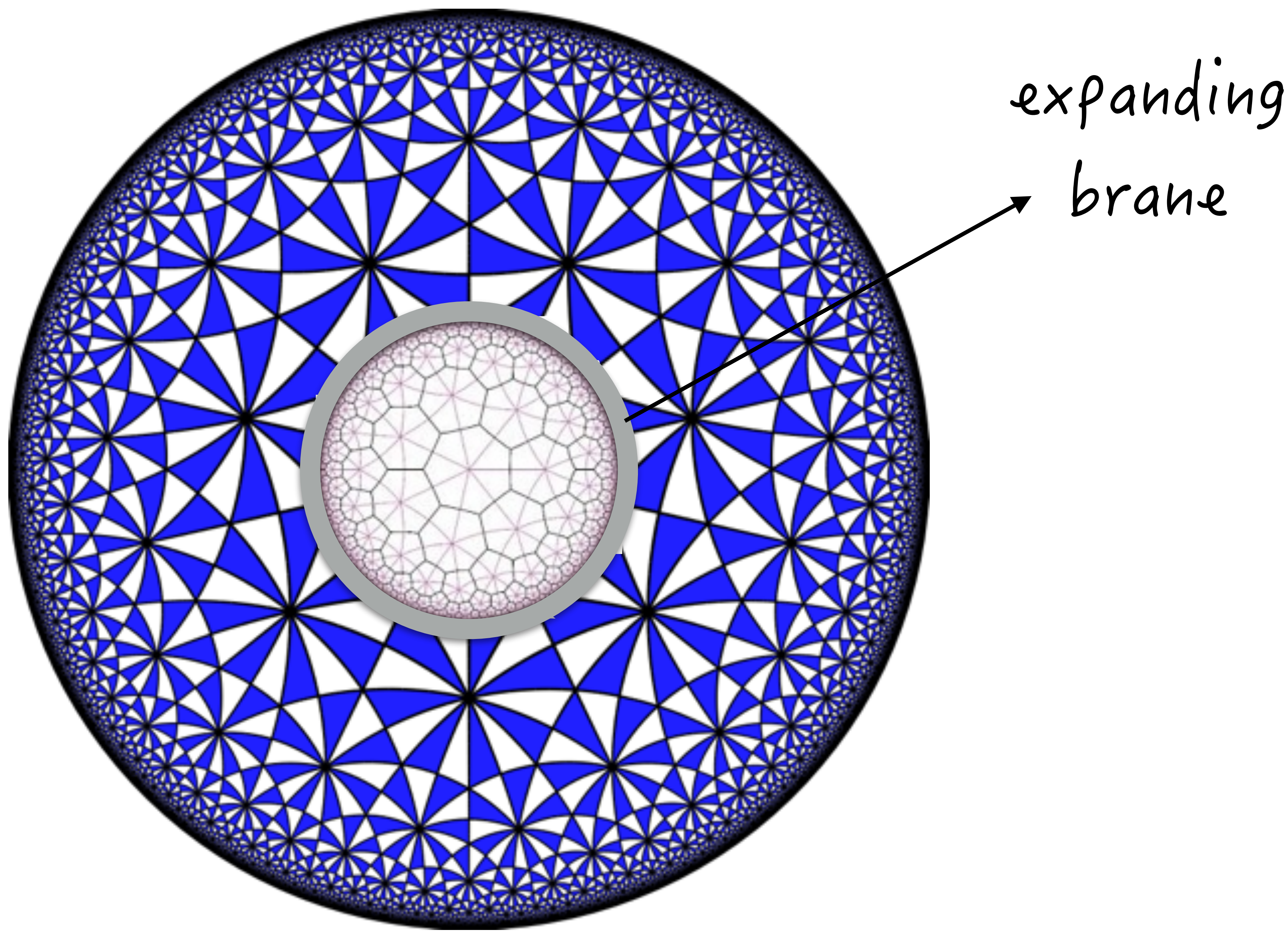
Nothing
AdS$_+$
$\rho = \infty$
$\tau = 0$
$\eta = \eta_\star \approx \pi$
$\chi = 0$
AdS$_-$
AdS$_+$
$\rho = \infty$
$\tau = 0$
$\eta = \eta_\star$
FRW
$\chi = 0$
AdS$_+$
$\rho = \infty$
$\tau = 0$



$$\frac{dC_{\rm UV}}{dt} + \frac{dC_{\rm IR}}{dt} < 0$$

Σ_{IR} grows at the expense of Σ_{UV}, but its induced metric has smaller radius of curvature

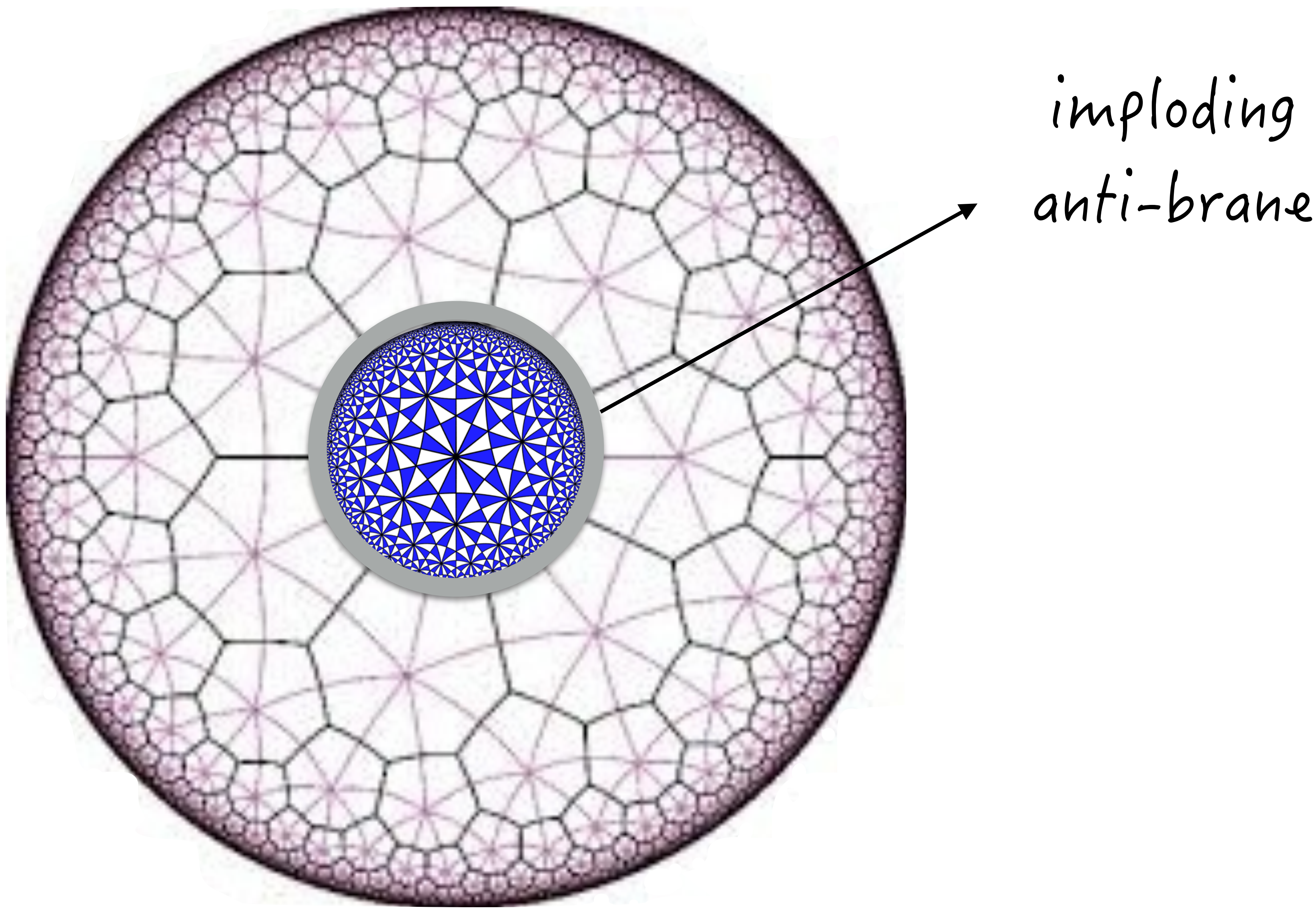
This time, the TN picture is that of nested HTNs ⋯



with time-dependent interface

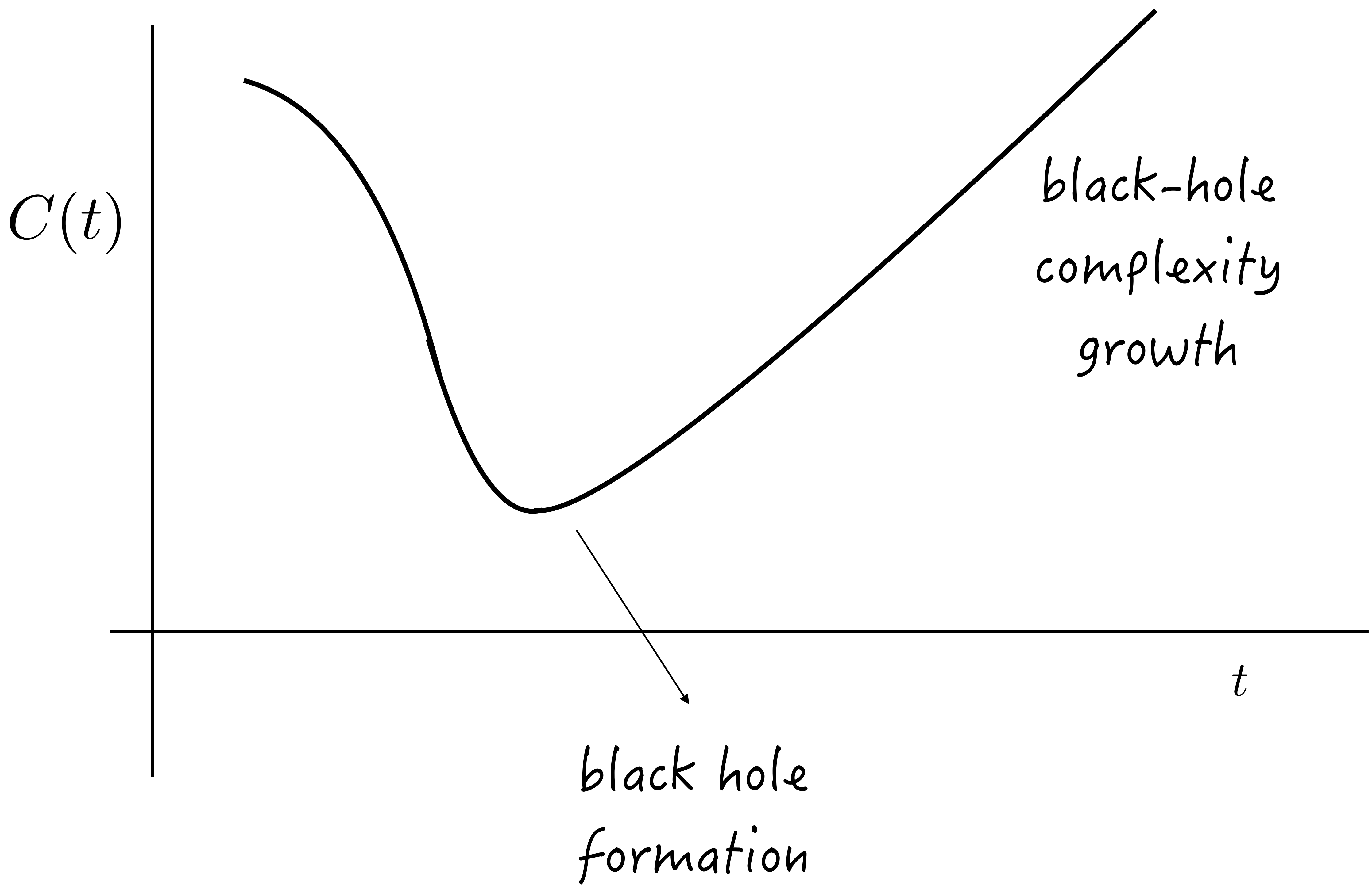
Define the appropriate tensors at the interface

What about the inverted case?



$$E \sim N\, V_{\mathrm{CFT}}\, \mu_{\max}^{d}$$

Complexity of antibrane imploding state



CONCLUSIONS

- Complexity of singularities is an important future question
- Some singularities engineered in AdS get a tentative complexity classification via C/V duality
- Deep IR singularities get growing complexity
- UV singularities get shrinking complexity
- Simple models for TN real-time evolution with holographic interpretation