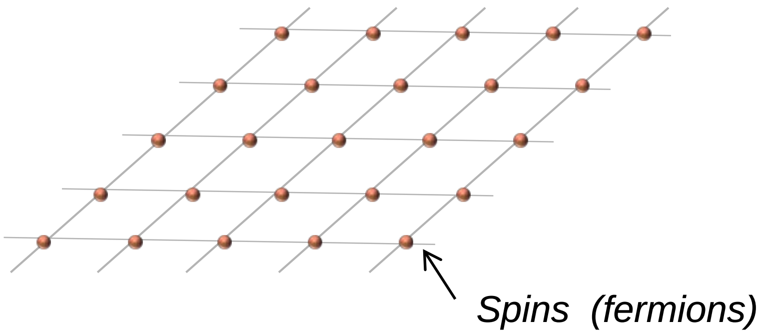
Bulk-Boundary Correspondence Tensor Networks

Max-Planck-Institut für Quantenoptik

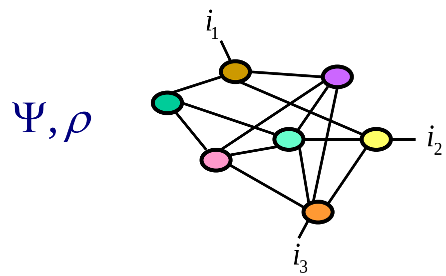
Conference on „Quantum Matter, Space-Time, and Quantum Information“
Yukawa Institute for Theoretical Physics,
University of Kyoto, June 14th, 2016

TENSOR NETWORKS

MANY-BODY QUANTUM SYSTEM



TENSOR NETWORK STATES

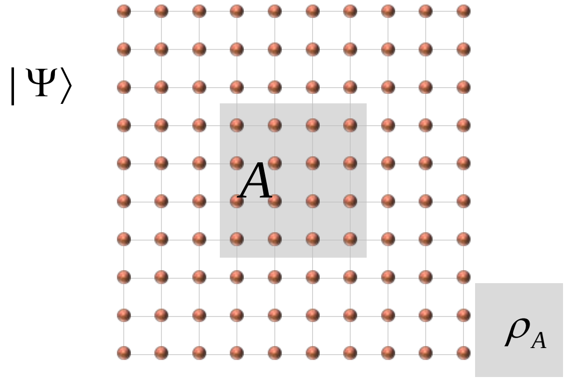


- Efficient description guided by entanglement
- Represent wide range of physical behavior
- Algorithms

OUTLINE

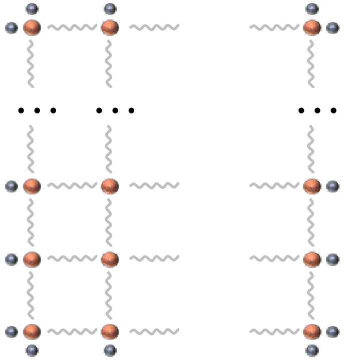
- Bulk-boundary correspondence in PEPS

JIC, Poilblanc, Schuch, and Verstraete, PRB 83, 245134 (2011)
JIC, D. Perez-Garcia, N. Schuch, F. Verstraete, arxiv:1606.00608



- Edge theories in PEPS

Yang, Lehman, Poilblanc, Acoley, Verstraete, JIC, Schuch, PRL **112**, 036402 (2014)

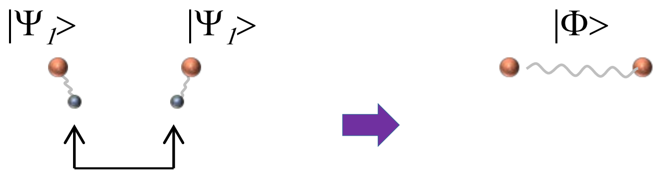


PROJECTED ENTANGLED-PAIR STATES (PEPS)

PEPS

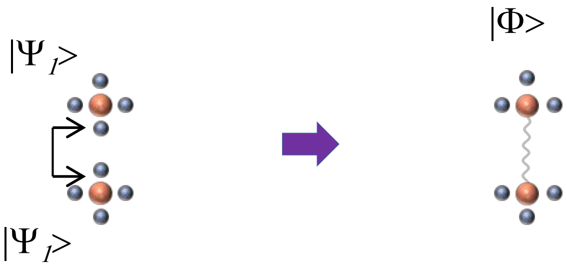
- Entanglement swapping

a)



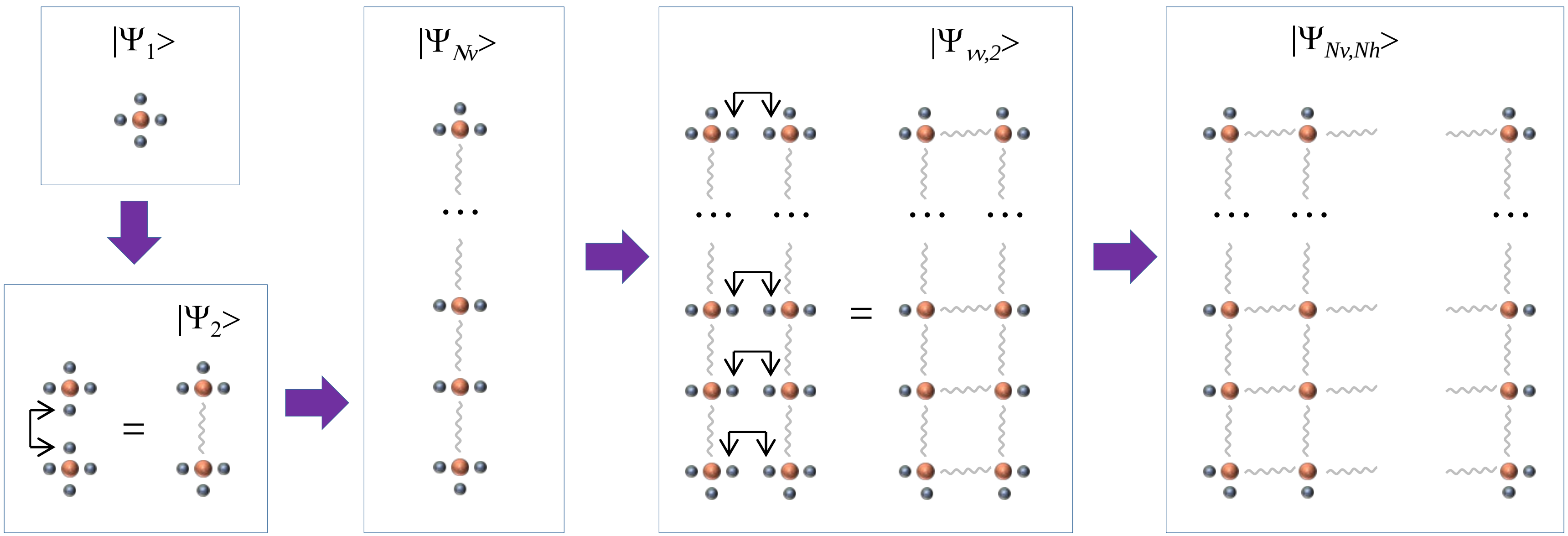
$$|\phi\rangle = \sum_{n=1}^{D} |n,n\rangle$$

b)



PEPS

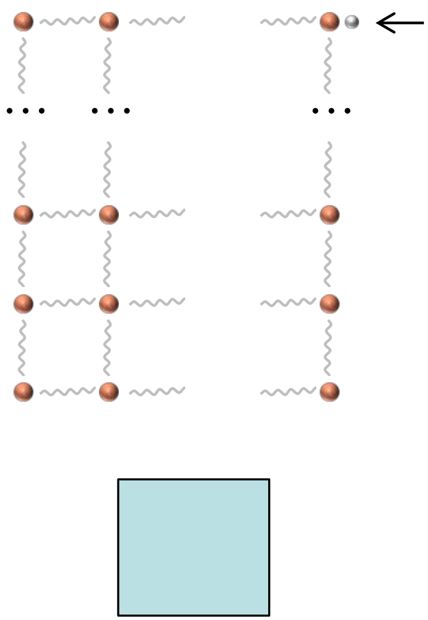
Verstraete, JIC (2004)



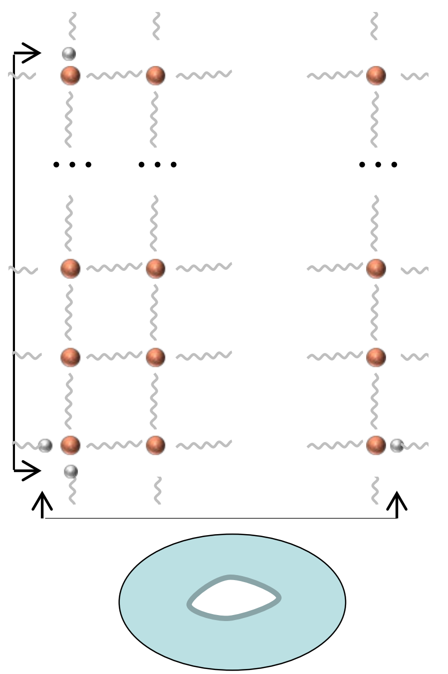
PEPS

Verstraete, JIC (2004)

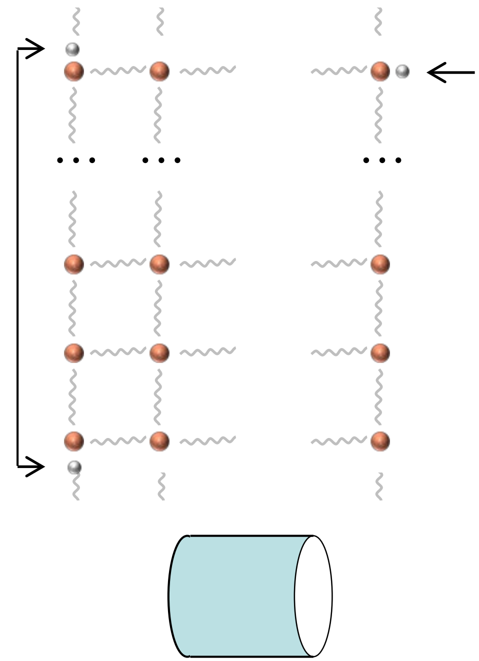
PLANE



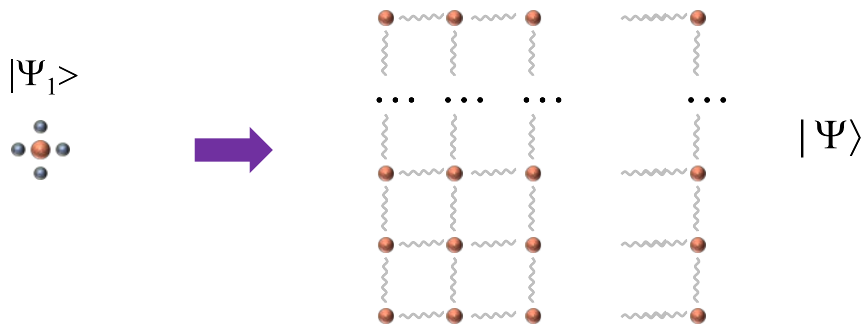
TORUS



CYLINDER



PEPS



- Other geometries and topologies

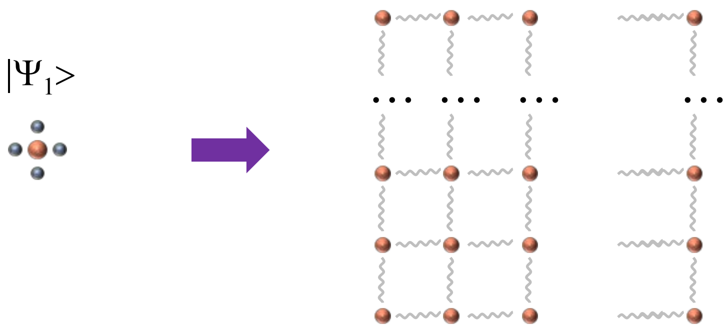
- Provide efficient descriptions for local theories

  Molnar, Schuch, Verstraete, JIC (2015)

PEPS

Verstraete, JIC (2004)

$|\Psi_1>$



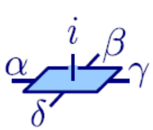
$|\Psi\rangle$

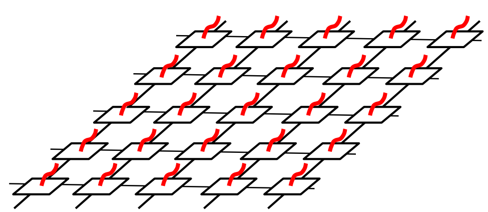
PEPS as a tensor network:

$$|\Psi_1\rangle = \sum A^i_{\alpha\beta\gamma\delta}\,|i;\alpha,\beta,\chi,\delta\rangle$$

Tensor network

$A^i_{\alpha\beta\gamma\delta}$

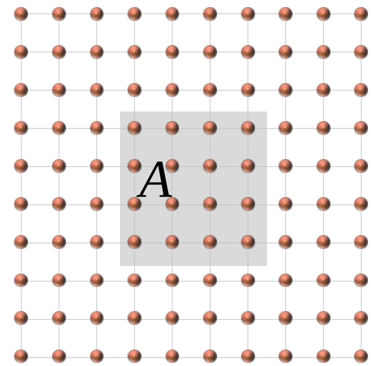




Easy to handle

PEPS
AREA LAW

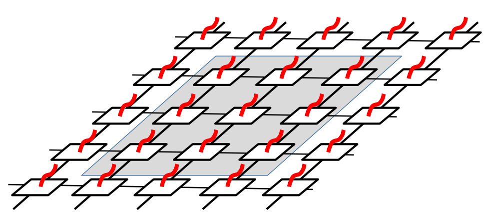
- Area law:

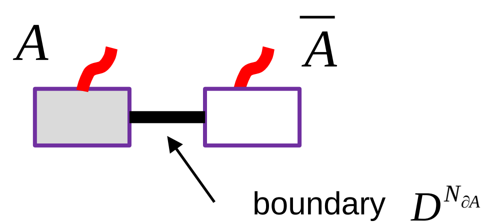


$$\rho_A = \mathrm{tr}\left[|\Psi\rangle\langle\Psi|\right]$$

$$S(\rho_A) \prec N_{\partial A}$$

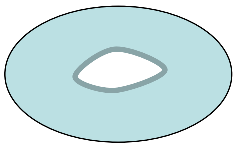
- All PEPS fulfilll area law: $S(\rho_A) \prec N_{\partial A} \log D$

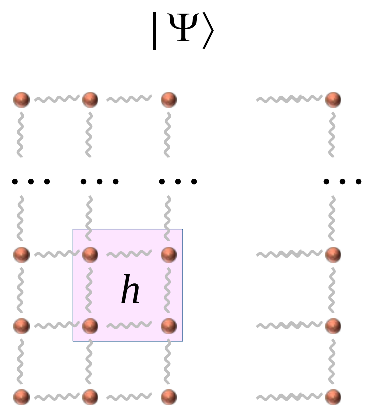


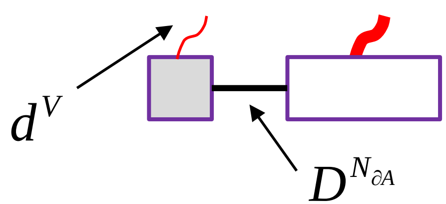


PEPS
PARENT HAMILTONIANS

- Torus:







PARENT HAMILTONIAN:

$$H = \sum_h h_n \geq 0$$

- Local
- Frustration-free:

$$h_n |\Psi\rangle = 0$$

- Ground state:

$$H |\Psi\rangle = 0$$

- Gapped

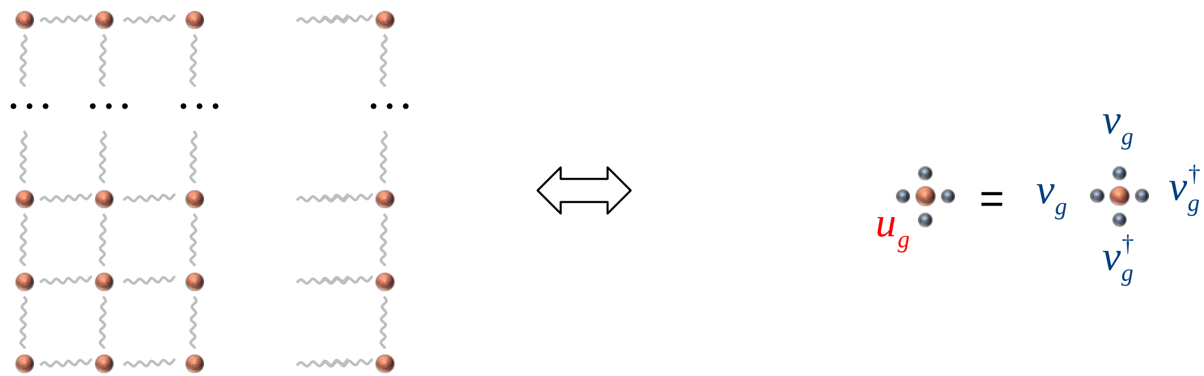
SYMMETRIES

TOPOLOGY

SYMMETRIES

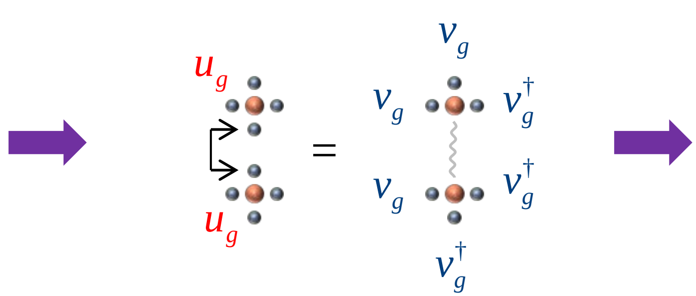
GLOBAL

Wolf, Perez-Garcia, Sanz, Verstraete, JIC (2008)
Perez-Garcia, Wolf, Gonzalez, JIC (2010)
Singh, Vidal (2012)



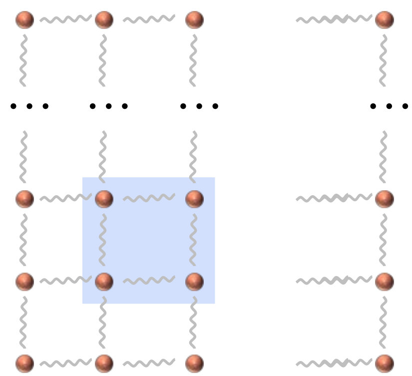
$$u_g^{\otimes N} | \Psi\rangle = | \Psi\rangle$$

$$g \in G$$



SYMMETRIES
LOCAL GAUGE

Tagliacozzo, Celi, Lewenstein (2014)
Haegeman, van Acoley, Schuch, JIC, Verstraete (2015)
Zohar, Walh, Burrello (2015)



$\Longleftrightarrow$

$|\Psi_1>$

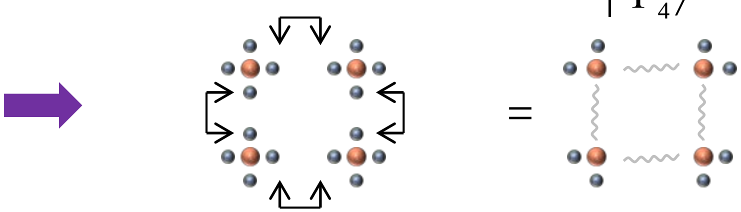
$$u_g \,|\Psi_1\rangle = v_g \otimes 1 \otimes v_g^\dagger \otimes 1 |\Psi_1\rangle = v_g \otimes 1 \otimes 1 \otimes v_g^\dagger \,|\Psi_1\rangle =$$

$$g \in G$$

$$u_g^{\otimes 4} \,|\Psi\rangle = |\Psi\rangle$$

$$g \in G$$

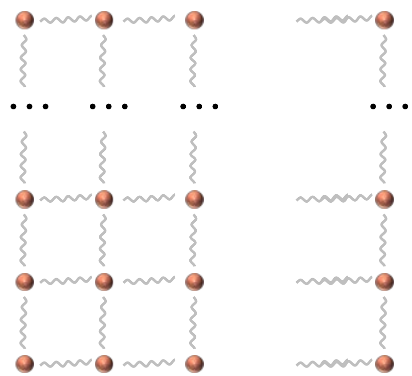
$|\Psi_4\rangle$



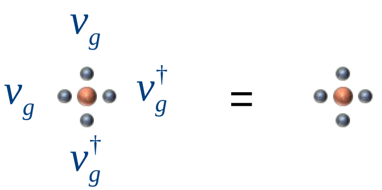
$$u_g^{\otimes 4} \,|\Psi_4\rangle = v_g v_g^\dagger \otimes \ldots \otimes \ldots |\Psi_4\rangle = |\Psi_4\rangle$$

TOPOLOGY

Schuch, JIC, Perez-Garcia (2010)
Zauner et al (2014)



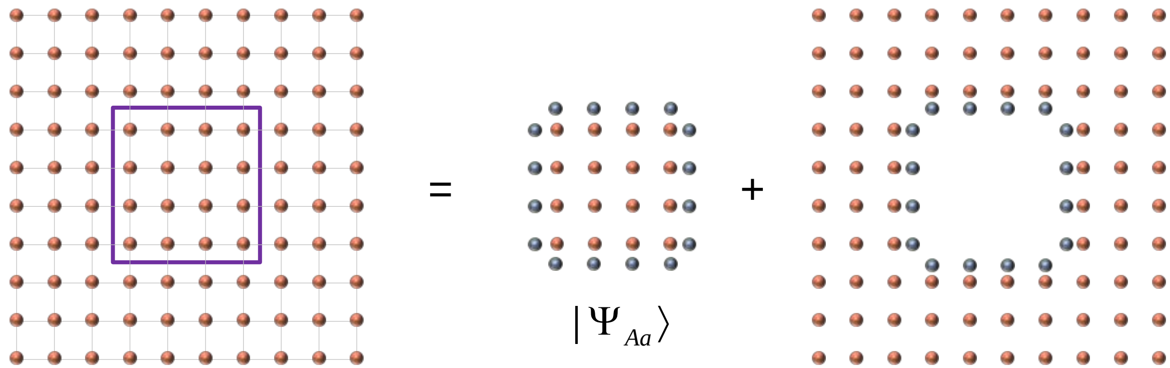
$\Longleftrightarrow$



$g \in G$

TOPOLOGY
STRINGS

Schuch, JIC, Perez-Garcia (2010)
Zauner et al (2014)

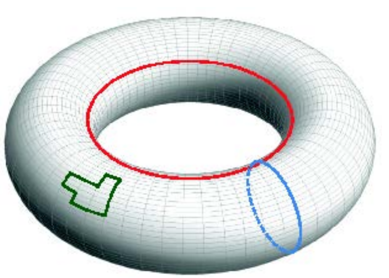


$|\Psi_{Aa}\rangle$ $|\Psi_{Bb}\rangle$

$|\Psi_{\odot}\rangle$

$$|\Psi_{\odot}\rangle = \langle \phi_{ab} | S | \Psi_{Aa}\rangle | \Psi_{Bb}\rangle$$

$$S = v_g^{\otimes C}$$



TOPOLOGY

STRINGS

Schuch, JIC, Perez-Garcia (2010)
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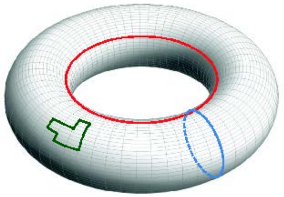
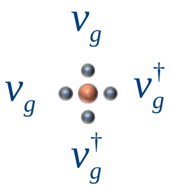
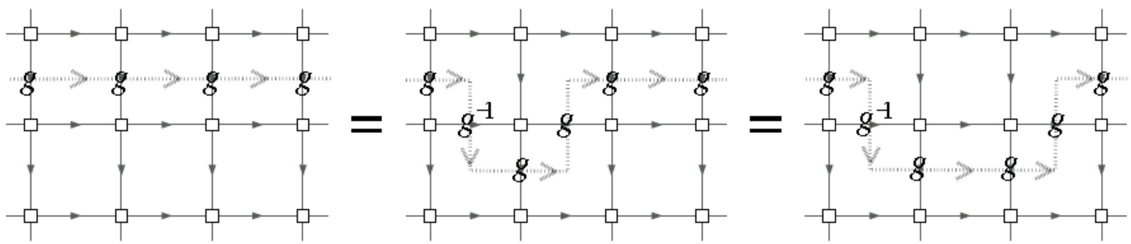


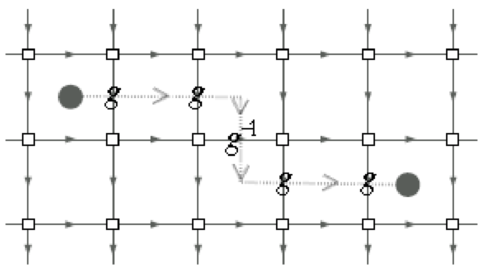


- Closed strings: Ground state



  - Ground state degeneracy
  - Locally indistinguishability

- Open strings: Excitations



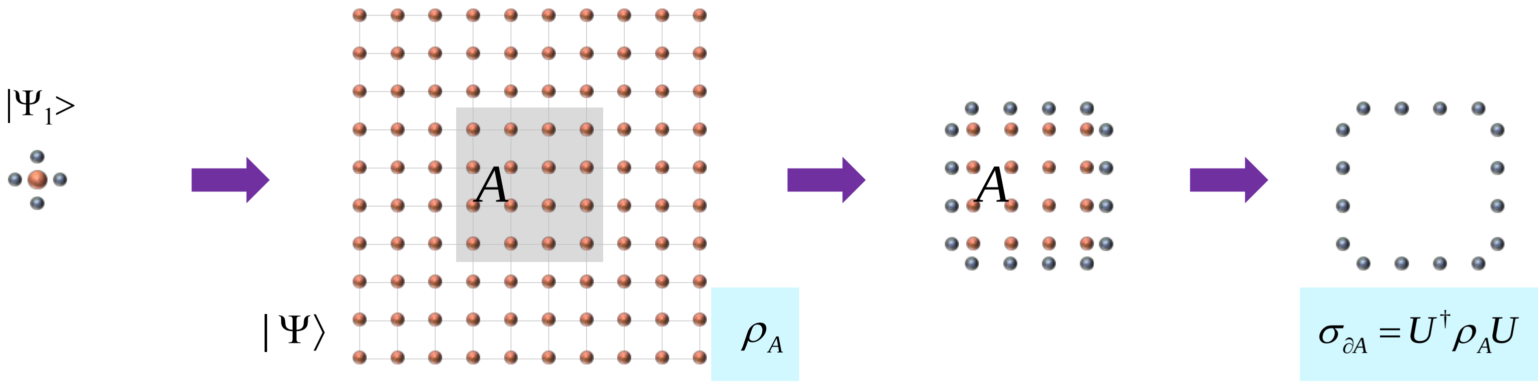
  - Braiding
  - Quantum computation

BULK-BOUNDARY CORRESPONDENCE

JIC, Poilblanc, Schuch, and Verstraete, PRB 83, 245134 (2011)
JIC, D. Perez-Garcia, N. Schuch, F. Verstraete, arxiv:1606.00608

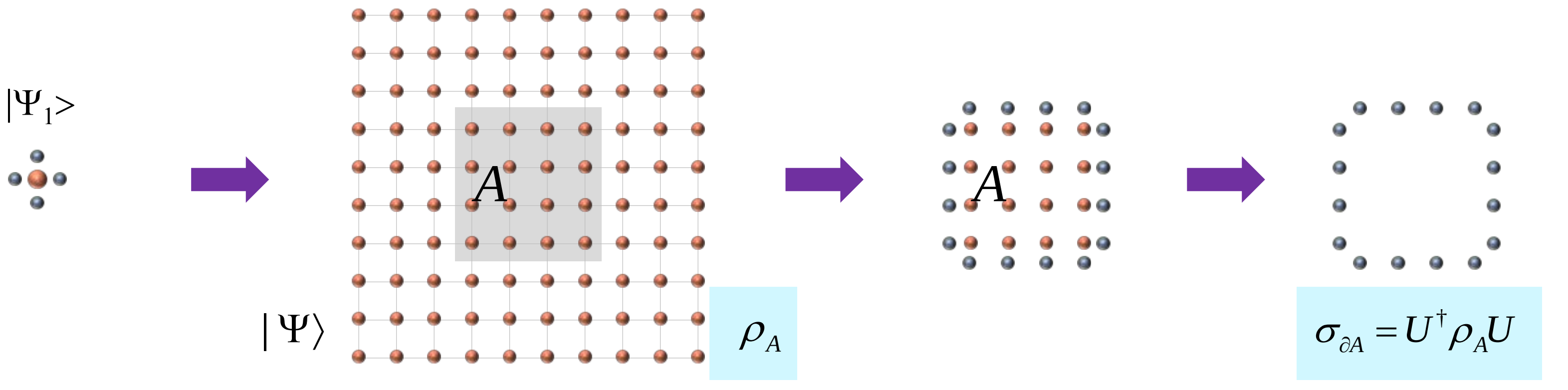
BULK-BOUNDARY CORRESPONDENCE

Poilblanc, Schuch, Verstreate, JIC (2011)

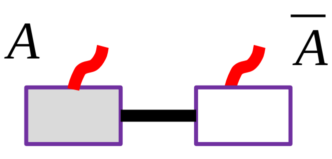


BULK-BOUNDARY CORRESPONDENCE

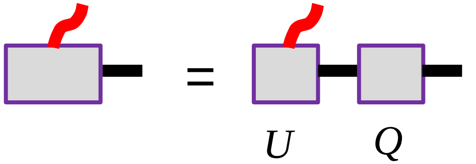
Poilblanc, Schuch, Verstreate, JIC (2011)



PROOF

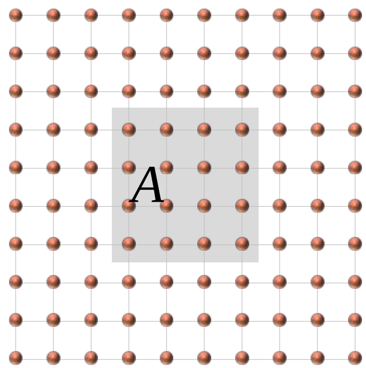


Polar decomposition: $M = UQ$



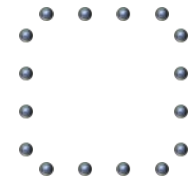
BULK-BOUNDARY CORRESPONDENCE
SYMMETRIES & TOPOLOGY

$|\Psi\rangle$



ρ_A

$\Leftrightarrow$



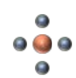
$\sigma_{\partial A} = U^{\dagger}\rho_A U$

- Symmetries:

$$u_g^{\otimes N}|\Psi\rangle = |\Psi\rangle$$
$$g \in G$$

$\Rightarrow$ $\sigma_{\partial A} = v_g^{\otimes N_{\partial A}\dagger}\sigma_{\partial A} v_g^{\otimes N_{\partial A}}$

- Topology:

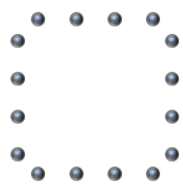


$\Rightarrow$ $\sigma_{\partial A} = v_g^{\otimes N_{\partial A}\dagger}\sigma_{\partial A} = \sigma_{\partial A} v_g^{\otimes N_{\partial A}}$

Restricts the subspace: topological correction to the area law

BULK-BOUNDARY CORRESPONDENCE
FIXED POINTS RENORMALIZATION

Cirac, Perez-Garcia, Schuch, Vestraete (2016)



$\sigma_{\partial A}$

Corse-grain

local commuting

$$\sigma_{\partial A} = e^{-H}\mathbf{P} = \mathbf{P}e^{-H}$$

Non-local projector
(topology)

RENORMALIZATION
FIXED POINTS

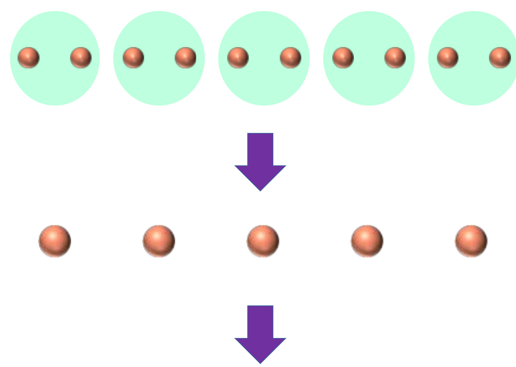
Verstraete, Latorre, Rico, Wolf, JIC (2004)

- Equivalence relation: $|\Psi\rangle \approx |\Phi\rangle$

  Two states are equivalent if they are connected by a local transformation

  $|\Psi\rangle$ $|\Phi\rangle$

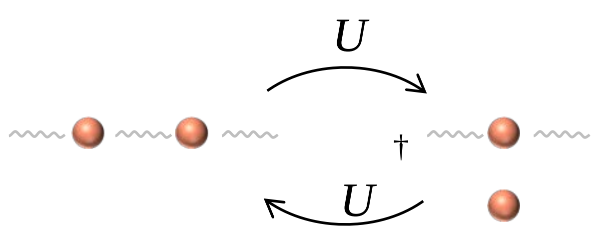
- Renormalization: Mapping among equivalence classes



FIXED POINTS RENORMALIZATION

Verstraete, Latorre, Rico, Wolf, JIC (2004)
Vidal (2005)

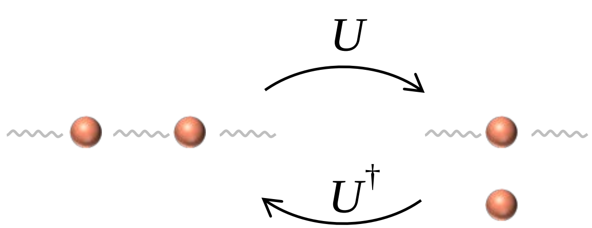
- Fixed points:
  - Classes: connected by a (local) unitary
  - Can be locally disentangled



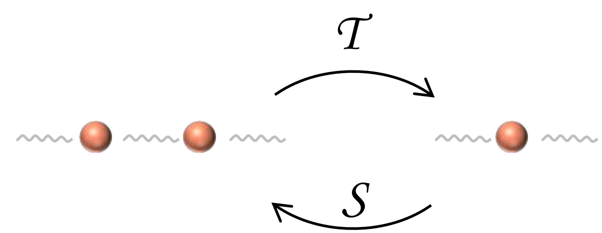
FIXED POINTS RENORMALIZATION

Cirac, Perez-Garcia, Schuch, Vestraete (2016)

PURE STATES



MIXED STATES

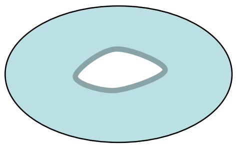


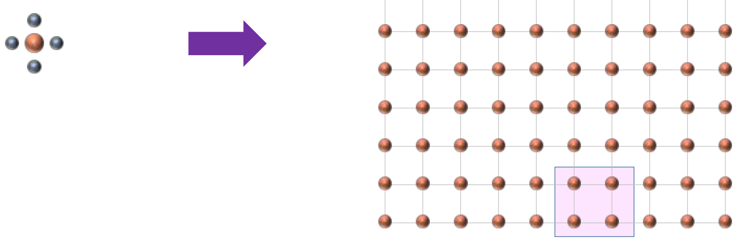
- Zero correlation length
- Saturate the area law for mutual information

EDGE THEORIES

Yang, Lehman, Poilblanc, Acoley, Verstraete, JIC, Schuch, PRL **112**, 036402 (2014)

- Torus:





PARENT HAMILTONIAN:

$$H = \sum_h h_n \geq 0$$

- Local
- Frustration-free:

$$h_n \,|\Psi\rangle = 0$$

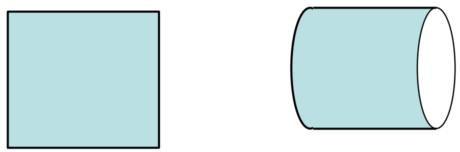
- Ground state:

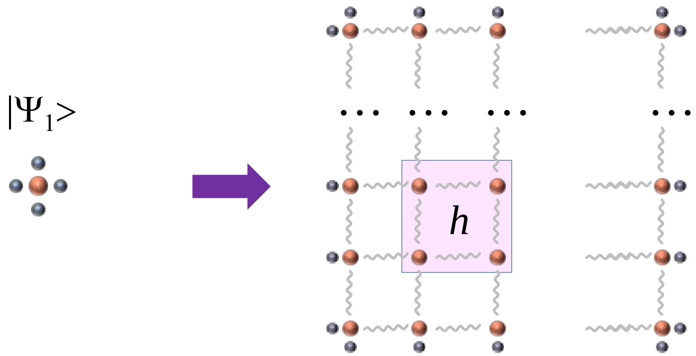
$$H\,|\Psi\rangle = 0$$

- Spectrum:

EDGE THEORY

- Spin system with a physical boundary





PARENT HAMILTONIAN:

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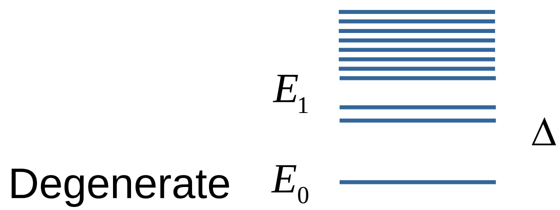
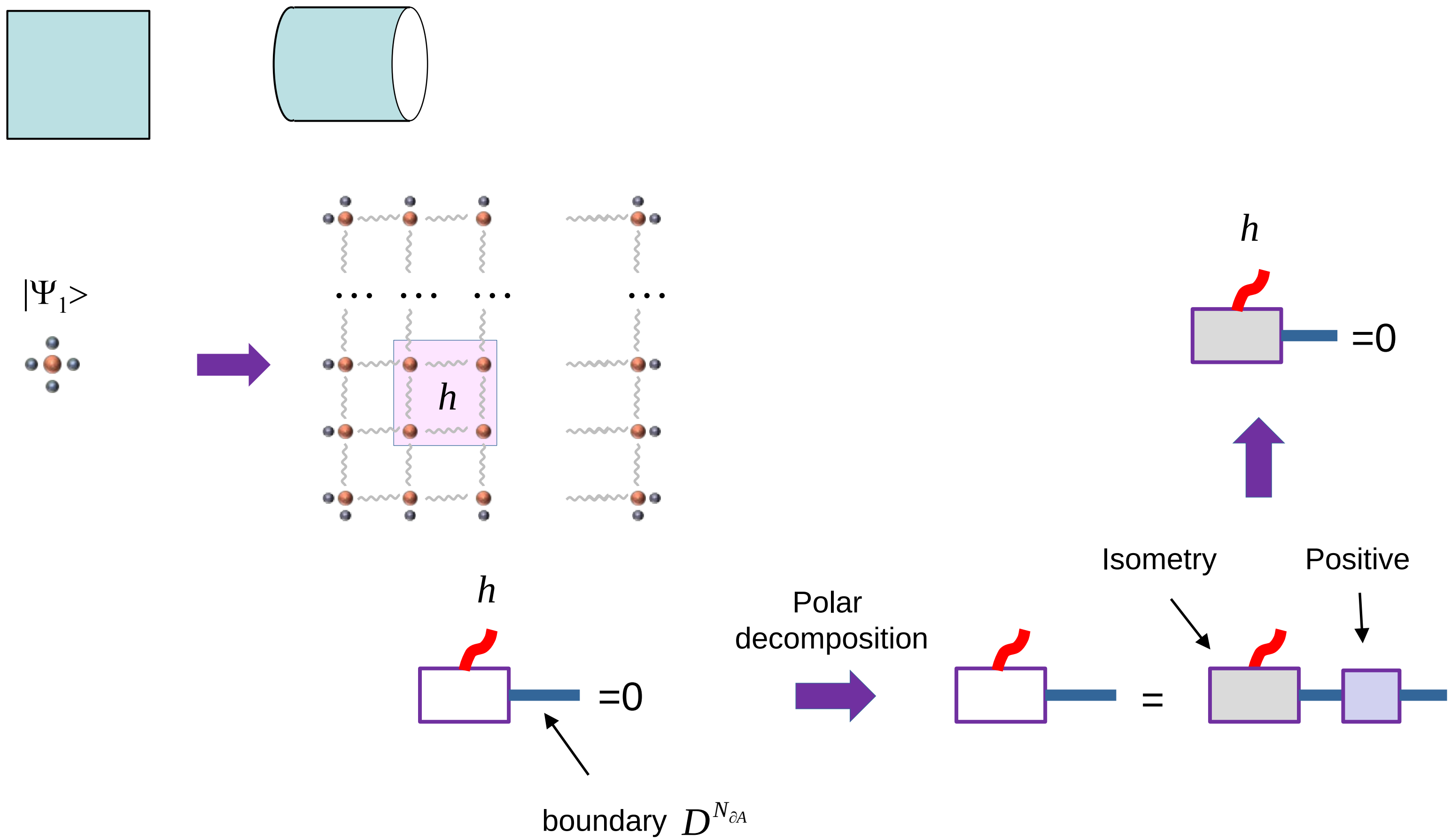
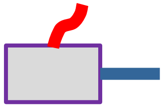


EDGE THEORY

- Spin system with a physical boundary



EDGE THEORY
LOW ENERGY EXCITATIONS

- Isometry:  =0

  - Maps bulk-boundary (cf AdS/CFT)

    Pastawski, Yoshida, Harlow, Preskill (2015)
    Hayden, Nezami, Qi, Thomas, Walther, Yang (2016)

  - Characterizes the GS subspace of *H*, $H|\Psi\rangle = 0$

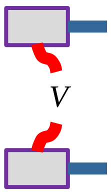
EDGE THEORY
LOW ENERGY EXCITATIONS

- Local perturbation in the bulk: $H' = H + \varepsilon V = H + \varepsilon \sum_{h} v_n$

  - Degenerate perturbation theory

$$H_{edge} = \varepsilon \mathbf{P}_0 \sum_n v_n \mathbf{P}_0 = \sum_{\vec{\alpha},\vec{\beta}} h_{\vec{\alpha},\vec{\beta}} \,|\Psi_{\vec{\alpha}}\rangle\langle\Psi_{\beta}|$$

$$\sum_{\vec{\alpha},\vec{\beta}} h_{\vec{\alpha},\vec{\beta}} \,|\alpha\rangle\langle\beta| = H_{\text{boundary}} =$$



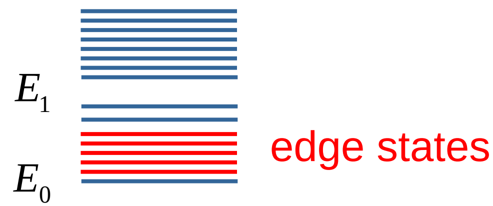
  - It can be interpreted as a Hamiltonian acting on the boundary

The isometry maps the bulk operator onto the boundary

EDGE THEORY

LOW ENERGY EXCITATIONS

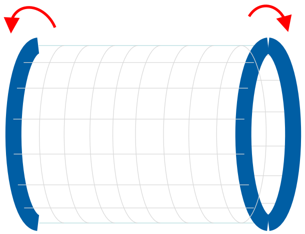
- Local perturbation in the bulk: $H' = H + \varepsilon V = H + \varepsilon \sum_{h} v_n$
  - Spectrum



- Global and gauge symmetries are preserved
- Topology gives rise to a local projector: superselection rule at the boundary

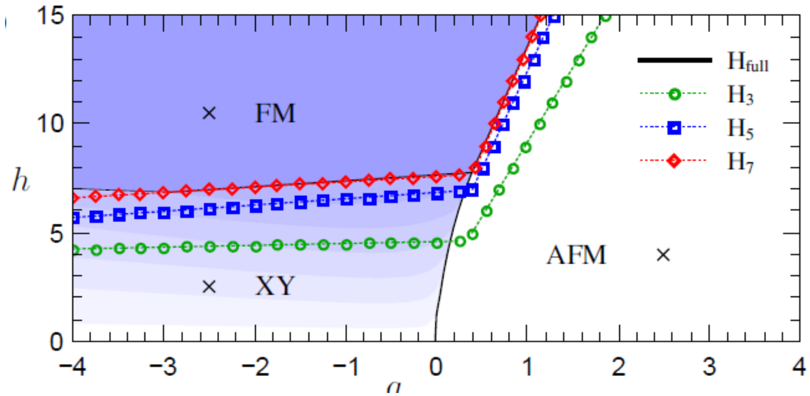
EDGE THEORY
MODEL

- AKLT model + bulk perturbations



$$V = \sum_{\langle ij \rangle} \left[J\, S_i \cdot S_j + g S_i^z S_j^z \right] + h \sum_i S_i^z$$

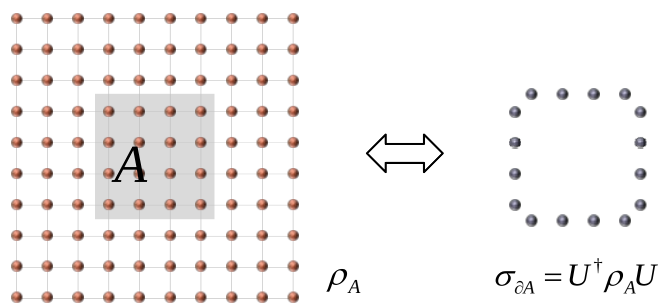
EDGE THEORY



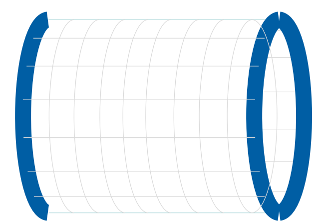
CONCLUSIONS & OUTLOOK

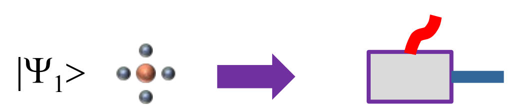
- PEPS offer efficient descriptions of many-body quantum systems
- Enconde symmetries and topology in a simple way

BULK-BOUNDARY



EDGES





- Isometry maps bulk operators onto the boundary
- Symmetries (global & gauge) are inherited
- Topology appear as a superselection rule (anomaly)

MPQ