Bulk-Boundary Correspondence
Tensor Networks

Conference on „Quantum Matter, Space-Time, and Quantum Information“
Yukawa Institute for Theoretical Physics,
University of Kyoto, June 14th, 2016
TENSOR NETWORKS

MANY-BODY QUANTUM SYSTEM

Spins (fermions)

TENSOR NETWORK STATES

\[ \Psi, \rho \]

- Efficient description guided by entanglement
- Represent wide range of physical behavior
- Algorithms
**OUTLINE**

- **Bulk-boundary correspondence in PEPS**
  
  JIC, Poilblanc, Schuch, and Verstraete, PRB 83, 245134 (2011)
  
  JIC, D. Perez-Garcia, N. Schuch, F. Verstraete, arxiv:1606.00608

  ![Bulk-boundary correspondence in PEPS](image)

- **Edge theories in PEPS**
  
  Yang, Lehman, Poilblanc, Acoley, Verstraete, JIC, Schuch, PRL 112, 036402 (2014)

  ![Edge theories in PEPS](image)
PROJECTED ENTANGLED-PAIR STATES (PEPS)
PEPS

- Entanglement swapping

\[ |\phi\rangle = \sum_{n=1}^{D} |n, n\rangle \]

\[ |\Psi_1\rangle \quad |\Psi_1\rangle \quad \rightarrow \quad |\Phi\rangle \]

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PEPS

Verstraete, JIC (2004)
PEPS

Verstraete, JIC (2004)
Provide efficient descriptions for local theories

Molnar, Schuch, Verstraete, JIC (2015)

- Other geometries and topologies
PEPS

Verstraete, JIC (2004)

|Ψ₁⟩

PEPS as a tensor network:

|Ψ₁⟩ = \sum A^i_{αβγδ} |i; α, β, χ, δ⟩

Tensor network

Easy to handle
Area law:

\[ \rho_A = \text{tr} [ \Psi \langle \Psi | ] \]

\[ S(\rho_A) < N_{\partial A} \]

All PEPS fulfill area law: \[ S(\rho_A) < N_{\partial A} \log D \]
Torus:

PARENT HAMILTONIANS:

**PARENT HAMILTONIAN:**

\[ H = \sum_{h} h_n \geq 0 \]

- Local
- Frustration-free:
  \[ h_n \left| \Psi \right\rangle = 0 \]
- Ground state:
  \[ H \left| \Psi \right\rangle = 0 \]
- Gapped
SYMMETRIES
TOPOLOGY
SYMMETRIES
GLOBAL

Wolf, Perez-Garcia, Sanz, Verstraete, JIC (2008)
Singh, Vidal (2012)

\[ u_g^{\otimes N} |\Psi\rangle = |\Psi\rangle \]

\[ g \in G \]
SYMMETRIES
LOCAL GAUGE

\[ |\Psi_1> = g G \in \text{SYMMETRIES} \]

\[ \Psi \otimes \Psi = \Psi \otimes \Psi = \Psi \otimes \Psi \]

\[ u_g |\Psi_1> = v_g \otimes 1 \otimes v_g^\dagger \otimes 1 |\Psi_1> = v_g \otimes 1 \otimes 1 \otimes v_g^\dagger |\Psi_1> = \]

\[ g \in G \]

\[ u_g^\otimes 4 |\Psi> = |\Psi> \]

\[ u_g^\otimes 4 |\Psi_4> = v_g v_g^\dagger \otimes ... \otimes ... |\Psi_4> = |\Psi_4> \]

Tagliacozzo, Celi, Lewenstein (2014)
Haegeman, van Acoley, Schuch, JIC, Verstraete (2015)
Zohar, Walh, Burrello (2015)
TOPOLOGY

Schuch, JIC, Perez-Garcia (2010)
Zauner et al (2014)

\[ g \in G \]
Schuch, JIC, Perez-Garcia (2010)
Zauner et al (2014)

\[ |\Psi_\otimes\rangle = \langle\phi_{ab}|S|\Psi_{Aa}\rangle|\Psi_{Bb}\rangle \]

\[ S = v_g^{\otimes C} \]
TOPOLOGY
STRINGS

Closed strings: Ground state

Open strings: Excitations

Ground state degeneracy
Locally indistinguishability

Braiding
Quantum computation

Schuch, JIC, Perez-Garcia (2010)
Zauner et al (2014)
BULK-BOUNDARY CORRESPONDENCE

JIC, Poilblanc, Schuch, and Verstraete, PRB 83, 245134 (2011)
JIC, D. Perez-Garcia, N. Schuch, F. Verstraete, arxiv:1606.00608
BULK-BOUNDARY CORRESPONDENCE

Poilblanc, Schuch, Verstreate, JIC (2011)

$|\Psi_i\rangle \rightarrow \rho_A \rightarrow \sigma_{\partial A} = U^\dagger \rho_A U$
BULK-BOUNDARY CORRESPONDENCE

Poilblanc, Schuch, Verstreate, JIC (2011)

\[ |\Psi_1\rangle \rightarrow \rho_A \rightarrow A \rightarrow 0 \]

\[ \sigma_{\partial A} = U\dagger \rho_A U \]

PROOF

Polar decomposition: \( M = UQ \)
Symmetries:

$$u^\otimes N_g |\Psi\rangle = |\Psi\rangle$$

$$g \in G$$

Topology:

$$\sigma_{\partial A} = v_g^{\otimes N_{\partial A}} \sigma_{A} v_g^{\otimes N_{\partial A}}$$

Restricts the subspace: topological correction to the area law
BULK-BOUNDARY CORRESPONDENCE
FIXED POINTS RENORMALIZATION

Cirac, Perez-Garcia, Schuch, Vestraete (2016)

Corse-grain

\[ \sigma_{\partial A} = e^{-H} P = Pe^{-H} \]

local commuting

Non-local projector (topology)
**Equivalence relation:** \( |\Psi\rangle \approx |\Phi\rangle \)

Two states are equivalent if they are connected by a local transformation

**Renormalization:** Mapping among equivalence classes

Fixed points:

- Classes: connected by a (local) unitary
- Can be locally disentangled

Vidal (2005)
Zero correlation length

Saturate the area law for mutual information
EDGE THEORIES

Yang, Lehman, Poilblanc, Acoley, Verstraete, JIC, Schuch, PRL 112, 036402 (2014)
**Torus:**

\[
\ket{\Psi_1}
\]

**Parent Hamiltonian:**

\[
H = \sum_h h_n \geq 0
\]

- Local
- Frustration-free:
  \[
h_n \ket{\Psi} = 0
\]
- Ground state:
  \[
H \ket{\Psi} = 0
\]
- Spectrum:
  \[
E_1 \quad \Delta \\
E_0
\]
EDGE THEORY

- Spin system with a physical boundary

![Diagram of a spin system with a physical boundary]

PARENT HAMILTONIAN:

\[ H = \sum_{h} h_n \geq 0 \]

- Local
- Frustration-free:
  \[ h_n \left| \Psi \right> = 0 \]
- Ground state:
  \[ H \left| \Psi \right> = 0 \]
- Spectrum:
  \[ E_1 \quad \Delta \]
  Degenerate \[ E_0 \]
Spin system with a physical boundary

$|\Psi_1> \rightarrow \text{Polar decomposition} = 0$

boundary $D_{N\omega}$
Isometry: $= 0$

Maps bulk-boundary (cf AdS/CFT)
- Pastawski, Yoshida, Harlow, Preskill (2015)
- Hayden, Nezami, Qi, Thomas, Walther, Yang (2016)

Characterizes the GS subspace of $H$, $H \lvert \Psi \rangle = 0$
Local perturbation in the bulk: \( H' = H + \varepsilon V = H + \varepsilon \sum_n v_n \)

Degenerate perturbation theory

\[
H_{\text{edge}} = \varepsilon \mathbf{P}_0 \sum_n v_n \mathbf{P}_0 = \sum_{\alpha, \beta} h_{\alpha, \beta} |\Psi^\alpha \rangle \langle \Psi^\beta | \\

= \sum_{\alpha, \beta} h_{\alpha, \beta} |\alpha \rangle \langle \beta | = H_{\text{boundary}} = V
\]

It can be interpreted as a Hamiltonian acting on the boundary.

The isometry maps the bulk operator onto the boundary.
Local perturbation in the bulk: \( H' = H + \varepsilon V = H + \varepsilon \sum_h v_n \)

- Spectrum

\[ E_0 \quad \text{edge states} \]

- Global and gauge symmetries are preserved
- Topology gives rise to a local projector: superselection rule at the boundary
AKLT model + bulk perturbations

\[ V = \sum_{\langle ij \rangle} [J \mathbf{S}_i \cdot \mathbf{S}_j + g S_i^z S_j^z] + h \sum_i S_i^z \]
CONCLUSIONS & OUTLOOK

- PEPS offer efficient descriptions of many-body quantum systems
- Encode symmetries and topology in a simple way

**BULK-BOUNDARY**

\[ \rho_A \]

**EDGES**

\[ \sigma_{\tilde{c}A} = U^\dagger \rho_A U \]

- Isometry maps bulk operators onto the boundary
- Symmetries (global & gauge) are inherited
- Topology appear as a superselection rule (anomaly)