Schwinger-Keldysh supersymmetry and a gauge theory of entropy

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YITP, 16 June 2016

based on:

FH, R. Loganayagam, M. Rangamani
[1510.02494], [1511.07809], [work in progress]

see also [1312.0610], [1412.1090], [1502.00636]

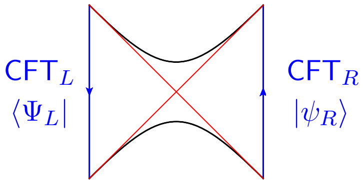
Summary

- This talk is about three statements:

  1. A satisfactory field theoretic understanding of **mixed state evolution** (in particular **EFT with dissipation**) is missing and desirable.
  2. Unitarity implies a **universal supergeometry** underlying any such field theory, which keeps track of the basic entanglement structure ('backbone') of the mixed initial state.
  3. (Near-)thermal EFTs have an **emergent $U(1)_T$ symmetry of gauged thermal translations.** This explains dissipation as a symmetry breaking, local entropy current, the second law etc.

Field theory and dissipation

- Many interesting and/or realistic systems are in **mixed states** due to tracing out part of $\mathcal{H}_{\text{tot}}$ ('environment')
- Example: low-energy EFT of thermal systems (fluids, black holes, ...)
  - UV/IR coupling ('entanglement')
  - **apparent non-unitarity:** dissipation, information loss, local entropy current, second law, ...
  - horizon, complementarity, entanglement, ...



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- **Task 1:** How to formulate QFT evolution of density matrices?
  - Well known aspect: **Schwinger-Keldysh** formalism
    - ⋆ Doubling of fields and symmetries to evolve $\langle \cdot |$ and $| \cdot \rangle$
    - ⋆ $S_{SK} = S[\Phi_R] - S[\Phi_L]$

Schwinger, Keldysh,

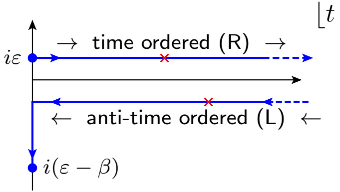
Feynman-Vernon, '60s

- **Task 2:** How to do RG on such systems?
  - Are the two copies going to interact? How?
  - Understand macroscopic irreversibility as a **symmetry breaking?**

Schwinger-Keldysh and unitarity

- SK generating functional:

$$\mathcal{Z}_{SK}[J_R, J_L] = \text{Tr}\left\{ U[J_R]\,\rho_{\text{initial}}\,U^\dagger[J_L] \right\}$$



- An interesting consequence of **unitarity:**

source alignment $\Rightarrow$ localization: $\mathcal{Z}_{SK}[J_R = J_L \equiv J] = \text{Tr}\,\rho_{\text{initial}}$

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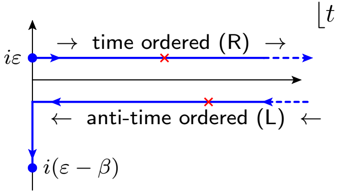

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change basis: $\mathbb{O}_R J_R - \mathbb{O}_L J_L = \mathbb{O}_{\text{av}} \underbrace{J_{\text{diff}}}_{\to 0} + \mathbb{O}_{\text{diff}} \underbrace{J_{\text{av}}}_{\to J}$ $\left(\text{av} \equiv \frac{R+L}{2}, \quad \text{diff} \equiv R - L \right)$

$\Rightarrow$ The sector of **difference operators** $\mathbb{O}_{\text{diff}} \equiv \mathbb{O}_R - \mathbb{O}_L$

decouples for any SK theory: $\langle \mathcal{T}_{SK}\, \mathbb{O}^{(1)}_{\text{diff}} \cdots \mathbb{O}^{(n)}_{\text{diff}} \rangle = 0$

  - Correlations of $\mathbb{O}_{\text{diff}}$ protected by (topological) symmetry

Schwinger-Keldysh and unitarity

- Can we cook up formalism where this localization is **manifest**?
- c.f. gauge theory:
  - characterize pure gauge modes using nilpotent, Grassmann-odd BRST charges: $[\text{pure gauge}] = Q_{\mathsf{BRST}}(\ldots) = \overline{Q}_{\mathsf{BRST}}(\ldots)$
  - **correlators of BRST-exact fields vanish**
- Efficient way to formulate SK localization: **SK BRST cohomology**
  - every operator $\hat{\mathbb{O}}$ represented by a quadruplet $\{\mathbb{O}_R, \mathbb{O}_L, \mathbb{O}_G, \mathbb{O}_{\bar{G}}\}$
  - BRST charges $\mathcal{Q}_{_{SK}}, \overline{\mathcal{Q}}_{_{SK}}$ define topological sector:

$$\begin{tikzcd}
 & \mathbb{O}_R, \mathbb{O}_L \arrow[dl, "\mathcal{Q}_{_{SK}}"] \arrow[dr, "\overline{\mathcal{Q}}_{_{SK}}"] & \\
\mathbb{O}_{_G} \arrow[dr, "\overline{\mathcal{Q}}_{_{SK}}"] & & \mathbb{O}_{_{\overline{G}}} \arrow[dl, "-\mathcal{Q}_{_{SK}}"] \\
 & \mathbb{O}_R - \mathbb{O}_L &
\end{tikzcd}$$

  - unitarity $\Rightarrow$ correlators of SK BRST-exact operators vanish

SK supergeometry

- Convenient way to ascertain topological structure: **superspace** $(x^\mu, \theta, \bar{\theta})$, where $\mathcal{Q}_{SK} = \partial_{\bar{\theta}}$ and $\overline{\mathcal{Q}}_{SK} = \partial_\theta$
- Quadrupling of fields $\Leftrightarrow$ lift fields to superfields:

$$
\begin{tikzcd}[column sep=small]
\mathbb{O}_{(S)} = \frac{\mathbb{O}_R+\mathbb{O}_L}{2} \arrow[r, bend left, "\overline{\mathcal{Q}}_{SK}"] \arrow[rr, bend right, "\mathcal{Q}_{SK}"'] & + \theta\,\mathbb{O}_{\overline{G}} \arrow[rr, bend left, "\mathcal{Q}_{SK}"] & + \bar{\theta}\,\mathbb{O}_G \arrow[r, bend right, "\overline{\mathcal{Q}}_{SK}"'] & + \bar{\theta}\theta\,(\mathbb{O}_R - \mathbb{O}_L)
\end{tikzcd}
$$

- This structure is **robust** under RG and **universal** for any unitary SK theory

- Note: for simple dissipative systems (e.g. Langevin theory of Brownian motion) the quadrupling is textbook material. BRST charges and superspace merely reformulation.

Supergeometry of low-energy SK theories

- **Low-energy SK EFT:** what are the symmetries, symmetry breakings, effective degrees of freedom Φ? *(see also Hong Liu's talk)*
- Many things as usual. New features:
  (1) Superspace, with quadrupling of naïve IR fields $\Phi \to \Phi_{(S)}$
  (2) Write topological field theory of initial correlations:

$$S_{SK}^{(\text{top})} = \int d^d x \, \left\{ \overline{\mathcal{Q}} \, , [\mathcal{Q} \, , \mathcal{L}(\Phi)] \right\} = \int d^d x \, d\theta \, d\bar{\theta} \, \mathcal{L}(\Phi_{(S)})$$

  (3) Then de-align sources $J_R \neq J_L$ to deform away from topological limit

Low energy, near-equilibrium: emergent $U(1)_T$

- We are particularly interested in SK evolution amongst the **class of mixed states whose low-energy dynamics is locally thermal**
  - I.e., UV modes are in thermal equilibrium
  - EFT of IR modes reflects this! (fluctuation-dissipation, second law, ...)

Proposal for implementing thermality in EFT

Invariance under **emergent gauge symmetry of thermal translations:** $U(1)_T$

[1510.02494]
[1502.00636]
[1412.1090]

- ⋆ C.f. Euclidean theory: translation invariance in thermal circle
- ⋆ Microscopic origin of $U(1)_T$: KMS condition
- ⋆ Two more BRST charges associated with $U(1)_T$
- ⋆ $U(1)_T$ current $=$ **entropy current** $+$ ghost terms
- ⋆ Apparent non-unitarity (dissipation) $\leftrightarrow$ **ghost non-decoupling?!**

Toy model: Langevin particle

- Consider Brownian motion of Langevin particle at $x(t)$:

$$-\mathsf{Eom} \equiv m\,\frac{d^2x}{dt^2} + \frac{\partial U}{\partial x} + \nu\,\frac{dx}{dt} = \mathbb{N}$$

- Martin-Siggia-Rose (MSR) construction:

Martin-Siggia-Rose '73

De Dominicis-Peliti '78

$$[dx]\int [d\mathbb{N}]\,\delta(\mathsf{Eom}+\mathbb{N})\,\mathsf{det}\left(\frac{\delta\mathsf{Eom}}{\delta x}\right)\,e^{i\,S_{\text{Gaussian noise}}[\mathbb{N}]}$$

$$= [dx]\int [df][d\overline{\psi}][d\psi]\;\mathsf{exp}\;i\int dt\left(f\,\mathsf{Eom} + i\,\nu\,f^2 + \overline{\psi}\left(\frac{\delta\mathsf{Eom}}{\delta x}\right)\psi\right)$$

- Can write this in superspace:

$$= [dx]\int [df][d\overline{\psi}][d\psi]\;\mathsf{exp}\;i\int dt\,d\theta\,d\bar{\theta}\left(\frac{m}{2}\left(\frac{dx_{(S)}}{dt}\right)^2 - U(x_{(S)}) - i\,\nu\,\mathcal{D}_\theta x_{(S)}\;\mathcal{D}_{\bar{\theta}} x_{(S)}\right)_{\mathsf{WZ}}$$

where $x_{(S)} = x + \theta\,\bar{\psi} + \bar{\theta}\,\psi + \bar{\theta}\theta\,f$

and $\mathcal{D}_\theta$ refers to $\mathcal{A} = \mathcal{A}_t\,dt + \mathcal{A}_\theta\,d\theta + \mathcal{A}_{\bar{\theta}}\,d\bar{\theta}$

Status and prospects

- **Step 1:** In simple examples (Langevin theory) this works beautifully (SUSY structure well-known; $U(1)_T$ symmetry completes the picture nicely)
  - Viscosity proportional to **CPT breaking order parameter** $\langle \mathcal{F}_{\theta\bar{\theta}} \rangle \neq 0$

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- **Step 2:** We are trying to **derive hydrodynamics** with this
  - Advantage of hydro: we gave complete solution and classification of transport $\Rightarrow$ very sharp goal for what the SK EFT has to achieve

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  - Have already written a SUSY EFT with thermal gauge symmetry, which describes **all of dissipative transport**

    [1511.07809], see also Crossley-Glorioso-Liu [1511.03646]
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  - W.i.p.: various other classes of transport also seem to work nicely
- **Step 3:** If hydro works, move on to dual **black holes**
  - SK doubling and ghosts: what will they teach us about **dissipation, complementarity, unitarity etc.** in gravity?

Summary

1. A satisfactory field theoretic understanding of **mixed state evolution** (in particular **EFT with dissipation**) is missing and desirable.

2. Unitarity implies a **universal supergeometry** underlying any such field theory, which keeps track of the basic entanglement structure ('backbone') of the mixed initial state.

3. (Near-)thermal EFTs have an **emergent $U(1)_T$ symmetry of gauged thermal translations.** This explains dissipation as a symmetry breaking, local entropy current, the second law etc.

- Or, for the mathematically inclined:
  - Near-thermal EFTs are deformations of TQFTs associated with the universal **balanced equivariant cohomology** of thermal translations.

    Vafa-Witten '94, Dijkgraaf-Moore '96

  - Hydrodynamics is a deformation of a gauged topological σ-model.

Further Details

$U(1)_T$ thermal gauge invariance

- Microscopic KMS condition:

$$e^{-i\delta_\beta}\mathbb{O}(t) \equiv \mathbb{O}(t-i\beta) \overset{\substack{\text{KMS}\\ \downarrow}}{=} \mathbb{O}(t)$$

- Macroscopically, ensure KMS by introducing gauge (super-)field for 'thermal translations'

$$\mathcal{A} = \mathcal{A}_a\, d\sigma^a + \mathcal{A}_\theta\, d\theta + \mathcal{A}_{\bar{\theta}}\, d\bar{\theta}$$

$$\mathcal{D}_\theta X^\mu_{(S)} \equiv \partial_\theta X^\mu_{(S)} + \mathcal{A}_\theta\, \pounds_\beta X^\mu_{(S)} \quad \text{(and so on)}$$

- $U(1)_T$ transformations act as thermal translations, e.g.:

$$\Phi_{(S)} \;\mapsto\; \Phi_{(S)} + \Lambda_{(S)}\, \pounds_\beta \Phi_{(S)}$$

Effective action for dissipation in fluids

- Fluids get their dynamics from σ-model maps:

$$X^\mu(\sigma): \quad d\text{-worldvolume} \to \text{phys. spacetime}$$

- Effective action for dissipative sector (at any order in ∇_μ):

$$S_{\text{eff}}^{\text{(dissipation)}} \sim \int_{\substack{\text{world}\\\text{volume}}} d^d\sigma \, d\theta \, d\bar{\theta} \, \frac{\sqrt{-\mathbb{g}^{(S)}}}{1 + \beta^a \, \mathcal{A}_a} \left(i \, \eta^{((ab)(cd))} \, \mathcal{D}_\theta \, \mathbb{g}_{ab}^{(S)} \, \mathcal{D}_{\bar{\theta}} \, \mathbb{g}_{cd}^{(S)} \right)$$

$$\Rightarrow \quad T^{ab} \sim i \, \mathfrak{F}_{\theta\bar{\theta}} \, \eta^{((ab)(cd))} \, \pounds_\beta \, \mathbb{g}_{ab} + \text{noise terms}$$

  - **Ghost bilinears** responsible for dissipation
  - $\langle \mathfrak{F}_{\theta\bar{\theta}} \rangle$: order parameter for dissipation
  - Can derive Jarzynski as SUSY Ward identity ($\Rightarrow$ Second Law)
  - Variation w.r.t. $\mathcal{A}_a$ gives entropy current