Schwinger-Keldysh supersymmetry
and a gauge theory of entropy

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YITP, 16 June 2016

based on:

FH, R. Loganayagam, M. Rangamani
[1510.02494], [1511.07809], [work in progress]

see also [1312.0610], [1412.1090], [1502.00636]
This talk is about three statements:

1. A satisfactory field theoretic understanding of mixed state evolution (in particular EFT with dissipation) is missing and desirable.

2. Unitarity implies a universal supergeometry underlying any such field theory, which keeps track of the basic entanglement structure (‘backbone’) of the mixed initial state.

3. (Near-)thermal EFTs have an emergent $U(1)_T$ symmetry of gauged thermal translations. This explains dissipation as a symmetry breaking, local entropy current, the second law etc.
Field theory and dissipation

- Many interesting and/or realistic systems are in **mixed states** due to tracing out part of $\mathcal{H}_{\text{tot}}$ (‘environment’)

- Example: low-energy EFT of thermal systems (fluids, black holes, ...)
  - UV/IR coupling (‘entanglement’)
  - **apparent non-unitarity:** dissipation, information loss, local entropy current, second law, ...
  - horizon, complementarity, entanglement, ...

\[
\begin{array}{c}
\text{CFT}_L \\
\langle \Psi_L \rangle
\end{array}
\quad \overset{\text{CFT}_R}{\leftrightarrow} \quad
\begin{array}{c}
|\psi_R\rangle
\end{array}
\]
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**Task 1:** How to formulate QFT evolution of density matrices?

  - Well known aspect: **Schwinger-Keldysh** formalism
    - Doubling of fields and symmetries to evolve $\langle \cdot |$ and $| \cdot \rangle$
    - $S_{SK} = S[\Phi_R] - S[\Phi_L]

**Task 2:** How to do RG on such systems?

  - Are the two copies going to interact? How?
  - Understand macroscopic irreversibility as a **symmetry breaking**?
Schwinger-Keldysh and unitarity

- **SK generating functional:**

\[ Z_{SK}[J_R, J_L] = \text{Tr} \left\{ U[J_R] \rho_{\text{initial}} U^\dagger[J_L] \right\} \]

- An interesting consequence of **unitarity:**

source alignment ⇒ localization:

\[ Z_{SK}[J_R = J_L \equiv J] = \text{Tr} \rho_{\text{initial}} \]

▶ Only sensitive to initial state correlations (entanglement structure)
Schwinger-Keldysh and unitarity

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\[ \mathcal{O}_R J_R - \mathcal{O}_L J_L = \mathcal{O}_{\text{av}} \underbrace{J_{\text{diff}}}_{\rightarrow 0} + \mathcal{O}_{\text{diff}} \underbrace{J_{\text{av}}}_{\rightarrow J} \quad \left( \text{av} \equiv \frac{R + L}{2}, \quad \text{diff} \equiv R - L \right) \]

- The sector of difference operators \( \mathcal{O}_{\text{diff}} \equiv \mathcal{O}_R - \mathcal{O}_L \) decouples for any SK theory:

\[ \langle T_{SK} \mathcal{O}_{\text{diff}}^{(1)} \cdots \mathcal{O}_{\text{diff}}^{(n)} \rangle = 0 \]

- Correlations of \( \mathcal{O}_{\text{diff}} \) protected by (topological) symmetry
Schwinger-Keldysh and unitarity

- Can we cook up formalism where this localization is manifest?
- c.f. gauge theory:
  - characterize pure gauge modes using nilpotent, Grassmann-odd BRST charges: 
    \[ \text{[pure gauge]} = Q_{\text{BRST}}(\ldots) = \overline{Q}_{\text{BRST}}(\ldots) \]
  - correlators of BRST-exact fields vanish

Efficient way to formulate SK localization: SK BRST cohomology

- every operator \( \hat{\mathcal{O}} \) represented by a quadruplet \( \{ \mathcal{O}_R, \mathcal{O}_L, \mathcal{O}_G, \mathcal{O}_{\overline{G}} \} \)
- BRST charges \( Q_{SK}, \overline{Q}_{SK} \) define topological sector:

\[
\begin{array}{c}
\mathcal{O}_R, \mathcal{O}_L \\
\mathcal{O}_G \\
\mathcal{O}_{\overline{G}}
\end{array}
\]

\[
\begin{array}{c}
Q_{SK} \\
\overline{Q}_{SK} \quad -Q_{SK} \\
\mathcal{O}_R - \mathcal{O}_L
\end{array}
\]

- unitarity \( \Rightarrow \) correlators of SK BRST-exact operators vanish
SK supergeometry

- Convenient way to ascertain topological structure: superspace \((x^\mu, \theta, \bar{\theta})\), where \(Q_{SK} = \partial_{\bar{\theta}}\) and \(\bar{Q}_{SK} = \partial_{\theta}\)

- Quadrupling of fields \(\iff\) lift fields to superfields:

\[
\mathcal{O}(S) = \mathcal{O}_R + \mathcal{O}_L + \theta \mathcal{O}_G + \bar{\theta} \mathcal{O}_G + \bar{\theta} \theta (\mathcal{O}_R - \mathcal{O}_L)
\]

- This structure is robust under RG and universal for any unitary SK theory

- Note: for simple dissipative systems (e.g. Langevin theory of Brownian motion) the quadrupling is textbook material. BRST charges and superspace merely reformulation.
**Low-energy SK EFT**: what are the symmetries, symmetry breakings, effective degrees of freedom $\Phi$?  

(see also Hong Liu’s talk)

Many things as usual. New features:

1. Superspace, with quadrupling of naive IR fields $\Phi \rightarrow \Phi(S)$
2. Write topological field theory of initial correlations:

$$S_{SK}^{(top)} = \int d^d x \{ \overline{Q} , [Q , \mathcal{L}(\Phi)] \} = \int d^d x \ d\theta \ d\bar{\theta} \ \mathcal{L}(\Phi(S))$$

3. Then de-align sources $J_R \neq J_L$ to deform away from topological limit
Low energy, near-equilibrium: emergent $U(1)_T$

- We are particularly interested in SK evolution amongst the class of mixed states whose low-energy dynamics is locally thermal
  - I.e., UV modes are in thermal equilibrium
  - EFT of IR modes reflects this! (fluctuation-dissipation, second law, ...)

Proposal for implementing thermality in EFT

Invariance under emergent gauge symmetry of thermal translations: $U(1)_T$

- C.f. Euclidean theory: translation invariance in thermal circle
- Microscopic origin of $U(1)_T$: KMS condition
- Two more BRST charges associated with $U(1)_T$
- $U(1)_T$ current = entropy current + ghost terms
- Apparent non-unitarity (dissipation) $\leftrightarrow$ ghost non-decoupling?!
Toy model: Langevin particle

- Consider Brownian motion of Langevin particle at $x(t)$:
  $\begin{align}  -\text{Eom} & \equiv m \frac{d^2 x}{dt^2} + \frac{\partial U}{\partial x} + \nu \frac{dx}{dt} = N \end{align}$

- Martin-Siggia-Rose (MSR) construction:
  $\begin{align}  [dx] \int [dN] \delta(\text{Eom} + N) \det \left( \frac{\delta\text{Eom}}{\delta x} \right) e^{iS_{\text{Gaussian noise}}} & \\
  = [dx] \int [df][d\bar{\psi}][d\psi] \exp i \int dt \left( f \text{Eom} + i \nu f^2 + \bar{\psi} \left( \frac{\delta\text{Eom}}{\delta x} \right) \psi \right) \end{align}$

- Can write this in superspace:
  $\begin{align}  = [dx] \int [df][d\bar{\psi}][d\psi] \exp i \int dt \ d\theta \ d\bar{\theta} \left( \frac{m}{2} \left( \frac{dx(S)}{dt} \right)^2 - U(x(S)) - i \nu D_\theta x(S) \ D_{\bar{\theta}} x(S) \right) \end{align}$

  where $x(S) = x + \theta \bar{\psi} + \bar{\theta} \psi + \theta\bar{\theta} f$
  and $D_\theta$ refers to $A = A_t \ dt + A_\theta \ d\theta + A_{\bar{\theta}} \ d\bar{\theta}$
Status and prospects

- **Step 1:** In simple examples (Langevin theory) this works beautifully (SUSY structure well-known; $U(1)_T$ symmetry completes the picture nicely)
  - Viscosity proportional to **CPT breaking order parameter** $\langle \mathcal{F}_{\theta\bar{\theta}} \rangle \neq 0$  
[1511.07809]
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- **Step 2:** We are trying to derive hydrodynamics with this
  - Advantage of hydro: we gave complete solution and classification of transport $\Rightarrow$ very sharp goal for what the SK EFT has to achieve [1412.1090], [1502.00636]
  - Have already written a SUSY EFT with thermal gauge symmetry, which describes **all of dissipative transport** [1511.07809], see also Crossley-Glorioso-Liu [1511.03646]
  - W.i.p.: various other classes of transport also seem to work nicely
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- **Step 3:** If hydro works, move on to dual **black holes**
  - SK doubling and ghosts: what will they teach us about **dissipation, complementarity, unitarity etc.** in gravity?
Summary

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2. Unitarity implies a **universal supergeometry** underlying any such field theory, which keeps track of the basic entanglement structure (‘backbone’) of the mixed initial state.

3. (Near-)thermal EFTs have an **emergent** $U(1)_T$ **symmetry of gauged thermal translations**. This explains dissipation as a symmetry breaking, local entropy current, the second law etc.

- Or, for the mathematically inclined:
  - Near-thermal EFTs are deformations of TQFTs associated with the universal **balanced equivariant cohomology** of thermal translations.
    - *Vafa-Witten ’94, Dijkgraaf-Moore ’96*
  - Hydrodynamics is a deformation of a gauged topological $\sigma$-model.
Further Details
$U(1)_T$ thermal gauge invariance

- Microscopic KMS condition:

$$e^{-i\delta \beta} \mathcal{O}(t) \equiv \mathcal{O}(t - i/\beta) \Downarrow \equiv \mathcal{O}(t)$$

- Macroscopically, ensure KMS by introducing gauge (super-)field for 'thermal translations'

$$\mathcal{A} = A_a \, d\sigma^a + A_\theta \, d\theta + A_{\bar{\theta}} \, d\bar{\theta}$$

$$\mathcal{D}_\theta X_\mu^{(S)} \equiv \partial_\theta X_\mu^{(S)} + A_\theta \, \mathcal{L}_\beta X_\mu^{(S)} \quad \text{(and so on)}$$

- $U(1)_T$ transformations act as thermal translations, e.g.:

$$\Phi^{(S)} \mapsto \Phi^{(S)} + \Lambda^{(S)} \, \mathcal{L}_\beta \Phi^{(S)}$$
Effective action for dissipation in fluids

- Fluids get their dynamics from $\sigma$-model maps:
  \[ X^\mu(\sigma): \text{ $d$-worldvolume} \rightarrow \text{phys. spacetime} \]

- Effective action for dissipative sector (at any order in $\nabla_\mu$):
  \[
  S^{(\text{dissipation})}_{\text{eff}} \sim \int_{\text{world volume}} d^d \sigma \, d\theta \, d\bar{\theta} \, \sqrt{-g^{(S)}} \frac{1}{1 + \beta^a A_a} \left( i \, \eta^{((ab)(cd))} \, \mathcal{D}_\theta g^{(S)}_{ab} \, \mathcal{D}_{\bar{\theta}} g^{(S)}_{cd} \right)
  \]
  \[ \Rightarrow \quad T^{ab} \sim i \, \mathcal{F}_{\theta \bar{\theta}} \, \eta^{((ab)(cd))} \, \mathcal{L}_\beta g_{ab} + \text{noise terms} \]

- **Ghost bilinears** responsible for dissipation
- $\langle \mathcal{F}_{\theta \bar{\theta}} \rangle$: order parameter for dissipation
- Can derive Jarzynski as SUSY Ward identity ($\Rightarrow$ Second Law)
- Variation w.r.t. $A_a$ gives entropy current