Subregion Duality in the Entanglement Wedge

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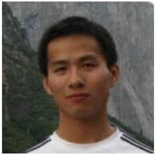

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- I will make use of a recent result relating bulk and boundary relative entropies, which I will review. Jafferis/Lewkowycz/Maldacena/Suh

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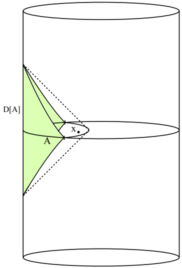

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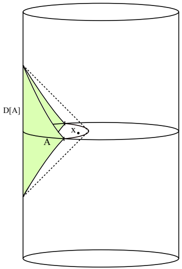

$$\phi(x)\Big|_{C_A} = \int_{D[A]} dX\ \hat{K}(x;X)\mathcal{O}(X) + O(1/N).$$

Here the region C_A is the *causal wedge of A*: Hubeny/Rangamani

$$C_A \equiv j^+[D(A)] \cap j^-[D(A)].$$

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- Indeed the construction will fail to reproduce bulk effective field theory unless the CFT has the appropriate properties: large N factorization and a large gap in the spectrum of single-trace primary operators.
- This isn't just kinematics!

Is the causal wedge the whole story? The RT formula $S_A = \frac{\mathrm{Area}(\gamma_A)}{4G}$ says no!

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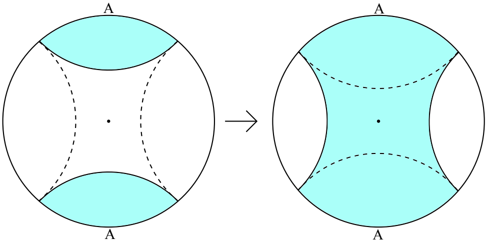

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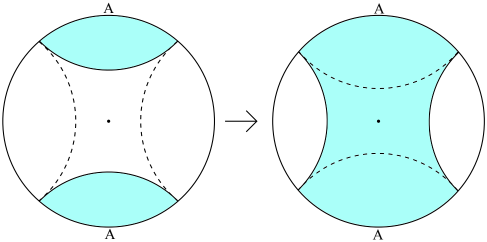

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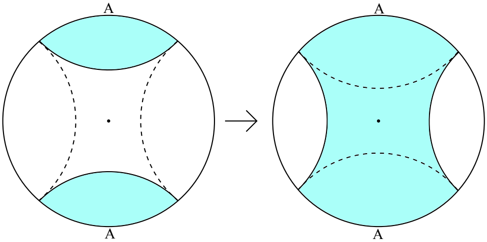

At least in this case, ρ_A has access to information beyond the causal wedge!
But how far can we go?

- Definition: say that Ξ is an achronal surface such that $\partial\Xi = A \cup \gamma_A$ (shaded blue in the figure). Then the *entanglement wedge of A* is defined as

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- Today we will prove it!

Quantum Error Correction

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- Indeed there must be states where any particular bulk operator reconstruction fails! Almheiri/Dong/Harlow
- Roughly speaking, bulk operators can be swallowed behind black hole horizons, and in such states they do not need to have effective field theory interpretations. This is the hack that AdS/CFT employs to allow a lower-dimensional theory to be equivalent to a higher-dimensional one.

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- One good example is obtained by taking the set of states in $\mathcal{N} = 4$ SYM theory on $\mathbb{S}^3 \times \mathbb{R}$ whose energies are at most $N^{1/4}$, and then taking the image of these states under conformal transformations: a "no black hole subspace".

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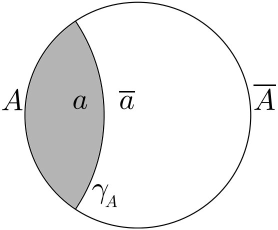
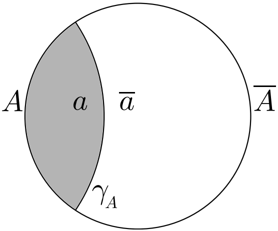
- More general subspaces are also possible, but we are then only able to reconstruct bulk fields which are at best "not too far" behind black hole horizons.



Using this language, we can formalize the entanglement wedge reconstruction conjecture:



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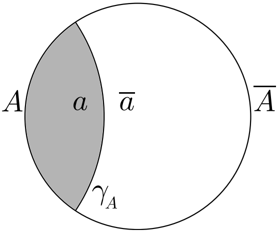

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- Then for any operator O_a on $\mathcal{H}_a$ we have an operator O_A on $\mathcal{H}_A$ such that for all $|\widetilde{\psi}\rangle \in \mathcal{H}_{code}$ we have:

$$O_A|\widetilde{\psi}\rangle = O_a|\widetilde{\psi}\rangle$$
$$O_A^\dagger|\widetilde{\psi}\rangle = O_a^\dagger|\widetilde{\psi}\rangle$$

- In proving this conjecture we can make use of a theorem from ADH, which shows that such an O_A can exist if and only if:

$$\forall X_{\overline{A}},\ \forall|\widetilde{\psi}\rangle,\ \langle\widetilde{\psi}|[X_{\overline{A}}, O_a]|\widetilde{\psi}\rangle = 0. \tag{1}$$

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- Claim: (1) follows from the quantum corrected RT formula, which in this language says that for all ρ on $\mathcal{H}_{code}$, we have:

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- Here $\mathcal{A}_{\mathrm{loc}}$ is some operator integrated on γ_A, which to leading order in G is $\frac{\mathrm{Area}(\gamma_A)}{4G}$.

- You might worry about the assumption of bulk factorization, this is indeed subtle, see Donnelly,Casini/Huerta/Rosabal, but it seems likely that including enough UV degrees of freedom near γ_A we can justify it Harlow. And actually we don't need to assume it if we work algebraically.

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- In fact this theorem also has a converse: any quantum error correcting code obeys a version of the RT formula! Harlow In general it involves the algebraic definition of entropy, and gives a "completely boundary" picture of how the formula works, to be contrasted with the "completely bulk" derivation of Lewkowycz/Maldacena,Faulkner/Lewkowycz/Maldacena.

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These are related by the "first law of entanglement":

$$
S(\rho + \delta\rho) - S(\rho) = \mathrm{Tr}(\delta\rho K_\sigma) + O(\delta\rho^2).
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We can apply this "first law" to both sides of the RT formula

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So in particular, $\rho_a = \sigma_a \Leftrightarrow \rho_A = \sigma_A$.

A Reconstruction Theorem

Now to finish the argument, we just need to show that indeed we have

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Woohoo!

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- Understand the higher-order corrections better, specifically insofar as they relate to backreaction.
- Algebraic reformulation (done!)
- This improves on HKLL both in scope (the whole entanglement wedge), and in that it avoids the nasty PDE issues I mentioned. But HKLL gives a *bulk* picture of what is going on, which is sorely lacking here.

ありがとうございました

Proof of the ADH theorem

Given an operator O on $\mathcal{H}_{code}$, how do we know that

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We can clearly mirror the operator O onto R, but can we then mirror it back onto A?

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$$\forall X_{\overline{A}},\ \forall|\widetilde{\psi}\rangle,\ \langle\widetilde{\psi}|[X_{\overline{A}}, O]|\widetilde{\psi}\rangle = 0$$

is sufficient (it is clearly necessary) for the existence of an O_A?
The basic idea is to consider the state

$$|\phi\rangle \propto \sum_i |i\rangle_R|\widetilde{i}\rangle_{A\overline{A}}.$$

We can clearly mirror the operator O onto R, but can we then mirror it back onto A?
What we need for this to work is that O_R preserves the Schmidt basis of $|\phi\rangle$ if we decompose it as $R\overline{A}$ and A, or in other words for $[O_R, \rho_{R\overline{A}}] = 0$.

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But this is precisely what the commutator condition ensures!