PERIMETER INSTITUTE FOR THEORETICAL PHYSICS

Entanglement Holography

Robert Myers

with de Boer, Haehl, Heller & Neiman

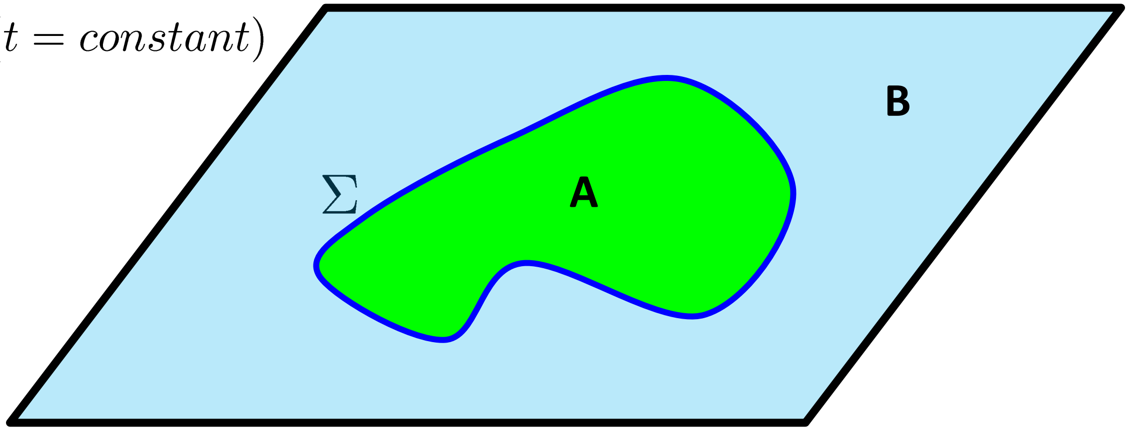
arXiv:1509.00113; arXiv:1606.03307

Entanglement Entropy

- general tool; divide quantum system into two parts and use entropy as measure of correlations between subsystems
- in QFT, typically introduce a (smooth) boundary or entangling surface Σ which divides the space into two separate regions
- integrate out degrees of freedom in "outside" region
- remaining dof are described by a density matrix ρ_A

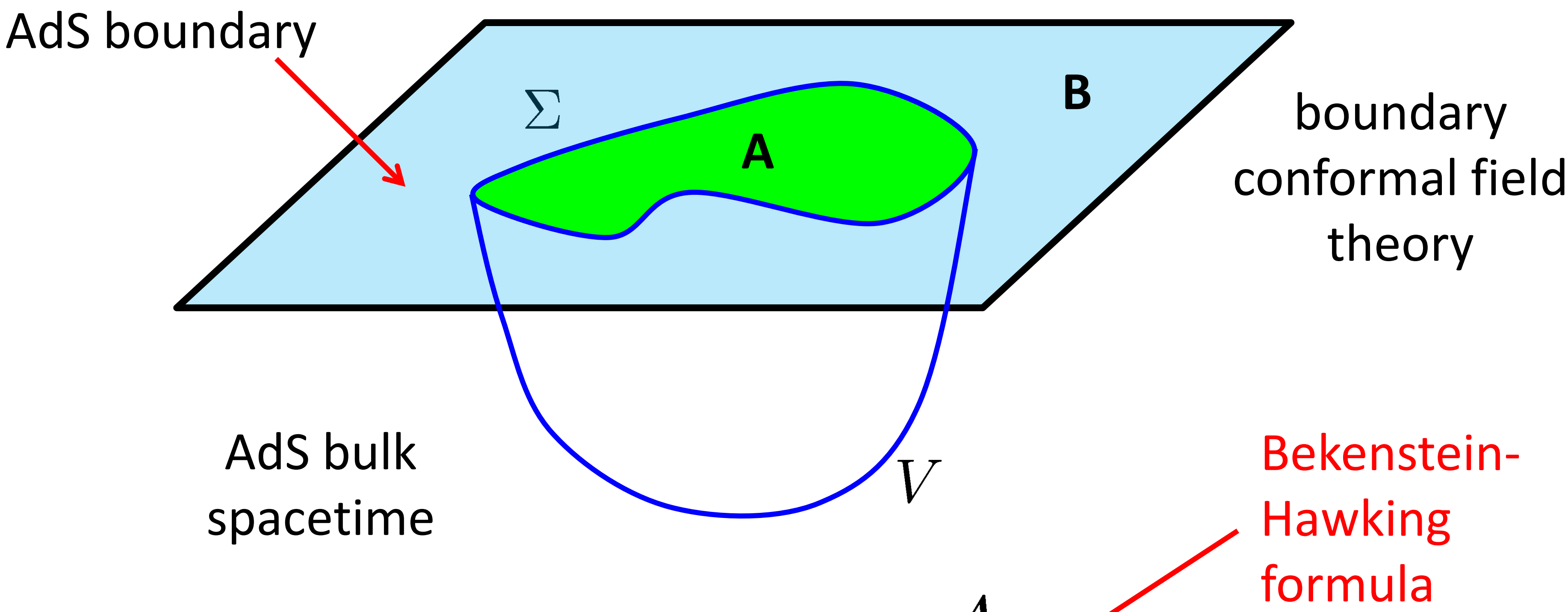
→ calculate von Neumann entropy: $S_{EE} = -Tr\left[\rho_A \log \rho_A\right]$

$(t = constant)$



Holographic Entanglement Entropy:

(Ryu & Takayanagi)



$$S(A) = \underset{V \sim A}{\mathrm{ext}} \frac{A_V}{4G_N}$$

- 2006 conjecture ⟶ many detailed consistency tests

(Ryu, Takayanagi, Headrick, Hung, Smolkin, RM, Faulkner, . . .)

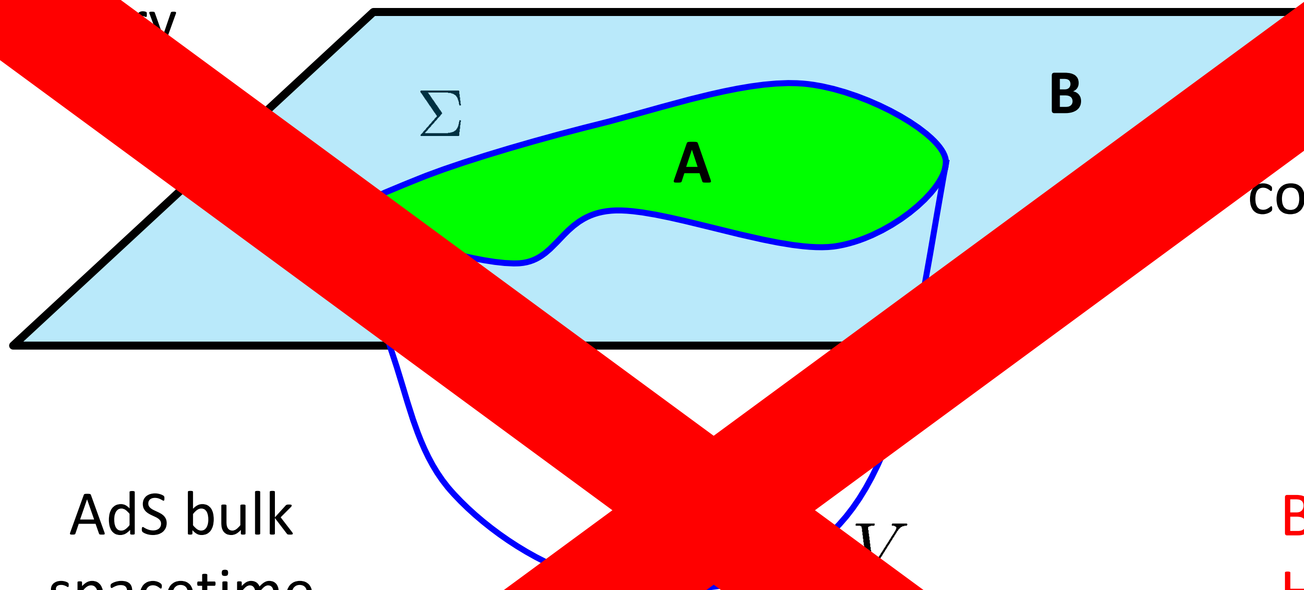
- 2013 proof (for static geometries)

(Maldacena & Lewkowycz)

H…graphic Entanglement Entropy:

(Ryu & Takaya…i)

AdS b…ry



…oundary conformal field theory

AdS bulk spacetime

Bekenstein-Hawking formula

$$\ldots(A) = \underset{V \sim A}{\mathrm{ext}} \; \frac{\ldots}{4G_N}$$

- 2006 c…ure → many detailed consistency tests…

(Ryu, Takayanagi, Headrick, Hung, Smolkin, RM, Fa… .)

- …3 proof (for static geometries)

(Maldacena & Lewko…)

Entanglement Holography:

- building on intuition and experience offered by EE in CFTs (and in AdS/CFT correspondence), propose reorganization of CFT in terms of new nonlocal observables

- find the emergence of a **new auxiliary geometry** as natural framework to describe any CFT – not relying on strong coupling or large # of dof

- may yield new insights into the structure of correlation functions, . . .

- for CFT's with conventional holographic duals, provides new observables based on extremal surfaces

- may give insight in the nonlocal nature of quantum gravity, bulk reconstruction, . . .

(see also Czech, Lamprou, McCandlish, Mosk & Sully: arXiv:1604.03110)

First Law of Entanglement

(Blanco, Casini, Hung & RM)

- entanglement entropy: $S(\rho_A) = -\mathrm{tr}(\rho_A \log \rho_A)$
- make a small perturbation of state: $\tilde{\rho} \ = \ \rho_A + \delta\rho$

$$\longrightarrow \quad \delta S = -\mathrm{tr}(\delta\rho \log \rho_A) - \underbrace{\mathrm{tr}(\rho_A\, \rho_A^{-1} \delta\rho)}_{= \mathrm{Tr}\,(\delta\rho) = 0} + O(\delta\rho^2)$$

$$= -\mathrm{tr}(\delta\rho \log \rho_A) + O(\delta\rho^2)$$

- modular (or entanglement) Hamiltonian: $\rho_A = \exp(-H_A)$

$$\delta S_A \ = \ \delta\langle H_A\rangle$$

"1st law" of entanglement entropy

- this is **the** 1st law for thermal states: $\rho_A = \exp(-H/T)$

"1st law" of entanglement entropy: $\delta S_A \;=\; \delta\langle H_A\rangle$

- generally H_A is "**nonlocal mess**" and flow is nonlocal/**not geometric**

$$H_A = \int d^{d-1}x\,\gamma_1^{\mu\nu}(x)\,T_{\mu\nu} + \int d^{d-1}x \int d^{d-1}y\,\gamma_2^{\mu\nu;\rho\sigma}(x,y)\,T_{\mu\nu}T_{\rho\sigma} + \cdots$$

$\longrightarrow$ hence usefulness of first law is very limited, in general

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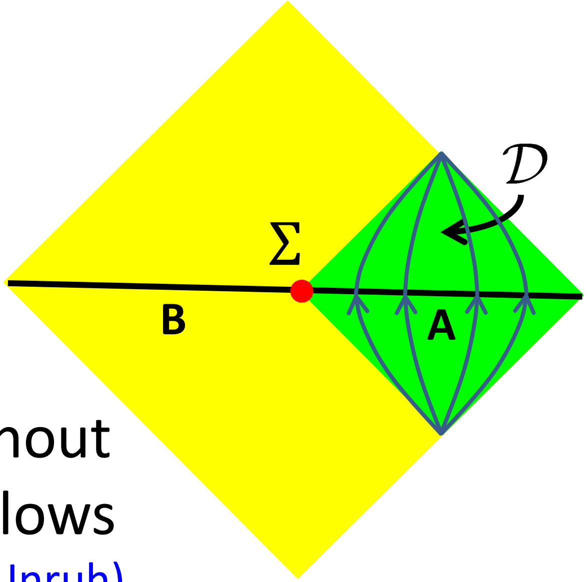
$\longrightarrow$ hence usefulness of first law is very limited, in general

- famous exception: **Rindler wedge**
- any relativistic QFT in Minkowski vacuum; choose $\Sigma = (\mathrm{x}=0, \mathrm{t}=0)$

$$H_A = 2\pi K \longleftarrow \text{boost generator}$$

$$= 2\pi \int_{A(x>0)} d^{d-2}y\,dx\,\left[\,x\;T_{tt}\,\right] + c'$$



- by causality, ρ_A and H_A describe physics throughout domain of dependence $\mathcal{D}$; eg, generate boost flows

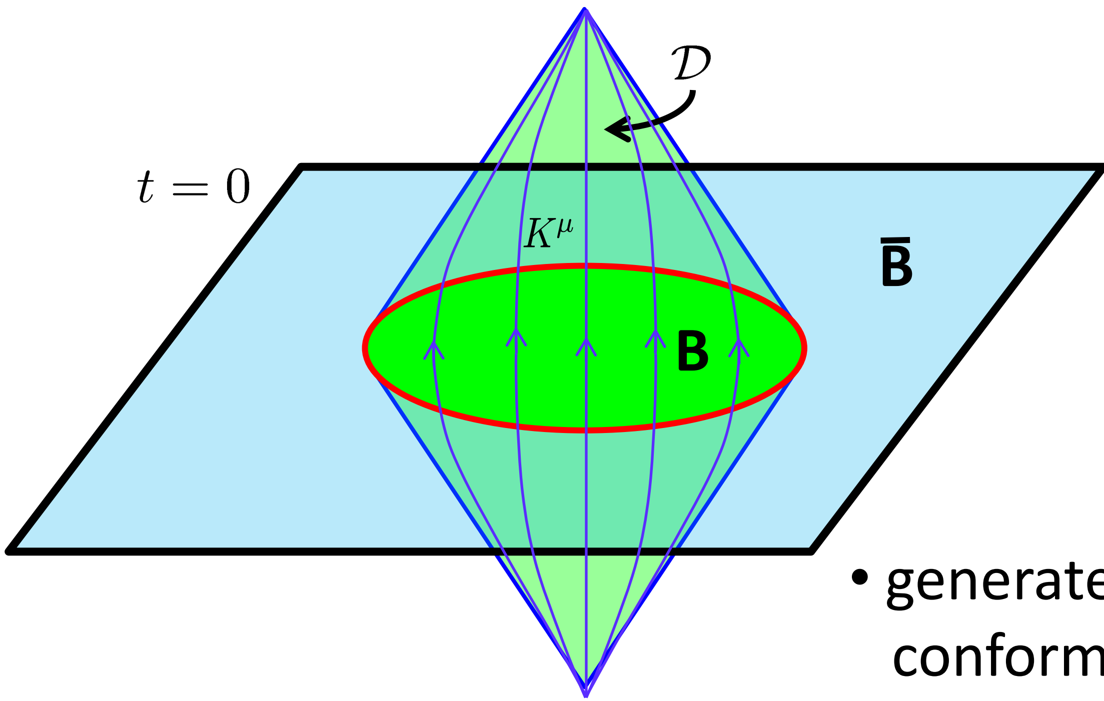
(Bisognano & Wichmann; Unruh)

“1st law” of entanglement entropy: $\delta S_A = \delta\langle H_A\rangle$

- **another exception**: CFT in vacuum of d-dim. flat space and entangling surface which is S^{d-2} with radius R

$$H_B = 2\pi\int_B d^{d-1}y\ \frac{R^2-|\vec{y}|^2}{2R}\,T_{tt}(\vec{y}) + c'$$

(Casini, Huerta & RM; Hislop & Longo)

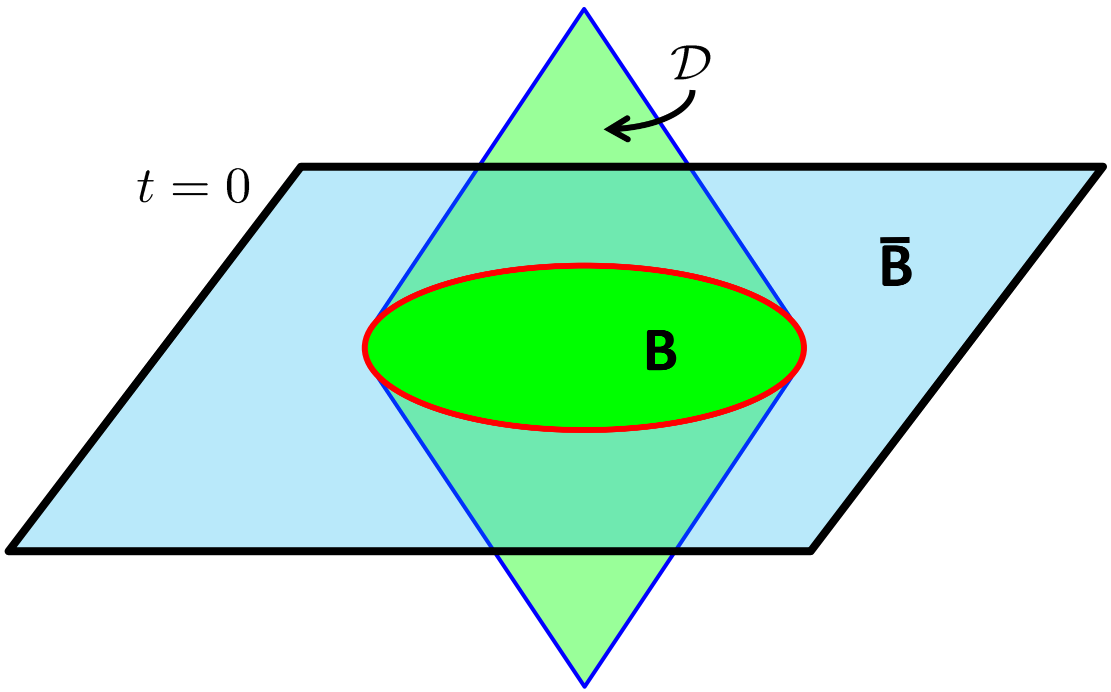


- generates flow along K^μ, conformal Killing vector

“1st law” of entanglement entropy: $\delta S_A \;=\; \delta\langle H_A\rangle$

- small excitations of CFT vacuum in d-dim. flat space and entangling surface which is S^{d-2} with radius R:

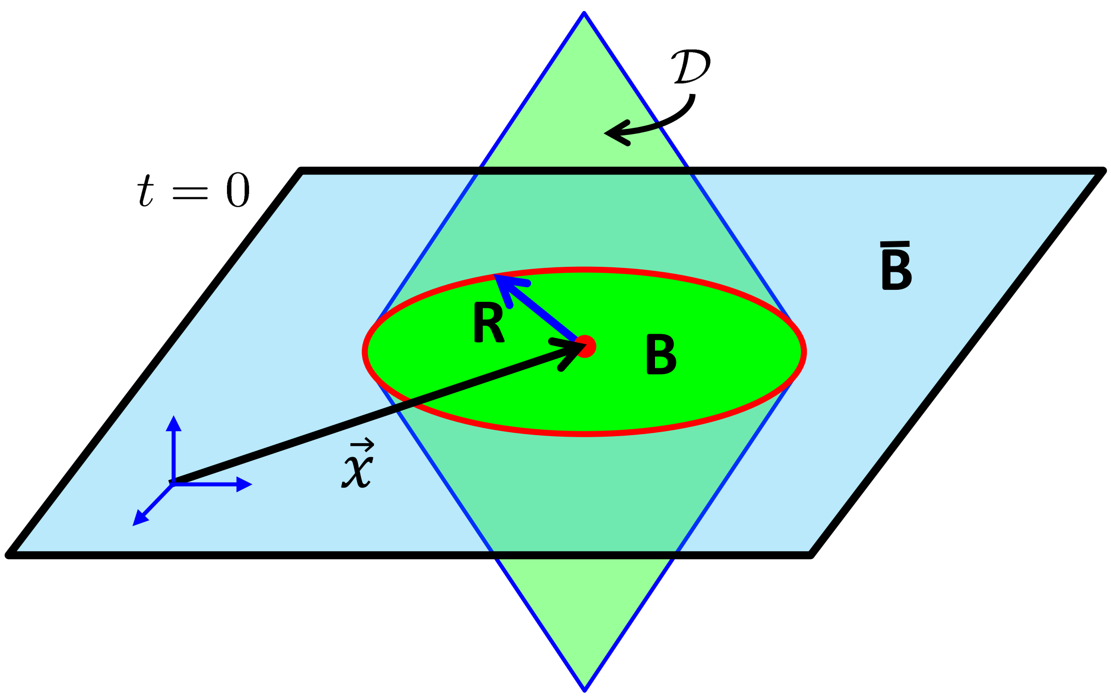
$$\delta S = \delta\langle H_B\rangle = 2\pi \int_B d^{d-1}y \;\frac{R^2 - |\vec{y}|^2}{2R}\,\langle T_{tt}(\vec{y})\rangle$$



“1st law” of entanglement entropy: $\delta S_A \;=\; \delta\langle H_A\rangle$

- small excitations of CFT vacuum in d-dim. flat space and entangling surface which is S^{d-2} with radius R:

$$\delta S(R,\vec{x}) = 2\pi \int_B d^{d-1}y \;\frac{R^2 - |\vec{y}-\vec{x}|^2}{2R}\,\langle T_{tt}(\vec{y})\rangle$$



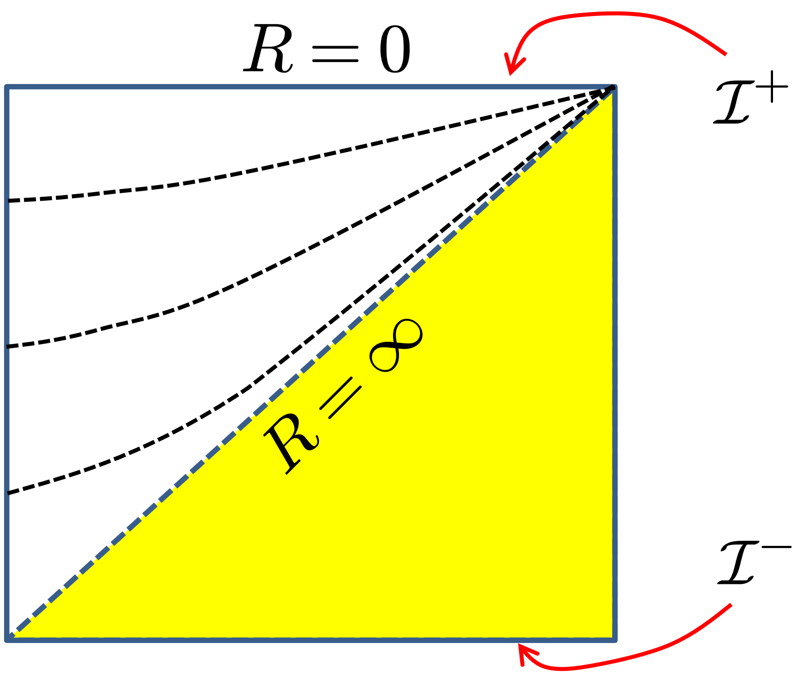
Entanglement Holography v1.0:

- small excitations of CFT vacuum in d-dim. flat space and entangling surface which is S^{d-2} with radius R:

$$\delta S(R,\vec{x}) = 2\pi \int_B d^{d-1}y \boxed{\frac{R^2 - |\vec{y}-\vec{x}|^2}{2R}} \langle T_{tt}(\vec{y})\rangle$$

- **boundary-to-bulk propagator in d-dim de Sitter space!**

(eg, see: Xiao 1402.7080)



$$ds^2 = \frac{L^2}{R^2}\left(-dR^2 + d\vec{x}^2\right)$$

radius plays role of **time**

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- **boundary-to-bulk propagator in d-dim de Sitter space!**

(eg, see: Xiao 1402.7080)

$$ds^2 = \frac{L^2}{R^2}\left(-dR^2 + d\vec{x}^2\right)$$

- straightforward to show δS satisfies wave equation in dS_d

$$\left(\nabla^2_{dS} - m^2\right)\,\delta S = 0 \qquad \text{with} \qquad m^2\,L^2 = -d$$

Entanglement Holography v1.0:

- de Sitter metric: $ds^2 = \frac{L^2}{R^2}\left(-dR^2 + d\vec{x}^2\right)$
- wave equation $\left(\nabla^2_{dS} - m^2\right)\,\delta S = 0$ with $m^2\,L^2 = -d$

$\longrightarrow$ 2 independent sol's: $\delta S \overset{R\to 0}{=} F(\vec{x})/R + f(\vec{x})\,R^d + \cdots$

$\Delta = -1$ $\qquad$ $\Delta = d$

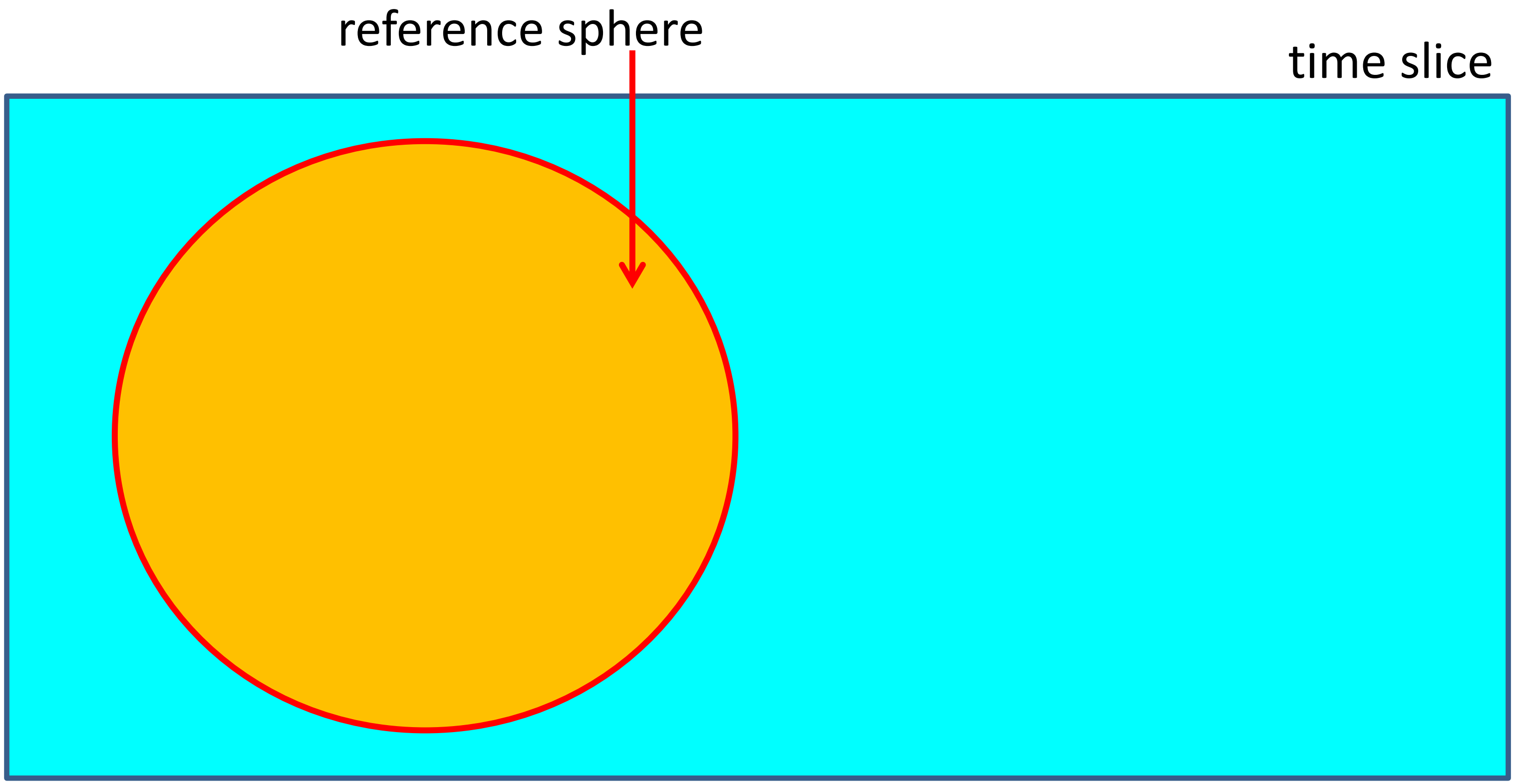
- "1[st] law" solution: $\delta S(R,\vec{x}) = 2\pi \int_B d^{d-1}y\,\frac{R^2 - |\vec{y}-\vec{x}|^2}{2R}\,\langle T_{tt}(\vec{y})\rangle$

$$\longrightarrow \qquad F(\vec{x}) = 0\;; \qquad f(\vec{x}) = \frac{\pi^{\frac{d+1}{2}}}{\Gamma\left(\frac{d+3}{2}\right)}\,\langle T_{tt}(\vec{x})\rangle$$

- $\langle T_{tt}\rangle$ sets δS at very small R and EE perturbations at larger scales determined by the **local Lorentzian propagation** into dS geometry
- $m^2\,L^2 = -d$: mass tachyonic! $\rightarrow$ above precisely removes the "non-normalizable" or unstable modes

New “holographic” coordinate is time-like. Really?

- geometry naturally gives partial ordering of spheres



(ordering of intervals for d=2 discussed by Czech, Lamprou, McCandlish & Sully)

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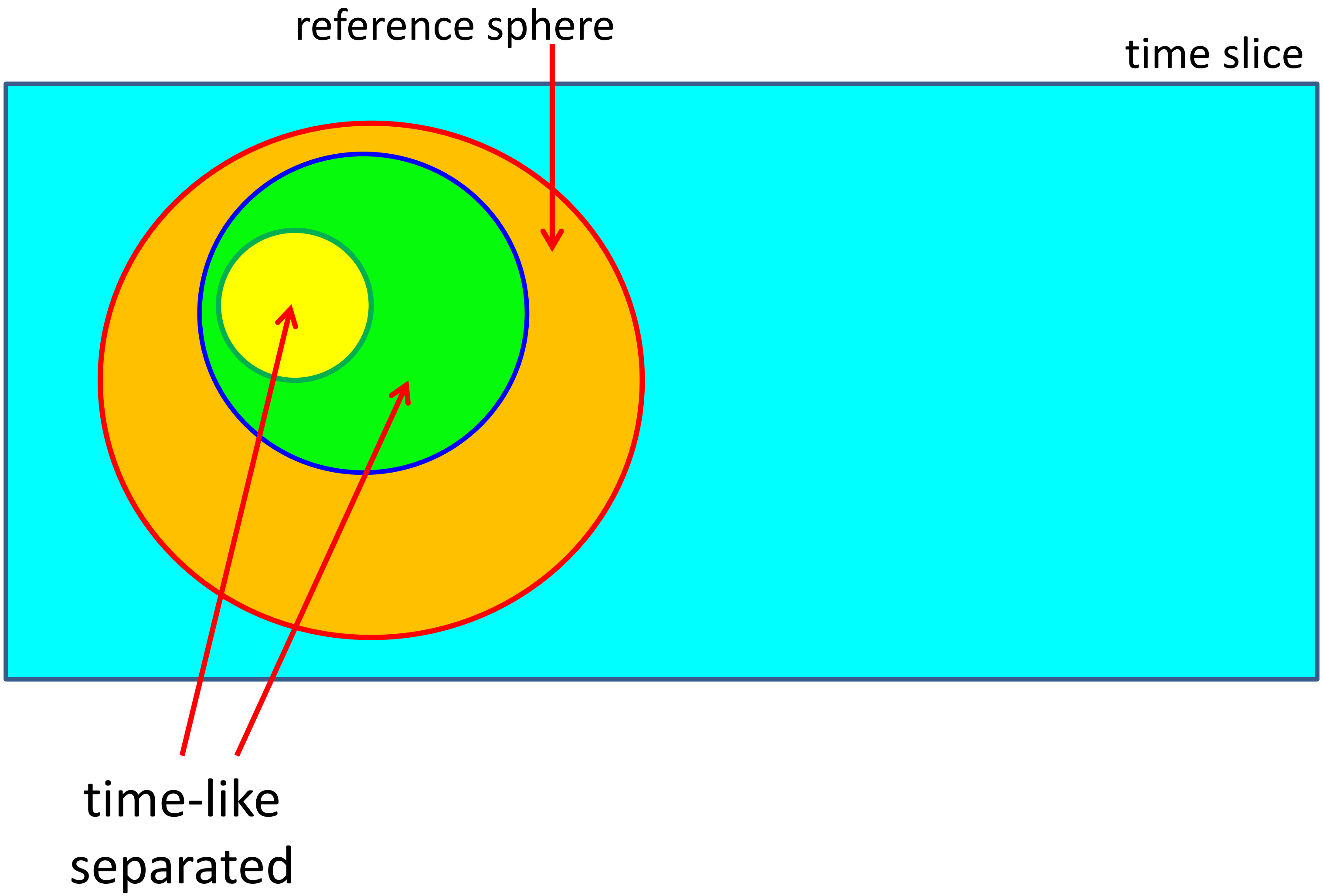

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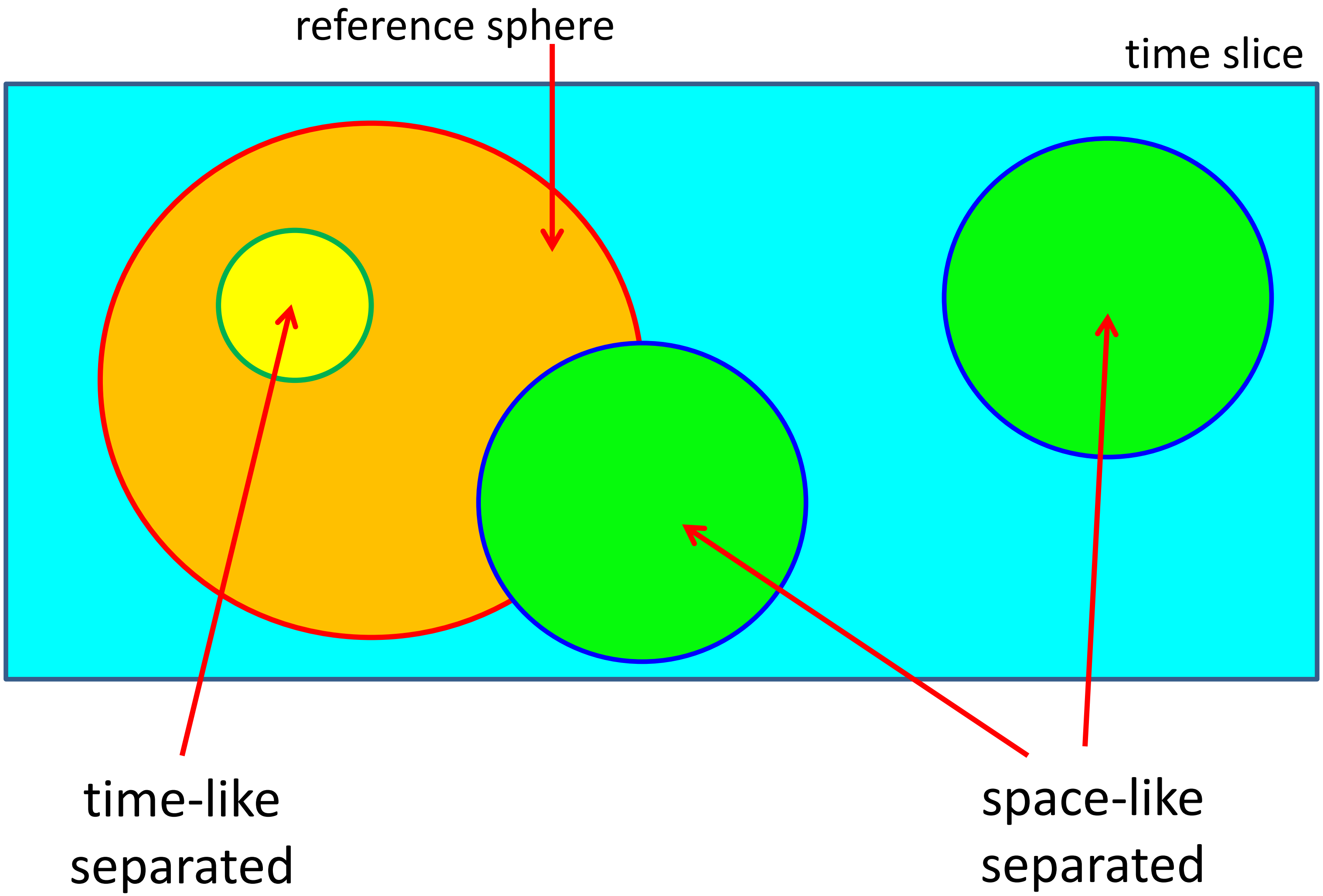

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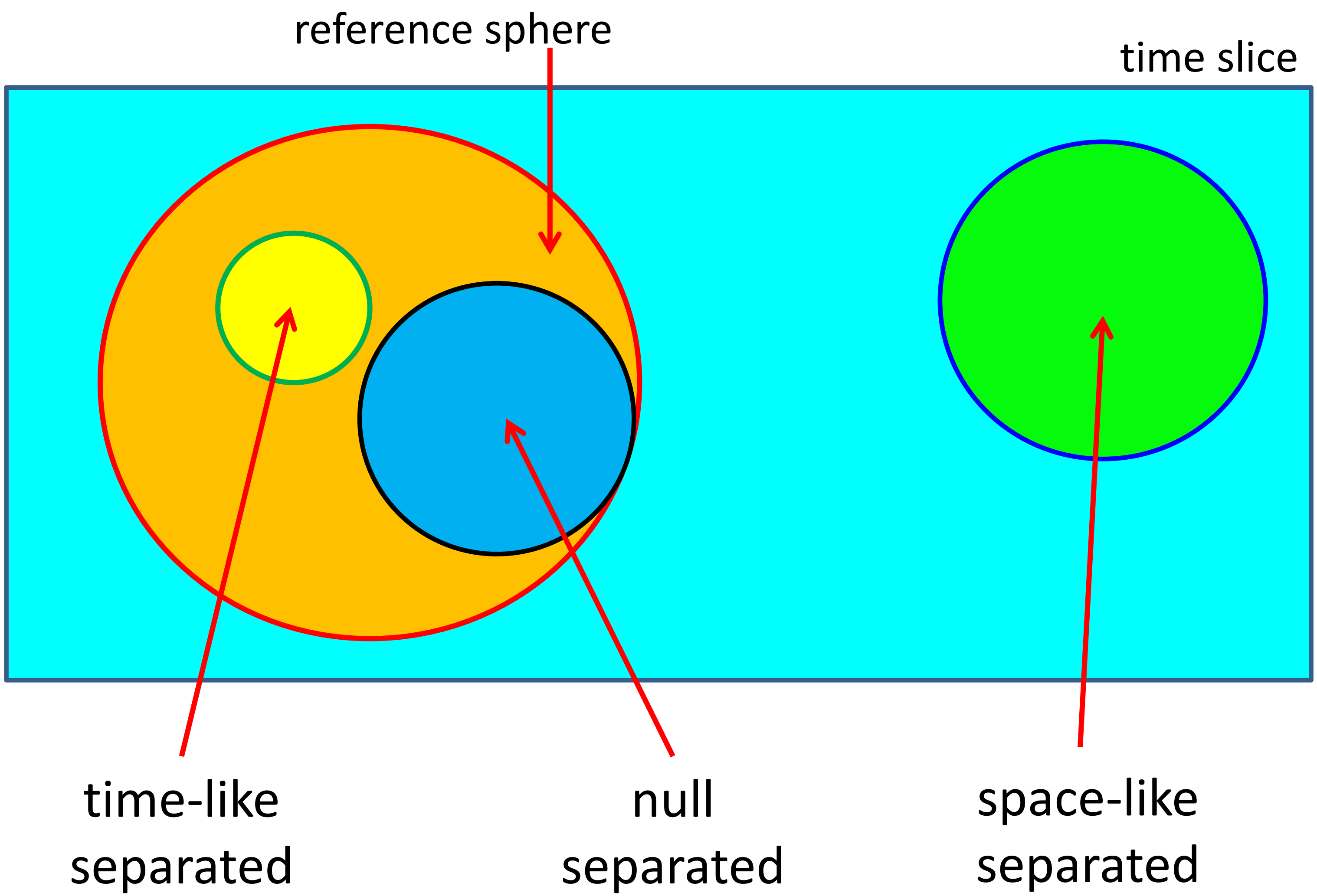
→ suggests auxiliary/holographic geometry should be Lorentzian



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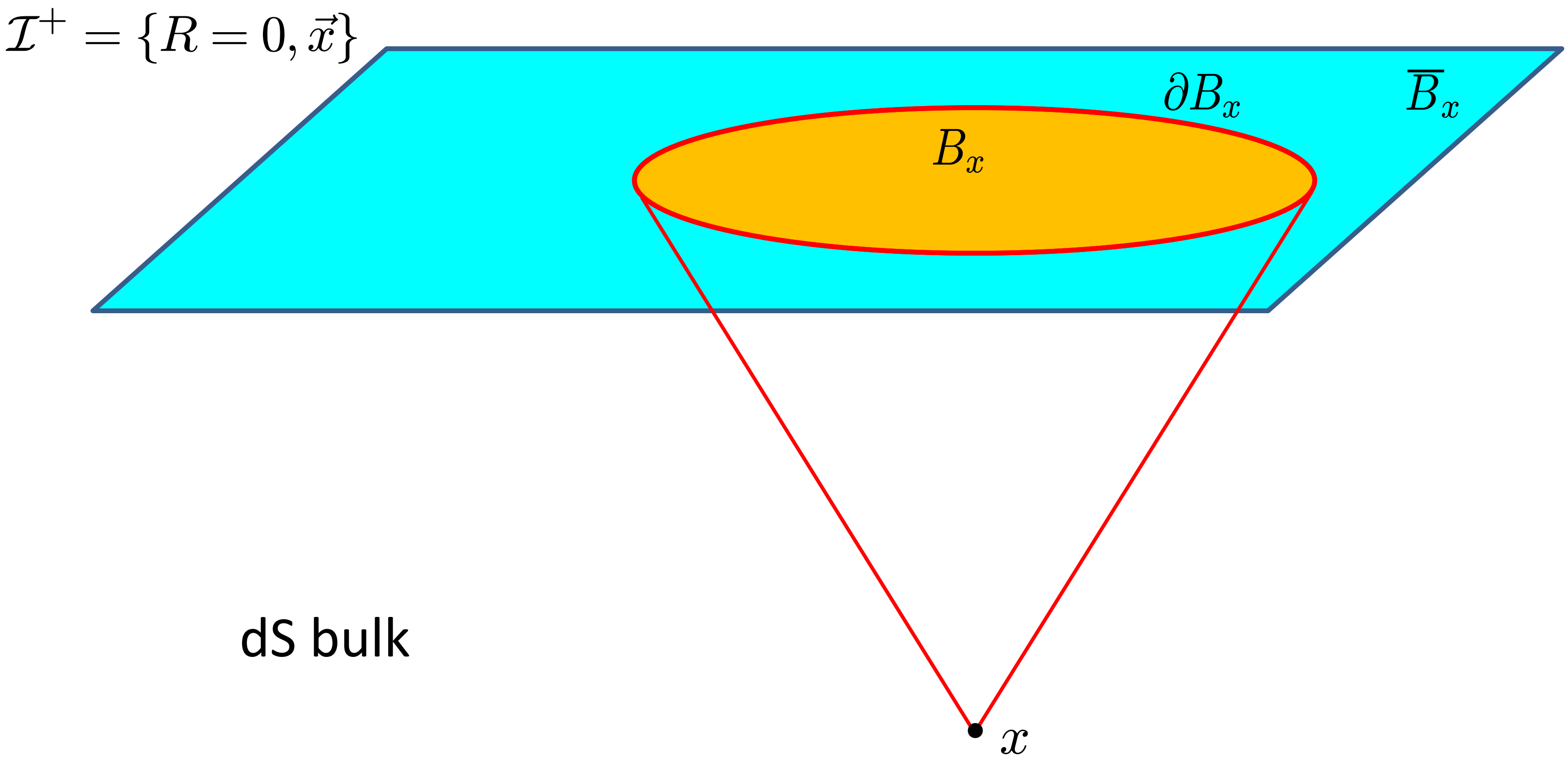
Mapping deSitter ↔ Balls?

- choose one of asymptotic boundaries of dS (eg, $\mathcal{I}^+$) ↔ time slice
- for any point x in bulk and send out future light cone to $\mathcal{I}^+$
- intersects $\mathcal{I}^+$ on a sphere and interior uniquely defines `dual' ball B_x



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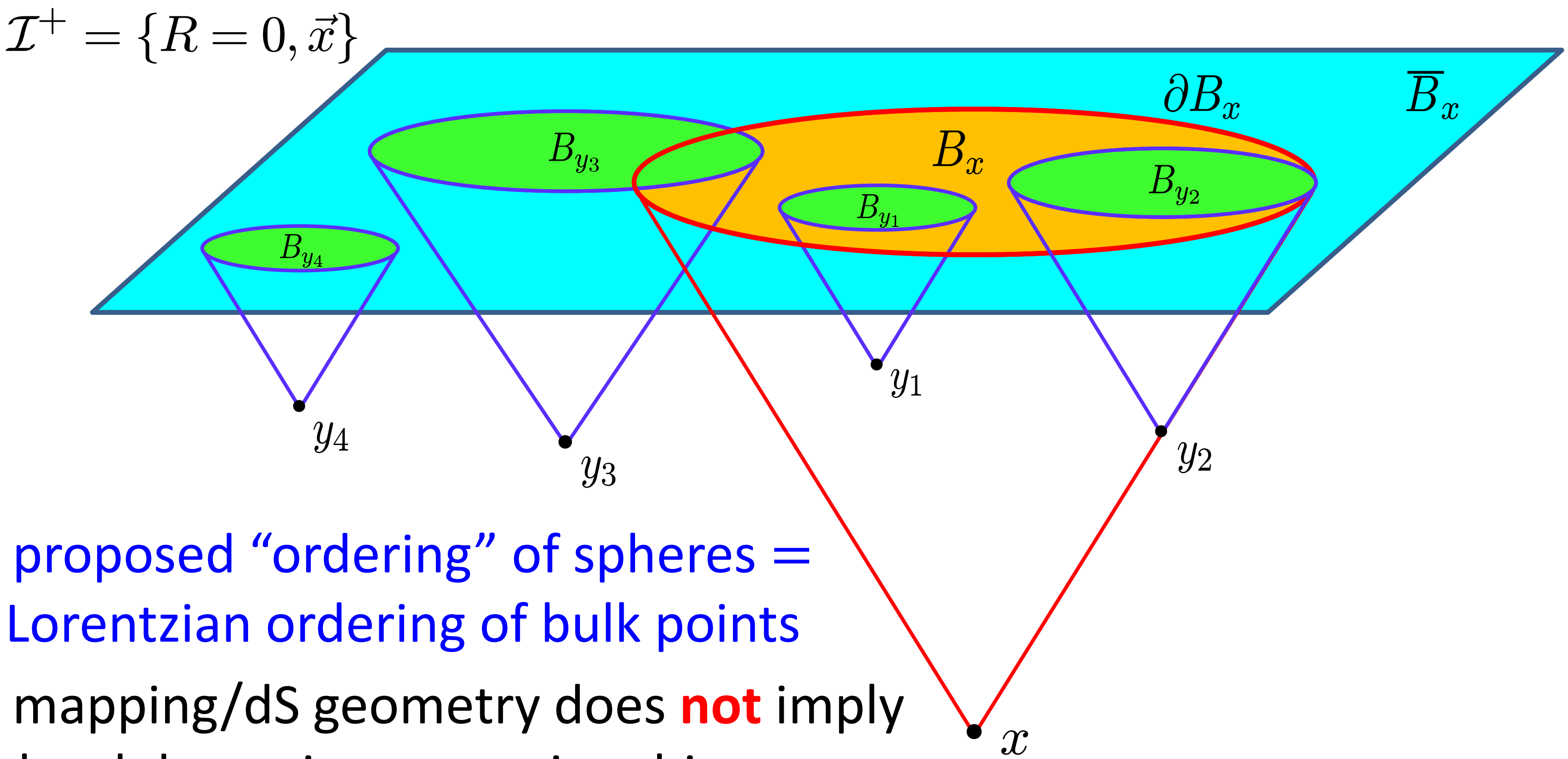


- proposed "ordering" of spheres = Lorentzian ordering of bulk points
- mapping/dS geometry does **not** imply local dynamics respecting this structure

Comments:

- same wave equation derived from AdS/CFT correspondence

Nozaki, Numasawa, Prudenziati& Takayanagi: arXiv:1304.7100
Bhattacharya, Takayanagi: arXiv:1308.3792

- Eg, linearized Einstein eqs in AdS$_4$ implied for holographic EE

$$\left[\frac{\partial^2}{\partial R^2} - \frac{1}{R}\frac{\partial}{\partial R} - \frac{3}{R^2} - \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2}\right] \delta S(t,x,y,R) = 0$$

- can be recast as d=3 deSitter wave equation:

$$\left[\underbrace{-\frac{R^3}{L^2}\frac{\partial}{\partial R}\left(\frac{1}{R}\frac{\partial}{\partial R}\right) + \frac{R^2}{L^2}\frac{\partial^2}{\partial x^2} + \frac{R^2}{L^2}\frac{\partial^2}{\partial y^2}}_{\text{d'Alembertian on dS}_3} + \underbrace{\frac{3}{L^2}}_{\text{mass term}}\right] \delta S(t,x,y,R) = 0$$

- here, we see equation readily extends to any d and follows purely from underlying conformal symmetry

Comments:

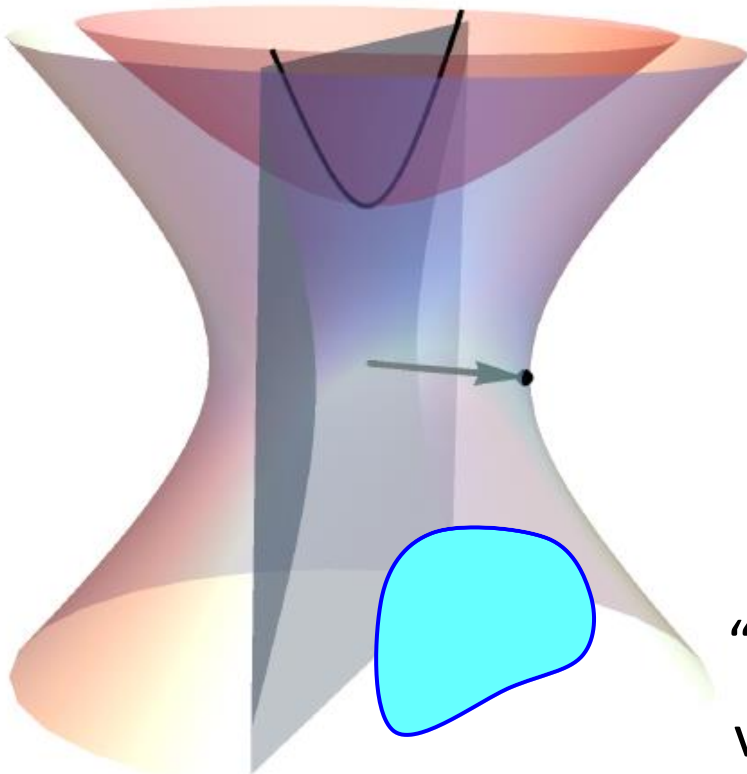
- deSitter geometry appears in recent discussions of integral geometry and the interpretation of MERA in terms of AdS_3/CFT_2

(Czech, Lamprou, McCandlish & Sully: arXiv:1505.05515; arXiv:1512.01548)

- consider space of intervals u<x<v on time slice of 2d holographic CFT

$\longleftrightarrow$ space of geodesics on 2d slice of AdS_3 $\longleftrightarrow$ pts in 2d de Sitter

AdS/CFT



$$ds^2 = L^2\,\frac{du\,dv}{(v-u)^2}$$

dS scale?

motivate the choice: $L^2 = \dfrac{c}{3}$

$$\longrightarrow\ ds^2 = \partial_u\partial_v S_0\,du\,dv$$

with $S_0 = \dfrac{c}{3}\log\dfrac{v-u}{\delta}$

"hole-ography":

volume in dS_2 = length in AdS_3 slice

Entanglement Holography v1.0 – Recap

- EE of excitations of CFT vacuum arranged in novel holographic manner
- δS satisfies wave equation in dS_d where scale plays the role of time

$$\left(\nabla^2_{dS} - m^2\right)\,\delta S = 0 \quad \text{with} \quad m^2 L^2 = -d$$

- $\langle T_{tt}\rangle$ sets δS at very small R and EE perturbations at larger scales determined by the local Lorentzian propagation into dS geometry

$\longrightarrow$ applies for any CFT in any d; relies only on the 1st law of entanglement; does **not** require strong coupling or large # dof

Question:

Is this only some "kinematic" constraint on entanglement in CFTs?

or

Is there a novel re-organization of CFT where nonlocal observables yield local field theory propagating in dS spacetime?

Question: Other dynamical fields in dS space?

Extension to Higher Spin Charges:

- CFT with conserved symmetric traceless currents $T_{\mu_1 \cdots \mu_s}$ with $s \geq 1$
- modular Hamiltonian is flux of $J^{(2)}_\mu = T_{\mu\nu} K^\nu$ through B where K^ν is conformal Killing vector that leaves ∂B invariant

$$\longrightarrow \quad H_B = \int d\Sigma^\mu \, J^{(2)}_\mu$$

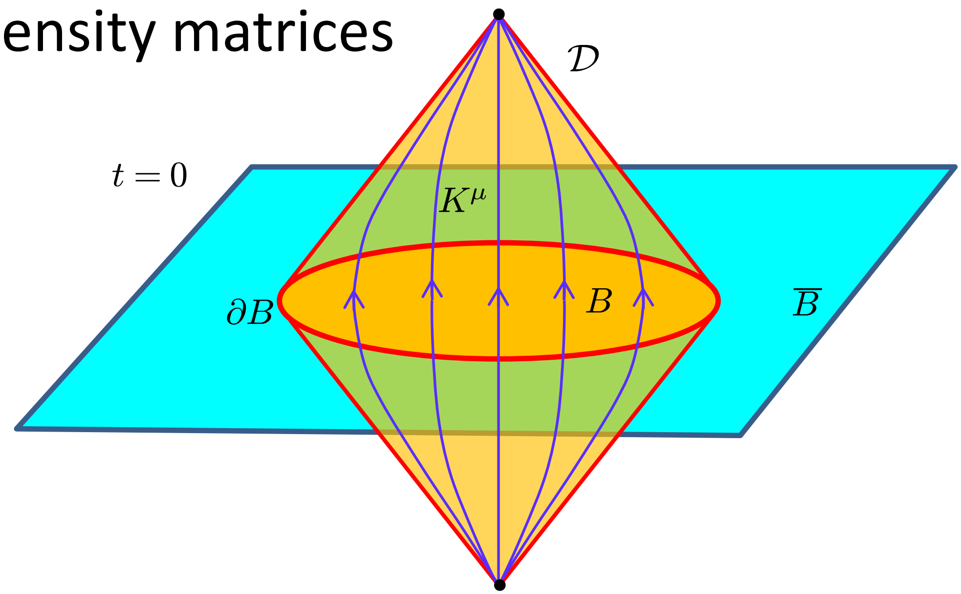
- extends to higher spin charges:

$$\delta Q^{(s)} = \int d\Sigma^\mu \, J^{(s)}_\mu \quad \text{with} \quad J^{(s)}_\mu = T_{\mu\mu_2\cdots\mu_s} K^{\mu_2} \cdots K^{\mu_s}$$

- appear in discussion of modified density matrices

$$\rho_B \sim \exp\left[-\sum \mu_s \, \delta Q^{(s)} \right]$$

(s≥3: Hijano & Kraus;
s=1: Belin, Hung etal)



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- on *t=0* slice, yields:

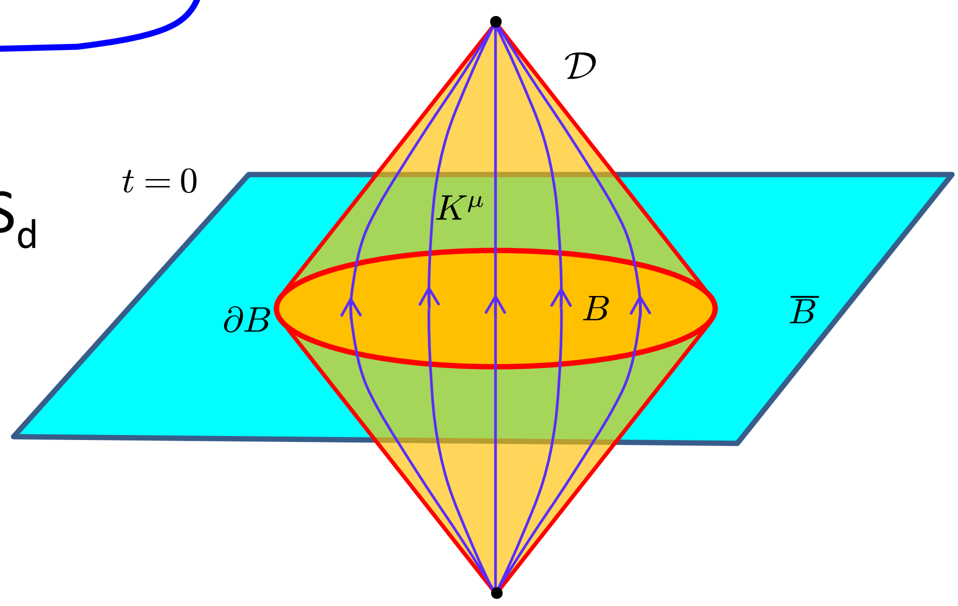
$$\delta Q^{(s)} = (2\pi)^{s-1} \int_B d^{d-1}y \underbrace{\left(\frac{R^2 - |\vec{x} - \vec{y}|^2}{2R} \right)^{s-1}}_{\text{bdry-to-bulk propagator for deSitter}} T_{tt\ldots t}(\vec{y})$$

- $\delta Q^{(s)}$ satisfies wave equation in $\mathrm{dS_d}$

$$\left(\nabla^2_{dS} - m^2\right) \, \delta Q^{(s)} = 0$$

with

$$m^2 \, L^2 = -(s-1)(d+s-2)$$



(see also Czech, Lamprou, McCandlish, Mosk & Sully: arXiv:1604.03110)

Question: What about time dependence in CFT?

- so far focused on single time slice; natural to consider perturbations of EE for all spheres throughout spacetime on any time slice & any frame

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Question: What about time dependence in CFT?

- so far focused on single time slice; natural to consider perturbations of EE for all spheres throughout spacetime on any time slice & any frame
- adopt group theoretic perspective of wave equation:

→ background for spheres on fixed time slice:

$$SO(1,d)/SO(1,d-1) \simeq \text{d-dim. deSitter space}$$

symmetries leaving time slice invariant

symmetries leaving sphere invariant

(see also Czech, Lamprou, McCandlish, Mosk & Sully: arXiv:1604.03110)

Entanglement Holography v2.0:

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symmetries leaving time slice invariant

symmetries leaving sphere invariant

$\longrightarrow$ background for spheres throughout spacetime:

$$SO(2,d)/\left[SO(1,d-1)\times SO(1,1)\right]$$

$\longrightarrow$ $2d$-dimensional space

(see also Czech, Lamprou, McCandlish, Mosk & Sully: arXiv:1604.03110)

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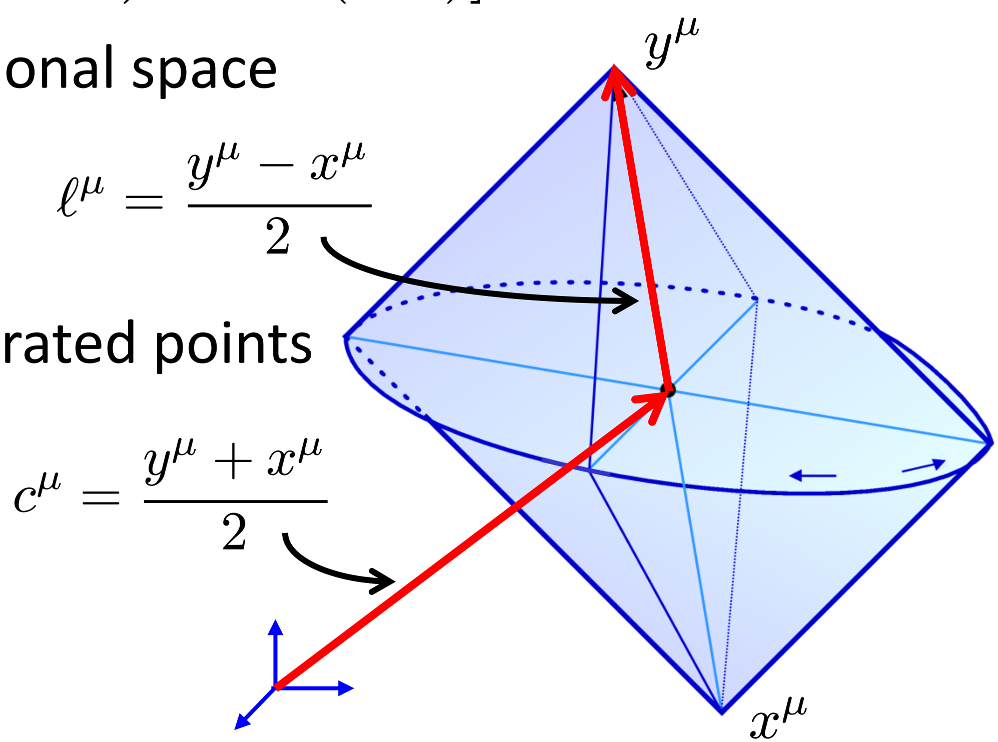
- moduli space of spheres
  = m.s. of causal diamonds
  = m.s. of pairs of time-like separated points

$$(y^\mu, x^\mu)$$



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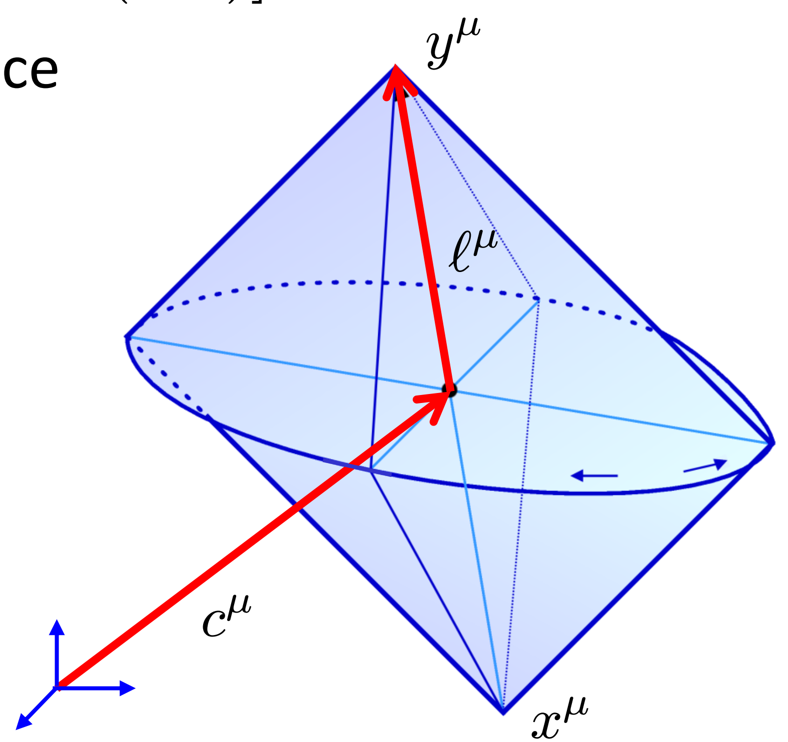

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$\longrightarrow$ $2d$-dimensional space

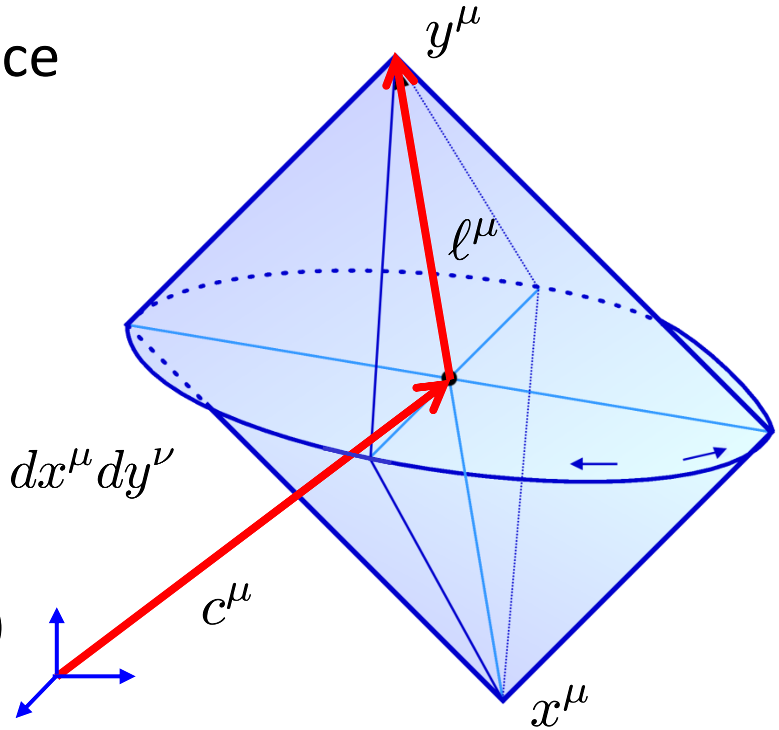
$\longrightarrow$ signature: (d,d)

too many times?!?!

need more eoms!?!?

- natural metric:

$$ds^2_\diamond = \frac{4L^2}{(x-y)^2}\left(-\eta_{\mu\nu}+\frac{2(x_\mu-y_\mu)(x_\nu-y_\nu)}{(x-y)^2}\right)dx^\mu dy^\nu$$
$$= -\frac{L^2}{\ell^2}\left(\eta_{\mu\nu}-\frac{2}{\ell^2}\ell_\mu\ell_\nu\right)\left(dc^\mu\, dc^\nu - d\ell^\mu\, d\ell^\nu\right)$$



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$\longrightarrow$ $2d$-dimensional space

$\longrightarrow$ signature: (d,d) $\longleftarrow$ too many times?!?! need more eoms!?!?

- special case: **d=2**

$$\begin{aligned} SO(2,2)/\left[SO(1,1)\times SO(1,1)\right] & \\ &= SO(2,1)/SO(1,1)\ \times\ SO(2,1)/SO(1,1) \\ &= dS_2\ \times\ dS_2 \end{aligned}$$

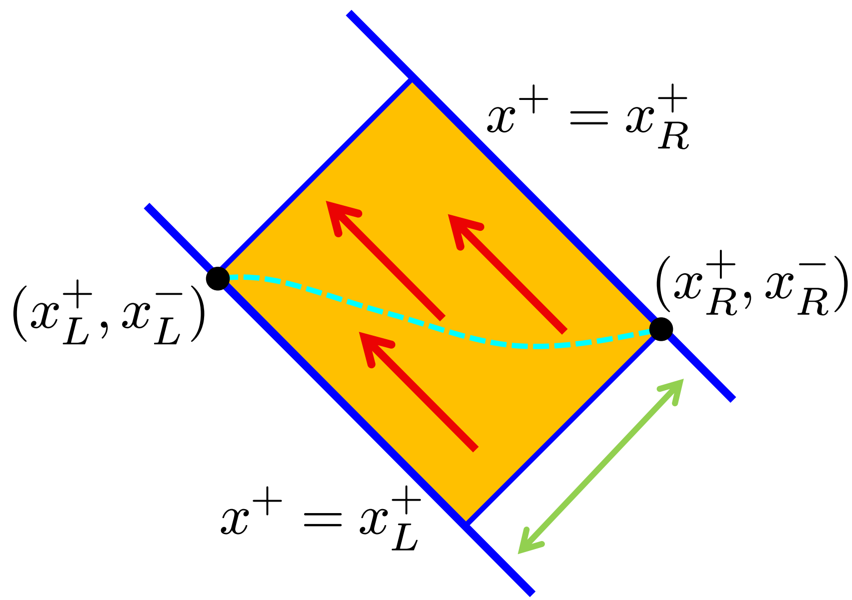
Need more eoms!?!?

- focus on d=2 CFT where found $dS_2 \times dS_2$
- natural to split δS into $\delta S_\pm$ = contributions of left/right-movers

eg, **in 1st law limit**: $\delta S_+ = 2\pi \int d\xi^+ \frac{(x_R^+ - \xi^+)(\xi^+ - x_L^+)}{x_R^+ - x_L^+} \langle T_{++}\rangle(\xi^+)$

$x^\pm = (x \pm t)/\sqrt{2}$

- $\delta S_\pm$ propagate on separate dS_2 geometries, eg, $ds^2 = L^2 \frac{dx_R^+\, dx_L^+}{(x_R^+ - x_L^+)^2}$



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- $\delta S_\pm$ propagate on separate dS_2 geometries, eg, $ds^2 = L^2 \frac{dx_R^+ \, dx_L^+}{(x_R^+ - x_L^+)^2}$

$\longrightarrow$ $\left(\nabla_+^2 - m_+^2\right) \, \delta S_+ = 0$ with $m_+^2 \, L^2 = -2$

implicitly: $\left(\nabla_-^2 - m_-^2\right) \, \delta S_+ = 0$ with $m_-^2 \, L^2 = 0$

- $\delta S_\pm$ propagate nontrivially on $dS_\pm$ and trivially on $dS_\mp$

$\longrightarrow$ two "standard" second-order wave equations

Question: What about interacting fields?

Beyond 1st Law: $\delta S_A \;\leq\; \delta\langle H_A\rangle$

- **specialize:** d=2; “conformally” excited states

$$w^+ = f_+(x^+) \quad \text{and} \quad w^- = f_-(x^-) \qquad\qquad x^\pm = (x\pm t)/\sqrt{2}$$

$$\longrightarrow \qquad \langle T_{++}\rangle(x^+) = \frac{c}{12}\left\{\frac{f_+'''}{f_+'} - \frac{3(f_+'')^2}{2(f_+')^2}\right\}$$

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- recall: $S = \lim_{n\to1}\frac{1}{1-n}\log \mathrm{tr}\,\rho^n \quad = \lim_{n\to1}\frac{1}{1-n}\log\langle\sigma_n\,\sigma_{-n}\rangle$

correlator of local primaries

- evaluate change of entropy under local conformal transformations

(Holzhey, Larsen & Wilczek; Calabrese & Cardy)

$$S(w_L^+,w_L^-;w_R^+,w_R^-) = S_+(f_+;w_L^+,w_R^+) + S_-(f_-;w_L^-,w_R^-)$$

with $S_+(f_+;w_L^+,w_R^+) = \dfrac{c}{12}\log\dfrac{\left(f_+(w_R^+)-f(w_L^+)\right)^2}{\delta^2\, f_+'(w_R^+)f_+'(w_L^+)}$

Beyond 1st Law: $\delta S_A \;\leq\; \delta\langle H_A\rangle$

- define: $\delta S_+(w_L^+, w_R^+) = S_+(f_+; w_L^+, w_R^+) \;-\; S_+(f_+(z)=z; w_L^+, w_R^+)$
- for finite shift of state, find nonlinear wave equation:

$$\nabla_+^2\,\delta S_+ = V'(\delta S_+) \qquad \text{with} \quad V'(\delta S_+) = \frac{c}{6L^2}\left[\exp\left(-\frac{12\,\delta S_+}{c}\right) - 1\right]$$

expected m^2 for d=2

$$= -\frac{2}{L^2}\,\delta S_+ \;+\; \frac{12}{c\,L^2}\,\delta S_+^2 + \cdots$$

(also implicitly: $\nabla_-^2\,\delta S_+ = 0$)

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interactions suppressed by central charge

(also implicitly: $\nabla_-^2\,\delta S_+ = 0$)

- **local** dynamics on auxiliary geometry!!

(see also: Beach, Lee, Rabideau & Van Raamsdonk: arXiv:1604.05308)

Beyond 1st Law: $\delta S_A \;\leq\; \delta\langle H_A\rangle$

- define: $\delta S_+(w_L^+, w_R^+) = S_+(f_+; w_L^+, w_R^+) \;-\; \underline{S_+(f_0; w_L^+, w_R^+)}$
- for finite shift of state, find nonlinear wave equation:

$$\nabla_+^2\, \delta S_+ = V'(\delta S_+) \qquad \text{with} \quad V'(\delta S_+) = \frac{c}{6L^2}\left[\exp\left(-\frac{12\,\delta S_+}{c}\right) - 1\right]$$

expected m² for d=2

$$= -\frac{2}{L^2}\,\delta S_+ \;+\; \frac{12}{c\,L^2}\,\delta S_+^2 + \cdots$$

interactions suppressed by central charge

(also implicitly: $\nabla_-^2\, \delta S_+ = 0$)

- **local** dynamics on auxiliary geometry!!
- choosing alternate reference state produces coordinate transformation on dS_2 geometry with $\tilde{w}_R^+ = f_0(w_R^+)$ and $\tilde{w}_L^+ = f_0(w_L^+)$

(see also: Asplund, Callebaut, Zukowski: arXiv:1604.02687; Beach, Lee, Rabideau & Van Raamsdonk: arXiv:1604.05308)

Beyond 1st Law: $\delta S_A \;\le\; \delta\langle H_A\rangle$

- d=2 higher spin CFT (use CS theory with 3d gauge fields & use Wilson line prescription for EE)

(deBoer & Jottar; Ammon, Castro & Iqbal; Hijano & Kraus, …)

$$\nabla^2\,\delta S + \frac{c}{6} - \frac{c}{6}\,\exp\left(-12\,\delta S/c\right)\,\cosh\left(72\,\delta Q^{(3)}/c\right) = 0$$

$$\nabla^2\,\delta Q^{(3)} + \frac{c}{12}\,\exp\left(-12\,\delta S/c\right)\,\sinh\left(72\,\delta Q^{(3)}/c\right) = 0$$

(+/– indices are suppressed)

- theory of two interacting scalar fields with **local** interactions
- appears to be related to Toda theory with same SL(3,R) symmetry

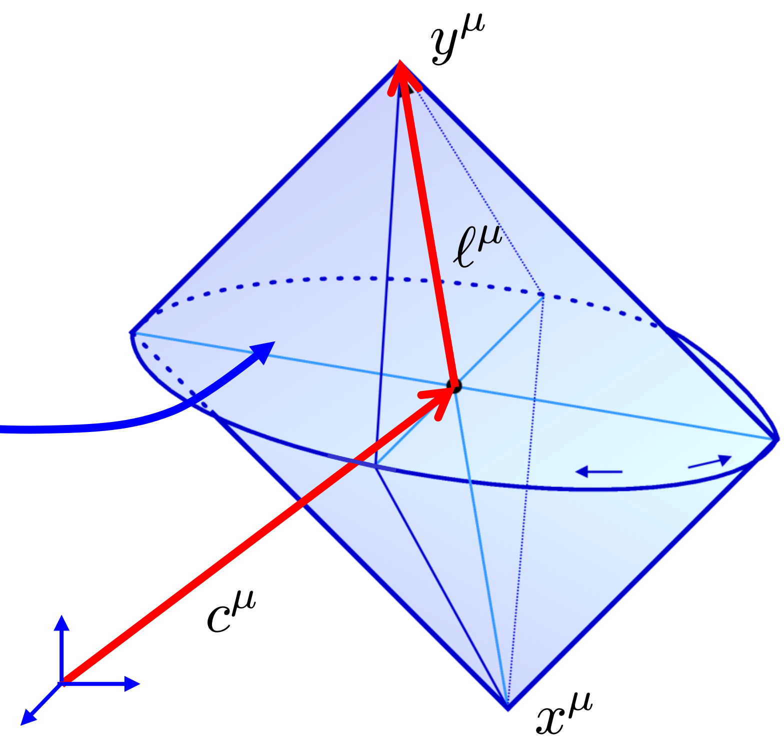
(see also Czech, Lamprou, McCandlish, Mosk & Sully: arXiv:1604.03110)

Beyond conserved currents:

- motivated by first law, define observables:

$$\delta Q(\mathcal{O};x,y) = C_{\mathcal{O}} \int_{D(x,y)} d^d\xi \left(\frac{(y-\xi)^2(\xi-x)^2}{-(y-x)^2} \right)^{\frac{1}{2}(\Delta_{\mathcal{O}}-d)} \langle \mathcal{O}(\xi) \rangle$$

integrate over entire
causal diamond D(x,y)



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- satisfies wave equation of moduli space:

$$(\nabla^2_{\diamond} - m^2_{\mathcal{O}})\, \delta Q(\mathcal{O}) = 0 \qquad \text{with} \quad m^2_{\mathcal{O}} L^2 = \Delta_{\mathcal{O}}(d-\Delta_{\mathcal{O}})$$

- reduces to known "charges" for conserved higher spin currents
- resummation of OPE contributions of $\mathcal{O}$ and all descendants

$\longrightarrow$ conformal blocks (Czech, Lamprou, McCandlish, Mosk & Sully)

- for holographic CFTs, bulk dual given by integral of extremal surface

$$\delta Q_{\text{holo}}(\mathcal{O};x,y) = \frac{C_{\mathcal{O}}}{8\pi G_N} \frac{\Gamma\left(\frac{\Delta_{\mathcal{O}}+2-d}{2}\right)\Gamma\left(\frac{\Delta_{\mathcal{O}}}{2}\right)}{\Gamma\left(\Delta_{\mathcal{O}}-\frac{d}{2}\right)} \int_{B(x,y)} d^{d-1}u\, \sqrt{h}\, \phi(u)$$

Beyond conserved currents:

- motivated by first law, define observables:

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- **need more eoms!?!?**

$$\Gamma_{abcd}(x,y)\,\delta Q(\mathcal{O};x,y) = \cancelto{0}{C_{\mathcal{O}} \int_{D(x,y)} d^d\xi \left(\frac{(y-\xi)^2(\xi-x)^2}{-(y-x)^2}\right)^{\frac{1}{2}(\Delta_{\mathcal{O}}-d)} \langle [\Gamma_{abcd}(\xi),\mathcal{O}(\xi)]\rangle}$$

where $J_{ab} =$ conformal generators with $a,b = -,0,1,\cdots,d-1,d$

- these constraints are not all independent; left with

$$12\,\Gamma_{-d\mu\nu} = 2\{M_{\mu\nu},D\} - \{P_\mu,Q_\nu\} + \{Q_\mu,P_\nu\}$$

Conclusions:

- EE of excitations of CFT vacuum arranged in novel "holographic" way
- δS satisfies wave equation on moduli space of causal diamonds

$$\left(\nabla_{\diamond}^2 - m^2\right)\,\delta S = 0 \qquad \text{with} \quad m^2\,L^2 = -2d$$

⟶ applies for any CFT in any d; relies only on the 1st law of entanglement; does **not** require strong coupling or large # dof

- extends to a variety of other nonlocal observables, as well as an interacting theory on moduli space for two dimensions

Question:

Is this only some "kinematic" constraint on entanglement in CFTs?

or

Is there a novel re-organization of CFT where nonlocal observables yield local field theory propagating in auxiliary spacetime?

Conclusions:

- EE of excitations of CFT vacuum arranged in novel "holographic" way
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Question:

Is this only some "kinematic" constraint on entanglement in CFTs?

Still lots to explore!!

Is there a n[illegible]bservables yield local field theory propagating in auxiliary spacetime?