GRAVITATIONAL PHYSICS FROM ENTANGLEMENT CONSTRAINTS

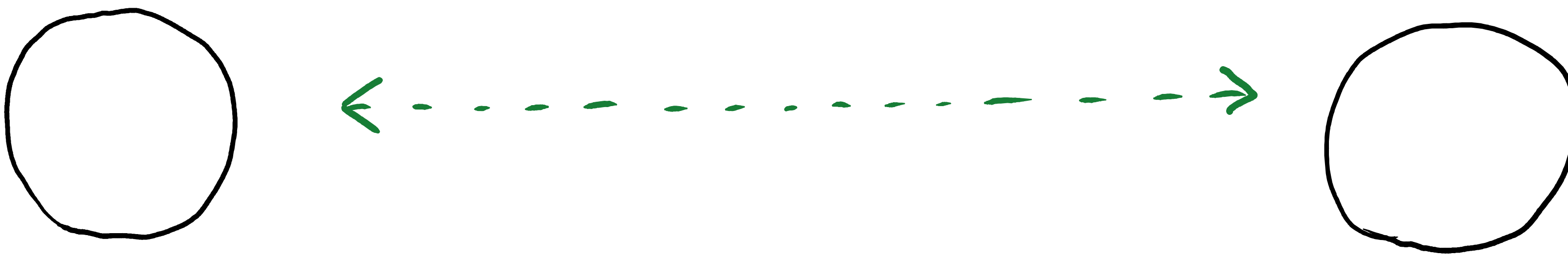
MARK VAN RAAMSDONK, UBC

KYOTO, JUNE 2016

AdS/CFT: CFT on ∂M $\longleftrightarrow$ Quant. grav. for asympt. AdS w. boundary ∂M

(some) CFT states

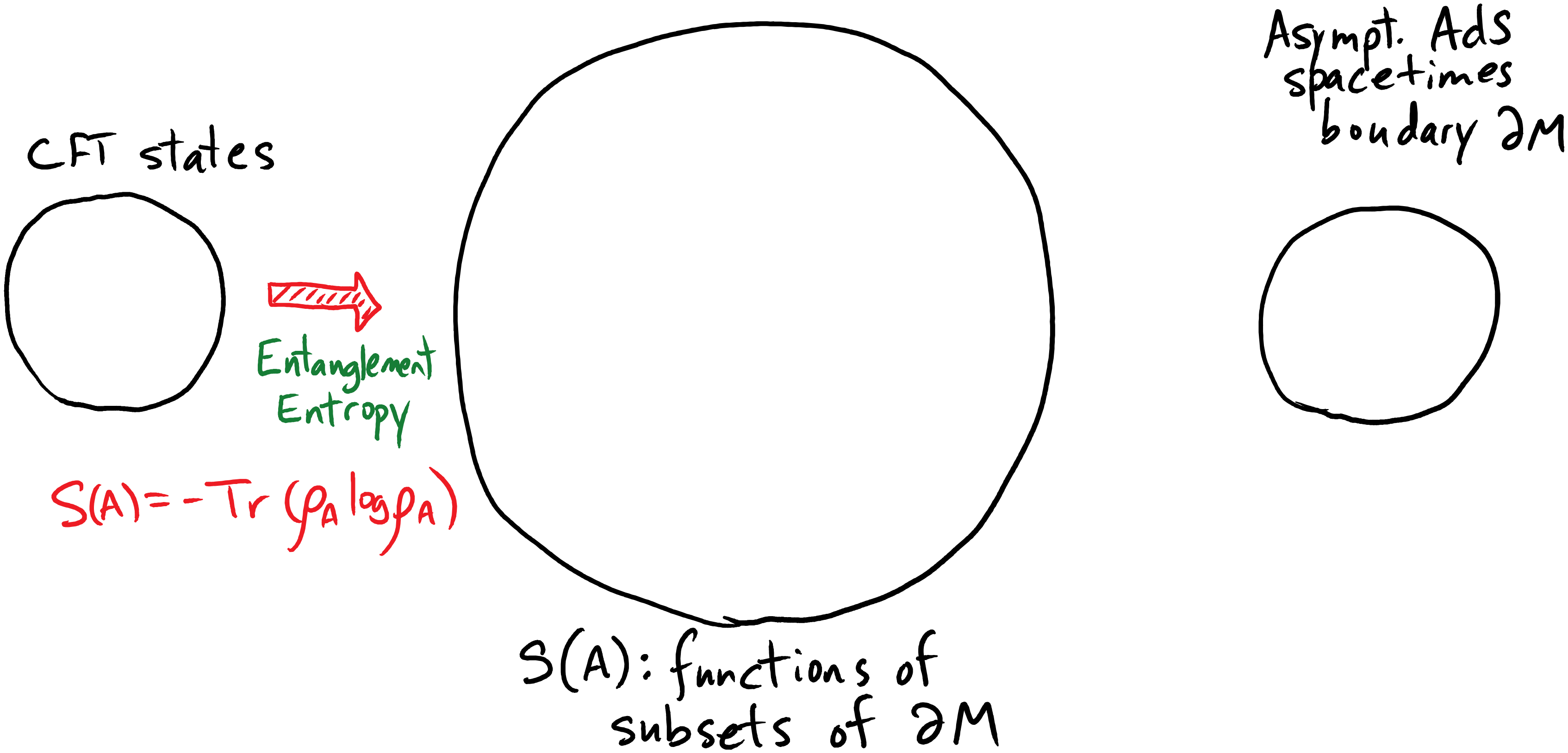
(some) Asympt. AdS spacetimes boundary ∂M



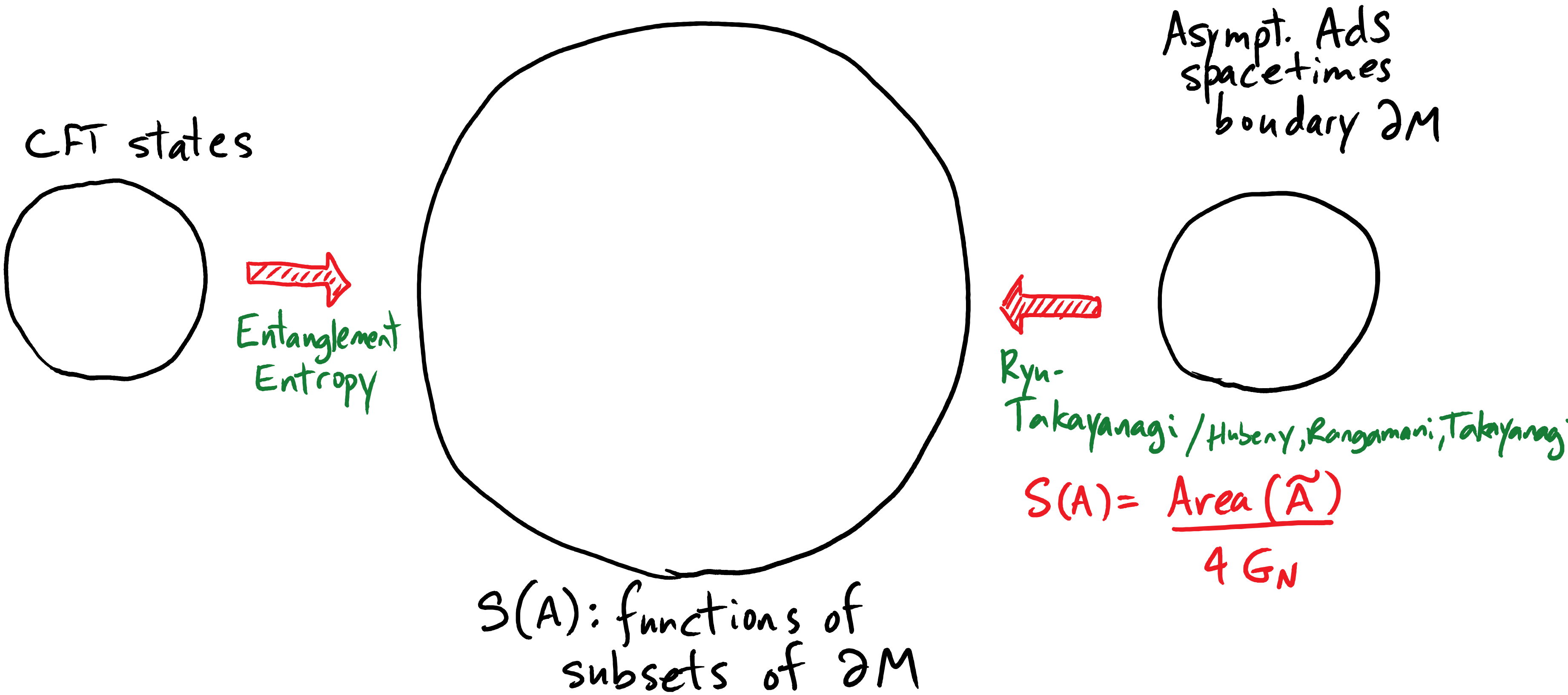
which ones?

which ones?

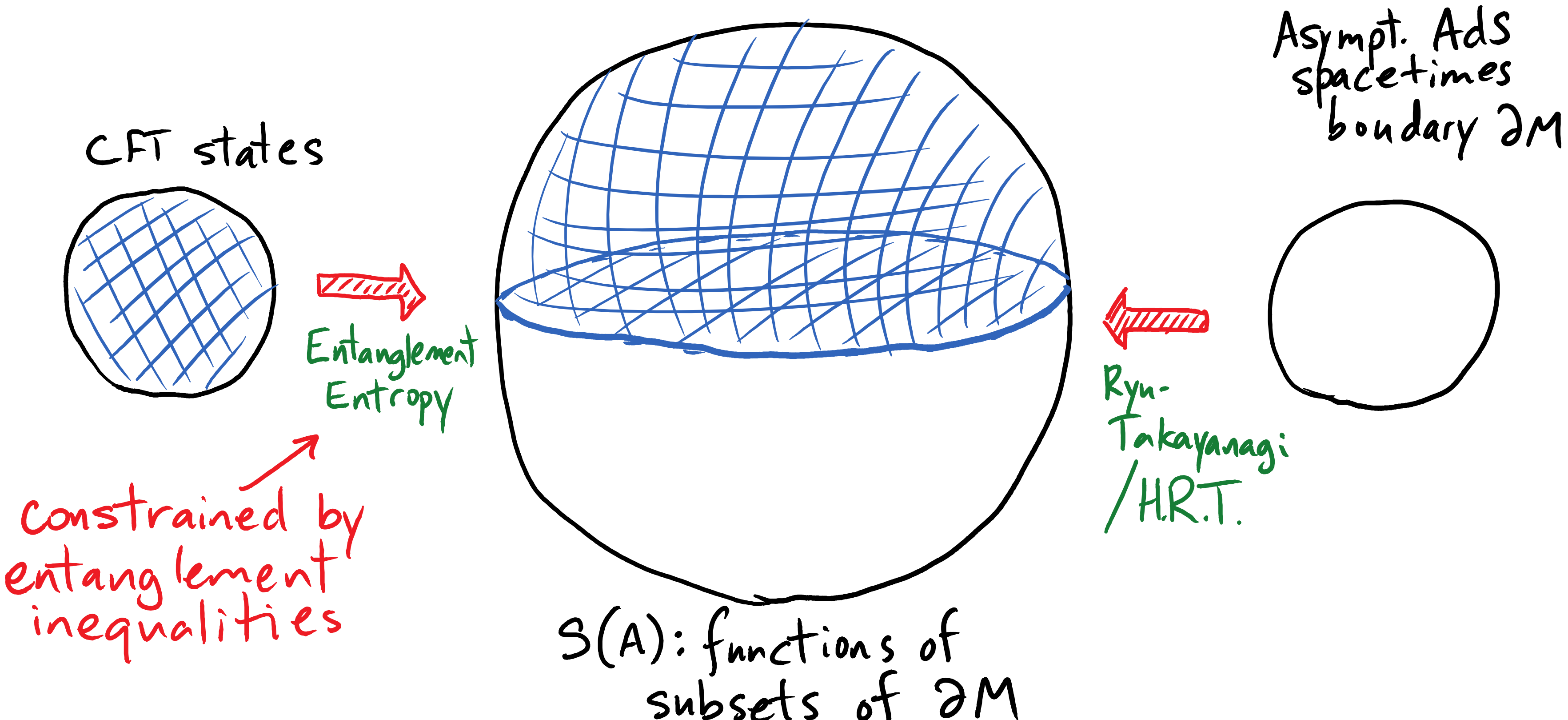
AdS/CFT: CFT on ∂M $\longleftrightarrow$ Quant. grav. for asympt. AdS w. boundary ∂M



AdS/CFT: CFT on ∂M $\longleftrightarrow$ Quant. grav. for asympt. AdS w. boundary ∂M



AdS/CFT: CFT on ∂M $\longleftrightarrow$ Quant. grav. for asympt. AdS w. boundary ∂M

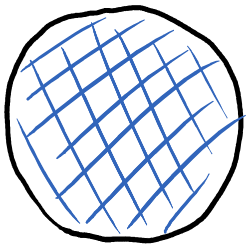


e.g.

$$S(A\cup B)+S(B\cup C)\geq S(B)+S(A\cup B\cup C)$$

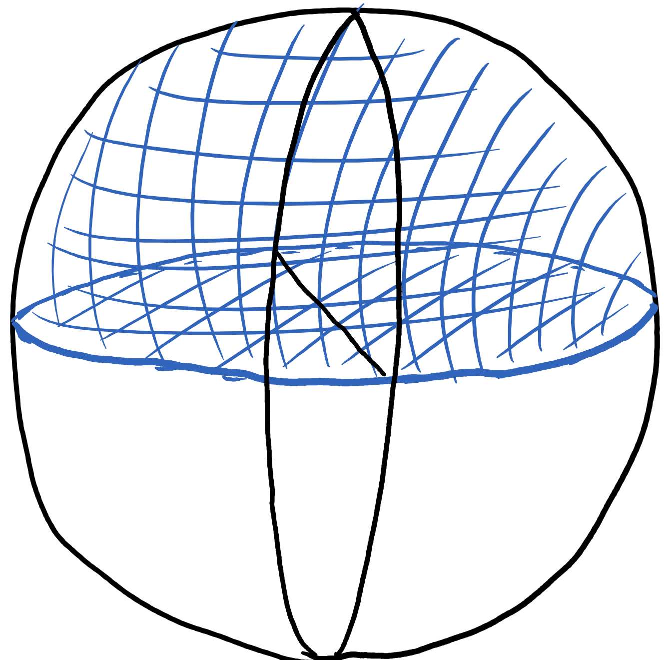
AdS/CFT: CFT on ∂M $\longleftrightarrow$ Quant. grav. for asympt. AdS w. boundary ∂M

CFT states



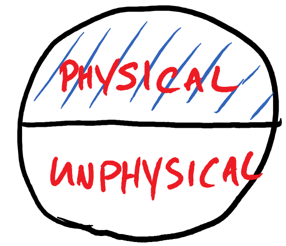
Entanglement Entropy

constrained by entanglement inequalities



$S(A)$: functions of subsets of ∂M

Asympt. AdS spacetimes boundary ∂M



Ryu-Takayanagi /H.R.T.

Which geometries M are allowed?

see: Lashkari
Rabideau
Sabella-Garnier
M.V.R.

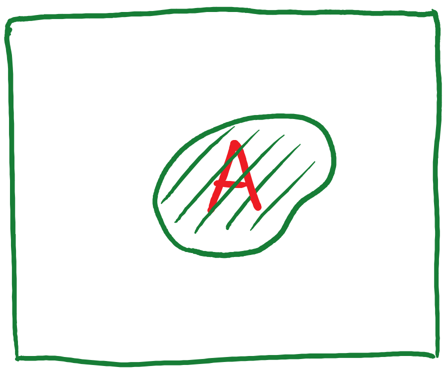
Gives <u>universal</u> constraints:

Spacetime M
with $S(A)$
violating constraints

$\longrightarrow$

Cannot correspond to
consistent state in
ANY UV complete theory
for which dual CFT obeys
RT/HRT.

#1 RELATIVE ENTROPY inequalities:

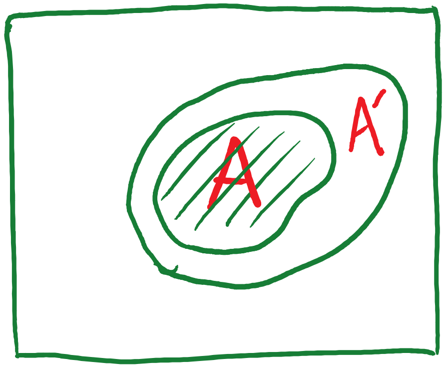


$$|\psi\rangle \longrightarrow \rho_A \qquad\qquad |vac\rangle \longrightarrow \sigma_A$$

$$S(\rho_A \| \sigma_A) \equiv \mathrm{Tr}\left(\rho_A \log \rho_A\right) - \mathrm{Tr}\left(\rho_A \log \sigma_A\right)$$

Measure of distinguishability : POSITIVE for $\rho_A \neq \sigma_A$

#1 RELATIVE ENTROPY inequalities:



$$|\psi\rangle \longrightarrow \rho_A \qquad |vac\rangle \longrightarrow \sigma_A$$

$$S(\rho_A \| \sigma_A) \equiv \mathrm{Tr}(\rho_A \log \rho_A) - \mathrm{Tr}(\rho_A \log \sigma_A)$$

Measure of distinguishability: POSITIVE for $\rho_A \neq \sigma_A$

Also MONATIONIC: $S(\rho_{A'} \| \sigma_{A'}) \geq S(\rho_A \| \sigma_A)$

for $A' \supset A$

What do relative entropy inequalities imply for $\mathcal{M}$?

Blanco, Casini, Hung, Myers

Have: $S(\rho_A \| \sigma_A) = \Delta\langle H_A \rangle - \Delta S_A$

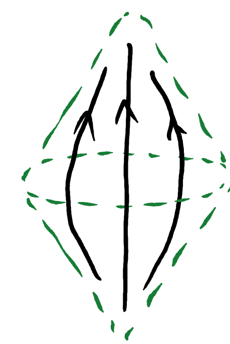
modular Hamiltonian

entanglement entropy

$H_A \equiv -\log \sigma_A$

Ball shaped, $\sigma_A = \sigma_A^{vac}$

$$= \int_B \xi^\mu \langle T_{\mu\nu} \rangle \varepsilon^\nu - \Delta S_B$$



Map inequalities to properties of dual spacetime for holographic $|\psi\rangle$.

Any CFT: $S(\rho_B \| \sigma_B) = 2\pi \int_B \xi^\mu \Delta \langle T_{\mu\nu} \rangle \varepsilon^\nu - \Delta S_B$

Assume $|\psi\rangle$ has entanglement calculated from some $\mathcal{M}$ via RT/HRT

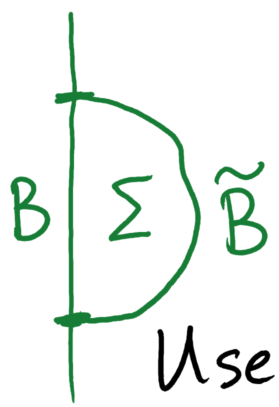
$$ds^2_{\mathcal{M}} = \frac{\ell_{AdS}^2}{z^2}\left[dz^2 + \Gamma_{\mu\nu}(z,x)\, dx^\mu dx^\nu\right]$$

Then:

$$S(\rho_B \| \sigma_B) = 2\pi \int_B \xi^\mu\, c \cdot \Gamma_{\mu\nu}^{(x)}\Big|_{z^d} \varepsilon^\nu - \frac{\Delta \, \mathrm{Area}(\tilde{B})}{4 G_N}$$

integral over B

integral over $\tilde{B}$



Use Wald technology to rewrite as integral over Σ

Start w. perturbative constraints: $|\psi(x)\rangle$, $|\psi(0)\rangle = |vac\rangle$

1st order: $S(\rho_B \| \sigma_B)\Big|_{\theta(\lambda)} = 0$

$\Longrightarrow \quad \delta E_B^{grav} = \delta S_B^{grav}$

$\Longrightarrow \quad \int_\Sigma (\text{something} \propto \text{Einstein tensor}) = 0$

w. McDermott, Lashkari
w. Faulkner, Guica, Hartman, Myers
w. Swingle

constaint = linearized Einstein eqns

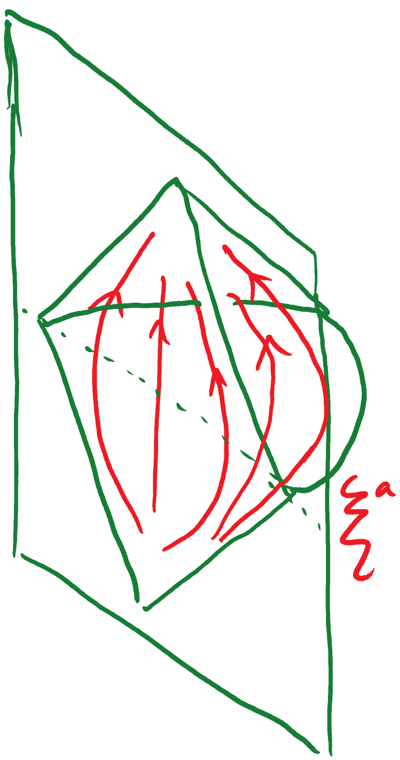
2nd order: $S(\rho_B(\lambda)\|\sigma_B)\big|_{\mathcal{O}(\lambda^2)} \equiv \langle \delta\rho_B, \delta\rho_B \rangle$

(w. Lashkari)

QUANTUM FISHER INFORMATION

Maps to: $W_\Sigma(g, \delta g, \mathcal{L}_\xi \delta g) \equiv \mathcal{E}(\delta g, \delta g)$

CANONICAL ENERGY



ζ_B extends to ξ_B: Killing vector of pure AdS = Rindler time

$$\mathcal{E}(\delta g, \delta g) \sim \int \xi^a T^{(2)}_{ab} \varepsilon_b + \mathcal{E}^{grav}_{perturbative}$$

= gravitational + matter energy associated w. ξ

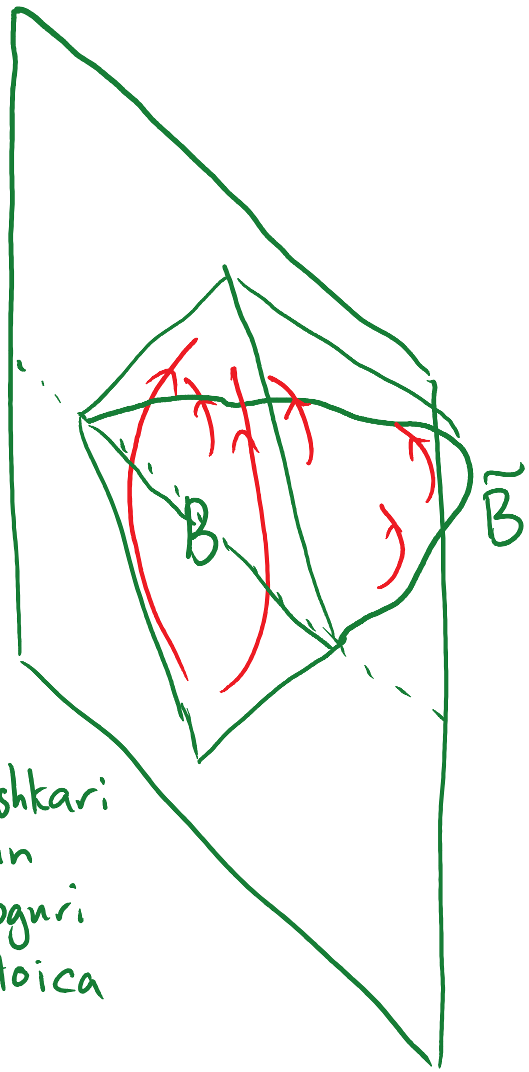
Positivity/monotonicity give energy conditions

Non-perturbative: $S(\rho_B \| \sigma_B)$ again maps to bulk integral.

→ Can interpret as energy associated w. vector field X

Require: $X \to \xi$, $\nabla_{(a} X_{b)} \to 0$ at B

$X \to 0$, $\nabla^a X^b \to 2\pi n^{ab}$ at $\tilde{B}$



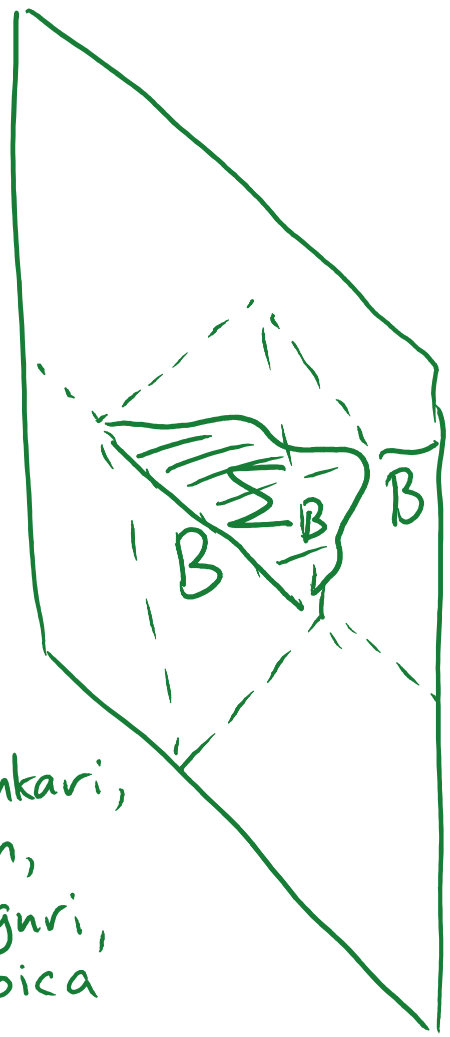
w. Lashkari
Lin
Ooguri
Stoica

Then:

$$S(\rho_B \| \sigma_B) = \int_\Sigma \underbrace{J_X}_{\text{Noether charge}} = H_X$$

Independent details of X!

New positive energy theorem for subsystems.



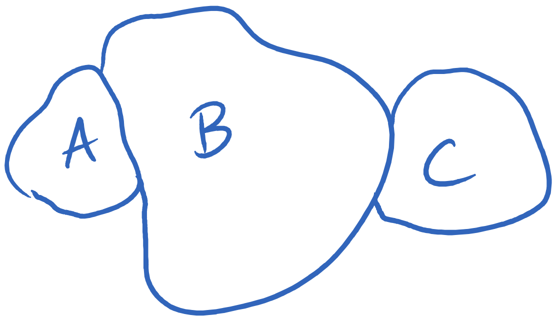
w. Lashkari, Lin, Ooguri, Stoica

Given any asympt. AdS M in consistent UV completion of Einstein grav + matter

For any $B \subset \partial M$ can define energy H_X associated w. Σ_B.

☆ H_X must be positive + increase w. increasing B ☆

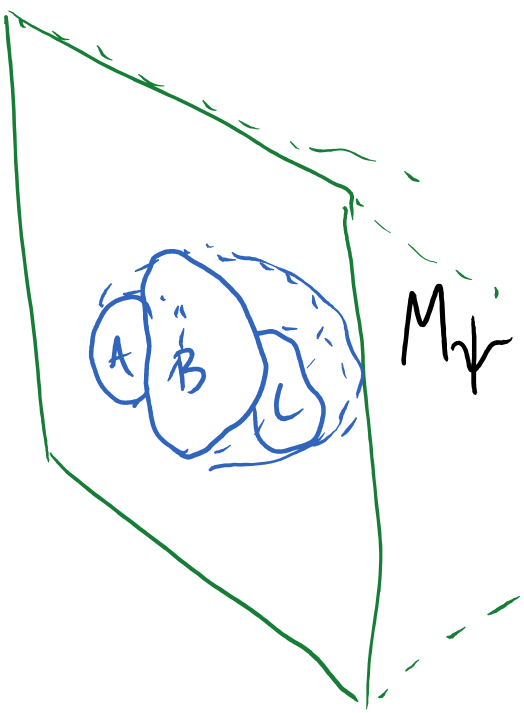
#2 Strong Subadditivity



$$S(AB) + S(BC) \geq S(B) + S(ABC)$$

$$\Downarrow$$

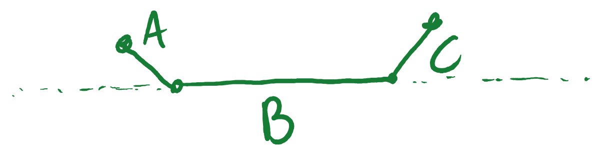
$$\text{Area}(\widetilde{AB}) + \text{Area}(\widetilde{BC}) \geq \text{Area}(\widetilde{B}) + \text{Area}(\widetilde{ABC})$$



What are necessary + sufficient conditions on M_ψ for this to be true?

Headrick+Takayanagi: automatic for static geometries, A,B,C in preferred spatial slice

BUT: nontrivial constraints by considering A,B,C in general slices.

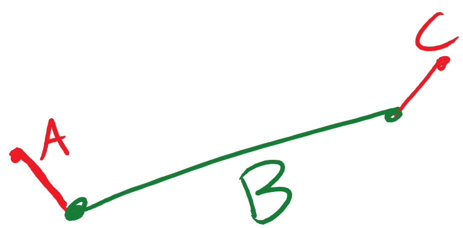


Wall: NEC is sufficient

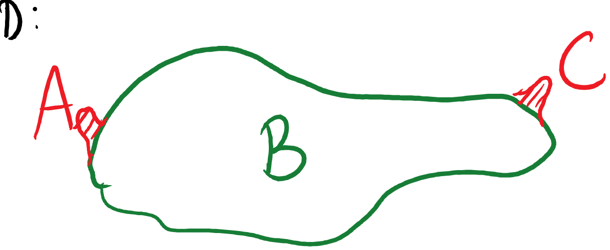
Useful simplification:

Only need to consider SSA for A, C infinitesimal, lightlike perturbations to B:

2D:



Higher D:



Combine SSA inequalities for these to obtain general SSA inequalities.

Dual of SSA for this case: some constraint localized to extremal surface $\tilde{B}$

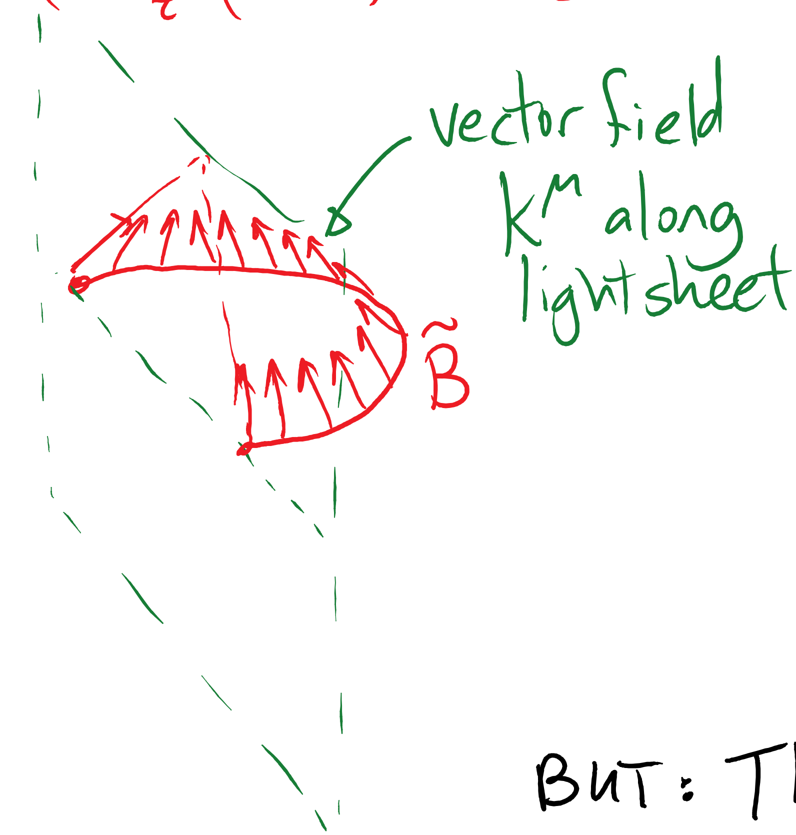
$$\int_{\tilde{B}} (\text{covariant}) \geq 0$$

Result for simplest case:

- w. Lashkari, Rabideau, Sabella-Garnier
- Bhattacharya, Hubeny, Rangamani, Takayanagi

2D, static, translation-invariant

$$ds^2 = \frac{1}{z^2}\left(dz^2 + f(z)dx^2 - g(z)dt^2\right)$$

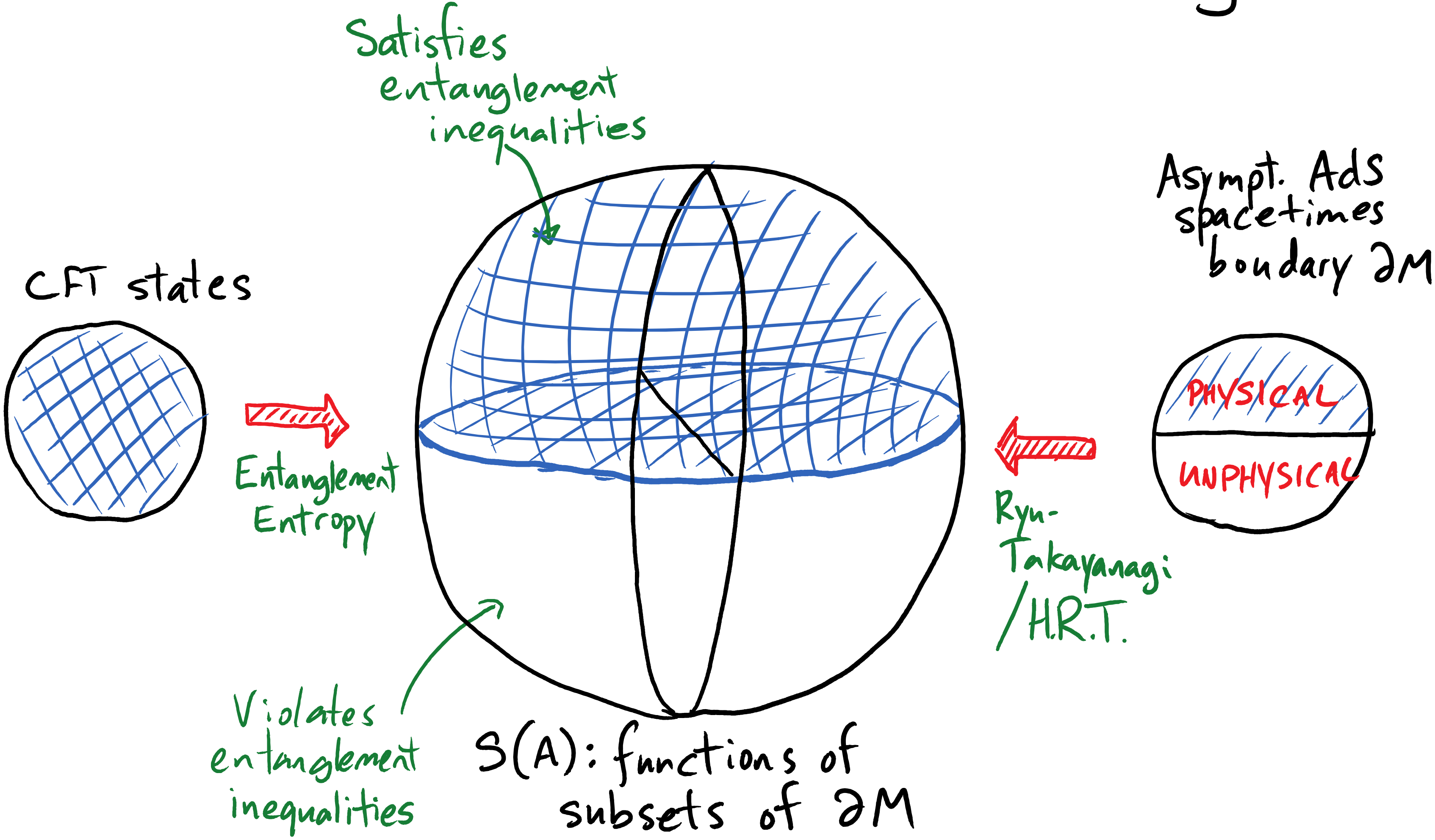


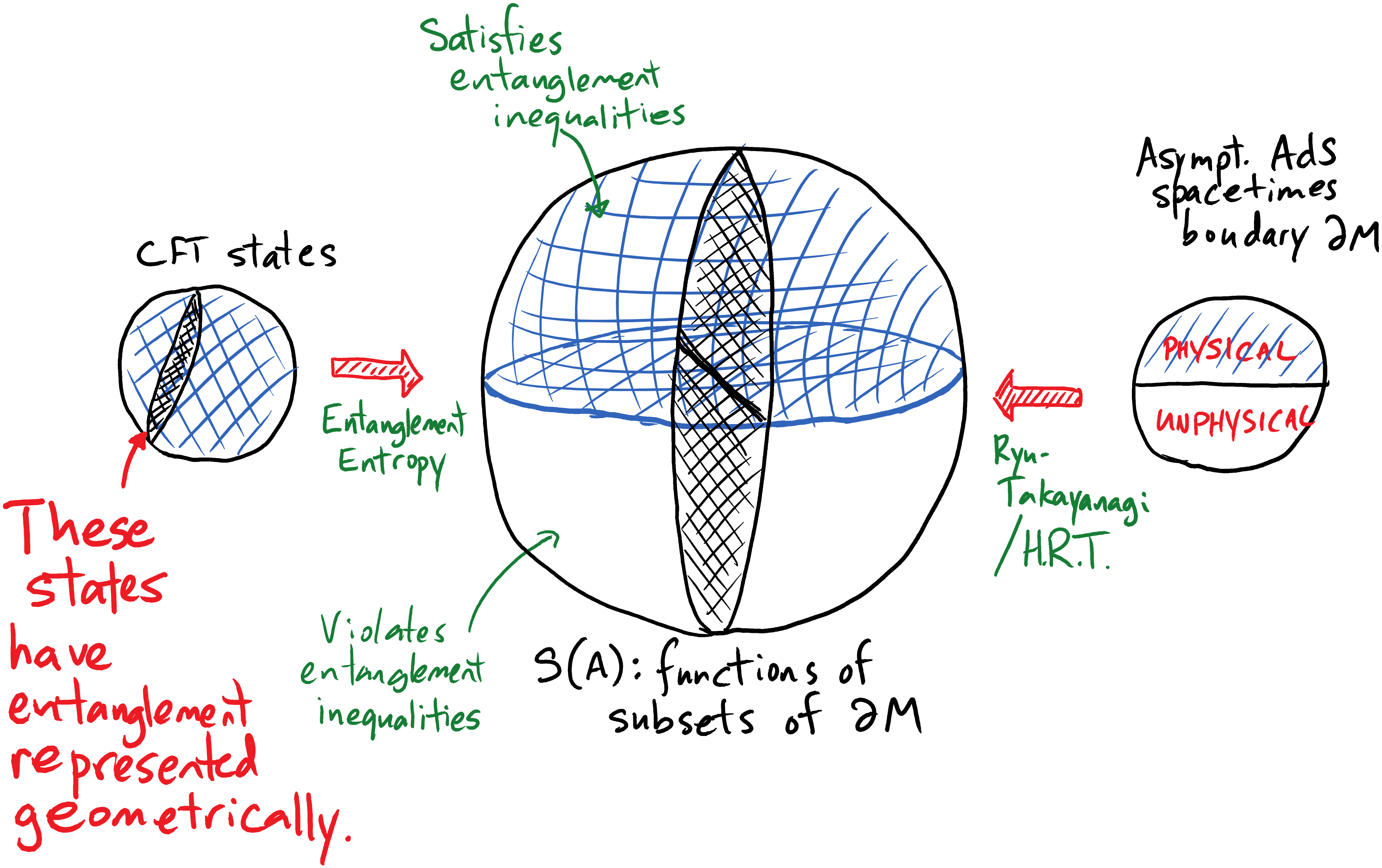
SSA

$$\int_{\tilde{B}} T_{\mu\nu} k^\mu k^\nu \geq 0$$

BUT: This can't be the general answer in higher dimensions

Another Direction: Which CFT states are we talking about?



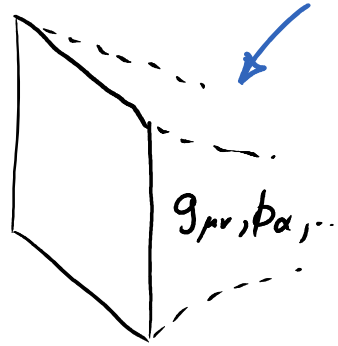


How can we characterize these in CFT?

Suppose

$$|\psi\rangle \xleftrightarrow{\text{dual to}} M_\psi$$

classical spacetime - encodes many observables of $|\psi\rangle$



solution of bulk E.O.M.

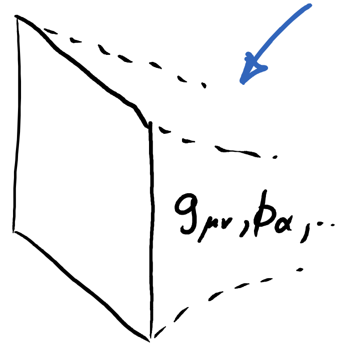
- determined by "initial" data at AdS boundary
- this corresponds to CFT one point functions

Suppose

$$|\psi\rangle \overset{\text{dual to}}{\longleftrightarrow} M_\psi$$

→ classical spacetime - encodes many observables of $|\psi\rangle$

solution of bulk E.O.M.



- determined by "initial" data at AdS boundary
- this corresponds to CFT one point functions

↓

Compute $\langle \mathcal{O}_\alpha \rangle$

↓

solve gravity problem to determine M_ψ

→ Compute nonlocal observables, entanglement entropy

* For holographic states, entanglement structure + nonlocal observables determined by local data *

$|\psi\rangle$

Compute $\langle O_\alpha \rangle$

ASSUME HOLOGRAPHIC

solve gravity problem to determine M_ψ

Compute nonlocal observables, entanglement entropy

$|\psi\rangle$

Compute $\langle \mathcal{O}_\alpha \rangle$

ASSUME HOLOGRAPHIC

solve gravity problem to determine M_ψ

Compute nonlocal observables, entanglement entropy

* For holographic states, entanglement structure + nonlocal observables determined by local data *

Question: what is this procedure in field theory?

How can we characterize the holographic states?

One idea: entropy maximization

Given $|\psi\rangle$ $\longrightarrow$ compute $\langle \theta_\alpha \rangle$ $\longrightarrow$ find ρ with same $\langle \theta_\alpha \rangle$ maximizing $S = -\mathrm{tr}(\rho \log \rho)$ $\longrightarrow$ Compute nonlocal observables

cf. Kelly+Wall

Current exploration: does this CFT procedure match gravity calculation for holographic $|\psi\rangle$?

If yes can say $|\psi\rangle_{\text{holographic}}$ are typical states in entropy-maximizing ensembles w. constrained one-point functions