GRAVITATIONAL PHYSICS
FROM ENTANGLEMENT CONSTRAINTS

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AdS/CFT: CFT on $\partial M \leftrightarrow$ Quant. grav. for asympt. AdS w. boundary $\partial M$

(some) CFT states

which ones?

(some) Asympt. AdS spacetimes boundary $\partial M$

which ones?
AdS/CFT: CFT on $\partial M \leftrightarrow$ Quant. grav. for asympt. AdS w. boundary $\partial M$

CFT states

$S(A) = -\text{Tr} (\rho_A \log \rho_A)$

Entropy

$S(A)$: functions of subsets of $\partial M$

Asympt. AdS spacetimes boundary $\partial M$
AdS/CFT: CFT on $\mathcal{D}M$ $\leftrightarrow$ Quant. grav. for asympt. AdS w. boundary $\partial M$

CFT states

Entanglement Entropy

$S(A)$: functions of subsets of $\partial M$

Asympt. AdS spacetimes boundary $\partial M$

Ryu-Takayanagi/Hubeny,Longo,Parigi,Takayanagi

$S(A) = \frac{\text{Area}(A')}{4\,G_N}$
AdS/CFT: CFT on $\mathcal{D}M$ $\leftrightarrow$ Quant. grav. for asympt. AdS w. boundary $\mathcal{D}M$

CFT states

 constrained by entanglement inequalities

$S(A)_{\text{functions of subsets of } \mathcal{D}M}$

$S(A \cup B) + S(B \cup C) \geq S(B) + S(A \cup B \cup C)$

Asympt. AdS spacetimes boundary $\mathcal{D}M$

Ryu-Takayanagi/HRT.
AdS/CFT: CFT on $\mathcal{D} M \leftrightarrow$ quant. grav. for asympt. AdS w. boundary $\mathcal{D} M$

CFT states

- Entanglement Entropy
- Constrained by entanglement inequalities

$S(A)$: functions of subsets of $\mathcal{D} M$

Asympt. AdS spacetimes boundary $\mathcal{D} M$

Ryu-Takayanagi / H.R.T.

Which geometries $M$ are allowed?

See: Lashkari Rabideau Sobella-Garnier MV.R.
Spacetime $M$ with $S(A)$ violating constraints →

Cannot correspond to consistent state in any UV complete theory for which dual CFT obeys RT/HRT.

Gives universal constraints:
#1 RELATIVE ENTROPY inequalities:

\[ |\psi\rangle \quad \text{and} \quad |\text{vac}\rangle \]

\[ \rho_A \quad \text{and} \quad \sigma_A \]

\[ S(\rho_{A|\sigma_A}) = \text{Tr}(\rho_A \log \rho_A) - \text{Tr}(\rho_A \log \sigma_A) \]

Measure of distinguishability: positive for \( \rho_A \neq \sigma_A \)
relative entropy inequalities:

\[ S(\rho_{A'}|\sigma_A) = \text{Tr}(\rho_A \log \rho_A) - \text{Tr}(\rho_A \log \sigma_A) \]

Measure of distinguishibility: positive for \( \rho_A \neq \sigma_A \)

Also monotonic: \( S(\rho_{A'}|\sigma_{A'}) \geq S(\rho_A|\sigma_A) \)

for \( A' \supset A \)
What do relative entropy inequalities imply for $M$?

Blanco, Casini, Han, Myers

Have:  

$$S(\rho_{\text{all}}|\sigma_A) = \Delta \langle H_A \rangle - \Delta S_A$$

modular Hamiltonian

$$H_A = -\log \sigma_A$$

Ball shaped,  

$$\sigma_A = \sigma_A^{\text{vac}}$$

=  

$$\int_B \xi^n \langle T^{\mu \nu} \rangle \xi^\nu - \Delta S_B$$

Map inequalities to properties of dual spacetime for holographic $|\Psi\rangle$. 
Any CFT:  \[ S(\rho_B | \sigma_B) = 2\pi \int_B \xi^m \Delta \langle T_{\mu \nu} \rangle \, \xi^\nu - \Delta S_B \]

Assume \( |\psi\rangle \) has entanglement calculated from some \( M \) via RT/HRT

\[ ds^2_M = \frac{l_{AdS}}{z^2} \left[ dz^2 + \Gamma_{\mu \nu}(z, x) dx^\mu dx^\nu \right] \]

Then:

\[ S(\rho_B | \sigma_B) = 2\pi \int_B \xi^m c \Gamma_{\mu \nu}(x) \, \xi^\nu - \frac{\Delta \text{Area}(B)}{4 \, 6N} \]

\[ \begin{array}{c}
\text{B} \\
\text{D} \end{array} \]

Use Wald technology to rewrite as integral over \( \Sigma \)
Start w. perturbative constraints: $|\psi(x)\rangle, |\psi(0)\rangle = |\text{vac}\rangle$

1st order: $S(\rho_B || \sigma_B) \bigg|_{\theta(x)} = 0$

$\Rightarrow \quad S E_{B}^{\text{grav}} = S S_{B}^{\text{grav}}$

$\Rightarrow \quad \int_{\Sigma} (\text{something} \propto \text{Einstein tensor}) = 0$

constant = linearized Einstein eqns

w. McDermott, Lashkari, Faulkner, Gnica, Hartman, Myers
w. Swingle
2nd order: 
\[ S(\rho_B(\lambda) \| \sigma_B) \] 
\[ \theta(\lambda^2) \equiv \langle \delta \rho_B, \delta \rho_B \rangle \]

Quantum Fisher Information

Maps to:

\[ W_\Sigma (g, \delta g, Z_\xi \delta g) \equiv \mathcal{E}(\delta g, \delta g) \]

Canonical Energy

\[ \xi_b \text{ extends to } \xi_b^a : \text{Killing vector of pure AdS = Rindler time} \]

\[ \mathcal{E}(\delta g, \delta g) \sim \int \xi_b^a \nabla_b \xi_a + \mathcal{E}_{\text{grav}} + \mathcal{E}_{\text{perturbative}} \]

= gravitational + matter energy associated w. \( \xi \)

Positivity/monotonicity give energy conditions
Non-perturbative: $S(p_{\text{bulk}})$ again maps to bulk integral.

→ Can interpret as energy associated with vector field $X$

Require: $X \to \xi$, $\nabla_a X_b \to 0$ at $B$

$X \to 0$, $\nabla^a X^b \to 2\pi n^{ab}$ at $B$

Then: Noether charge

$S(p_{\text{bulk}}) = \int_{\Sigma} J_x = H_x$

Independent details of $X$!
New positive energy theorem for subsystems.

Given any asympt. Ads $M$ in consistent UV completion of Einstein gravity matter

For any $B \subset M$ can define energy $H_x$ associated w. $\Sigma_B$.

* $H_x$ must be positive & increase w. increasing $B$. 

w/Lashkari, Lin, Ooguri, Stoica
#2  Strong Subadditivity

\[
S(AB) + S(BC) \geq S(B) + S(ABC)
\]

\[
\text{Area } (\overline{AB}) + \text{Area } (\overline{BC}) \geq \text{Area } (B) + \text{Area } (\overline{ABC})
\]

What are necessary and sufficient conditions on \( M_f \) for this to be true?
Headrick & Takayanagi: automatic for static geometries, \(A, B, C\) in preferred spatial slice

But: nontrivial constraints by considering \(A, B, C\) in general slices.

Wall: NEC is sufficient
Useful simplification:

Only need to consider SSA for $A, C$ infinitesimal, lightlike perturbations to $B$:

2D:

Higher D:

Combine SSA inequalities for these to obtain general SSA inequalities.

Dual of SSA for this case: some constraint localized to extremal surface $\tilde{B}$

$$\int_{\tilde{B}} (\text{covariant}) \geq 0$$
Result for simplest case:

2D, static, translation-invariant

\[ ds^2 = \frac{1}{z^2} \left( dz^2 + f(z) dx^2 - g(z) dt^2 \right) \]

\[ \int_{B} \nabla \cdot k = 0 \]

Vector field \( k^m \) along light sheet

But: This can't be the general answer in higher dimensions
Another direction: Which CFT states are we talking about?

CFT states

S(A): functions of subsets of $\partial M$

Entanglement Entropy

Satisfies entanglement inequalities

Asympt. AdS spacetimes boundary $\partial M$

Ryu-Takayanagi/HRT.

Physical

Unphysical
How can we characterize these in CFT?
Suppose $|\psi\rangle \xrightarrow{\text{dual to}} M_{\psi}$

- classical spacetime - encodes many observables of $|\psi\rangle$

solution of bulk E.O.M.
- determined by "initial" data at AdS boundary
- this corresponds to CFT one point functions
Suppose $|\psi\rangle$ dual to $M_\psi$ classical spacetime - encodes many observables of $|\psi\rangle$

Compute $\langle 0 | \alpha \rangle$

solve gravity problem to determine $M_\psi$

compute nonlocal observables, entanglement entropy

solution of bulk E.O.M.
- determined by "initial" data at AdS boundary
- this corresponds to CFT one point functions
\[ |\Psi\rangle \]

Compute \( \langle D\alpha \rangle \)

Assume Holographic

solve gravity problem to determine \( M_{\Psi} \)

\( \Rightarrow \) compute nonlocal observables, entanglement entropy

For holographic states, entanglement structure + nonlocal observables determined by local data.
For holographic states, entanglement structure of nonlocal observables determined by local data.

Question: what is this procedure in field theory?

How can we characterize the holographic states?

Compute $\langle O_\alpha \rangle$.

Assume holographic.

Solve gravity problem to determine $M_\Psi$.

Compute nonlocal observables, entanglement entropy.
One idea: entropy maximization

Given $|\Psi\rangle$ $\rightarrow$ compute $\langle \Theta_0 \rangle$ $\rightarrow$ find $\rho$ with same $\langle \Theta_0 \rangle$ maximizing $S = - \text{tr}(\rho \log \rho)$

c.f. Kelly & Wall

Current exploration: does this CFT procedure match gravity calculation for holographic $|\Psi\rangle$?

If yes can say $|\Psi\rangle_{\text{holographic}}$ are typical states in entropy-maximizing ensembles w. constrained one-point functions