Holographic Yang-Mills-Chern-Simons Defects

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Why (YM)-CS Holography with/without Defects?

\[ S_A = -\frac{1}{4g^2_{3d}} \int d^3x \text{Tr}(F^{\mu\nu}F_{\mu\nu}) + \frac{k}{4\pi} \int \text{Tr} (A \wedge F) \]

Confinement/Topological

Level-Rank Duality

\[ SU(N)_k \leftrightarrow U(k)_{-N,-N} \]

FQHE / Edge States

\[ \sigma_{xy} = \frac{e^2}{h} \]
\[ \nu = \frac{1}{2k+1} \quad U(1)_{2k+1} \]
\[ \nu = \frac{1}{2} \quad SU(2)_2 \]

[Aharony 1512.00161]

[Fradkin-Nayak-Tsvelik-Wilczek ‘98]
Chern-Simons Level-Changing Defects

Two regions w/different level:

\[ S_{CS} = \frac{k_1}{4\pi} \int_{y<0} \omega_3(A) + \frac{k_2}{4\pi} \int_{y>0} \omega_3(A) \]

Gauge transformation:

\[ \delta S_{CS} = \frac{k_1 - k_2}{4\pi} \int_{y=0} Tr(\lambda F) \]

level-changing defect

For k fundamental chiral fermions trapped on defect:

\[ S_F = \int d^2x \psi_i^\dagger (i\partial_+ + A_+) \psi^i \]

Gauge Anomaly

Fermion effective action:

\[ \delta S_F = \frac{k}{4\pi} \int d^2x Tr(\lambda F) \]

For k = k_2 - k_1: [Laughlin '81, Callan-Harvey '85]

total action is gauge invariant
Holographic YMCS & Defect D7 Branes

\[ - \frac{1}{4g_{3d}^2} \int d^3 x Tr_{SU(N)} F^2 \]

\[ \frac{k}{4\pi} \int d^3 x Tr_{SU(N)} A \wedge F \]

\[ g_{3d}^2 = g_s M_{KK} \]

Fujita+Li+Ryu+Takayanagi 0901.0924

Fujita, Melby-Thompson, Meyer, Sugimoto 1601.00525
Holography of Defect D7 Branes

Main focus:

\[ u = u_* \]

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Holography of Defect D7 Branes

### Main focus:

- **U(1) × U(1)**
- **$\psi$**
- **$N$ D3 branes**

### Table:

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Holography of Defect D7 Branes

Main focus:

\[ \mathcal{U}(1) \supset \mathcal{U}(N) \quad \sigma_{xy} = \frac{N}{k} \frac{e^2}{h} \]

Fractional (?) QH Sample

[Fujita+Li+Ryu+Takayanagi 0901.0924]
Holography of Defect D7 Branes

Single D7 wrapped on $S^5$, extended in $x^M = (t, x^1, u)$, $S^5$ singlet states only

Bosonic fields: $y(x^M), \tau(x^M), a_M(x^M)$

Fields at right-hand defect (left-hand defect has opposite chirality):

<table>
<thead>
<tr>
<th>operator $J_-$</th>
<th>$\Delta$</th>
<th>source $a_+$</th>
<th>vev $a_-$</th>
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<td>$O_y \sim \psi_+ \psi_-$</td>
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<td>$a_+$</td>
<td>$a_-$</td>
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<tr>
<td>$O_+ \sim \psi_+ F^+<em>y \psi</em>-$</td>
<td>3</td>
<td>$y$</td>
<td>$y$</td>
</tr>
<tr>
<td>$O_- \sim \psi_+ F^2_y \psi_-$</td>
<td>5</td>
<td>$a_-$</td>
<td>$a_+$</td>
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</table>

Asymptotic form ($w = u/u_0$):

- $y$: $y_0 - \frac{Rw^3}{4w^4} + \ldots$
- $a_-$: $a_-^{(4)} w^4 + (\ldots) + a_-^{(0)} + \ldots$
- $a_+$: $a_+^{(0)} + a_+^{(-4)} w^{-4} + \ldots$

[Harvey+Royston 0804.2854]

(F statistical SU(N) gauge field strength)

[Harvey, Melby-Thompson, Meyer, Sugimoto]
Topology vs. Confinement

Topological Wilson Loop

Level-Rank Duality

\[ SU(N)_k \leftrightarrow U(k)_{-N,-N} \]

[Fujita+Li+Ryu+Takayanagi 0901.0924]
One Point Functions & Anomaly

(1) Scalar condensate:
\[ \langle O_y \rangle \sim \frac{\partial S}{\partial L} \sim \langle \psi_+^\dagger F_{+y} \psi_- \rangle \]

Known?

(2) Current:
\[ \langle J^+ \rangle = -\frac{N}{4\pi} a_{-}^{(0)} \]
\[ \partial_+ \langle J_- \rangle = \frac{N}{2\pi} \partial_- A_+ \]

Chiral anomaly

(3) Chiral edge bosons:
\[ a_{\pm}^{(0)} \sim \partial_{\pm} \phi \]
\[ A_+ = 0 \quad \Rightarrow \quad \partial_+ \phi = 0 \]

(4) Dim. 5 operator:
\[ \langle O_+ \rangle = \frac{\delta S_{\text{total}}^{\text{S-s.}}}{\delta C_-} = -\frac{N}{4\pi} \frac{c_+}{8\pi \alpha'} \frac{u_*^4}{R^8} e^{-\xi} \quad \Rightarrow \quad \langle O_- - O_+ \rangle \sim -\frac{N}{4\pi} \frac{u^6 e^{2c_0}}{R^{16}} e^{-(2M_{KK})^2 L} \]

Known?
Edge Chiral Symmetry Breaking

\[ \langle J_+ \rangle \sim \partial_+ \phi' \quad \text{and} \quad \langle J_- \rangle \sim \partial_- \phi \]

Chiral currents: Flat part of the connection \[ a^{(0)} = d\phi \]

Shift in \( \phi + \phi' \) doesn’t change \[ \int_{L}^{R} a^{(0)} = \int_{L}^{R} d\phi = \phi - \phi' \]

Shift in \( \phi - \phi' \) does \( \text{NG boson} \) (“Pion”, as in [Sakai-Sugimoto])

(lifted by quantum effects)

(restored at \( T >> M_{\text{KK}} \))
Edge Chiral Symmetry Breaking

\[ \langle \psi^\dagger_L \mathcal{P} e^{i \int_R^L A \psi_R} \rangle = e^{-\mathcal{S}^{F1}_{\text{ren}}} \]

[Aharony+Kutasov 0803.3547]

**Known?**

**Edge mode**

\[ \text{= Nambu-Goldstone boson of broken chiral symmetry} \]
Correlations and Phase Transitions

\[ l_c \sim L_{\text{KK}} \]
Correlations and Phase Transitions

Dimension 5 Correlations

Large $l > 1/\text{Gap}$: adjacent correlations

Small $l < 1/\text{Gap}$: distant correlations

Known? (vanish at $T > \text{gap}$)

(F statistical gauge field)
Correspondence between FQHE and 2D QCD

(1) Basically 2+1 dimensional holographic domain wall fermions
   = 2D holographic QCD with all its features (chiral symmetry
   breaking, Goldstone modes, anomaly)

(2) By compactifying and T-dualizing perpendicular to the defect, the
    system becomes the holographic dual to 2D QCD of [Yee-Zahed 1103.6286]

(3) LEEA of (abelian) FQH States: U(1) CS theory

    LEEA of single flavored 2D QCD:
    U(1) DBI-CS Theory on a D7 brane [Yee-Zahed 1103.6286]

• Anyonic quasiparticle (couples with unit charge to the statistical
  gauge field) \sim Fundamental quark (baryon number charge 1/N)
  \sim Fundamental string ending on D7 brane

• Electron \sim Baryon vertex (bound state of N quarks has baryon
  number charge (electric charge) 1)
Conclusions/Outlook

- YM-CS theory with level-changing defects in Holography

- Similarities to FQH samples

- Chiral Symmetry Breaking between Edges

- VEVs involving edge fermions & statistical gauge field, Transitions between samples, Dim. 5 correlations

- Edge transport, Impurity scattering, Relation to 2D QCD, Backreaction, Tunneling of anyons/electrons, …

- Non-abelian (YM)CS in strongly coupled systems

\[ \sigma_{xy}^{U(1) \subset U(N)} = \frac{N}{k} \frac{e^2}{h} \]

[Fujita+Li+Ryu+Takayanagi 0901.0924]