

# Holographic Yang-Mills- Chern-Simons Defects

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[arXiv.org](https://arxiv.org/abs/1601.00525) > [hep-th](https://arxiv.org/abs/1601.00525) > [arXiv:1601.00525](https://arxiv.org/abs/1601.00525)

# Why (YM)-CS Holography with/without Defects?

$$S_A = -\frac{1}{4g_{3d}^2} \int d^3x \text{Tr}(F^{\mu\nu} F_{\mu\nu}) + \frac{k}{4\pi} \int \text{Tr}(A \wedge F)$$

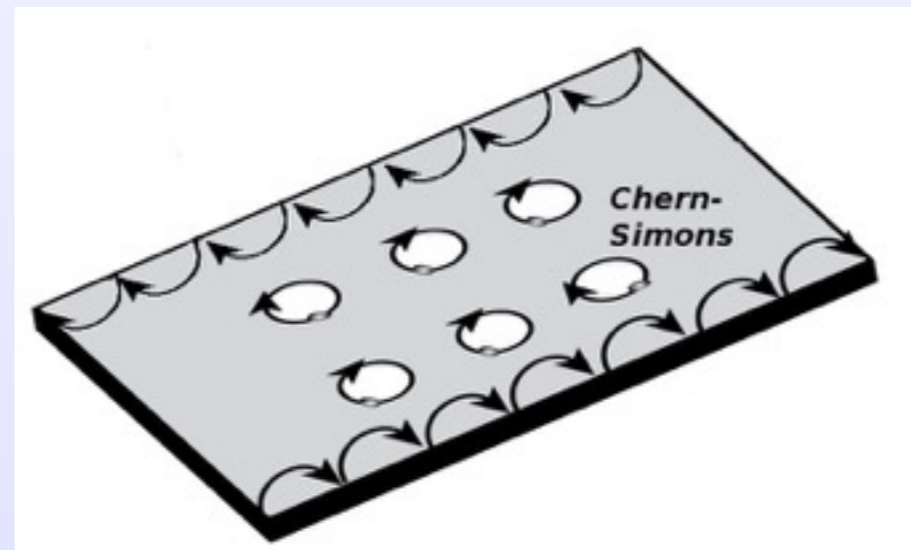
Confinement/  
Topological

Level-Rank  
Duality

$$SU(N)_k \leftrightarrow U(k)_{-N, -N}$$

[Aharony 1512.00161]

FQHE / Edge States

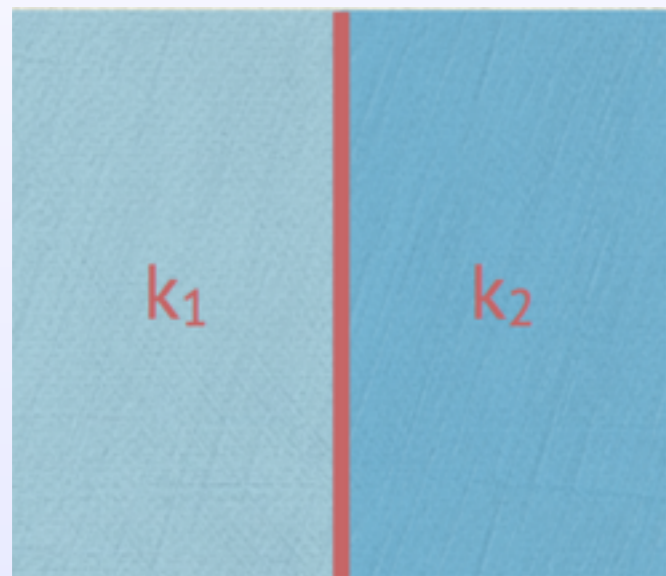


$$\begin{aligned} \sigma_{xy} &= \nu \frac{e^2}{h} \\ \nu &= \frac{1}{2k+1} \quad U(1)_{2k+1} \\ \nu &= \frac{1}{2} \quad SU(2)_2 \end{aligned}$$

[Fradkin-Nayak-Tsvetlik-Wilczek '98]

# Chern-Simons Level-Changing Defects

Two regions w/different level:



level-changing defect

$$S_{CS} = \frac{k_1}{4\pi} \int_{y < 0} \omega_3(A) + \frac{k_2}{4\pi} \int_{y > 0} \omega_3(A)$$

Gauge transformation:

$$Tr(A \wedge dA + \frac{2}{3} A \wedge A \wedge A)$$

$$\delta S_{CS} = \frac{k_1 - k_2}{4\pi} \int_{y=0} Tr(\lambda F)$$

**not gauge invariant**

For k fundamental chiral fermions trapped on defect:

$$S_F = \int d^2x \psi_i^\dagger (i\partial_+ + A_+) \psi^i$$

Gauge Anomaly

Fermion effective action:

$$\delta S_F = \frac{k}{4\pi} \int d^2x Tr(\lambda F)$$

For  $k = k_2 - k_1$ :

[Laughlin '81, Callan-Harvey '85]

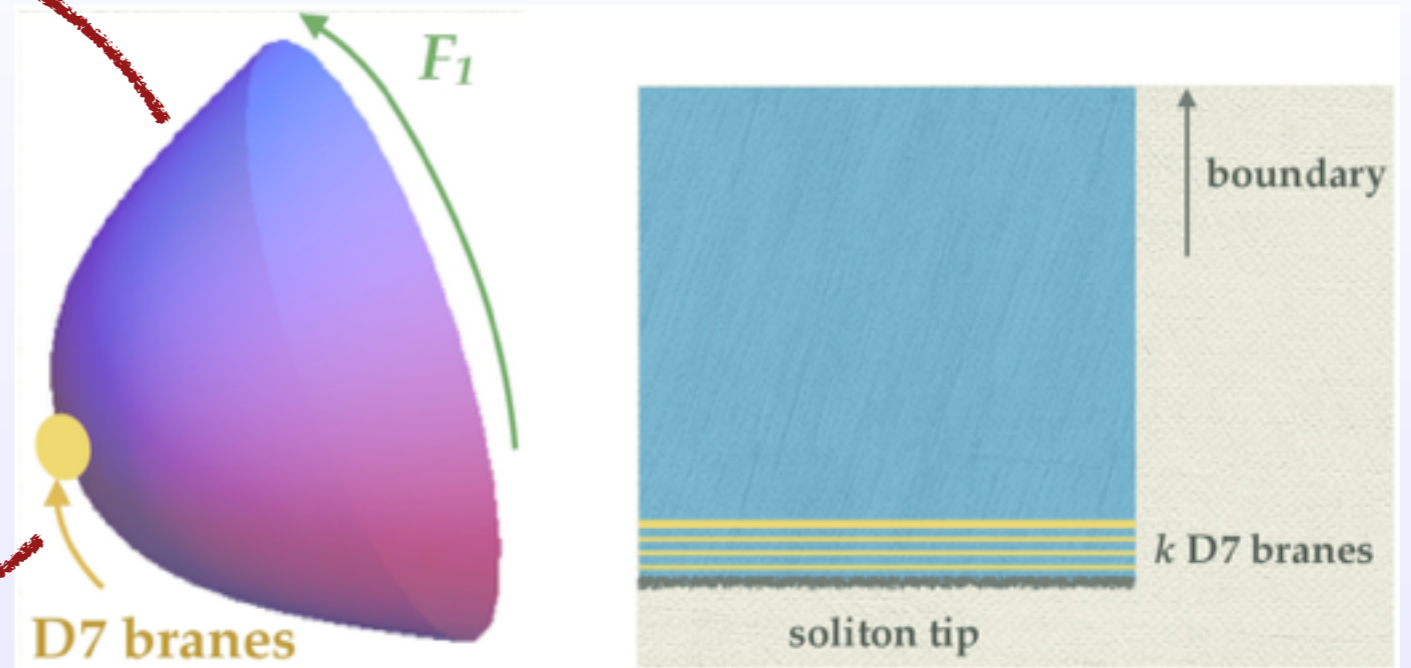
**total action is gauge invariant**

# Holographic YMCS & Defect D7 Branes

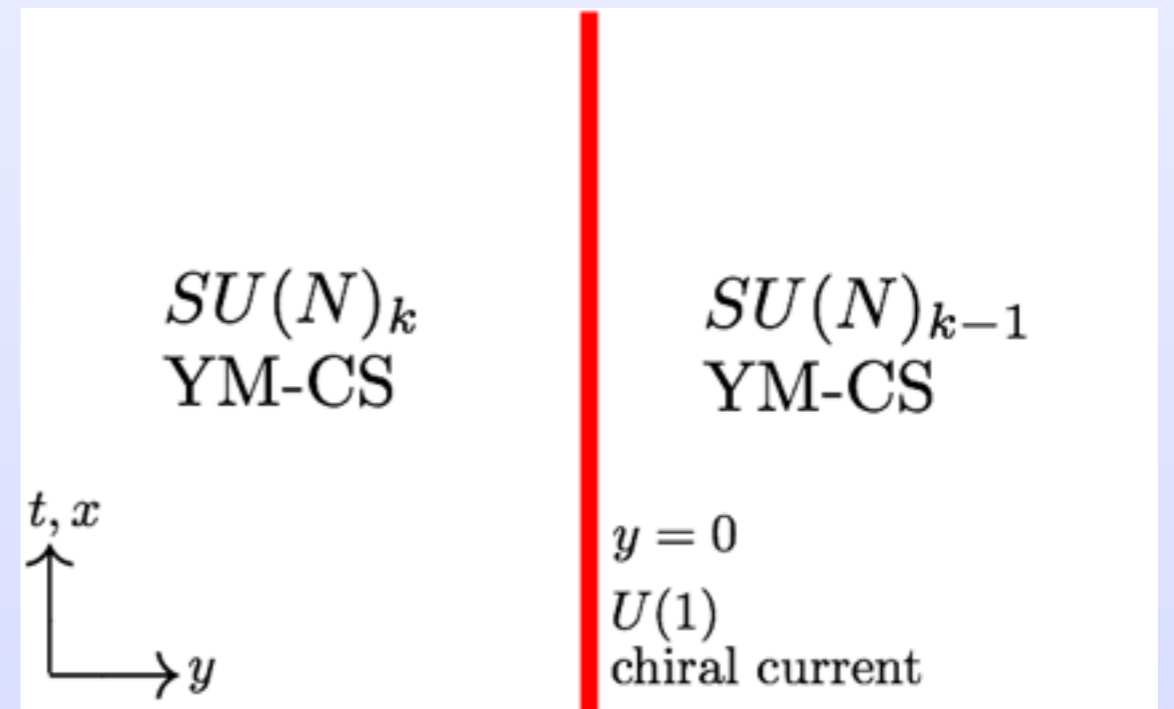
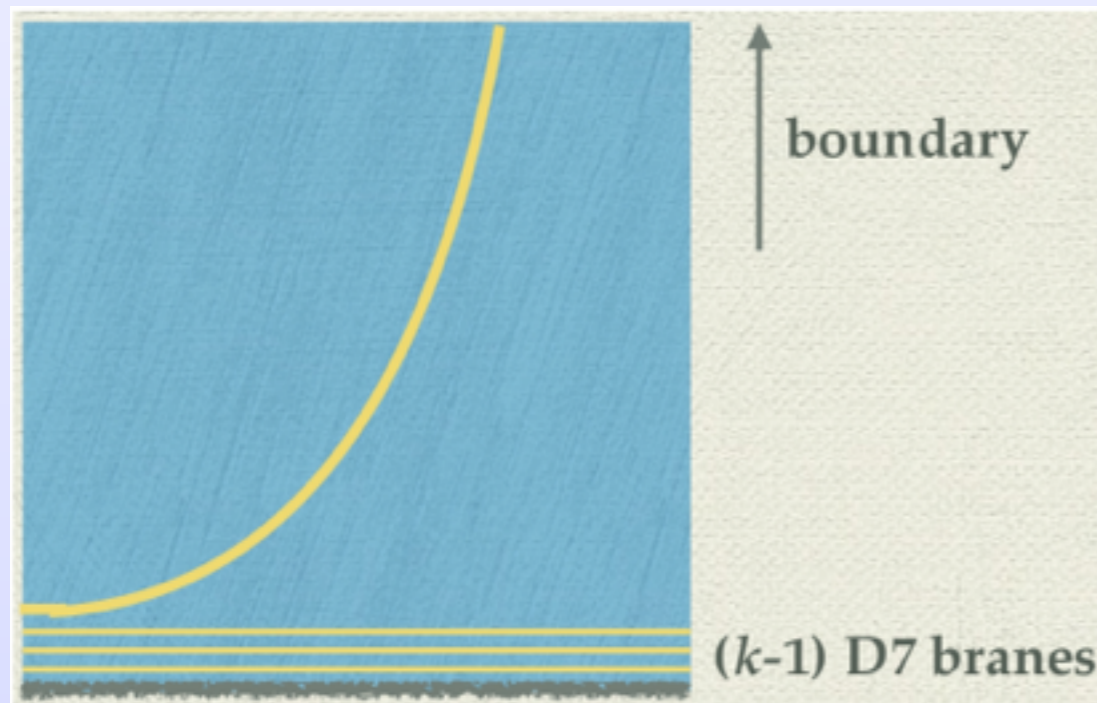
$$-\frac{1}{4g_{3d}^2} \int d^3x \text{Tr}_{SU(N)} F^2$$

$$\frac{k}{4\pi} \int d^3x \text{Tr}_{SU(N)} A \wedge F$$

$$g_{3d}^2 = g_s M_{\text{KK}}$$



Fujita+Li+Ryu+Takayanagi 0901.0924

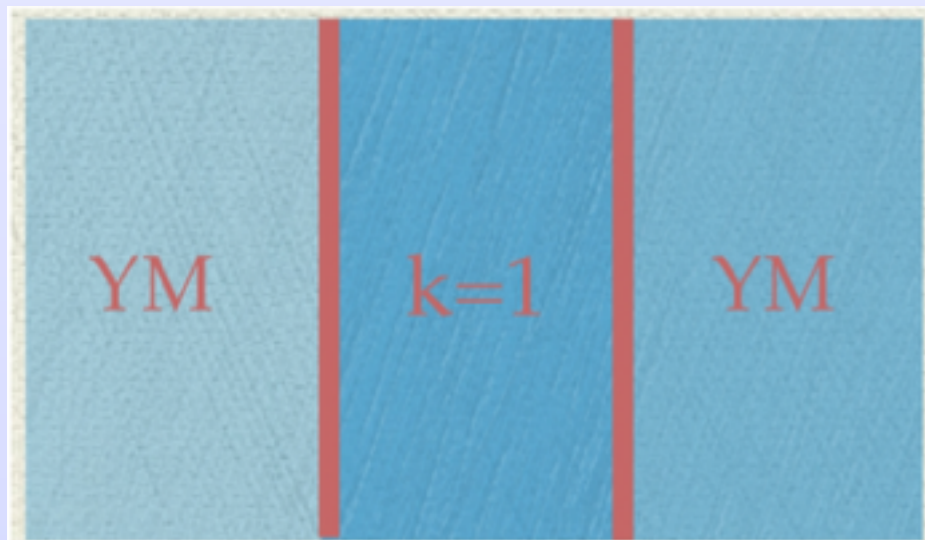
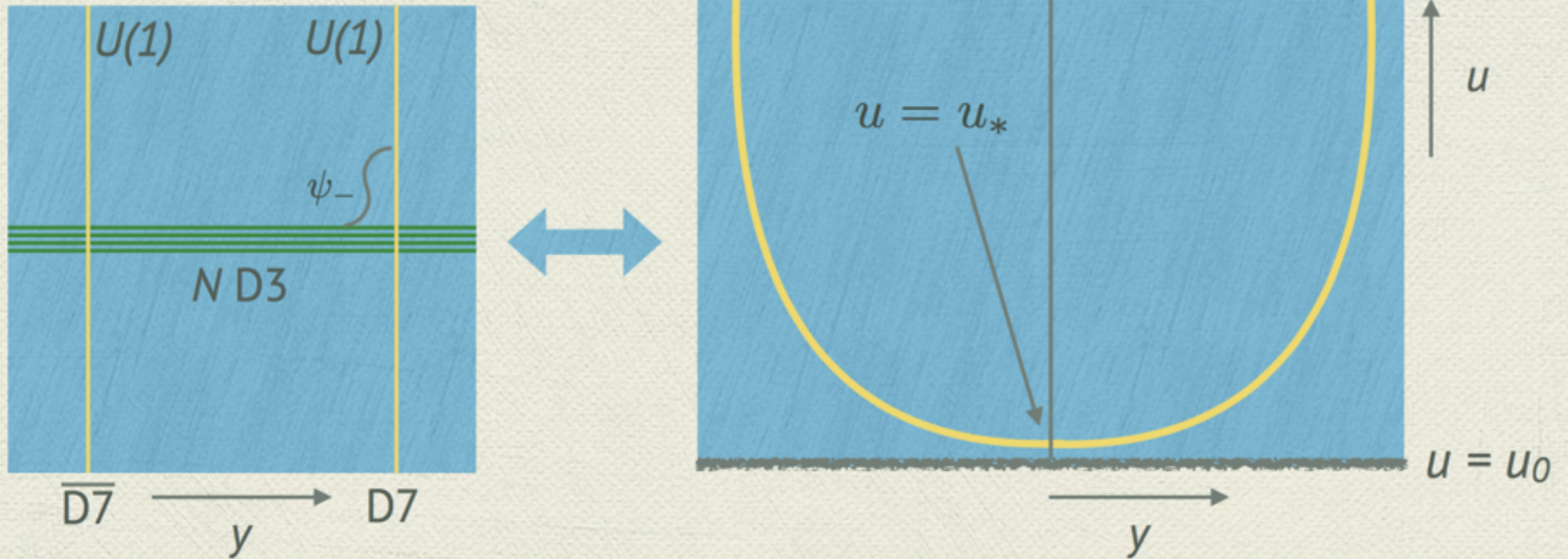


Fujita, Melby-Thompson, Meyer, Sugimoto 1601.00525

$$S_\psi = \int_{y=0} d^2x \psi_-^\dagger (i\partial_+ + A_+) \psi_-$$

# Holography of Defect D7 Branes

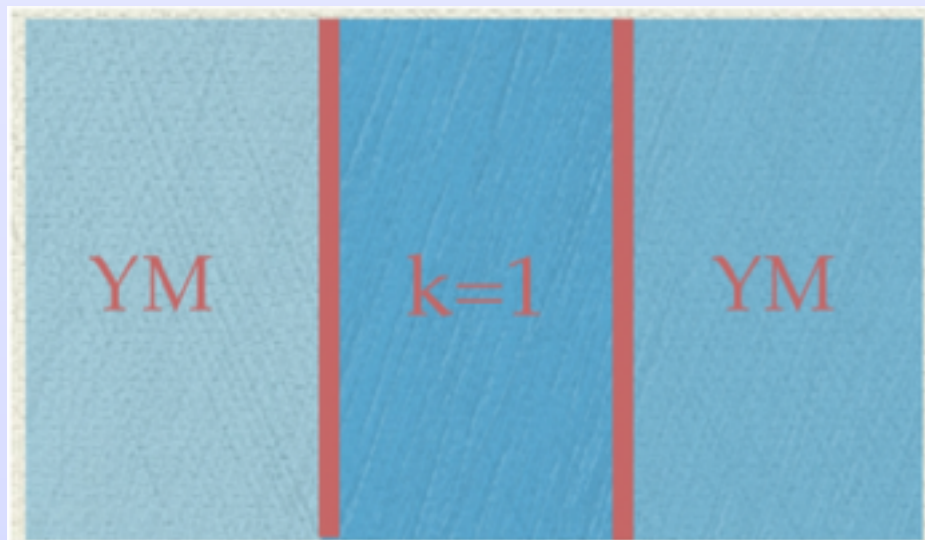
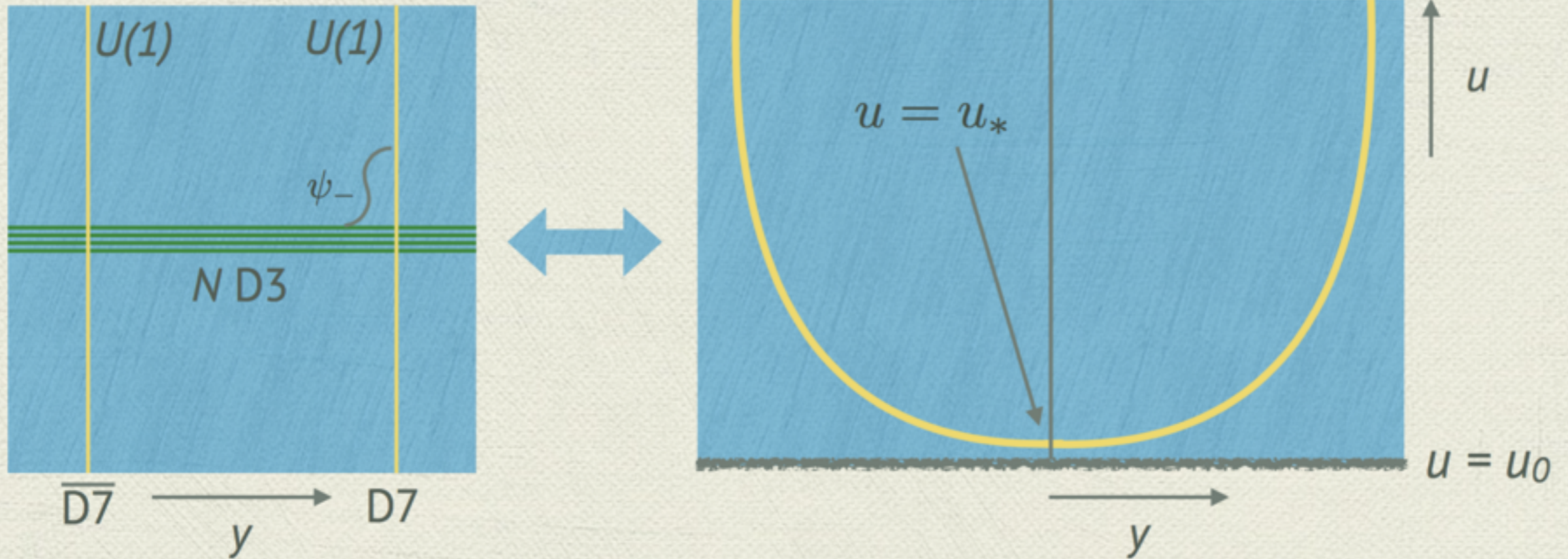
Main focus:



	0	1	2	3	4	5	6	7	8	9
D3	x	x	x	x						
D7	x	x			x	x	x	x	x	x
$\bar{D}7$	x	x			x	x	x	x	x	x

# Holography of Defect D7 Branes

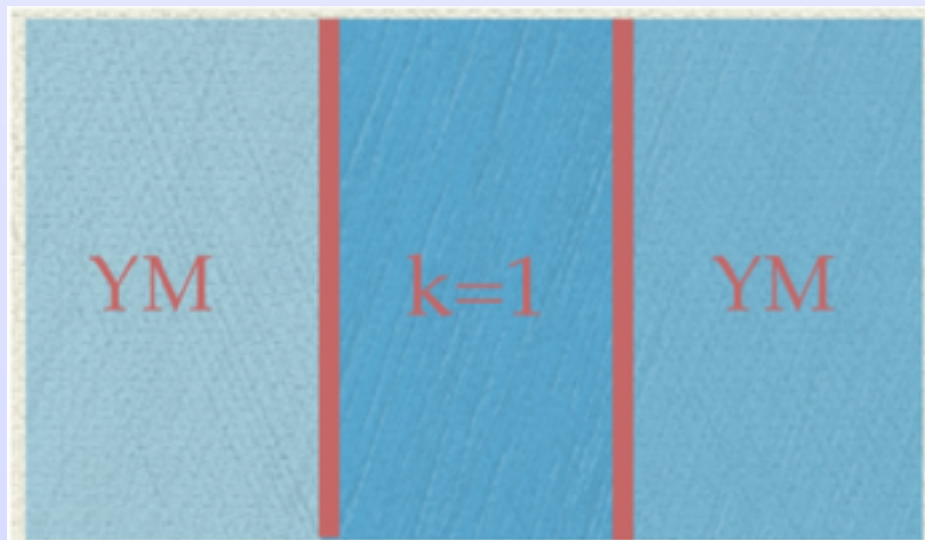
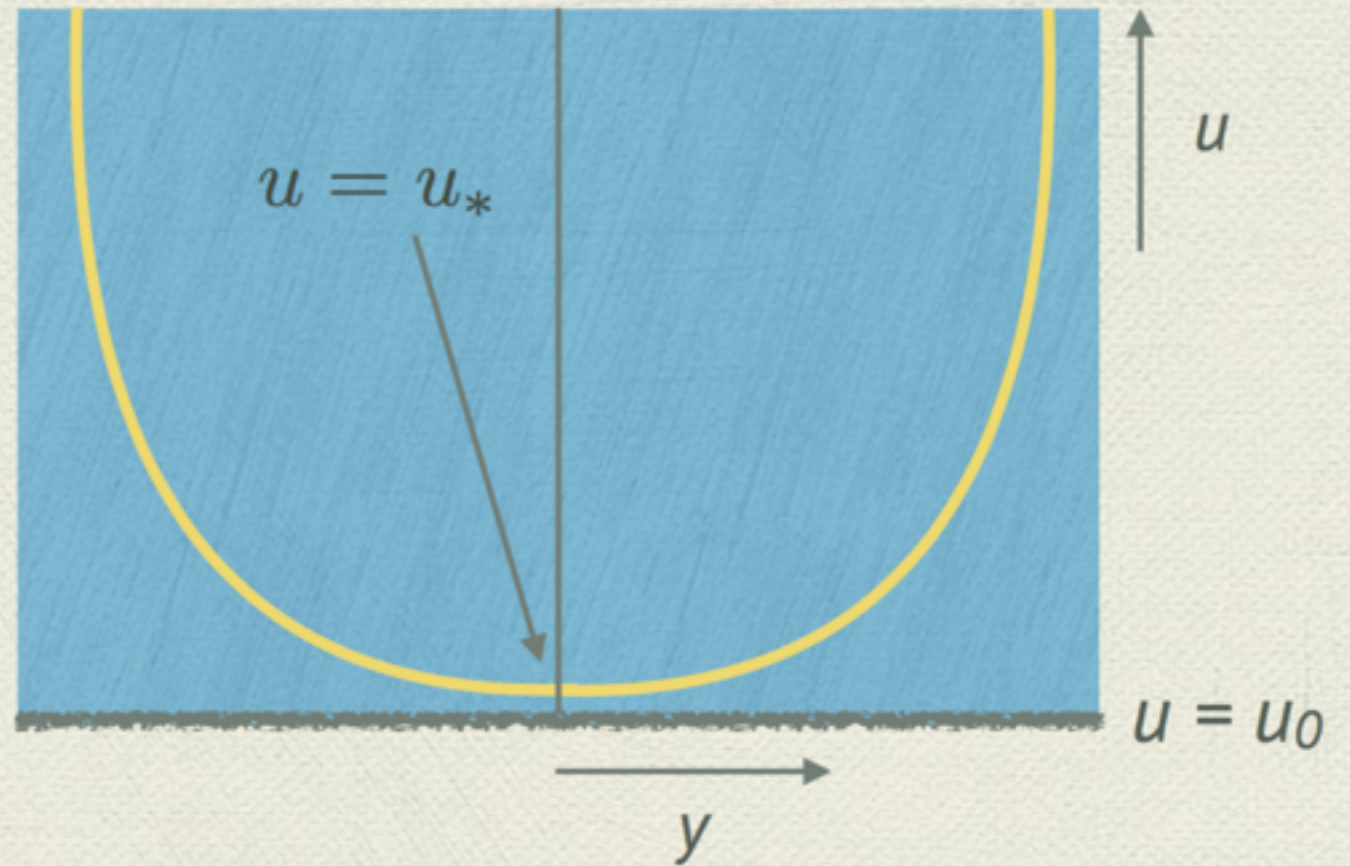
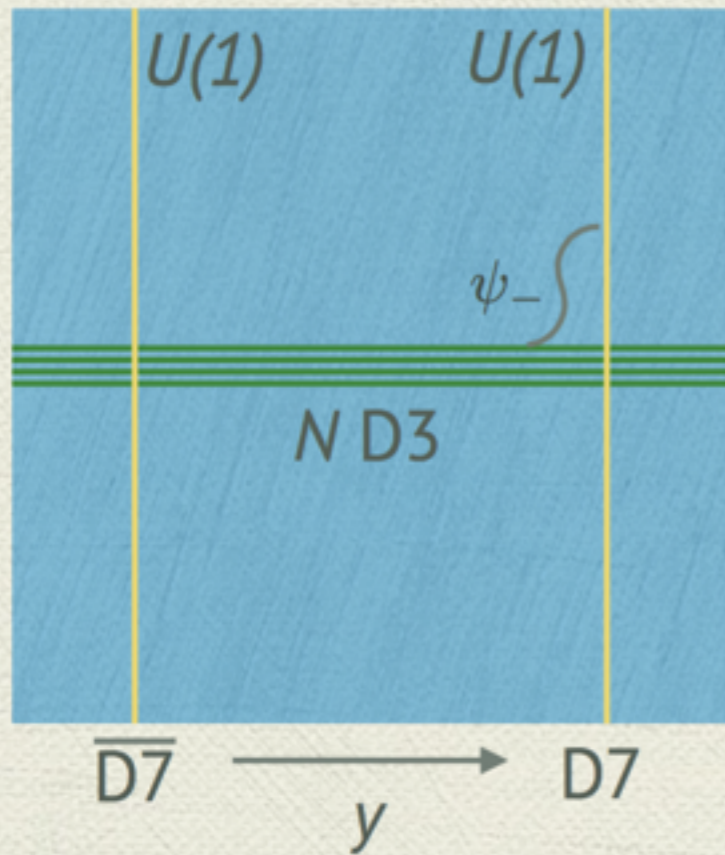
Main focus:



	$t$	$x$	$y$	$\tau$	$r$	$S_5$	$S_5$	$S_5$	$S_5$	$S_5$
D3	x	x	x	x						
D7	x	x	$y(r)$	$\tau(r)$	x	x	x	x	x	x
$\bar{D}7$	x	x	$y(r)$	$\tau(r)$	x	x	x	x	x	x

# Holography of Defect D7 Branes

Main focus:

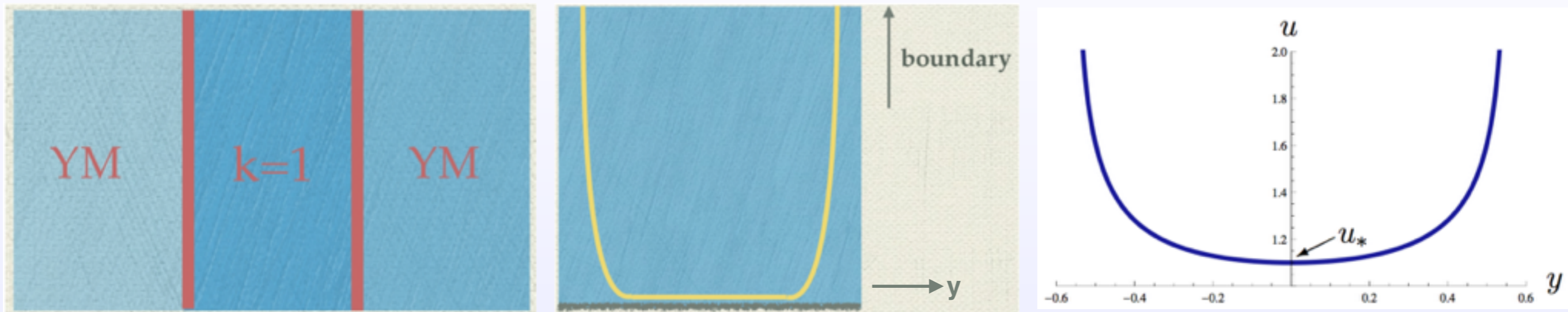


Fractional (?) QH Sample

$$\sigma_{xy}^{U(1) \subset U(N)} = \frac{N}{k} \frac{e^2}{h}$$

[Fujita+Li+Ryu+Takayanagi 0901.0924]

# Holography of Defect D7 Branes



Single D7 wrapped on  $S^5$ , extended in  $x^M = (t, x^1, u)$ ,  $S^5$  singlet states only

Bosonic fields:  $y(x^M)$ ,  $\tau(x^M)$ ,  $a_M(x^M)$

Fields at right-hand defect (left-hand defect has opposite chirality):

operator	$\Delta$	source	vev
$J_- \sim \psi_-^\dagger \psi_-$	1	$a_+$	$a_-$
$\mathcal{O}_y \sim \psi_-^\dagger F_{+y} \psi_-$	3	$y$	$y$
$\mathcal{O}_+ \sim \psi_-^\dagger F_{+y}^2 \psi_-$	5	$a_-$	$a_+$

[Harvey+Royston 0804.2854]

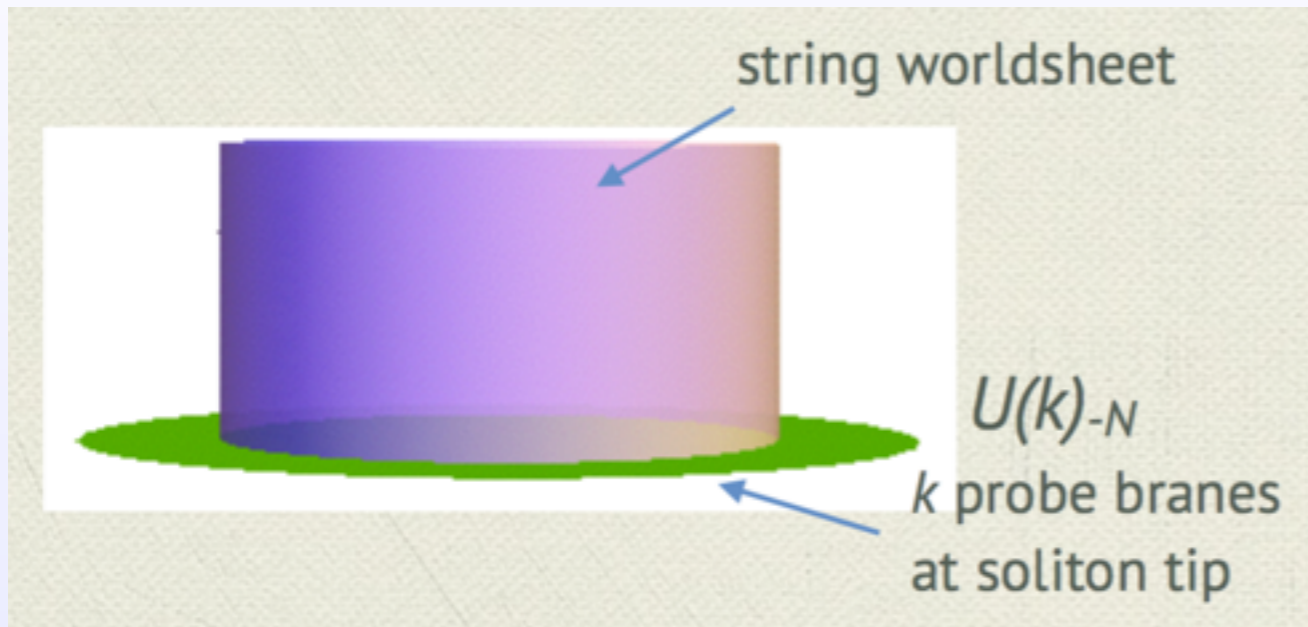
Field	Asymptotic form ( $w = u/u_0$ )
$y$	$y_0 - \frac{Rw^3}{4w_*^4} + \dots$
$a_-$	$a_-^{(4)} w^4 + (\dots) + a_-^{(0)} + \dots$
$a_+$	$a_+^{(0)} + a_+^{(-4)} w^{-4} + \dots$

[Fujita, Melby-Thompson, Meyer, Sugimoto]

(F statistical SU(N) gauge field strength)



# Topology vs. Confinement

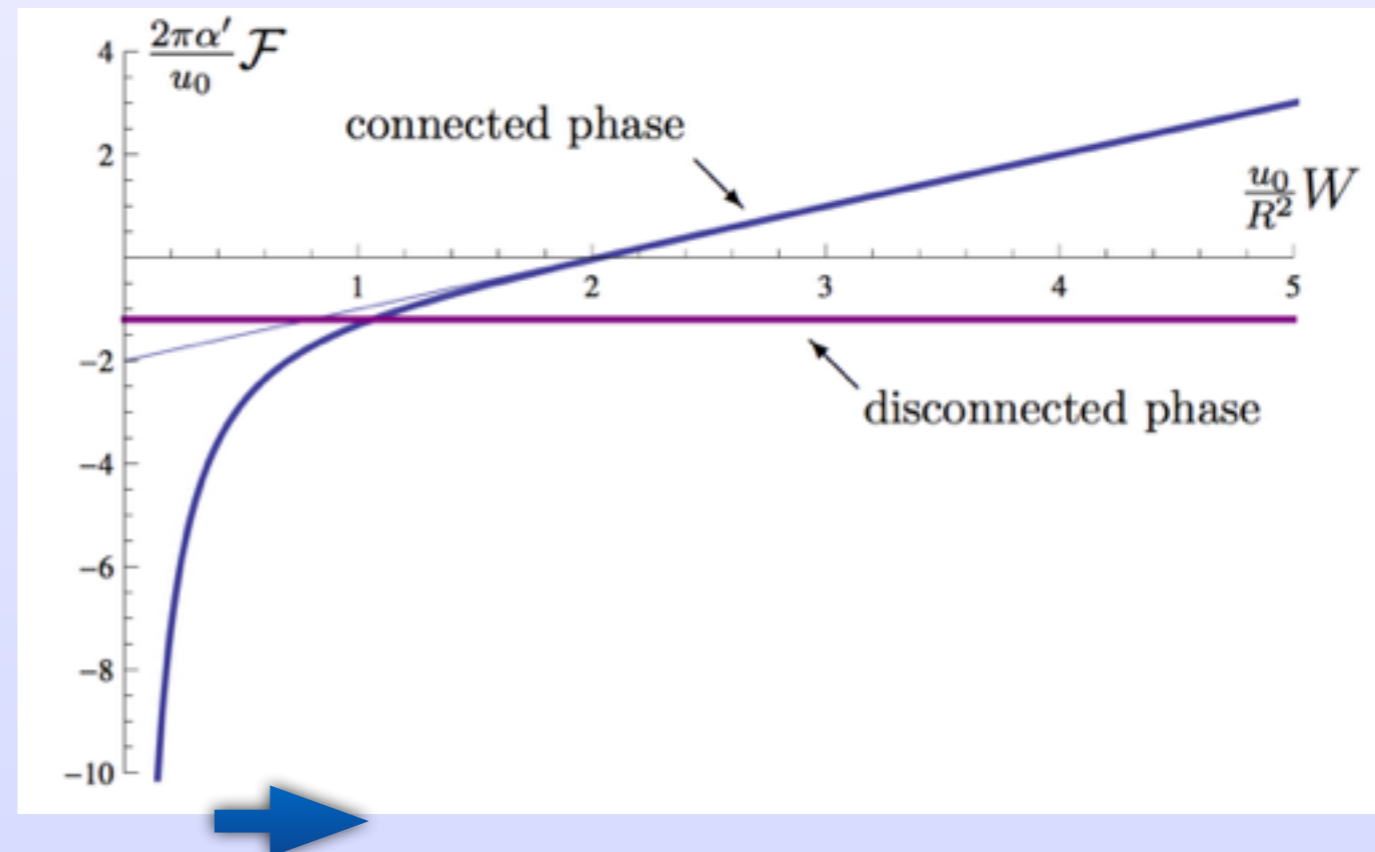
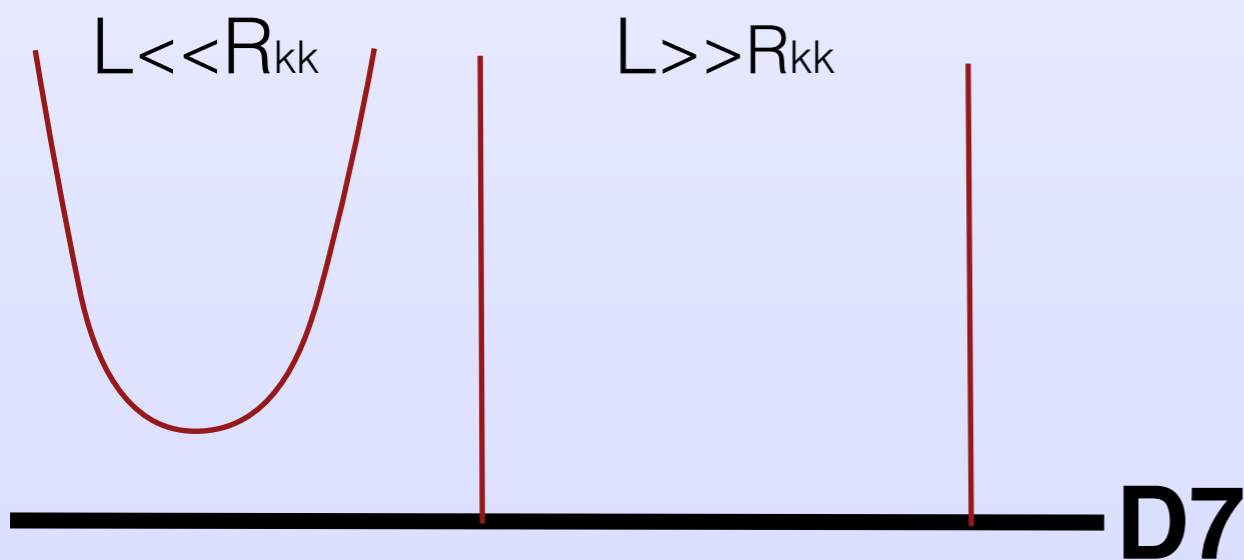


Topological Wilson Loop

Level-Rank Duality

$$SU(N)_k \leftrightarrow U(k)_{-N, -N}$$

[Fujita+Li+Ryu+Takayanagi 0901.0924]

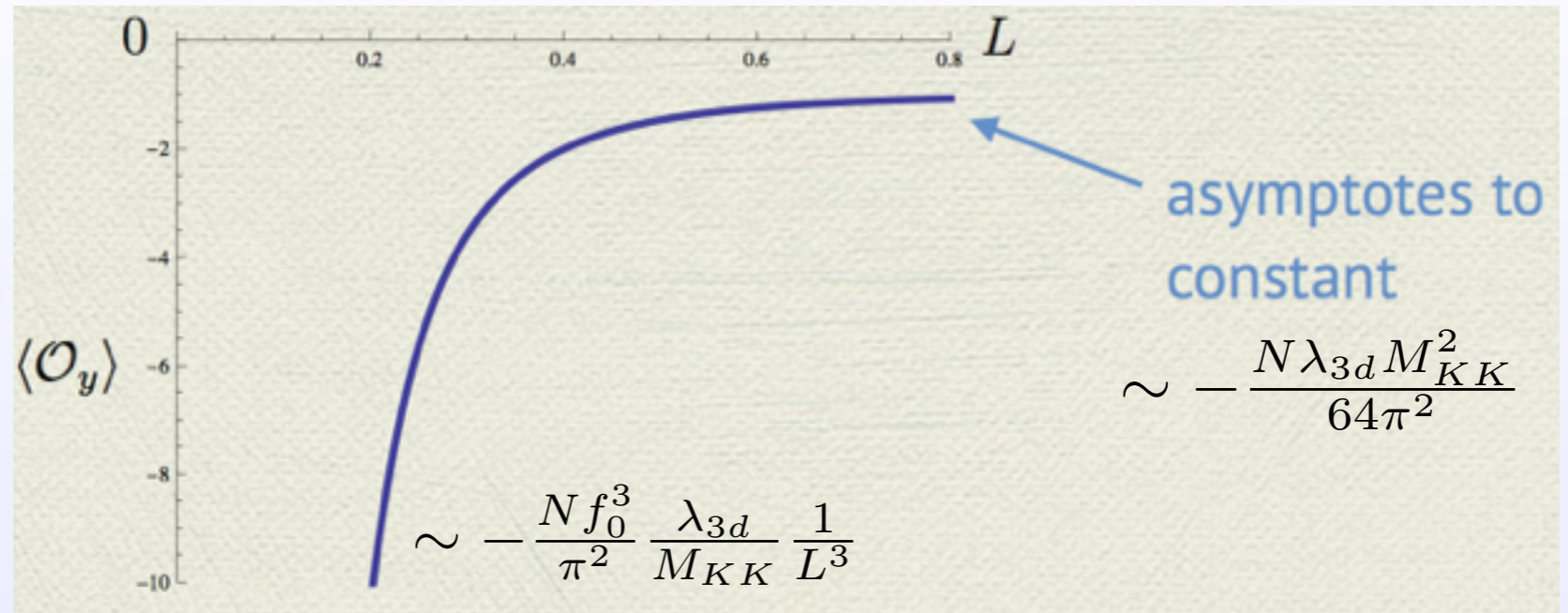


# One Point Functions & Anomaly

(1) Scalar condensate:

$$\langle \mathcal{O}_y \rangle \sim \frac{\partial S}{\partial L} \sim \langle \psi_-^\dagger F_{+y} \psi_- \rangle$$

**Known?**



(2) Current:

$$\langle J^+ \rangle = -\frac{N}{4\pi} a_-^{(0)}$$

$$\partial_+ \langle J_- \rangle = \frac{N}{2\pi} \partial_- \mathcal{A}_+$$

**Chiral anomaly**

(3) Chiral edge bosons:  $a_\pm^{(0)} \sim \partial_\pm \phi$

$$\mathcal{A}_+ = 0 \quad \Rightarrow \quad \partial_+ \phi = 0$$

(4) Dim. 5 operator:

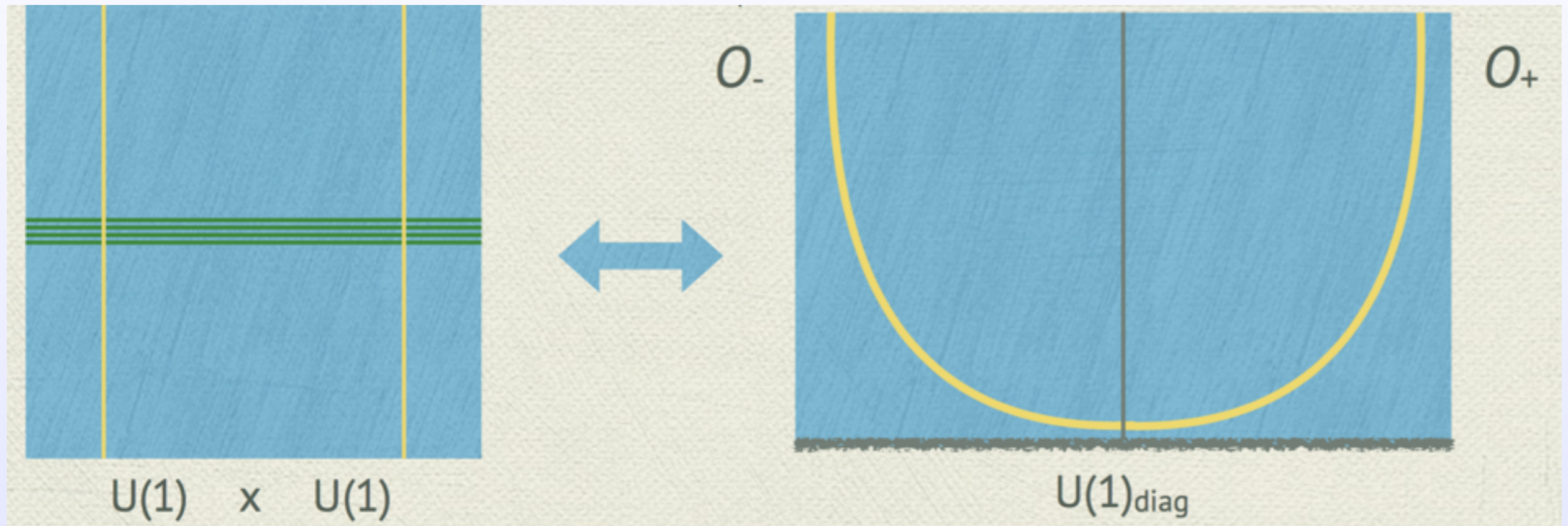
$$\langle \mathcal{O}_+ \rangle = \frac{\delta S_{total}^{o.s.}}{\delta \mathcal{C}_-} = -\frac{N}{4\pi} \frac{c_+}{8\pi\alpha'} \frac{u_*^4}{R^8} e^{-\xi} \quad \Rightarrow \quad \langle \mathcal{O}_- \mathcal{O}_+ \rangle \simeq -\frac{N}{4\pi} \frac{u_0^8 e^{2c_0}}{R^{16}} e^{-(2M_{KK})2L}$$

**Known?**

# Edge Chiral Symmetry Breaking

$$\langle J_+ \rangle \sim \partial_+ \phi'$$

$$\langle J_- \rangle \sim \partial_- \phi$$



Chiral currents: Flat part of the connection  $a^{(0)} = d\phi$

Shift in  $\phi + \phi'$  doesn't change  $\int_L^R a^{(0)} = \int_L^R d\phi = \phi - \phi' \rightarrow U(1)_{\text{diag}}$

Shift in  $\phi - \phi'$  does  $\rightarrow$  NG boson ("Pion", as in **[Sakai-Sugimoto]**)



(lifted by quantum effects)

(restored at  $T \gg M_{\text{KK}}$ )

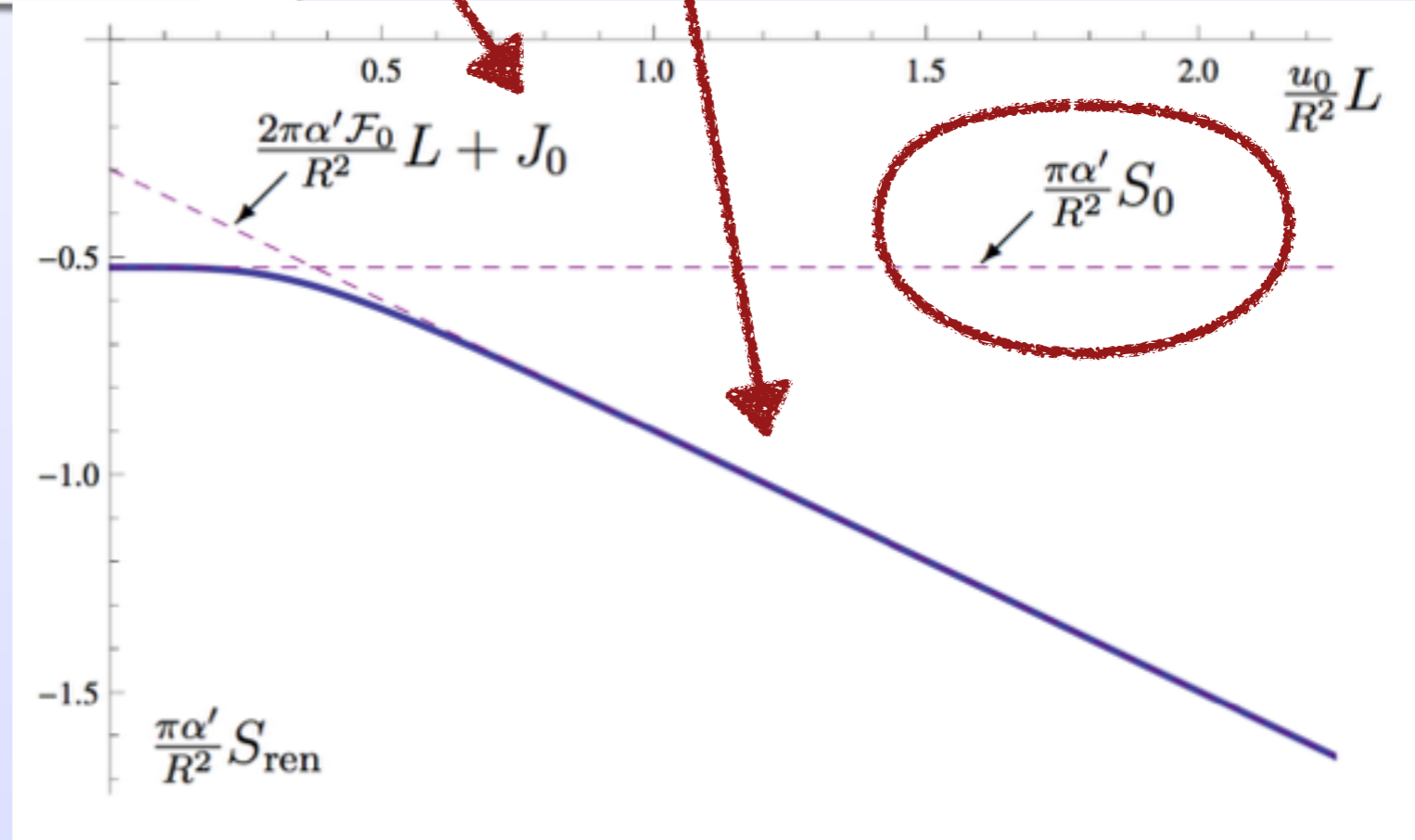
# Edge Chiral Symmetry Breaking



$$\langle \psi_L^\dagger \mathcal{P} e^{i \int_R^L A} \psi_R \rangle = e^{-S_{\text{ren}}^{F1}}$$

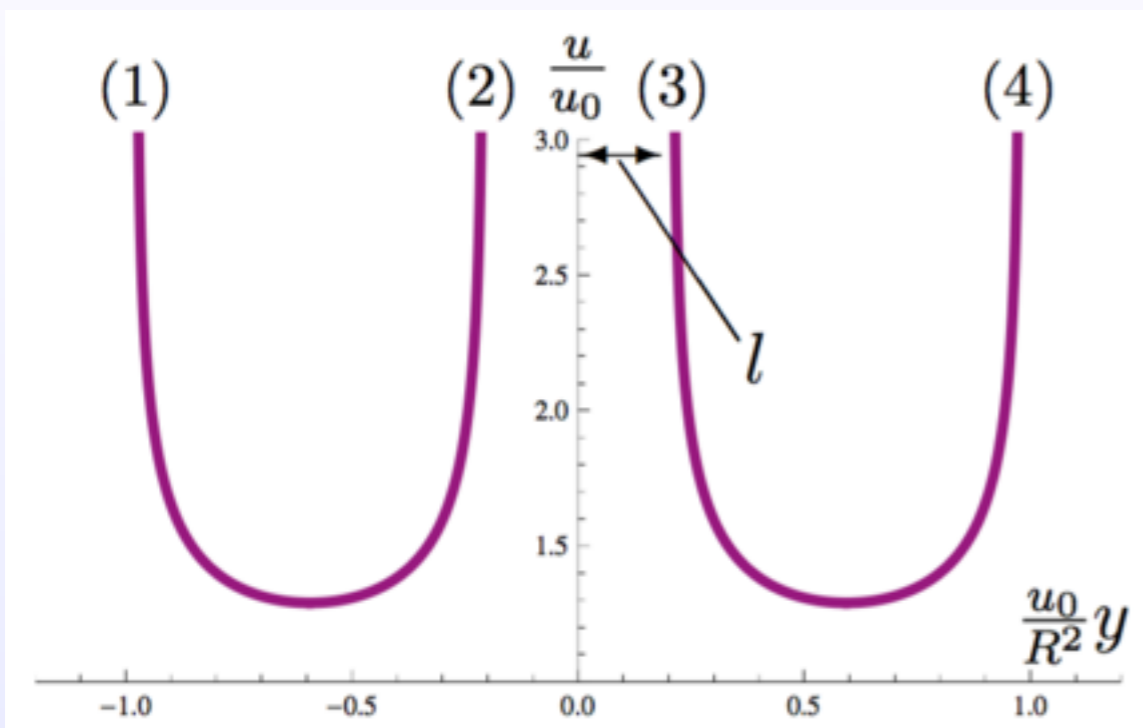
[Aharony+Kutasov 0803.3547]

Edge mode  
=  
Nambu-Goldstone  
boson of broken  
chiral symmetry



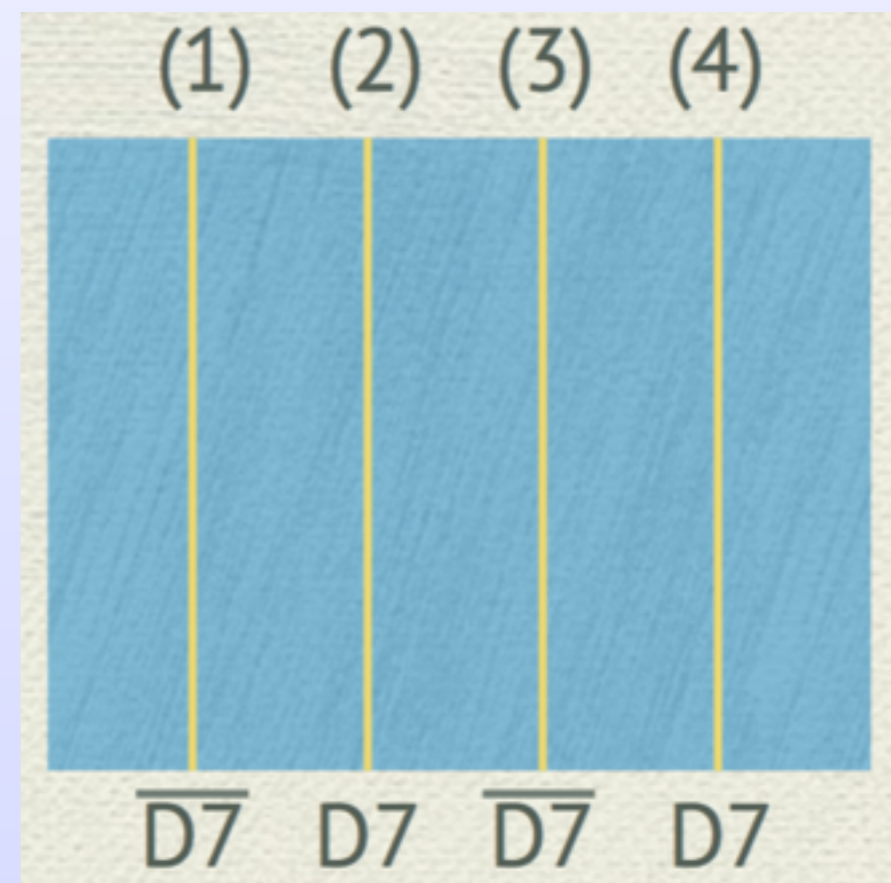
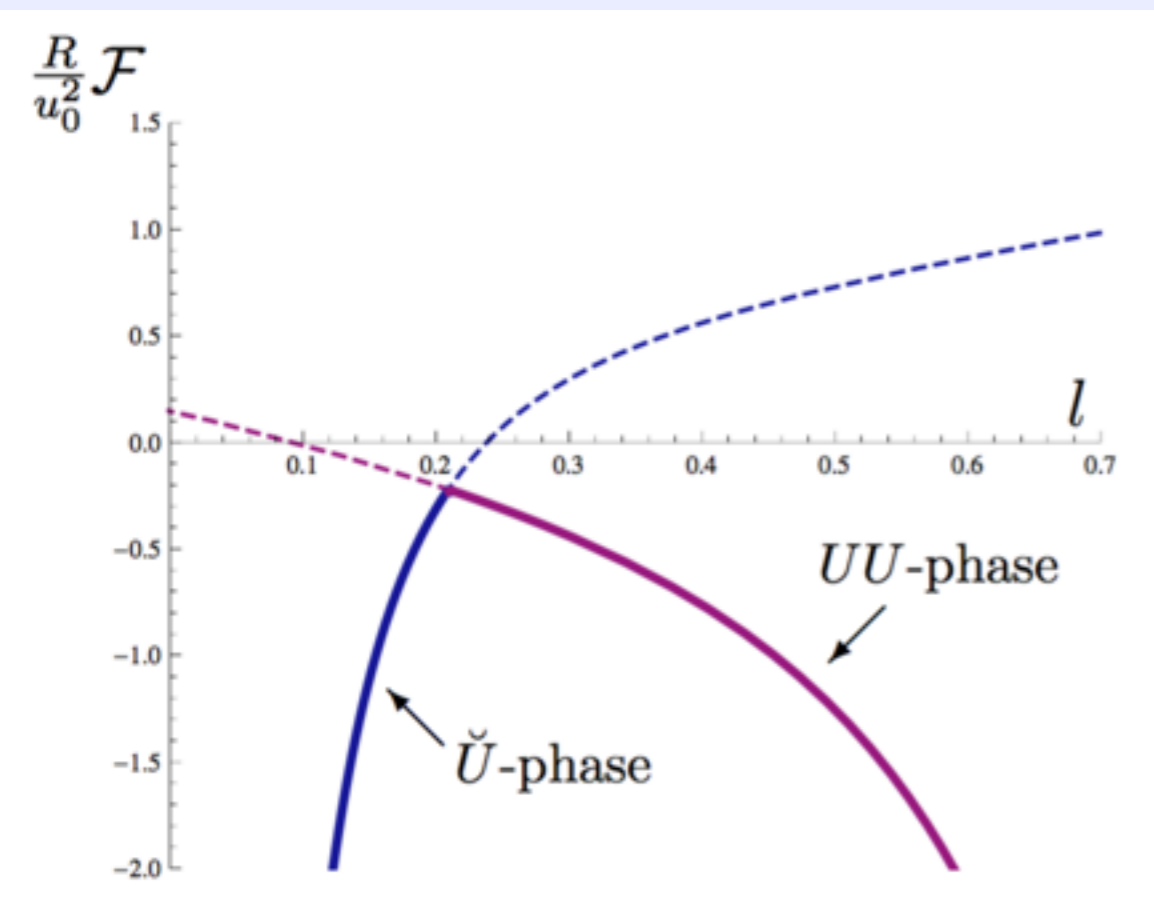
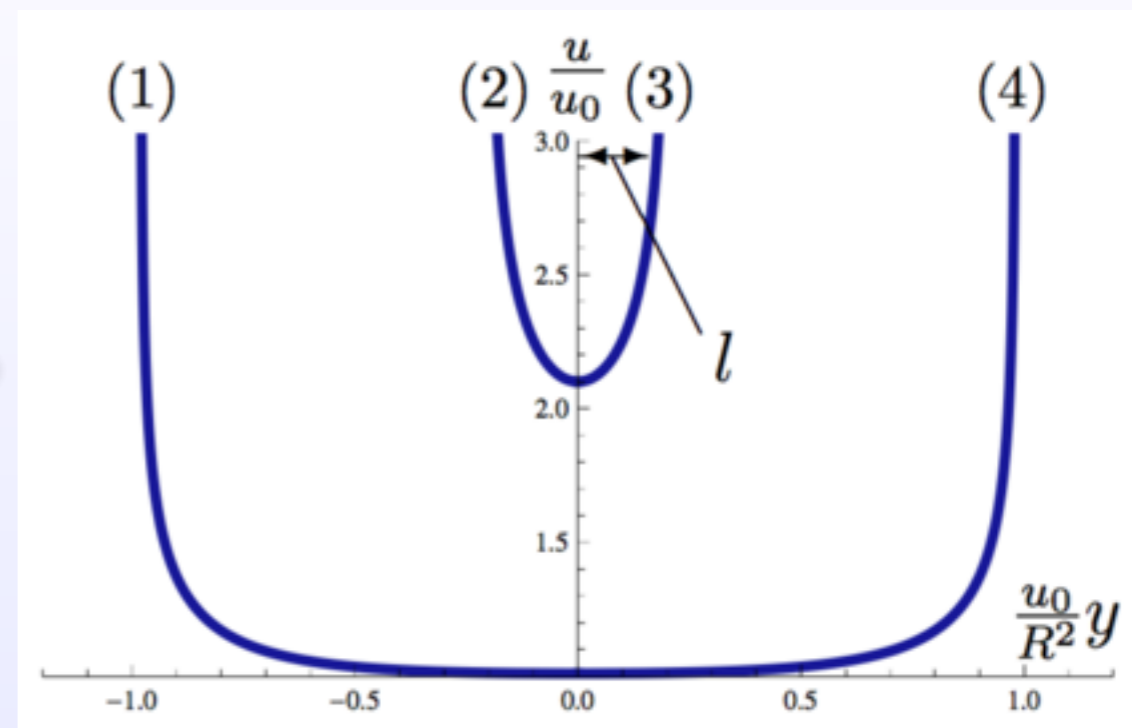
Known?

# Correlations and Phase Transitions

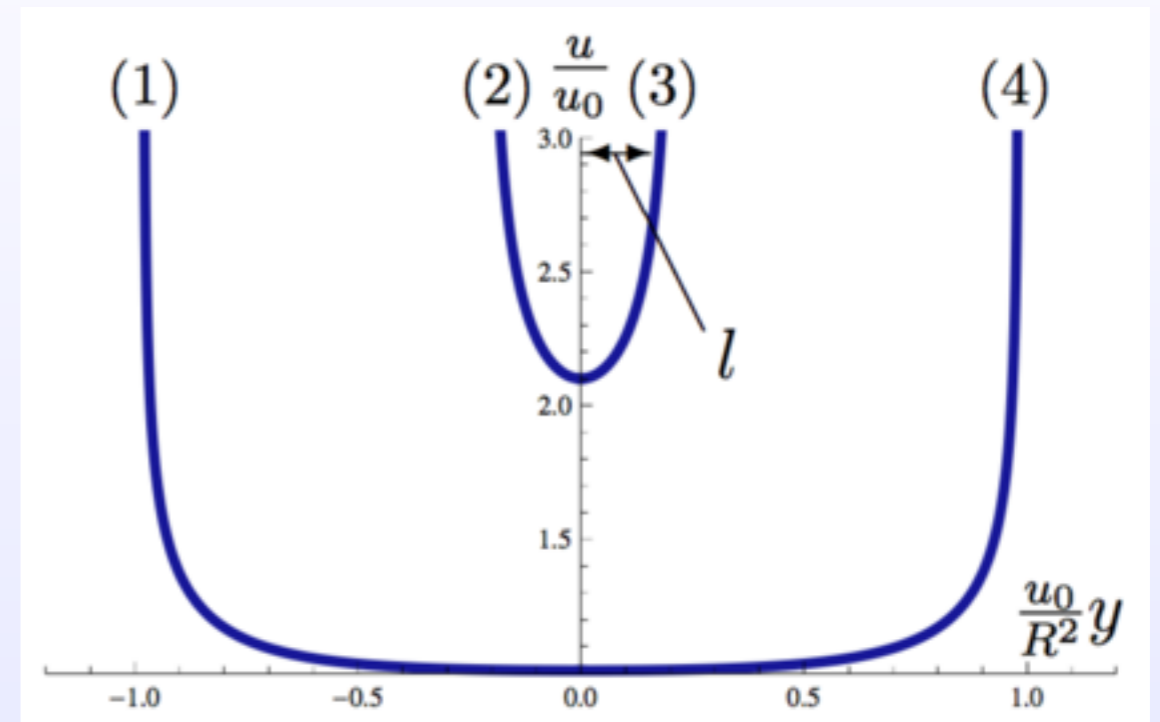
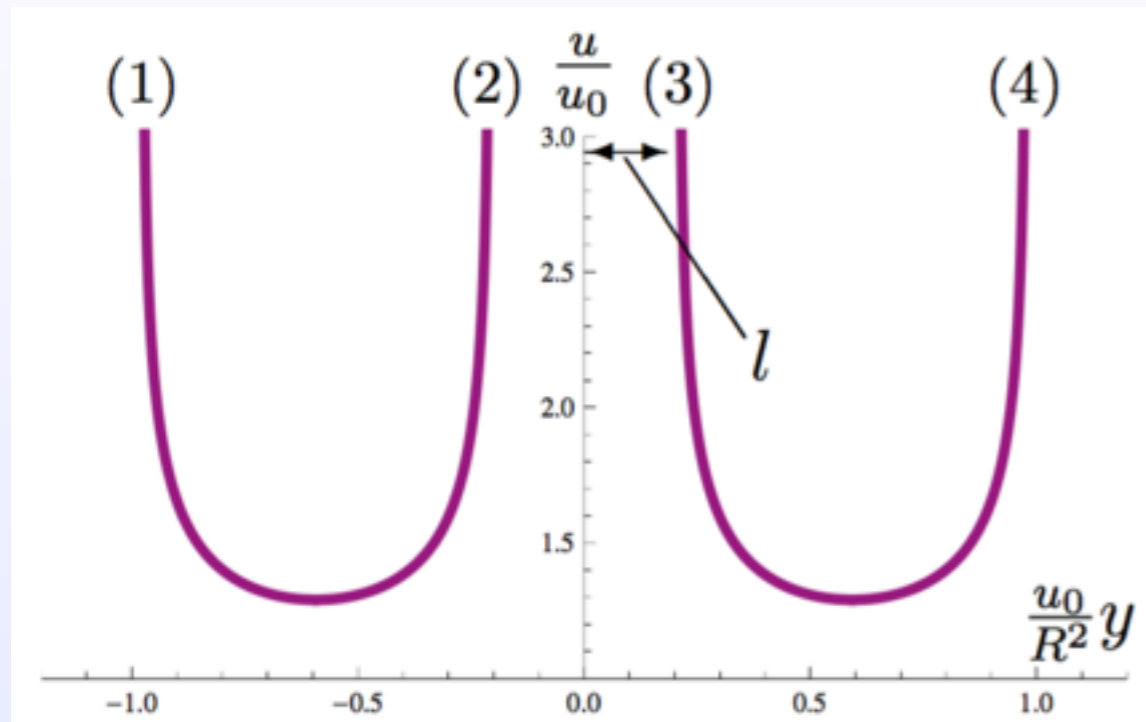


$l_c \sim L_{KK}$

↔



# Correlations and Phase Transitions



## Dimension 5 Correlations

$$\mathcal{O}_{\pm} \sim \psi_{\mp}^{\dagger} F_{\pm y}^2 \psi_{\mp}$$

Large  $l > 1/\text{Gap}$ : adjacent correlations  $\langle \mathcal{O}_{-}^{(1)} \mathcal{O}_{+}^{(2)} \rangle \neq 0$   $\langle \mathcal{O}_{-}^{(3)} \mathcal{O}_{+}^{(4)} \rangle \neq 0$

Small  $l < 1/\text{Gap}$ : distant correlations  $\langle \mathcal{O}_{+}^{(2)} \mathcal{O}_{-}^{(3)} \rangle \neq 0$   $\langle \mathcal{O}_{-}^{(1)} \mathcal{O}_{+}^{(4)} \rangle \neq 0$

**Known?**

(F statistical gauge field)

(vanish at  $T > \text{gap}$ )

# Correspondence between FQHE and 2D QCD

- (1) Basically 2+1 dimensional holographic domain wall fermions = 2D holographic QCD with all its features (chiral symmetry breaking, Goldstone modes, anomaly)
- (2) By compactifying and T-dualizing perpendicular to the defect, the system becomes the holographic dual to 2D QCD of **[Yee-Zahed 1103.6286]**

(3) LEEA of (abelian) FQH States: U(1) CS theory

LEEA of single flavored 2D QCD:

U(1) DBI-CS Theory on a D7 brane **[Yee-Zahed 1103.6286]**

- Anyonic quasiparticle (couples with unit charge to the statistical gauge field) ~ Fundamental quark (baryon number charge  $1/N$ )  
~ Fundamental string ending on D7 brane
- Electron ~ Baryon vertex (bound state of N quarks has baryon number charge (electric charge) 1)

# Conclusions/Outlook

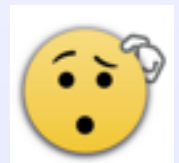
- YM-CS theory with level-changing defects in Holography

- Similarities to FQH samples

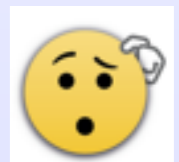
$$\sigma_{xy}^{U(1) \subset U(N)} = \frac{N}{k} \frac{e^2}{h}$$

[Fujita+Li+Ryu+Takayanagi 0901.0924]

- Chiral Symmetry Breaking between Edges



- VEVs involving edge fermions & statistical gauge field,  
Transitions between samples, Dim. 5 correlations



- Edge transport, Impurity scattering, Relation to 2D QCD,  
Backreaction, Tunneling of anyons/electrons, ...

- Non-abelian (YM)CS in strongly coupled systems

