Entanglement vs Nonlocality

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Work in progress with
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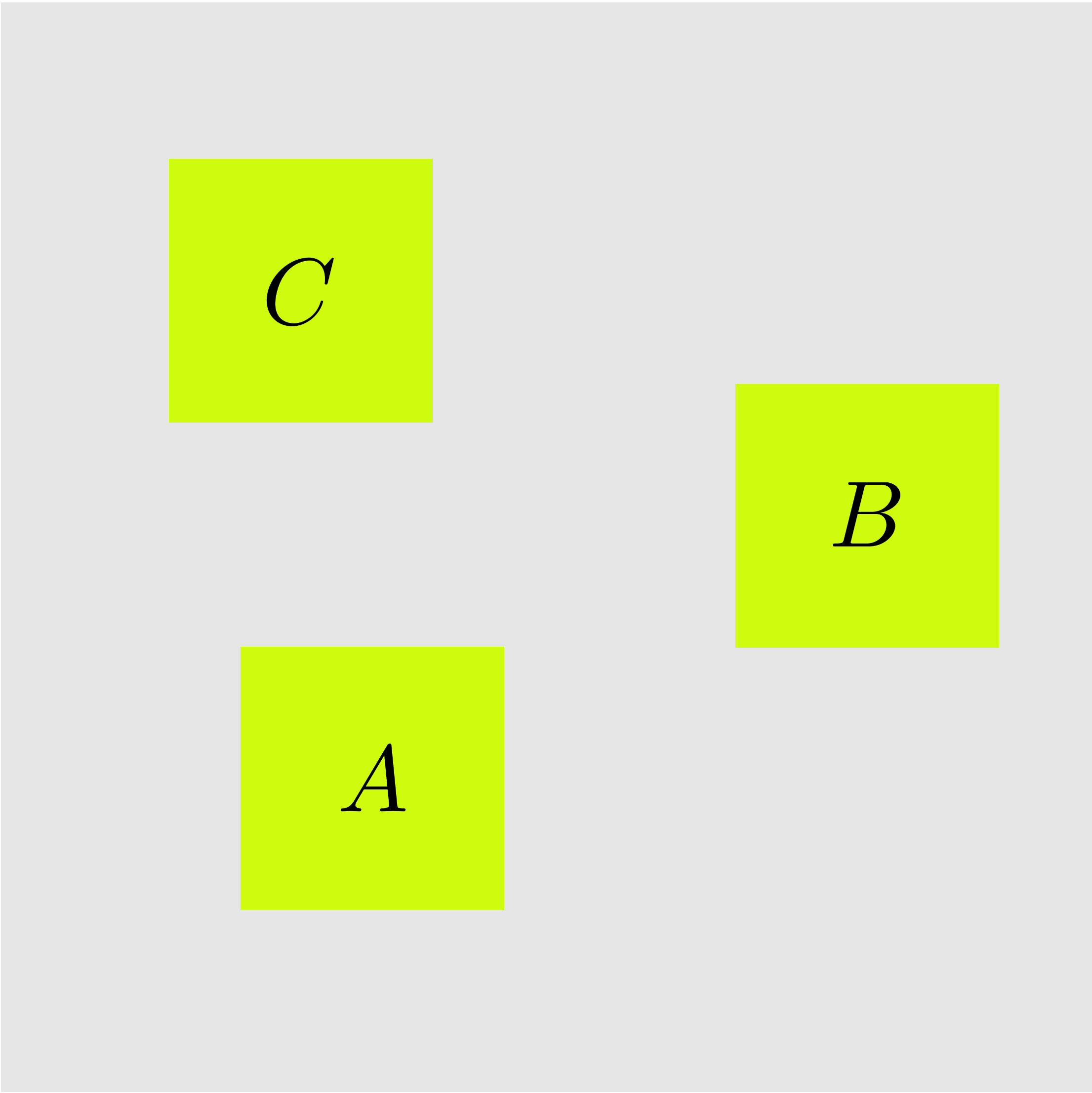
$$t = 0$$



What is the nature of **quantum** correlations between **A-B** or **A-B-C** ?

$$\mathcal{H}_A \otimes \mathcal{H}_B$$

$$\{\rho^i_{AB} = \rho^i_A \otimes \rho^i_B\} \xrightarrow{\lambda} \rho_{AB} = \sum_\lambda p(\lambda)\rho^\lambda_A \otimes \rho^\lambda_B$$

By definition a state is **entangled** if it is **not separable**

The definition can naturally be extended to the **multipartite case**

Measures involving combinations of entropies like the **mutual information** or **I3** do not work because they mix classical and quantum correlations.

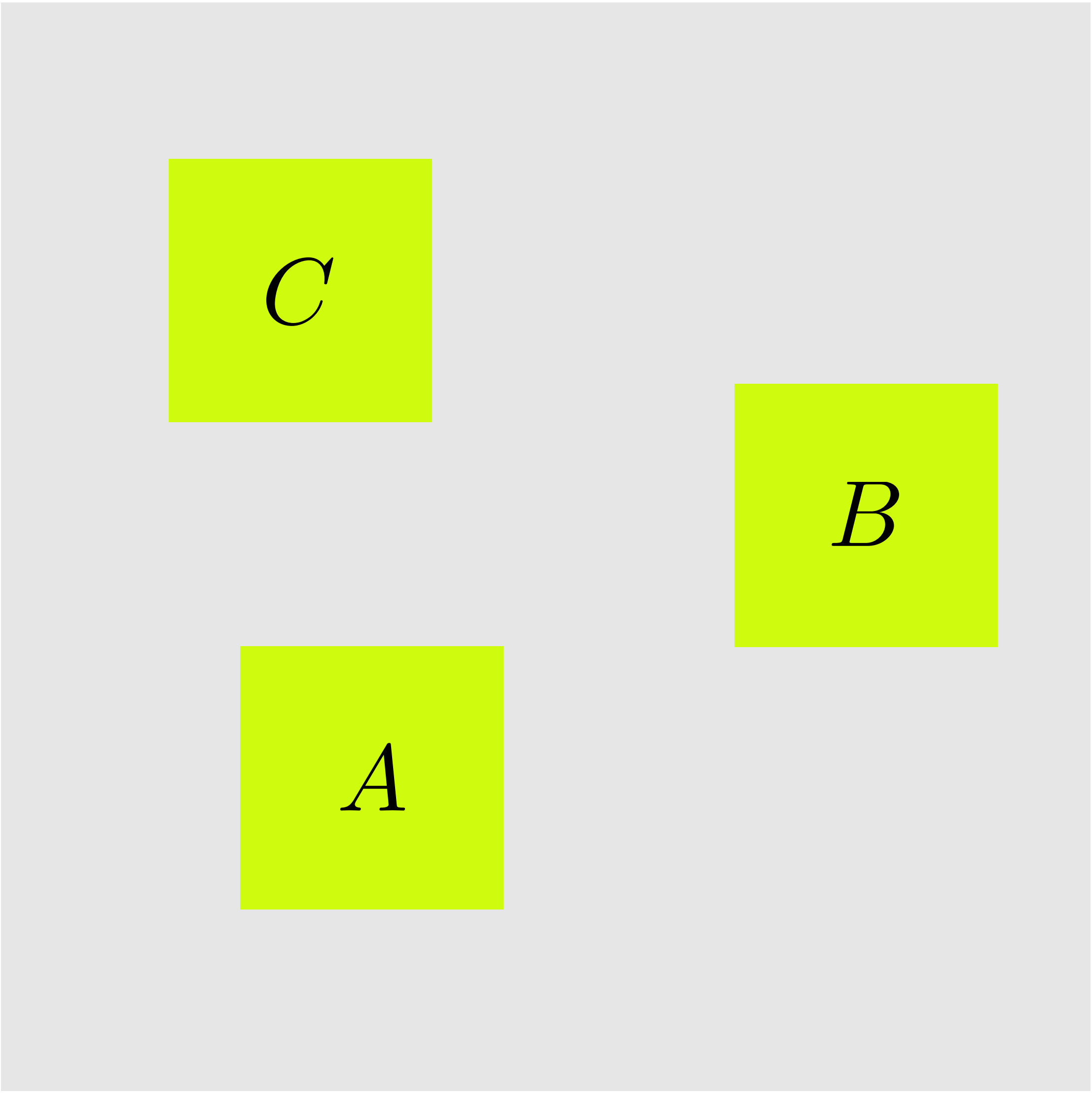
Measures like **distillable entanglement** and **entanglement cost** are very hard to compute even for few qubits.

For the bipartite case one can use the **negativity**, but in field theory it notoriously difficult to compute even in the vacuum of a 1+1 dim CFT. (Coser, Tonni, Calabrese ‘15)

For **multipartite entanglement** the situation is even worst, as for mixed states there is no proposed computable measure, even for qubits.

For **gauge theories**, issue with tensor product structure.

$t = 0$



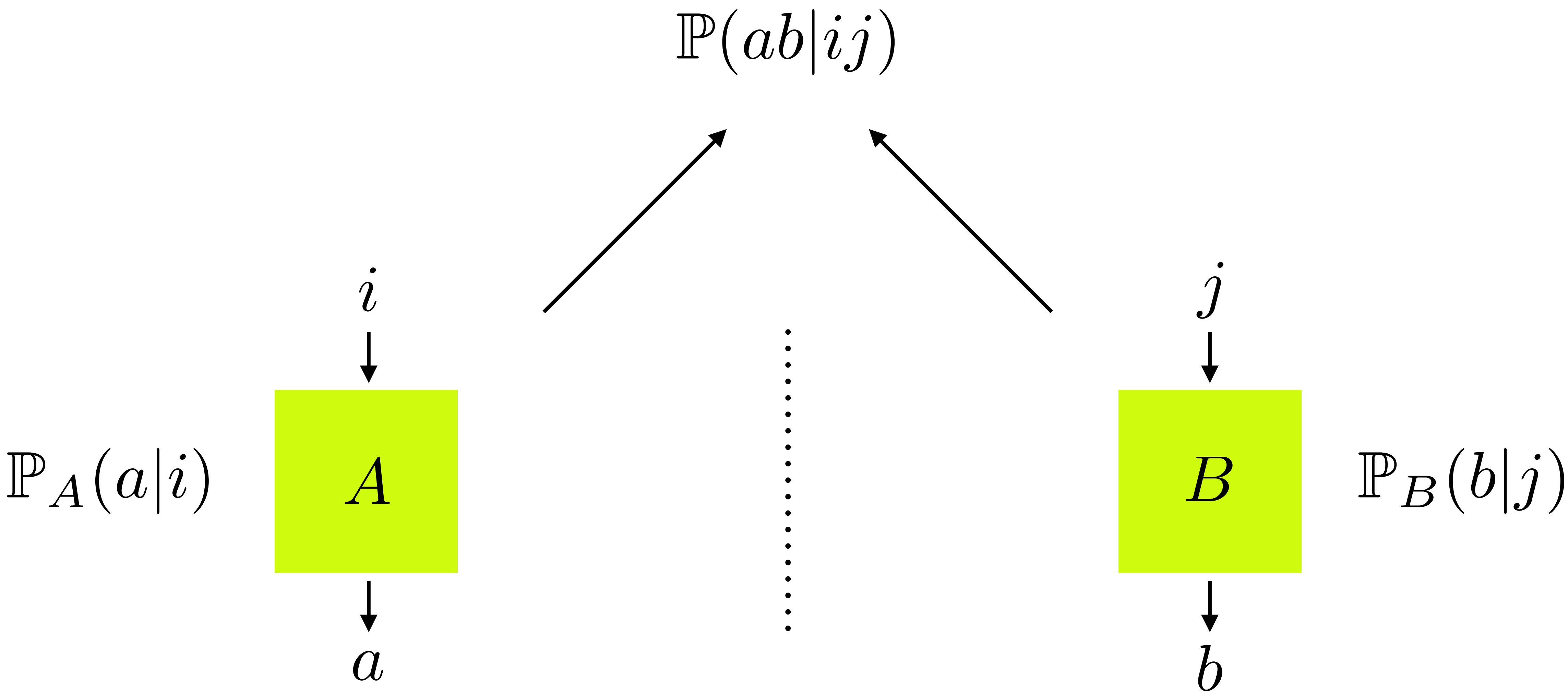
Other ways to look at quantum correlations ?

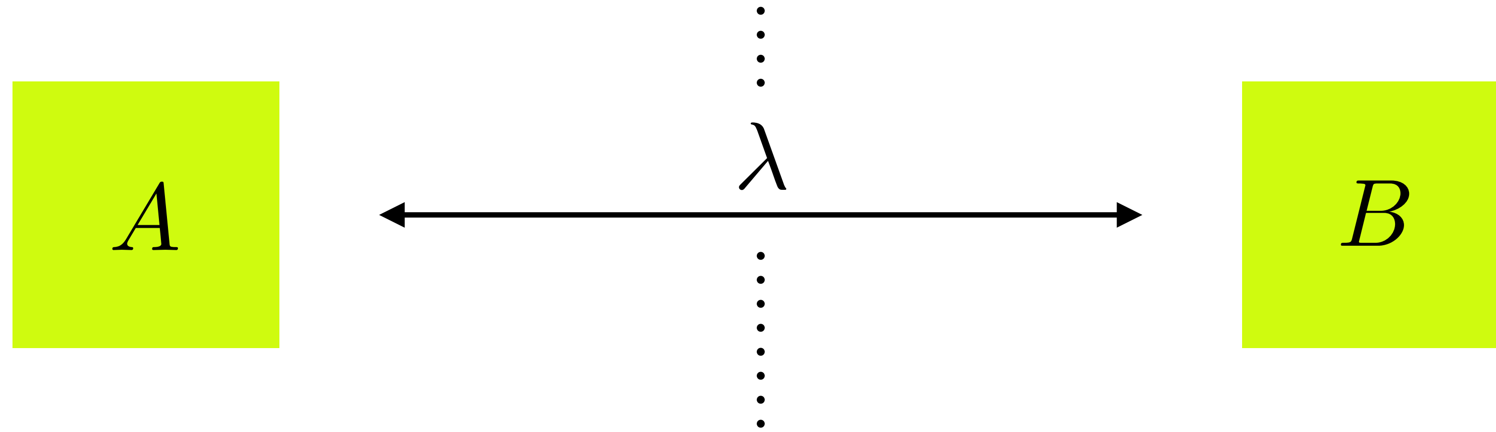
no-signaling conditions

(Popescu, Rohrlich '94)

$$\mathbb{P}_A(a|i) = \sum_b \mathbb{P}(ab|ij) = \sum_b \mathbb{P}(ab|ij') \qquad \forall\, a, i, j, j'$$

$$\mathbb{P}_B(b|j) = \sum_a \mathbb{P}(ab|ij) = \sum_a \mathbb{P}(ab|i'j) \qquad \forall\, b, i, i', j$$





$$\mathbb{P}(ab|ij) = \sum_{\lambda} p(\lambda)\mathbb{P}_A(a|\underset{\substack{\uparrow\\ j}}{i}\lambda)\mathbb{P}_B(b|\underset{\substack{\uparrow\\ i}}{j}\lambda)$$ **locality!**

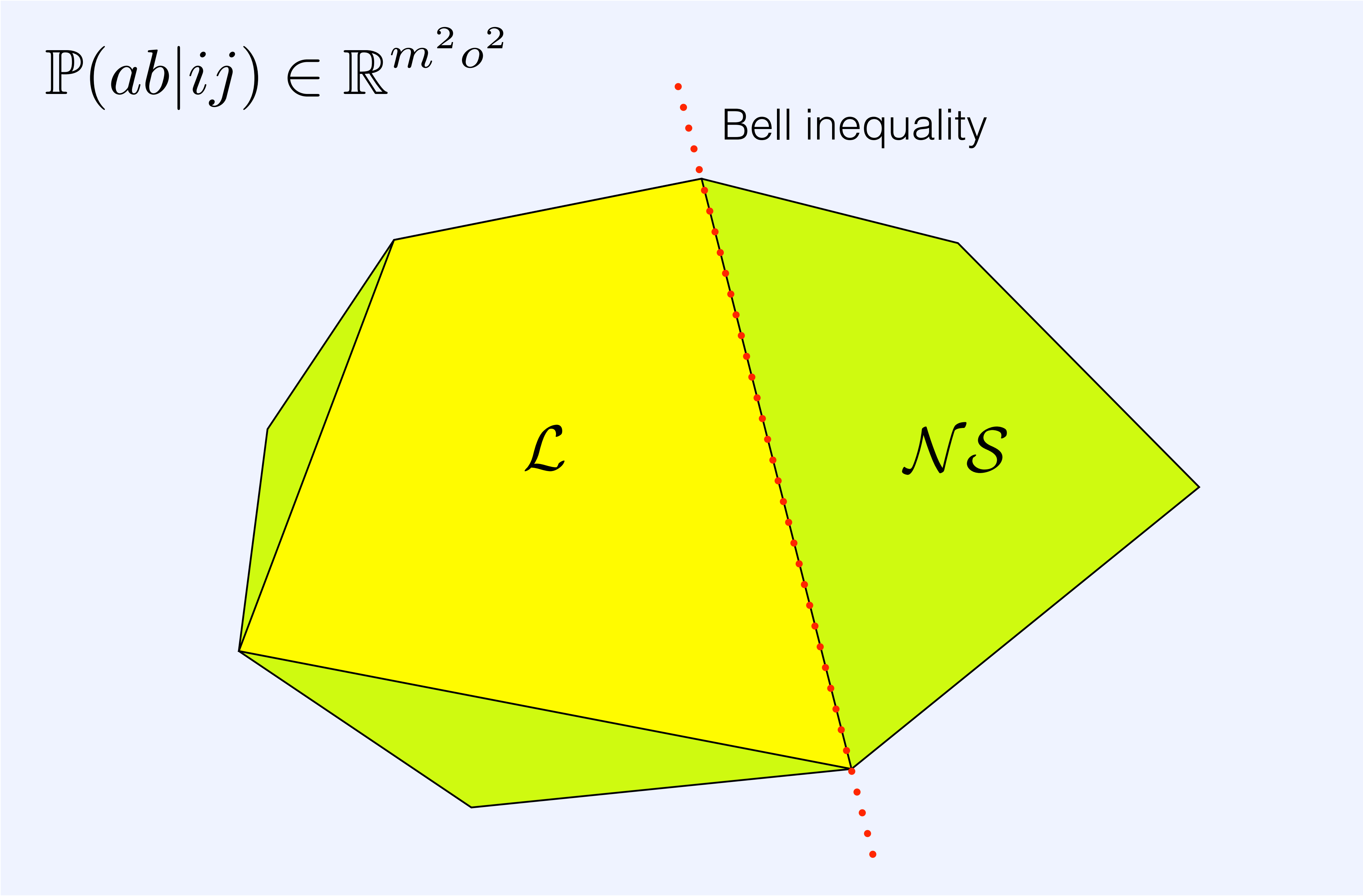
If the probability distribution does not admit this form, it is **nonlocal**

As for entanglement, the definition can be extended to a **multipartite setting**

The definition does **not require a tensor product structure!**

Even more generally, it is completely **theory-independent!**

$$(2, m, o) \qquad i, j \in \{1...m\} \qquad a, b \in \{1...o\}$$



$$\sum_{abij} S^{ab}_{ij}\, \mathbb{P}(ab|ij) \leq S_k$$

$$\mathbb{P}(ab|ij) = \langle\psi|\,\Pi_{a|i}\Pi_{b|j}\,|\psi\rangle$$

QM: $\Pi_{a|i}\Pi_{b|j} \equiv \Pi_{a|i} \otimes \Pi_{b|j}$ $\qquad \mathcal{Q} = \{\mathbb{P}^{\mathrm{QM}}(ab|ij)\}$

$$\Downarrow \quad \cancel{\Uparrow} \qquad\qquad\qquad \rotatebox{90}{$\subseteq$}$$

QFT: $\left[\Pi_{a|i}, \Pi_{b|j}\right] \equiv 0$ $\qquad \mathcal{Q}' = \{\mathbb{P}^{\mathrm{QFT}}(ab|ij)\}$

$\mathcal{Q} \overset{?}{\subset} \mathcal{Q}'$ **Tsirelson problem**

Separable states are local

This follows immediately from the construction that uses a shared random variable.

Nonlocal states are not separable, i.e. entangled

Indeed Bell inequalities (BI) are used in experiments to detect entanglement, they are entanglement witnesses.

But some entangled states are local!

There are known families of states which are non-separable (and hence entangled) but nevertheless only locally correlated.

How "strong" can quantum nonlocality be?

In the simplest scenario (2,2,2), the local polytope is specified by a single BI, the CHSH inequality.

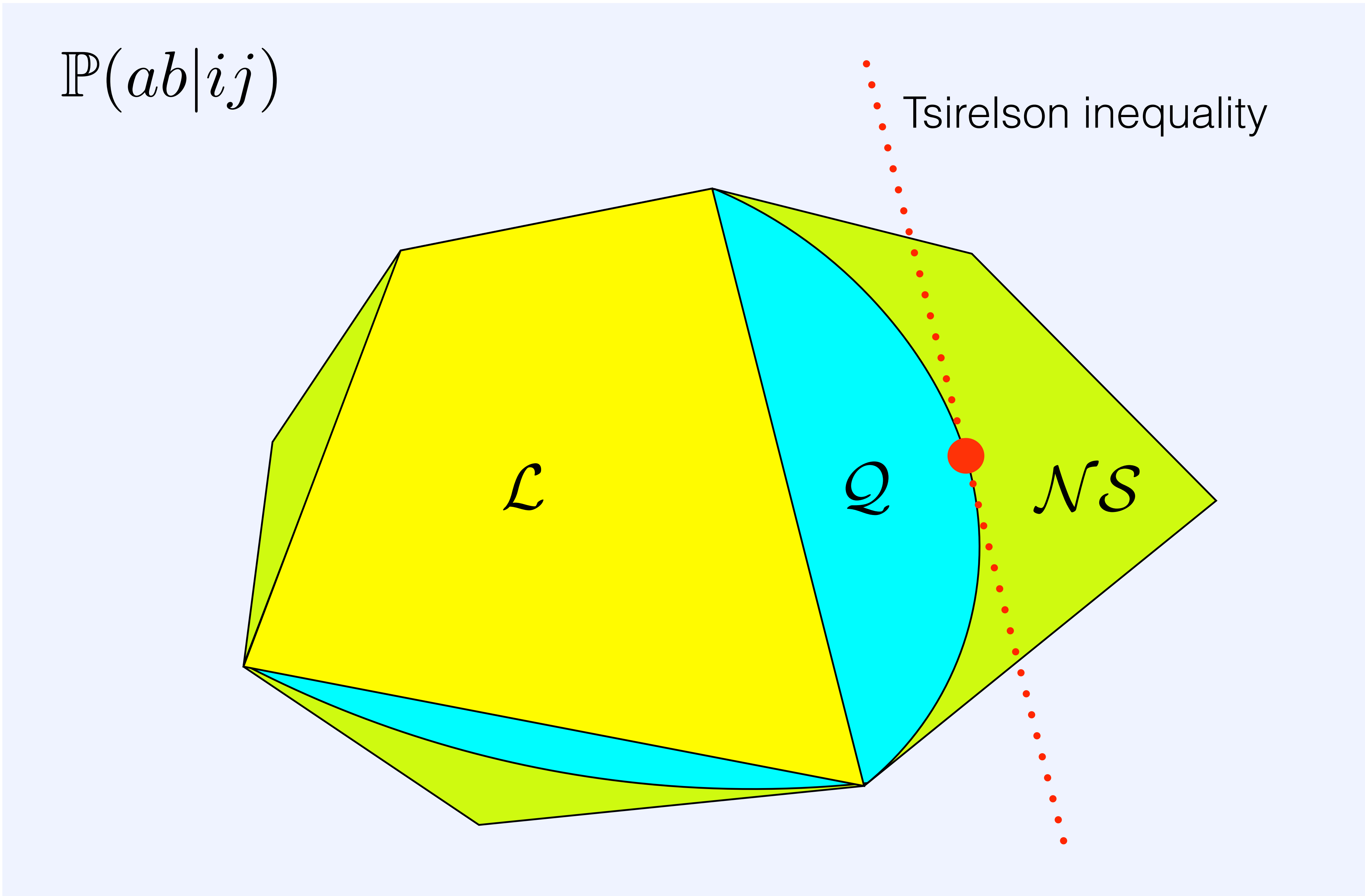
$$\langle \mathcal{B} \rangle \leq 2 \quad (\mathcal{L})$$

$$\langle \mathcal{B} \rangle \leq 2\sqrt{2} \quad (\mathcal{Q})$$ **Tsirelson bound**

(Tsirelson '80)

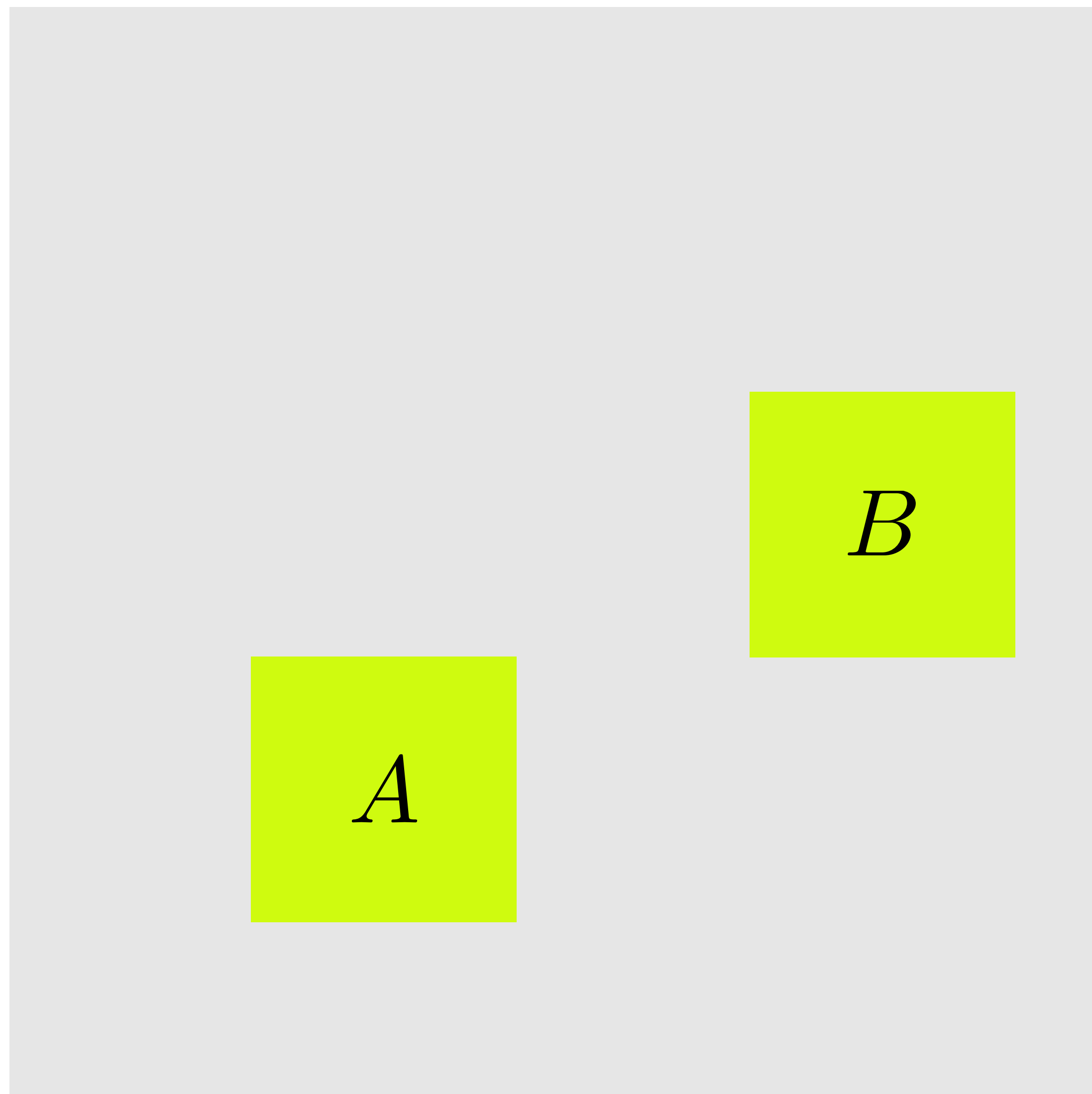
Are no-signaling and nonlocality sufficient to define QM? **No!** (Popescu, Rohrlich '94)

$$\langle B \rangle \leq 4 \quad (\mathcal{NS})$$



The space of quantum behaviours is **not a polytope** and it can be specified only by an infinite number of inequalities.

$$t = 0$$



$(2, 2, o)$ $\qquad$ $i, j \in \{1, 2\}$ $\qquad$ $a, b \in \{1 ... o\}$

We can assume that "o" is still finite (some lattice regularisation) but typically it would be infinite and we want to be able to take the continuum limit. Now in this scenario choose a BI

$$\sum_{abij} S^{ab}_{ij}\, \mathbb{P}(ab|ij) \leq S_k$$

We want to construct "experiments" that could be used to detect its violation, so we need to rewrite this inequality in terms of expectation values of appropriately chosen observables.

For the (2,2,2) scenario the answer is obvious and indeed this is how the CHSH inequality is usually presented:

$$\langle O_1 \tilde{O}_1 \rangle + \langle O_1 \tilde{O}_2 \rangle + \langle O_2 \tilde{O}_1 \rangle - \langle O_2 \tilde{O}_2 \rangle \leq 2$$

For an arbitrary BI in the more general scenario (2,2,o) there is also a trivial construction. Each probability that appears in the inequality can be obtained as the expectation value of the corresponding projector:

$$\mathbb{P}(ab|ij) \to \langle \psi | \, \Pi_{a|i} \Pi_{b|j} \, | \psi \rangle$$

This transformation maps the original BI to itself, but in a different scenario

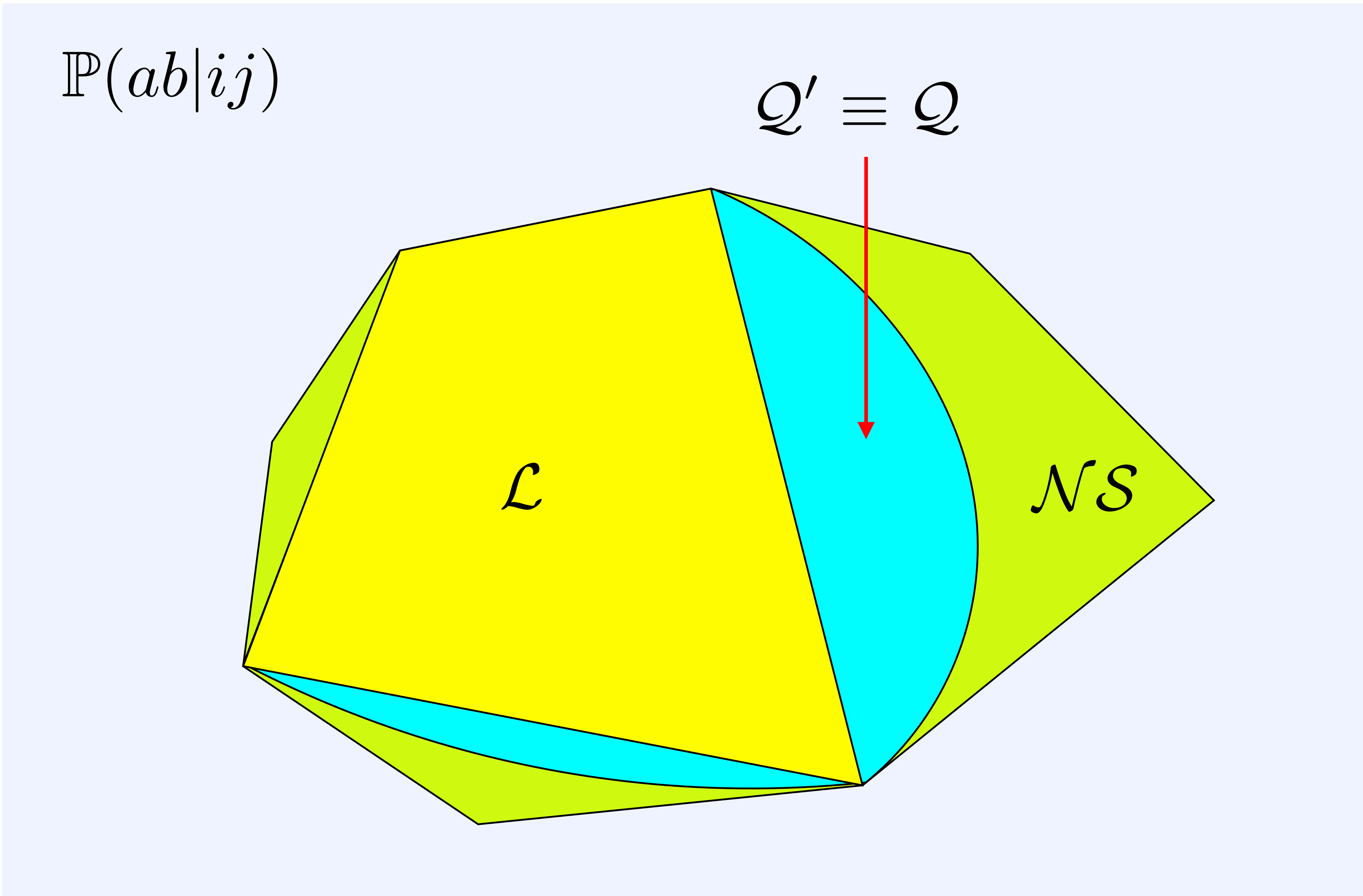
$$(2, 2, o) \to (2, 4o^2, 2)$$

In the case where “o” is infinite, this construction implies that in order to be able to detect a violation, infinitely many measurements are necessary. We want instead to rewrite the original inequality within the original scenario (2,2,o), using expectation values of observables with a spectrum containing “o” different eigenvalues.

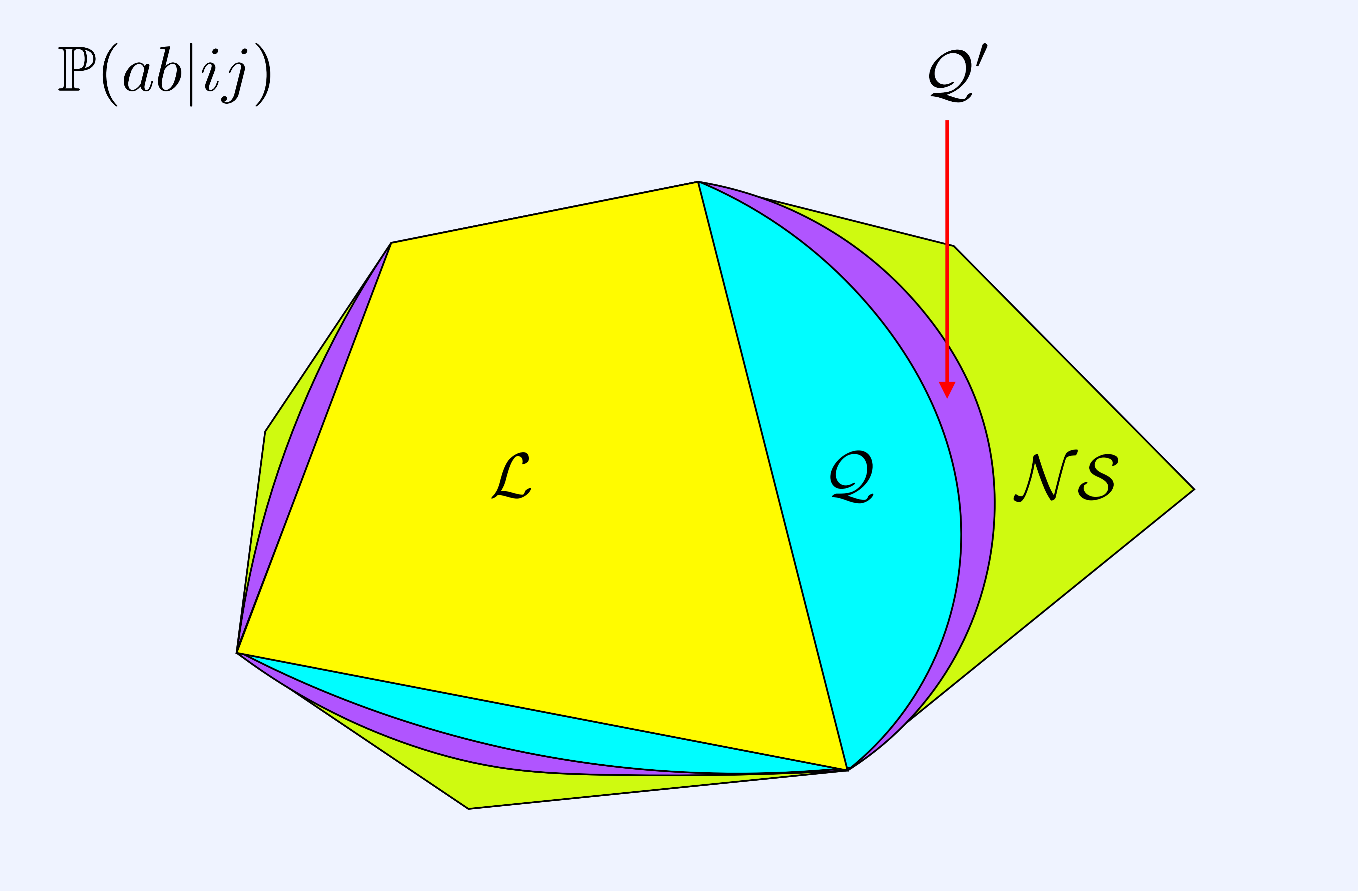
$$\sum_i \alpha_i \langle O_i \rangle + \sum_j \tilde{\alpha}_j \langle \tilde{O}_j \rangle + \sum_{ij} \beta_{ij} \langle O_i \tilde{O}_j \rangle \leq S_k$$

A BI then imposes a set of constraints on the spectrum of the observables that should be used to detect its violation.

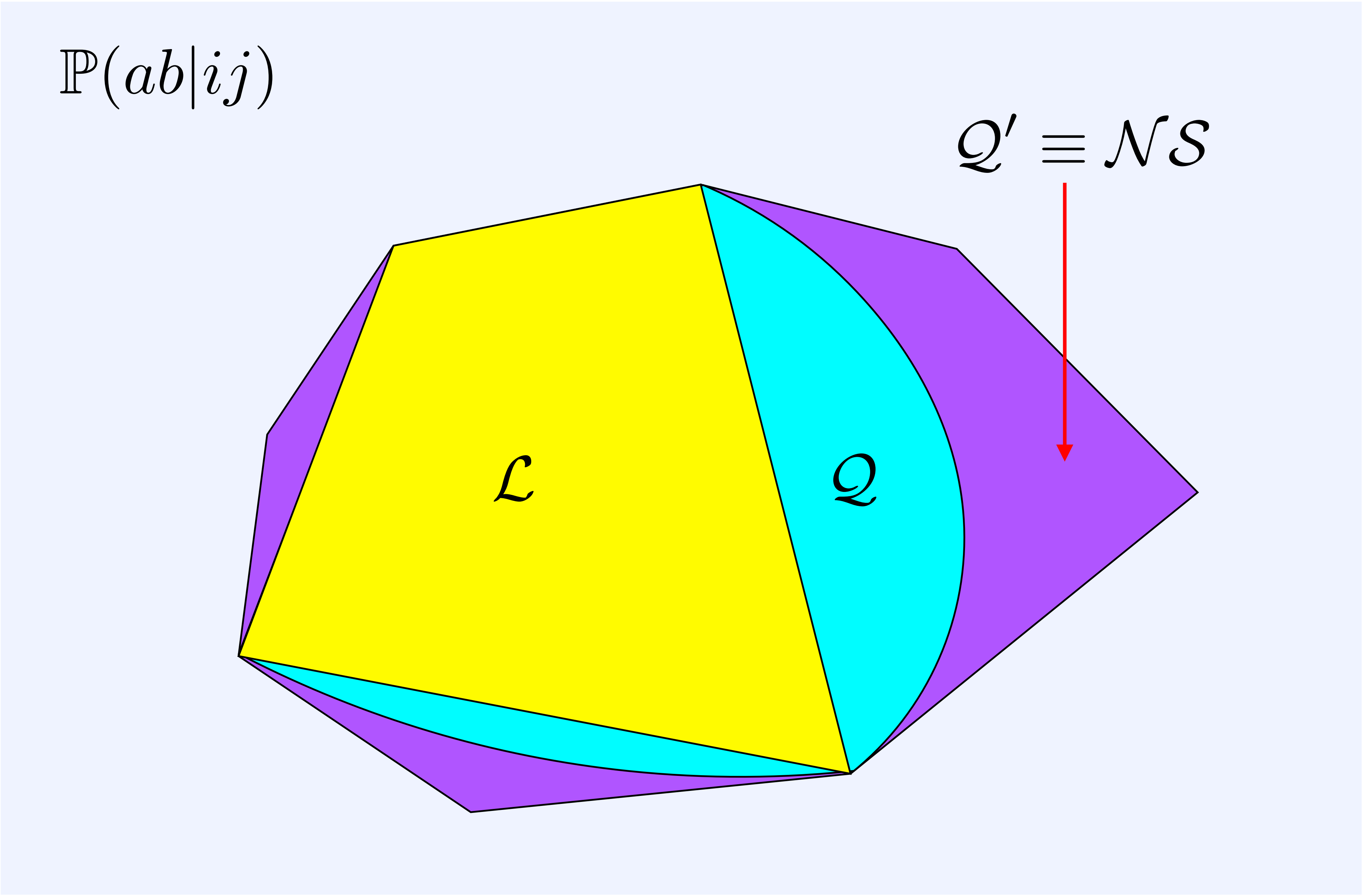
Is there always a solution to this problem and what are the properties of these operators? A formulation in terms of AQFT seems natural in this context, but ultimately one would hope to translate the construction into a more “practical” formulation where one can investigate the possible violation of BI by computing correlation functions.



$\mathbb{P}(ab|ij)$
$\mathcal{Q}' \equiv \mathcal{Q}$
$\mathcal{L}$
$\mathcal{NS}$



$\mathbb{P}(ab|ij)$
$\mathcal{Q}'$
$\mathcal{L}$
$\mathcal{Q}$
$\mathcal{NS}$



Are causality, nonlocality and covariance sufficient to “define” relativistic QFT?