Measurement-based formulation of quantum heat engine
and
optimal performance of quantum heat engines

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Research Theme:

Information theoretical analysis for heat engines;

Data compression:

Cryptography:

Heat Engine:

What is the merit?
Merits of information theoretical approach

Merit 1: A new sight helps us to find hidden problems in previous formulations.

Result 1: Measurement based formulation for quantum heat engine

Merit 2: A new mathematical tool helps us to try unsolved problems.

Result 2: Asymptotic analysis for the optimal performance of quantum heat engine
Two formulations for work extraction from quantum system

**Operational scenario: Internal unitary model**

\[ U := T \rightarrow \exp \left( \sum_{i} \lambda_{i}^{(0)} \right) \]

\[ \langle W \rangle := \text{tr} \left[ \rho \Lambda_{I} \right] \]

When we request the detectability of work, the time evolution of \( I \) cannot be unitary.

**Autonomous scenario:**

**Total unitary model**

\[ \left[ \rho I, H_{I} + H_{E_{X}} \right] = 0 \]

\[ \langle W \rangle := \text{tr} \left[ \Lambda_{E_{X}} \left( \rho_{E_{X}} \right) H_{E_{X}} - \rho_{E_{X}} H_{E_{X}} \right] \]

or deterministic work extraction

\[ \rho_{E_{X}} \rightarrow \sigma^{\rho_{I}}_{E_{X}} \]

References:
“Detectability” of work

The work extraction translates the energy ...

As the minimal request for the “useful form,” we demand the following;

We can detect the amount of the extracted energy only by measuring the external system.

Can Internal-unitary model satisfy this request?
A hint from quantum communication

A well-known fact in quantum communication field;

“When a system evolves approximately unitarily, it is difficult to obtain information from the system.”

The time evolution is the more close to unitary,

\[
\rho I \quad \Lambda_I(\rho_I) \\
\rho E_X \quad \sigma^\rho I E_X
\]

This state is the more independent of \( \rho_I \)

In fact, we can derive a trade-off relation for the internal unitary-model.
Trade-off relation

Whenever the set \( \{ V, \rho_E \} \) satisfies
\[
b(\Lambda_I(\rho_I), U \rho_{I} U^\dagger) < \epsilon \quad \text{for any } \rho_I,
\]
then,
\[
b(\sigma_{E_X}^{\rho_I}, \Lambda_I(\rho_I)) < \epsilon
\]

When we request the detectability of work, the time evolution of \( I \) cannot be unitary.…. 

- The Brues distance expresses the limit of the distinguishability of the states with the POVM measurements.
- Whenever \( \Lambda_I \approx U \), the measurements on \( E_X \) can not distinguish the final states of \( E_X \) whose initial state of \( I \) are different.
- Because the energy gain changes from its minimum to its maximum during the initial state of \( I \) changes, we can not know the energy gain of \( E_X \) by the measurements on \( E_X \), after all.


So we need another operational formulation!
Reconsider about “definition of work”

In macroscopic heat engine...

In quantum heat engine...

So, work extraction from quantum system
=Discernable change of macro system caused by quantum system
=measurement process!
Measurement-based formulation

In general, an arbitrary measurement process is described as a set of completely positive (CP) maps;

$$\mathcal{E}_A \otimes I_B (\rho_{AB})$$

is positive.

So, we formulate the work extraction as a set of CP maps;

$$\rho_I \rightarrow \left\{ \mathcal{E}_j, w_j \right\} \frac{\mathcal{E}_j(\rho_I)}{p_j} \quad \text{with} \quad p_j := \text{tr}[\mathcal{E}_j(\rho_I)]$$

$\mathcal{E}_j$ is completely positive (CP) map, and $w_j$ is a function of $j$.

$$\sum_j p_j w_j = \text{tr}[\rho_I H_I - \sum_j \mathcal{E}_j(\rho_I) H_I] \iff \text{energy conservation condition}$$

Our formulation is consistent with total unitary model, because of direct-indirect measurement relation.

So, now we have an operational scenario which is consistent with the autonomous scenario.
Merits of information theoretical approach

Merit 1: A new sight helps us to find hidden problems in previous formulations.

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Researches for the optimal performance of heat engines

In Thermodynamics;

The optimal performances for macroscopic heat engines are given.

In Statistical mechanics (and quantum information);

In i.i.d. case, i.e., \( \rho I = (\rho \beta)^{\otimes n} \), the optimal performances are given.


We clarify here in i.i.d case, by asymptotic theory in data compression.


Heat engine with finite-size heat baths

Operational form:

$$W(\{\mathcal{E}_j\}) := \sum \mathcal{E}_j (\rho_{H\beta_L}) w_j = \text{tr}[(\rho_{H\beta_L} - \rho_{\text{fin} H}) (H_H + H_L)]$$

$$Q_H(\{\mathcal{E}_j\}) := \text{tr}[\hat{Q}_H(\rho_{H\beta_L} - \rho_{\text{fin} H}) H_H]$$

We measure here.

We can add arbitrary catalytic system.

Let us define the optimal efficiency as follows:

\[ \eta_{\text{opt}}[\beta_H, \beta_L, Q_n] := \sup_{\{\mathcal{E}_j, w_j\} : Q(\{\mathcal{E}_j, w_j\} = Q_n)} \eta(\{\mathcal{E}_j, w_j\}) \]

When the particles of each heat bath do not interact each others, i.e.,

\[ \rho_{\beta_H \beta_L} = \left( \rho_{\beta_H|H_H} \otimes \rho_{\beta_L|H_L} \right) \otimes n \]

The optimal efficiency satisfies

\[ \eta_{\text{opt}}[\beta_H, \beta_L, Q_n] = 1 - \frac{\beta_H}{\beta_L} - C_1 \frac{Q_n}{n} - C_2 \frac{Q_n^2}{n^2} + O \left( \frac{Q_n}{n^2} \right) \]

We can easily compute the coefficients;

Our results give computable approximation of optimal efficiency even when \( n = 10^4, 10^8, 10^{12} \ldots \)
Evaluation of the quality of energy gain

We can give a concrete dynamics satisfying the asymptotic expansion of the efficiency, and the followings;

\[ A_{BH} = \frac{\Delta E_{BH}}{\Delta S_{BH}} = O(1) \]
\[ A_{BL} = \frac{\Delta E_{BL}}{\Delta S_{BL}} = O(1) \]
\[ A_W = \frac{\Delta E_W}{\Delta S_W} \leq O\left(\frac{\log n}{Q_n}\right) \]

When \[ Q_n = an^b \quad 0 < b < 1 \]
\[ A_W \to 0 \quad (n \to \infty) \]
holds.

Optimal process is an example of translation from “heat” to “work”.

Summary 1/2:

As another operational formulation, we propose a measurement-based formulation, which is consistent with the autonomous formulation.

We find that this model is not consistent with the autonomous formulation by considering the detectability of work.

As another operational formulation, we propose a measurement-based formulation, which is consistent with the autonomous formulation.
Summary 2/2:

We give an asymptotic expansion of optimal efficiency.

\[ \eta_{\text{opt}}[\beta_H, \beta_L, Q_n] = 1 - \frac{\beta_H}{\beta_L} - C_1 \frac{Q_n}{n} - C_2 \frac{Q_n^2}{n^2} + O \left( \frac{Q_n}{n^2} \right) \]

We give the optimal process as a concrete dynamics. It is a good example of translation from “heat” to “work.”

\[ A_{B_H} = O(1) \quad A_{B_H} = O(1) \quad A_W \leq O \left( \frac{\log n}{Q_n} \right) \]