Measurement-based formulation of quantum heat engine
and
optimal performance of quantum heat engines

arXiv:1405 .6457 (2014)

arXiv:1504 .06150 (2015)

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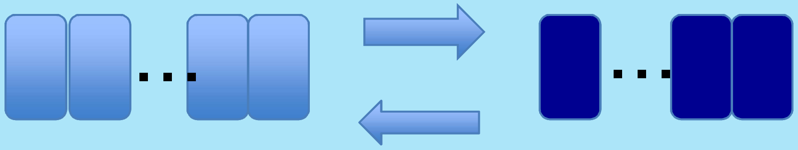
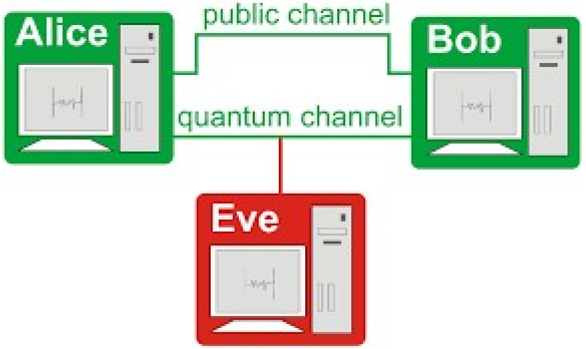
Research Theme:

Information theoretical analysis for heat engines;

Data compression :

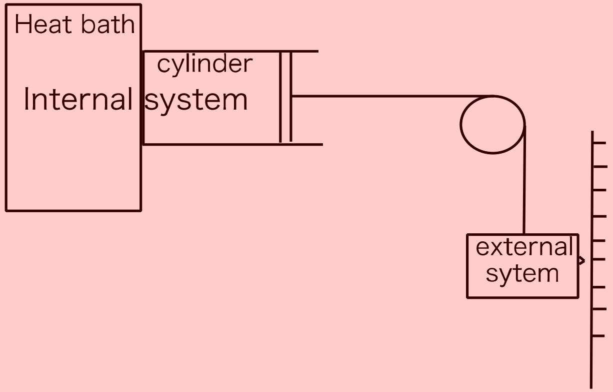


Cryptography:



Apply!

Heat Engine :



What is the merit ?

Merits of information theoretical approach

Merit 1: A new sight helps us to find hidden problems in previous formulations.

⇨ Result 1: Measurement based formulation for quantum heat engine

Merit 2: A new mathematical tool helps us to try unsolved problems.

⇨ Result 2: Asymptotic analysis for the optimal performance of quantum heat engine

Two formulations for work extraction from quantum system

Operational scenario: Internal unitary model

*I:*Baths an

When we request the ditectability of work, the time evolution of *I* cannot be unitary….

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$$U := T_{\rightarrow} \exp$$

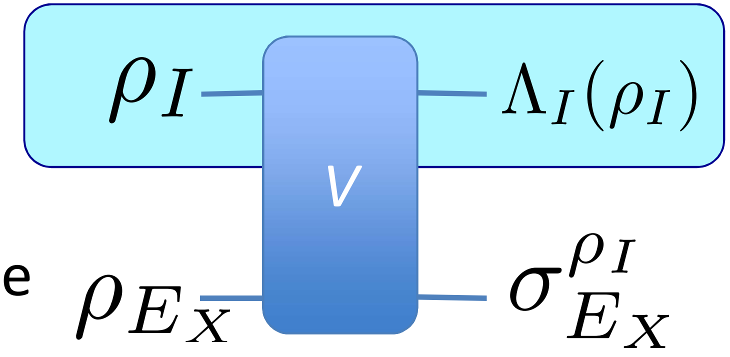
$$\langle W \rangle := \mathrm{tr}[\rho_I H_I(\lambda(0))$$

J. Kurchan, arXiv:cond-mat/0007360(2000).

H. Tasaki, arXiv:cond-mat/0009244 (2000).

T. Sagawa and M. Ueda, Phys. Rev. Lett, **100** 080403 (2008).

Autonomous scenario: Total unitary model



Ex: work storage

$$[V, H_I + H_{E_X}] = 0$$

$$\langle W \rangle := \mathrm{tr}[\Lambda_{E_X}(\rho_{E_X}) H_{E_X} - \rho_{E_X} H_{E_X}]$$ or deterministic work extraction

F. G. S. L. Brandao, *et. al.*, arXiv:1305.5278, (2013).

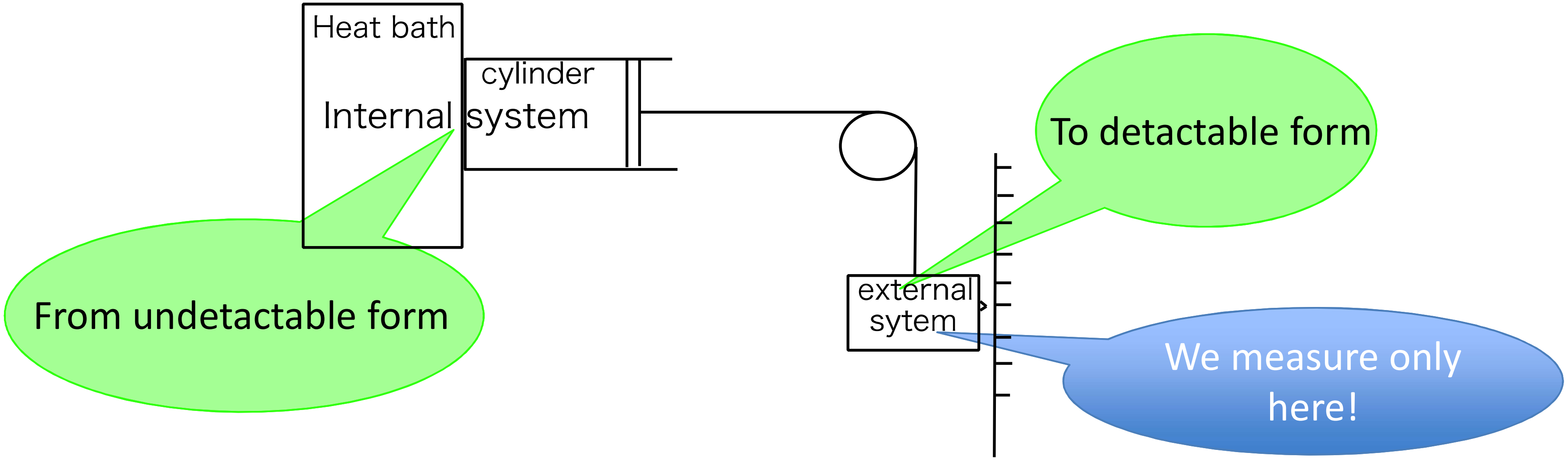
P. Skrzypczyk, *et. al* Nat. Comm. **5**, 4185, (2014)

H. Tasaki, arXiv:1511.01999 (2015).

M. Horodecki and J. Oppenheim, Nat. Commun. **4**, 2059 (2013).

“Detectability” of work

The work extraction translates the energy …



To detactable form

From undetactable form

We measure only here!

As the minimal request for the “useful form,” we demand the following;

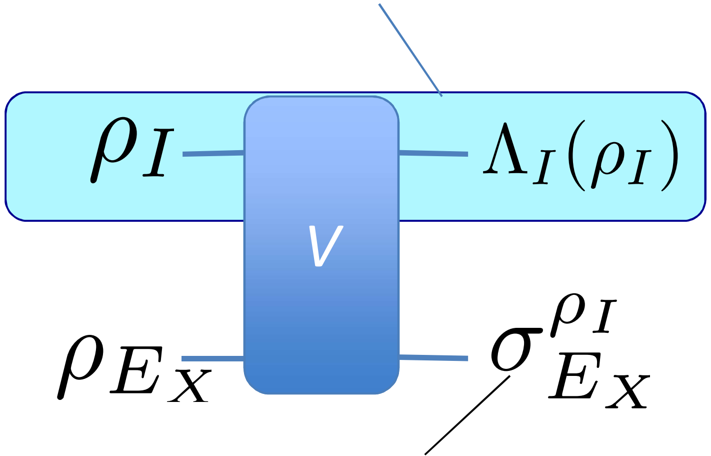
> We can detect the amount of the extracted energy only by measuring the external system.

Can Internal-unitary model satisfy this request?

A hint from quantum communication

A well-known fact in quantum communication field;

"When a system evolutes approximately unitarily, it is difficult to obtain information from the system."



The time evolution is the more close to unitary,

This state is the more independent of ρ_I

In fact, we can derive a trade-off relation for the internal unitary-model.

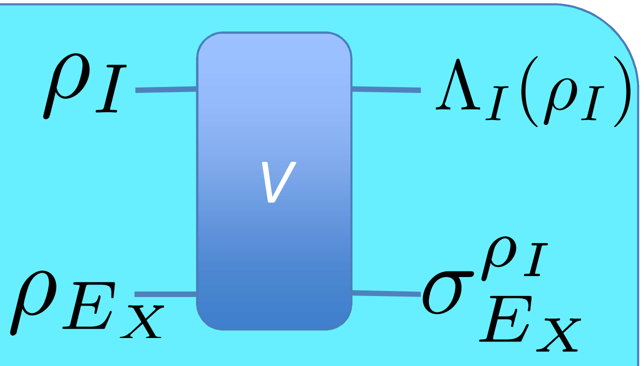
Trade-off relation

Whenever the set $\{V, \rho_{E_X}\}$ satisfies

$b(\Lambda_I(\rho_I), U\rho_I U^\dagger) < \epsilon$ for any ρ_I,

then,

$b(\sigma$



When we request the ditectability of work,
the time evolution of *I* cannot be unitary….

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➡ So we need another operational formulation!

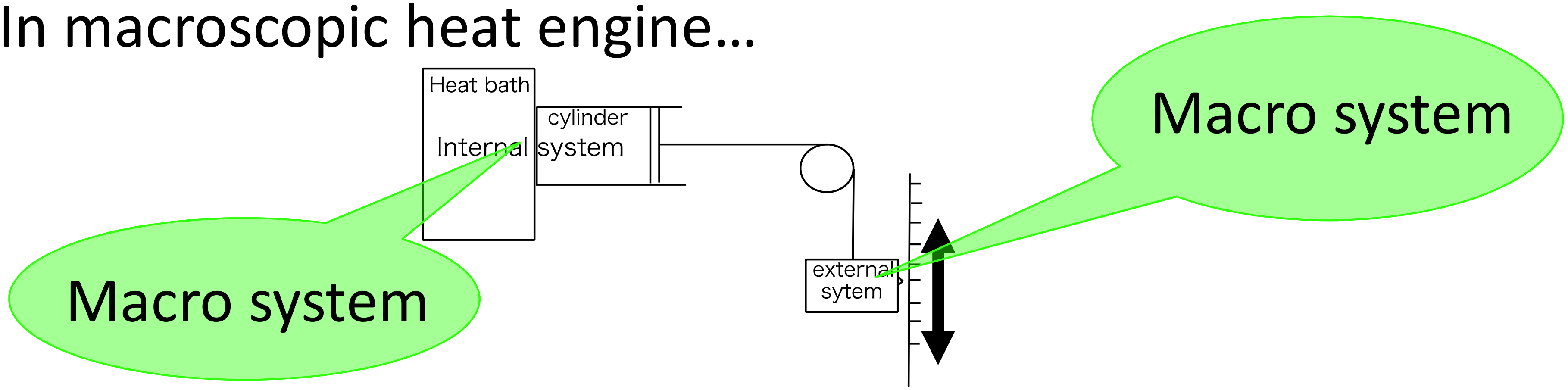
- The E
  distin
  nts.

◆ Whenever $\Lambda_I \approx U$, the measurements on *E*x can not distinguish the final states of *E*x whose initial state of *I* are different.

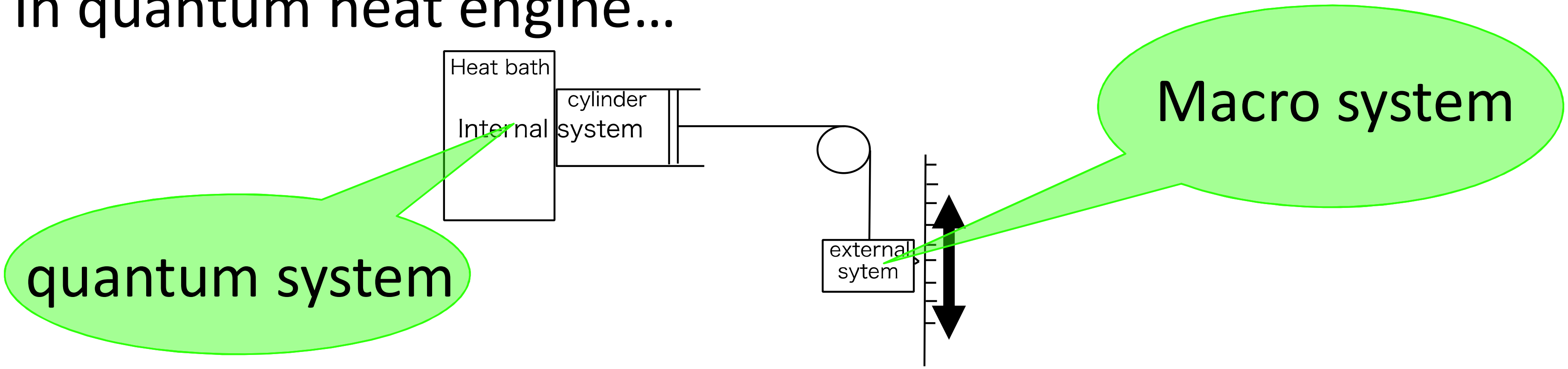
◆ Because the energy gain changes from its minimum to its maximum during the initial state of *I* changes, we can not know the energy gain of *E*x by the measurements on *E*x, after all.

Reconsider about “definition of work”

In macroscopic heat engine…



In quantum heat engine…



So, work extraction from quantum system
=Discernable change of macro system caused by quantum system =measurement process!

Measurement-based formulation

In general, an arbitrary measurement process is described as a set of completely positive (CP) maps;

CP map on system *A;* For arbitrary positive ρ_{AB},

$$\mathcal{E}_A \otimes I_B(\rho_{AB})$$

is positive.

So, we formulate the work extraction as a set of CP maps;

$$\rho_I \text{---} \boxed{\{\mathcal{E}_j, w_j\}} \text{---} \frac{\mathcal{E}_j(\rho_I)}{p_j} \quad \text{with} \quad p_j := \mathrm{tr}[\mathcal{E}_j(\rho_I)]$$

$\mathcal{E}_j$ is completely positeve (CP) map, and w_j is a function of *j*.

$$\sum_j p_j w_j = \mathrm{tr}[\rho_I H_I - \sum_j \mathcal{E}_j(\rho_I) H_I]$$ ⇦energy conservation condition

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Relation to the total unitary models

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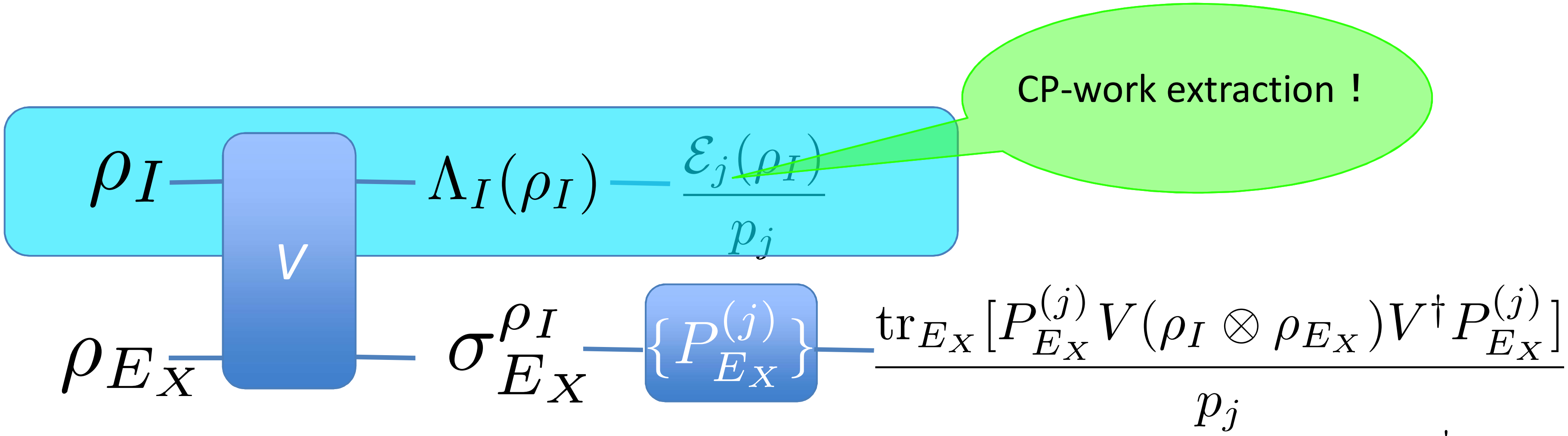
Our formulation is consistent with total unitary model, because of direct-indirect measurement relation.



$$\mathrm{tr}[H_I \rho_I] + \mathrm{tr}[H_{E_X} \rho_{E_X}] = \mathrm{Tr}[(H_I + H_{E_X}) U (\rho_I \otimes \rho_{E_X}) U^\dagger].$$

So, now we have an operational scenario which is consistent with the autonomous scenario.

Merits of information theoretical approach

Merit 1: A new sight helps us to find hidden problems in previous formulations.

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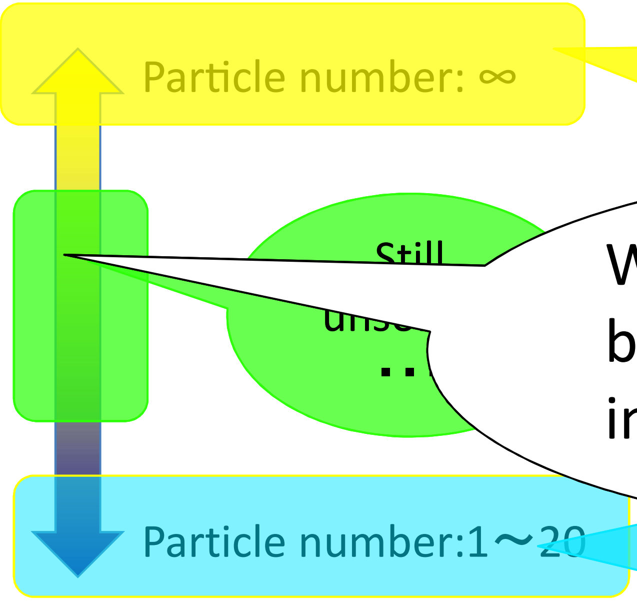
⇨ Result 2: Asymptotic analysis for the optimal performance of quantum heat engine

Researches for the optimal performance of heat engines

In Thermodynamics;

The optimal performances for macroscopic heat engines are given.

In Statistical mechanics(and quantum information);



Particle number: ∞

In i.i.d. case, i.e., $\rho_I = (\rho_\beta)^{\otimes n}$,
the optimal performances are given.

arXiv:cond-mat/0009244 (2000).
mat/0007360(2000).
om., 4185, (2014).

Still
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We clarify here in i.i.d case,
by asymptotic theory
in data compression.
arXiv:1405 .6457 (2014)

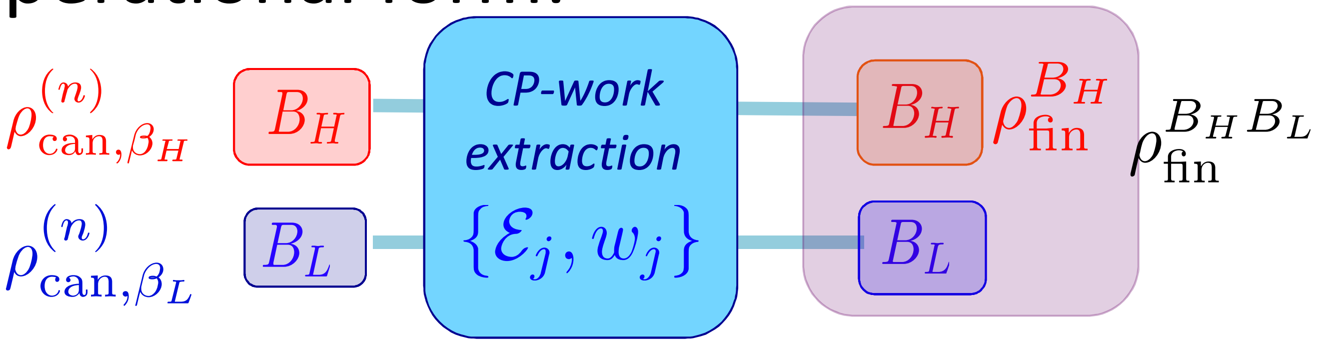
Particle number:1～20

optimal
performances are given.

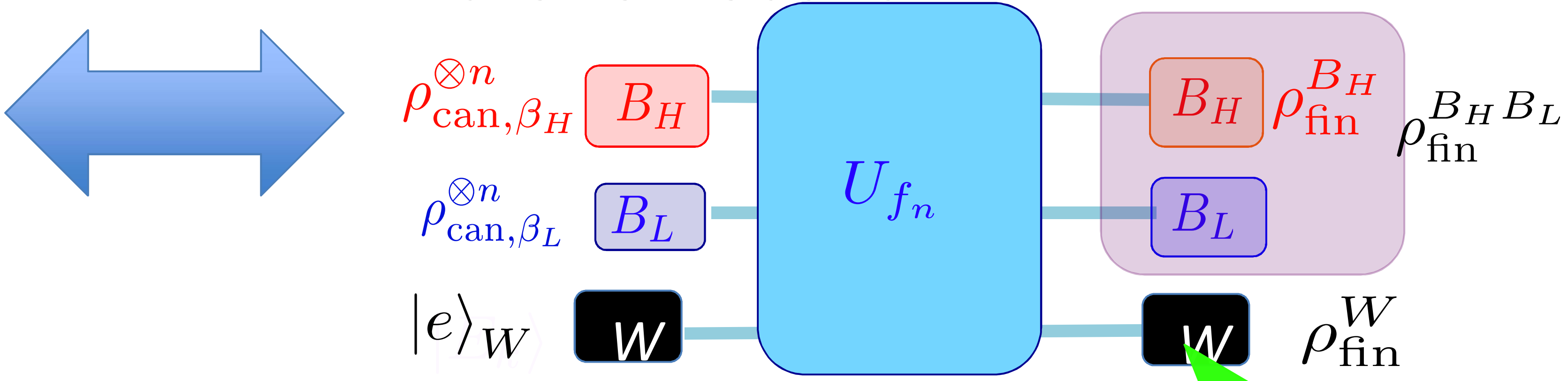
M. Horodecki and J. Oppenheim, Nat. Comm. 4, 2059 (2013).
F. G. S. L. Brandao, *et. al.,* PNAS, 112,3215(2015).

Heat engine with finite-size heat baths

Operational form:



Autonomous form:



We measure here.

$$W(\{\mathcal{E}_j\}) := \sum_j \mathcal{E}_j(\rho_{\beta_H \beta_L}) w_j = \mathrm{tr}[(\rho_{\beta_H \beta_L} - \rho^{HL}_{\mathrm{fin}})(H_H + H_L)]$$

$$Q_H(\{\mathcal{E}_j\}) := \mathrm{tr}[(\rho_{\beta_H \beta_L} - \rho^{HL}_{\mathrm{fin}}) H_H]$$

We can add arbitrary catalytic system.

Asymptotic evaluation of optimal efficiency

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Let us define the optimal efficiency as follows;

$$\eta_{\mathrm{opt}}[\beta_H,\beta_L,Q_n] := \sup_{\{\mathcal{E}_j,w_j\}:Q(\{\mathcal{E}_j,w_j\}=Q_n)} \eta(\{\mathcal{E}_j,w_j\})$$

When the particles of each heat bath do not interact each others, i.e.,

$$\rho_{\beta_H\beta_L} = (\rho_{\beta_H|H_h} \otimes \rho_{\beta_L|H_l})^{\otimes n}$$

The optimal efficiency satisfies

$$\eta_{\mathrm{opt}}[\beta_H,\beta_L,Q_n] = 1 - \frac{\beta_H}{\beta_L} - C_1\frac{Q_n}{n} - C_2\frac{Q_n^2}{n^2} + O\left(\frac{Q_n}{n^2}\right)$$

We can easily compute the coefficients;

Our results give computable approximation of optimal efficiency even when $n = 10^4, 10^8, 10^{12}\ldots$

Evaluation of the quality of energy gain

We can give a concrete dynamics satisfying the asymptotic expansion of the efficiency, and the followings;

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$$A_{B_H} = \frac{\Delta E_{B_H}}{\Delta S_{B_H}} = O(1)$$

$$A_{B_L} = \frac{\Delta E_{B_L}}{\Delta S_{B_L}} = O(1)$$

$$A_W = \frac{\Delta E_W}{\Delta S_W} \leq O\left(\frac{\log n}{Q_n}\right)$$

When $Q_n = an^b \quad 0 < b < 1\,,$

$$A_W \to 0 \ \ (n \to \infty)$$ holds.

Optimal process is an example of translation from “heat” to “work”.

Summary 1/2:

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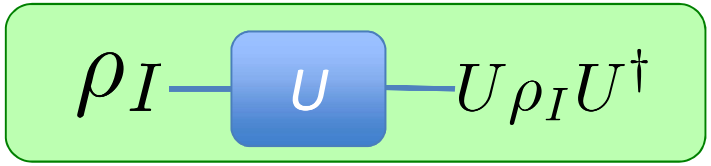
As operational formulation of quantum heat engines, “internal unitary model” has been widely used.



$$U := T_{\rightarrow}\exp[-i\int H_I(\lambda(t))dt]$$

We find that this model is not consistent with the autonomous formulation by considering the detectability of work.

As another operational formulation, we propose a measurement-based formulation, which is consistent with the autonomous formulation.

Summary 2/2:

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We give an asymptotic expansion of optimal efficiency.

$$\eta_{\mathrm{opt}}[\beta_H, \beta_L, Q_n] = 1 - \frac{\beta_H}{\beta_L} - C_1 \frac{Q_n}{n} - C_2 \frac{Q_n^2}{n^2} + O\left(\frac{Q_n}{n^2}\right)$$

We give the optimal process as a concrete dynamics.
It is a good example of translation from “heat” to “work.”

$$A_{B_H} = O(1) \qquad A_{B_H} = O(1) \qquad A_W \leq O\left(\frac{\log n}{Q_n}\right)$$