QUANTUM QUENCH AND CRITICAL BEHAVIOR : SCALING AT ANY RATE

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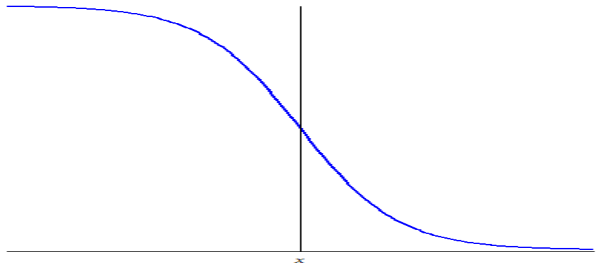
QUANTUM QUENCH

- Suppose we have a Hamiltonian system where a parameter in the Hamiltonian depends on time – attaining constant values at early and late times



- Following standard practice – I will call this a quantum quench, regardless of the rate of change.
- Question : Starting with some nice initial state (e.g. the vacuum), what is the nature of the final state ?

- This is of course a standard problem in physics.
- It is a difficult problem – unless one can apply perturbation theory or adiabatic approximation.
- In recent years, this problem has been studied intensely both theoretically and experimentally using cold atom systems
- Among many other issues, this problem is interesting for two key questions

  - Thermalization : Does the system reach a steady state resembling a thermal state ?
  - Critical Phenomena : If the quench involves a critical point, what kind of universal signature is carried by subsequent time evolution ?

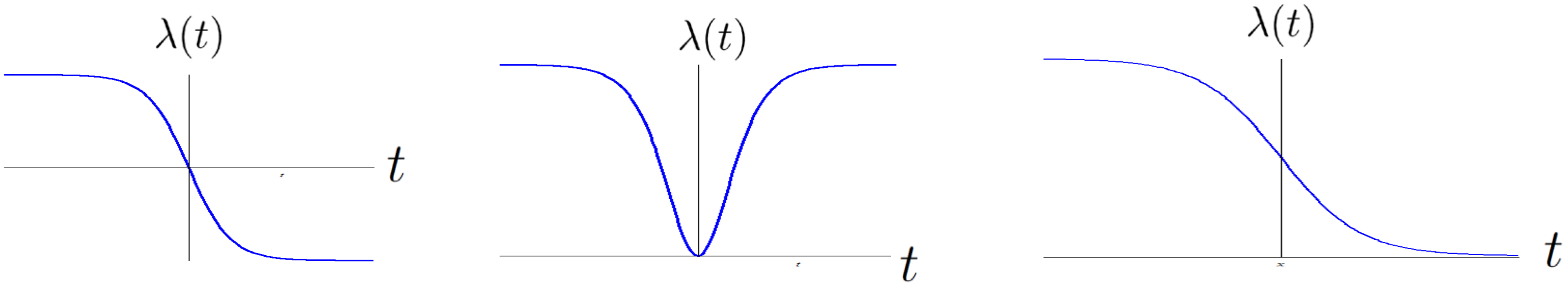
- In this talk I will concentrate on the second aspect.

Quantum Quench and Critical Points

- We will consider a theory with a time dependent coupling

$$S = S_{critical} - \int dt \int d^{d-1}x \; \lambda(t)\mathcal{O}(\vec{x},t) \qquad \lambda(t) = \lambda_0 F(t/\delta t)$$

- The function $\lambda(t)$ goes to constant values at early and late times, and crosses or touches zero somewhere in between



- The systems starts off in its ground state.

Slow Quench : Kibble-Zurek

- When the quench is slow compared to the initial mass gap,

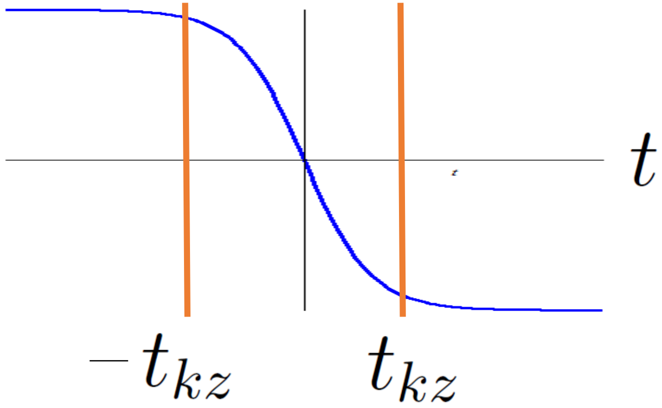
$$\delta t \gg \lambda_0^{-1/(d-\Delta)}$$

- And near $t=0$ the coupling behaves as

$$\lambda(t) = \lambda_0 (t/\delta t)^r$$

- Kibble-Zurek : Initial evolution is adiabatic. If t_{kz} denotes the time where adiabaticity breaks, the instantaneous correlation length at that time $\xi_{kz} = \xi(t_{kz})$ is the only scale in the problem.

To the crudest approximation the system becomes diabatic at this time



- A better description is in terms of scaling functions

(*de Grandi, Polkovnikov and Sandvik*; *Chandran, Erez, Gubser and Sondhi*)

$$< \mathcal{O}(t) > \sim \xi_{kz}^{-\Delta} f(t/t_{kz})$$

$$< \mathcal{O}(\vec{x},t)\mathcal{O}(\vec{x}',t') > \sim \xi_{kz}^{-2\Delta} g(\frac{|\vec{x}-\vec{x}'|}{\xi_{kz}}, \frac{|t-t'|}{t_{kz}})$$

where the time and length scales are determined in terms of the equilibrium critical exponents

$$t_{kz} \sim (\frac{\delta t}{\lambda_0^{1/r}})^{\frac{z\nu}{z\nu+1}} \qquad \xi_{kz} = \xi(t_{kz}) = (\frac{\delta t}{\lambda_0^{1/r}})^{\frac{\nu}{z\nu+1}}$$

- In systems with quasiparticles, scaling of e.g. defect densities known to work for low dimensions *(c.f. deGrandi, Gritsev, Polkovnikov*). Some experimental evidence as well. Nothing much known for strongly coupled systems.

- The instantaneous energy gap is given by

$$E_{gap}(t) \sim [\lambda_0 (\frac{t}{\delta t})^r]^{z\nu}$$

- The time at which adiabaticity fails is given by Landau criterion

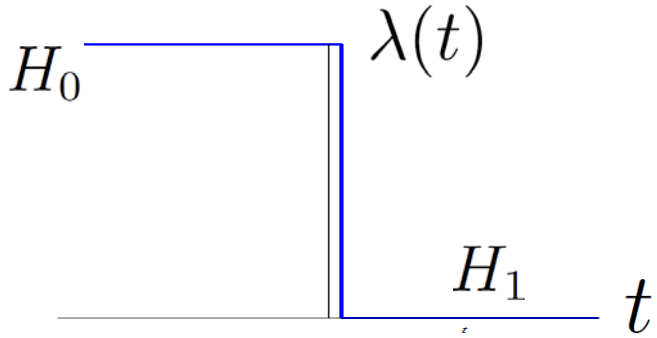
$$[\frac{1}{E_{gap}(t)^2} \frac{dE_{gap}(t)}{dt}]_{t=t_{KZ}} \sim 1$$

- This immediately leads to $t_{kz} \sim (\frac{\delta t}{\lambda_0^{1/r}})^{\frac{z\nu}{z\nu+1}}$

- Apart from some specific models this kind of argument is almost "state of the art".
- Can holography help ?

Instantaneous Quench

- Universal behavior is also known for instantaneous quench from a gapped phase to a critical point, where the state at the time of quench evolves according to the new constant Hamiltonian



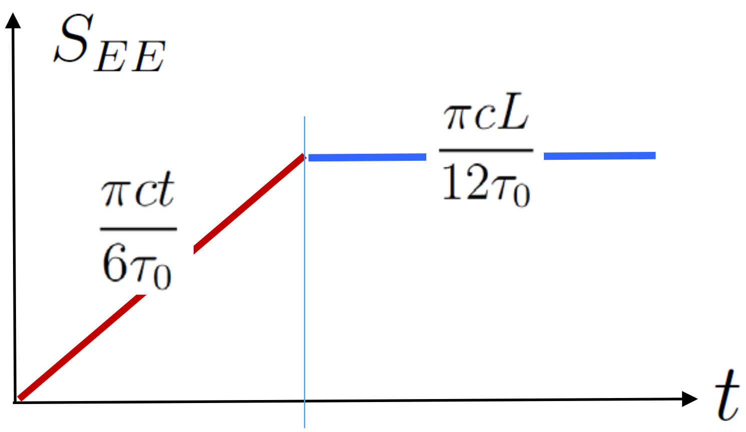
- *Calabrese and Cardy* argued that the quench state can be approximated by a state related boundary state in the final conformal field theory.

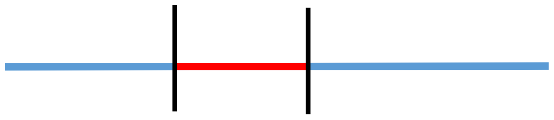
$$|\psi_0 > \sim \exp[-\tau_0 H_1] |B >$$

- In 1+1 dimensions methods of boundary conformal field theory can be used to obtain universal expressions for correlation functions, e.g.

$$< A(t) > \sim \exp[-\frac{\pi\Delta}{2\tau_0}t] \qquad\qquad \frac{\tau^{(1)}_{relax}}{\tau^{(1)}_{relax}} = \frac{\Delta^{(2)}}{\Delta^{(1)}}$$

- Ratios of relaxation times of different operators are universal.
- Another interesting result concerns the growth of entanglement entropy of a region of size L





- Generalization to spin or charge quench. (*Caputa, Mandal and Sinha*; *Mandal, Sinha and Sorokhaibam)*

New Scaling from Holography

- Over the last few years, considerable insight into such universal properties have been obtained using the AdS/CFT correspondence.
- Along the way, holographic methods have led to new universal behavior in a regime of fast, smooth quench

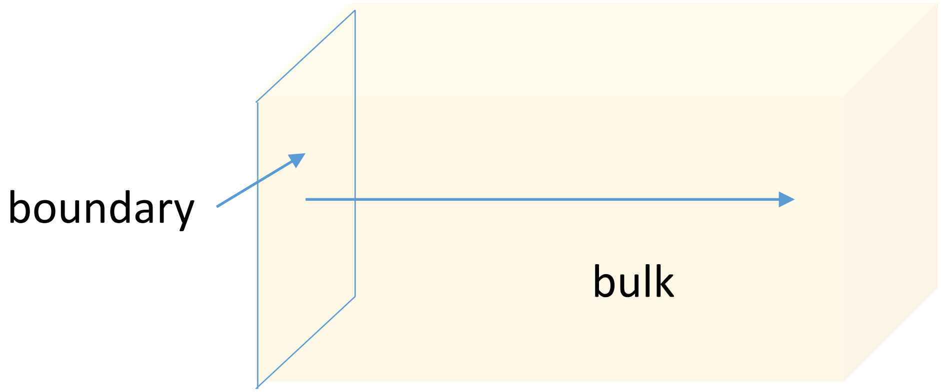
$$\Lambda_{UV}^{-1} \ll \delta t \ll m_{gap}^{-1} \qquad < \mathcal{O} > \sim (\delta t)^{d-2\Delta} \quad < \mathcal{E} > \sim (\delta t)^{d-2\Delta} \to \infty$$

- As we will argue, this new universality is in fact a general property of any field theory of the form we are considering, rather than some special property of field theories which are holograms of a theory of gravity.

The General Setup

- In the AdS/CFT correspondence – for cases where there is a regime where the bulk is weakly coupled – the problem of quantum quench becomes that of time dependent boundary conditions

$$
\begin{array}{ccc}
\text{scalar} & \mathcal{O}(\vec{x},t) \leftrightarrow \phi(\vec{x},t,z) & \text{scalar field} \\
\text{vector current} & J_\mu(\vec{x},t) \leftrightarrow A_\mu(\vec{x},t,z) & \text{gauge field} \\
\text{EM tensor} & T_{\mu\nu}(\vec{x},t,z) \leftrightarrow h_{\mu\nu}(\vec{x},t,z) & \text{metric perturbation}
\end{array}
$$



- Near the boundary one has an asymptotic expansion

$$\phi(\vec{x},t,z) \sim z^{d-\Delta}[\lambda(\vec{x},t) + O(z^2)] + z^{\Delta}[A(\vec{x},t) + O(z^2)]$$

- The integration function $\lambda(\vec{x},t)$ is the coupling for the deformation

$$S = S_{CFT} - \int d^{d-1}x dt\ \lambda(\vec{x},t)\mathcal{O}(\vec{x},t)$$

- While the integration function $A(\vec{x},t)$ gives the response

$$< \mathcal{O}(\vec{x},t) >= A(\vec{x},t)$$

- As has been extensively studied, thermalization is signaled by formation of horizons.
- Here we will study quantum quench involving a critical point – concentrating on universal scaling behavior

  (1) Identify a bulk theory which is dual to a critical point.

  (2) Impose time dependent boundary conditions on the bulk field which is dual to a relevant operator which crosses or approaches the critical point.

  (3) Calculate the response.

Holographic Kibble-Zurek

This has been studied both at zero and non-zero temperature

- *P. Basu and S.R.D., JHEP 1201 (2012) 103; P. Basu, D. Das, S.R.D. & T. Nishioka, JHEP 1301 (2013) 107; P. Basu, D. Das, S.R.D. & K. Sengupta, JHEP 1311 (2013) 186; S.R.D. & T. Morita, JHEP 1501 (2015) 084.*
- *J. Sonner, A. del Campo and W. Zurek, Nat. Comm. 6 (2015) 7405; P. Chesler, A. Garcia-Garcia and H. Liu, Phys. Rev. X (2015) 021015*

These works lead to a set of general lessons for usual critical points

- The equilibrium critical behavior (typically leading to condensation of a scalar operator $\mathcal{O}$) is caused by the presence of a zero mode of the dual bulk scalar.

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- In the critical region, a power series expansion in the rate δt^{-1} breaks down, but a new expansion in $\delta t^{-\kappa}$ appear, where κ is typically fractional, and determined by the critical exponents.

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- In the critical region, a power series expansion in the rate δt^{-1} breaks down, but a new expansion in $\delta t^{-\kappa}$ appear, where κ is typically fractional, and determined by the critical exponents.
- To leading order in this expansion the dynamics is dominated by the zero mode – this immediately reduces the PDE to a ODE satisfied by the zero mode. This has scaling solutions.

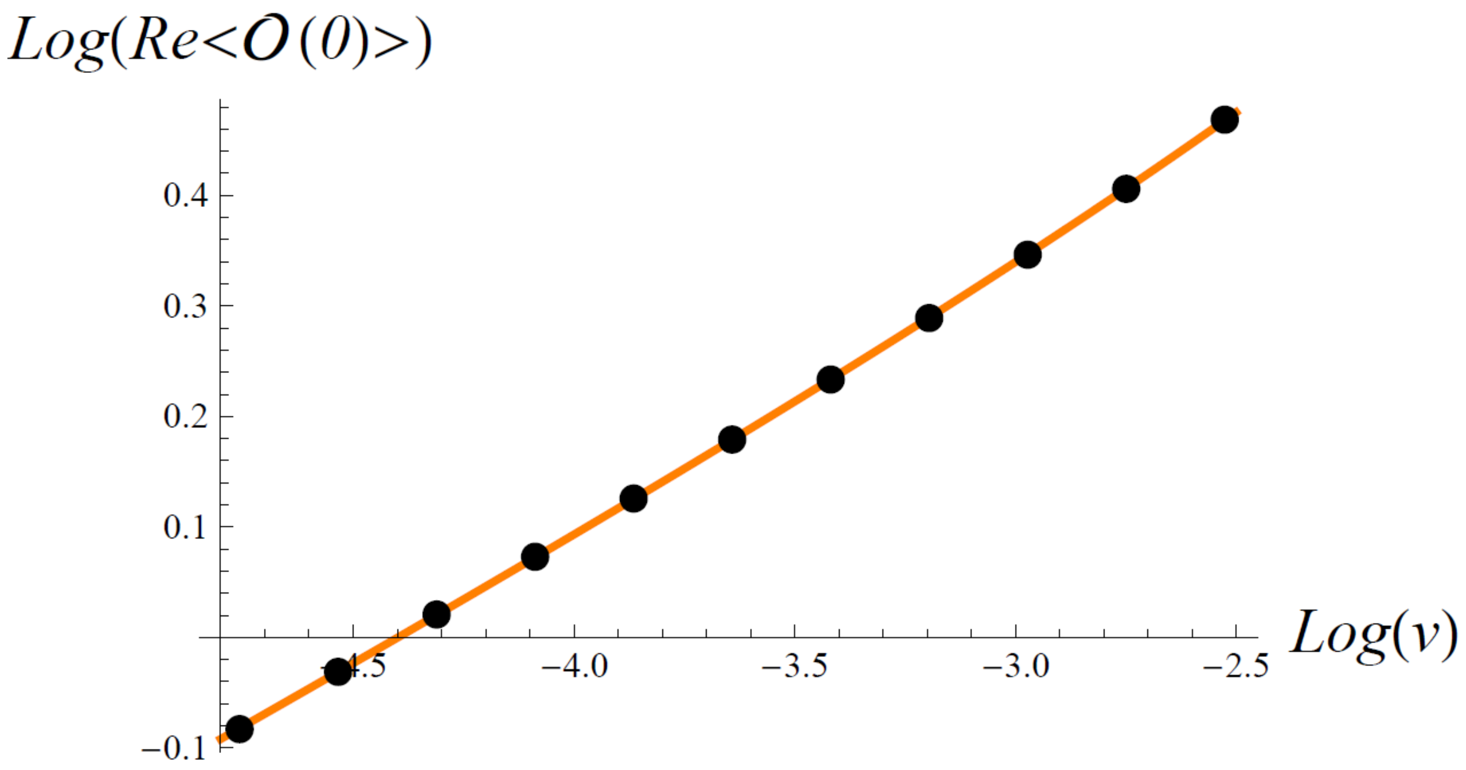
$$< \mathcal{O} >\sim (\delta t)^{-1/5} F(t/t_{kz})$$

$$< \rho(t) >\sim \delta t^{-2/5} G(t/t_{kz})$$

$$< T_{\mu\nu}(t) >\sim \delta t^{-2/5} H_{\mu\nu}(t/t_{kz})$$

T=0 Holographic superconductor :

P. Basu, D. Das, S.R.D. & T. Nishioka



Scaling of the order parameter at t=0. Here v = δt^{-1}
Model : T=0 Holographic Superconductor

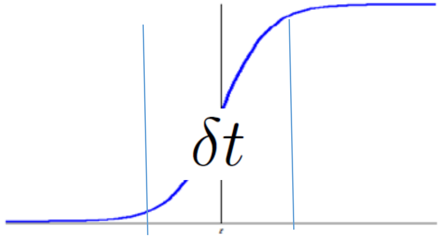
Holographic Fast Quench

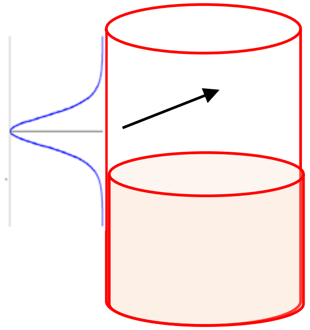
- *Buchel, Lehner, Myers, van Niekerk* : performed holographic analysis of a quench

$$S = S_{CFT} - \int dt \int d^{d-1}x \ \lambda(t)\mathcal{O}(\vec{x}, t)$$

- where

$$\delta t \ll (\lambda_0)^{-\frac{1}{d-\Delta}}$$





- Start with AdS and then turn on a time dependent boundary condition for a scalar with an appropriate mass. They found a set of universal results

$$< \mathcal{O} >\sim (\delta t)^{d-2\Delta} \qquad < \mathcal{E} >\sim (\delta t)^{d-2\Delta}$$

- The energy density $\mathcal{E}$ was consistent with the Ward Identity

$$\frac{d}{dt} < \mathcal{E} >= \frac{d\lambda(t)}{dt} < \mathcal{O} >$$

- The same behavior appears to hold for changes of expectation values for a reverse quench – i.e. a quench to a critical point.
- The result also holds for thermal quenches which start with a thermal state, i.e. a black brane in the bulk.
- When $2\Delta > d$ these results are puzzling since the expectation values appear to diverge as $\delta t \to 0$, while there seems to be a well defined limit of an instantaneous quench.

$$< \mathcal{O} > \sim (\delta t)^{d-2\Delta} \to \infty \qquad < \mathcal{E} > \sim (\delta t)^{d-2\Delta} \to \infty$$

- Could this be a property of those special strongly coupled field theories which have gravity duals ?
- In the following we will investigate such quenches directly in quantum field theories – starting with arbitrary quench rates and performing the limit of fast quenches.

- S.R.D., D. Galante and R.C. Myers, Phys.Rev.Lett 112 (2014) 171601

  JHEP 1502 (2015) 167

  JHEP 1508 (2015) 073

  JHEP 1606 (2016) XXX

- D. Das, S.R.D., D. Galante, R.C. Myers and K. Sengupta – arXiv:1606.XXXX

START WITH THE OTHER EXTREME
: FREE FIELD THEORIES AND LATTICE MODELS WHICH
REDUCE TO FREE THEORIES

THEN WORK OUR WAY UP TO GENERAL RESULTS IN ARBITRARY INTERACTING THEORIES

Exactly solvable Quench in Free Fields

- We will consider free field theories with time dependent masses

$$S = -\int dt \int d^{d-1}x \; \frac{1}{2}[(\partial\phi)^2 + m^2(t)\phi^2]$$

$$S = \int dt \int d^{d-1}x \; [i\bar{\psi}\gamma^\mu\partial_\mu + M(t)]\psi$$

- Find mass profiles for which the quantum dynamics is exactly solvable for all quench rates. Here are some examples

bosons $\qquad m^2(t) = A + B\tanh(t/\delta t) \qquad m^2(t) = m_0^2 + \frac{m^2}{\cosh^2 t/\delta t}$

fermions $\qquad M(t) = C + D\tanh(t/\delta t)$

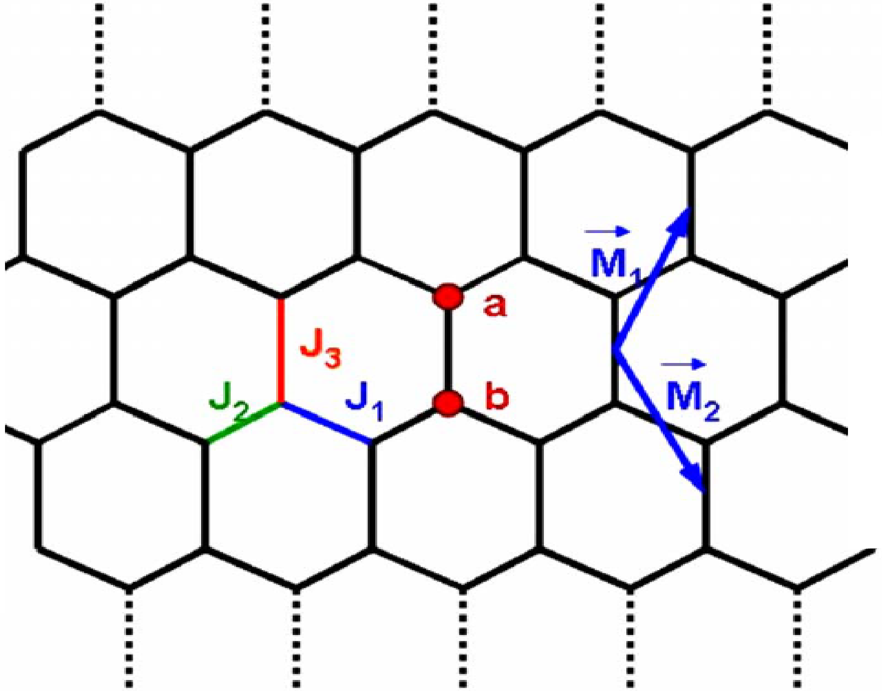
Solvable quenches in Lattice Models

- Ising chain in one dimension with Hamiltonian with time dependent transverse field

$$H = -\sum_n \left[g(t)\sigma^3(n) + \sigma^1(n)\sigma^1(n+1) \right]$$

- In equilibrium, i.e. a constant coupling this has a critical point at g=1.

- The Kitaev model on a honeycomb lattice.



$$H_{Kitaev} = \sum_{j+l=\text{even}} \left[J_1 \sigma^{(1)}_{j,l} \sigma^{(1)}_{j+1,l} + J_2 \sigma^{(2)}_{j,l} \sigma^{(2)}_{j-1,l} + J_3 \sigma^{(3)}_{j,l} \sigma^{(3)}_{j,l+1} \right]$$

(j, l) denote the column and row indices of a site

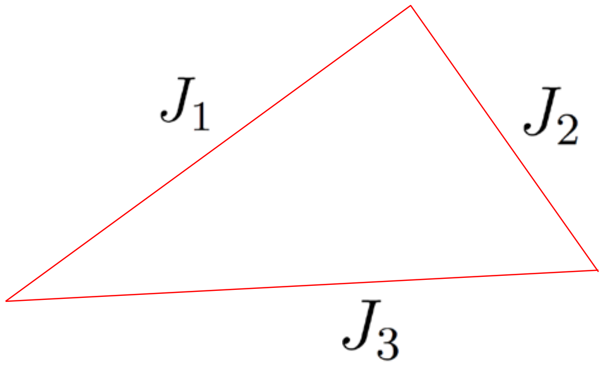
- This model is critical over a whole surface in the coupling constant plane. The gapless points correspond to momenta

$$k_1 \equiv \vec{k} \cdot \vec{M}_1 \qquad k_2 \equiv \vec{k} \cdot \vec{M}_2 \qquad \vec{M}_1 = \frac{\sqrt{3}}{2}\hat{i} + \frac{3}{2}\hat{j} \qquad \vec{M}_2 = \frac{\sqrt{3}}{2}\hat{i} - \frac{3}{2}\hat{j}$$

- Which satisfy the following relationships with the couplings

$$J_1 \sin k_1 = J_2 \sin k_2$$

$$\cos k_1 = \frac{J_3^2 + J_1^2 - J_2^2}{2J_3 J_1} \qquad \cos k_2 = \frac{J_3^2 + J_2^2 - J_1^2}{2J_3 J_2}$$



Perform quench by $J_3(t)$ keeping J_1 and J_2 constant

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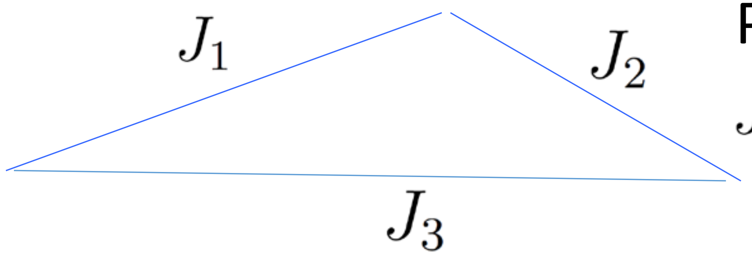


Perform quench by $J_3(t)$ keeping J_1 and J_2 constant

Scalars with time dependent mass

- For a free scalar with time dependent mass

$$\boxed{S = -\int dt \int d^{d-1}x \, \frac{1}{2}[(\partial\phi)^2 + m^2(t)\phi^2]} \qquad \boxed{m^2(t) = A + B\tanh(t/\delta t)}$$

- The mode expansion is

$$\phi = \int \frac{d^{d-1}k}{(2\pi)^{(d-1)/2}} \left(a_{\vec{k}}\, u_{\vec{k}} + a^\dagger_{\vec{k}}\, u^*_{\vec{k}} \right)$$

$$u_{\vec{k}} = \frac{1}{\sqrt{2\omega_{\rm in}}} \exp\left(i\vec{k}\cdot\vec{x} - i\omega_+ t - i\omega_- \delta t \log(2\cosh t/\delta t) \right) \times$$
$${}_2F_1\left(1 + i\omega_-\delta t, i\omega_-\delta t; 1 - i\omega_{\rm in}\delta t; \frac{1+\tanh(t/\delta t)}{2}\right),$$

$$\omega_{\rm in} = \sqrt{\vec{k}^2 + m^2(A-B)}\,,$$
$$\omega_{\rm out} = \sqrt{\vec{k}^2 + m^2(A+B)}\,,$$
$$\omega_\pm = (\omega_{\rm out} \pm \omega_{\rm in})/2\,.$$

- The "in" vacuum is given by $a_{\vec{k}}|0>= 0$

Comes from explicit solution

- The response is

$$\langle\phi^2\rangle \equiv \langle in,0|\phi^2|in,0\rangle = \frac{1}{2(2\pi)^{d-1}}\int\frac{d^{d-1}k}{\omega_{\rm in}}\,|{}_2F_1|^2$$

- This, of course, is UV divergent : we need to subtract counter-terms.

$$\langle\phi^2\rangle_{\rm ren} \equiv \frac{\Omega_{d-2}}{2(2\pi)^{d-1}}\int dk\left(\frac{k^{d-2}}{\omega_{\rm in}}\,|{}_2F_1|^2 - f_{\rm ct}(k,m(t))\right)$$

- These counter-terms will involve $m(t)$ as well as its time derivatives – similar to renormalization of field theories in a curved space-time. It turns out that these counter-terms can be obtained – with exact coefficients – by considering the divergent pieces of the adiabatic expansion of $\langle\phi^2\rangle$.

The adiabatic expansion is good for slow changes. Our quench rate is always much smaller than the UV scale –

$$\Lambda_{UV}^{-1} \ll \delta t \ll m_{gap}^{-1}$$

explains why high momentum behavior of the adiabatic expansion agrees with the exact answer

The Fast Quench Limit

- In the fast quench limit $m\delta t \ll 1$ it is possible to derive analytic expressions for the leading order results.

$$\text{odd} \quad d \geq 5 \quad \langle \phi^2 \rangle_{\text{ren}} = (-1)^{\frac{d-1}{2}} \frac{\pi}{2^{d-2}} \boxed{\partial_t^{d-4} m^2(t)} + O\left(\delta t^{6-d}\right)$$

$$\text{even} \quad d \geq 4 \quad \langle \phi^2 \rangle_{\text{ren}}^{(d)} = (-1)^{d/2} \boxed{\log(\mu\delta t)} \boxed{\frac{\partial_t^{d-4} m^2(t)}{2^{d-3}}} + \cdots$$

$$d = 3 \qquad \langle \phi^2 \rangle_{\text{ren}} = -\frac{m}{4\pi} - \boxed{\frac{m^2 \delta t}{16} \log\left(\frac{1 - \tanh t/\delta t}{2}\right)}$$

$$d = 4 \qquad < \phi^2 >_{\text{ren}} = \boxed{\frac{m^2}{4}[1 + \tanh(t/\delta t)] \log(\mu\delta t) + \cdots}$$

- Since the conformal dimension of the quenched operator is $\Delta = d - 2$ these are consistent with the scaling law

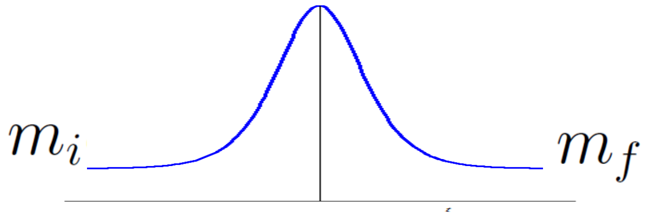
$$< \mathcal{O} >\sim (\delta t)^{d-2\Delta}$$

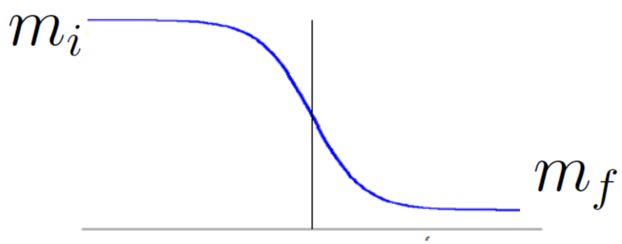
upto logarithmic factors for even dimensions. These logarithmic factors were also present in the holographic calculations.

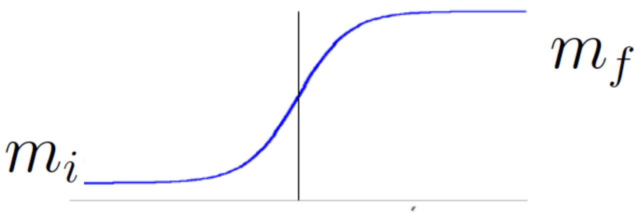
- Similarly the energy density scales as

$$< \mathcal{E} >\sim (\delta t)^{d-2\Delta}$$

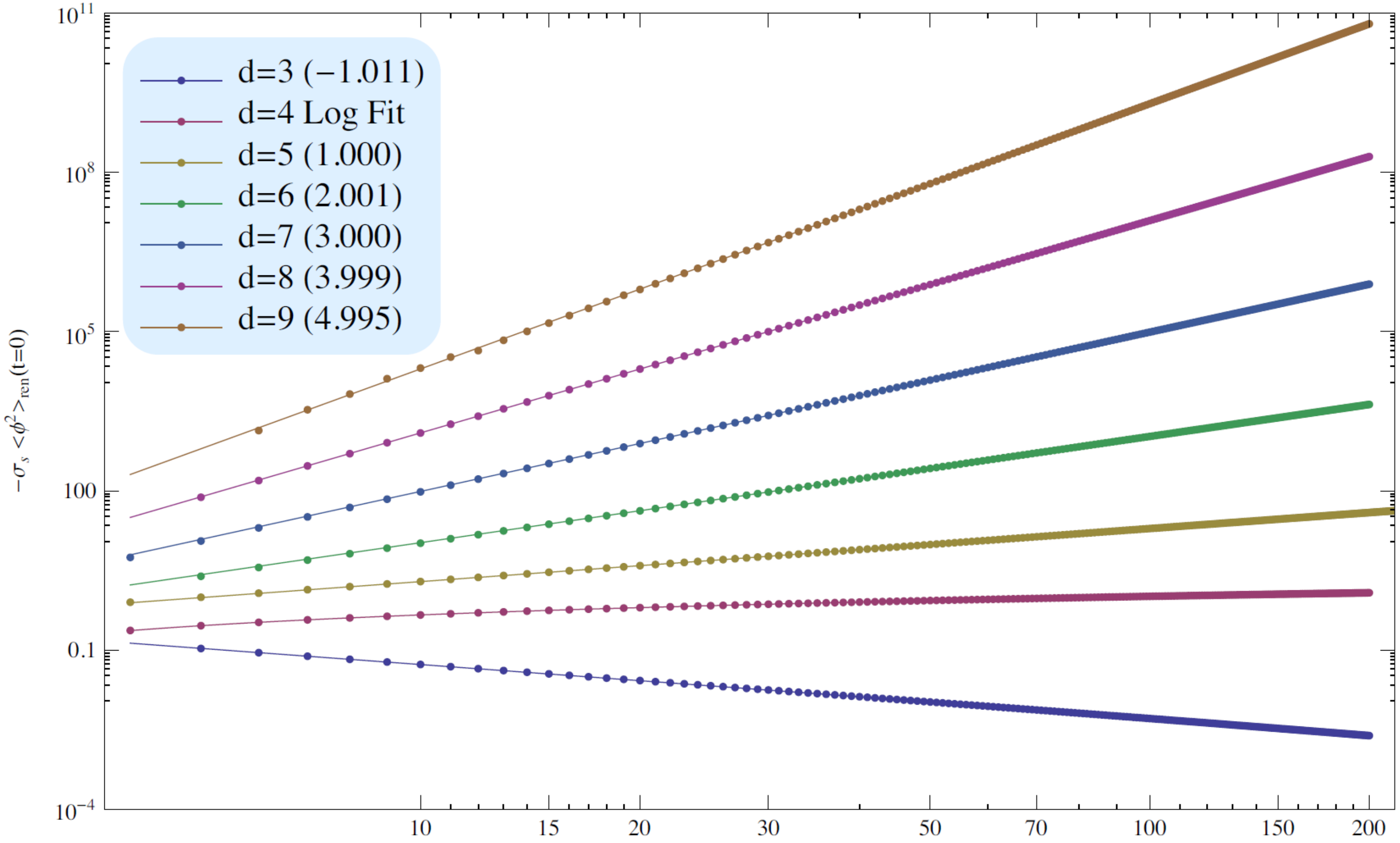
- The scaling behavior holds for several other kinds of quenches so long as the quench rate is fast compared to all physical mass scales in the problem, and holds for fermionic quenches







$$m_i \delta t, m_f \delta t \ll 1$$



$$< \mathcal{O} >\sim (\delta t)^{d-2\Delta}$$

Expectation value $\langle \phi^2 \rangle_{\rm ren}(t=0)$ as a function of the quench times δt
slope of the linear fit in each case is shown in brackets

- The results are similar for free fermions with a time dependent mass
- They also hold for other quench protocols
- This scaling law seems to be valid for

  (1) Strongly coupled theories with gravity duals

  (2) Free field theories

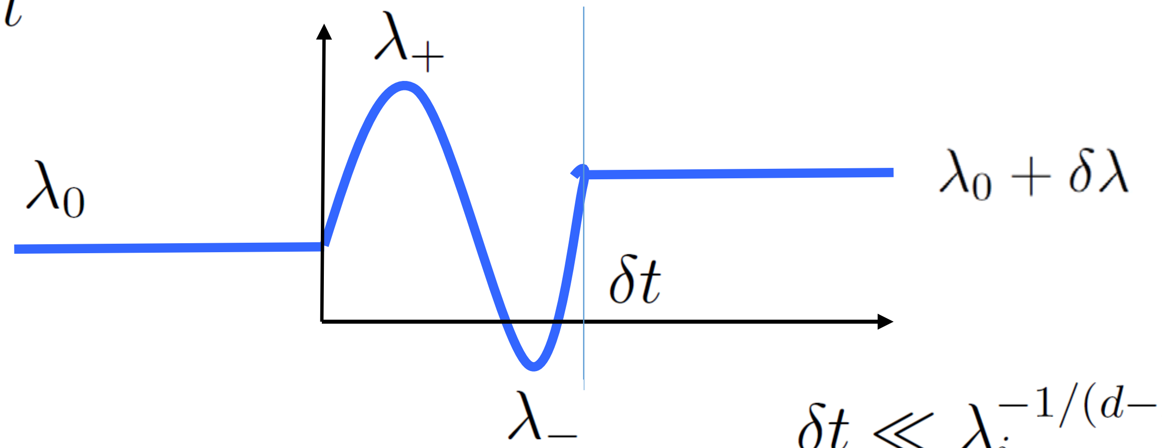
Could this be a general result ?

A General Result

- Now consider a general interacting CFT deformed by some relevant operator with a time dependent coupling

$$S = S_{CFT} - \int dt \int d^{d-1}x \; \lambda(t)\mathcal{O}(\vec{x},t) \qquad \lambda(t) = \lambda_0 F(t/\delta t)$$

- The coupling is at some constant value λ_0 for $t < 0$. At $t = 0$ it smoothly turns on and changes with time in a time interval δt, and becomes a constant quickly after $t = \delta t$



$$\delta t \ll \lambda_i^{-1/(d-\Delta)}, \delta\lambda \qquad i = 0, \pm$$

- Start computing $< \mathcal{O} >$ in perturbation theory. The first few terms are

$$\boxed{\begin{aligned}\langle\mathcal{O}(\vec{x},t)\rangle-\langle\mathcal{O}(\vec{x},t)\rangle_{\lambda_0} &= -\delta\lambda\int_0^t dt' F\left(t'/\delta t\right)\int d^{d-1}\vec{x}' G_{R,\lambda_0}\left(\vec{x}-\vec{x}',t-t'\right)\\ &+\frac{\delta\lambda^2}{2}\int_0^t dt' F\left(t'/\delta t\right)\int d^{d-1}\vec{x}'\int_0^t dt'' F\left(t''/\delta t\right)\int d^{d-1}\vec{x}''\, K_{\lambda_0}\left(t',\vec{x}';t'',\vec{x}'';t,\vec{x}\right)\end{aligned}}$$

- Where $G_{R,\lambda_0}(\vec{x},t)$ is the retarded Green's function of the initial theory

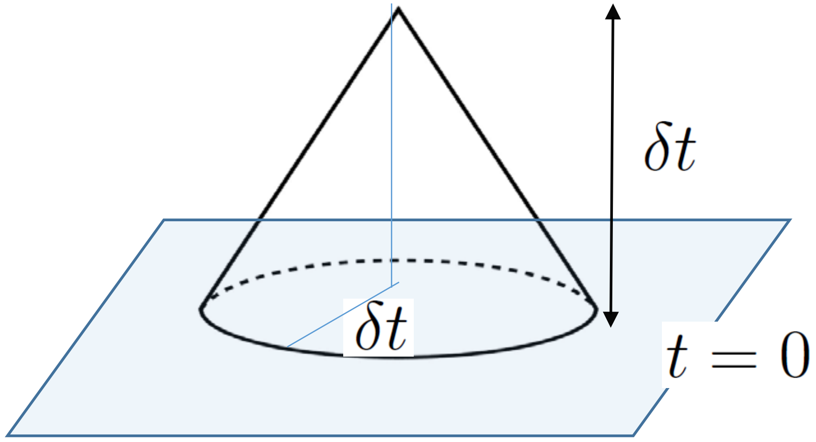
$$G_{R,\lambda_0}(\vec{x},t) = i\theta(t) < 0|[\mathcal{O}(\vec{x},t),\mathcal{O}(0,0)]|0>_{\lambda_0}$$

- And $K_{\lambda_0}\left(t',\vec{x}';t'',\vec{x}'';t,\vec{x}\right)$ is a three point function.
- All quantities above are renormalized quantities.

- Consider the first term

$$\int_0^t dt' F\left(t'/\delta t\right) \int d^{d-1}\vec{x}' G_{R,\lambda_0}\left(\vec{x}-\vec{x}', t-t'\right)$$

- While the integration over $\vec{x}'$ has been written as over entire space, causality implies that only the region $|\vec{x}-\vec{x}'| \leq t$ has a non-trivial contribution.
- Now suppose we want to calculate the quantity at $t = \delta t$, right at the end of the quench. Then both the space and time intervals which appear in the integral are at most of size δt



- Recall that the scale δt is smaller than all other physical scales in the problem, in particular the scale associated with the deformation of the CFT by λ_0

- Therefore the Green's function $G_{R,\lambda_0}\left(\vec{x}-\vec{x}',t-t'\right)$ is basically the Green's function of the UV conformal field theory.

$$\boxed{G_{R,\lambda_0}(\vec{x}'-\vec{x},t'-t)\sim G_{R,CFT}(\vec{x}'-\vec{x},t'-t)} \qquad |\vec{x}'-\vec{x}|,|t'-t|\ll(\lambda_0)^{-1/(d-\Delta)}$$

- This means that in the leading contribution, the only scale which appears in the integral is δt

- This leads to an expression for the response which is of the form

$$\langle \mathcal{O}_\Delta(t) \rangle_{\rm ren} - \langle \mathcal{O}_\Delta(t) \rangle_{{\rm ren},\lambda_0} = (\delta t)^{-\Delta} \left[b_1(t/\delta t)\, g + b_2(t/\delta t)\, g^2 + \cdots \right]$$

- Where we have introduced a dimensionless coupling

$$g \equiv \delta\lambda \delta t^{d-\Delta}$$

- In the fast quench limit this dimensionless coupling is small, so that to leading order we get the universal scaling

$$< \mathcal{O} > \sim (\delta t)^{d-2\Delta}$$

- There is a similar argument for the energy density produced. Since the coupling remains constant after δt this quantity is in fact the net energy produced.
- Note that the result is completely general and depends only on the properties of the conformal field theory in the UV.
- A more detailed understanding of this general result appears in *Berentsein and Miller*

The limit of instantaneous quench

- Why is there a divergence when $\delta t \to 0$ for $d \geq 4$?
- We have been dealing with renormalized quantities. This is because we have been interested in $\Lambda_{UV}^{-1} \ll \delta t \ll m_{phys}^{-1}$.
- Strictly speaking, an instantaneous quench means that δt is smaller than all scales in the problem, including Λ_{UV}^{-1}. So the renormalized local quantities are not expected to yield answers in this limit.
- However there should be some IR quantities which should have a limit.
- One way to examine the connection in detail is to look at UV finite quantities.
- We will now go back to the free field examples.

- One such quantity is the excess energy

$$\delta\mathcal{E} = \mathcal{E}(t \to \infty) - \mathcal{E}_{ground,\lambda_1}$$

- This is UV finite for any finite δt , since the additional counter-terms needed to render $\mathcal{E}$ finite involve time derivatives of $m^2(t)$ - these all go to zero at late times.

$d=2$	$\delta\mathcal{E}^{\delta t\to 0} = \frac{m^2}{16\pi} + c_1 m^4(\delta t)^2 + \cdots$	$\delta\mathcal{E}^{instant} = \frac{m^2}{16\pi}$

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$d=3$	$\delta\mathcal{E}^{\delta t \to 0} = \frac{m^3}{24\pi} + c_2 m^4 \delta t + \cdots$	$\delta\mathcal{E}^{instant} = \frac{m^3}{24\pi}$

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$d = 4$	$\delta\mathcal{E}^{\delta t\to 0} = c_3 m^4 \log(m\delta t) + O(\delta t)$	$\delta\mathcal{E}^{instant} = \infty$
$d \geq 5$	$\delta\mathcal{E}^{\delta t\to 0} = c_4 m^4 \delta t^{4-d} + \cdots$	$\delta\mathcal{E}^{instant} = \infty$

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- Thus for $d \geq 4$, i.e. $2\Delta > d$ the $\delta t \to 0$ limit is divergent : this is also the case when the instantaneous excess energy has a UV divergence.

- These results are for free fields.
- For general interacting theories conjecture : fast quenches with $2\Delta < d$ behave rather differently from $2\Delta > d$, regardless of the dimensionality.

(1) For $2\Delta < d$ one expects that there is a smooth limit of the excess energy density to instantaneous quench – and the sub-leading quantity scales in a universal fashion.

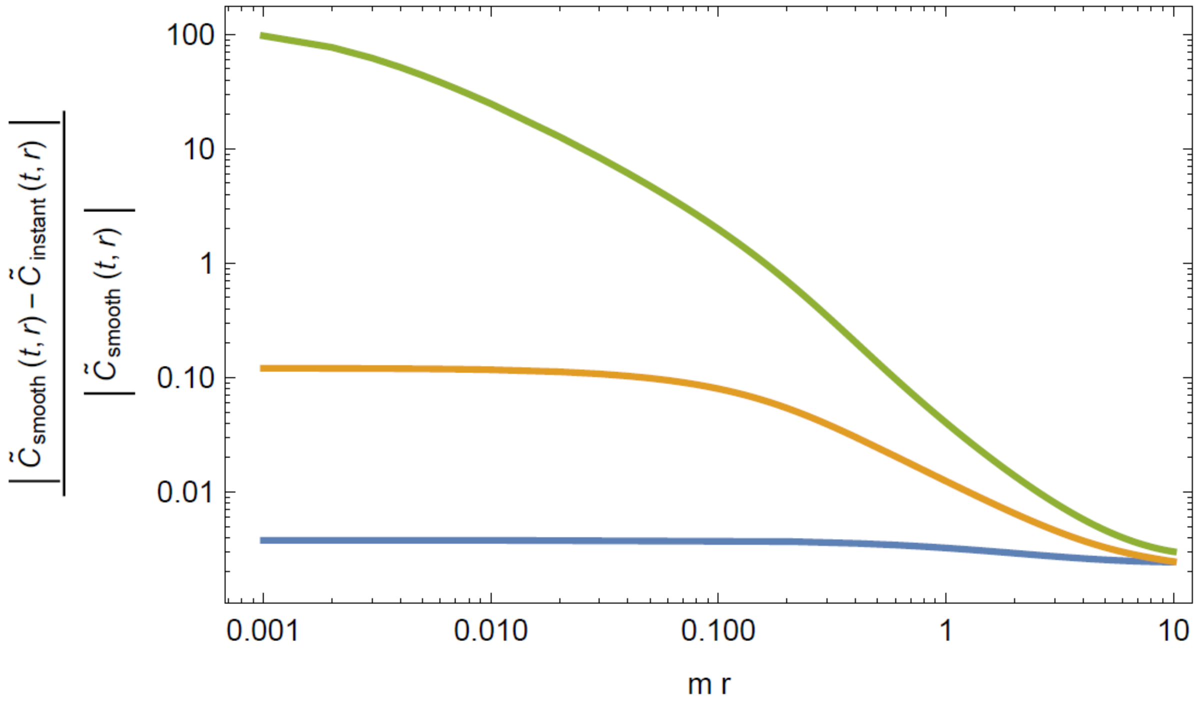
(2) For $2\Delta > d$ the excess energy diverges.

- This has implications for the applicability of Calabrese-Cardy states as approximations of quench states.

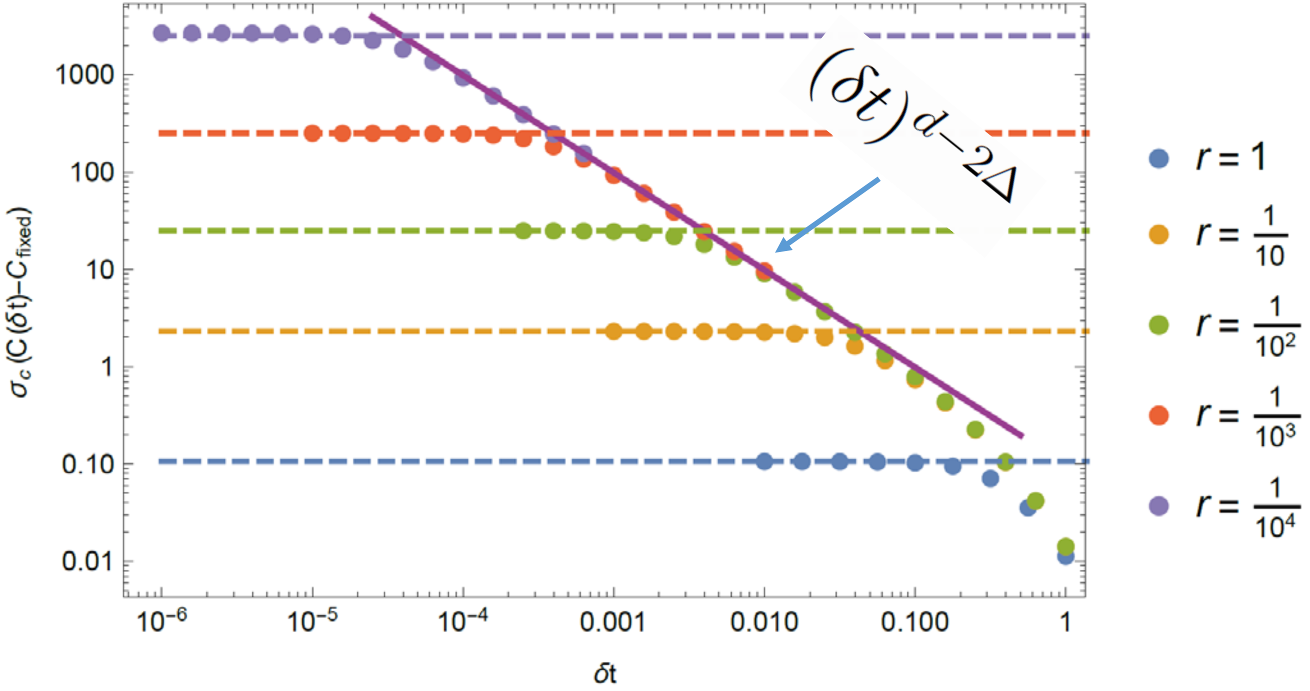
- Another UV finite object is the equal time correlation function of the field at finite spatial separations

$$C(t,\vec{r}) \equiv \langle \phi(t,\vec{r})\phi(t,\vec{0})\rangle$$

- For large enough separations and late times one expects that this agrees with the result of an instantaneous quench – since the momenta which contribute are smaller than the UV scale. That is indeed correct



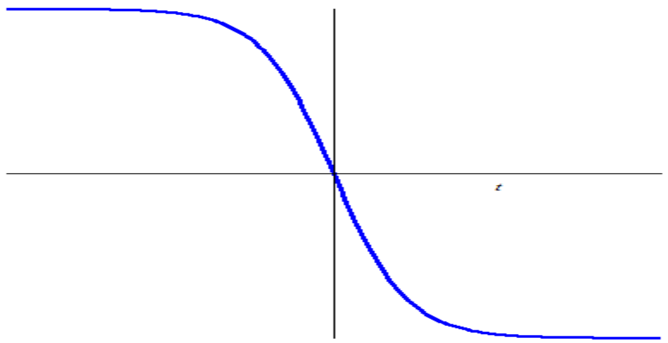
- The most interesting behavior, however, happens at **early times**.
- For a given r the correlator becomes independent of δt for $r \gg \delta t$
- However for $r < \delta t < 1/m$ the correlator scales with δt exactly as the renormalized expectation value, i.e. as $(\delta t)^{d-2\Delta}$



Correlation function at t=0, with the constant mass correlator subtracted. The purple line is the scaling behavior.

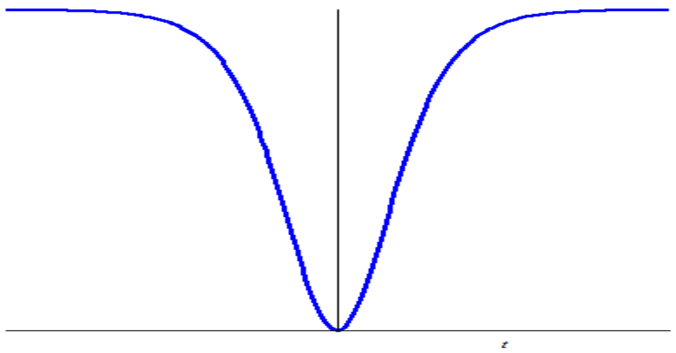
From slow to fast quench

- How does the scaling relations for slow quench go over to those for fast quench ? Is there a phase transition ?
- For free field theories this can be investigated in detail:



$$M(t) = C + D\tanh(t/\delta t)$$

Fermions



$$m^2(t) = m_0^2 \tanh^2(t/\delta t)$$

Bosons

- Much of the Kibble-Zurek can be now studied analytically in a slow quench rate regime

(1) For slow rates scaling consistent with Kibble-Zurek

(2) Time dependence governed by scaling functions

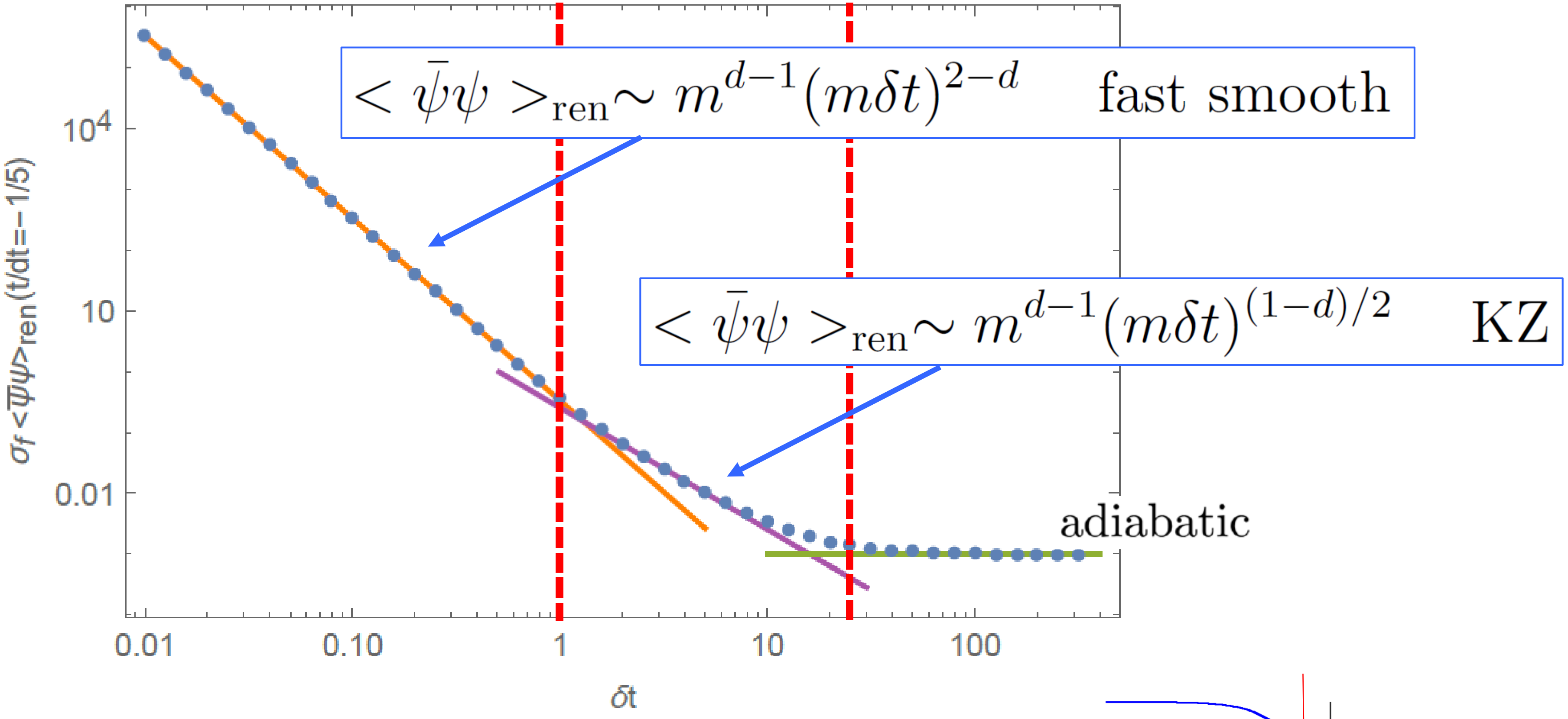
$$< \mathcal{O}(t) >\sim \xi_{kz}^{-\Delta} f(t/t_{kz})$$

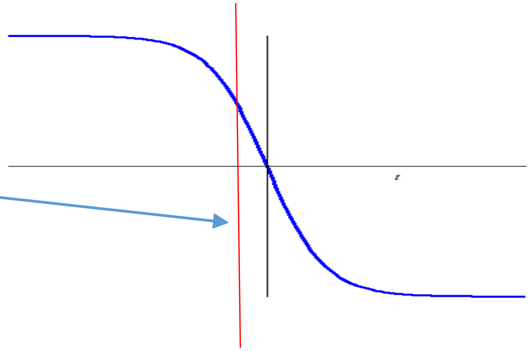
- For the protocols we have studied

$$t_{\mathrm{KZ}} = \sqrt{\delta t/m}\,.$$

$$\langle\bar{\psi}\psi\rangle_{ren} \simeq \frac{1}{t_{\mathrm{KZ}}^{d-1}} = \left(\frac{m}{\delta t}\right)^{\frac{d-1}{2}}$$

$$\langle\phi^2\rangle_{ren} \simeq \frac{1}{t_{\mathrm{KZ}}^{d-2}} = \left(\frac{m}{\delta t}\right)^{\frac{d-2}{2}}$$





Results for $\langle\bar{\psi}\psi\rangle_{ren}$ in $d = 5$ at time $t = -\frac{1}{5}\delta t.$

Connecting to Lattice Models

- So far, our results have been for continuum field theory – where the quench rate is always slow compared to the UV scale.
- It is interesting to connect this to the regime of quench rates of the order of the cutoff where we expect a saturation – and identify the regime where the fast quench universal scaling starts working.
- We have found exactly solvable quench protocols for some lattice models.

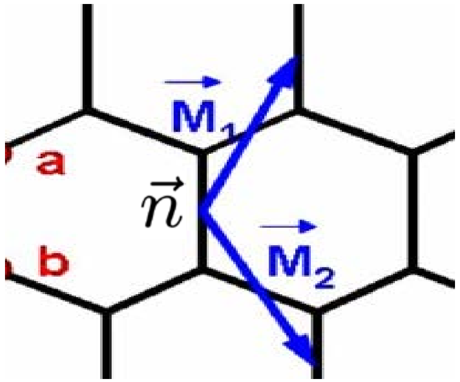
(Diptarka Das, S.R.D., D. Galante, R.C. Myers and K. Sengupta – to appear)

Ising and Kitaev

- Both the transverse field Ising model and the Kitaev honeycomb model can be expressed in terms of fermions.

$$H_{Ising} = -\sum_n \left[g(t) c_n^\dagger c_n + (c_n^\dagger - c_n)(c_{n+1}^\dagger - c_n) \right]$$

$$H_{Kitaev} = i \sum_{\vec{n}} [J_1 \; b_{\vec{n}} a_{\vec{n}-\vec{M}_1} + J_2 \; b_{\vec{n}} a_{\vec{n}+\vec{M}_2} + J_3 D_{\vec{n}} b_{\vec{n}} a_{\vec{n}}]$$



- $c_n, a_{\vec{n}}, b_{\vec{n}}$ are standard fermion operators. $D_{\vec{n}}$ is an operator which commutes with the Hamiltonian, and takes values ± 1 independently at each site. The ground state has $D_{\vec{n}} = 1$ for all the sites.
- If we quench the system starting in the ground state, the value of $D_{\vec{n}}$ does not change – so we can set $D_{\vec{n}} = 1$

- Both these models can be then written in the form

$$\boxed{H = \int d^{d-1}k(2\pi)^{(d-1)}\psi^\dagger(\vec{k})\left[-m(k,t)\sigma_3 + G(k)\sigma_1\right]\psi(\vec{k})} \qquad \psi(k) = \begin{pmatrix}\psi_1(k)\\ \psi_2(k)\end{pmatrix}$$

	d	$m(k,t)$	$G(k)$
Ising	2	$g(t) - \cos k$	$\sin k$
Kitaev	3	$-J_3(t) - J_1\cos k_1 - J_2\cos k_2$	$J_1\sin k_1 - J_2\sin k_2$

- The solutions to the Heisenberg equations of motion are of the form

$$U(\vec{k},t) = \begin{pmatrix}-i\partial_t + m(\vec{k},t)\\ -G(\vec{k})\end{pmatrix}\phi(\vec{k},t) \qquad V(\vec{k},t) = \begin{pmatrix}G(\vec{k})\\ i\partial_t + m(\vec{k},t)\end{pmatrix}\phi^\star(\vec{k},t)$$

- Where the scalar function $\phi(\vec{k},t)$ satisfies

$$\boxed{\partial_t^2\phi + i(\partial_t m(\vec{k},t))\phi + [(G(\vec{k}))^2 + (m(\vec{k},t))^2]\phi = 0}$$

The idea is to find physically interesting profiles of the couplings for which this equation can be solved exactly in terms of special functions

- This can indeed be done if the time dependence of the couplings can be chosen to be of the form

$$\begin{pmatrix} g(t) \\ J_3(t) \end{pmatrix} = a + b \tanh(t/\delta t)$$

- This leads to the time dependent mass functions

$$m(k,t) = A(k) + B \tanh(t/\delta t)$$

	$A(k)$	B
Ising	$a - \cos k$	b
Kitaev	$-a - J_1 \cos k_1 - J_2 \cos k_2$	$-b$

- Note that the mass functions are now dependent on both the momenta and time – in position space these are space-time dependent masses.
- Nevertheless the problem can be solved exactly.

- The solutions which are purely positive frequency at early times – the "in" solutions – can be now obtained in terms of the solution for the auxilliary scalar field

$$\phi_{in}(\vec{k},t) = \exp[-i\omega_+(\vec{k})t - i\omega_-(\vec{k})\delta t \log(2\cosh(t/\delta t))]$$
$$ {}_2F_1[1 + i\omega_-(\vec{k})\delta t + iB\delta t, i\omega_-(\vec{k})\delta t - iB\delta t; 1 - \omega_{in}(\vec{k})\delta t; \frac{1}{2}(1 + \tanh(t/\delta t))]$$

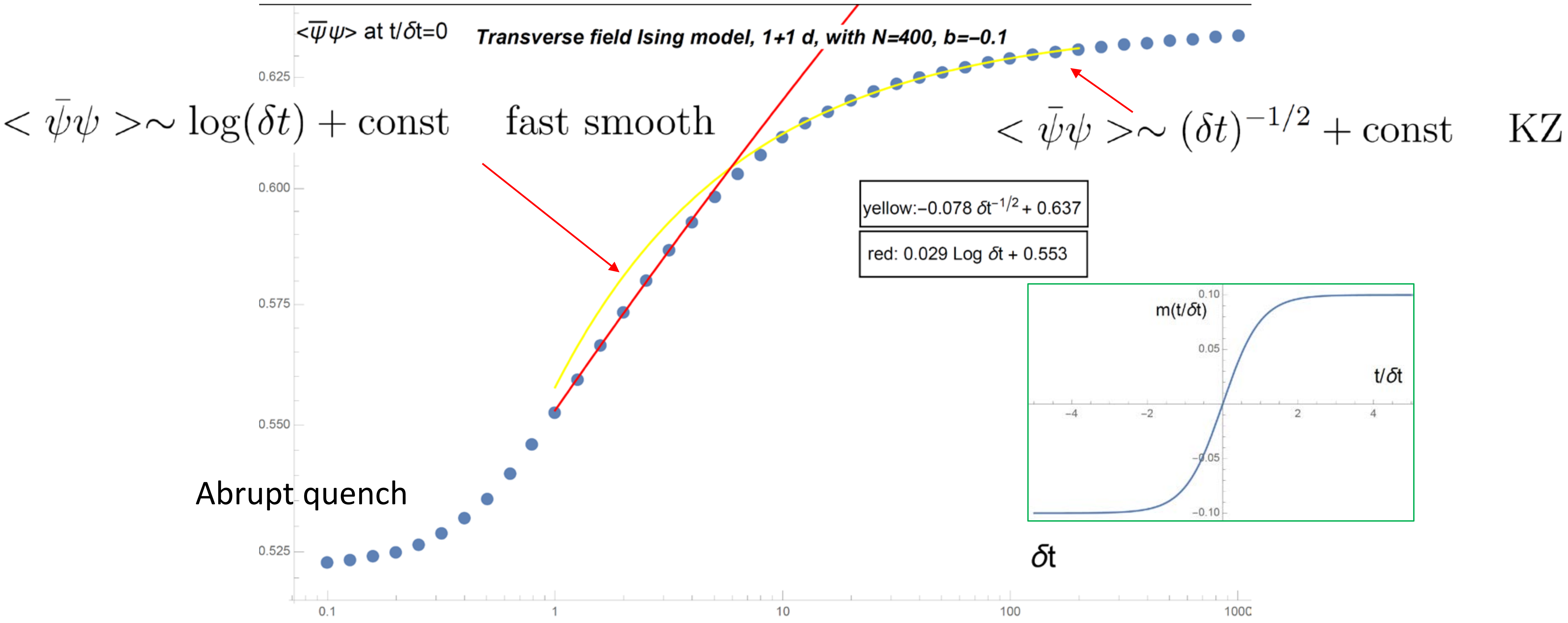
$$\omega_{in} = \sqrt{G(\vec{k})^2 + (A(\vec{k}) - B)^2} \qquad \omega_{out} = \sqrt{G(\vec{k})^2 + (A(\vec{k}) + B)^2} \qquad \omega_\pm = \frac{1}{2}(\omega_{out} \pm \omega_{in})$$

- The final mode expansion is then

$$\psi(\vec{k},t) = \frac{1}{|G(k)|}\sqrt{\frac{\omega_{in} + m_{in}}{2\omega_{in}}}\left[a_{in}(\vec{k})U_{in}(\vec{k},t) + b_{in}^\dagger(-\vec{k})V_{in}(-\vec{k},t)\right]$$

- For Ising we need to impose $a_{in}(\vec{k}) = b_{in}(\vec{k})$
- We want to then compute expectation values in the "in" vacuum

$$a_{in}(\vec{k})|0>_{in} = b_{in}(\vec{k})|0>_{in} = 0$$

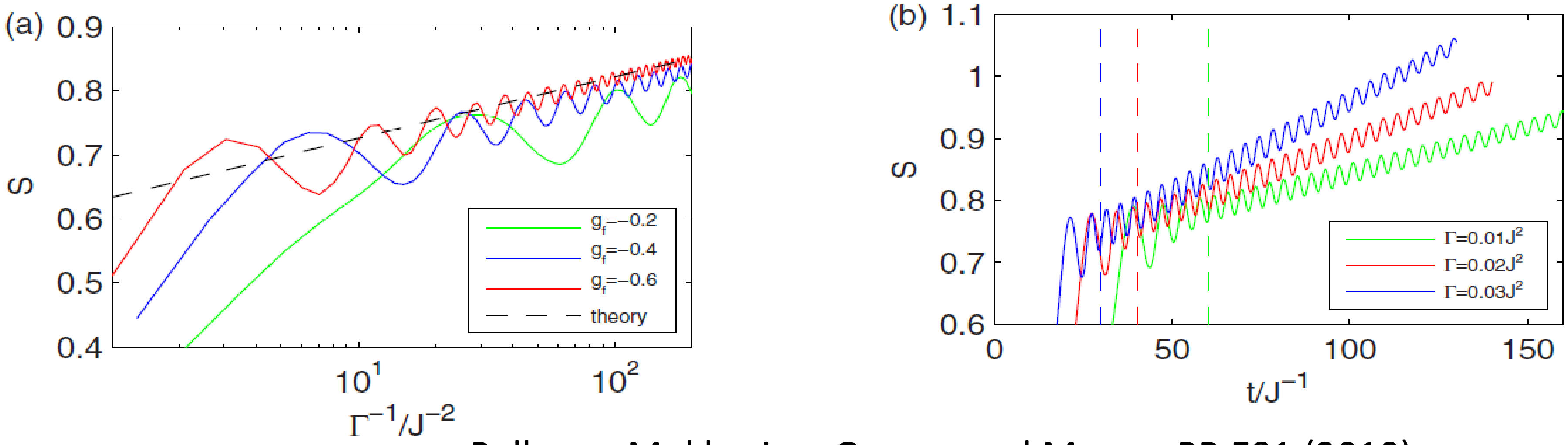


$<\overline{\psi}\psi>$ at $t/\delta t=0$
Transverse field Ising model, 1+1 d, with N=400, b=−0.1
$< \bar{\psi}\psi >\sim \log(\delta t) + \mathrm{const}$
fast smooth
$< \bar{\psi}\psi >\sim (\delta t)^{-1/2} + \mathrm{const}$
KZ
yellow: $-0.078\ \delta t^{-1/2} + 0.637$
red: $0.029\ \mathrm{Log}\ \delta t + 0.553$
$m(t/\delta t)$
$t/\delta t$
Abrupt quench
δt
0.625
0.600
0.575
0.550
0.525
0.1
1
10
100
1000
0.10
0.05
−0.05
−0.10
−4
−2
2
4

- The Kitaev model has two features which makes it different and interesting
    - It is a model in 2 space dimensions
    - There is a critical surface rather than a critical point
- If we quench from a point in the gapped phase to some point on the critical surface one might have thought that the scaling which one gets would be appropriate to that in 2+1 dimensions.
- This is not what happens. One gets 1+1 dimensional scaling both in the fast and in the slow quench regimes
- This is because the terms in the Hamiltonian which are responsible for exciting the system depend on only one combination of the momenta and are independent of the orthogonal combination.
- The low momentum expansion of the Hamiltonian around a point at the edge of the critical surface is anisotropic – in the continuum limit the effective speed of light in one of the directions vanishes.

Entanglement Entropy

- One interesting question relates to the behavior of the entanglement entropy as a function of the quench rate- does this display some kind of scaling behavior ?
- This has not been studied very much – but there has been some numerical work in the transverse field Ising model in the Kibble-Zurek regime.



Pollman, Mukherjee, Greene and Moore, PR E81 (2010)

- Work in progress on understanding this for any rate – both in field theory and holography.

Outlook

- It is interesting that thinking about this problem holographically has led to a set of new results in field theories, regardless of holography.
- These results indicate that there is a window of quench rates which lies between slow changes and instantaneous changes where a universal scaling different from Kibble-Zurek should hold in early time behavior.
- Since cold atom experiments can now-a-days engineer some lattice models, these results can be possibly tested experimentally !!

THANK YOU