Black Holes @ Large D

Things we've learned so far

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RE + R Suzuki + K Tanabe 2013, 2014, 2015, 2016

RE + Grumiller + Tanabe 2013

RE + Shiromizu + Suzuki + Tanabe + Tanaka 2015

RE + Izumi + Luna + Suzuki + Tanabe 2016

see also *Asnin+Gorbonos+Hadar+Kol+Levi+Miyamoto* 2007

A different formulation

S Bhattacharyya + S Minwalla + R Mohan + A Saha
2015

S Bhattacharyya + M Mandlik + S Minwalla + S Thakur
2015

same concepts but rather different implementation

Large D for AdS/CMT

RE + Tanabe 2013: holographic superconductivity

García-García + Romero-Bermúdez 2015: holosucon, entanglement entropy

Andrade + Gentle + Withers 2016: Drude peaks

Herzog + Spillane + Yarom 2016: Riemann problem

1/# expansions

$\sim$ local degrees of freedom at a point

Large N, eg SU(N) gauge theory

also vector model, Potts model...

Large *c* in 2D CFTs

1/# expansions

$\sim$ connections between nearby points
= directions out of a point: **Large D**

Large coordination number in a lattice
quantum fluctuations average out→Mean Field Theory

CFT_D
conformal blocks solvable (good down to D=2), MFT good

Hydrodynamics
Burgers turbulence (?)

Large D expansion and quantum entanglement?

– Short-distance quantum fluctuations strongly enhanced

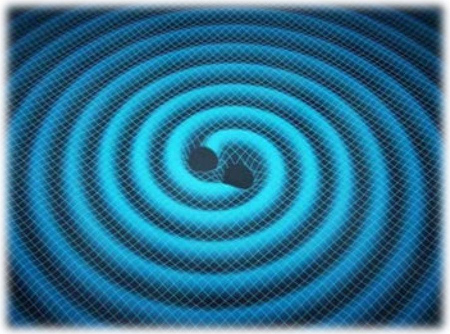
– Long-distance quantum fluctuations average out

Dual to behavior of gravitational field at large D

General Relativity

The Perfect Theory

$$R_{\mu\nu} = 0$$



No scale

No parameter

Fiendish complexity

D-diml General Relativity

Well-defined for all D

Many problems can be formulated keeping D arbitrary

→ D = continuous parameter

→ expand in 1/D

D-diml General Relativity

Large D:

Keeps essential physics of D=4

∃ black holes

∃ gravitational waves

BH in D dimensions

$$ds^2 = -\left(1 - \left(\frac{r_0}{r}\right)^{D-3}\right) dt^2 + \frac{dr^2}{1 - \left(\frac{r_0}{r}\right)^{D-3}} + r^2 d\Omega_{D-2}$$

$$\Phi(r) \sim \left(\frac{r_0}{r}\right)^{D-3} \qquad \nabla\Phi|_{r_0} \sim \frac{D}{r_0} \gg \frac{1}{r_0}$$

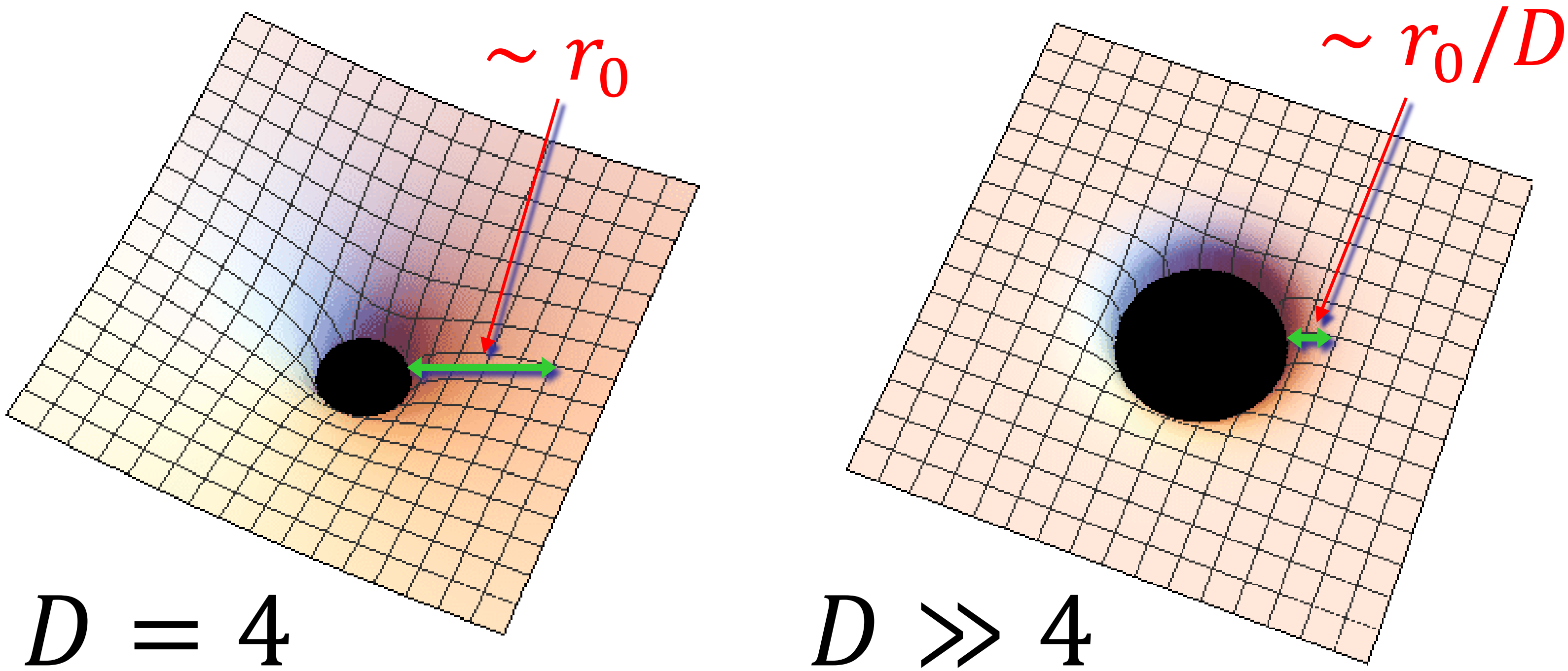
Thing we've learned:

Large D introduces new, parametrically separated scales

$$r_0 \gg \frac{r_0}{D}$$

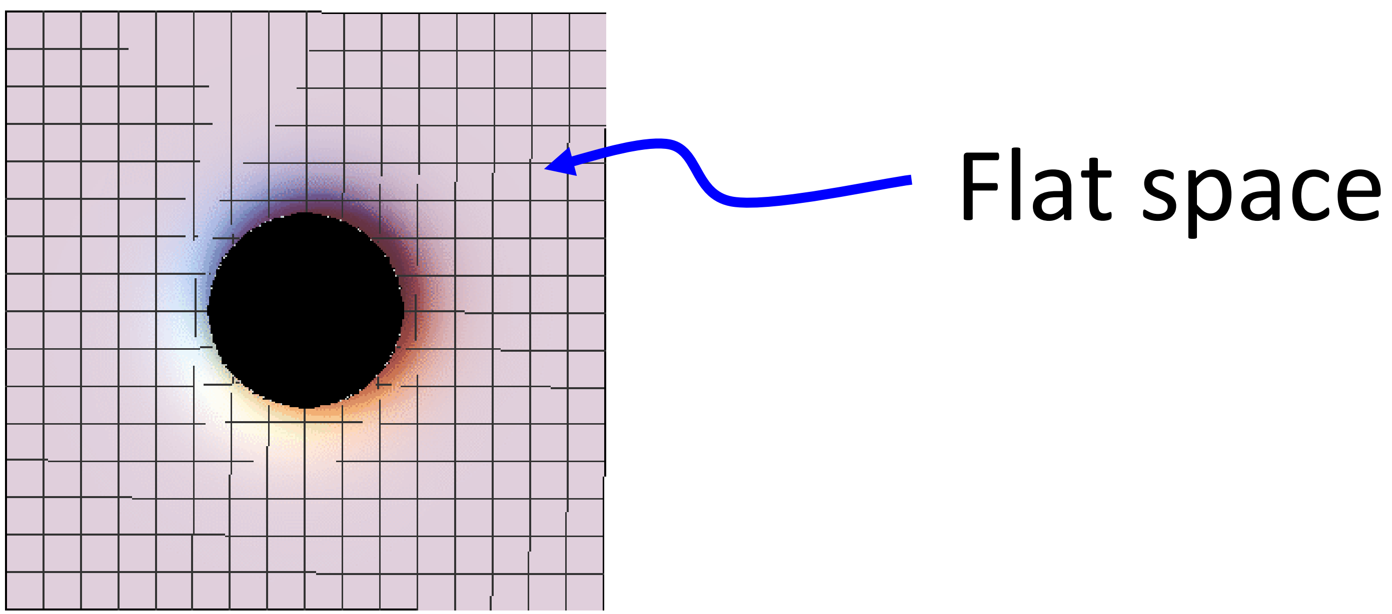
In black branes, ∃ more scales:

$$\lambda_{instab} \sim \frac{r_0}{\sqrt{D}} \qquad c_{sound} \sim \frac{1}{\sqrt{D}}$$

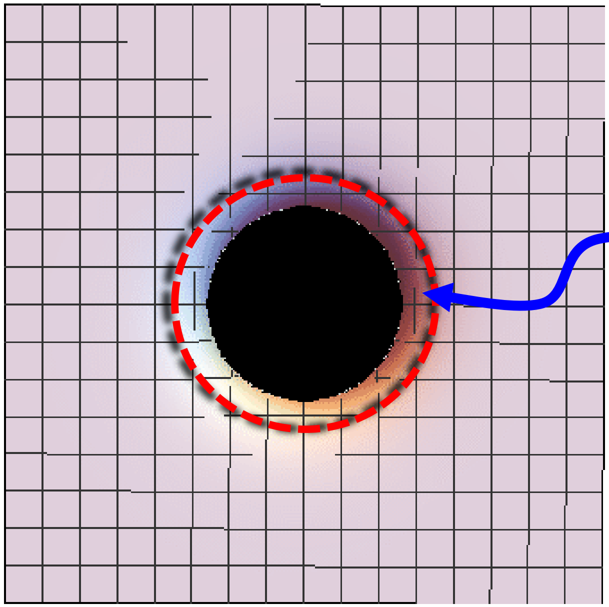


$\sim r_0$
$\sim r_0/D$
$D = 4$
$D \gg 4$

$$r > r_0 \quad \Rightarrow \left(\frac{r_0}{r}\right)^{D-3} \rightarrow 0$$



$$\left(\frac{r_0}{r}\right)^{D-3} = \mathcal{O}(1) \Leftrightarrow r - r_0 \lesssim \frac{r_0}{D}$$



$r - r_0 \sim \frac{r_0}{D}$

non-trivial
gravitational field

Thing we've learned:

$\exists$ well-defined, universal near-horizon geometry

Take $D \to \infty$ keeping finite $\left(\frac{r}{r_0}\right)^{D-3}$

Near-horizon geometry

$$\left(\frac{r}{r_0}\right)^{D-3} \equiv \cosh^2\rho$$

$$ds_{nh}^2 \rightarrow \frac{4r_0^2}{D^2}\left(-\tanh^2\rho \; dt^2 + d\rho^2\right)$$

$$+ r_0^2 \left(\cosh\rho\right)^{4/D} d\Omega_{D-2}^2$$

Small fluctuations of black hole horizon

Quasinormal modes

Thing we've learned:

Most QN modes have high frequencies

$$\omega \sim D/r_0$$

featureless oscillations of a hole in space

<u>A few long-lived QN modes localized in near-horizon region</u>

$$\omega \sim 1/r_0$$

Decoupled from far-zone

They capture interesting horizon dynamics

Black hole perturbations

Quasinormal modes of Schw-(A)dS bhs

Gregory-Laflamme instability of black branes

Ultraspinning instability of rotating bhs

All solved analytically

to several orders in 1/D

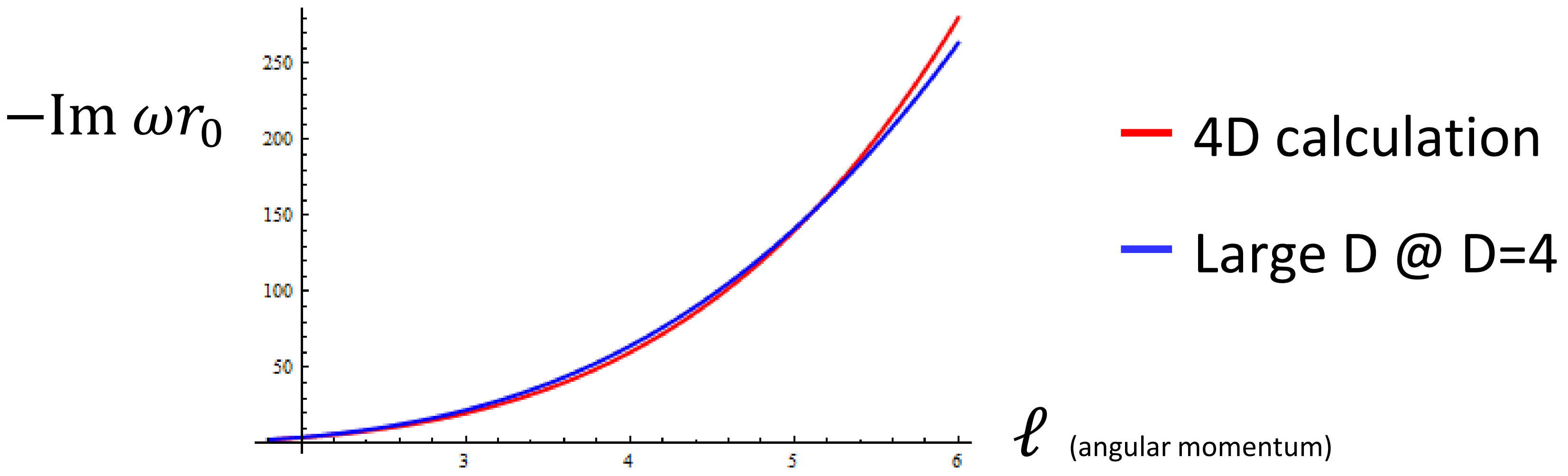
Thing we've learned:

Large D can be a very good approximation

for moderate, even small D

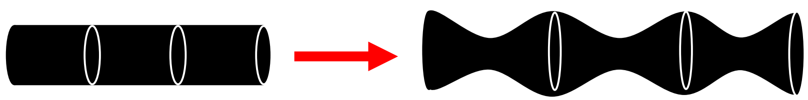
Quasinormal frequency of Schw bh in $D = 4$

(vector-type)



Calculation up to $\frac{1}{D^3}$: 6% accuracy in $D = 4$

Threshold mode of black string in $D = n + 4$



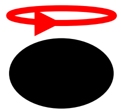
$$k_{GL} = \sqrt{n}\left(1 - \frac{1}{2n} + \frac{7}{8n^2} + \left(2\zeta(3) - \frac{25}{16}\right)\frac{1}{n^3} + \left(\frac{363}{128} - 5\zeta(3)\right)\frac{1}{n^4} + \mathcal{O}(n^{-5})\right)$$

$$k_{GL}|_{n=2} = 1.238$$

1.269 (numerical)

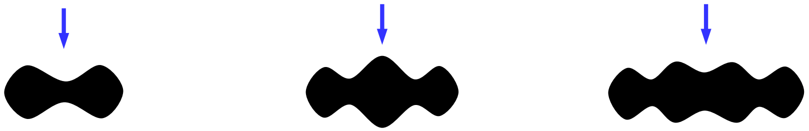
2.4% accuracy

Ultraspinning bifurcations of Myers-Perry black holes



Dias et al

Numerical: $\frac{a}{r_+} = 1.77, \quad 2.27, \quad 2.72 \ldots$ (D=8)



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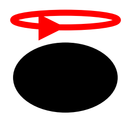

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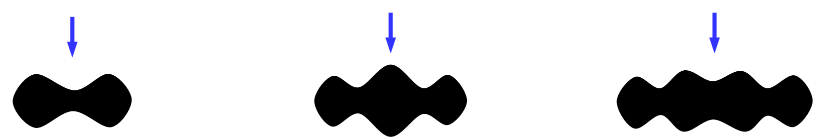


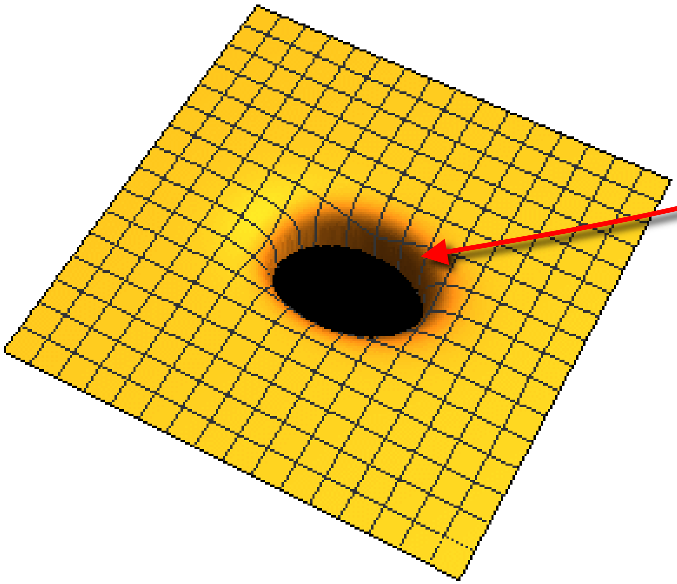
Large D: $\dfrac{a}{r_+} = \sqrt{3}, \quad \sqrt{5}, \quad \sqrt{7}, \ldots$

Suzuki+Tanabe

Error $\lesssim 2.7\%$

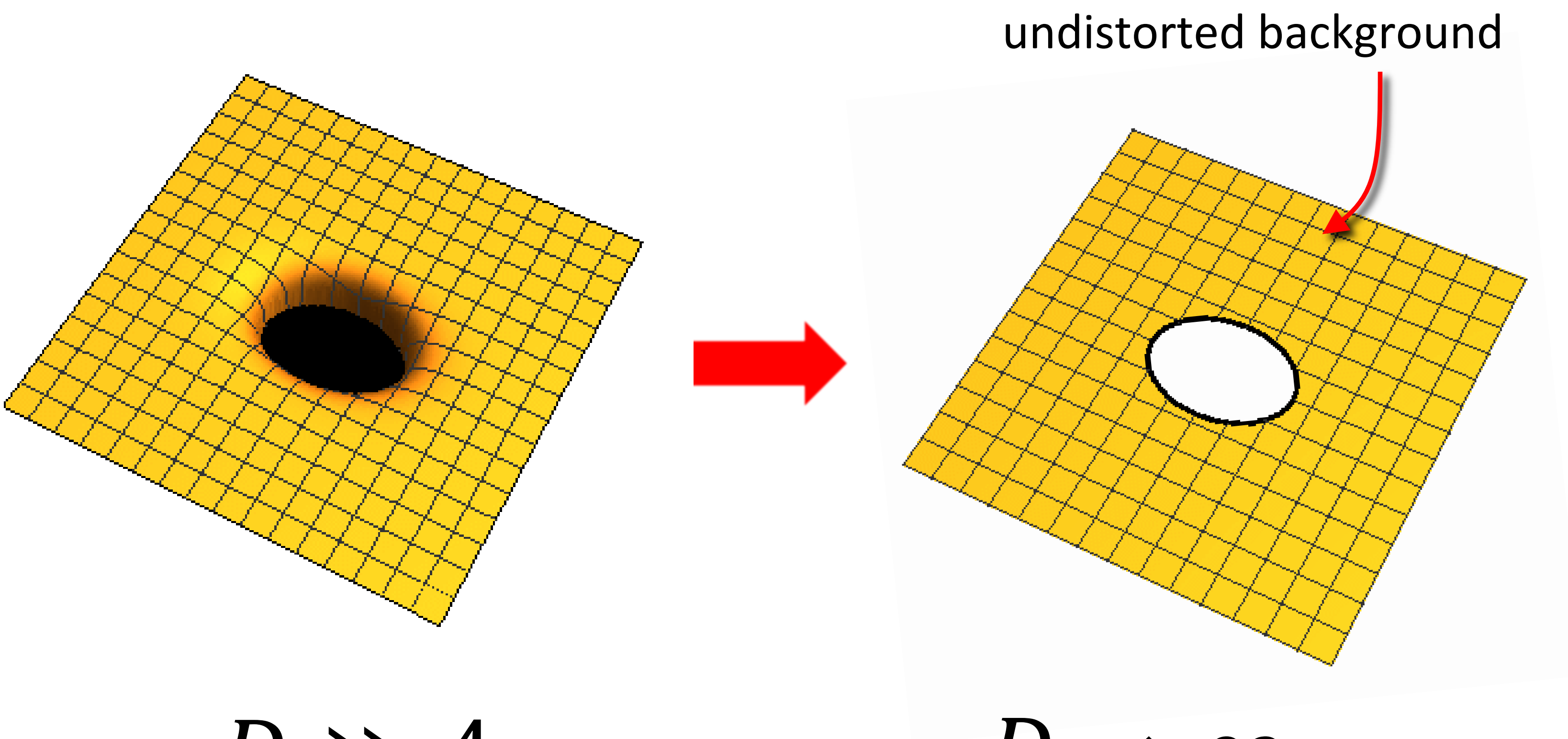
Non-linear effective theory of black hole fluctuations

All the black hole physics is concentrated here



$D \gg 4$

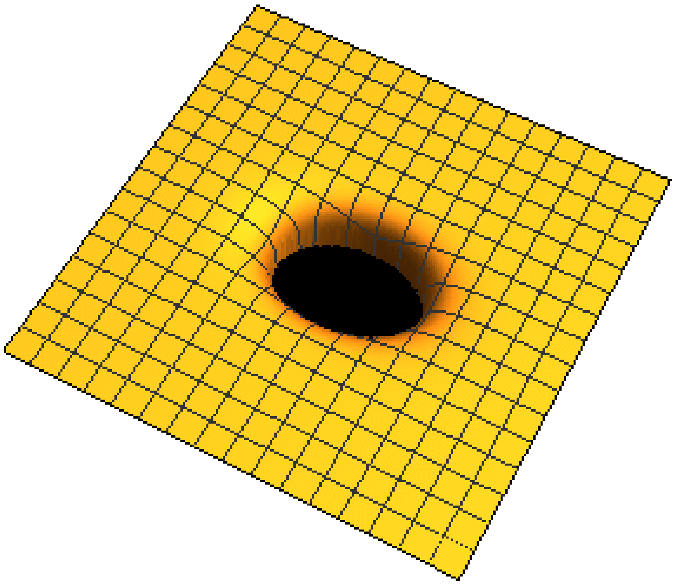
Replace bh → Surface (‘membrane’)



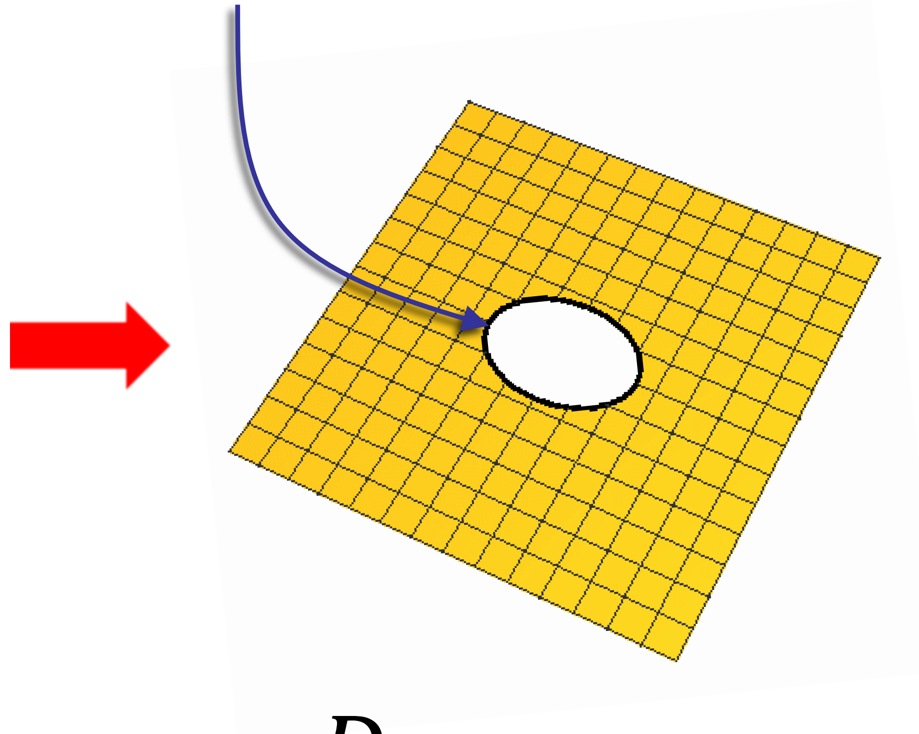
$D \gg 4$

$D \to \infty$

What's the dynamics of this membrane?



$D \gg 4$



$D \to \infty$

Solve Einstein equations in near-horizon

→ *Effective membrane theory*

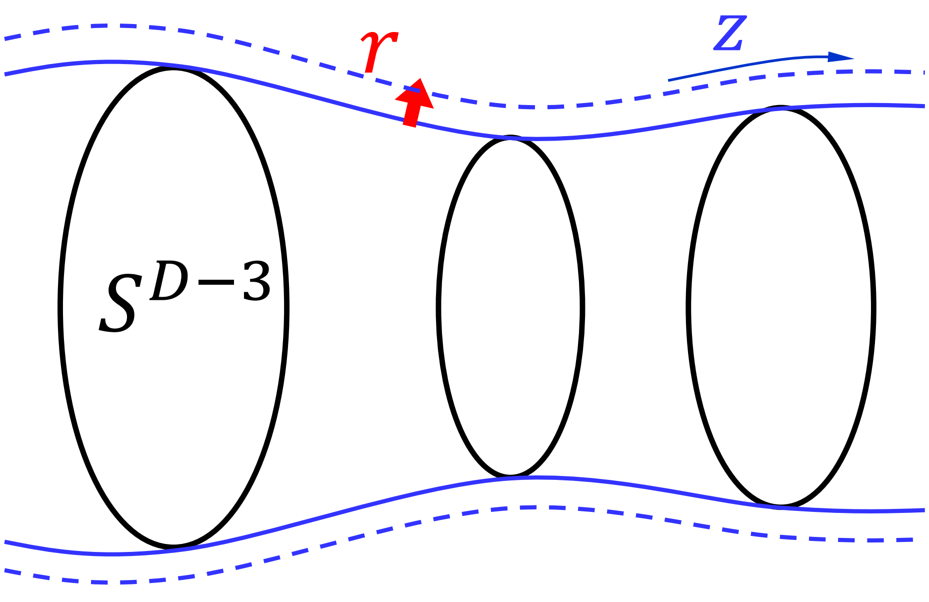
Non-linear effective theory of lightest

quasinormal modes

Gradient hierarchy

⊥ Horizon: $\partial_r \sim D$

∥ Horizon: $\partial_z \sim 1$ (or $\sim \sqrt{D}$)



Stationary solution

Soap-bubble equation (redshifted)

$$K = 2\gamma\kappa$$

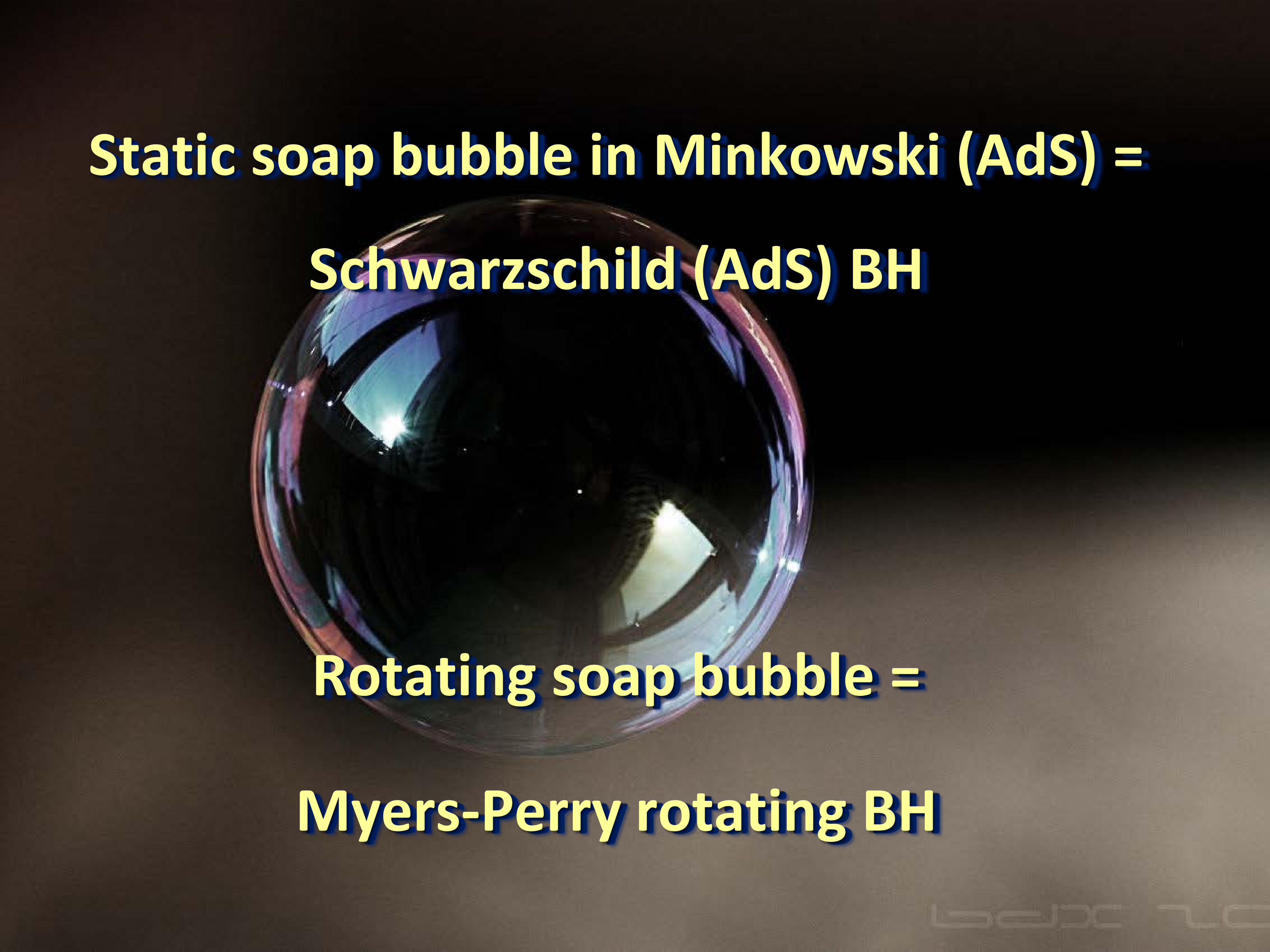
K = trace **extrinsic curvature** of membrane

γ = **redshift** factor on membrane

Lorentz boost from rotation

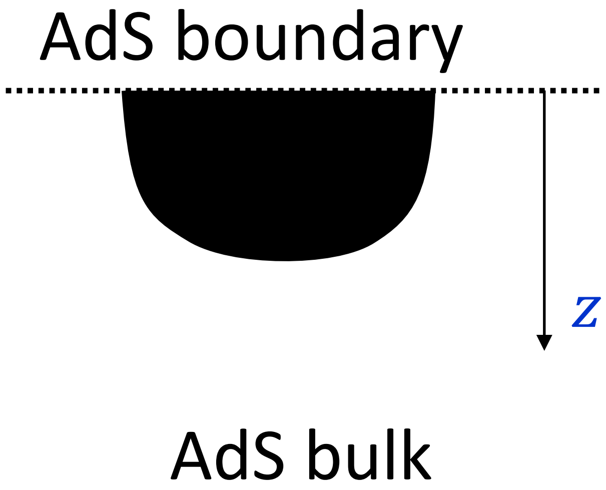
+ gravitational redshift from background

κ = **surface gravity**


Static soap bubble in Minkowski (AdS) =
Schwarzschild (AdS) BH
Rotating soap bubble =
Myers-Perry rotating BH

Black droplets

Black hole at boundary of AdS



dual to

CFT in BH background

Numerical solution:

Figueras+Lucietti+Wiseman

Black droplets

RE+Tanaka to appear

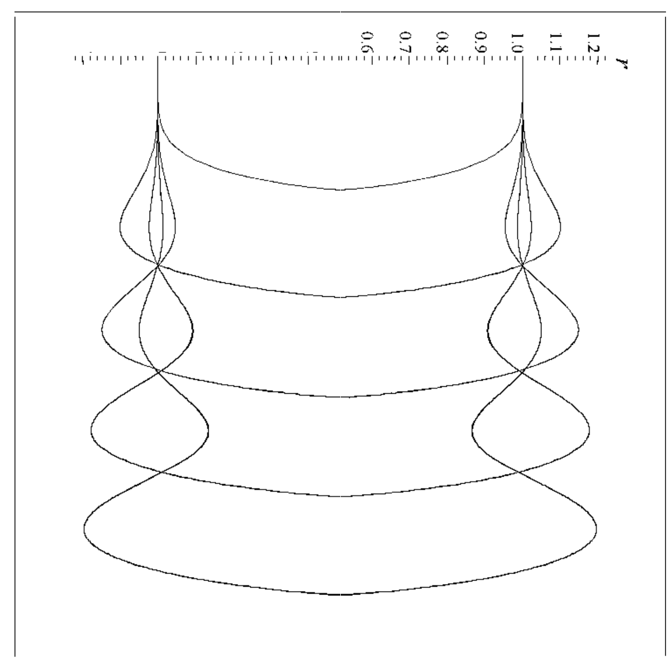
Large D analysis

simple ODE

Sequence of droplets

⇩

Sequence of states $\langle T_{\mu\nu} \rangle_{CFT}$



Time-dependence:
Effective theory of black branes

(Asymp Flat or AdS)

Solve Einstein equations for a neutral black brane

$$ds^2 = 2dt\,dr + r^2\left(-\boxed{A}\,dt^2 - \frac{2}{D}C_i dz^i dt + \frac{1}{D}G_{ij}dz^i dz^j\right)$$

$$\boxed{A = 1 - \frac{\rho(t,z^i)}{r^D}}$$

Horizon at $r^D = \rho(t,z^i)$

$$C_i = \frac{p_i(t,z)}{r^D}$$

$$p_i = \rho v_i + \partial_i \rho$$

$$G_{ij} = \delta_{ij} + \frac{1}{D}\frac{p_i(t,z)p_j(t,z)}{\rho(t,z)r^D}$$

$v_i(t,z^j)$= velocity along brane

Effective equations

effective fields $\rho\left(t, z^i\right)$, $v_i(t, z^j)$

$$\partial_t \rho + \partial_i(\rho v^i) = 0$$

$$\partial_t(\rho v_i) + \partial^j \left(\pm\rho\, \delta_{ij} + \rho v_i v_j - 2\,\rho\, \partial_{(i} v_{j)} - \rho\, \partial_{ij}^2 \ln\rho\right) = 0$$

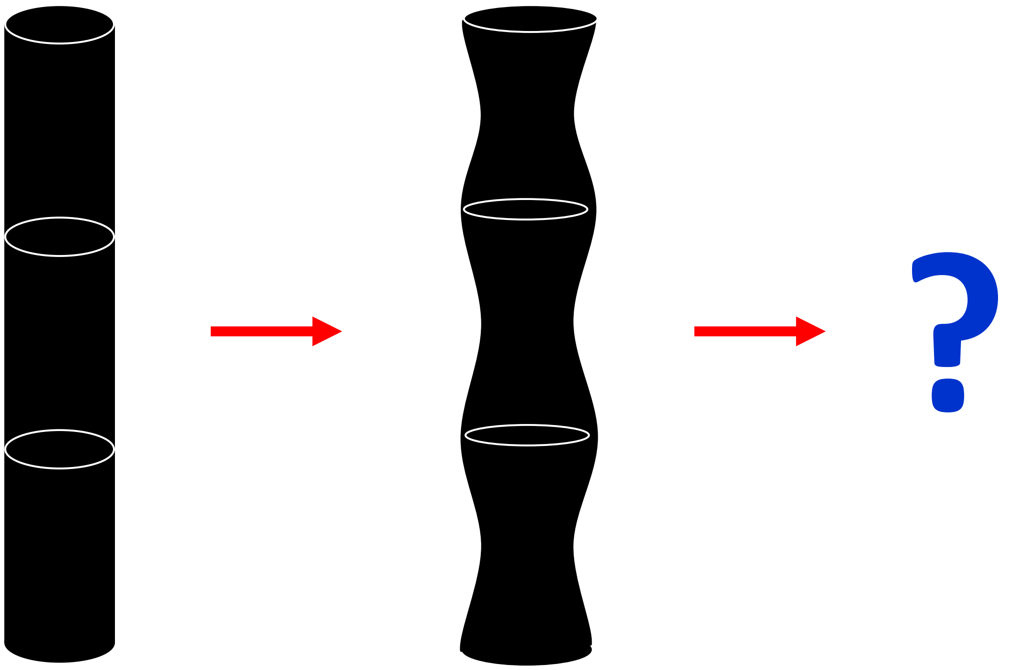
pressure viscosity

Hydrodynamics* truncates exactly

Can study phenomena at finite wavelengths

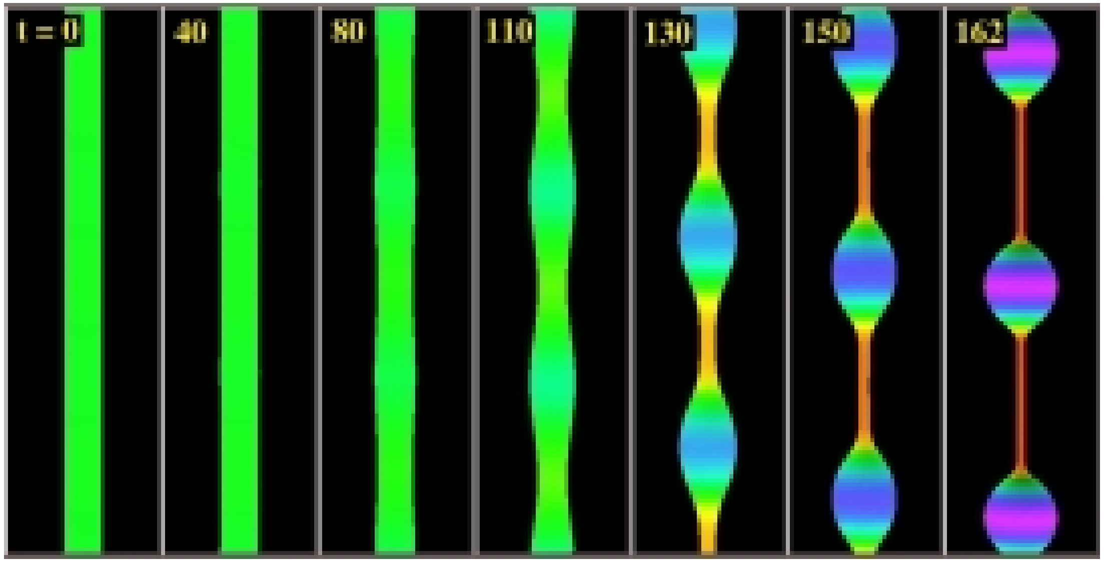
* non-relativistic: $c_{sound} \sim 1/\sqrt{D}$

Black String Instability: Evolution and Endpoint



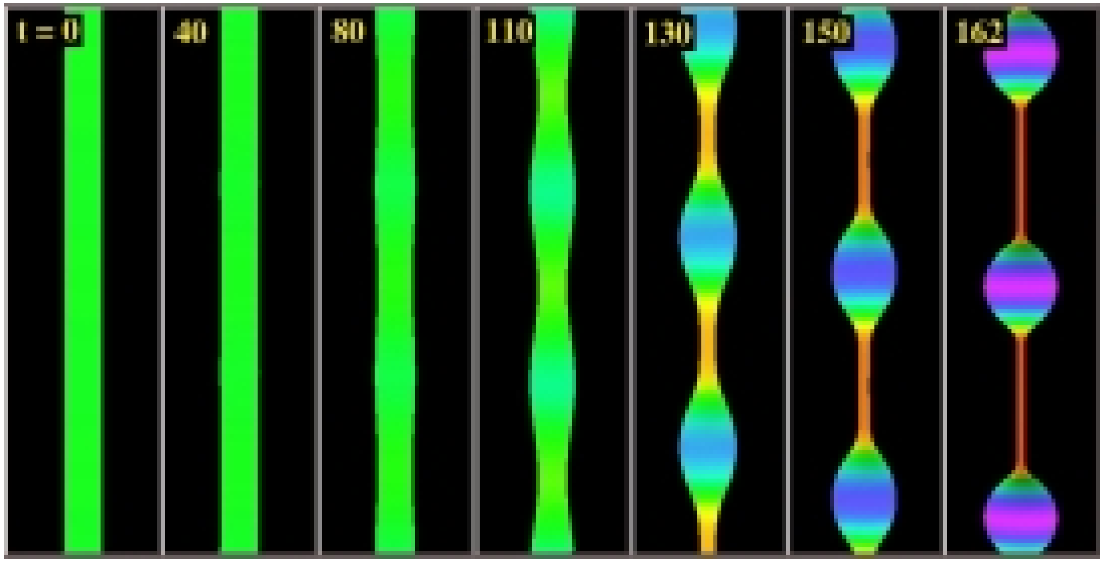
Gregory+Laflamme

5-dimensional black string



Lehner and Pretorius

5-dimensional black string



Lehner and Pretorius

100 000 CPU hours

2 months on 100 processors

Large D: evolve these 1+1 equations

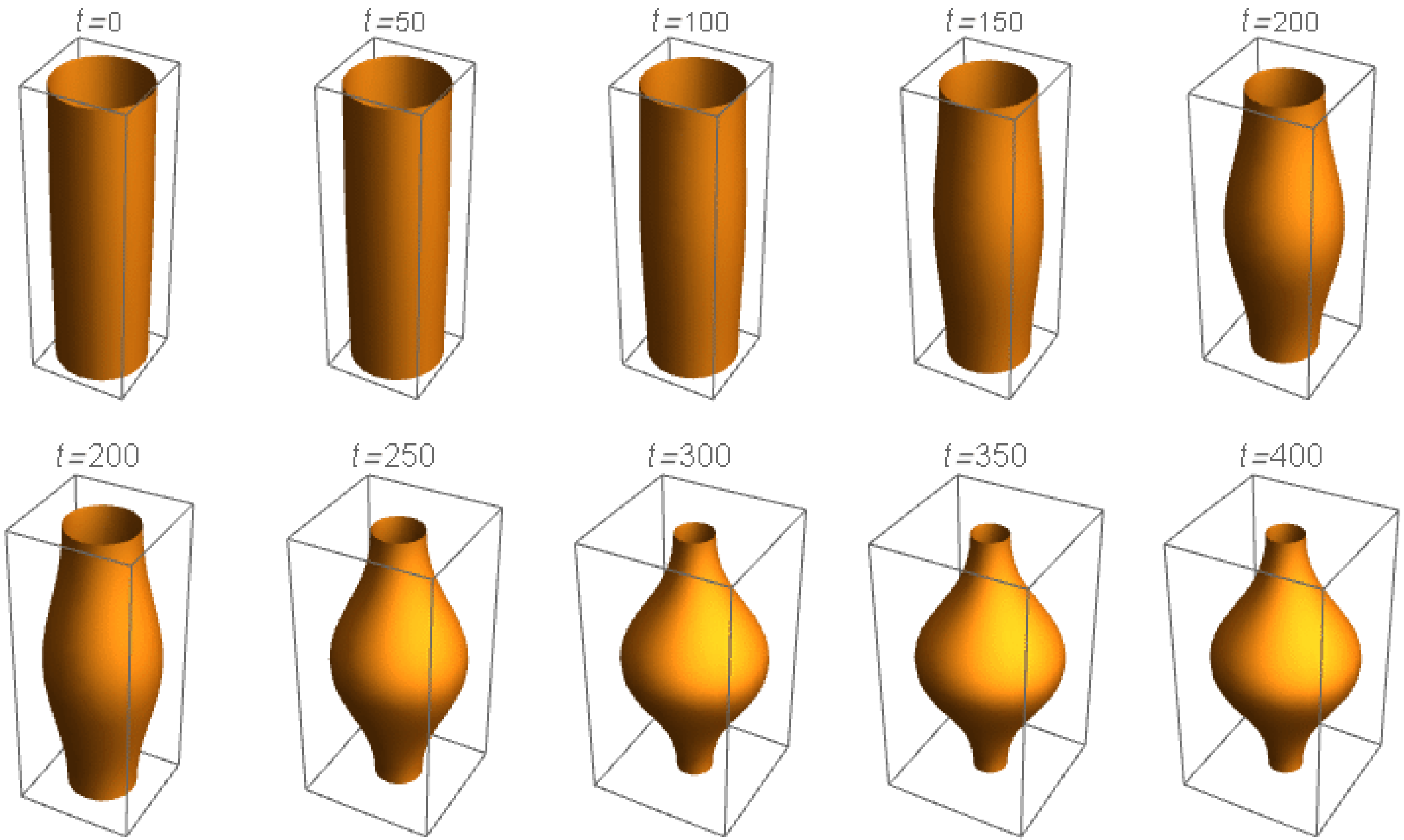
$$\partial_t \rho(t,z) - \partial_z^2 \rho(t,z) = -\partial_z p(t,z)$$

$$\partial_t p(t,z) - \partial_z^2 p(t,z) = \partial_z \rho(t,z) - \partial_z \left(\frac{p^2}{\rho}\right)$$

Code for black string evolution

```
eq1 = ∂_t m[t, z] - ∂_{z,z} m[t, z] + ∂_z p[t, z];
eq2 = ∂_t p[t, z] - ∂_{z,z} p[t, z] - ∂_z m[t, z] + ∂_z (p[t, z]^2 / m[t, z]);
pde = {eq1 == 0, eq2 == 0};
tmax = 400; k = .98; Mstep = .1;
L = 2 π / k;
δm = 0.01 Exp[-4 z^2];
δp = 0;
icbc = {m[0, z] == 1 + δm, p[0, z] == δp, m[t, -L/2] == m[t, L/2], p[t, -L/2] == p[t, L/2]};
NDSolve[{pde, icbc}, {m, p}, {t, 0, tmax}, {z, -L/2, L/2}, MaxStepSize -> Mstep]
```

Takes $O(1)$ seconds to produce a solution

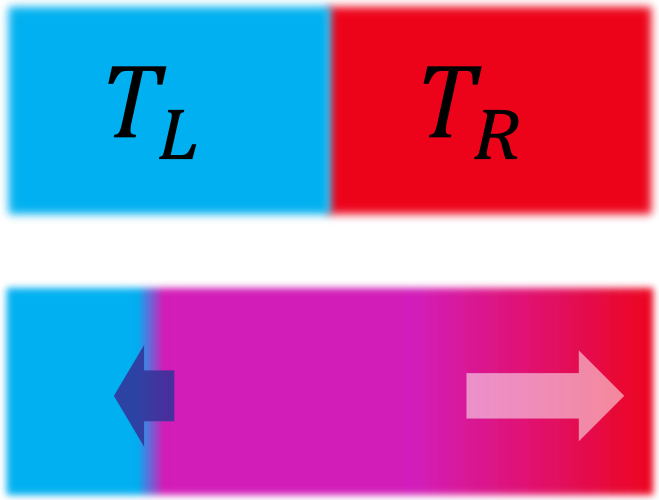


Endpoint: **stable non-uniform** black string

Horowitz+Maeda

Another application of this Eff theory (AdS)

Riemann problem *Herzog+Spillane+Yarom*



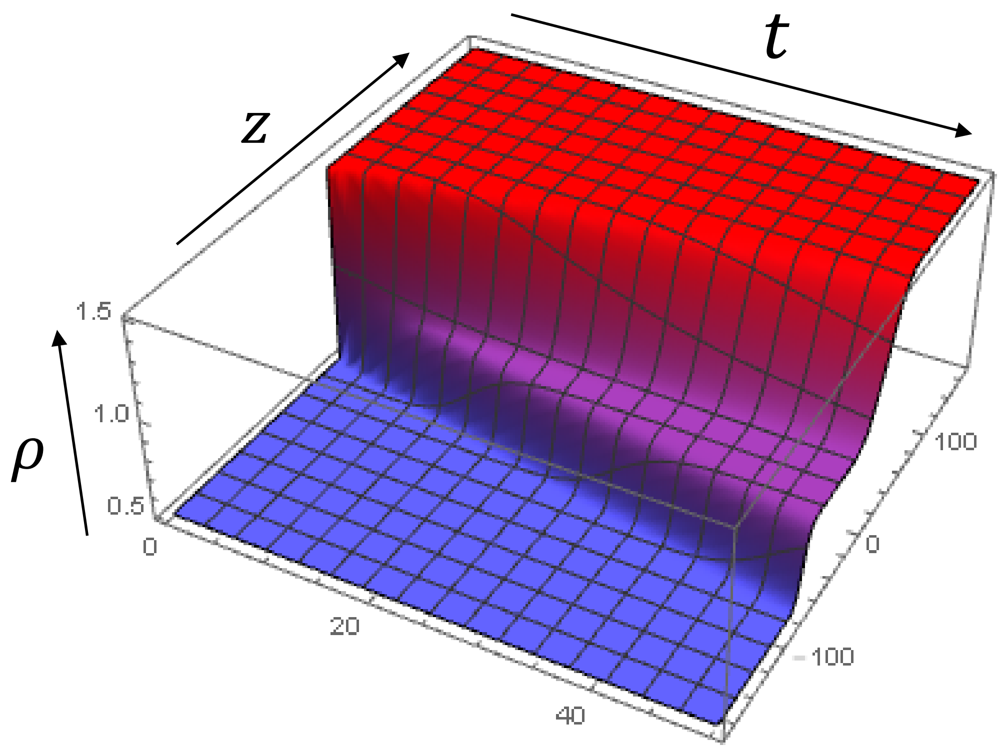
$t = 0$

$t > 0$

shock waves / rarefaction waves

Solved in black brane dual @ D=∞

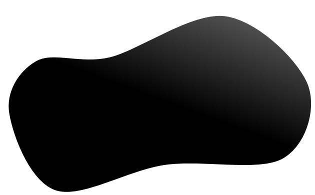
Herzog+Spillane+Yarom

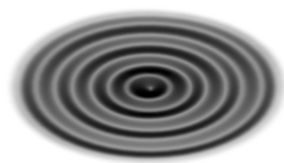


shock waves / rarefaction waves
on a black brane horizon

The main thing we've learned so far

Large D is very efficient for describing and solving **horizon deformations and fluctuations**





大きに有り難う

Thank you

$$ds^2 = \frac{N^2(r,x)}{D^2} dr^2 + g_{ab}(r,x)dx^a dx^b$$

Einstein equations (w/ Λ)

$$K^a_b = \frac{D}{N} g^{ac} \partial_r g_{cb}$$ → def: extrinsic curvature

$$\frac{D}{N}\partial_r K^a_b + K\, K^a_b = R^a_b + \frac{\delta^a_b}{L^2} - \frac{\nabla^a \nabla_b N}{N}$$ → radial evolution eqn

$$R - K^2 + K^\mu_\nu K^\nu_\mu + \frac{(D-1)(D-2)}{L^2} = 0$$

$$\nabla_\nu K^\nu_\mu - \nabla_\mu K = 0$$

→ r-independent constraints

Effective equations

effective fields $\rho(t,z)$, $v_i(t,z)$

$$\partial_t \rho + \partial_i(\rho v^i) = 0$$

$$\partial_t(\rho v_i) + \partial^j \left(\rho v_i v_j - 2\,\rho \partial_{(i} v_{j)}\right) = -\rho \partial_i(\sqrt{-g_{tt}} K(\rho))$$

$K(\rho)$ = extrinsic curvature with radius ρ

Effective equations

continuity equation (energy conservation)

$$\partial_t \rho + \partial_i (\rho v^i) = 0$$

$$\partial_t (\rho v_i) + \partial^j \left(\rho v_i v_j - 2\, \rho \partial_{(i} v_{j)} \right) = -\rho \partial_i (\sqrt{-g_{tt}} K(\rho))$$

compressible viscous fluid

ρ= energy (mass) density

Effective equations

$$\partial_t \rho + \partial_i(\rho v^i) = 0$$

extrinsic curvature with radius ρ

$$\partial_t(\rho v_i) + \partial^j \left(\rho v_i v_j - 2\,\rho \partial_{(i} v_{j)}\right) = -\rho \partial_i(\sqrt{-g_{tt}} K(\rho))$$

ρ= local radius of membrane

Effective **static** equations

$$\partial_t \rho + \partial_i(\rho v^i) = 0$$

extrinsic curvature with radius ρ

$$\partial_t(\rho v_i) + \partial^j\left(\rho v_i v_j - 2\,\rho \partial_{(i} v_{j)}\right) = -\rho \partial_i\left(\sqrt{-g_{tt}} K(\rho)\right)$$

0

Elastic equation: $\sqrt{-g_{tt}} K$ =constant

can extend to stationary

Hydro-elastic complementarity

$$\partial_t(\rho v_i) + \partial^j \left(\rho v_i v_j - 2\,\rho\partial_{(i} v_{j)}\right) = -\rho\partial_i(\sqrt{-g_{tt}}K(\rho))$$

evolve dynamically as fluids,

settle down as elastic membranes

Critical dimension for non-uniform black strings

$$D = D_* \simeq 13.5$$

E Sorkin

$D < D_*$: weak non-uniformity decreases area

$\Rightarrow$ ~~non-uniform bl-st endpoint~~

$D > D_*$: weak non-uniformity increases area

$\Rightarrow$ possible endpoint at non-uniform bl-st