Tales from the Edge: Boundary Terms and Entanglement Entropy

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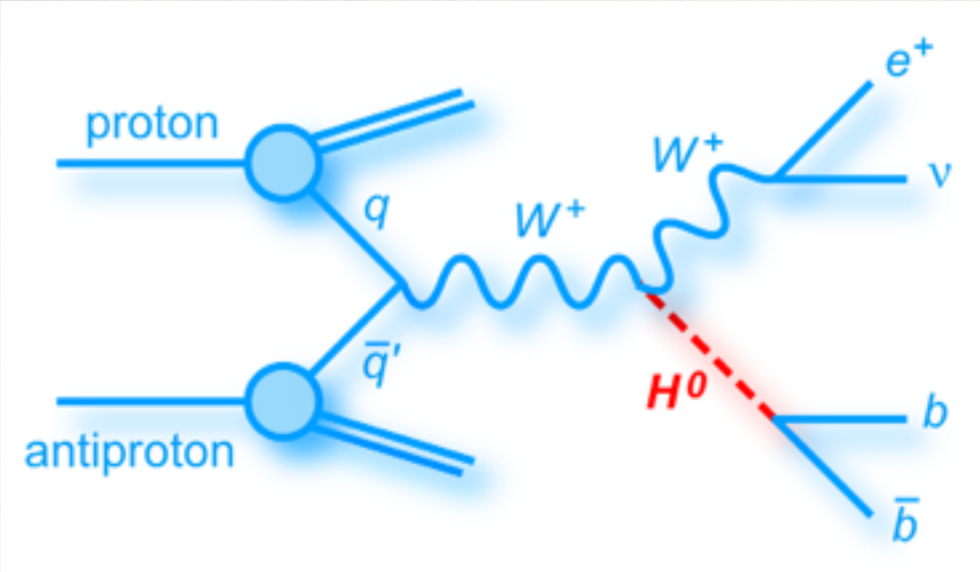
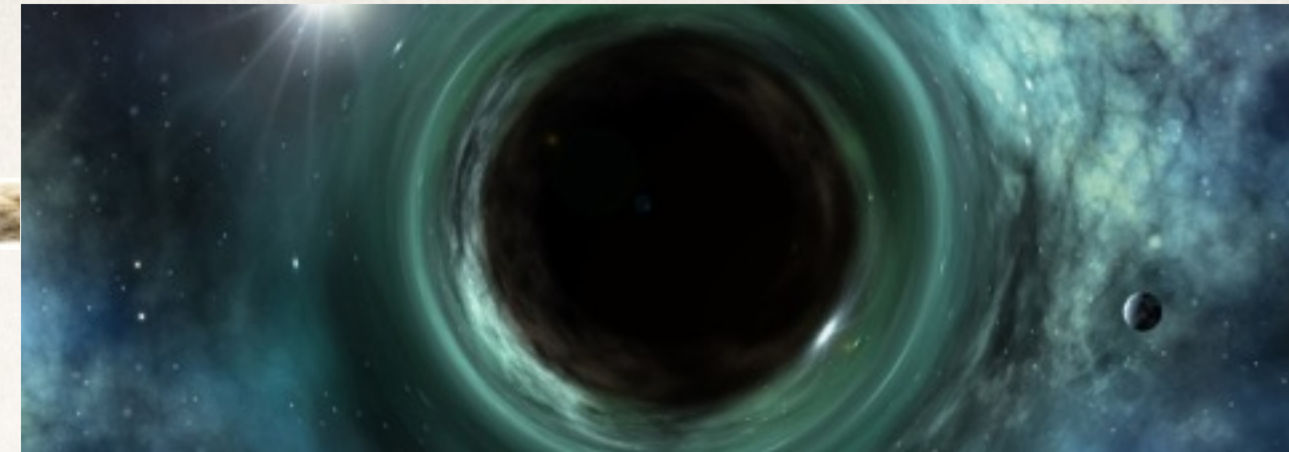
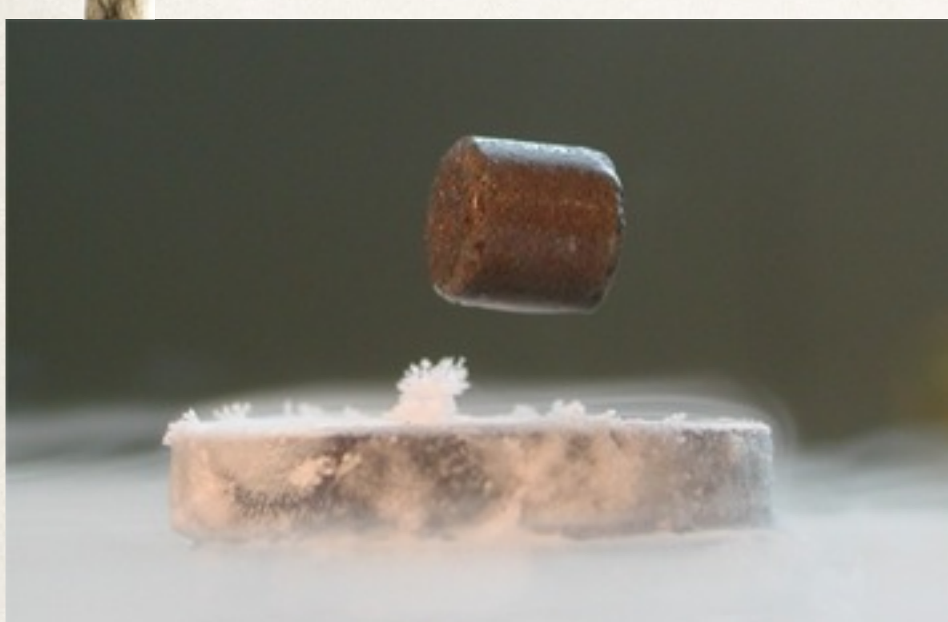
May 30, 2016

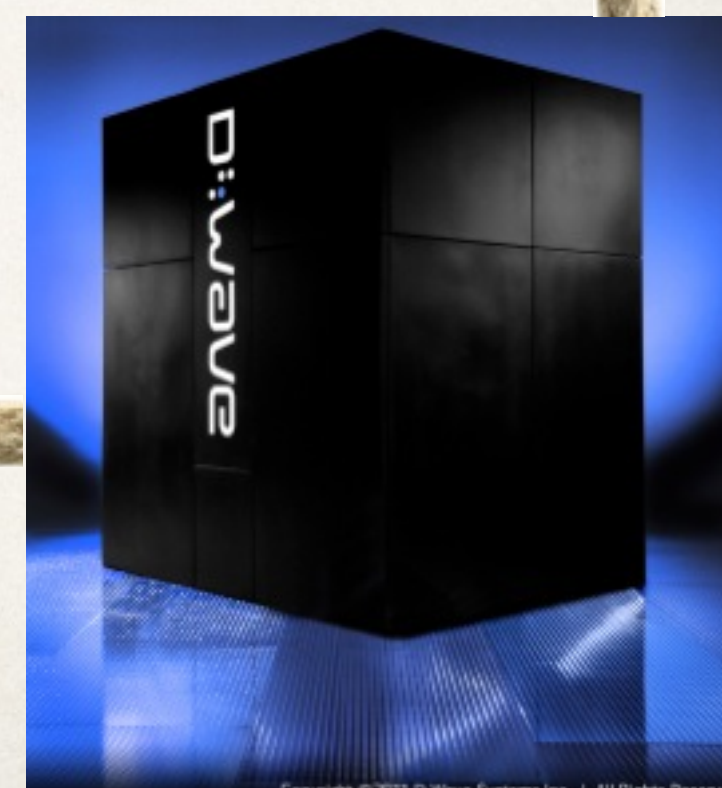




For a nonlocal, nonobservable, ultraviolet cut-off dependent quantity, entanglement entropy has become surprisingly important in theoretical physics today.



A Unifying Theme



Why is It Important?

- Quantum information, communication and computation — measure of entanglement in quantum systems
- Condensed matter physics — order parameter for exotic phase transitions (Osborne-Nielsen 2002, Vidal et al. 2003)
- Quantum field theory (QFT) — measure of renormalization group flow (a and c theorems) (Casini-Huerta 2006, 2012)
- Gravity — relations to black hole entropy (Bombelli et al. 1986, Srednicki 1993); Bekenstein bound (Casini 2008)
- String theory — Ryu-Takayanagi (2006) formula and AdS/CFT ties QFT and gravity aspects together.

Two Tales from the Edge

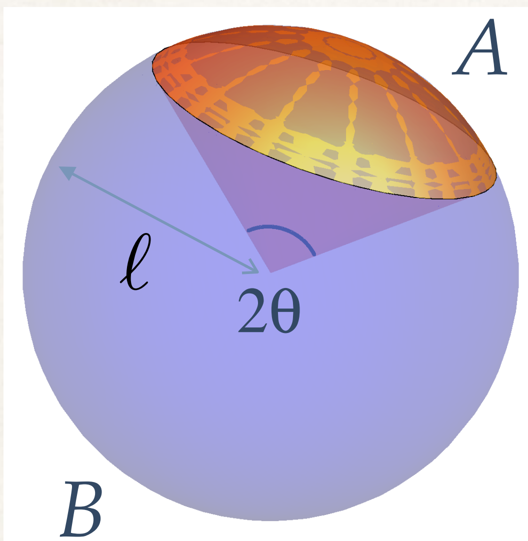
For conformal field theories (CFTs)

- Thermal corrections to entanglement entropy (work with M. Spillane, J. Nian, R. Vaz, and J. Cardy).

- Universal contributions to entanglement entropy at zero temperature (work with K.-W. Huang and K. Jensen).

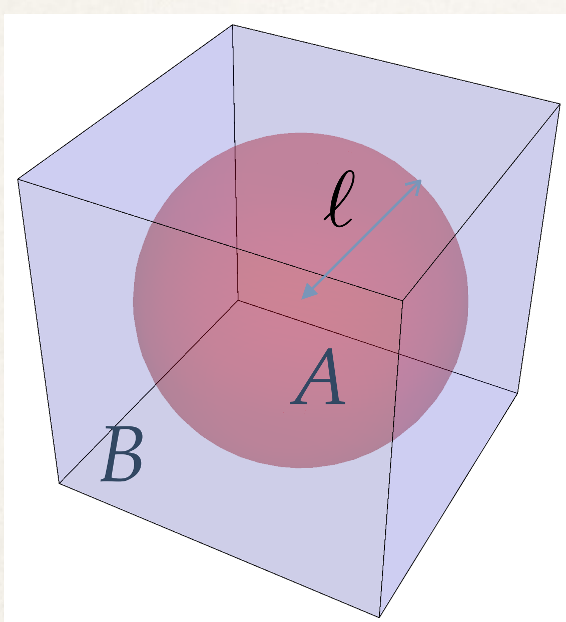
Moral: The importance of boundary terms.

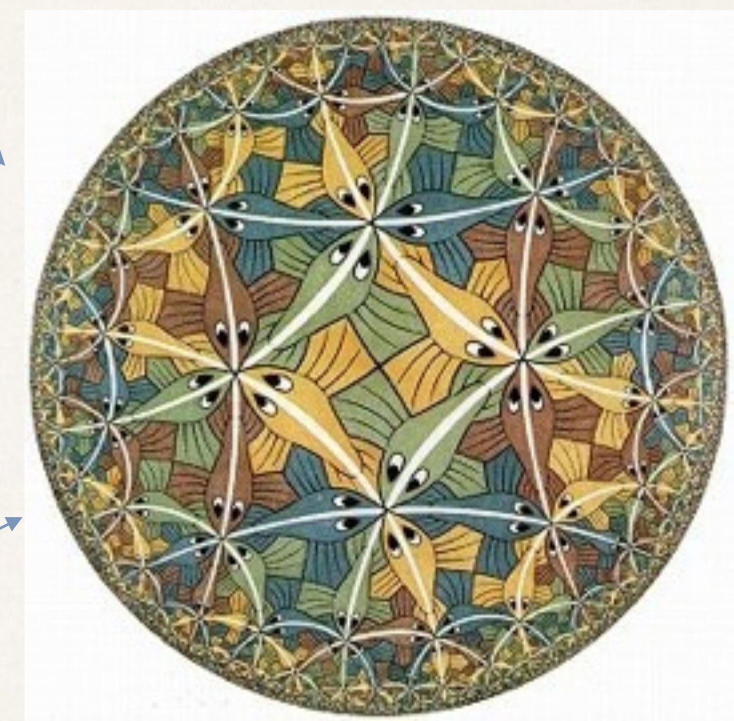
Trick for Calculating EE of CFTs



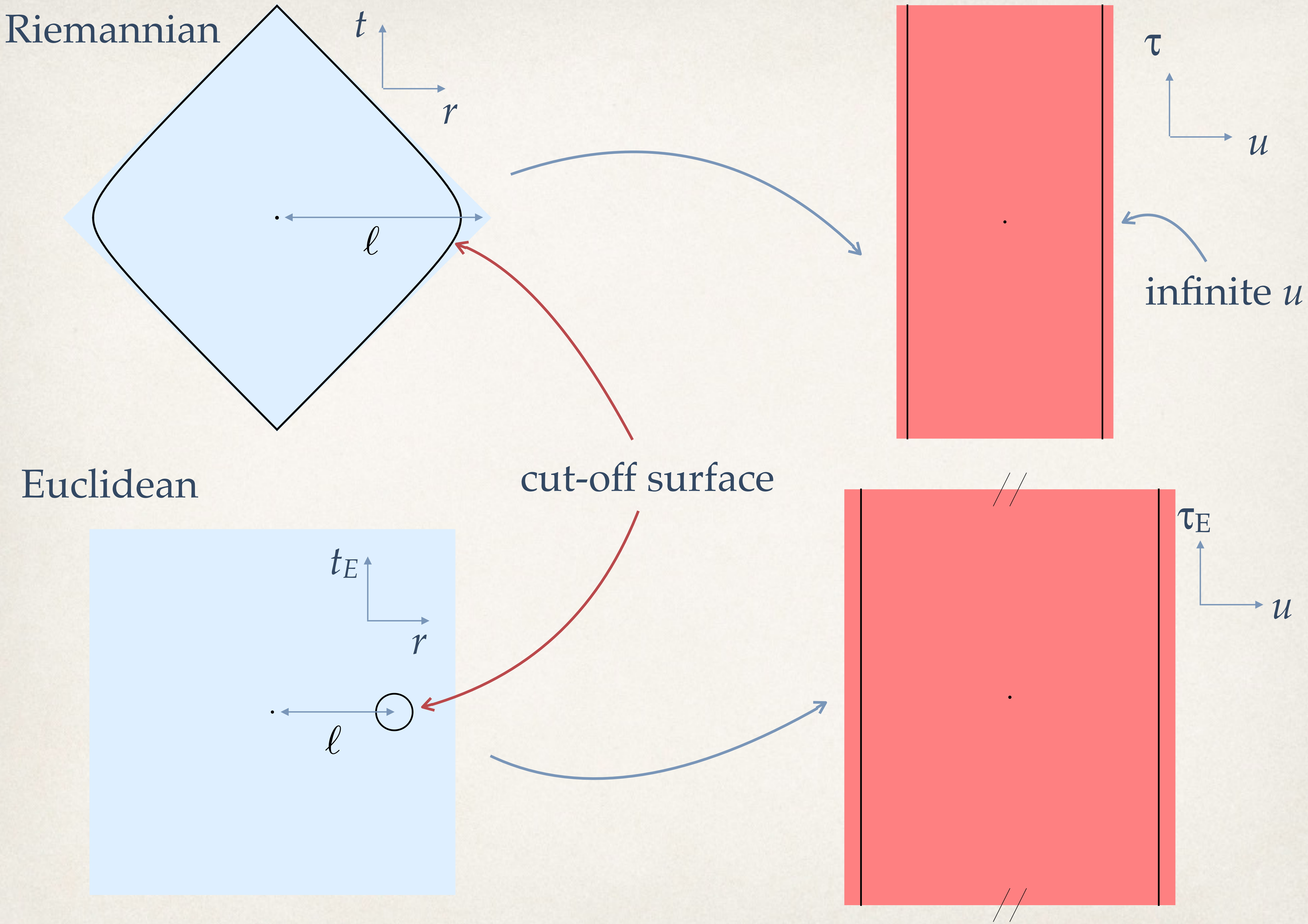
Has $T = 1/2\pi\ell$

conformal
transformation





For caps on spheres and balls in flat space,
"A" gets mapped to all of hyperbolic space.


Riemannian
t
r
ℓ
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u
infinite u
Euclidean
cut-off surface
t_E
r
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τ_E
u

Map to Hyperbolic Space

- Density matrix on hyperbolic space is thermal: $\beta = 2\pi\ell$

$$\rho = \frac{e^{-\beta H}}{\mathrm{tr}\, e^{-\beta H}}$$

H called the modular Hamiltonian

- $\rho_A = U^{-1} \rho U$ for some unitary operator U.

- EE invariant under U implies thermal entropy of hyperbolic space is EE. (see e.g. Casini-Huerta-Myers 2011)

A Tale from the Edge

Thermal Corrections?

The initial density matrix is
not that of a pure state!

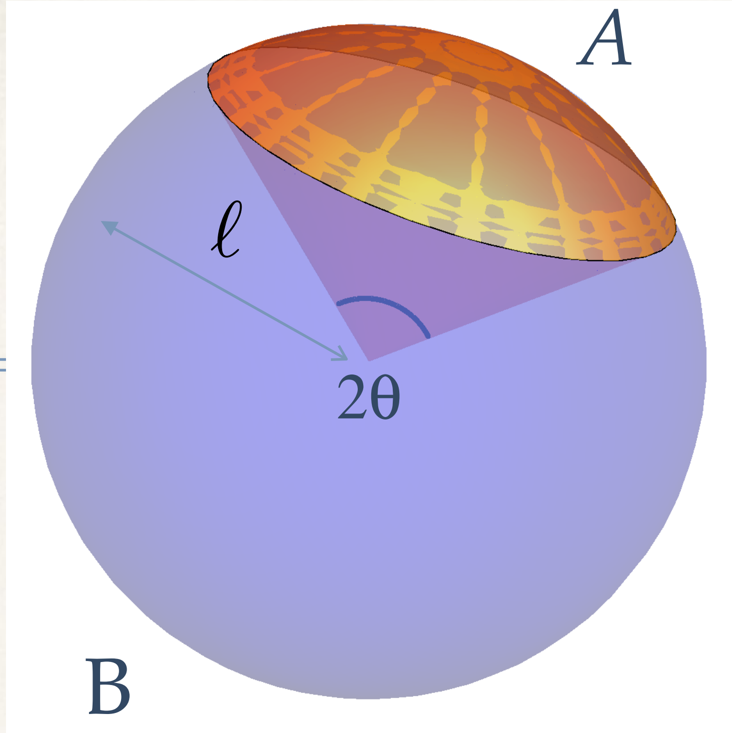
$$\rho(T) = \frac{e^{-H/T}}{\mathrm{tr}(e^{-H/T})}$$

Entanglement entropy measures some combination
of thermal entropy and quantum entanglement.

Why bother with thermal effects?

- Nice to be able to remove them.
- Lessons to be learned from EE in non-traditional contexts.
- Connection to black hole physics.

A Universal Result



In the $\ell T \ll 1$ limit, for a cap A of opening angle 2θ on the S^3,

$$S_E(A,T) - S_E(B,T) = 2\pi g m \ell \cot(\theta) e^{-m/T} + o(e^{-m/T})$$

(Herzog 2014)

m is the mass gap, $\sim 1/\ell$
g is the degeneracy of the 1st excited state

- Turns out to be true for any CFT in any dimension!
- Subleading in a large N expansion.
- The exp($-m/T$) Boltzmann suppression should be true of any gapped QFT (Herzog-Spillane 2012).

Where does it come from?

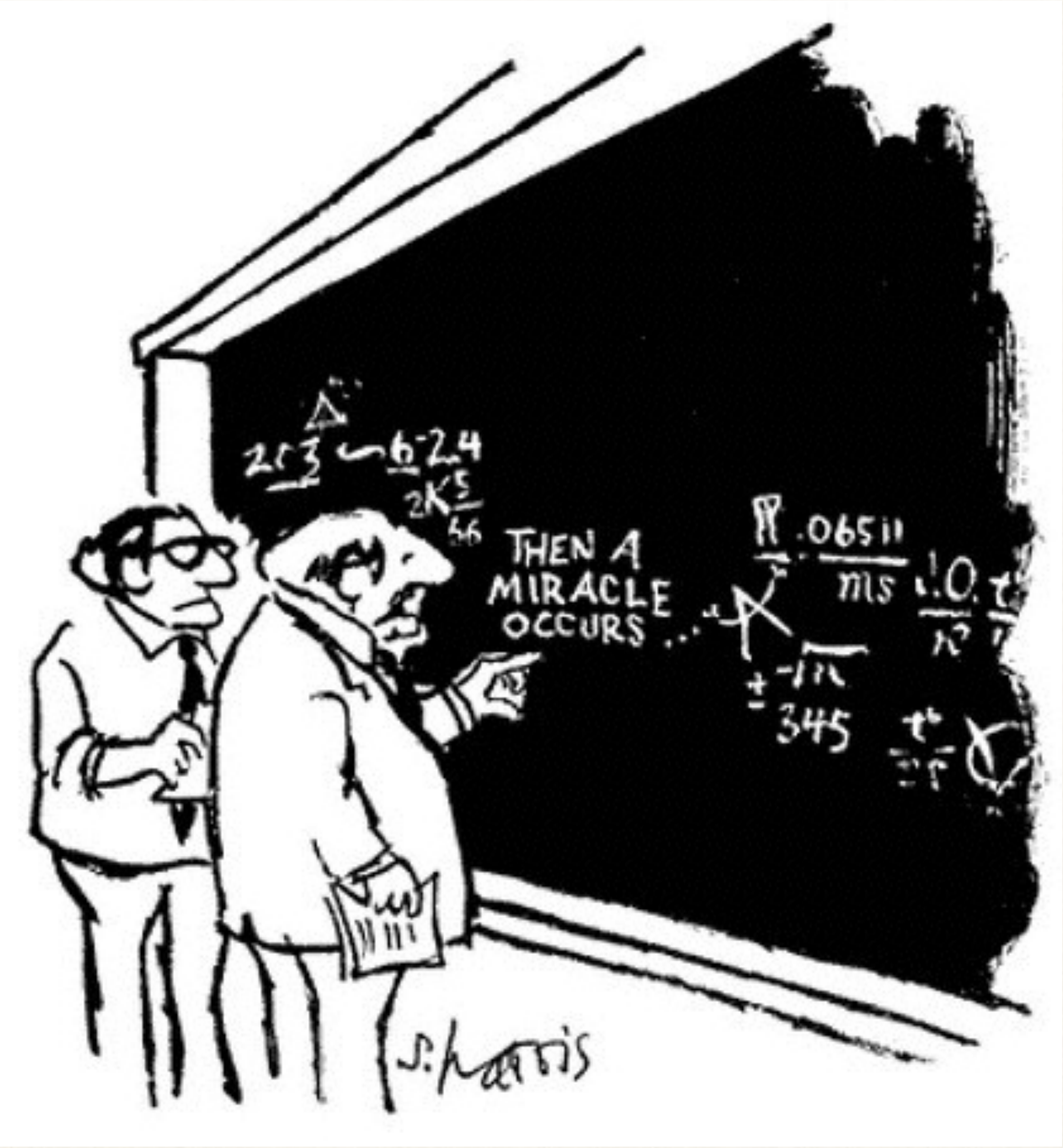
Start with a thermal density matrix $$\rho(T) = \frac{e^{-H/T}}{\mathrm{tr}(e^{-H/T})}$$

(That ρ is mixed means we're not really measuring quantum entanglement.)

Make a small *T* perturbative expansion

Need to calculate $\langle \psi(x)\psi(y) \log \rho_A(0) \rangle$

where $\psi(x)$ creates the first excited state.


THEN A
MIRACLE
OCCURS . . .
S. Harris

A Special Trick for CFTs

For CFTs and "A" a cap on a sphere, $-\log \rho_A(0)$

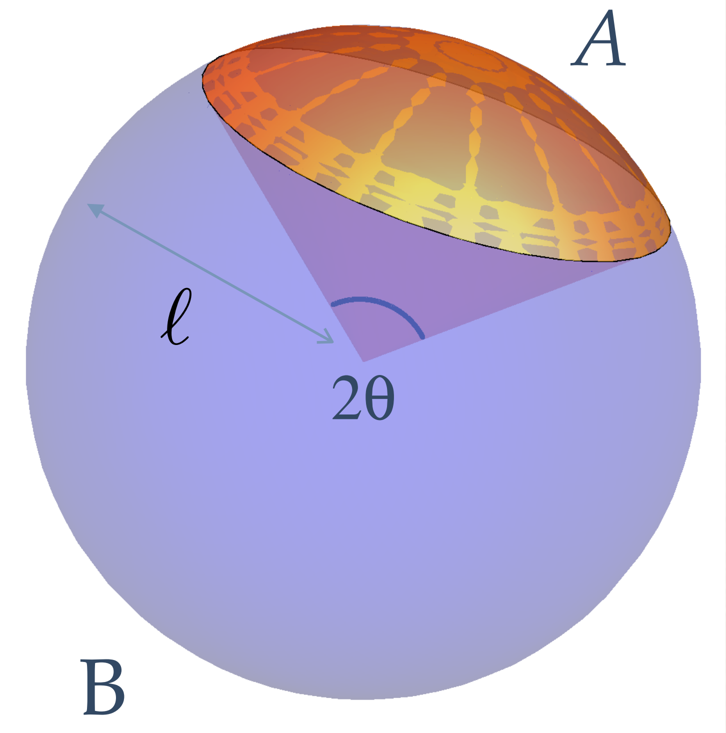
is unitarily related to the Hamiltonian on hyperbolic space.

H is the integral of the *tt* component
of the stress-energy tensor $T_{\mu\nu}$.

$$\langle \psi(x)\psi(y) \log \rho_A(0)\rangle \to \langle \psi(x)\psi(y) T_{\mu\nu}(0)\rangle$$

Three point functions involving the stress tensor in CFTs are constrained by symmetry to take relatively simple forms.

Related Result Not Quite Right



From the modular Hamiltonian method

$$S_E(A,T) - S_E(A,0) = gm\ell I_d(\theta)e^{-m/T} + \dots$$

where

$$I_d(\theta) = 2\pi \frac{\mathrm{Vol}(S^{d-2})}{\mathrm{Vol}(S^{d-1})} \int_0^{\theta_0} \frac{\cos\theta - \cos\theta_0}{\sin\theta_0} \sin^{d-2}\theta \, d\theta$$

But for a scalar field, it turns out other methods match $I_{d-2}(\theta)$.

WHAT'S GOING ON!?!

A Resolution

Claim: The modular Hamiltonian should be defined with nonsingular Robin or Neumann boundary conditions.

- Sometime the naive modular Hamiltonian may be self-adjoint with bad (singular) boundary conditions.
- Sometimes the naive modular Hamiltonian can be improved by a boundary term to a modular Hamiltonian with good (non-singular) boundary conditions
- This problem and resolution occurs for both the conformally coupled scalar and for 4d gauge fields.

Half Space Entanglement

Stress tensor for a conformally coupled scalar field

$$T_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi - \frac{1}{2}g_{\mu\nu}(\partial\phi)^2 - \xi(\partial_\mu\partial_\nu - g_{\mu\nu}\partial^2)\phi^2$$

Naively, the modular Hamiltonian is

$$H_\xi = 2\pi\int_{x^1>0} \mathrm{d}^{d-1}x\, x^1\, T_{00}(x,\xi) = H_0 - 2\pi\xi\int_{x^1=0}\mathrm{d}^{d-2}x\,\phi^2(x)$$

Zero modes of H have boundary behavior

$$\phi = a + b\log(x^1) + \dots$$

The Robin condition for H_ξ means b will be nonzero!

The Neumann condition for H_0 allows $b = 0$.

Lee, Lewkowycz, Perlmutter, Safdi (2014);
Casini, Mazitelli, Teste (2014)

The boundary term

Claim: This boundary counter-term appears in the hyperbolic space computation as

$$\Delta H = 2\pi\xi \int_{\partial H^{d-1}} \mathrm{d}^{d-2}x\sqrt{\gamma}\,\phi^2$$

and it is precisely what the doctor ordered to fix the discrepancy in the thermal correction story and send

$$I_{d-2}(\theta) \to I_d(\theta)$$

Boundary Terms in AdS/CFT

Holo RG for a scalar ϕ generically requires at least the boundary term

$$\int_{\partial AdS} \sqrt{\gamma}\phi^2$$

and often also $\int_{\partial AdS} \sqrt{\gamma}\phi\Box\phi$

Holo RG for the metric requires a host of boundary terms

$$\int_{\partial AdS} \sqrt{\gamma}K \;,\; \int_{\partial AdS} \sqrt{\gamma} \,,\, \int_{\partial AdS} \sqrt{\gamma}R \,,\; \int_{\partial AdS} \sqrt{\gamma}R^2, \quad \text{etc.}$$

Boundary Terms in AdS/CFT

Holo RG for a scalar ϕ generically requires at least the boundary term

$$\int_{\partial AdS} \sqrt{\gamma}\phi^2$$

and often also $\int_{\partial AdS} \sqrt{\gamma}\phi\Box\phi$

crucial for understanding
an apparent discrepancy
for the thermal corrections story

Holo RG for the metric requires a host of boundary terms

$$\int_{\partial AdS} \sqrt{\gamma}K \ , \ \int_{\partial AdS} \sqrt{\gamma} \, , \int_{\partial AdS} \sqrt{\gamma}R \, , \ \int_{\partial AdS} \sqrt{\gamma}R^2 , \quad \text{etc.}$$

Boundary Terms in AdS/CFT

Holo RG for a scalar ϕ generically requires at least the boundary term

$$\int_{\partial AdS} \sqrt{\gamma}\phi^2$$

Think about it as the boundary term for the 2d Euler character. A higher dimensional analog will be key for the zero temperature story

and often also $\int_{\partial AdS} \sqrt{\gamma}\phi\Box\phi$

Holo RG for the metric requires a host of boundary terms

$$\int_{\partial AdS} \sqrt{\gamma}K \,, \int_{\partial AdS} \sqrt{\gamma} \,, \int_{\partial AdS} \sqrt{\gamma}R \,, \int_{\partial AdS} \sqrt{\gamma}R^2 , \quad \text{etc.}$$

Boundary Terms in AdS/CFT

Holo RG for a scalar ϕ generically requires at least the boundary term

$$\int_{\partial AdS} \sqrt{\gamma}\phi^2$$

likely important too

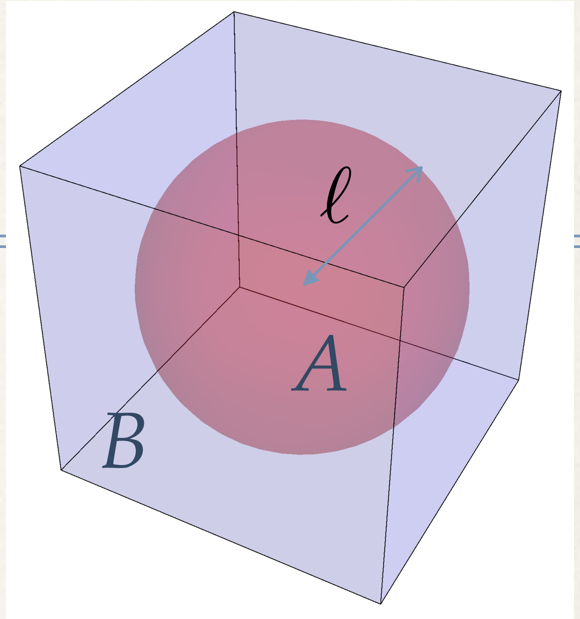
and often also $\displaystyle\int_{\partial AdS} \sqrt{\gamma}\phi\Box\phi$

Holo RG for the metric requires a host of boundary terms

$$\int_{\partial AdS} \sqrt{\gamma}K \;,\; \int_{\partial AdS} \sqrt{\gamma} \,,\, \int_{\partial AdS} \sqrt{\gamma}R \,,\; \int_{\partial AdS} \sqrt{\gamma}R^2, \quad \text{etc.}$$

A Second Tale from the Edge

Universal contributions to EE at zero T



There is a "universal" contribution to EE that is proportional to "a" anomaly coefficient in $\langle T^\mu_\mu \rangle$.

$$\langle T^\mu{}_\mu \rangle = \underbrace{\sum_j c_j I_j}_{\text{Weyl curvature invariants}} - (-1)^{d/2} \frac{4a}{d!\,\mathrm{Vol}(S^d)} \underbrace{E_d}_{\text{Euler density}} + \mathrm{D}_\mu J^\mu$$

Weyl curvature invariants

Euler density

UV cutoff

$$S_E = \alpha \frac{\mathrm{Area}(\partial A)}{\delta^{d-2}} + \ldots + 4a(-1)^{d/2} \ln \frac{\delta}{\ell} + \ldots$$

2×Euler character of sphere.

(Solodukhin 2008; Casini-Huerta-Myers 2011)

First Take

Map the ball to a manifold with a single scale ℓ,
say $H_{d-1} \times S^1$ of the previous story or dS.

For such a manifold

$$\ell \frac{d}{d\ell} W \sim \int T^\mu_\mu \, \mathrm{d}^d x \sim a\chi$$

$$\Rightarrow W \sim a\chi \log(\ell/\epsilon)$$

$$\Rightarrow S_E \sim (\beta \frac{d}{d\beta} - 1) W$$

Works for dS (Casini-Huerta-Myers (2011)), but not for $H_{d-1} \times S^1$.

One problem CHM ran into is that E_d vanishes for $H_{d-1} \times S^1$.

Can we Succeed where CHM failed: 2D Case

We want to deduce an effective action $W[g_{\mu\nu}]$ from the trace anomaly

$$\langle T^\mu_\mu \rangle = \frac{c}{24\pi} R$$

According to Polchinski, in the presence of a boundary, the most general form for the anomalous variation is

$$\delta_\sigma W = -\frac{c}{24\pi}\left[\int_M \mathrm{d}^2 x \sqrt{g} R\, \delta\sigma + 2\int_{\partial M} \mathrm{d}y \sqrt{\gamma} K\, \delta\sigma\right]$$

K here is the trace of the extrinsic curvature.

The Euler characteristic for a 2d manifold with boundary!

The 2d effective action.

We want to integrate $\delta_\sigma W$.
In fact the best I can do is determine a difference:

$$\mathcal{W}[g_{\mu\nu}, e^{-2\tau} g_{\mu\nu}] \equiv W[g_{\mu\nu}] - W[e^{-2\tau} g_{\mu\nu}]$$

The answer is

$$\mathcal{W} = -\frac{c}{24\pi}\left[\int_M \mathrm{d}^2x\sqrt{g}\left(R[g_{\mu\nu}]\tau - (\partial\tau)^2\right) + 2\int_{\partial M} \mathrm{d}y\sqrt{\gamma}\, K[g_{\mu\nu}]\tau\right]$$

Various methods:
1) guess work
2) dimensional regularization
3) integral formula

Dimensional Regularization

Define $\widetilde{W}[g_{\mu\nu}]$ in $n = 2 + \epsilon$ dimensions.

$$\widetilde{W}[g_{\mu\nu}] \equiv -\frac{c}{24\pi(n-2)}\left[\int_M \mathrm{d}^n x\,\sqrt{g}\,R + 2\int_{\partial M} \mathrm{d}^{n-1}y\,\sqrt{\gamma}\,K\right]$$

Then

$$\mathcal{W}[g_{\mu\nu}, e^{-2\tau}g_{\mu\nu}] = \lim_{n\to 2}\left(\widetilde{W}[g_{\mu\nu}] - \widetilde{W}[e^{-2\tau}g_{\mu\nu}]\right)$$

Trick employed by Brown and Cassidy (1977).
Relies on nice transformation properties of *R* under Weyl scaling.

under $g_{\mu\nu} \to e^{-2\tau}g_{\mu\nu}$, $\sqrt{g}R \to e^{(2-n)\tau}\sqrt{g}R + \text{total derivative}$

Entanglement of an Interval

- Consider an interval with endpoints u and v on the z plane along with the following map to the cylinder with coordinate w:

$$e^{2\pi w/\beta} = \frac{z-u}{z-v}$$

$$\Rightarrow \tau = -\frac{1}{2}\ln\left[\frac{\beta}{2\pi}\left(\frac{1}{v-z} - \frac{1}{u-z}\right)\right] + c.c.$$

- The cylinder has a periodic Euclidean time coordinate.

- The reduced density matrix on the interval is mapped to the thermal density matrix on the cylinder with inverse temperature β.

Plan of Attack

$$S_E = \beta\langle H\rangle - W_{\rm cyl}$$

Can be obtained from Schwarzian derivative which in turn can be derived from varying

$$\mathcal{W}[g_{\mu\nu}, e^{-2\tau}g_{\mu\nu}]$$

with respect to the metric.

Think of this term as

$$\mathcal{W}[\delta_{\mu\nu}, e^{-2\tau}\delta_{\mu\nu}] - \widetilde{W}[\delta_{\mu\nu}]$$

Assembling the Pieces

$$\beta\langle H\rangle \sim \frac{c}{6}\ln\frac{|v-u|}{\delta}$$

$$\mathcal{W}[\delta_{\mu\nu}, e^{-2\tau}\delta_{\mu\nu}]|_{\rm bulk} \sim \frac{c}{6}\ln\frac{|v-u|}{\delta}$$

Comes from regulating infinite volume of the cylinder

$$\mathcal{W}[\delta_{\mu\nu}, e^{-2\tau}\delta_{\mu\nu}]|_{\rm boundary} \sim -\frac{c}{3}\ln\frac{|v-u|}{\delta}$$

τ multiplying *K* in the effective action

$$-\widetilde{W}[\delta_{\mu\nu}] \sim \frac{c}{3}\ln\frac{|v-u|}{\delta}$$

Dim reg of extrinsic curvature

$$S_E \sim \frac{c}{3}\ln\frac{|v-u|}{\delta}$$

Holzhey, Larsen, Wilczek (1994)

Remarks about 2d

- Two ways of picking apart the answer.
  - EE comes from bulk terms on the cylinder.
  - EE comes purely from $\widetilde{W}[\delta_{\mu\nu}]$
- One can use $\mathcal{W}[\delta_{\mu\nu}, e^{-2\tau}\delta_{\mu\nu}]$ for three purposes:
  - to derive Schwarzian derivative
  - to compute the EE
  - to compute the Rényi entropies $S_n \sim \frac{c}{6}\left(n + \frac{1}{n}\right) \ln \frac{|v-u|}{\delta}$

Anomaly Action in General

"a" contribution to trace anomaly comes from the Euler character χ

$$\delta_\sigma W = (-1)^{d/2} 2a\chi(M) + \ldots$$

$$= (-1)^{d/2} \frac{4a}{d!\,\mathrm{Vol}(S^d)} \left(\int_M \mathcal{E}_d \delta\sigma - \int_{\partial M} \mathcal{Q}_d \delta\sigma \right) + \ldots$$

Euler density

CS like term

Then for dim reg, define

$$\widetilde{W}[g_{\mu\nu}] = (-1)^{d/2} \frac{4a}{(n-d)d!\,\mathrm{Vol}(S^d)} \left(\int_M \mathcal{E}_{n,d} - \int_{\partial M} \mathcal{Q}_{n,d} \right)$$

and $\quad \mathcal{W}[g_{\mu\nu}, e^{-2\tau} g_{\mu\nu}] = \lim_{n\to d} \left(\widetilde{W}[g_{\mu\nu}] - \widetilde{W}[e^{-2\tau} g_{\mu\nu}] \right)$

4d effective action

Euler density

Einstein tensor

$$\mathcal{W}[g_{\mu\nu}, e^{-2\tau} g_{\mu\nu}] = \frac{a}{(4\pi)^2} \int_M \mathrm{d}^4x \sqrt{g} \left[\tau E_4 + 4E^{\mu\nu}(\partial_\mu \tau)(\partial_\nu \tau) + 8(\mathrm{D}_\mu \partial_\nu \tau)(\partial^\mu \tau)(\partial^\nu \tau) + 2(\partial\tau)^4\right]$$
$$- \frac{a}{(4\pi)^2} \int_{\partial M} \mathrm{d}^3y \sqrt{\gamma} \left[\tau Q_4 + 4(K\gamma^{\alpha\beta} - K^{\alpha\beta})(\partial_\alpha \tau)(\partial_\beta \tau) + \frac{8}{3}\tau_n^3\right]$$

CS like term: only place τ appears w/out a derivative in the bry

normal derivative of τ

Bulk term figured in Komargodski-Schwimmer proof of the "a"-theorem

Boundary term is a new result.

6d effective action (bulk)

$$\mathcal{W}[g_{\mu\nu}, e^{-2\tau} g_{\mu\nu}]_{(\text{Bulk})} =$$

$$\frac{a}{3(4\pi)^3} \int_M \mathrm{d}^6x \sqrt{g} \Big\{ -\tau E_6 + 3E^{(2)}_{\mu\nu} \partial^\mu\tau\partial^\nu\tau + 16 C_{\mu\nu\rho\sigma} (\mathrm{D}^\mu\partial^\rho\tau)(\partial^\nu\tau)(\partial^\sigma\tau)$$
$$+ 16 E_{\mu\nu} \left[(\partial^\mu\tau)(\partial^\rho\tau)(\mathrm{D}_\rho\partial^\nu\tau) - (\partial^\mu\tau)(\partial^\nu\tau)\Box\tau\right] - 6R(\partial\tau)^4$$
$$-24(\partial\tau)^2(\mathrm{D}\partial\tau)^2 + 24(\partial\tau)^2(\Box\tau)^2 - 36(\Box\tau)(\partial\tau)^4 + 24(\partial\tau)^6 \Big\} \ .$$

where

$$E^{(2)\mu\nu} \equiv g^{\mu\nu} E_4 + 8R^\mu_\rho R^{\rho\nu} - 4R^{\mu\nu}R + 8R_{\rho\sigma}R^{\mu\rho\nu\sigma} - 4R^\mu{}_{\rho\sigma\tau}R^{\nu\rho\sigma\tau} \ ,$$
$$C_{\mu\nu\rho\sigma} \equiv R_{\mu\nu\rho\sigma} - g_{\mu\rho}R_{\nu\sigma} + g_{\mu\sigma}R_{\nu\rho}$$

Reproduces a result from Elvang, Freedman, Hung, Kiermaier, Myers, Theisen (2012).

6d effective action (conformally flat)

$$\mathcal{W}[\delta_{\mu\nu}, e^{-2\tau}\delta_{\mu\nu}] = -\frac{a}{16\pi^3}\int_M \mathrm{d}^6x\sqrt{g}\left\{2(\partial\tau)^2(\partial_\mu\partial_\nu\tau)^2 - 2(\partial\tau)^2(\Box\tau)^2 + 3\Box\tau(\partial\tau)^4 - 2(\partial\tau)^6\right\}$$

$$-\frac{a}{3(4\pi)^3}\int_{\partial M}\mathrm{d}^5y\sqrt{\gamma}\Big[-\tau Q_6[\delta_{\mu\nu}] + 48P^\alpha_\beta(\partial_\alpha\tau)(\partial^\beta\tau) + 3Q_4[\delta_{\mu\nu}](\mathring{\mathrm{D}}\tau)^2$$
$$+ 48K^{\alpha\beta}(\mathring{\Box}\tau)(\mathring{\mathrm{D}}_\alpha\partial_\beta\tau) + 24K(\mathring{\mathrm{D}}_\alpha\partial_\beta\tau)^2 - 48K_{\alpha\gamma}(\mathring{\mathrm{D}}^\beta\partial^\alpha\tau)(\mathring{\mathrm{D}}^\gamma\partial_\beta\tau)$$
$$- 24K(\mathring{\Box}\tau)^2 - 32K(\mathring{\mathrm{D}}\tau)^2\mathring{\Box}\tau - 16K(\partial^\alpha\tau)(\partial^\beta\tau)(\mathring{\mathrm{D}}_\alpha\partial_\beta\tau)$$
$$+ 16K_{\alpha\beta}(\partial^\alpha\tau)(\partial^\beta\tau)\mathring{\Box}\tau + 32K_{\alpha\beta}(\mathring{\mathrm{D}}^\alpha\partial^\beta\tau)(\mathring{\mathrm{D}}\tau)^2 + 12K\tau_n^4$$
$$+ 12K(\mathring{\mathrm{D}}\tau)^4 + 24K(\mathring{\mathrm{D}}\tau)^2\tau_n^2 + 48(\mathring{\Box}\tau)(\mathring{\mathrm{D}}\tau)^2(\tau_n) + 16(\mathring{\Box}\tau)(\tau_n^3)$$
$$- 24(\mathring{\mathrm{D}}\tau)^2\tau_n^3 - 36\tau_n(\mathring{\mathrm{D}}\tau)^4 - \frac{36}{5}\tau_n^5\Big]$$

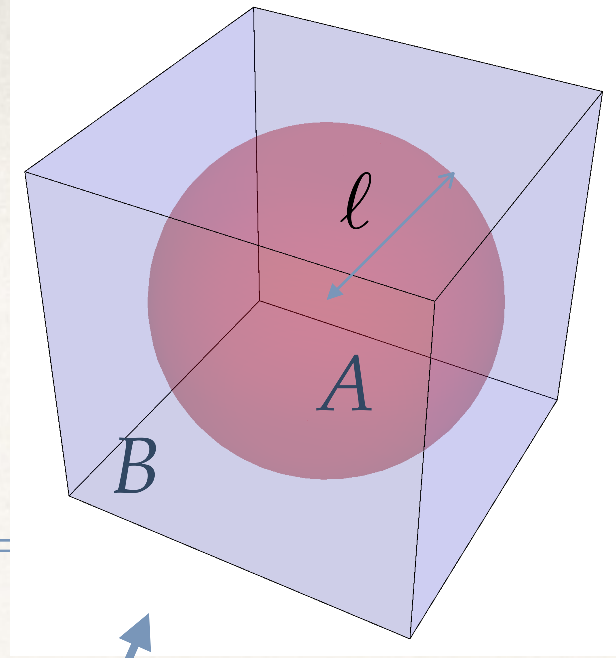
only τ
in the bry

where

$$P^\alpha_\beta \equiv \left(K^2 - \mathrm{tr}(K^2)\right)K^\alpha_\beta - 2KK^{\alpha\gamma}K_{\beta\gamma} + 2K_{\gamma\delta}K^{\alpha\gamma}K^\delta_\beta$$

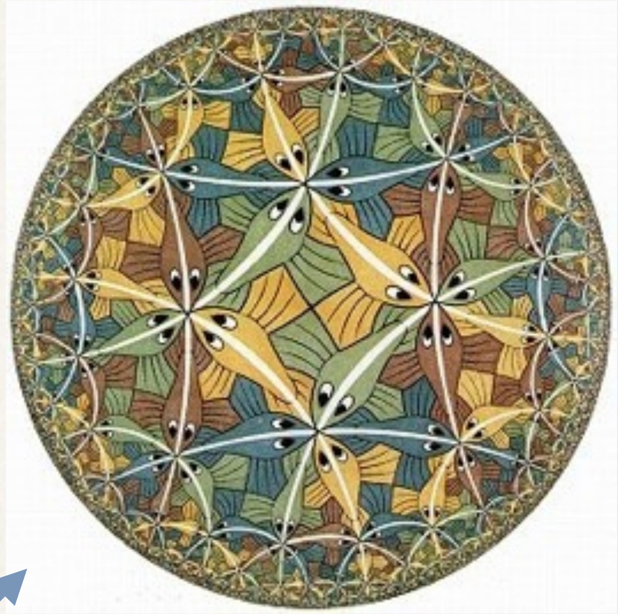
The boundary term is a new result.

EE of the Ball



flat space

$$
\begin{aligned}
\mathrm{d}s^2 &= -\mathrm{d}t^2 + \mathrm{d}r^2 + r^2 \mathrm{d}\Omega^2_{d-2} \ , \\
&= e^{2\sigma}\left[-\mathrm{d}T^2 + \ell^2(\mathrm{d}u^2 + \sinh^2 u \, \mathrm{d}\Omega^2_{d-2})\right]
\end{aligned}
$$



$S^1 \times H^{d-1}$

where

$$e^{-\sigma} = \cosh u + \cosh T/\ell$$

$$S_E = \beta\langle H\rangle + \mathcal{W}[\delta_{\mu\nu}, e^{-2\sigma}\delta_{\mu\nu}] - \widetilde{W}[\delta_{\mu\nu}]$$

Assembling the Pieces: 4d

$$\beta\langle H\rangle \sim -\frac{3}{2} a \ln\frac{\ell}{\delta}$$

$$\mathcal{W}[\delta_{\mu\nu}, e^{-2\sigma}\delta_{\mu\nu}]|_{\text{bulk}} \sim \left(\frac{3}{2} - 4\right) a \ln\frac{\ell}{\delta}$$

$$\mathcal{W}[\delta_{\mu\nu}, e^{-2\sigma}\delta_{\mu\nu}]|_{\text{boundary}} \sim 4a \ln\frac{\ell}{\delta}$$

$$-\widetilde{W}[\delta_{\mu\nu}] \sim -4a \ln\frac{\ell}{\delta}$$

$$S_E \sim -4a \ln\frac{\ell}{\delta}$$

Assembling the Pieces: 6d

$$\beta\langle H\rangle \sim \frac{5}{4}a\ln\frac{\ell}{\delta}$$

$$\mathcal{W}[\delta_{\mu\nu}, e^{-2\sigma}\delta_{\mu\nu}]|_{\text{bulk}} \sim \left(-\frac{5}{4}+4\right)a\ln\frac{\ell}{\delta}$$

$$\mathcal{W}[\delta_{\mu\nu}, e^{-2\sigma}\delta_{\mu\nu}]|_{\text{boundary}} \sim -4a\ln\frac{\ell}{\delta}$$

$$-\widetilde{W}[\delta_{\mu\nu}] \sim 4a\ln\frac{\ell}{\delta}$$

$$S_E \sim 4a\ln\frac{\ell}{\delta}$$

Technical Problem

Why can't I give you the story in general dimension?

Order of limits issue
(fixing the metric before or after
taking the n to d limit)

I have not been able to evaluate $\widetilde{W}[g_{\mu\nu}]$
for $S^1 \times H^{d-1}$ reliably.

Computing $\mathcal{W}[g_{\mu\nu}, e^{-2\sigma} g_{\mu\nu}]$ becomes harder as dimension increases.

Point of View #1

We can make an invariant distinction between $\mathcal{W}[\delta_{\mu\nu}, e^{-2\tau}\delta_{\mu\nu}]|_{\text{boundary}}$ and $\mathcal{W}[\delta_{\mu\nu}, e^{-2\tau}\delta_{\mu\nu}]|_{\text{bulk}}$.

Then $\beta\langle H\rangle + \mathcal{W}[\delta_{\mu\nu}, e^{-2\tau}\delta_{\mu\nu}]|_{\text{bulk}}$ computes the EE

while $\mathcal{W}[\delta_{\mu\nu}, e^{-2\tau}\delta_{\mu\nu}]|_{\text{boundary}} - \widetilde{W}[\delta_{\mu\nu}]$ comes purely from flat space and vanishes

Somewhat nicer — clean separation:
Maps a problem in flat space to a problem in hyperbolic space.

Point of View #2

$-\widetilde{W}[\delta_{\mu\nu}]$ computes the EE and all the other terms cancel.

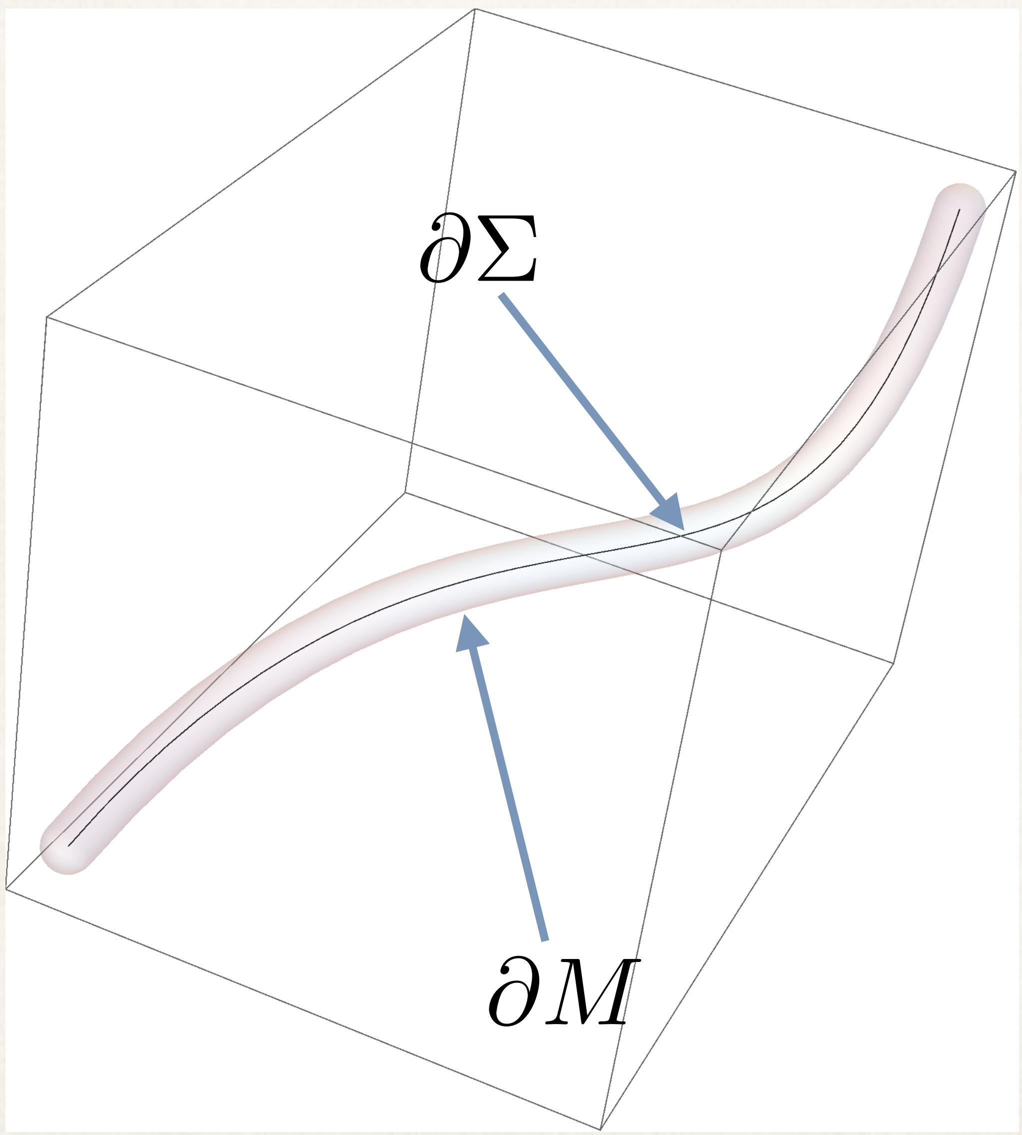
Consistent with Solodukhin's result in 4d
that the "a" contribution to the EE is proportional
to χ of the entangling surface.

$$S_E \sim \ldots + (-1)^{d/2} 2a\, \chi(\partial A)\, \ln \frac{\delta}{\ell} + \ldots$$

Somewhat discouraging:
We tried to map the problem to hyperbolic space
but somehow never got away from flat space.

A Failed Idea

Try to use $\widetilde{W}[\delta_{\mu\nu}]$ to calculate other central charges in the EE.



Deduce EE associated to Σ from boundary part of $\widetilde{W}[\delta_{\mu\nu}]$ evaluated on ∂M .

Only works for the ``*a*'' central charge.

Final Remarks

- For certain types of entanglement entropy, mapping to hyperbolic space is a useful tool.
- Hyperbolic space has a boundary, and the boundary has important effects.
    - Thermal corrections.
    - Log contribution to the zero T EE.

Thanks to my collaborators

- Michael Spillane (grad student)
- Kuo-Wei Huang (grad student)
- Tatsuma Nishioka (U. Tokyo)
- John Cardy (Oxford)
- Jun Nian (grad student)
- Ricardo Vaz (grad student)
- Kristan Jensen (SF State)

(a chronological order)