Supersymmetric Rényi entropy

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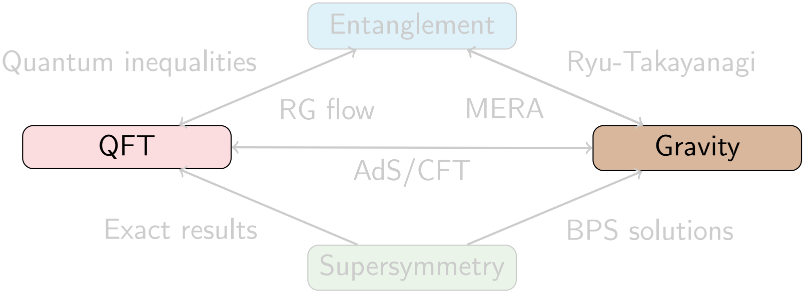
Introduction



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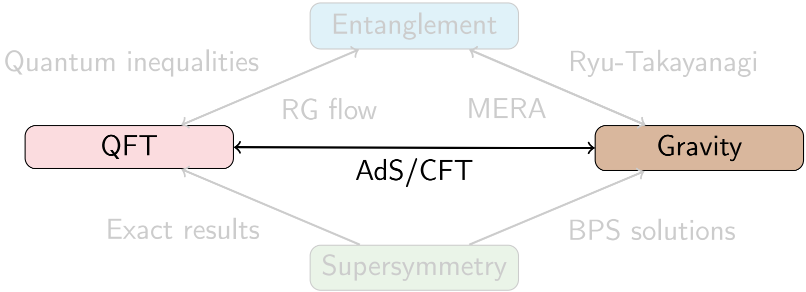
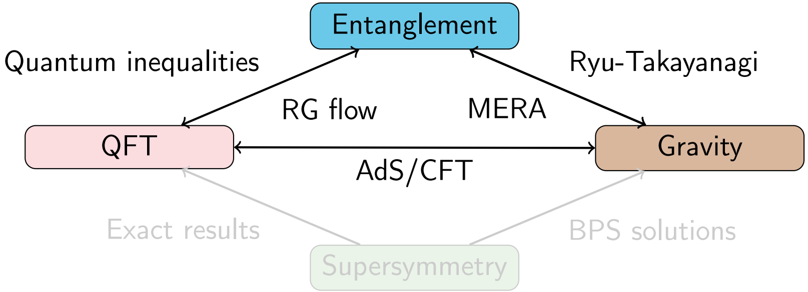
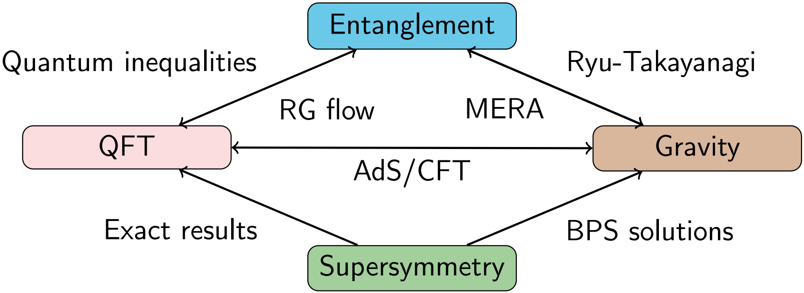
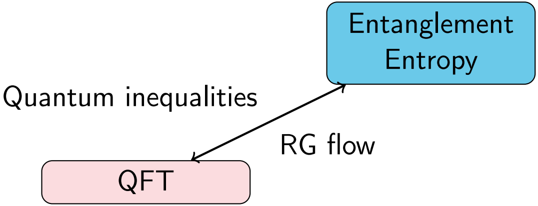



Entanglement
Quantum inequalities
Ryu-Takayanagi
RG flow
MERA
QFT
Gravity
AdS/CFT
Exact results
BPS solutions
Supersymmetry

Introduction



RG flow in QFT



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- Construct a monotonic function $c(\text{Energy})$ of the energy scale
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  - The a-theorem in (3+1) dimensions [Cardy 88, Komargodski-Schwimmer 11]
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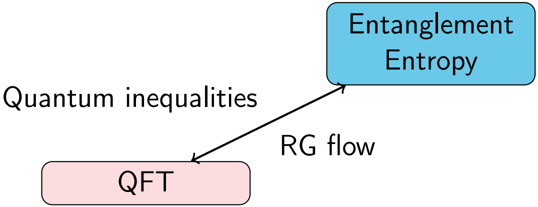

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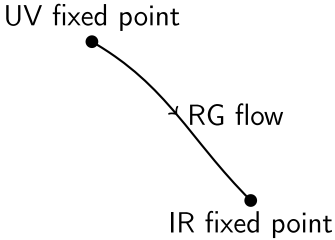

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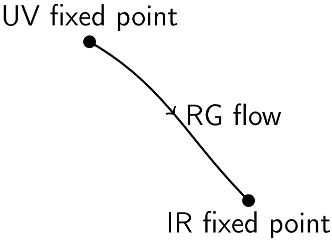

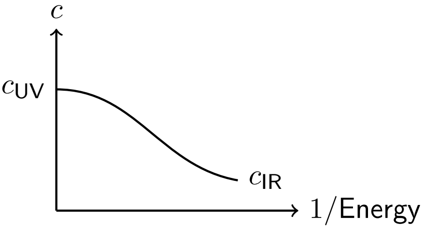

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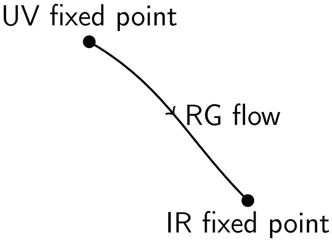

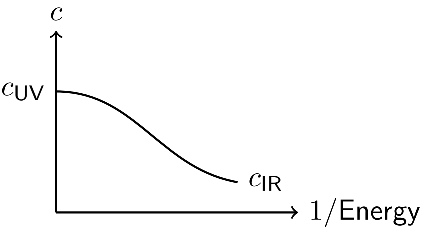

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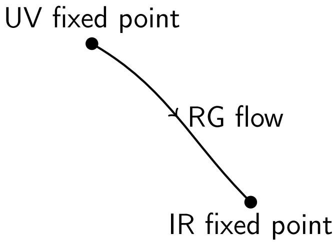

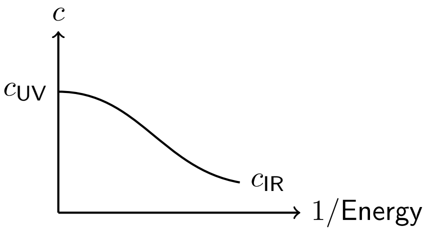

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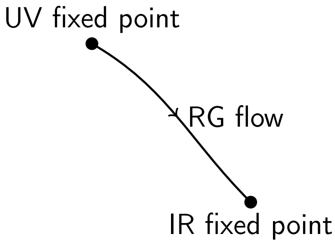

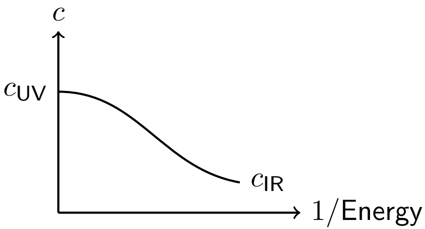

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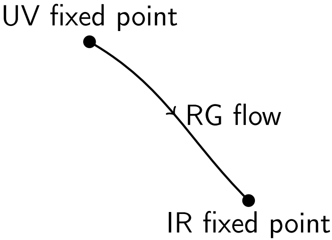

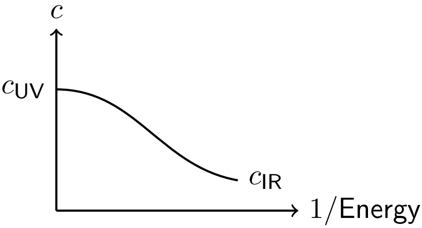

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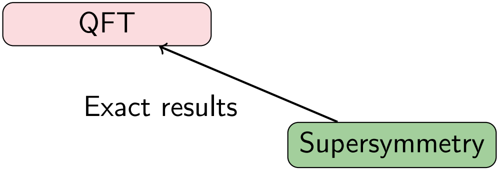
Supersymmetry and exact results in QFT



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  - Supersymmetric indices [Witten 82, Romelsberger 05, Kinney-Maldacena-Minwalla-Raju 05, $\cdots$]
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- Supersymmetry reduces an infinite-dimensional measure of the path integral to a finite one (supersymmetric localization)

- Partition functions on a compact manifold $\mathcal{M}$ typically become a matrix model:

$$Z[\mathcal{M}] \sim \int dM\, f(M)$$

($M : N \times N$ matrix for $U(N)$ gauge theories)

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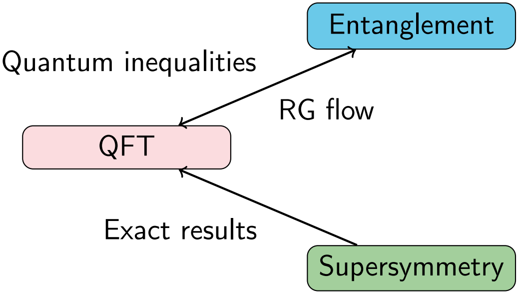

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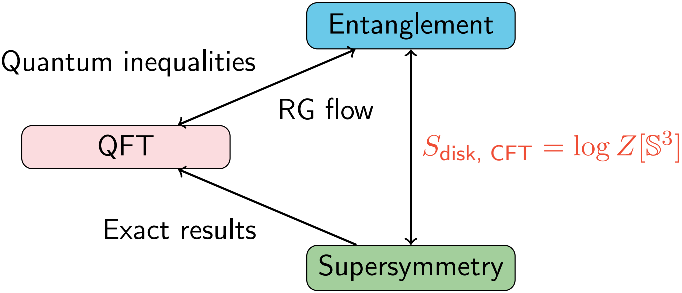
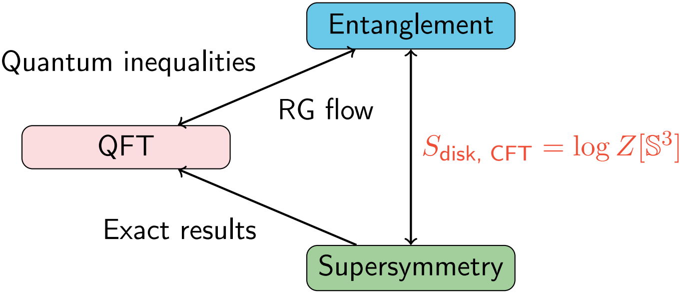




Main question

Can we calculate Rényi entropies of a disk exactly for SUSY gauge theories?

Outline

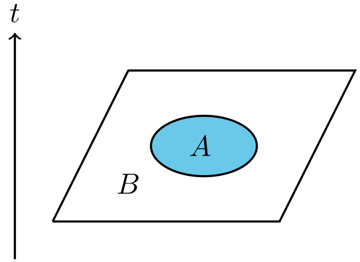
1 Entanglement entropy in QFT

2 Supersymmetric Rényi entropy

3 Holographic supersymmetric Rényi entropy

Definition of entanglement entropy

- Divide a system to A and $B = \bar{A}$: $\mathcal{H}_{tot} = \mathcal{H}_A \otimes \mathcal{H}_B$



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$$S_A = -\mathrm{tr}_A \rho_A \log \rho_A$$

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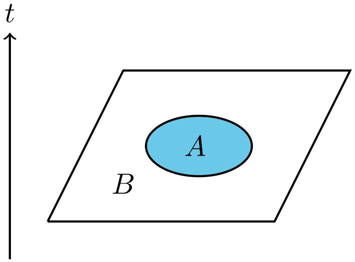

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Path integral representation

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$$[\rho_A]_{ab} = \langle \phi_a^A | \rho_A | \phi_b^A \rangle$$

$|\phi_{a,b}^A\rangle$: states in $\mathcal{H}_A$

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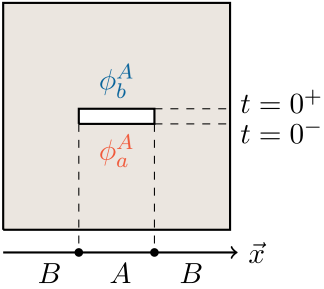
$$[\rho_A]_{ab} = \langle \phi_a^A | \rho_A | \phi_b^A \rangle$$

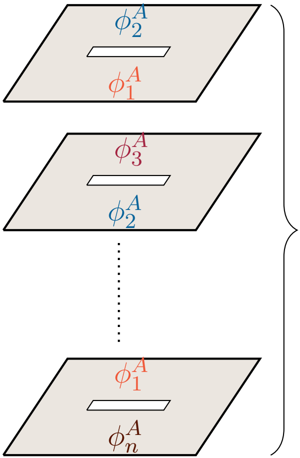
$$= \frac{1}{Z_1}$$



$|\phi_{a,b}^A\rangle$: states in $\mathcal{H}_A$ (= boundary conditions on region A)

The n-fold cover $\mathcal{M}_n$

$$\mathrm{tr}_A \rho_A^n = \frac{1}{(Z_1)^n}$$



n copies $\equiv Z_n$

Replica trick

- As a one-parameter extension

n^{th} Rényi entropy

$$S_n = \frac{1}{1-n} \log \mathrm{tr}_A \, \rho_A^n$$

- It indeed reduces to entanglement entropy

$$S_A = \lim_{n\to 1} S_n = -(\partial_n - 1) \log Z_n \Big|_{n=1}$$

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Z_n: the partition function on $\mathcal{M}_n$ $(= Z[\mathcal{M}_n])$

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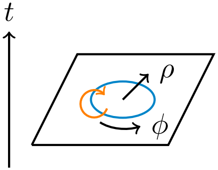

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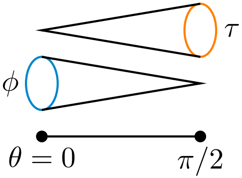

$\mathbb{R}^3$

$$ds^2 = dt^2 + d\rho^2 + \rho^2 d\phi^2$$

$\mathbb{S}^3$

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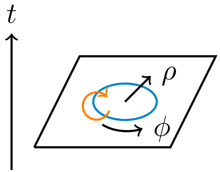

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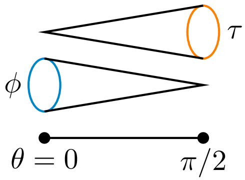

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For CFT$_3$ [Casini-Huerta-Myers 11]

$$Z_n = Z[\mathbb{S}^3_n]$$

$\mathbb{S}^3_n$: n-fold cover of $\mathbb{S}^3$ $\quad (\tau \sim \tau + 2\pi n)$

Rényi entropy for CFT

The Rényi entropy of a disc for CFT

$$S_n = \frac{1}{1-n} \log \frac{Z[\mathbb{S}^3_n]}{(Z[\mathbb{S}^3])^n}$$

- For free fields, $Z[\mathbb{S}^3_n]$: one-loop determinant [Klebanov-Pufu-Sachdev-Safdi 11]
- For SUSY gauge theories, $Z[\mathbb{S}^3]$ $(n=1)$ can be obtained by localization [Kapstin-Willet-Yaakov 09, Jafferis, Hama-Hosomichi-Lee 10]

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SUSY on singular space

- SUSY is broken on the singular space $\mathbb{S}^3_n$

$$ds^2 = d\theta^2 + n^2 \sin^2\theta d\tau^2 + \cos^2\theta d\phi^2 \ , \quad \tau \sim \tau + 2\pi$$

- To recover SUSY, turn on the $U(1)_R$ symmetry b.g. gauge field in $\mathcal{N} = 2$ theories

$$A^{(R)} = \frac{n-1}{2} d\tau$$

- The definition of RE should be modified if SUSY is preserved! (c.f. charged Rényi entropies [Belin-Hung-Maloney-Matsuura-Myers-Sierens 13])

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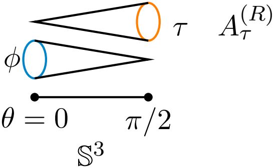

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SUSY on singular space

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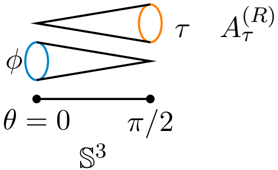

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SUSY partition function on $\mathbb{S}_n^3$

$$Z^{\mathrm{susy}}[\mathbb{S}_n^3] = Z^{\mathrm{susy}}[\mathbb{S}_b^3]$$

$\mathbb{S}_b^3$: squashed three-sphere with squashing parameter $b = \sqrt{n}$

$Z^{\mathrm{susy}}[\mathbb{S}_b^3]$ obtained by [Hama-Hosomichi-Lee 11, Imamura-Yokoyama 11]

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Large-N limit

$$S_n^{\text{susy}} = \frac{3n+1}{4n} S_1$$

Twist operator representation

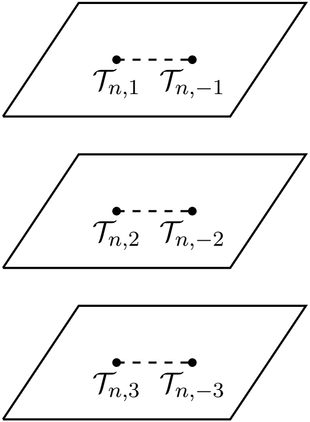
- A free field partition function Z_n decomposes into the n-copies

$$Z_n = \prod_{k=1}^{n} Z_{n,k}$$

- Using twist operators $\mathcal{T}_{n,k}$

$$Z_{n,k} = \langle \mathcal{T}_{n,k}(\partial A) \rangle$$

where $\mathcal{T}_{n,k}\phi \sim e^{2\pi i k/n}\phi$



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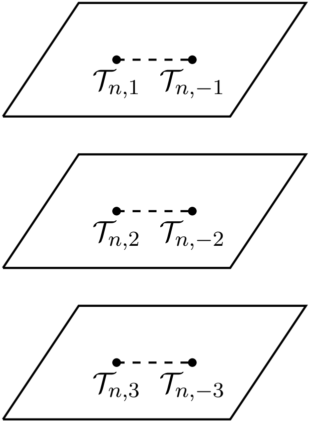

Twist operator in supersymmetric Rényi entropy

- In d dimensions, twist operators for Rényi entropy are dim ∂A $= d - 2$ dimensional objects

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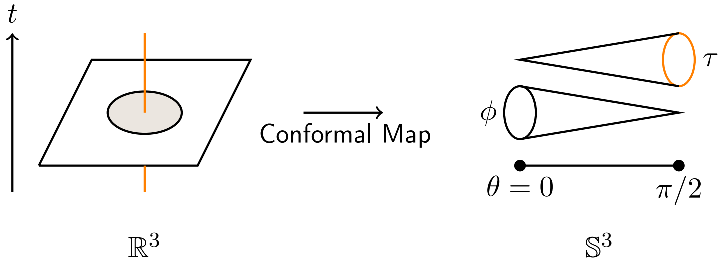
Adding Wilson loop

- The quark insertion is equivalent to the Wilson loop [Lewkowycz-Maldacena 13]



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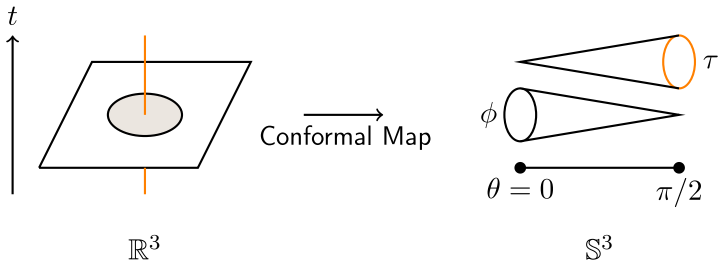

The variation of RE by the loop

$$S_{W,n} = \frac{1}{1-n}\left(n \log |\langle W \rangle_1| - \log |\langle W \rangle_n|\right)$$

Wilson loop in the large-N [TN 14]

- $1/6$-BPS Wilson loop in ABJM on $\mathbb{S}^3_n$

$$\log\langle W\rangle_n = \frac{\pi(n+1)}{2}\sqrt{2\lambda} + O(\log N)$$

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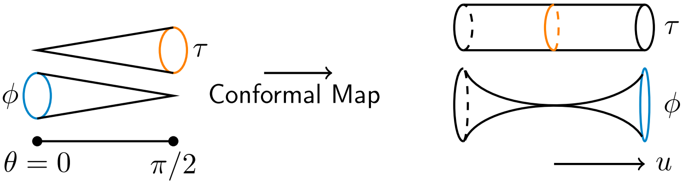
- The variation does not depend on the Rényi parameter n!

The SRE of the Wilson loop

$$S^{\text{susy}}_{W,n} = \frac{\pi}{2}\sqrt{2\lambda}$$

Outline



$\mathbb{S}^3$

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$\mathbb{S}^1 \times \mathbb{H}^2$

$ds^2 = d\tau^2 + du^2 + \sinh^2 u d\phi^2$

The n-fold cover has $\tau \sim \tau + 2\pi n$

The dual gravity solution

- The $1/2$-BPS $U(1)$ charged topological AdS$_4$ black hole in $\mathcal{N}=2$ gauged SUGRA

$$ds^2 = f(r)d\tau^2 + \frac{dr^2}{f(r)} + r^2 ds^2_{\mathbb{H}^2}$$

$$A = Q\left(\frac{1}{r} - \frac{1}{r_H}\right) d\tau$$

The horizon is at $r = r_H$ where $f(r_H) = 0$

- The temperature:

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- Using the holographic free energy,

$$F_n = -\log Z_n = \frac{(n+1)^2}{4n}\frac{\pi}{2G_4}$$

Holographic supersymmetric Rényi entropy

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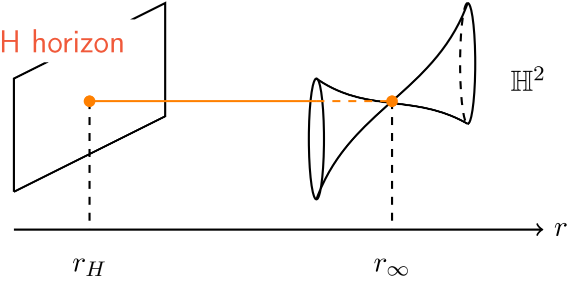
Agrees with the large-N result! [Huang-Rey-Zhou 14, TN 14]

- The fundamental string dual to the Wilson loop



- This configuration reproduces the large-N result! [TN 14]

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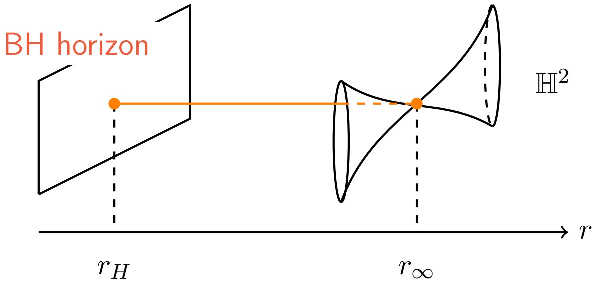

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Summary

- SUSY has to be broken for the Rényi entropies with $n \neq 1$
- A new observable, supersymmetric Rényi entropy, is introduced
- The holographic duals of the supersymmetric Rényi entropies are given by the BPS charged topological AdS black holes

Future direction

- Can SRE be defined in other dimensions?
  - $2d\ \mathcal{N}=(2,2)$ [Giveon-Kutasov 15], [Mori 15]
  - $4d\ \mathcal{N}=2$ [Huang-Zhou 14], [Crossley-Dyer-Sonner 14]
  - $5d\ \mathcal{N}=1$ [Alday-Richmond-Sparks 14], [Hama-TN-Ugajin 14]
  - $6d\ \mathcal{N}=(2,0)$ [Nian-Zhou 15], [Zhou 15]

- Boundary SRE or squashed SRE?

- Entangling surface as a surface operator? [TN-Yaakov, WIP]