1.

オッペンハイマー

Refferences
- Horodecki $^{\otimes 3}$
- Plenio and Virmani Entanglement measures
- Preskil's notes (ch 4, ch 10)

- Entanglement as a resource theory (RT)
  ↳ measures of entanglement

- is it entangled?

- ~~to~~ local time reversal

- monogamy

Resource

Alice Bob

- teleportation
- private message
* - win non-local games (Bell's thm)
- super dense coding

Is entanglement a kind stuff. (like charge)
← conserveする量

Quantify entanglement

pure states : Yes

mixed states : ?

Resource Theory

$\mathcal{C}$: class of operation ⟶ S: free state

このoperationでつくり出る全てのstate

$\rho \notin S$

こっちのopはCに入って入ってない

$\rho_1 \rightarrow \rho_{21} \rightarrow \rho_3$

more variabe

$\sigma^{\otimes k_1}$ $\sigma^{\otimes k_2}$ $\sigma^{\otimes k_3}$

~~σ: maximally~~

Resource States

eg
σ

$\mathcal{C}$: LOCC Λ_{AB}

LO
1) $U_A \otimes \mathbb{1}_B$, $\mathbb{1}_A \otimes U_B$ ← local unitary op
2) M_A, M_B ← ~~これの定義なに~~? measurement
3) $\rho_{AB} \rightarrow \rho_{AB} \otimes \rho_A$, $\rho_{AB} \rightarrow \rho_{AB} \otimes \rho_{B'}$ ← 自由度を加える
4) $\rho_{ABA'D'} \rightarrow \rho_{AB}$

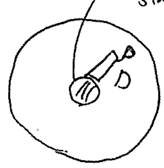
CC 5) classical communication
← measurementに依った communication

$$E(\rho_{AB}) \geq E(\Lambda_{AB}(\rho_{AB}))$$

why? ~~this~~ a definition (convenc se

Entanglement is amonotone under LOCC

Seperable state



purestate : $|\psi_A\rangle \otimes |\psi'_B\rangle$ ← Pure entangled state can't be written like this

mixedstate : $\sum p_i \rho_A^i \otimes \rho_B^i$

← tensorの和で作れる

~~(sepa~~ (sep)erable state ...

$|\psi^\pm\rangle = \frac{1}{\sqrt{2}}[|01\rangle \pm |10\rangle]$

ρ_{AB} $\quad \sum_i p_i |i\rangle\langle i| \otimes \rho_A^i \xrightarrow[\text{measure}]{} \sum_i p_i \otimes \rho_A^i \otimes \rho_B^i$

↑ p_i measureに依して ρ_B^i をつくる。

mixed entangle states ≠ sep

$$\rho_{AB} = p|\psi^+\rangle_{AB}\langle\psi^+|_{AB} + (1-p)|\psi^-\rangle_{AB}\langle\psi^-|_{AB}$$

entangled unless $p=\frac{1}{2}$

$$E^{distance}(\rho) = \inf_{\sigma\in S} D^{istance}(\rho\|\sigma)$$

Relavite entropy of ~~relation~~ entanglement ↙ free stateに対して free energyみたい。

$$E^{rel}(\rho) := \inf_{\sigma\in sep} D(\rho\|\sigma)$$

$D(\rho\|\sigma) = tr\rho\ln\rho - tr\rho\ln\sigma$

↖ Ground state は常に sepstate?

$$\rho_{AB} = \frac{1}{2}|0\rangle_A\langle 0| \otimes |1\rangle_B\langle 1| + \frac{1}{2}|1\rangle_A\langle 1|_A \otimes |0\rangle_B\langle 0| \qquad p=\frac{1}{2}$$

$p=1$ $\quad \rho_{AB} = |\psi^+\rangle_{AB}\langle\psi^+|$

$p=0$ $\quad \rho_{AB} = |\psi^-\rangle_{AB}\langle\psi^-|$

$E^{distance}$ is a monotone if $D^{istace}(\Lambda(\rho)\|\Lambda(\sigma)) \leq D^{istance}(\rho\|\sigma)$

σ がsep state ならば Λ(σ)もsep state

$\Lambda(\sigma) \in S$

$$E^{distance}(\Lambda(\rho)) \leq E^{distance}(\rho)$$

$D(\Lambda(\rho) : \Lambda(\sigma)) \le D(\rho|\sigma)$

dis (AP) $\le D(\Lambda(\sigma) : \Lambda(\sigma)) \le \inf_{\sigma \in S} D(\rho|\sigma)$

Pure state E.T $|\psi_{AB}\rangle$

$S(\rho_A) = S(\rho_B) = -\mathrm{Tr}\,\rho_A \ln_2 \rho_A$ Von Neumann entropy

Concentration $D = \frac{k}{n}$

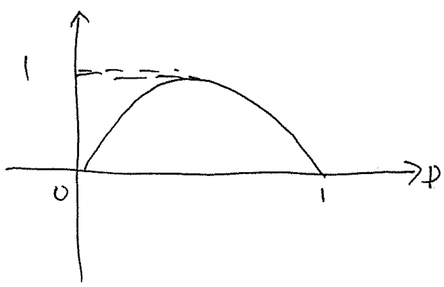
~~$|\varphi\rangle^{\otimes n}_{AB}$~~ $|\varphi\rangle = \sqrt{p}|00\rangle + \sqrt{1-p}|11\rangle$

$|\varphi\rangle^{\otimes n}_{AB} \longrightarrow |\psi^+\rangle^{\otimes k}_{AB}$ ~~n is greater than k?~~

これの Reverse

$\frac{k}{n} = S(\rho_A)$

qubitので定義より $0 < \frac{k}{n} \le 1$



Dilution $|\psi^+\rangle^{\otimes k} \longrightarrow |\varphi\rangle^{\otimes n}$ $E_f = \frac{k}{n}$

$|\psi^+\rangle^{\otimes k} \xleftarrow{\text{concentration}} |\varphi\rangle^{\otimes n}$

$|\psi^+\rangle^{\otimes k} \xrightarrow{\text{dilution}} |\varphi\rangle^{\otimes n}$

Entanglement monotone

$D \le E \le E_f$

$E(\psi^+) = 1$ extensive $E(\varphi^{\otimes n}) = nE(\varphi)$

claim $E(\varphi) = S(\rho)$

3 Mixed ~~state~~ entungement theory

ではこれ.

$|\psi^+\rangle \rightarrow \rho \not\Rightarrow D=0$ ↙

oppen 2-1

C : LOCC

$|\psi^-\rangle = \frac{1}{\sqrt{2}}[|0\rangle|1\rangle - |1\rangle|0\rangle]$

← Bell state

S : $\rho_{AB} = \sum p_i \rho_A^i \otimes \rho_B^i$

pure $S(\rho_A)$

ppt ~~#~~ criteria

mixed important entanglement measures

monogamy of E

~~$E^\infty(\rho) = \lim_{n\to\infty} E(\rho^{\otimes n})$~~

$E(|\psi^-\rangle^{\otimes k}) = k$

\# of EPR pairs

$D \leq E^\infty(\rho) = \lim_{n\to} \frac{E(\rho^{\otimes n})}{n} \leq E^c \sim |\psi\rangle^{\otimes nE_c} \xrightarrow{LOCC} \rho_{AB}^{\otimes n}$

$E(\rho^{\otimes n}) \geq E(\Lambda_{LOCC}(\rho^{\otimes n})) = E(|\psi^-\rangle_{AB}^{\otimes nD}) = nD$

⇓

$\lim_{n\to\infty} \frac{E(\rho^{\otimes n})}{n} \geq D$

~~任意~~ $|\varphi\rangle^{\otimes n} \longleftrightarrow |\psi^-\rangle^{\otimes k}$

$D = 0$, $E^c \gg 1$ bound entanglement

ppt criterion

$\Lambda(\rho) = \rho'$ $\quad Tr\rho' = Tr\rho = 1$ ~~(TP)~~ (TP)

physical condition ⇐ CPTP

部分系のeigen valuesをpositive

$\Lambda(\rho) = $ positive, CP ⟺ $\underline{\mathbb{1}_A \otimes \Lambda_B(\rho_{AB}) = \text{positive}}$

2.

$$\Lambda(p_1\rho_1+p_2\rho_2)=p_1\Lambda(\rho_1)+p_2\Lambda(\rho_2)$$

P: $\Lambda(\rho)$ = positive

CP: $\mathbb{1}_A\otimes\Lambda_B(\rho)$ = position

transpose $\quad \rho \leftarrow$ positive not CP map

partial transpose ← Alice か ~~p bod~~ Bob の index のをいれかえる.

⇒ Eigen value は正のまま

~~$|4^-\rangle\langle 4^-|$~~

$$\rho^{\Gamma_A}_{a_ia_jb_ib_j}=\rho_{a_ja_ib_ib_j}$$

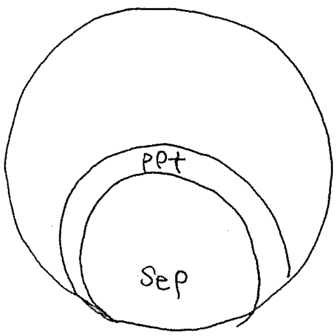
$$|\Psi^-\rangle\langle\Psi^-| = \begin{pmatrix} 0 & & & \\ & 1 & -1 & \\ & -1 & 1 & \\ & & & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 0 & & & -1 \\ & 1 & 0 & \\ & 0 & 1 & \\ -1 & & & 0 \end{pmatrix}$$

(rows/columns labelled 00, 01, 10, 11)

$$\mathbb{1}_A\otimes\Lambda_B\,\rho_{sep}=\sum_i p_i\,\rho_A^i\otimes\Lambda(\rho_B^i)=\sum_i p_i\,\rho_A^i\otimes\sigma_B^i$$

($\Lambda \geq 0$, $\sigma \geq 0$)

$\rho_{AB}^{\Gamma_A}$ if not positive then is entangled.



PPT states have D=0

↑

positive partitial traspose

$$N(\rho_{AB})=\log\|\rho_{AB}^{\Gamma_A}\|_1,\qquad \|\ \|_1=\sum|\lambda_i|$$

↑ ↑

Negative eigen value が無いと そもそも $\sum|\lambda_i|=1$ なので

$N(\rho_{AB})=1$. なので $N(\rho_{AB})=0$.

3.

$D : \rho_{AB}^{\otimes n} \xrightarrow{LOCC} |\psi\rangle^{\otimes}$

~~Dsup~~

~~Dim~~ $D = \lim_{n\to} \sup_{LOCC} k$

$E^c \; |\psi\rangle^{\otimes n} \longrightarrow \rho^{\otimes k}$

$\rho_{AB} = \sum p_i \, |\psi^i_{AB}\rangle \langle \psi^i_{AB}|$

$\{ \mathcal{E} = \{ p_i ; |\psi^i\rangle \} \}$

$E^f = \inf_{\mathcal{E}} \sum p_i S(\rho_A^i)$

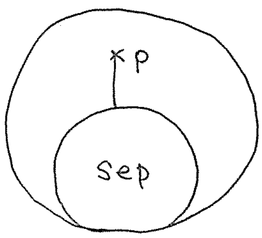
↖定義的に?

$E^c = \lim_{n\to\infty} \frac{E^f(\rho^{\otimes n})}{n}$

$\rho_{AB} = \frac{1}{2} |\psi^-\rangle_{AB} \langle \psi^-| + \frac{1}{2} |\psi^+\rangle_{AB} {}_{AB}\langle \psi^+|$

$= \frac{1}{2} |01\rangle\langle 01| + \frac{1}{2} |10\rangle\langle 01|$ ← E^fをみたす decomposition

$E^R(\rho_{AB}) = \inf_{\sigma \in sep} D(\rho_{AB} \| \sigma_{AB})$



↓

$D(\rho_{AB} \| \rho_A \otimes \rho_B) = I(A,B)$ E^Rで $P_{i=j} = 1$ $i \neq j$ $P_i = 0$ のとき.

$= S(A) + S(B) - S(AB)$

$\mathrm{tr}_E \, \rho_{ABE} = \rho_{AB}$

4

$$E^{sq}(\rho_{AB}) = \lim \inf_{E} \frac{1}{2} I(A:B|E)$$

$$\mathrm{tr}_E \, \rho_{ABE} = \rho_{AB}$$

$$I(A:B|E) = S(A|E) + S(B|E) - S(AB|E)$$

$$S(x|y) = S(xy) - S(y)$$

$$E^{sq}(\rho_{A_1B_1} \otimes \rho_{A_2B_2}) = E^{sq}(\rho_{A_1E_1}) + E^{sq}(\rho_{A_2B_2})$$

Reduction critelia

if $S(B) \gtrsim S(AB)$ then ρ_{AB} entangled

$S(A) > S(AB)$

thermal Entropy $S_{th}(A) \lesssim S_{th}(AB)$ なのに.

~~だけど~~

~~$S_A($ $S(A) <$~~ .

$$\overrightarrow{D} = \sup_{\Lambda_{LOCC}} S(\rho_B) - S(\Lambda_A \otimes \mathbb{1}_B \rho_{AB})$$

$S(A|B)$ can be negative

$|\psi_{AB}\rangle \rightarrow$ ~~$|\psi_{AB}\rangle \otimes_B |\psi_E\rangle$~~

$|\psi_{AB}\rangle \otimes |\psi_E\rangle$ を加えても、~~物量~~

情報量も physical な量は変わらない.

ある[illegible]全てがわかる.

$\rho_{AB} = \mathrm{tr}_E \, |\psi_{ABE}\rangle\langle\psi_{ABE}|$

5.

Bell basis

$$|\Phi^{\pm}\rangle = \frac{1}{\sqrt{2}}\left[\,|00\rangle \pm |11\rangle\,\right]$$

$$\rho_{AB} = \sum P_1 |\Psi^-\rangle\langle\Psi^-| + P_2 |\Psi^+\rangle\langle\Psi^+| + P_3 |\Phi^+\rangle\langle\Phi^+| + P_4 |\Phi^-\rangle\langle\Phi^-|$$

~~E(E%~~

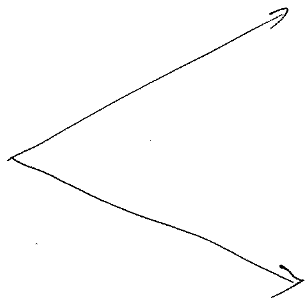
$$E(A:B_1) + E(A:B_2) \leq E(A:B_1 B_2)$$

$$E_f(A:B) + \vec{I}(A:B_2) \leq S(A)$$

↑ Quantum E

↑ classical correlation

↑ ~~some bound~~. ~~Entan~~ Entropy



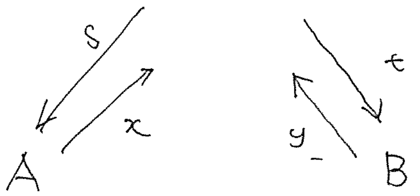
$$\vec{I} := \sup_{M(B_2)} I(A, B_2)$$

Oppen 3-1

Bell's thm : ~~teleportation~~ CHSH game

John Watrous

teleportation



勝つ条件

$x \oplus y = s \cdot t$
↑
mod2

QM7.12
x_0、x_1 とはっきり[illegible]
↓ えなく[illegible]

s	t	st	$x_s \oplus y_t$
0	0	0	$x_0 \oplus y_0$
0	1	0	$x_0 \oplus y_1$
1	0	0	$x_1 \oplus y_0$
1	1	1	$x_1 \oplus y_1$

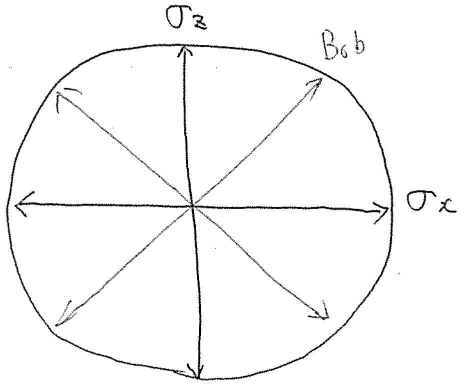
1 0

うまく x_0、y_0、x_1、y_1 をとると $p_{min} = \frac{3}{4}$ にとれる. →

$$p_{min} \leq \frac{3}{4}$$

$$p^0 = \frac{1}{2} + \frac{1}{2\sqrt{2}} \simeq 88\%$$

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}\left[|00\rangle + |11\rangle\right]$$



$\bar{A}_0$ $\bar{A}_1$

$\bar{B}_0$ $\bar{B}_1$

$A_0, A_1 \in \pm 1$

$B_0, B_1 \in \pm 1$

$$\beta = |A_0(B_0 + B_1) + A_1(B_0 - B_1)| = 2$$

~~in Q β~~

in QM $\beta = 2\sqrt{2}$

$\langle \beta \rangle \leq 2$

$$\langle A_0 B_0 \rangle + \langle A_0 B_1 \rangle + \langle A_1 B_0 \rangle - \langle A_1 B_1 \rangle$$

$A_0 = \sigma_x$

$A_1 = \sigma_z$

$B_0 = \frac{\sigma_x + \sigma_z}{\sqrt{2}}$

$B_1 = \frac{\sigma_x - \sigma_z}{\sqrt{2}}$

Quantum telepotation LOCC

$|\varphi_c\rangle = \alpha|0\rangle + \beta|1\rangle$

A $|\Phi^+\rangle_{AB}$ B

$$|\Phi^\pm\rangle_{AB} = \frac{1}{\sqrt{2}}\left[|00\rangle \pm |11\rangle\right]_{AB}$$

$$|\psi^\pm\rangle_{AB} = \frac{1}{\sqrt{2}}\left[|01\rangle \pm |10\rangle\right]$$

ACでAliceは measure

↙おくりたい情報

$$|\varphi\rangle_c \otimes |\Phi\rangle_{AB} = \left[\alpha|0\rangle + \beta|1\rangle\right]_c \otimes \left[|00\rangle + |11\rangle\right]_{AB}\frac{1}{\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}}\left[|00\rangle_{AC}\alpha|0\rangle_B + |01\rangle_{AC}\alpha|1\rangle_B + |10\rangle\beta|1\rangle_B + |11\rangle_{AC}\beta|1\rangle_B\right.$$

$$= \left[|\Phi^+\rangle + |\Phi^-\rangle\right]_{AC}\alpha|0\rangle_B + \left[|\psi^+\rangle + |\psi^-\rangle\right]\alpha|1\rangle \cdots$$

$$\frac{1}{2}\left[|\Phi_{AC}^+\rangle \otimes \left[\alpha|0\rangle + \beta|1\rangle\right]_B + |\Phi^-\rangle_{AC} \otimes \left[\alpha|0\rangle - \beta|1\rangle\right]\right.$$

$$+ |\psi^+\rangle_{AC}\left[\alpha|1\rangle + \beta|0\rangle\right]_B$$

$$\left. + |\psi^-\rangle_{BC} \otimes \left[\alpha|1\rangle - \beta|0\rangle\right]_B\right]$$

— ~~えられる状態~~

えられる情報