Consistency between Luscher's method and HAL method for two-baryon systems on the lattice

Takumi Doi

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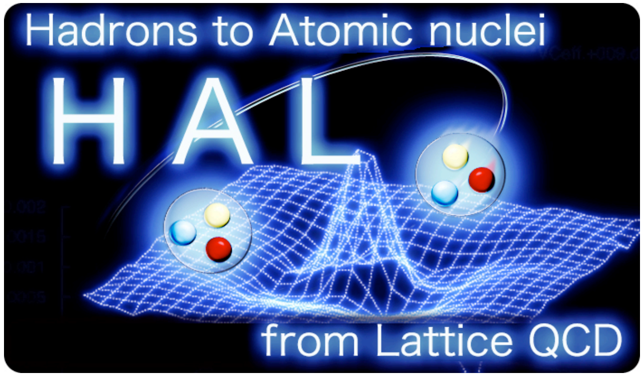
for HAL QCD Collaboration



Y. Akahoshi, S. Aoki, T. Miyamato, K. Sasaki (YITP)
T. Aoyama (KEK)
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Takumi Iritani, T. Sugiura (RIKEN)
F. Etminan (Univ. of Birjand)
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T. Inoue (Nihon Univ.)

Interactions on the Lattice

- Direct method (*a la* Luscher's method)
  - Phase shift & B.E. from temporal correlation in finite V

M.Luscher, CMP104(1986)177
CMP105(1986)153
NPB354(1991)531

- HAL QCD method
  - "Potential" from spacial (& temporal) correlation
  - Phase shift & B.E. by solving Schrodinger eq in infinite V

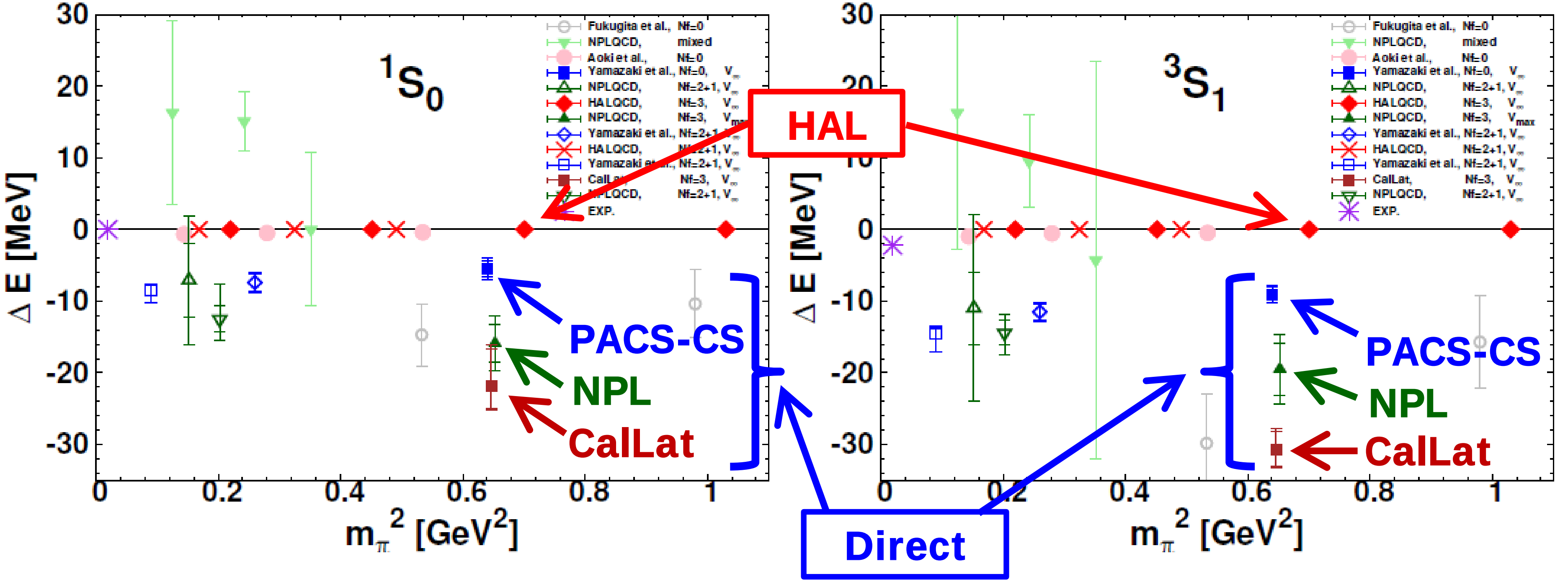
Ishii-Aoki-Hatsuda, PRL99(2007)022001, PTP123(2010)89
HAL QCD Coll., PTEP2012(2012)01A105

$$\psi(r) \simeq A \frac{\sin(kr - l\pi/2 + \delta(k))}{kr}$$

Direct method vs HAL method
(NN @ heavy quark masses)

HAL method (HAL) : **unbound**
Direct method (PACS-CS (Yamazaki et al.) / NPL / CalLat): **bound**

“di-neutron” “deuteron”



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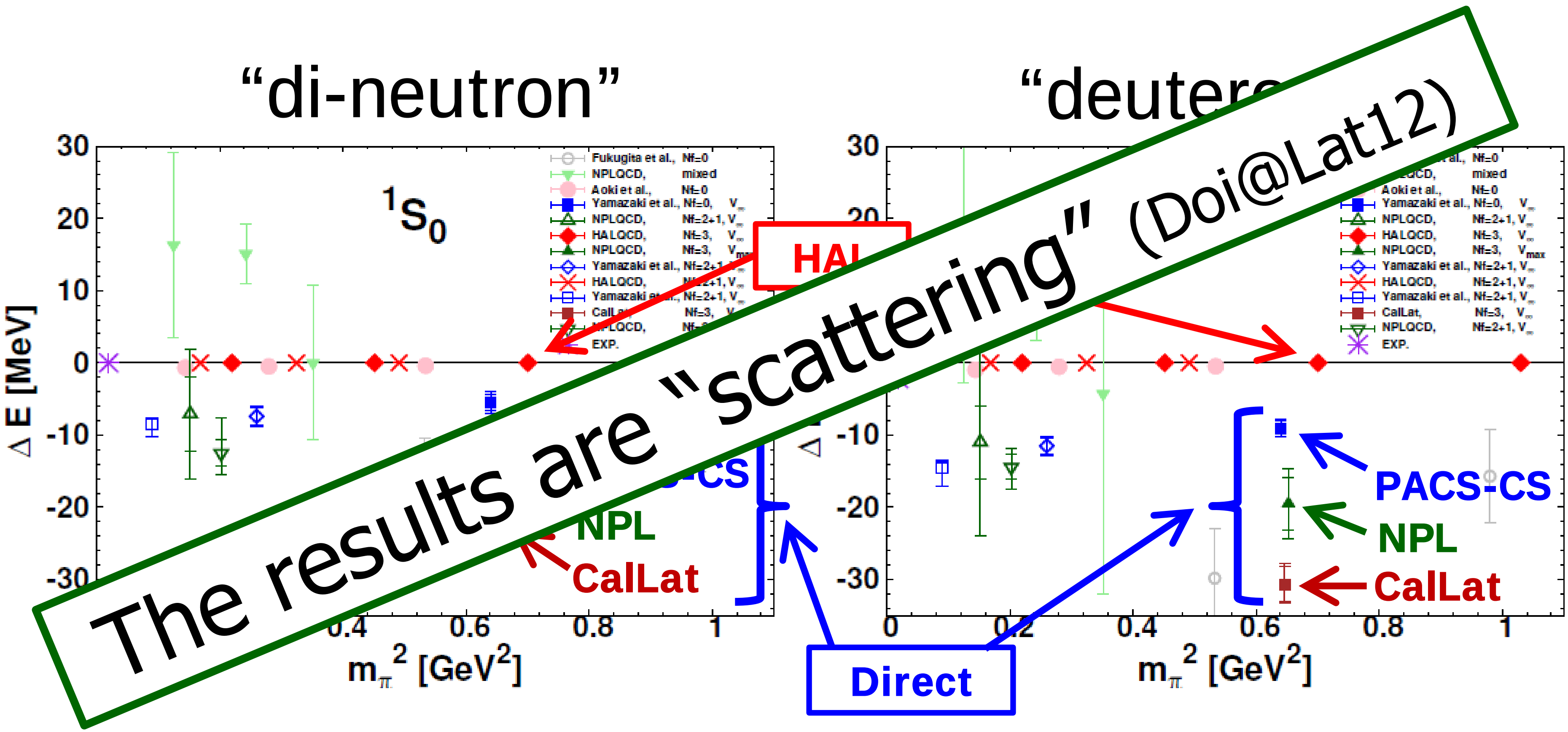
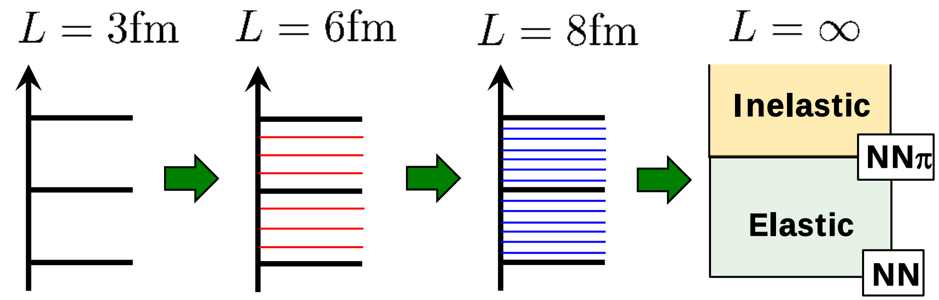


The Challenge in multi-baryons on the lattice

Existence of elastic scatt. states

➔ (almost) No Excitation Energy

➔ LQCD method based on G.S. saturation impossible



Signal/Noise issue

"Sign Problem"

$$S/N \sim \exp[-\mathbf{A} \times (\mathbf{m_N} - \mathbf{3/2m_\pi}) \times \mathbf{t}]$$

Parisi, Lepage(1989)

L=8fm @ physical point $(E_1 - E_0) \simeq 25\text{MeV} \Longrightarrow t > 10\text{fm}$

$$S/N \sim 10^{-32}$$

Direct method: plateau fitting at t ~ 1fm ➔ excited states give "noises"

HAL method: t-dep formalism to extract "signal" from all elastic states

Examine the reliability of the Direct method (w/ plateau fitting)

LQCD data: $\Xi\Xi(^1S_0)$ @ $m\pi$=0.51GeV

wall source & smeared source

Same confs in Yamazaki et al.(‘12)

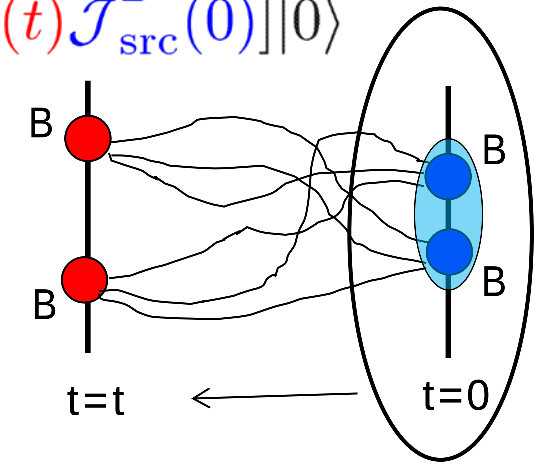
T. Iritani et al. (HAL Coll.) JHEP1610(2016)101

T. Iritani et al. (HAL Coll.) PRD96(2017)034521

Operator dependence in the direct method

$$R(t) = C_{2B}(t)/(C_B(t))^2, \quad C_B(t) = \langle 0|\mathrm{T}[\mathcal{J}^B_{\mathrm{sink}}(t)\overline{\mathcal{J}}^B_{\mathrm{src}}(0)]|0\rangle$$

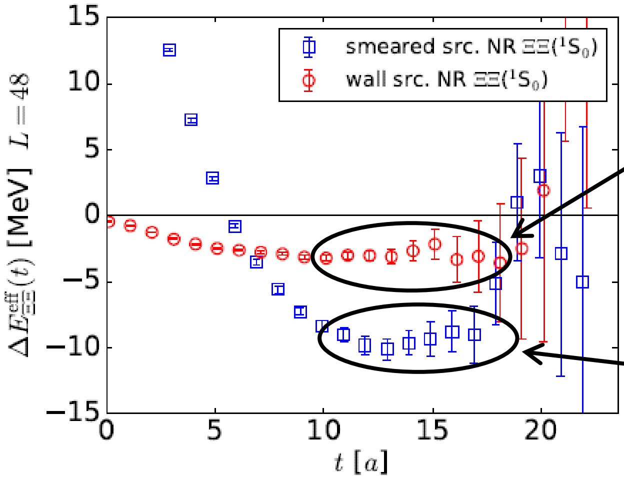
$$C_{2B}(t) = \langle 0|\mathrm{T}[\mathcal{J}^{2B}_{\mathrm{sink}}(t)\overline{\mathcal{J}}^{2B}_{\mathrm{src}}(0)]|0\rangle$$



- The results should be indep of sink/src op

- Reality: The results are dependent on src op !

T. Iritani et al. JHEP10(2016)101



plateau from **wall quark src**

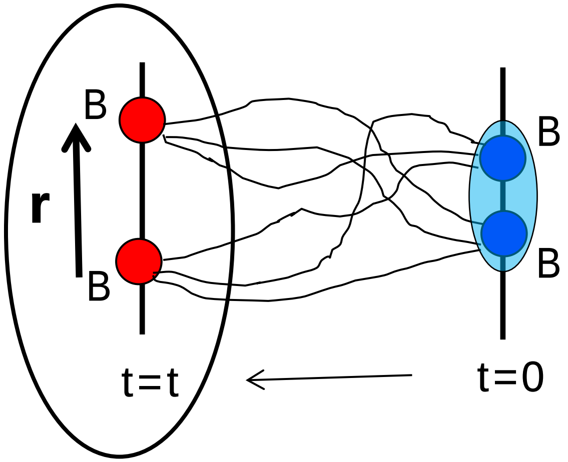
inconsistent

plateau from **smeared quark src**

- "Objection" from direct method groups
  - wall src has large inelastic state contaminations in single-baryon

Operator dependence in the direct method

$$C_{2B}(t) = \langle 0|\mathrm{T}[\mathcal{J}^{2B}_{\mathrm{sink}}(t)\overline{\mathcal{J}}^{2B}_{\mathrm{src}}(0)]|0\rangle$$

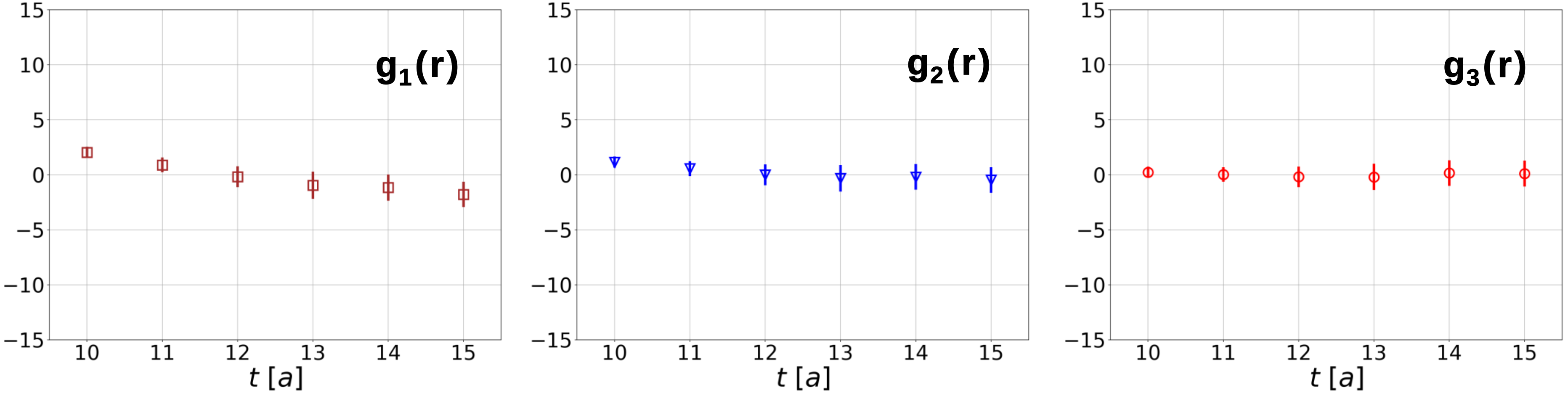


Study sink op dep
w/ smeared src tuned in single-baryon

$$\mathcal{J}^{2B}_{\mathrm{sink}} = \sum_{\vec{r}} g(r) \sum_{\vec{x}} B(\vec{r}+\vec{x})B(\vec{x})$$

Usual direct method: g(r)=1 only

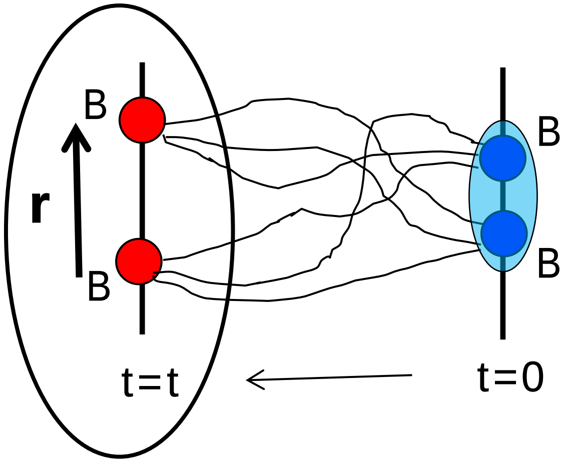
Effective Energy shift ΔE



All plateaux “look” reliable

Operator dependence in the direct method

$$C_{2B}(t) = \langle 0|\mathrm{T}[\mathcal{J}^{2B}_{\mathrm{sink}}(t)\overline{\mathcal{J}}^{2B}_{\mathrm{src}}(0)]|0\rangle$$



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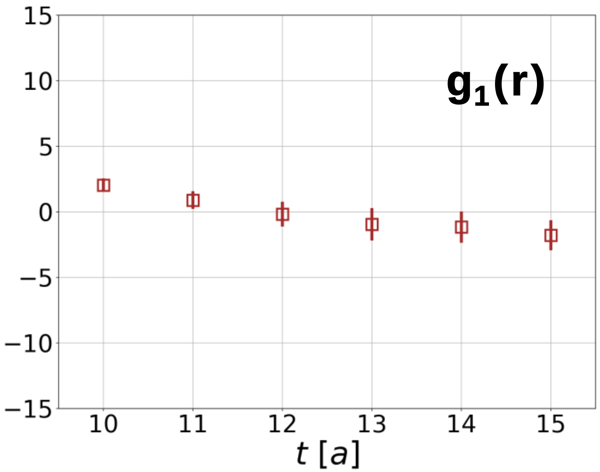
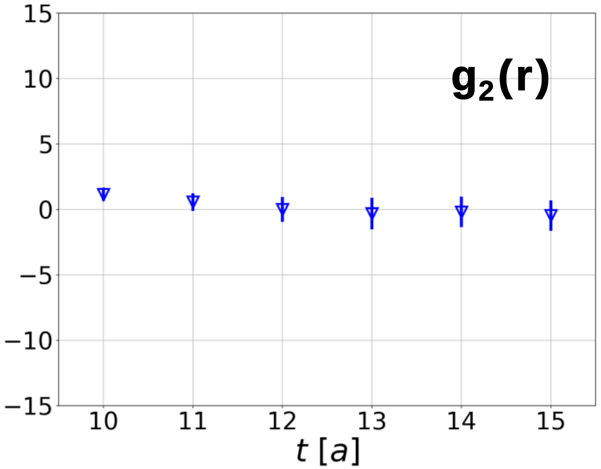
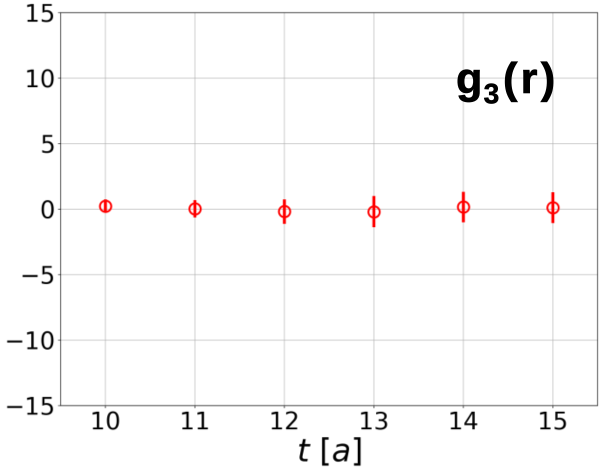




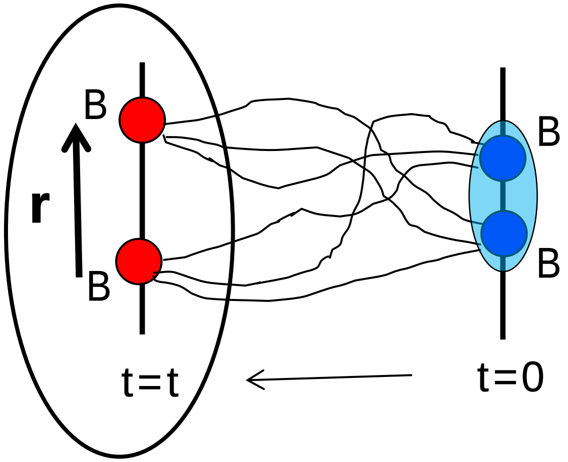


All plateaux "look" reliable

In reality, I shift data vertically "by hand"

Operator dependence in the direct method

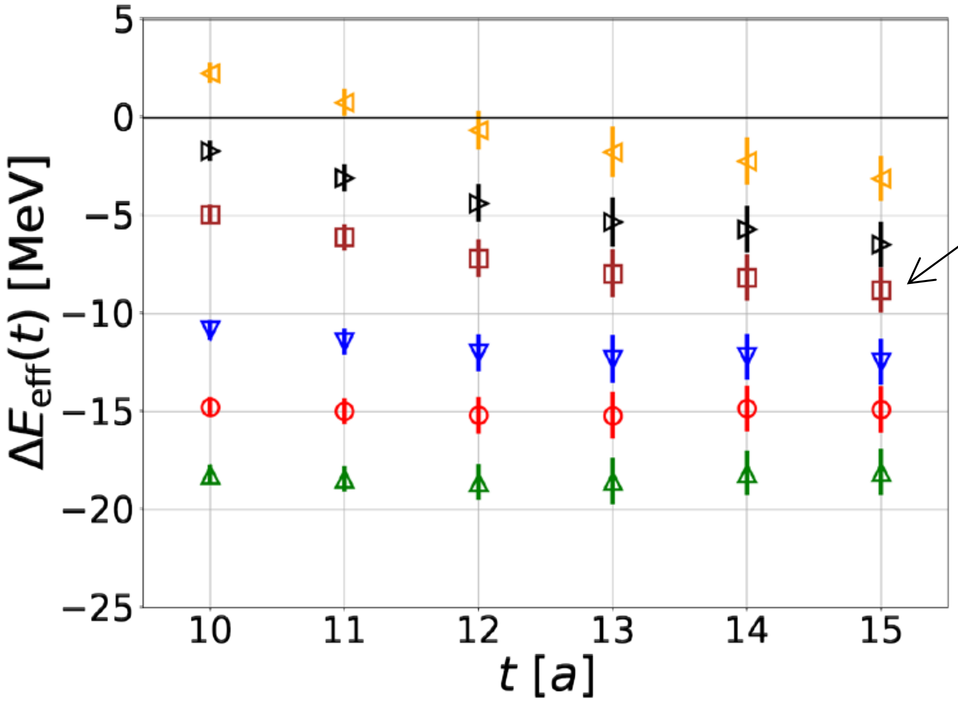
$$C_{2B}(t) = \langle 0 | \mathrm{T}[\mathcal{J}^{2B}_{\mathrm{sink}}(t) \overline{\mathcal{J}}^{2B}_{\mathrm{src}}(0)] | 0 \rangle$$



Study sink op dep
w/ smeared src tuned in single-baryon

$$\mathcal{J}^{2B}_{\mathrm{sink}} = \sum_{\vec{r}} g(r) \sum_{\vec{x}} B(\vec{r}+\vec{x}) B(\vec{x})$$

Usual direct method: g(r)=1 only



No predictive power in direct method w/ naïve plateau fitting !

T. Iritani et al. JHEP10(2016)101 + update

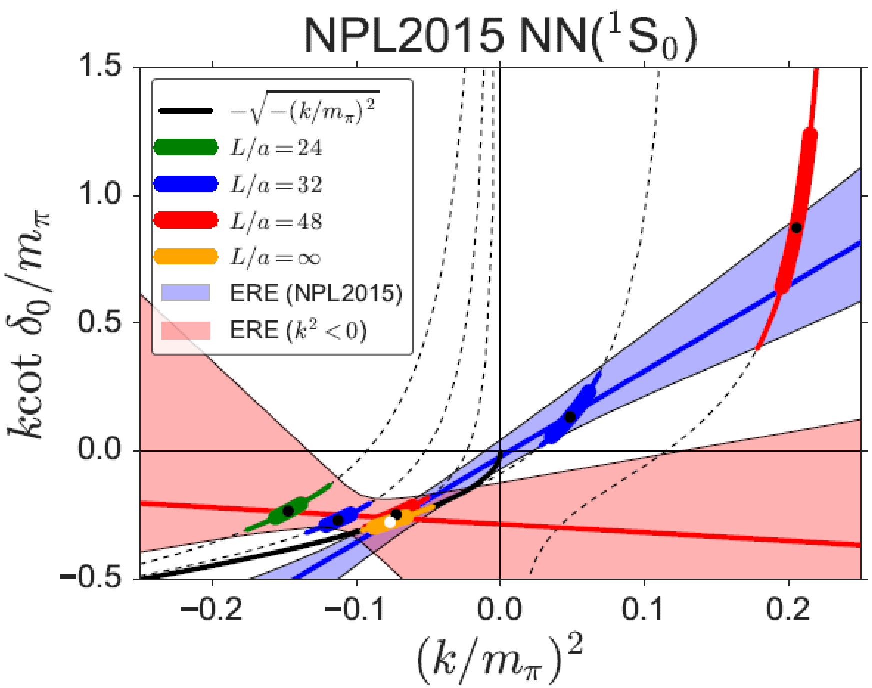
“Normality Check” for results from direct method

T. Iritani et al. (HAL Coll.) PRD96(2017)034521

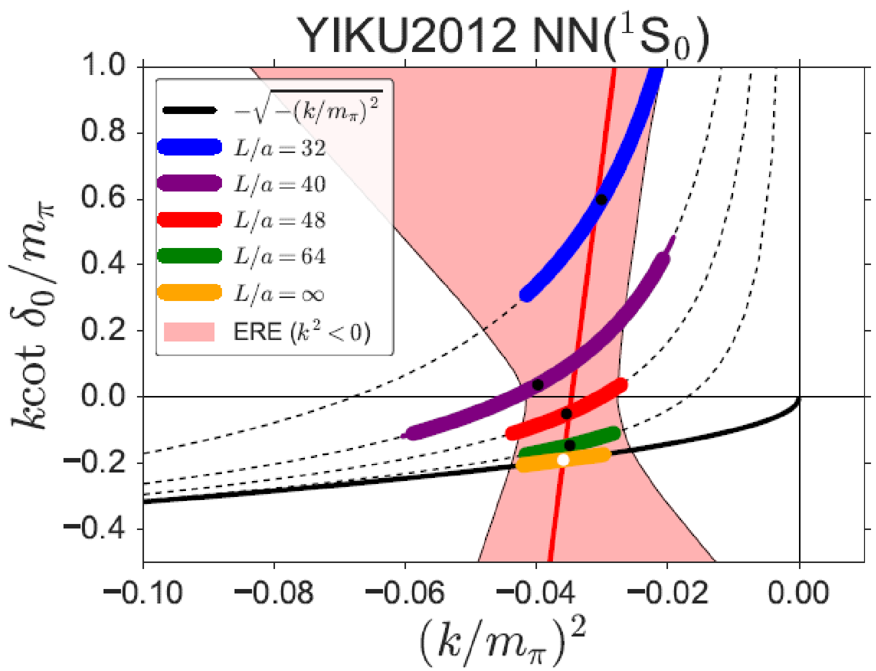
$k \cot \delta(k)$ vs k^2 plot

ERE: $k \cot \delta(k) = \frac{1}{\mathbf{a}} + \frac{1}{2} \mathbf{r}\, k^2 + \cdots$

Data from NPL Coll. (‘15)



Data from Yamazaki et al (‘12)



Inconsistent ERE

Unphysical pole residue

FAILED

Singular behaviors

$r \simeq \pm\infty$

Examine the reliability of the HAL QCD method

Convergence of the derivative expansion of potential

Contaminations from inelastic states

T. Iritani et al. (HAL) PRD99(2019)014514

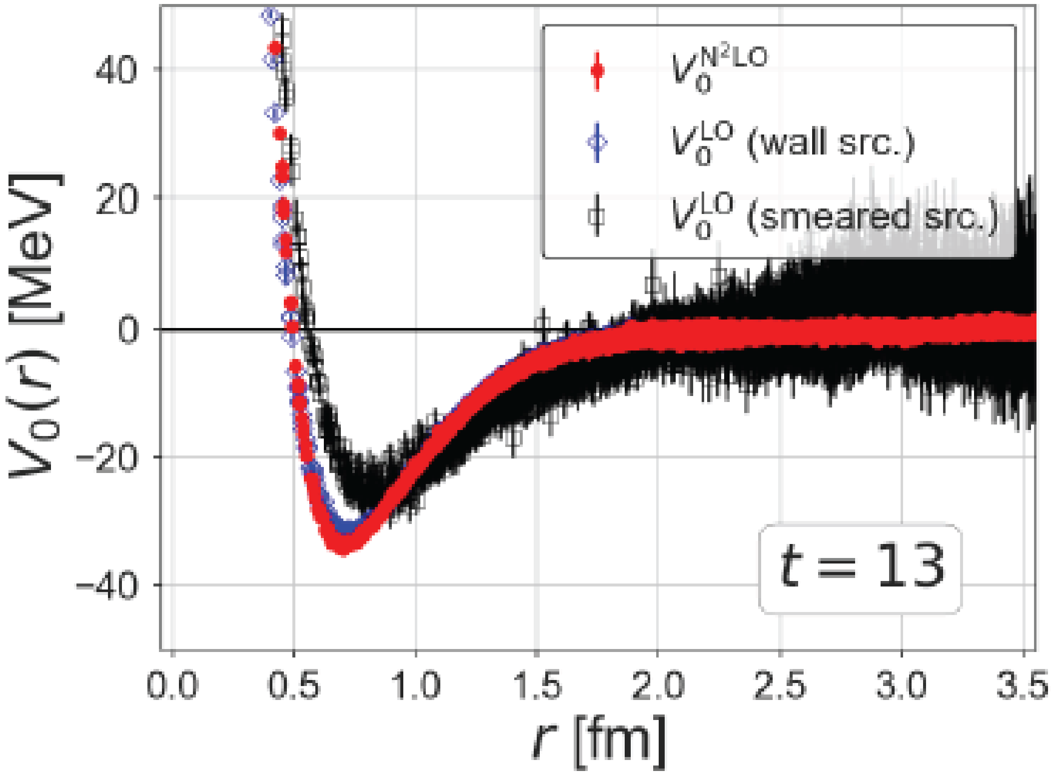
Higher Order Approximation (N^2LO) (2)

$$U(r,r') \simeq \left[V_0^{\mathrm{N^2LO}}(r) + V_2^{\mathrm{N^2LO}}(r)\nabla^2\right]\delta(r-r')$$

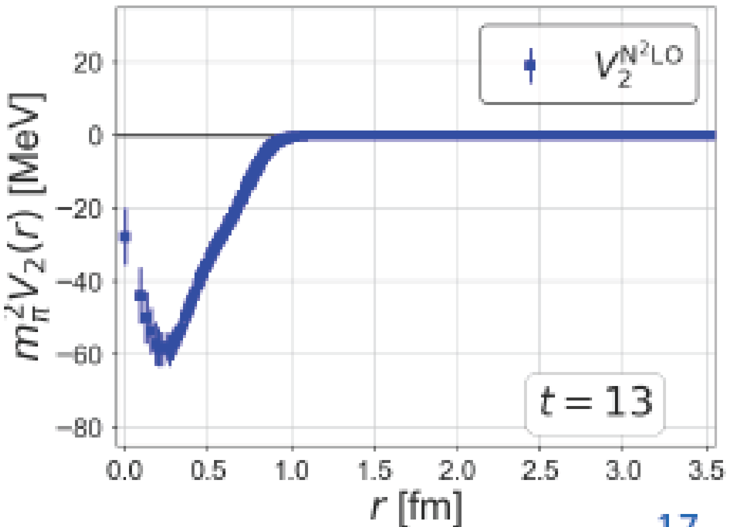
wall src. –> small $\mathbf{V_2\nabla^2}$ **correction**
smeared src. –> large $\mathbf{V_2\nabla^2}$ **correction**

$$V_2(r)\nabla^2 R^{\mathrm{wall/smear}}(r)$$

dep. on shape of **R**

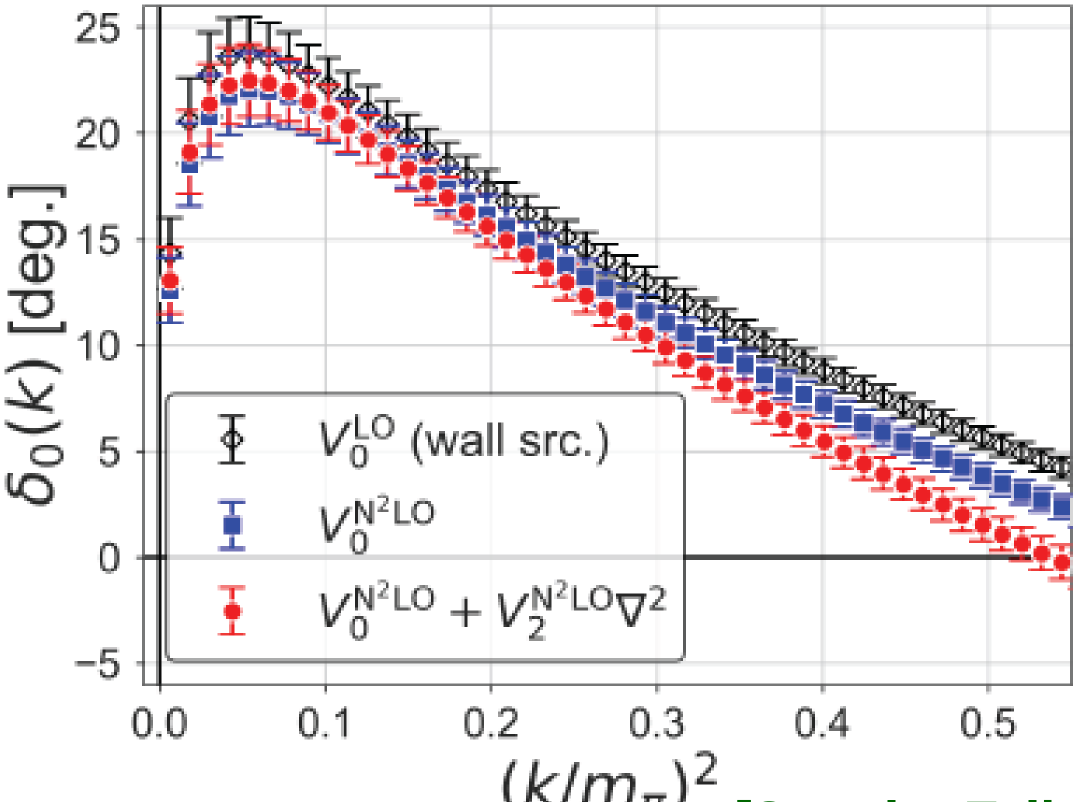


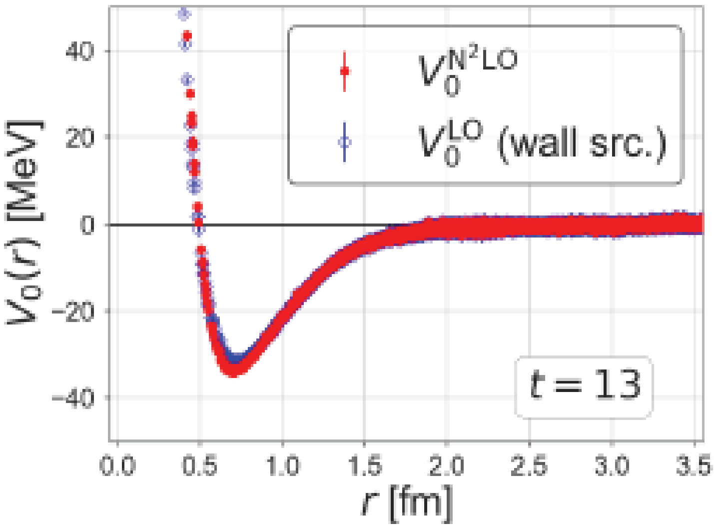
V₂-pot.

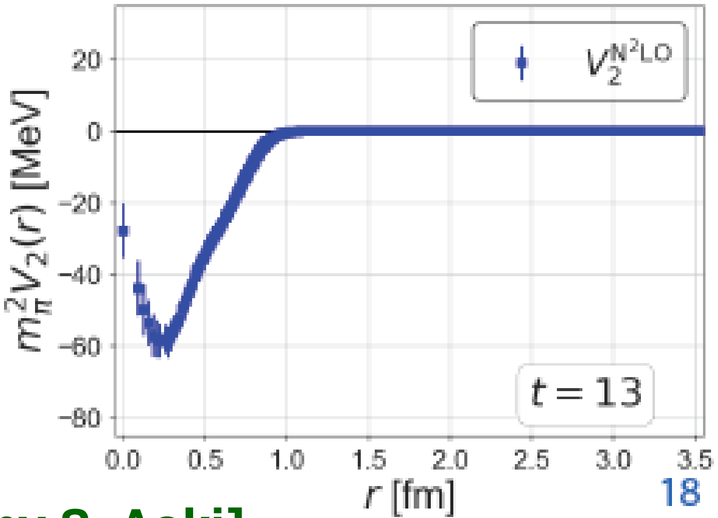


Phase Shift and Uncertainties in Velocity Expansion

- **Wall src. LO approx.** (standard of HAL QCD studies) **works well at low energy.**
- **V_2 correction** at high energy

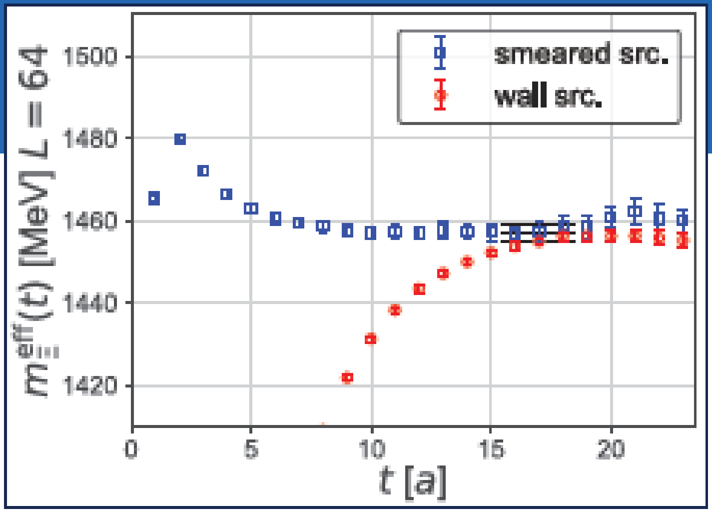






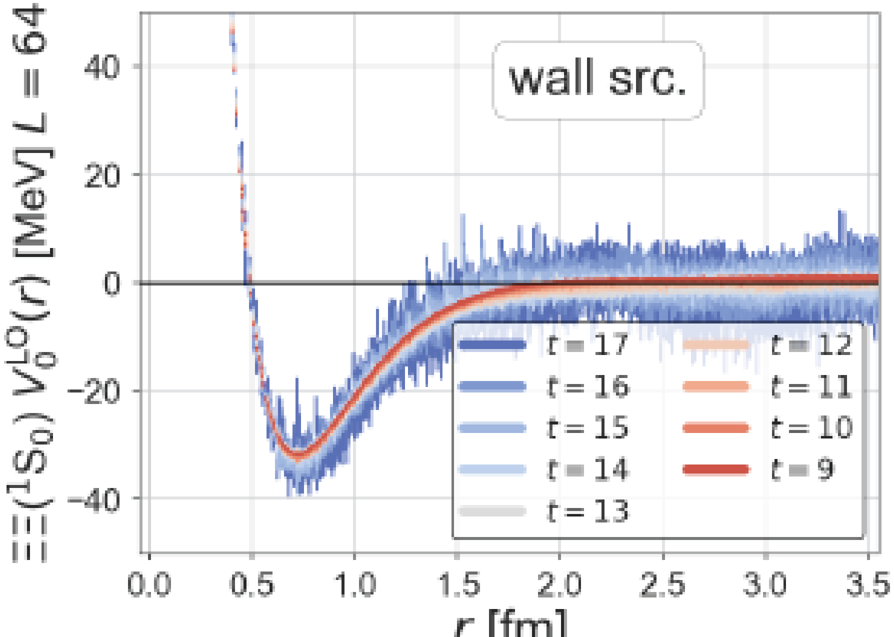
[See also Talk by S. Aoki]

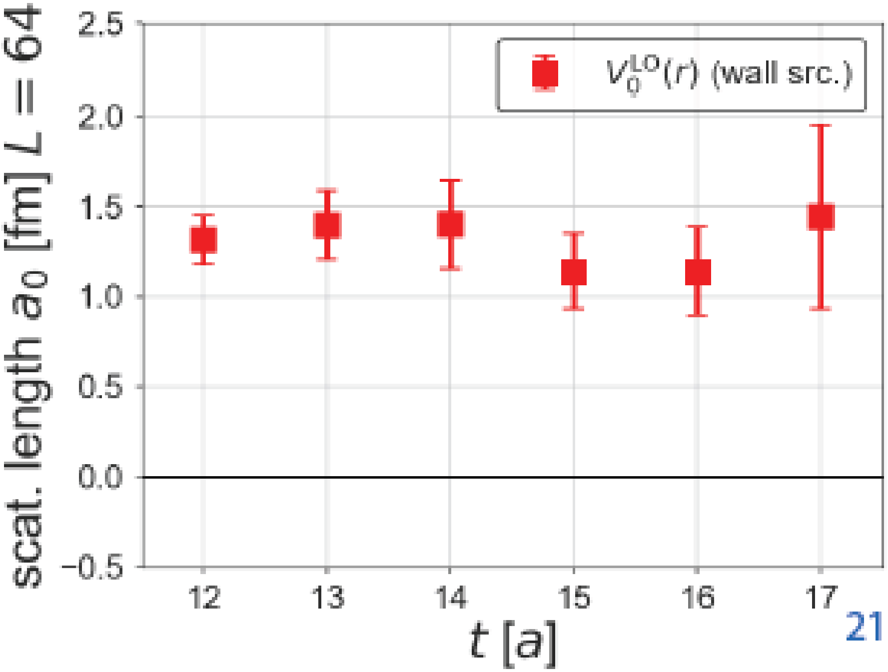
t-dep. of the Wall src.



single saturation is later than smeared src.

potential & observable are stable even at early time

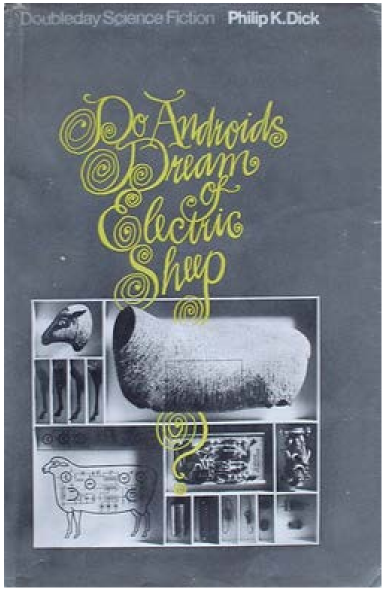


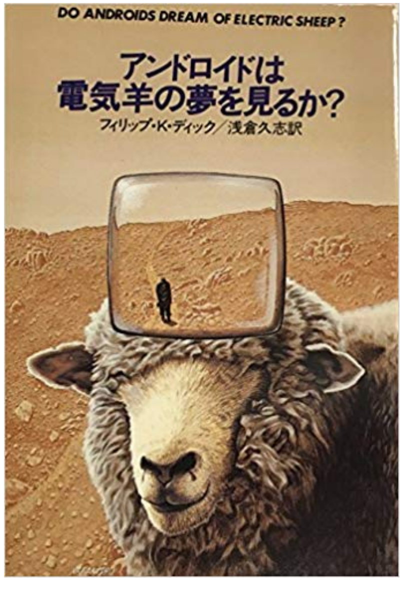


Understand how the direct method leads to unreliable results

T. Iritani et al. (HAL) JHEP03(2019) 007

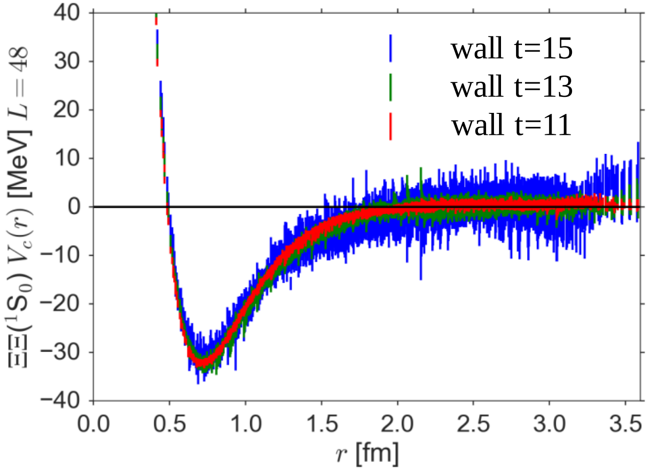
<u>*Do Plateaux Dream of the Ground State ?*</u>





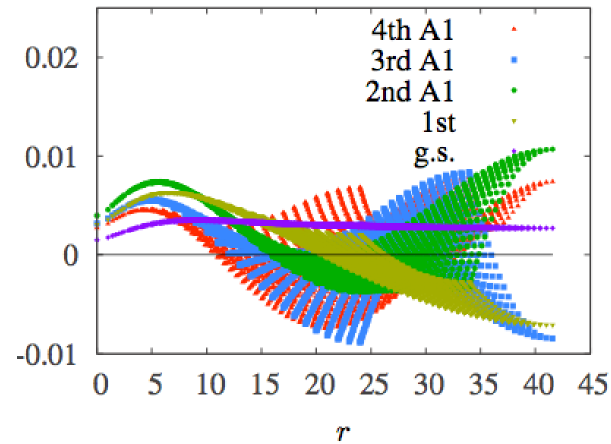
Understand the origin of “pseudo-plateaux”

Potential



Solve Schrodinger eq. in Finite V

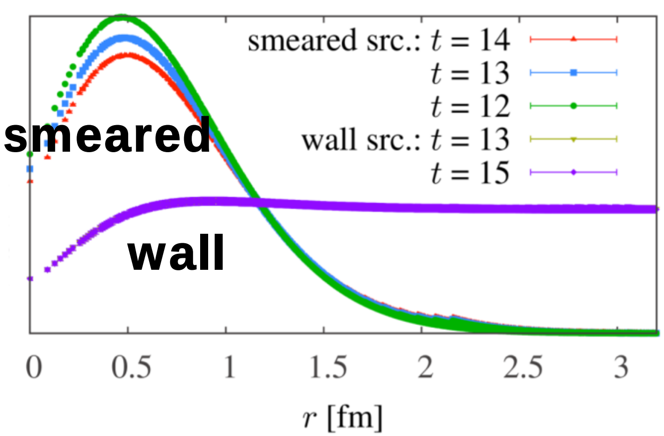
Eigen-wave functions



Eigen-energies

n-th A1	ΔE_n [MeV]
0	-2.58(1)
1	52.49(2)
2	112.08(2)
3	169.78(2)
4	224.73(1)

NBS correlator R(r,t)



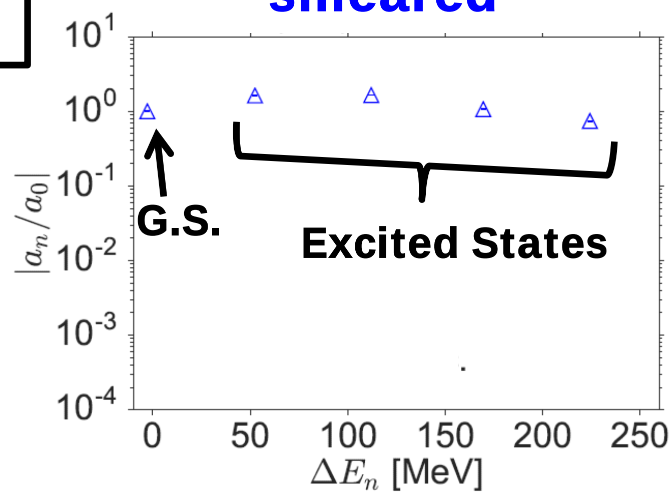
Decompose NBS correlator to each eigenstates

Decompose NBS correlator
to each eigenstates

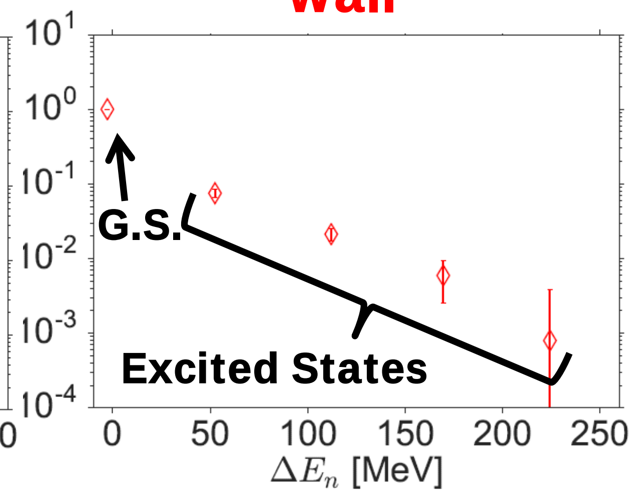
NBS correlator R(r,t)

Contribution from each (excited) states (@ t=0)

smeared



wall



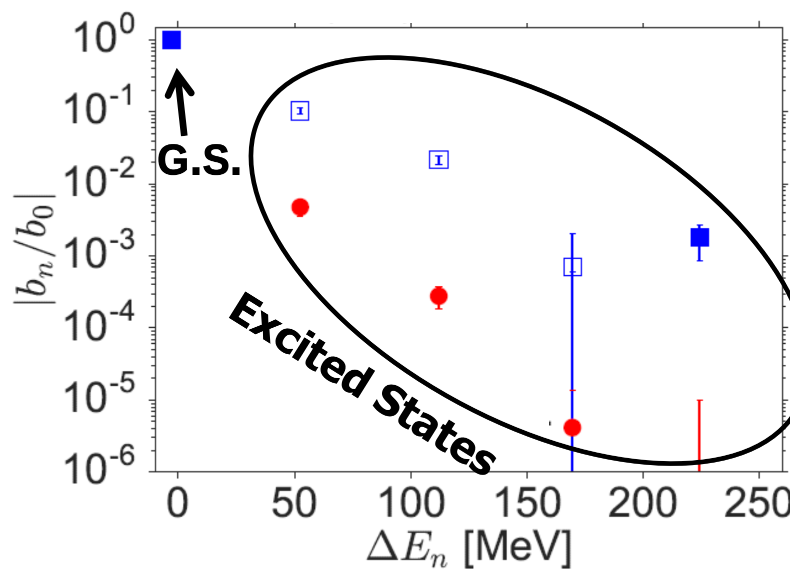
excited states NOT suppressed

excited states suppressed

Temporal-correlator
$R(t) = \Sigma_r R(r,t)$

(R(t) w/ smeared has been used in Luscher's method)

Contribution from each (excited) states (@ t=0)

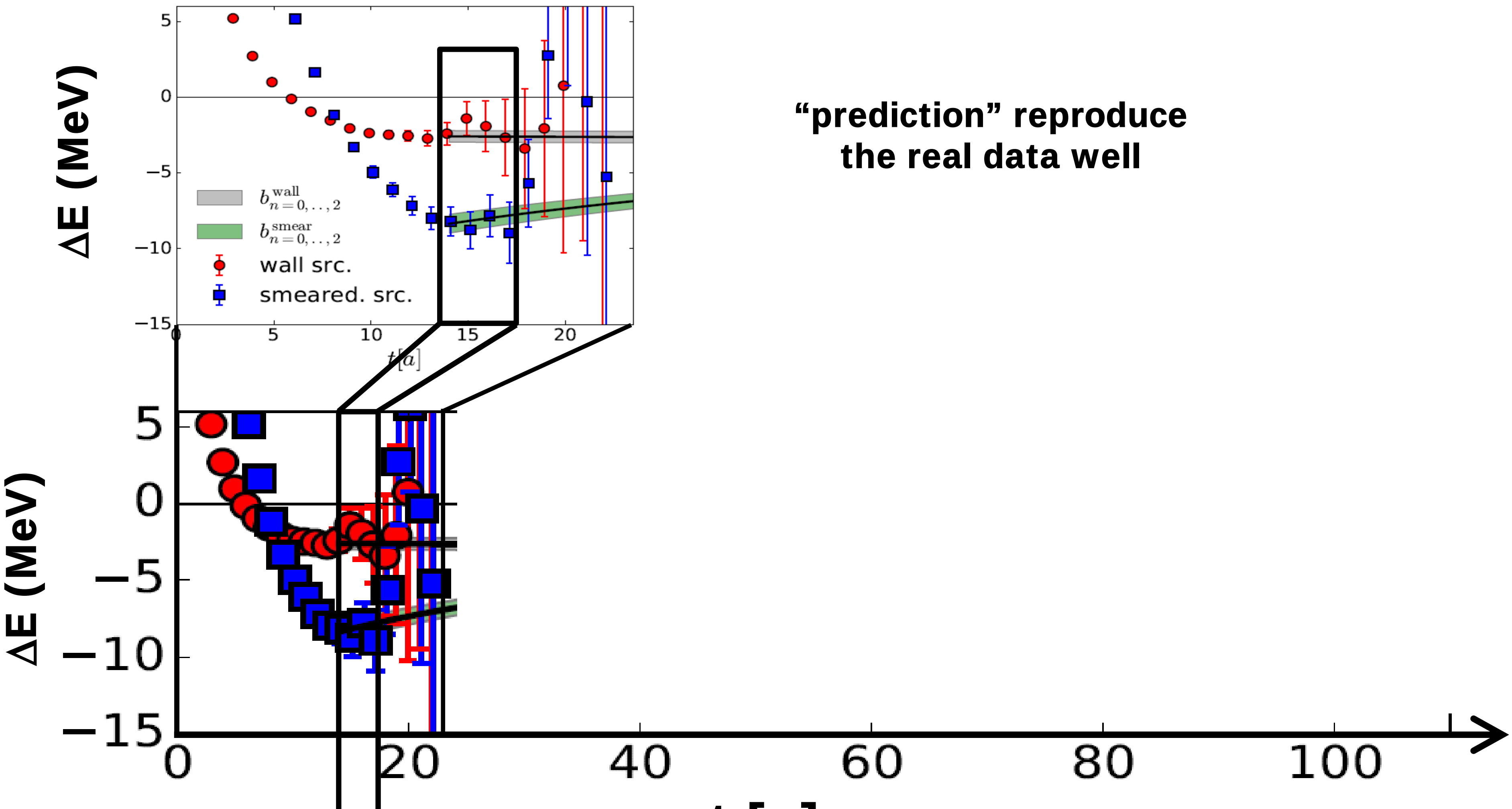


Blue: smeared

Red: wall

Understand the origin of "pseudo–plateaux"

We are now ready to "predict" the behavior of m(eff) of ΔE at any "t"



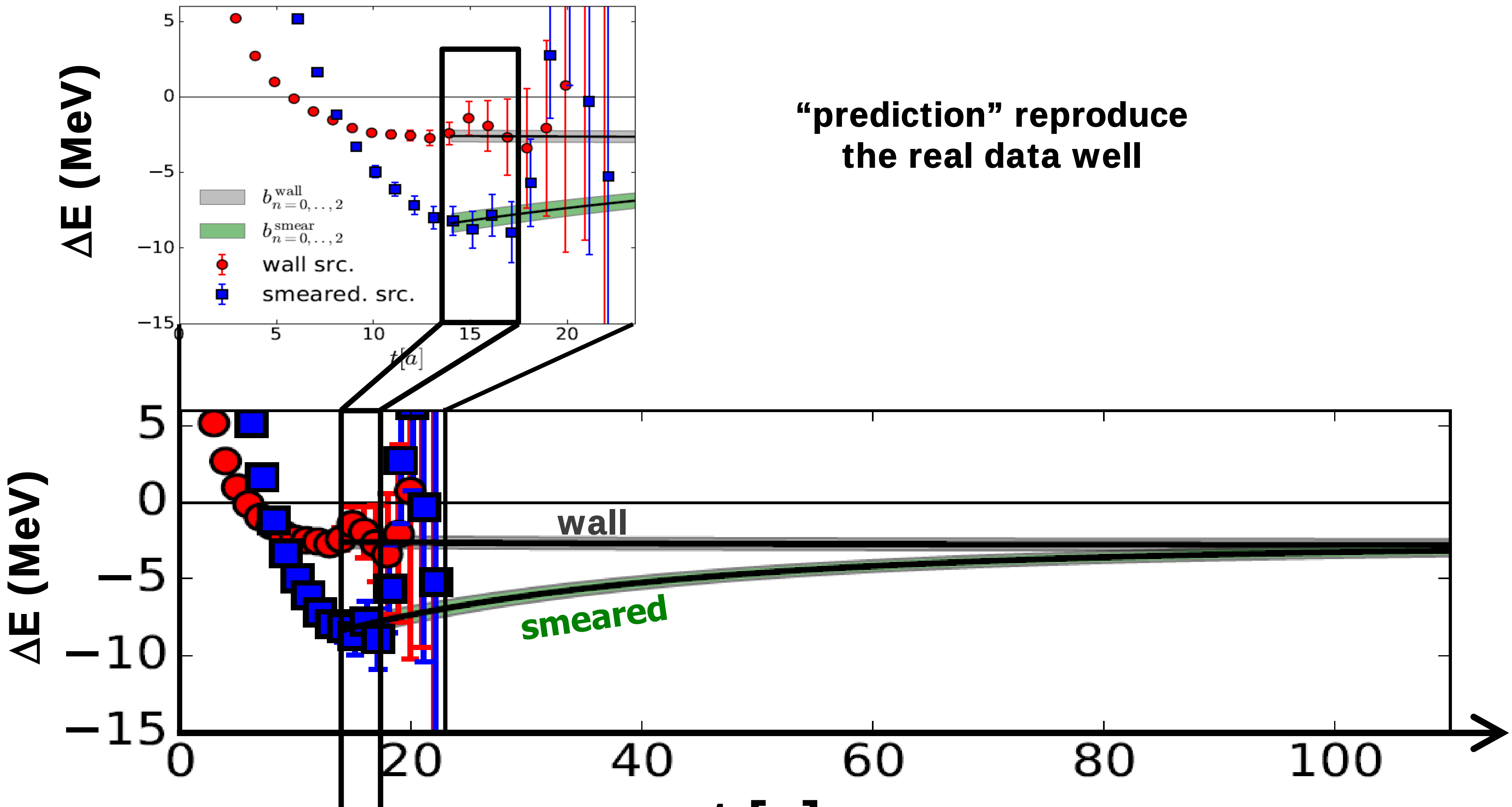
"pseudo-plateaux"
at t ~ 1fm

"real plateau"
at t ~ 10fm

(E_1-E_0=50MeV)

Understand the origin of “pseudo−plateaux”

We are now ready to “predict” the behavior of m(eff) of ΔE at any “t”



“prediction” reproduce the real data well

“pseudo-plateaux” at t ~ 1fm

“real plateau” at t ~ 10fm

(E_1-E_0=50MeV)

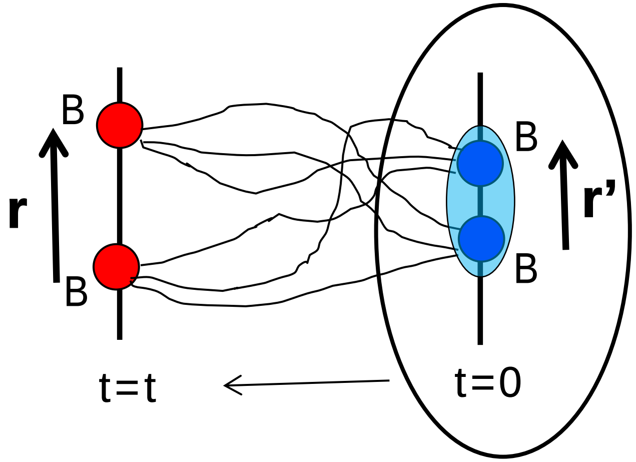
Ideal and real of "optimized" smeared src

Smeared src:

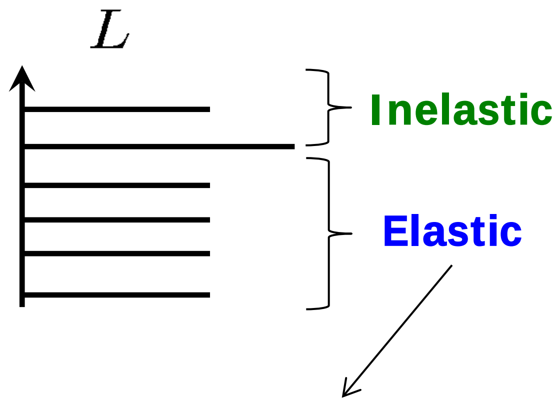
Optimized to suppress **1-body inelastic states**

Recall the real challenge for two-baryon systems:

➔ Noises from **2-body elastic excited states**



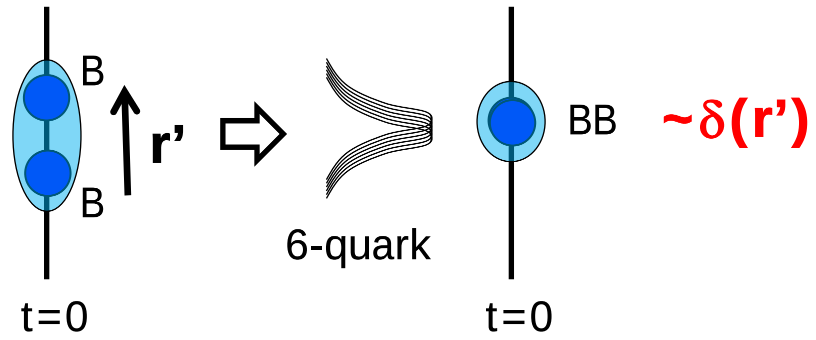
➔ Traditional smeared src is NOT optimized for two-body systems !



$$\sim B(\vec{p'})B(-\vec{p'}),\ \ \vec{p'} = (2\pi/L)\vec{n}$$

Detailed implementation of smeared src

all 6-quarks are smeared at the same spacial point



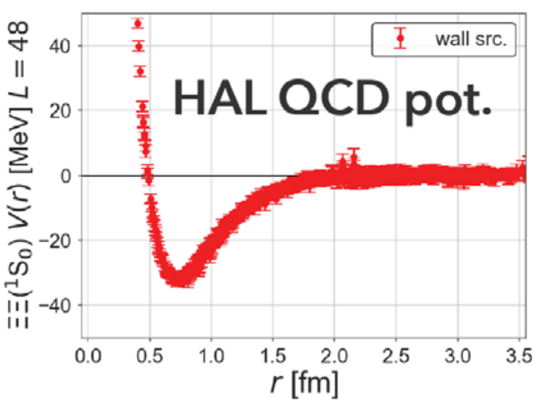
➔ Large contaminations from 2-body elastic excited states are "rather natural"

Consistency between Luscher's method and HAL method

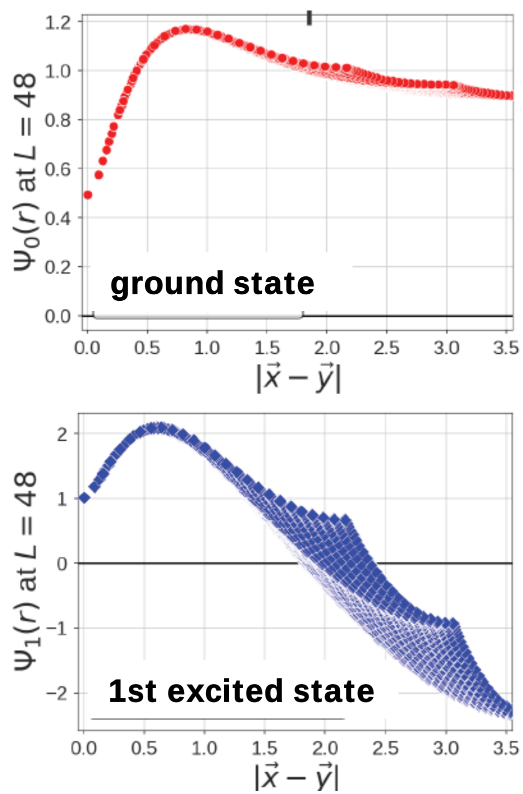
T. Iritani et al. (HAL Coll.) JHEP03(2019) 007

Operator optimized for 2-body system by HAL

- HAL method ➔ HAL pot ➔ 2-body wave func. @ finite V
- 2-body wave func. ➔ optimized operator
  - Applicable for sink and/or src op : Here we apply for sink op
- While utilizing info by HAL, formulation is Luscher's method

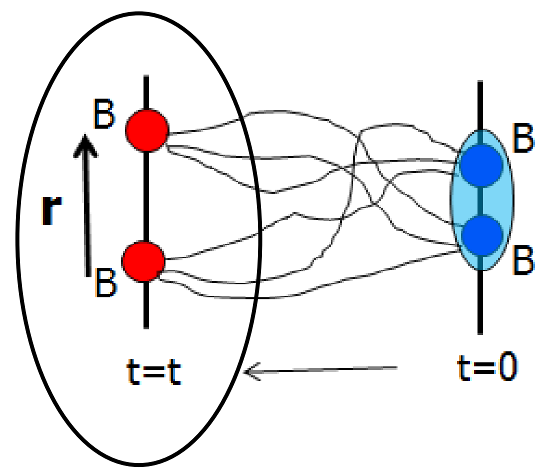


wave func. ψ(r)



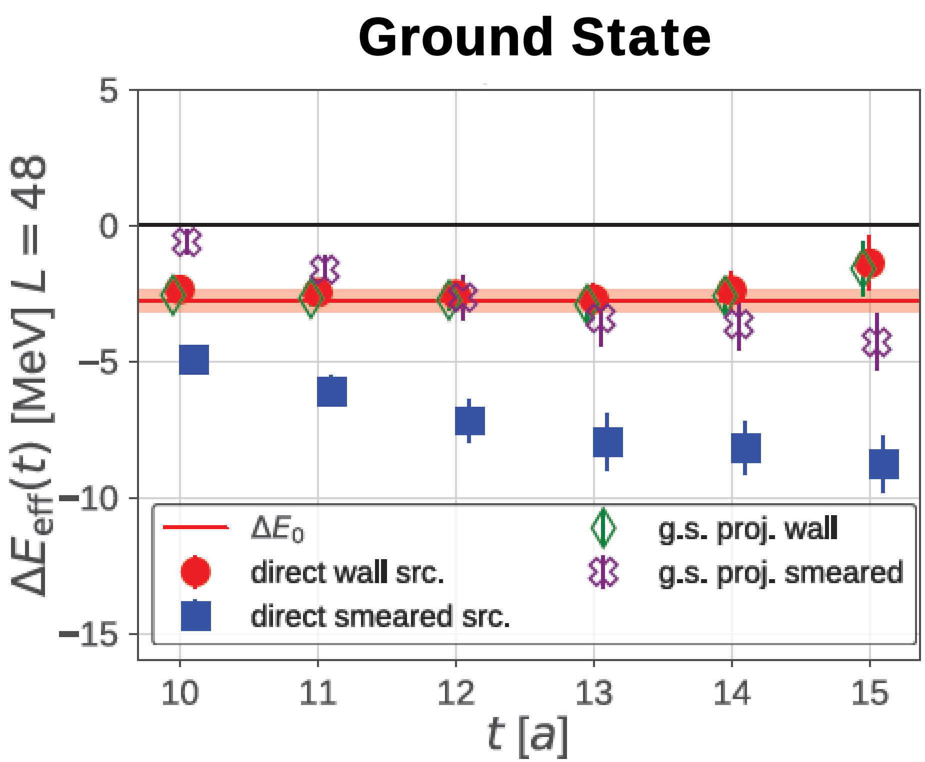
HAL-optimized sink op

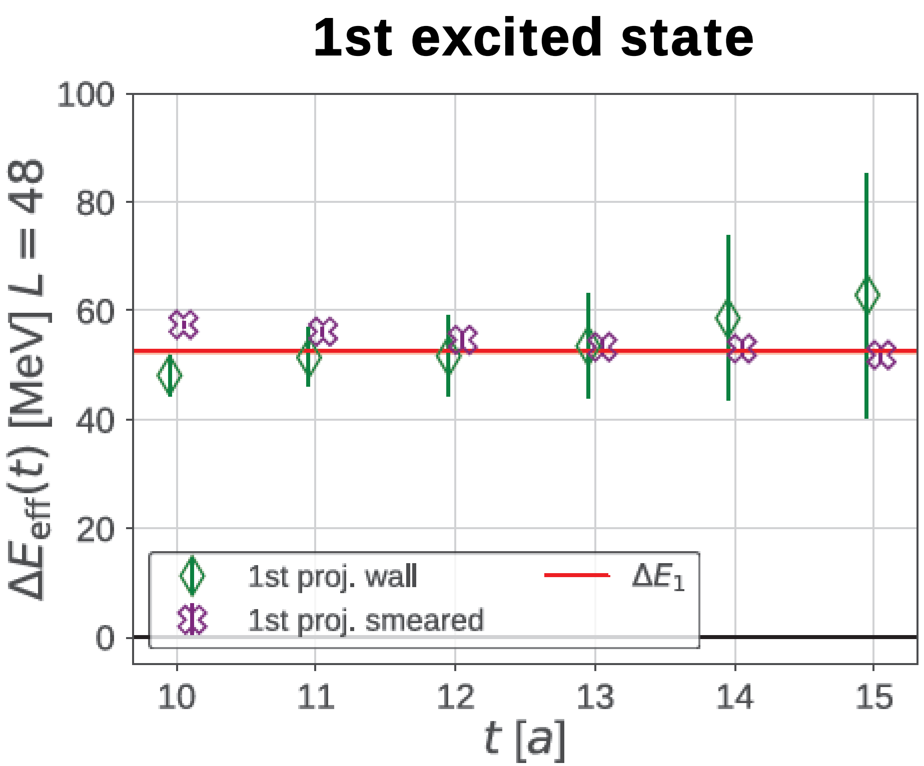
$$\mathcal{J}^{2B}_{\rm sink} = \sum_{\vec{r}} \psi^\dagger(r) \sum_{\vec{x}} B(\vec{r}+\vec{x})B(\vec{x})$$



Effective energy shift ΔE from “HAL-optimized op”

HAL-optimized sink op ➔ projected to each state ➔ “True” plateaux



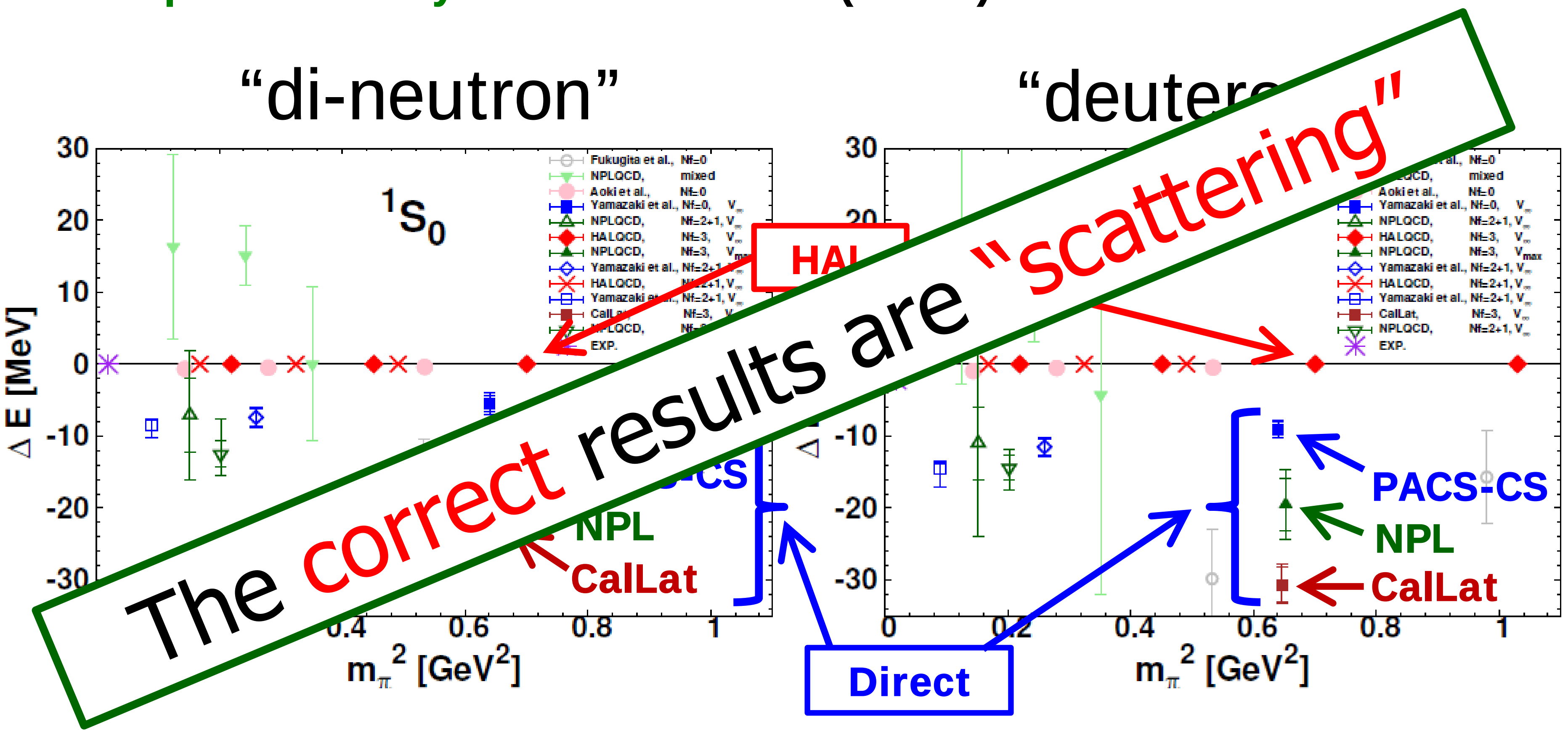


HAL QCD pot = Lushcer’s method w/ proper projection

≠ Direct method w/ naïve plateau fitting

Direct method vs HAL method
(NN @ heavy quark masses)

HAL method (HAL) : unbound
Direct method (PACS-CS (Yamazaki et al.) / NPL / CalLat): bound
Improved calc by Luscher's method (Mainz) : unbound



Summary: Are Luscher's method and HAL method consistent ?

- **(Were seemingly) NO,** because …
  - Direct method w/ naïve plateau fitting is unreliable
  - "plateau-like structure" strongly depends on sink / src op

- **YES !**
  - True plateau by the projection to the eigenstate
    - Info from HAL is necessary for the proper projection
  - HAL method = Luscher's method (≠ Direct method)
    - Useful to examine possible systematics
  - Necessary procedure w/ Luscher's method

  ➔Variational method to identify each ground/excited state

    - Talks by A. Hanlon, B. Hoerz