Decuplet-Decuplet interaction and recent development of partial wave decomposition on lattice

Shinya Gongyo (RIKEN)

SG, K.Sasaki + (HAL QCD Coll.), PRL 120 (2018) 212001
T. Miyamoto, et al. (HAL QCD Coll.), in preparation

HAL QCD Collaboration

K.Sasaki(YITP), S. Aoki (YITP), Y. Akahoshi (YITP),
T. Doi (RIKEN), F. Etiminan (Birjand U.),
T. Hatsuda (RIKEN), Y. Ikeda (YITP), T. Inoue (Nihon U.),
T. Iritani (RIKEN), N. Ishii (RCNP), T. Miyamoto (YITP), H. Nemura (RCNP)

Apr. 24, 2019@FLQCD

Outline

First part: Dec-Dec interaction from lattice QCD

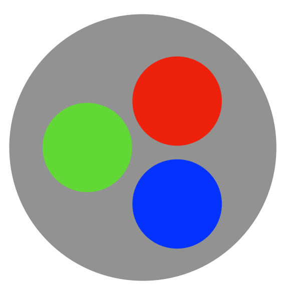
- Introduction:
  Dibaryon candidates and model studies
- Results at heavy quark masses for $\Delta\Delta(^7S_3)$
- Results at (almost) physical quark masses for $\Omega\Omega(^1S_0)$

Second part: Partial wave decomposition on lattice

- fixed-r method
- Misner's method
- numerical test and application to $\Lambda_c N$ system

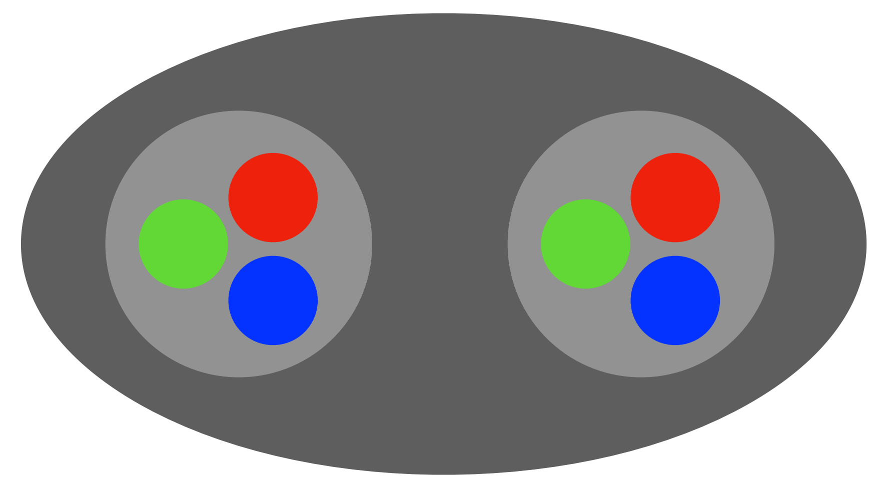
Introduction

Baryon (B=1)



Proton, Neutron,
Lambda, Omega,…

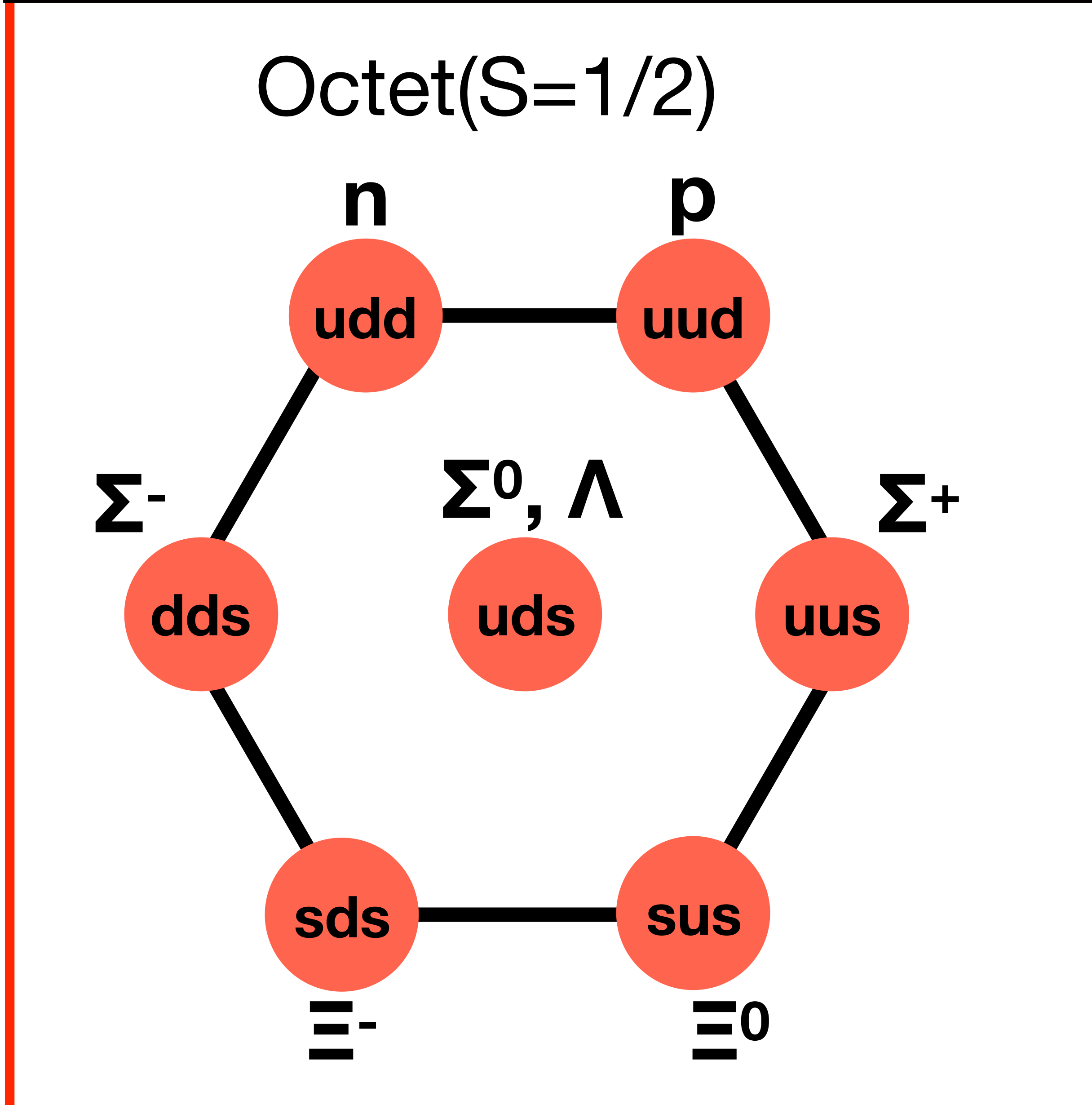
Dibaryon (B=2)

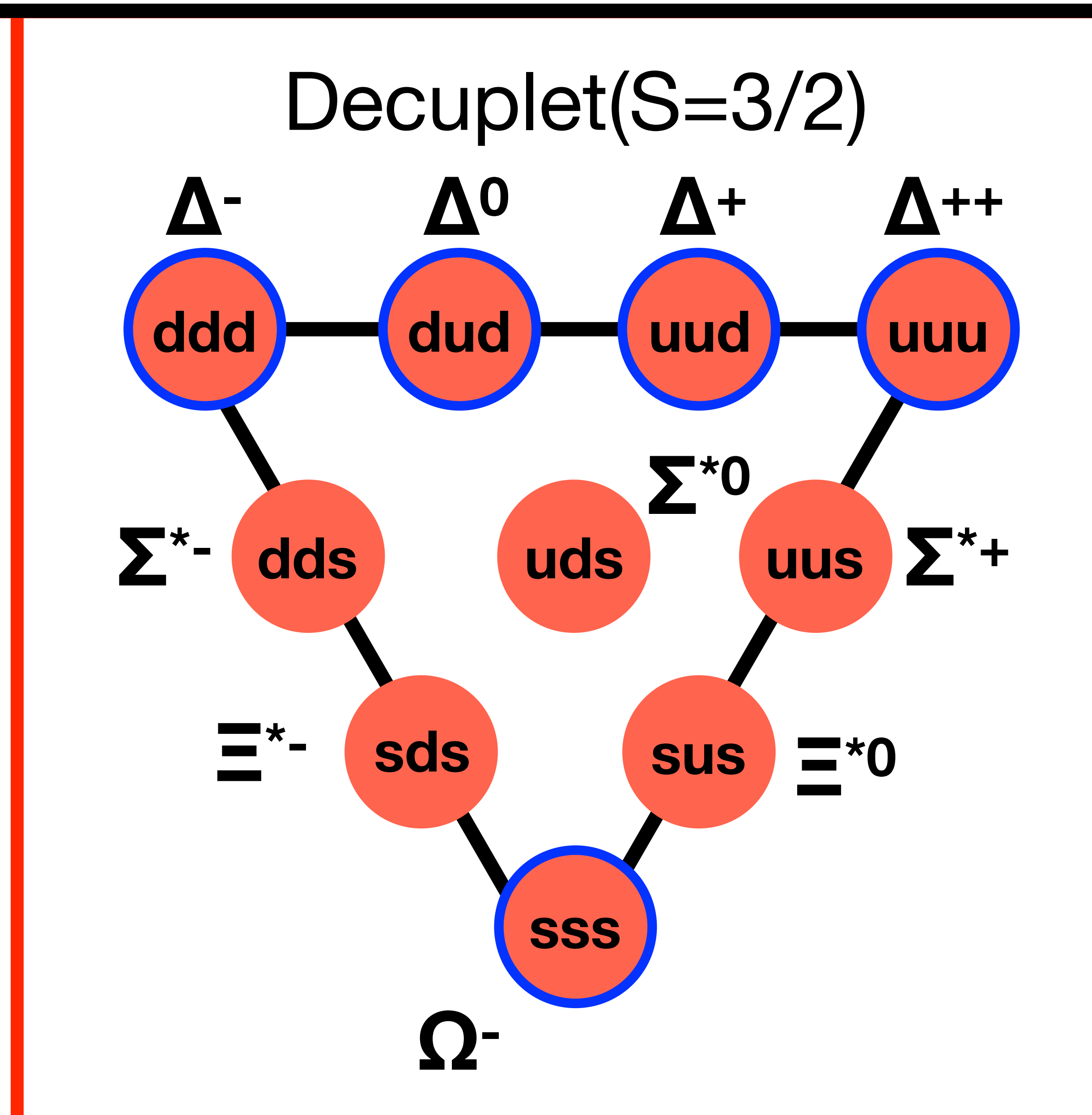


Deuteron
observed in 1930s
+ d*(2380) resonance

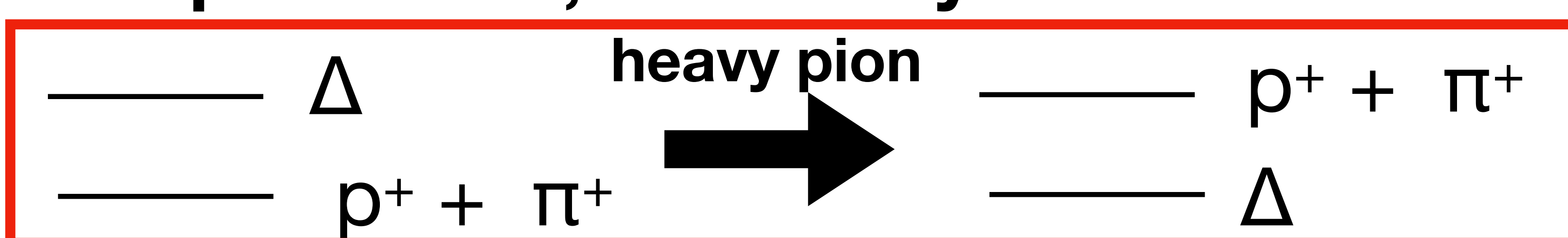
Dibaryon = two baryon bound state or resonance

Our lattice simulation
ΔΔ => heavy pion
ΩΩ => phys. pt.

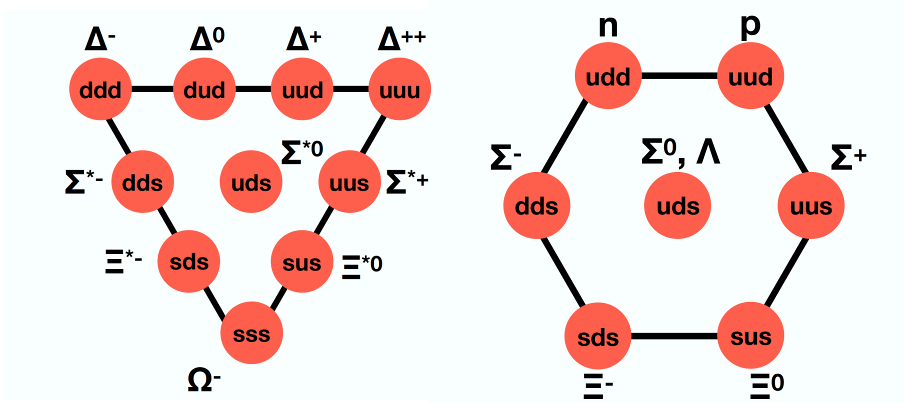




In decuplet baryons, only Ω is stable under strong decay.
In the case of heavier pion mass, Delta baryons become stable.



Introduction: SU(3) classification for Dibaryon candidates (B=2)



Jaffe (1977)

H-dibaryon(J=0)

1) octet-octet system

$$8 \otimes 8 = 27 \oplus 8_s \oplus \boxed{1} \oplus \boxed{\overline{10}} \oplus 10 \oplus 8_a$$

Deuteron(J=1)

2) decuplet-octet system

NΩ system and NΔ system (J=2)

$$10 \otimes 8 = 35 \oplus \boxed{8} \oplus 10 \oplus 27$$

Goldman et al (1987)
Dyson, Xuong (1964)

3) decuplet-decuplet system

$$10 \otimes 10 = \boxed{28} \oplus 27 \oplus 35 \oplus \boxed{\overline{10}}$$

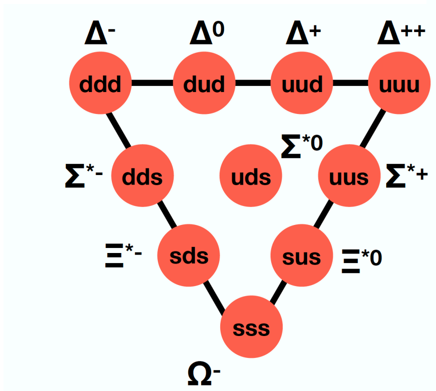
ΩΩ system (J=0)

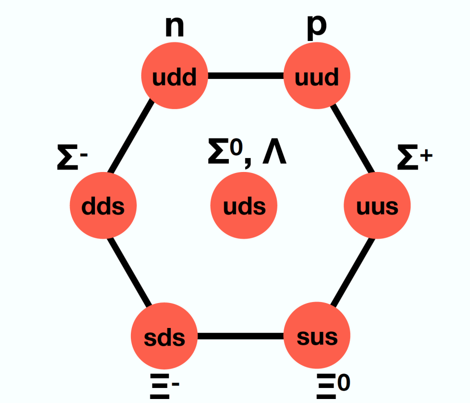
Zhang et al(1997)

ΔΔ system (J=3)

Dyson, Xuong (1964)
Kamae, Fujita(1977)
Oka, Yazaki(1980)

Introduction: SU(3) classification for Dibaryon candidates (B=2)





Jaffe (1977)

H-dibaryon(J=0)

1) octet-octet system

$$8 \otimes 8 = 27 \oplus 8_s \oplus \boxed{1} \oplus \boxed{\overline{10}} \oplus 10 \oplus 8_a$$

Deuteron(J=1)

2) decuplet-octet system

NΩ system and NΔ system (J=2)

$$10 \otimes 8 = 35 \oplus \boxed{8} \oplus 10 \oplus 27$$

Goldman et al (1987)

Dyson, Xuong (1964)

d*(2380) resonance by Kamae et al, 1975 WASA@COSY, 2009

3) decuplet-decuplet system

$$10 \otimes 10 = \boxed{28} \oplus 27 \oplus 35 \oplus \boxed{\overline{10}}$$

ΩΩ system (J=0)

ΔΔ system (J=3)

Zhang et al(1997)

Dyson, Xuong (1964)
Kamae, Fujita(1977)
Oka, Yazaki(1980)

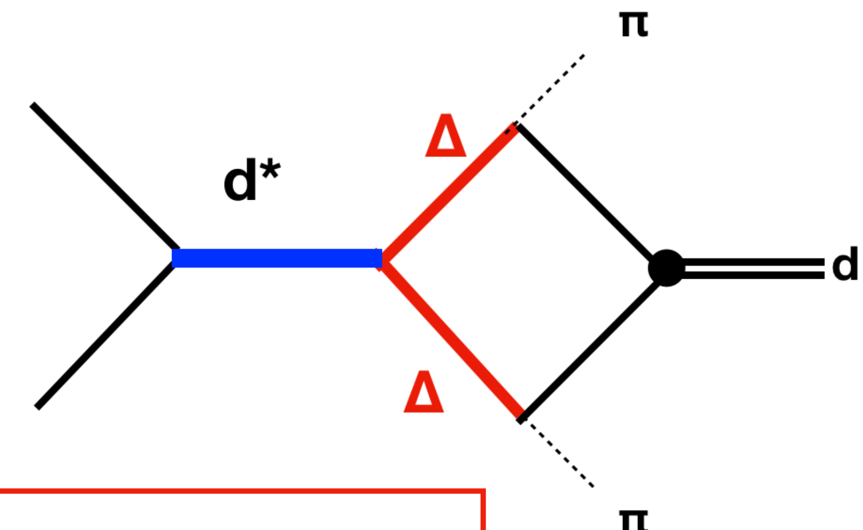
d*(2380) resonance

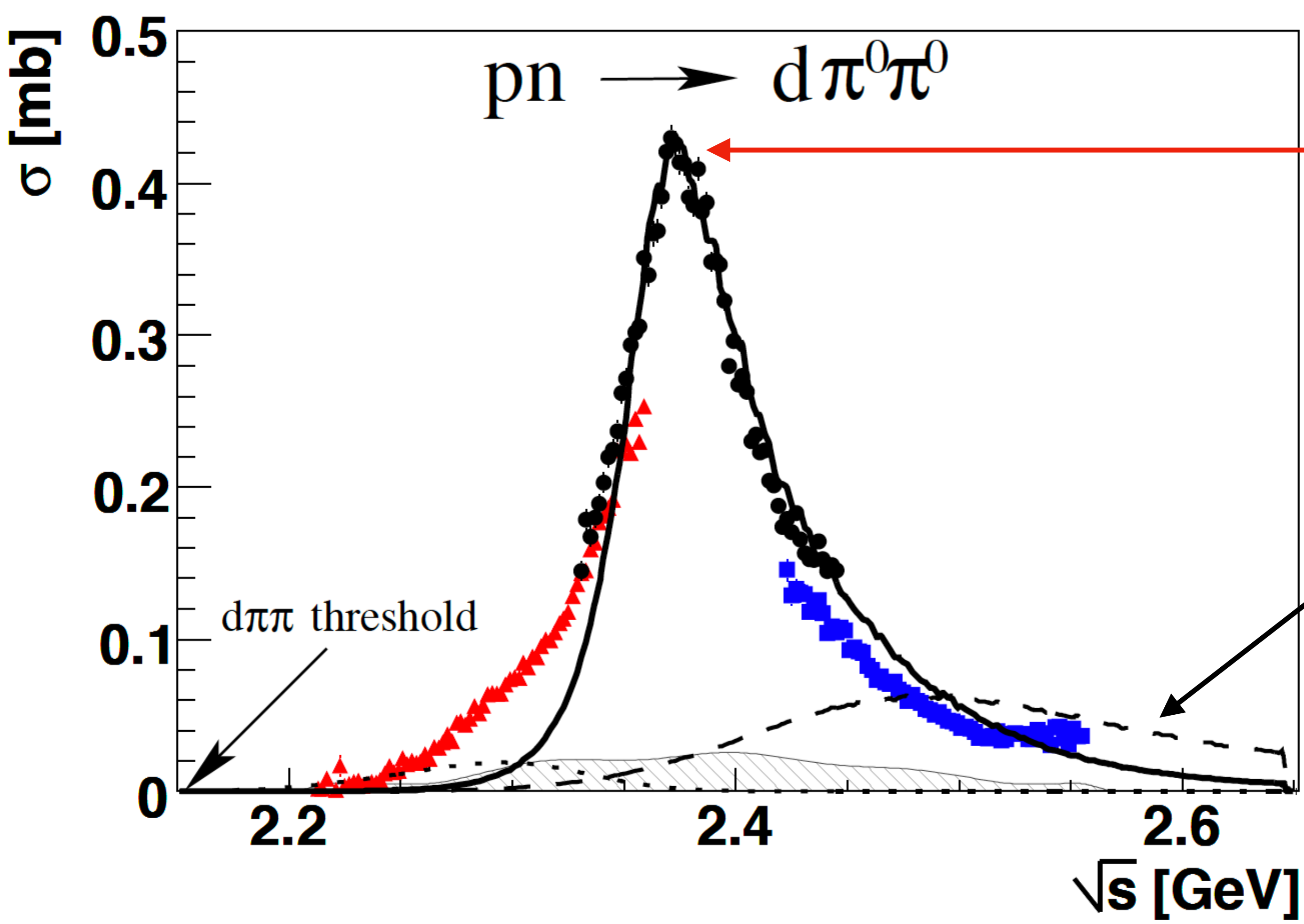
WASA@COSY, PRL 106, 242302 (2011)

d* (2380) observed by WASA@COSY col.

$$p + n(d) \rightarrow d + \pi^0 + \pi^0 (+p_{\text{spectator}})$$

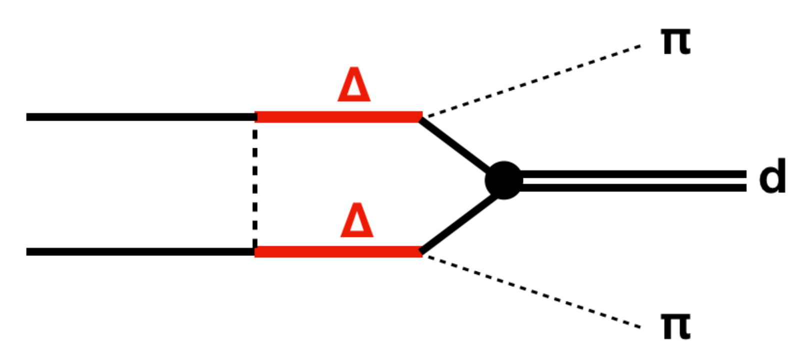
m~ 2.38 GeV, Γ ~ 70 MeV, $J^\pi = 3^+$, I=0



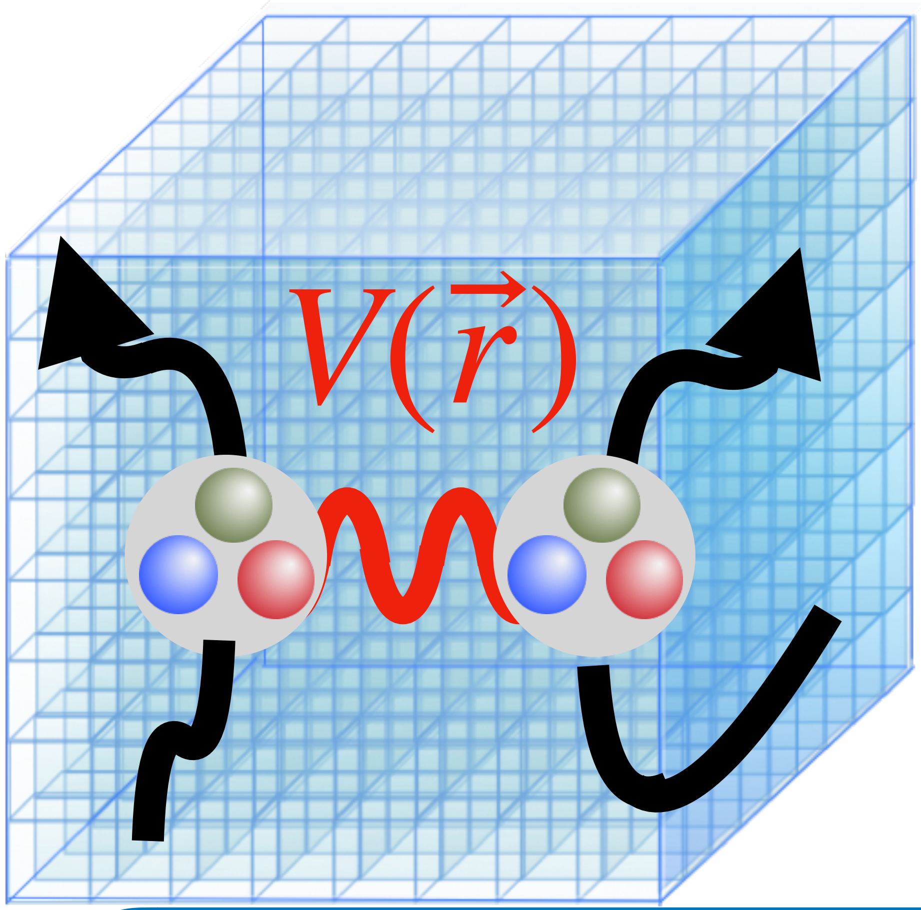


d* resonance
m~2.38 GeV
Γ~70 MeV

ΔΔ contributions



Baryon-Baryon interaction from lattice QCD
-HAL method-



Aoki, Hatsuda, Ishii, PTP123, 89 (2010)

c.f. anothor method: Luscher's direct method

Nambu-Bethe-Salpeter (NBS) w.f.

$$\Psi_n(\vec{r})\, e^{-E_n t} = \sum_{\vec{x}} \langle 0 | B_1(t, \vec{r}+\vec{x}) B_2(t, \vec{x}) | E_n \rangle$$

Local operators B_1&B_2 for decuplet baryons

$$B_1, B_2 \to \; D_{\mu\alpha} = \epsilon_{abc} \left(q^{aT} C \gamma_\mu q^b \right) q^c_\alpha$$

Schroedinger type equation is satisfied

$$\left(\vec{p}_n^2 + \nabla^2 \right) \Psi_n(\vec{r}) = 2\mu \int d\vec{r'} U(\vec{r}, \vec{r'}) \Psi_n(\vec{r'})$$

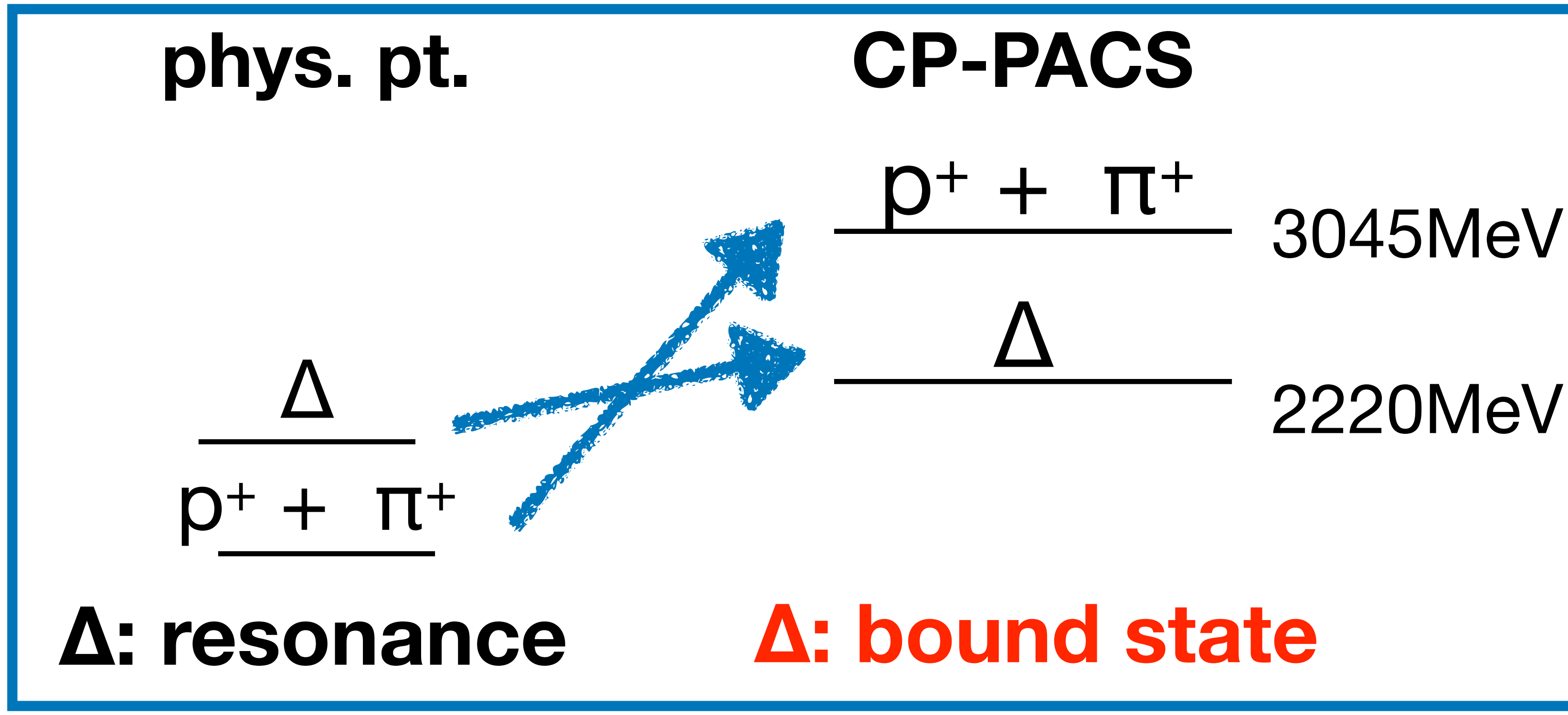
non-local pot.

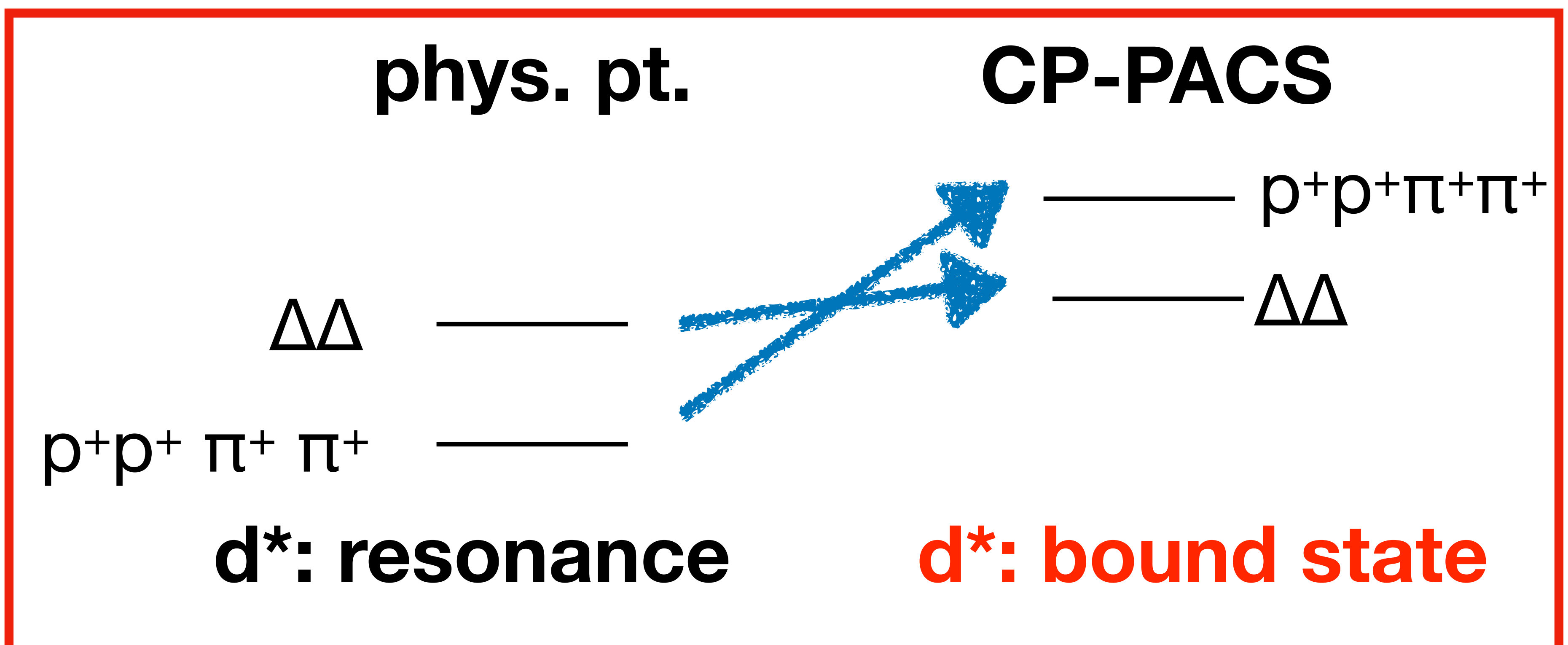
The potential is extracted from this equation

I. $\Delta\Delta$ system with J=3

Nf = 2+1 full QCD with L = 1.93fm, SU(3) limit (CP-PACS Conf)

	[MeV]
m_{ps}	1015
m_{oct}	2030
m_{dec}	2220





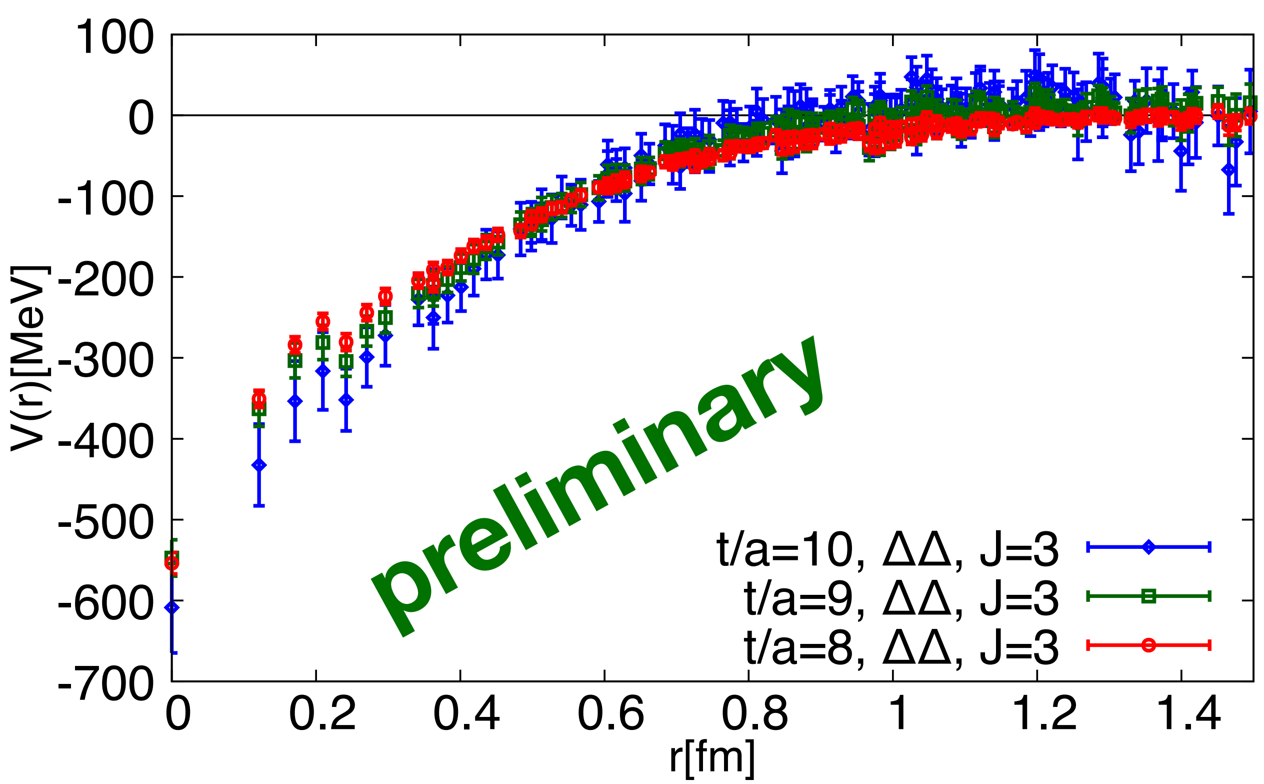
SG and K. Sasaki

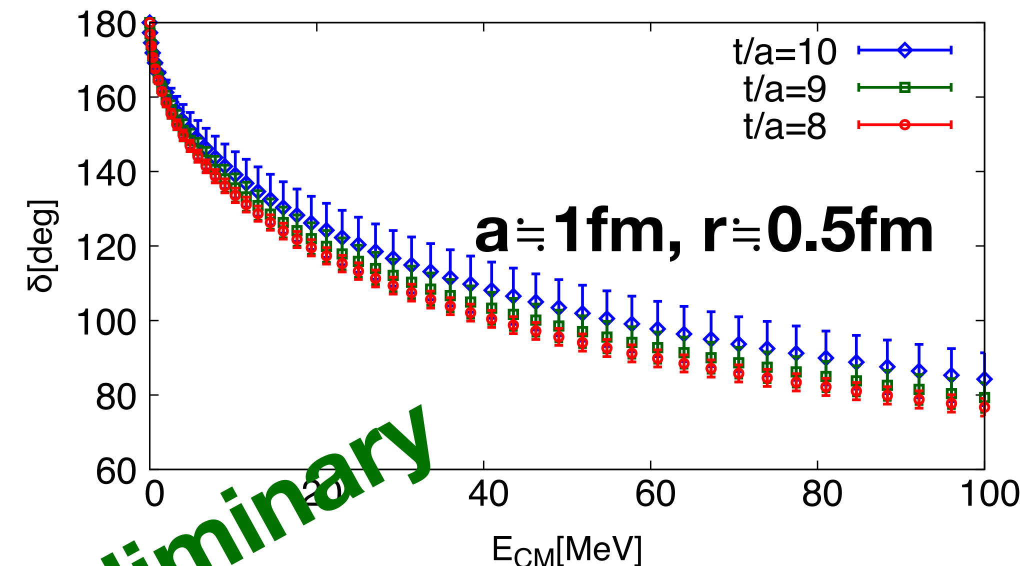
$\overline{10}$ plet in decuplet-decuplet system

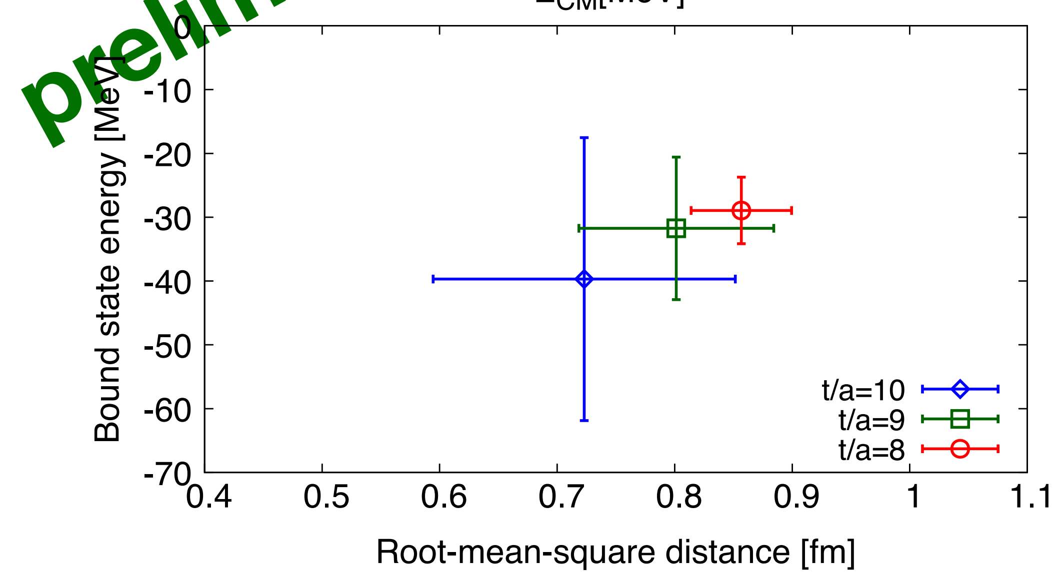
Nf = 2+1 full QCD with L = 1.93fm, m_π=1015MeV, SU(3) limit

ΔΔ in $J^p(I) = 3^+(0)$

$m_\Delta \simeq 2225\text{MeV}$







We assume that
decay to NN($^3D_3$) is neglected

- In short range, there is no repulsive core
- Deep bound state is found

d* is supported from lattice QCD

II. ΩΩ system

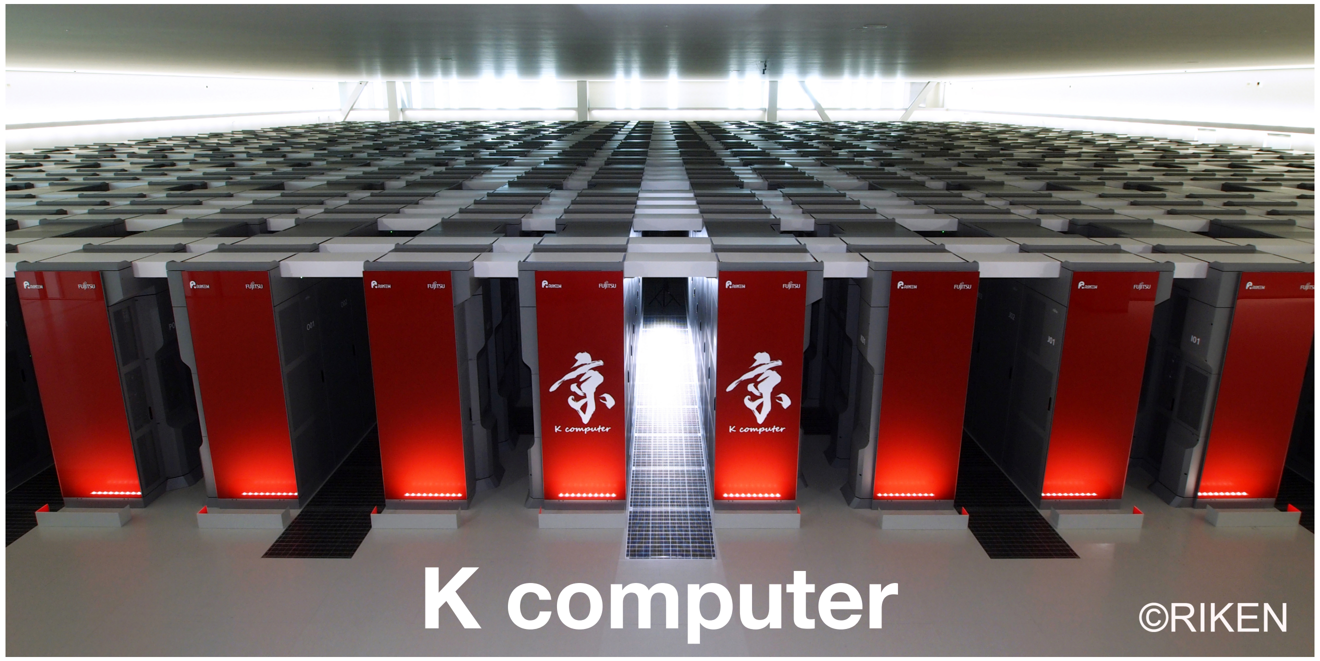
Numerical Setup at (almost) physical mass

2+1 flavor gauge configurations

- Iwasaki gauge action & O(a) improved Wilson quark action
- a= 0.0846 [fm], a^{-1} = 2333 [MeV]
- 96^3x96 lattice, L = 8.1[fm]
- 400 confs x 48 source positions x 4 rotations

Wall source is employed. only S-wave state is produced.

	[MeV]	phys.
π	146	8%
K	525	6%
N	964	3%
Ω	1712	2%

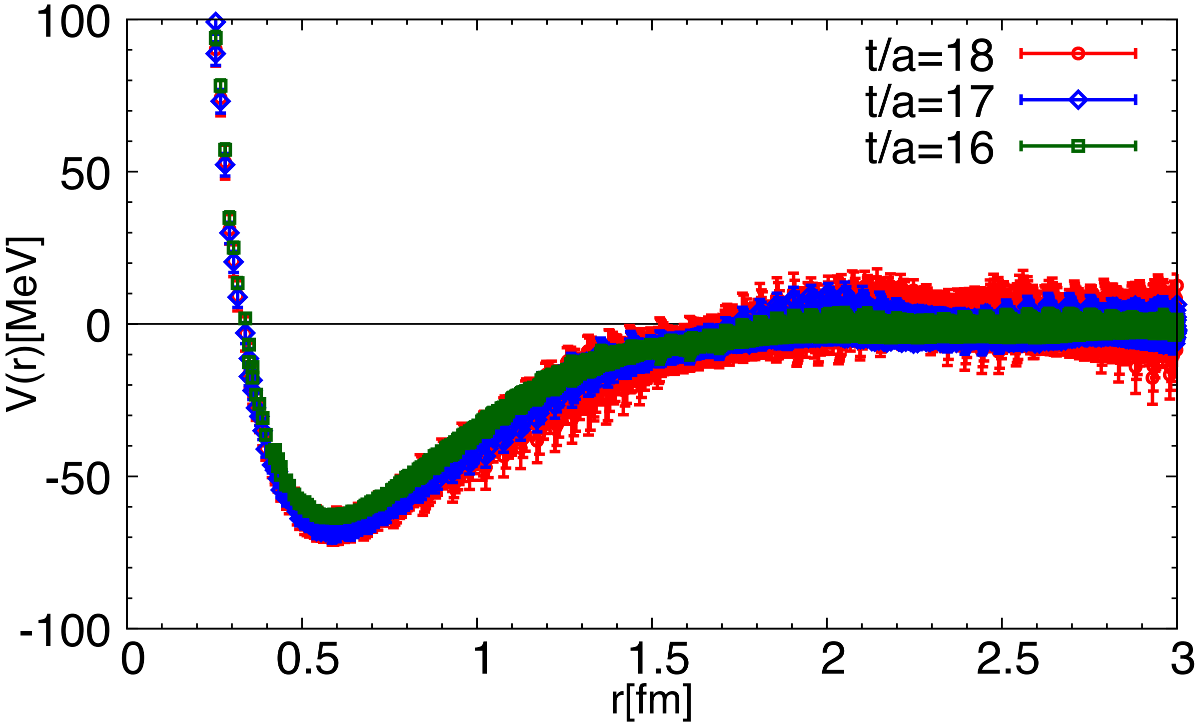


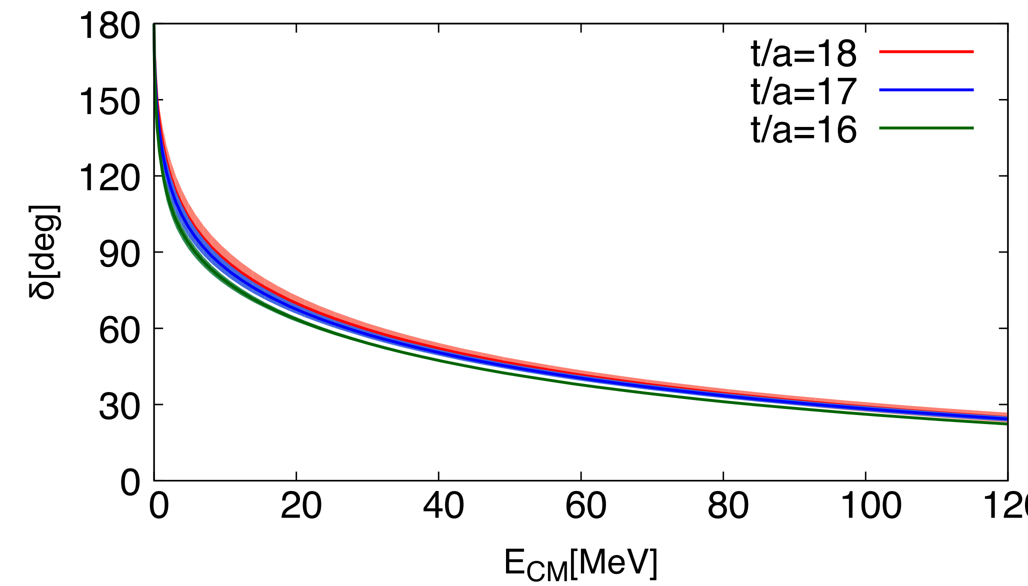
SG, K.Sasaki + (HAL QCD Coll.), PRL 2018

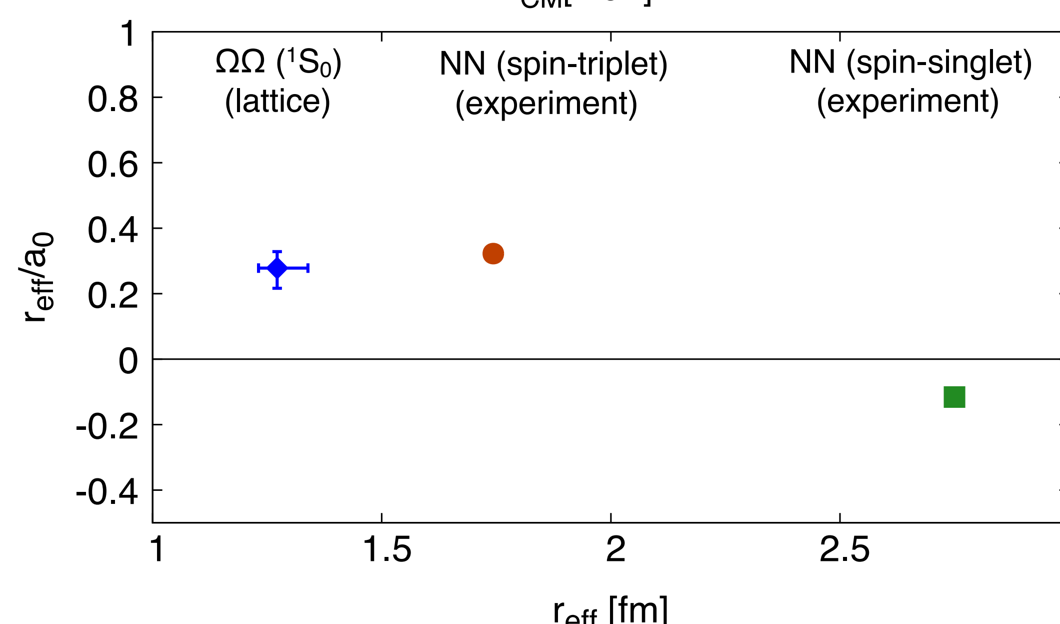
ΩΩ in *J* =*0*

“most strange dibaryon”

3)Nf=2+1 full QCD with L = 8.1fm, m_π= 146MeV







$$a_0^{(\Omega\Omega)} = 4.6(6)(^{+1.2}_{-0.5}) \quad \text{fm},$$

$$r_{\text{eff}}^{(\Omega\Omega)} = 1.27(3)(^{+0.06}_{-0.03}) \quad \text{fm}.$$

- **Short range repulsive core and attractive pocket are found**
- **Phase shift shows the presence of a bound state**
- **The state is very close to the unitary region (r/a<1)**

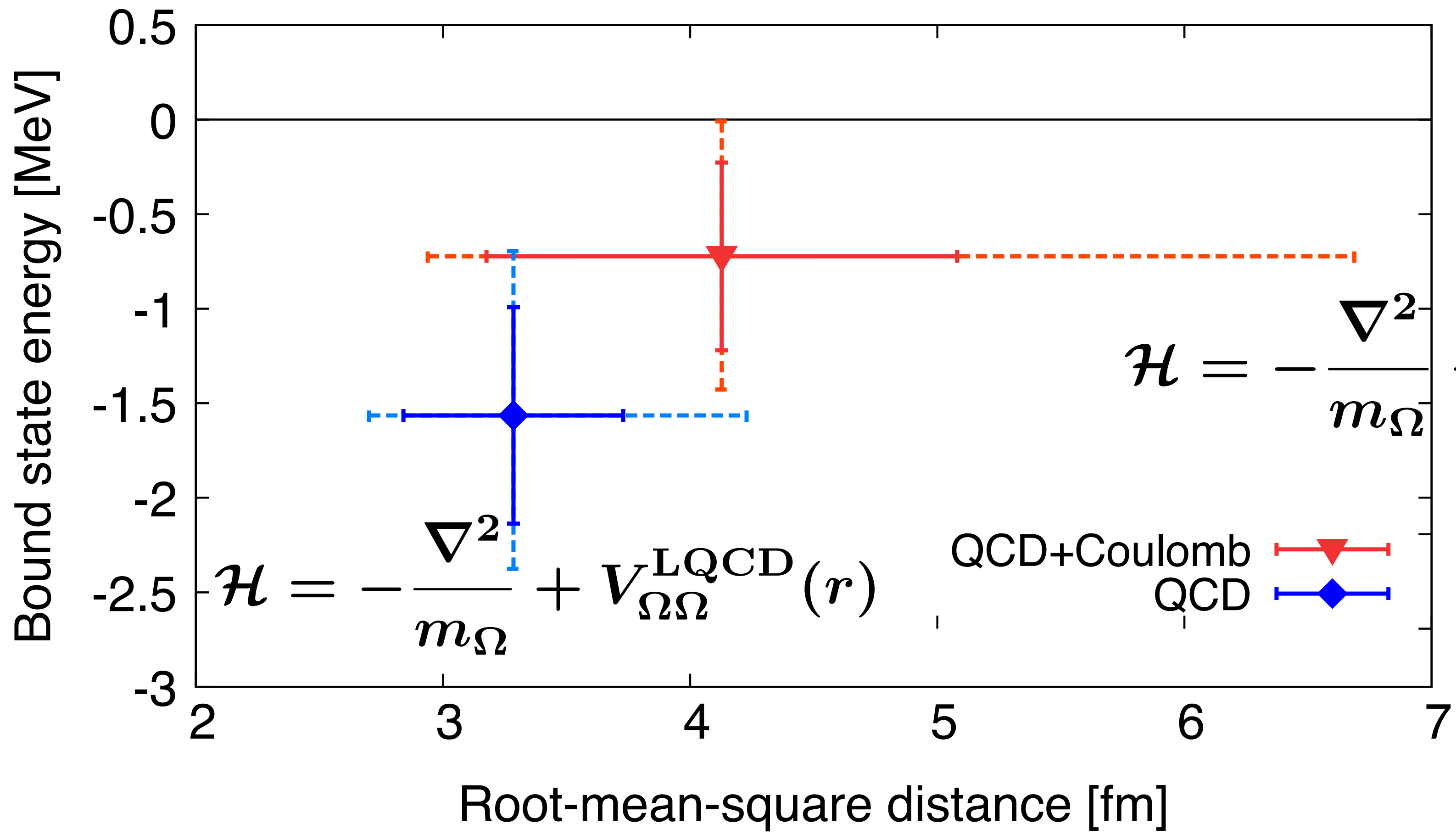
SG and K. Sasaki et.al.(HAL), PRL(2018)

ΩΩ in *J* =0

Binding energy and the Coulomb effect

“most strange dibaryon”

Q=-1



$$\mathcal{H} = -\frac{\nabla^2}{m_\Omega} + V^{\rm LQCD}_{\Omega\Omega}(r) + \frac{\alpha}{r}$$

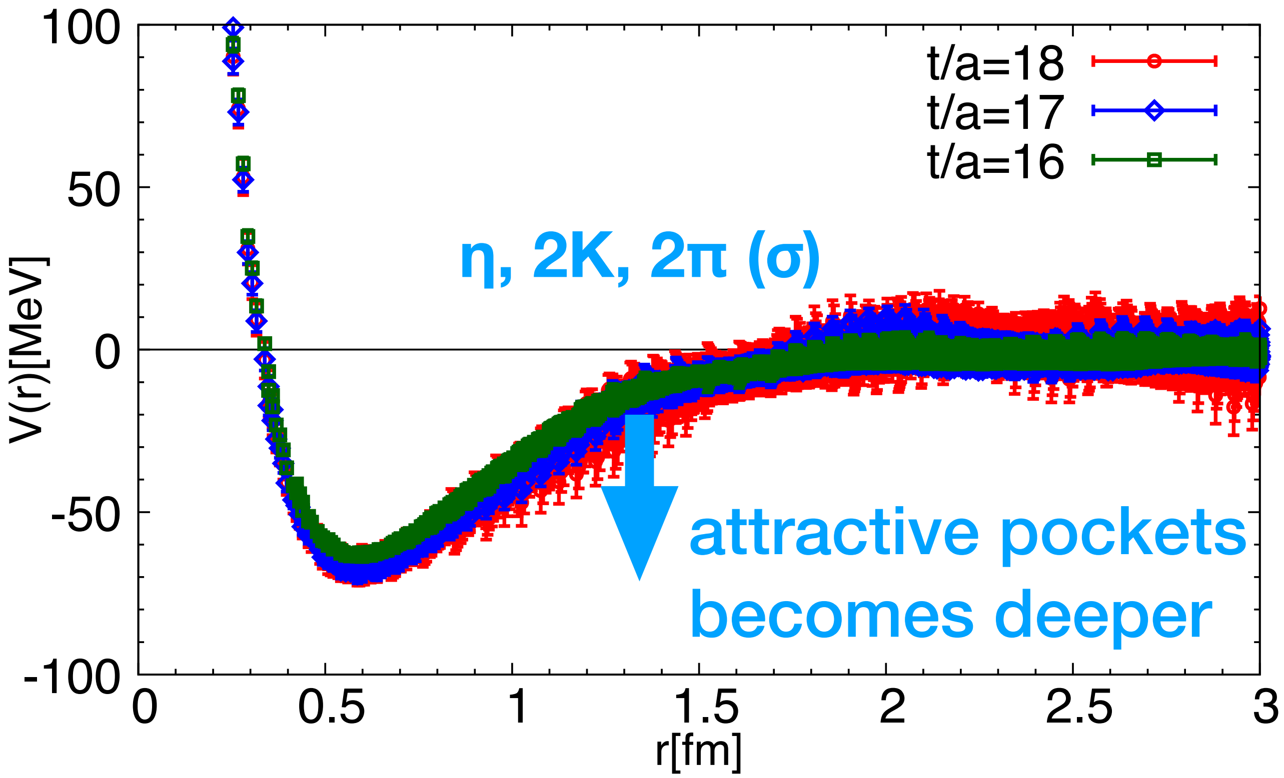
$$\mathcal{H} = -\frac{\nabla^2}{m_\Omega} + V^{\rm LQCD}_{\Omega\Omega}(r)$$

$$(B^{\rm (QCD)}_{\Omega\Omega}, B^{\rm (QCD+Coulomb)}_{\Omega\Omega}) = (1.6(6){\rm MeV}, 0.7(5){\rm MeV})$$

Conservative estimate at exact phys. pt.

m_{π}=146 MeV -> 135 MeV, m_{Ω}= 1712MeV -> 1672 MeV

B.E. ↑



V.S.

B.E. ↓

$$\mathcal{H} = \boxed{-\frac{\nabla^2}{m_\Omega}} + V_{\Omega\Omega}^{\mathrm{LQCD}}(r)$$

kinetic energy is increasing
-> B.E. is reduced

conservative estimate:
only change the mass of kinetic term

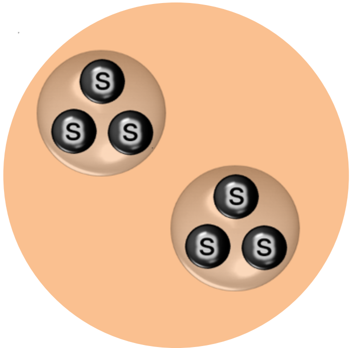
$$(B_{\Omega\Omega}^{(\mathrm{QCD})}, B_{\Omega\Omega}^{(\mathrm{QCD+Coulomb})}) = (1.6(6)\mathrm{MeV}, 0.7(5)\mathrm{MeV})$$
$$\rightarrow (1.3(5)\mathrm{MeV}, 0.5(5)\mathrm{MeV})$$

These changes are within errors

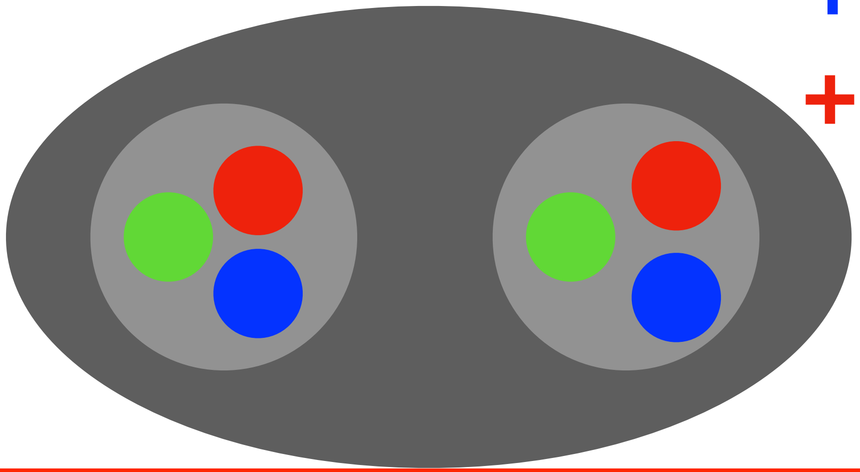
Summary in first part

- heavy pion masses:
  ΔΔ interaction in $^7S_3$
  - shows only attractive region
  - bound state in J=3 channel (=d* resonance)
- physical pion masses:
  ΩΩ interaction in $^1S_0$
  - short range repulsive and attractive pocket
  - a very shallow bound state [di-Omega]

di-Omega



Dibaryon (B=2)



Deuteron(1930s)

+ d*(2380) resonance ***<= supported***

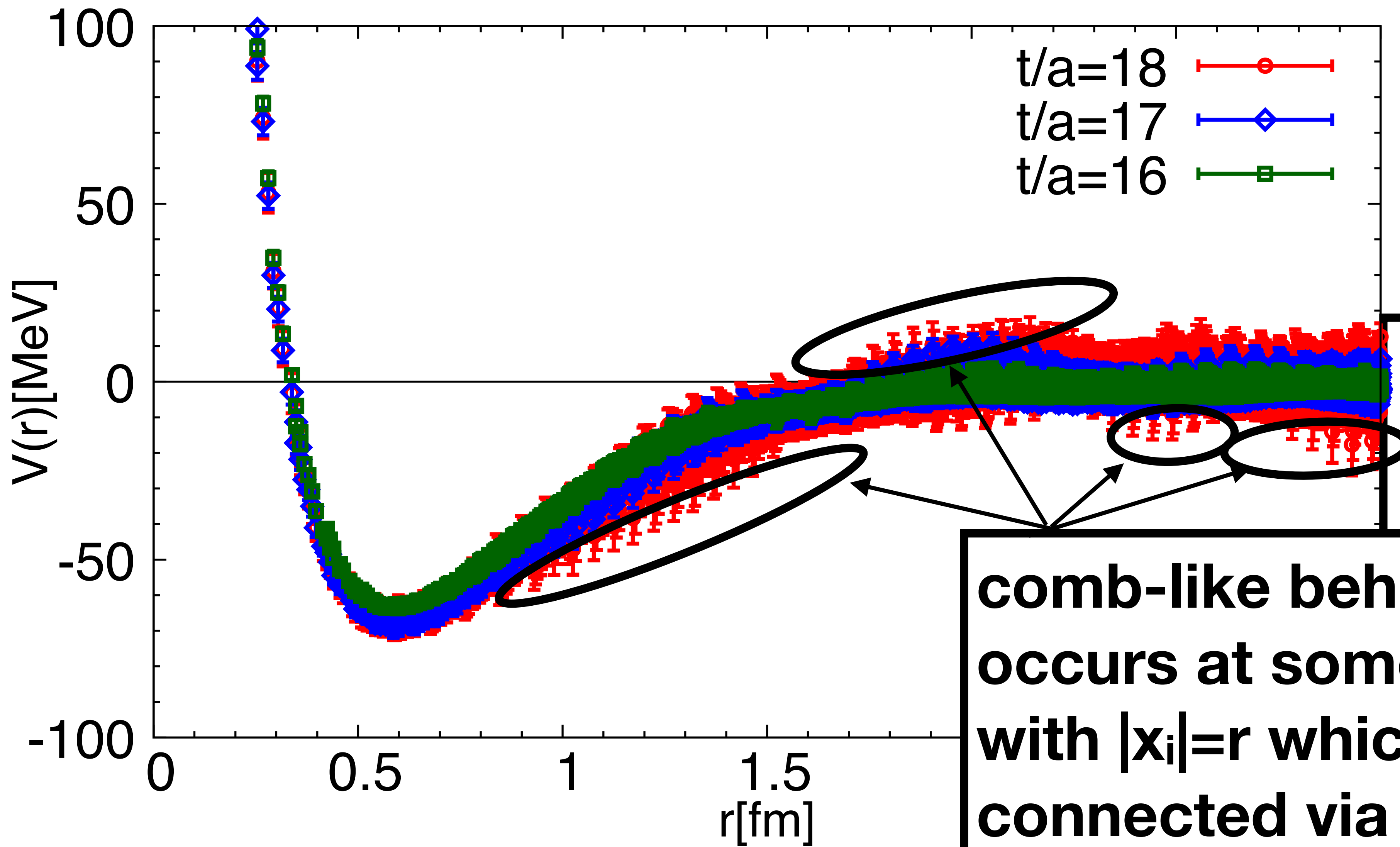
+ di-Omega (bound) ***<= predicted***

found in future HIC ?
(LHC RUN3/FAIR/J-PARC)

Recent development of partial wave decomposition on lattice

T. Miyamoto, et al. (HAL QCD), in preparation

Origin of comb-like behavior

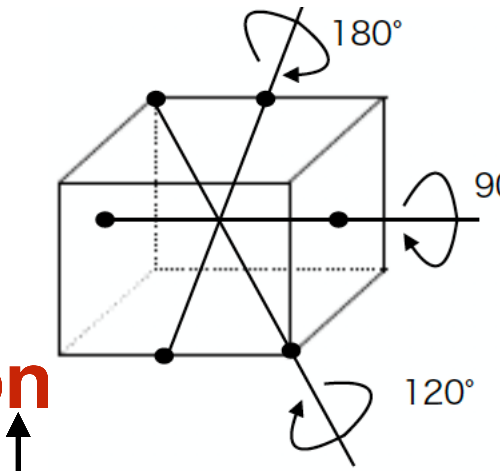


comb-like behavior occurs at some points with $|x_i|=r$ which cannot be connected via cubic rotation.
ex) (1,2,2), (3,0,0)

If higher partial wave components were negligible,
the wave function and its potential should have been isotropic.

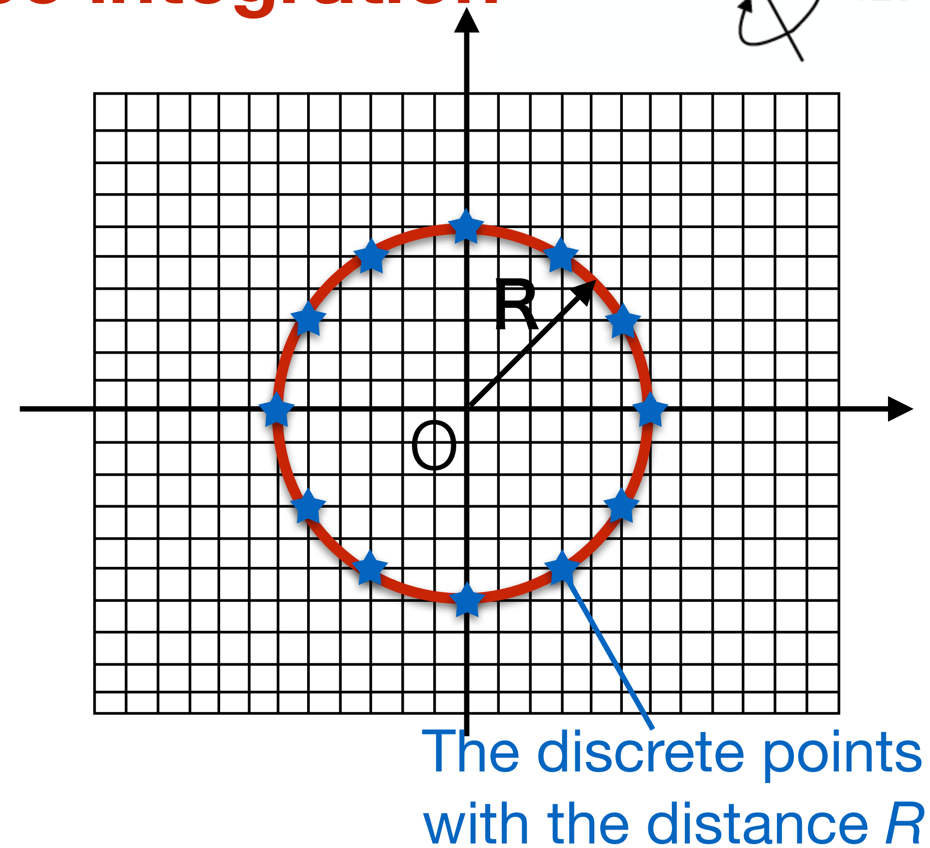
The comb-like behavior = higher partial wave contributions

$$\psi_{\rm NBS}(\vec{x}) \xrightarrow{\text{S-wave projection}} \psi^{\rm L=0}_{\rm NBS}(\vec{x})$$



➡ **Continuum space O(3,R): Spherical surface integration**

$$\psi^{\rm L=0}(R) = \int_S d\Omega\ Y^*_{00}(\theta,\phi)\psi(\vec{x}; r=R)$$



➡ **Discrete space O(3,Z): A_1^+ projection**

Cubic rotation average + Parity average

$$\psi^{A_1^+}(\vec{x}) \equiv P^{A_1^+}\psi(\vec{x}) = \frac{1}{48}\sum_{g\in O_h}\psi(g^{-1}\vec{x})$$

A_1^+ representation includes also $l \geq 4$

ℓ	P	A_1	A_2	E	T_1	T_2
0 (S)	+	1	0	0	0	0
1 (P)	−	0	0	0	1	0
2 (D)	+	0	0	1	0	1
3 (F)	−	0	1	0	1	1
4 (G)	+	1	0	1	1	1
5 (H)	−	0	0	1	2	1
6 (I)	+	1	1	1	1	2

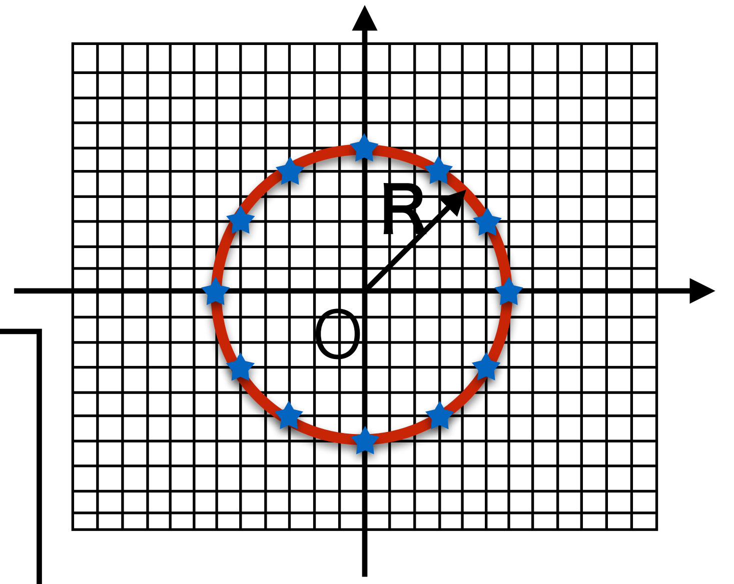
At 2 points x_1, x_2 **s.t.** $|x_1| = |x_2| = R$

which cannot be connected via cubic rotation

$$\psi^{A_1^+}(\vec{x}_1) \neq \psi^{A_1^+}(\vec{x}_2)$$

Using the different values, we can extract each component from A_1^+ projected NBS wave function.

Naive treatment:
Decomposition at fixed r



After A_1^+ projection

$$\begin{aligned}\psi^{A_1^+}(\vec{x}) &\equiv P^{A_1^+}\psi(\vec{x}) \\ &= Y_{00}^{A_1^+}(\theta,\phi)g_{00}(r) + \sum_{m=0,\pm 4} Y_{4m}^{A_1^+}(\theta,\phi)g_{4m}(r) + \cdots,\end{aligned}$$

$$Y_{00}^{A_1^+}(x,y,z) = Y_{00}(x,y,z) = \frac{1}{\sqrt{4\pi}},$$

$$Y_{40}^{A_1^+}(x,y,z) = \frac{7}{8\sqrt{\pi}}\frac{x^4+y^4+z^4-3(x^2y^2+y^2z^2+z^2x^2)}{r^4}, \quad Y_{4,+4}^{A_1^+}(x,y,z) = Y_{4,-4}^{A_1^+}(x,y,z) = \sqrt{\frac{5}{14}}Y_{40}^{A_1^+}(x,y,z)$$

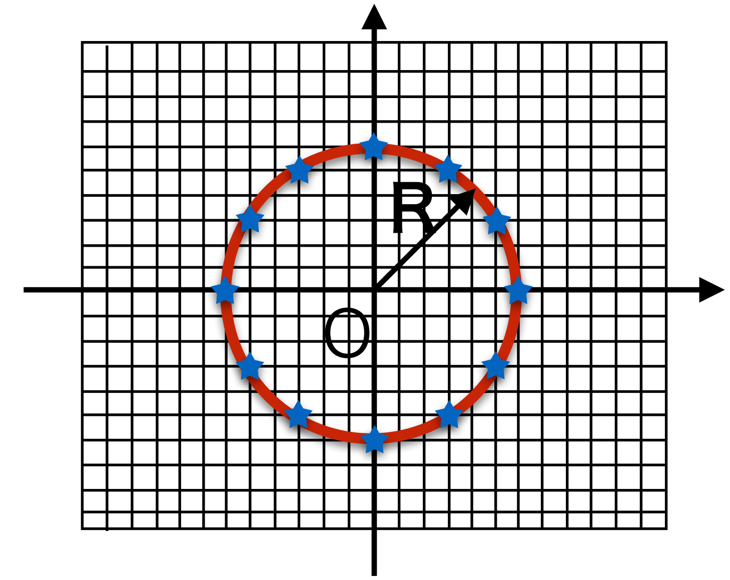
Suppose $l \geq 6$ components are neglected.

At x_1, x_2 , the eq. is written as

$$\begin{pmatrix}\psi^{A_1^+}(\vec{x}_1) \\ \psi^{A_1^+}(\vec{x}_2)\end{pmatrix} = \begin{pmatrix}Y_{00}^{A_1^+} Y_{40}^{A_1^+}(\vec{x}_1) \\ Y_{00}^{A_1^+} Y_{40}^{A_1^+}(\vec{x}_2)\end{pmatrix}\begin{pmatrix}g_{00}(R) \\ g_4(R)\end{pmatrix} \qquad g_4(r) \equiv g_{40}(r) + \sqrt{\frac{5}{14}}(g_{44}(r) + g_{4-4}(r))$$

$g_{00}(R), g_4(R)$ are obtained

Naive treatment: Decomposition at fixed r



In general case: spherical functions up to Y_{n0}

Consider N points s.t. $|x_1| = |x_2| = \cdots = |x_N| = R$

$$\begin{pmatrix} \psi^{A_1^+}(\vec{x}_1) \\ \vdots \\ \psi^{A_1^+}(\vec{x}_N) \end{pmatrix} = \begin{pmatrix} Y_{00}^{A_1^+} & Y_{40}^{A_1^+}(\vec{x}_1) & Y_{60}^{A_1^+}(\vec{x}_1) & \cdots \\ \vdots & \vdots & \vdots & \vdots \\ Y_{00}^{A_1^+} & Y_{40}^{A_1^+}(\vec{x}_N) & Y_{60}^{A_1^+}(\vec{x}_N) & \cdots \end{pmatrix} \begin{pmatrix} g_{00}(R) \\ g_4(R) \\ g_6(R) \\ \vdots \end{pmatrix}$$

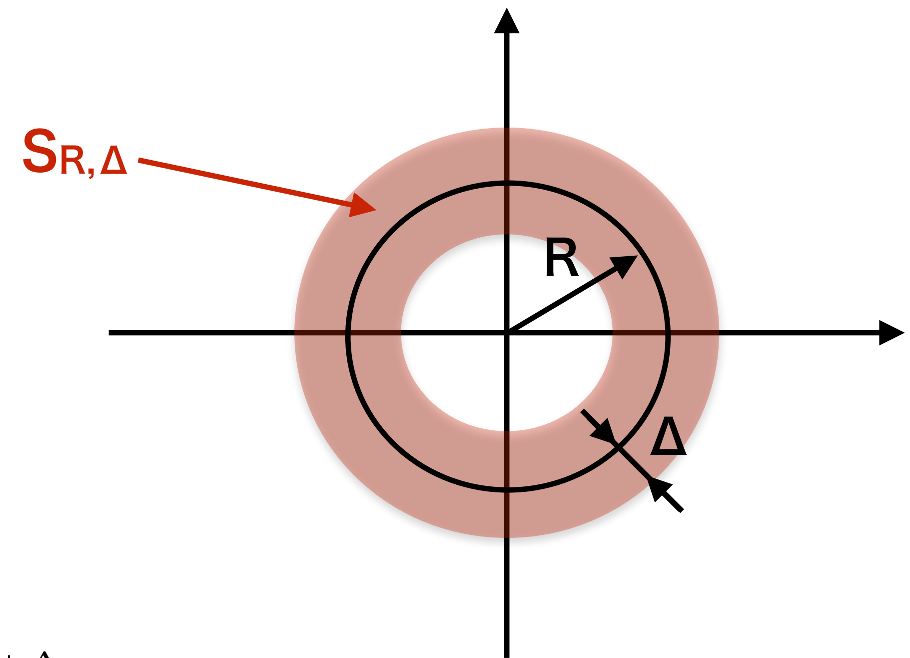
- **Using SVD, the components g_l are extracted from N points**
- **At least # points (N) ≧ # spherical functions (n)**
- **#points (N) at fixed r is not large.**

Misner's method in continuum space

Charles. W. Misner, Class. Quantum Grav. **21** (2004) S243-S247

To overcome this problem, we utilize points inside a spherical shell.

Let us first consider continuum space.



A complete orthonormal set of functions on the shell $S_{R,\Delta} = \{\vec{x} \mid R-\Delta \le |\vec{x}| \le R+\Delta\}$

$$\mathcal{Y}^{R,\Delta}_{nlm}(r,\theta,\phi) \equiv G^{R,\Delta}_n(r) Y_{lm}(\theta,\phi)$$

$$G^{R,\Delta}_n(r) \equiv P_n\left(\frac{r-R}{\Delta}\right)\frac{1}{r}\sqrt{\frac{2n+1}{2\Delta}}$$

Legendre polynomial

$$\int_{S_{R,\Delta}} d^3x\, \overline{\mathcal{Y}^{R,\Delta}_{nlm}(\theta,\phi)}\mathcal{Y}^{R,\Delta}_{n'l'm'}(r,\theta,\phi) = \delta_{nn'}\delta_{ll'}\delta_{mm'}$$

$$\int_{R-\Delta}^{R+\Delta} dr\, r^2\, G^{R,\Delta}_n(r) G^{R,\Delta}_m(r) = \delta_{nm}$$

n	$P_n(x)$
0	1
1	x
2	$\frac{1}{2}(3x^2-1)$
3	$\frac{1}{2}(5x^3-3x)$
4	$\frac{1}{8}(35x^4-30x^2+3)$
5	$\frac{1}{8}(63x^5-70x^3+15x)$
6	$\frac{1}{16}(231x^6-315x^4+105x^2-5)$
7	$\frac{1}{16}(429x^7-693x^5+315x^3-35x)$
8	$\frac{1}{128}(6435x^8-12012x^6+6930x^4-1260x^2+35)$
9	$\frac{1}{128}(12155x^9-25740x^7+18018x^5-4620x^3+315x)$
10	$\frac{1}{256}(46189x^{10}-109395x^8+90090x^6-30030x^4+3465x^2-63)$

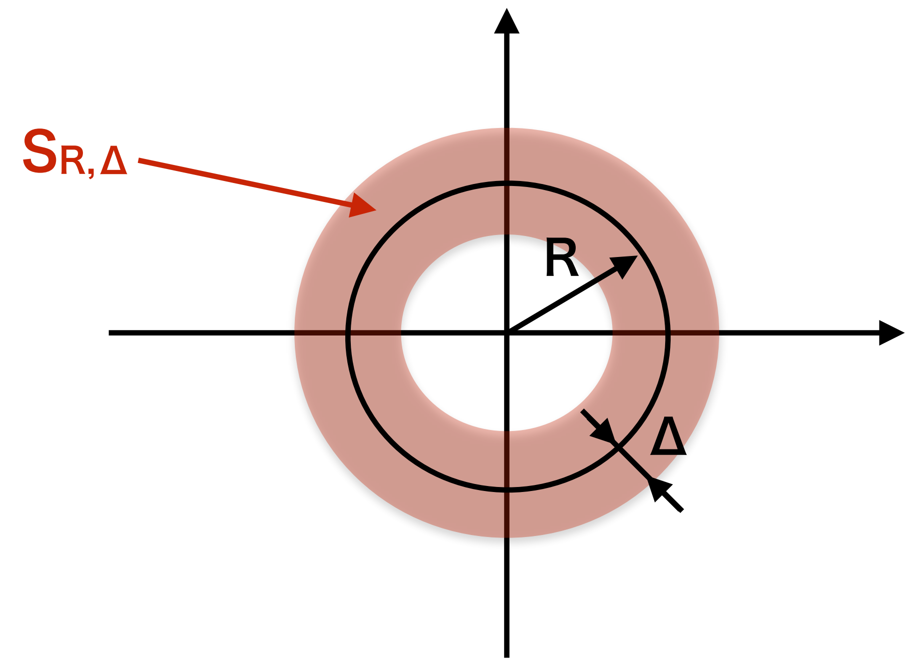
Misner's method in continuum space

Charles. W. Misner, Class. Quantum Grav. **21** (2004) S243-S247

Inside the shell, the wave function is expanded by

$$\psi(r,\theta,\phi) = \sum_{n=0}^{\infty}\sum_{l=0}^{\infty}\sum_{m=-l}^{l} c_{nlm}^{R,\Delta}\, \mathcal{Y}_{nlm}^{R,\Delta}(r,\theta,\phi)$$

$$= \sum_{l=0}^{\infty}\sum_{m=-l}^{l} g_{lm}(r) Y_{lm}(\theta,\phi)$$



$c_{nlm}^{R,\Delta}$ **is determined by the integration over the shell:**

$$c_{nlm}^{R,\Delta} = \int_{S_{R,\Delta}} d^3x\, \overline{\mathcal{Y}_{nlm}^{R,\Delta}(r,\theta,\phi)}\, \psi(r,\theta,\phi)$$

$$S_{R,\Delta} = \{\vec{x} \mid R-\Delta \le |\vec{x}| \le R+\Delta\}$$

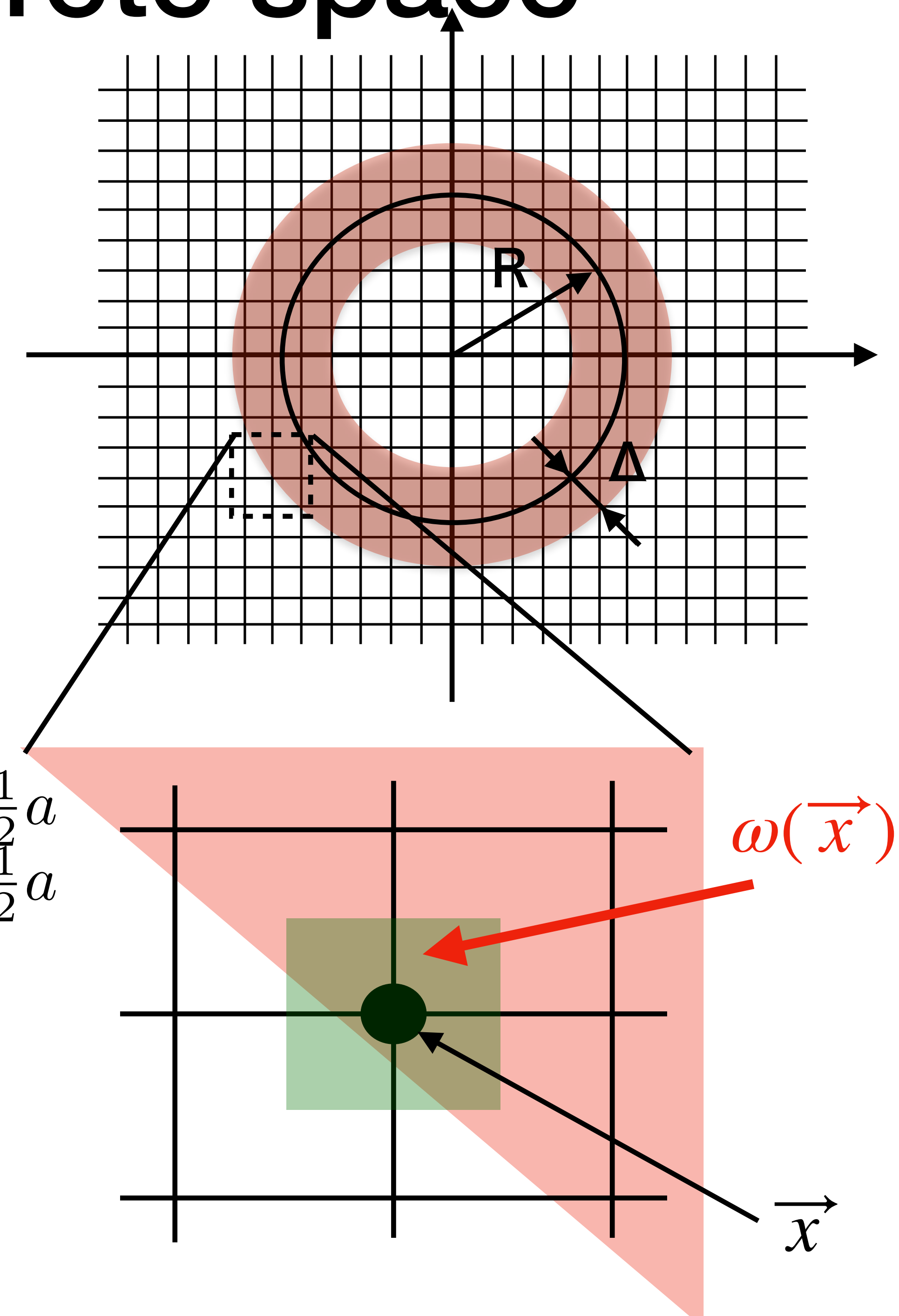
$$\mathcal{Y}_{nlm}^{R,\Delta}(r,\theta,\phi) \equiv G_n^{R,\Delta}(r) Y_{lm}(\theta,\phi)$$

The components of the partial wave inside the shell are obtained by

$$g_{lm}(r) = \sum_{n=0}^{\infty} c_{nlm}^{R,\Delta}\, G_n^{R,\Delta}(r)$$

Misner's method in discrete space

Charles. W. Misner, Class. Quantum Grav. **21** (2004) S243-S247



The volume integration is replaced by

$$\int_{S_{R,\Delta}} d^3x \Longrightarrow \sum_{\vec{x}} \omega^{R,\Delta}(\vec{x})$$

An approximate choice of the weight function

$$\omega^{R,\Delta}(\vec{x}) = \begin{cases} a^3 & \text{for } |r-R| < \Delta - \frac{1}{2}a \\ 0 & \text{for } |r-R| > \Delta + \frac{1}{2}a \\ a^2\left(\Delta + \frac{1}{2}a - |R-r|\right) & \text{otherwise} \end{cases}$$

<u>overlap region between the shell and a lattice cube</u>

The inner product on lattice

$$\langle f|g\rangle_{S_{R,\Delta}} \equiv \sum_{\vec{x}} \omega^{R,\Delta}(\vec{x})\ \overline{f(\vec{x})}\ g(\vec{x})$$

Misner's method in discrete space

Charles. W. Misner, Class. Quantum Grav. **21** (2004) S243-S247

Because of finite points, orthonormality is broken:

$$\langle \mathcal{Y}^{R,\Delta}_{nlm} | \mathcal{Y}^{R,\Delta}_{n'l'm'} \rangle_{S_{R,\Delta}} \neq \delta_{n,n'} \delta_{l,l'} \delta_{m,m'}$$

$$\langle \mathcal{Y}^{R,\Delta}_{A} | \mathcal{Y}^{R,\Delta}_{B} \rangle_{S_{R,\Delta}} = \mathcal{G}_{AB}$$

Dual basis

$$\tilde{\mathcal{Y}}^{R,\Delta}_{A}(\vec{x}) \equiv \sum_{B}{}' \mathcal{Y}^{R,\Delta}_{B}(\vec{x})\ \mathcal{G}^{-1}_{BA}$$

$$A = n, l, m,\ B = n', l', m'$$

$$\sum_{B}{}' \equiv \sum_{n=0}^{n_{max}} \sum_{l=0}^{l_{max}} \sum_{m=-l}^{l}$$

To get $\mathcal{G}^{-1}_{BA}$, the restriction of summation (l_{max}, n_{max}) is introduced.

This satisfies orthonormality for $l \leqq l_{max}$, $n \leqq n_{max}$:

$$\langle \tilde{\mathcal{Y}}^{R,\Delta}_{A} | \mathcal{Y}^{R,\Delta}_{B} \rangle_{S_{R,\Delta}} = \sum_{C}{}' \mathcal{G}^{-1}_{AC} \langle \mathcal{Y}^{R,\Delta}_{C} | \mathcal{Y}^{R,\Delta}_{B} \rangle_{S_{R,\Delta}} = \sum_{C}{}' \mathcal{G}^{-1}_{AC} \mathcal{G}_{CB} = \delta_{AB}$$

Misner's method in discrete space

Charles. W. Misner, Class. Quantum Grav. **21** (2004) S243-S247

Suppose that the components higher than $l_{\max}$, $n_{\max}$ are negligibly small:

$$\psi(\vec{x}) \simeq \sum_{n=0}^{n_{\max}} \sum_{l=0}^{l_{\max}} \sum_{m=-l}^{l} c_{nlm}^{R,\Delta}\, \mathcal{Y}_{nlm}^{R,\Delta}(r,\theta,\phi)$$

$c_{nlm}^{R,\Delta}$ **are obtained from**

$$c_{nlm}^{R,\Delta} = \langle \tilde{\mathcal{Y}}_{nlm}^{R,\Delta} | \psi \rangle_{S_{R,\Delta}}.$$

Components of partial wave expansion in the shell are

$$g_{lm}(r) \simeq \sum_{n=0}^{n_{\max}} c_{nlm}^{R,\Delta} G_n^{R,\Delta}(r), \quad R-\Delta < r < R+\Delta$$

Using this form, Laplacian can be calculated analytically

$$\vec{\nabla}^2 g_{lm}(r) = \sum_{n=0}^{n_{\max}} c_{nlm}^{R,\Delta} \frac{1}{r} \frac{\partial^2}{\partial r^2} \left[r G_n^{R,\Delta}(r) \right]$$

$$G_n^{R,\Delta}(r) \equiv P_n\left(\frac{r-R}{\Delta}\right) \frac{1}{r} \sqrt{\frac{2n+1}{2\Delta}}$$

Misner's method vs fixed-r method

Zero-shell limit (fixed-r limit) for Misner method

$$A = n, l, m$$

$$\mathcal{Y}^{R,\Delta}_{nlm}(r,\theta,\phi) \equiv G^{R,\Delta}_{n}(r) Y_{lm}(\theta,\phi)$$

$$\mathcal{G}_{AA'} \equiv \langle \mathcal{Y}^{R,\Delta}_{A} | \mathcal{Y}^{R,\Delta}_{A'} \rangle_{S_{R,\Delta}} \to G_{lm,l'm'} \equiv \langle Y_{lm} | Y_{l'm'} \rangle$$

$$\langle f|g \rangle_{S_{R,\Delta}} = \sum_{\vec{x}} \omega^{R,\Delta}(\vec{x})\ \overline{f(\vec{x})}\ g(\vec{x}) \to \langle f|g \rangle_{|\vec{x}|=R} = \sum_{|\vec{x}|=R} \overline{f(\vec{x})}\ g(\vec{x})$$

Dual basis $$\tilde{\mathcal{Y}}^{R,\Delta}_{A}(\vec{x}) \to \tilde{Y}_{lm}(\theta,\phi) \equiv \sum_{l',m'} Y_{l'm'}(\theta,\phi)\ G^{-1}_{l'm',lm}$$

$$g_{lm} = \langle \tilde{Y}_{lm} | \psi \rangle_{|\vec{x}|=R}$$

$$\begin{pmatrix} \psi^{A_1^+}(\vec{x}_1) \\ \vdots \\ \psi^{A_1^+}(\vec{x}_N) \end{pmatrix} = \begin{pmatrix} Y^{A_1^+}_{00} & Y^{A_1^+}_{40}(\vec{x}_1) & Y^{A_1^+}_{60}(\vec{x}_1) & \cdots \\ \vdots & \vdots & \vdots & \vdots \\ Y^{A_1^+}_{00} & Y^{A_1^+}_{40}(\vec{x}_N) & Y^{A_1^+}_{60}(\vec{x}_N) & \cdots \end{pmatrix} \begin{pmatrix} g_{00}(R) \\ g_4(R) \\ g_6(R) \\ \vdots \end{pmatrix}$$

Fixed-r method

Misner's method vs fixed-r method

Zero-shell limit (fixed-r limit) for Misner method

$$A = n, l, m$$

$$\mathcal{Y}^{R,\Delta}_{nlm}(r,\theta,\phi) \equiv G^{R,\Delta}_{n}(r) Y_{lm}(\theta,\phi)$$

$$\mathcal{G}_{AA'} \equiv \langle \mathcal{Y}^{R,\Delta}_{A} | \mathcal{Y}^{R,\Delta}_{A'} \rangle_{S_{R,\Delta}} \rightarrow G_{lm,l'm'} \equiv \langle Y_{lm} | Y_{l'm'} \rangle$$

$$\langle f | g \rangle_{S_{R,\Delta}} = \sum_{\vec{x}} \omega^{R,\Delta}(\vec{x}) \ \overline{f(\vec{x})} \ g(\vec{x}) \rightarrow \langle f | g \rangle_{|\vec{x}|=R} = \sum_{|\vec{x}|=R} \overline{f(\vec{x})} \ g(\vec{x})$$

Dual basis $\tilde{\mathcal{Y}}^{R,\Delta}_{A}(\vec{x}) \rightarrow \tilde{Y}_{lm}(\theta,\phi) \equiv \sum_{l',m'} Y_{l'm'}(\theta,\phi) \ G^{-1}_{l'm',lm}$

$$g_{lm} = \langle \tilde{Y}_{lm} | \psi \rangle_{|\vec{x}|=R}$$

$$\Leftrightarrow \begin{pmatrix} \psi^{A_1^+}(\vec{x}_1) \\ \vdots \end{pmatrix} = \begin{pmatrix} Y^{A_1^+}_{00} & Y^{A_1^+}_{40}(\vec{x}_1) & Y^{A_1^+}_{60}(\vec{x}_1) & \cdots \\ \vdots & \vdots & \vdots & \vdots \end{pmatrix} \begin{pmatrix} g_{00}(R) \\ g_4(R) \\ \vdots \end{pmatrix}$$

Misner's method= extension of fixed-r method to include points inside shell

test calculation 1: check the decomposition

Ex)

$$\psi_0(r) \equiv 2 - e^{-\frac{r^2}{60}}$$

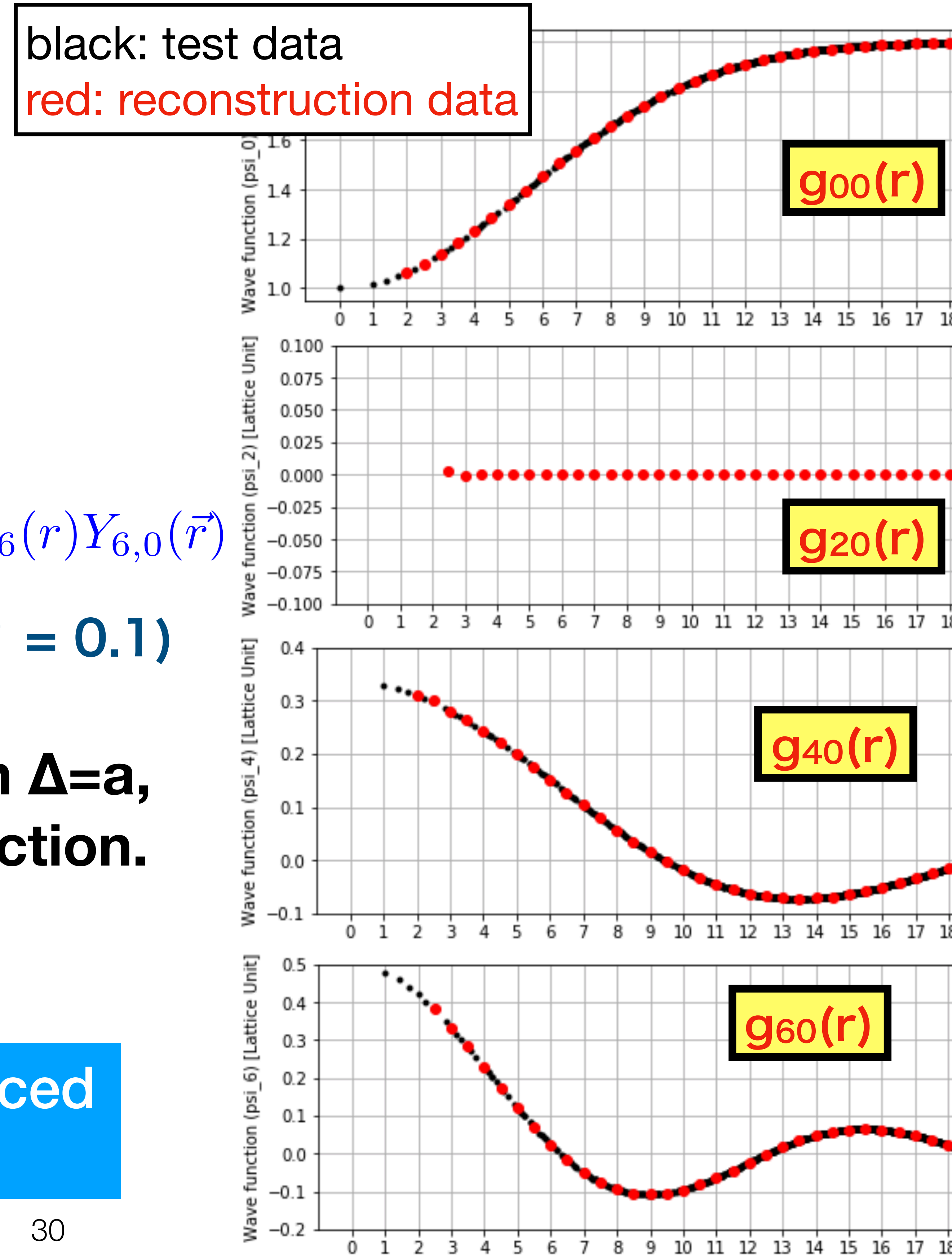
$$\psi_4(r) \equiv \frac{\sin(r/3)}{r}$$

$$\psi_6(r) \equiv \frac{\sin(r/2)}{r}$$

$$\psi(\vec{r}) \equiv \psi_0(r) Y_{0,0}(\vec{r}) + \alpha \psi_4(r) Y_{4,0}(\vec{r}) + \beta \psi_6(r) Y_{6,0}(\vec{r})$$

$(\alpha = 0.2, \ \beta = 0.1)$

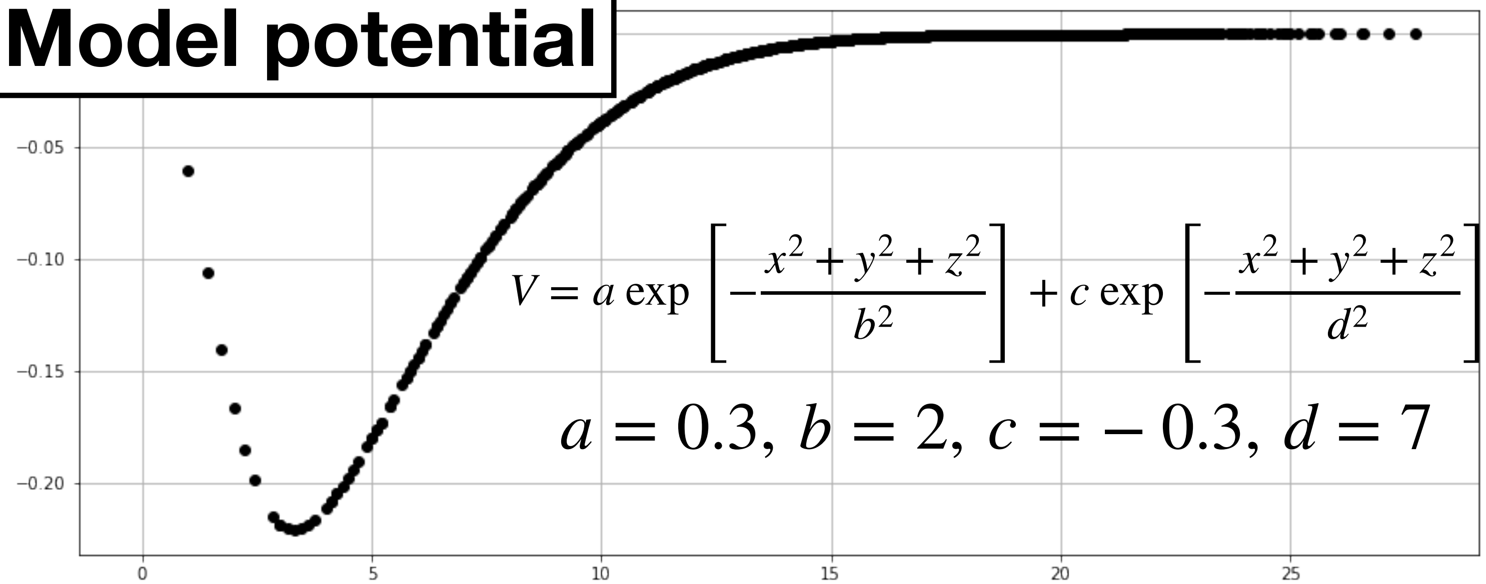
We apply Misner's method with Δ=a, n_{max}=2, l_{max}=6 to this wave function.



All components were reproduced by Misner's method

test calculation 2: solve Hamiltonian

Model potential



$$V = a\exp\left[-\frac{x^2+y^2+z^2}{b^2}\right] + c\exp\left[-\frac{x^2+y^2+z^2}{d^2}\right]$$

$$a = 0.3,\ b = 2,\ c = -0.3,\ d = 7$$

1. $\left[H_0 + V(r)\right]\psi_n^{A_1^+} = E\psi_n^{A_1^+}$

2. $\psi_n^{A_1^+} \to \psi_{n,l}(l = 0,4,\ldots)$

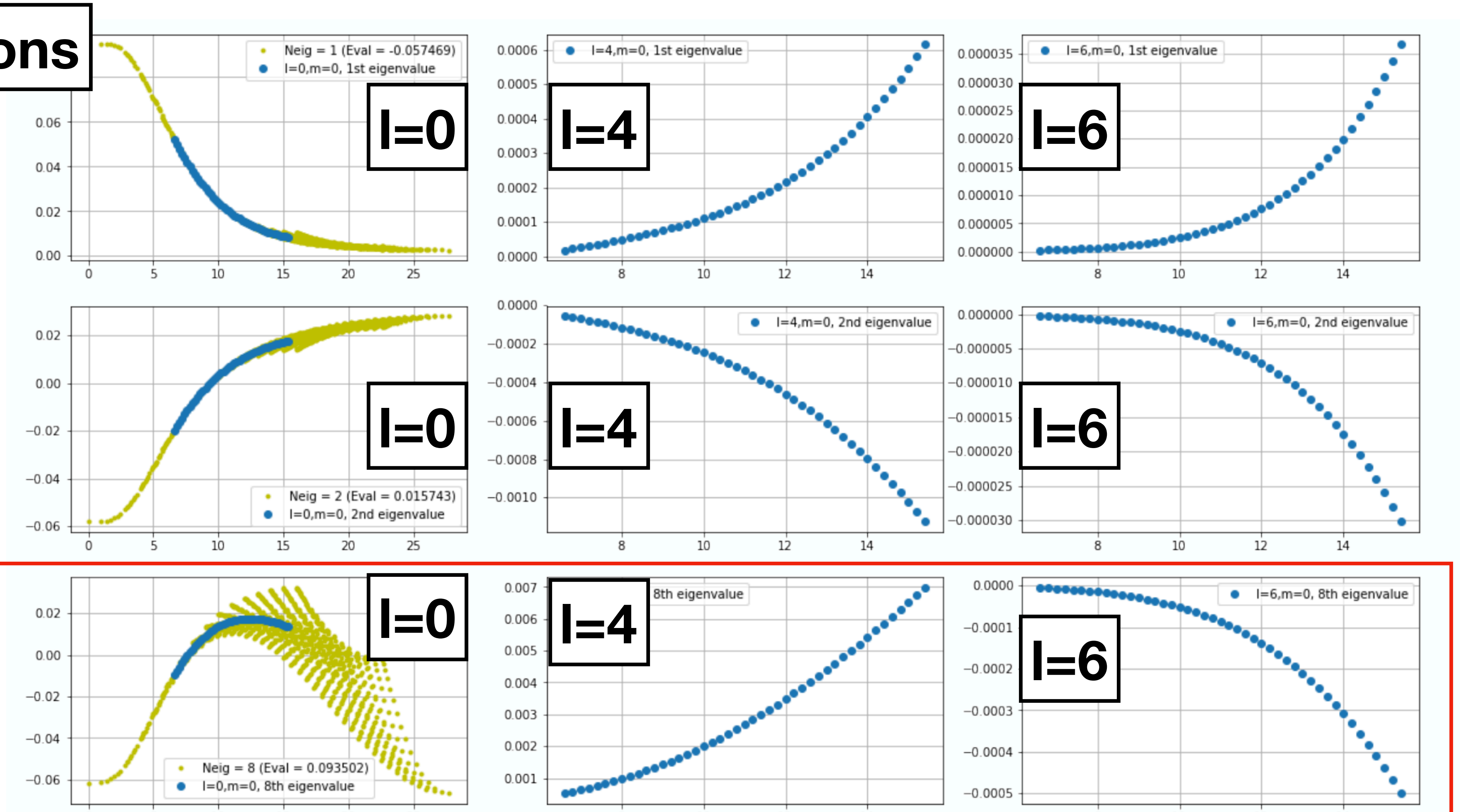
3. $V(r) = \dfrac{\left[E - H_0\right]\psi_{n,l}}{\psi_{n,l}}$

wave functions

1st

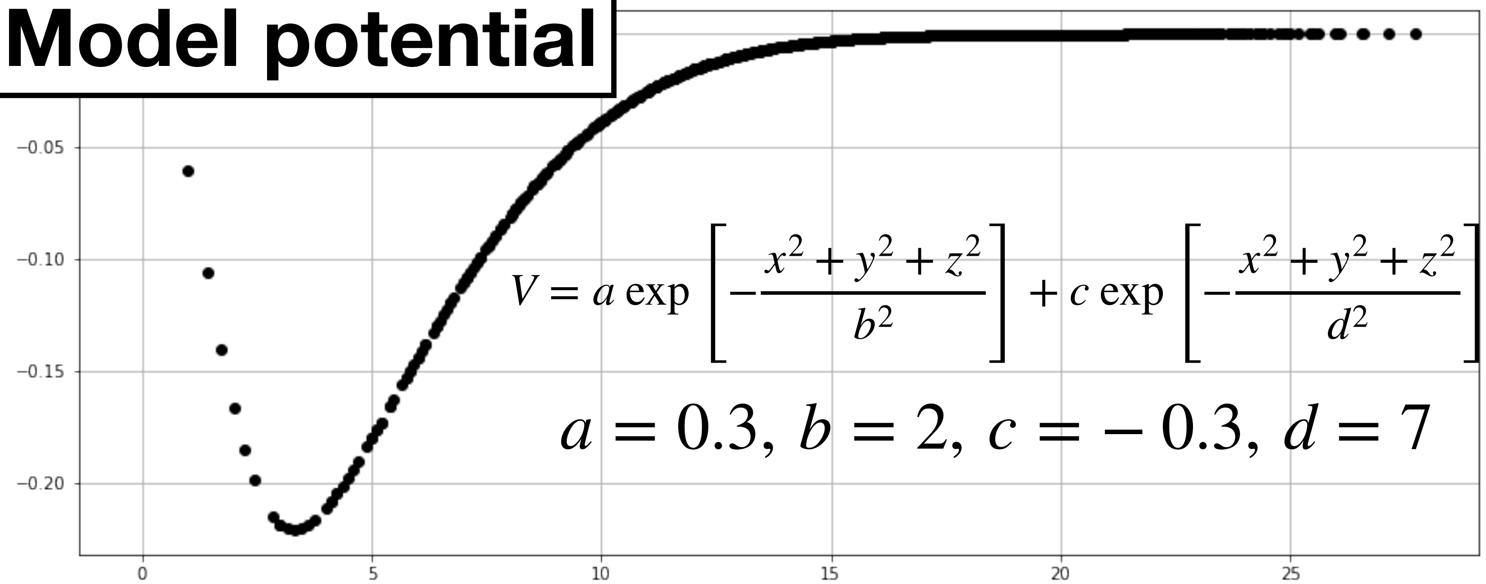
2nd

8th



test calculation 2: solve Hamiltonian

Model potential

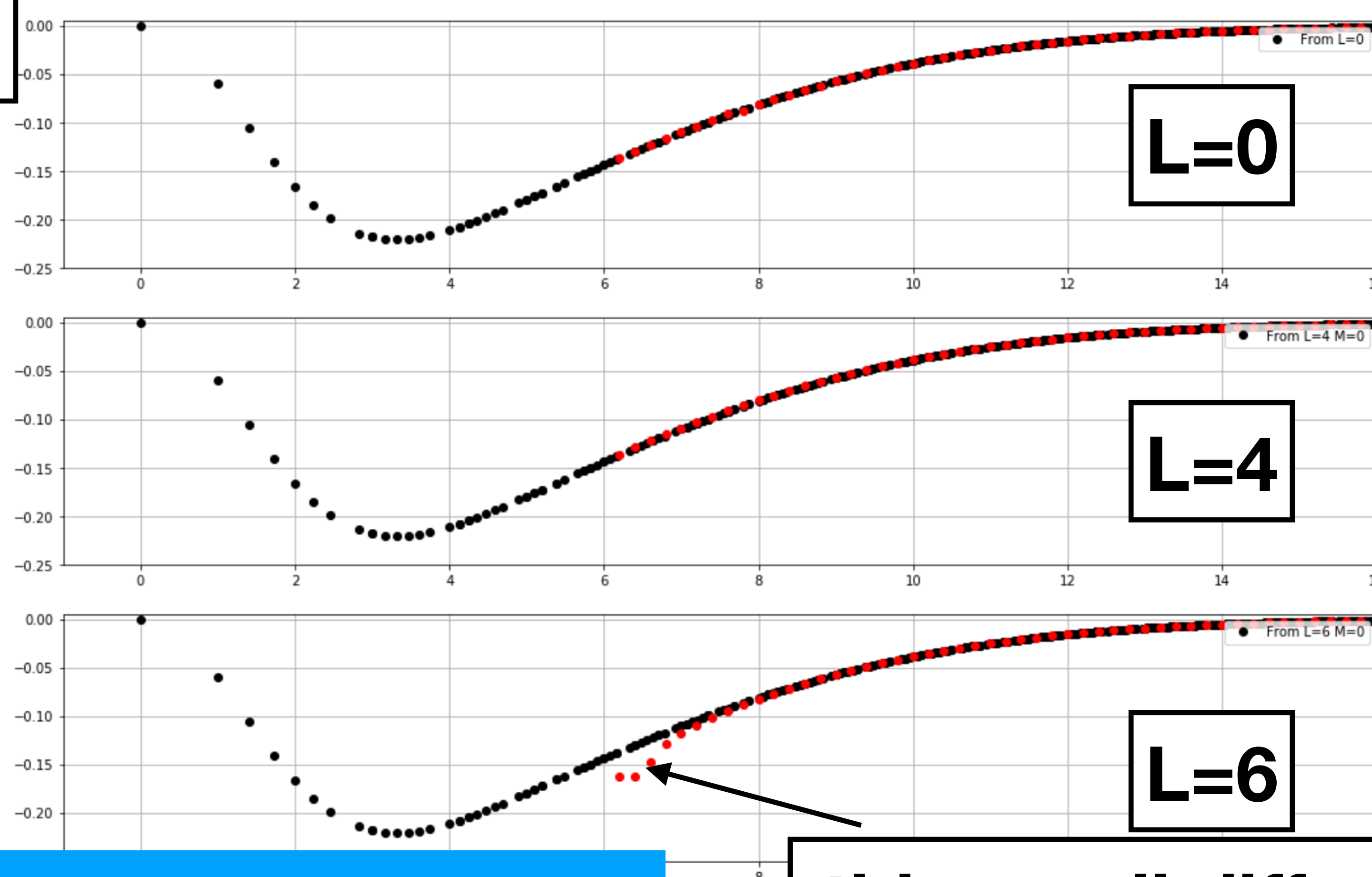


1. $\left[H_0 + V(r)\right] \psi_n^{A_1^+} = E \psi_n^{A_1^+}$

2. $\psi_n^{A_1^+} \rightarrow \psi_{n,l}(l = 0,4,\ldots)$

3. $V(r) = \dfrac{\left[E - H_0\right] \psi_{n,l}}{\psi_{n,l}}$

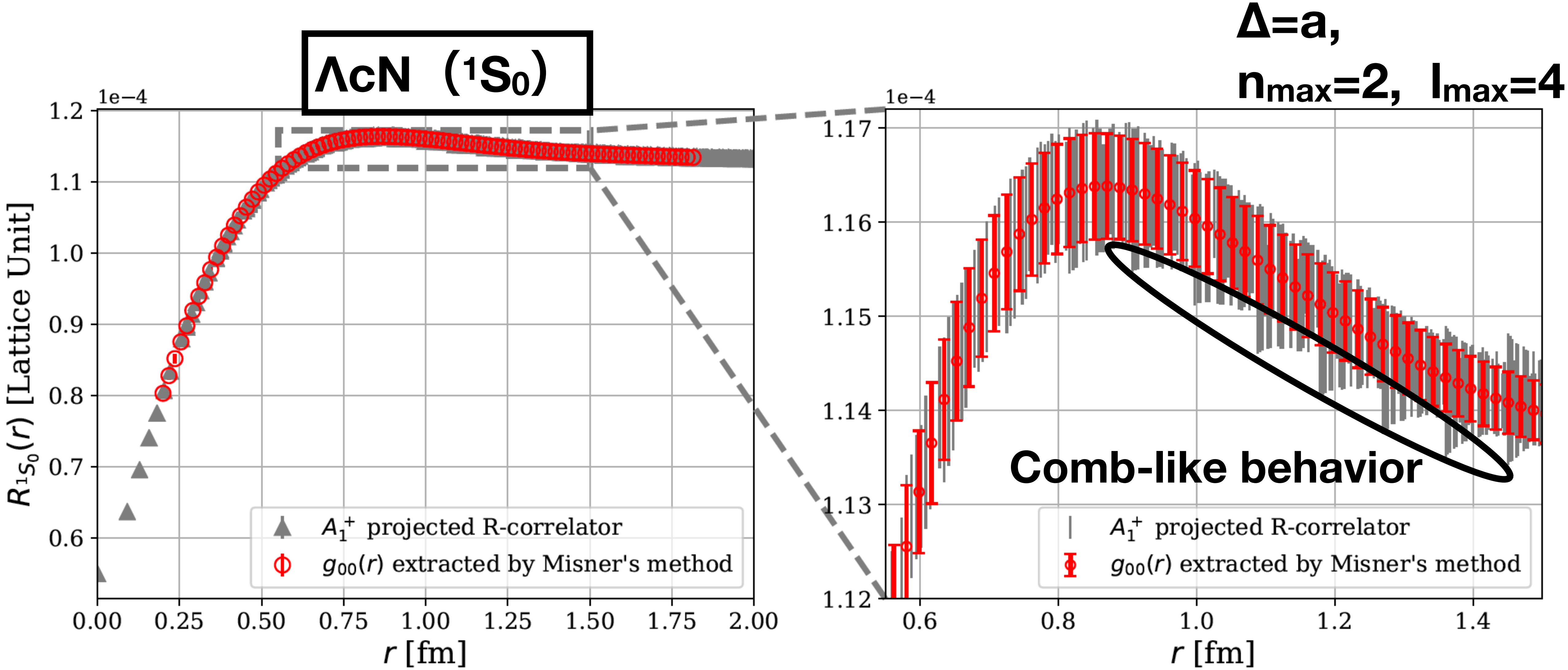
reconstruction



Even l≧4 components reproduce model potential

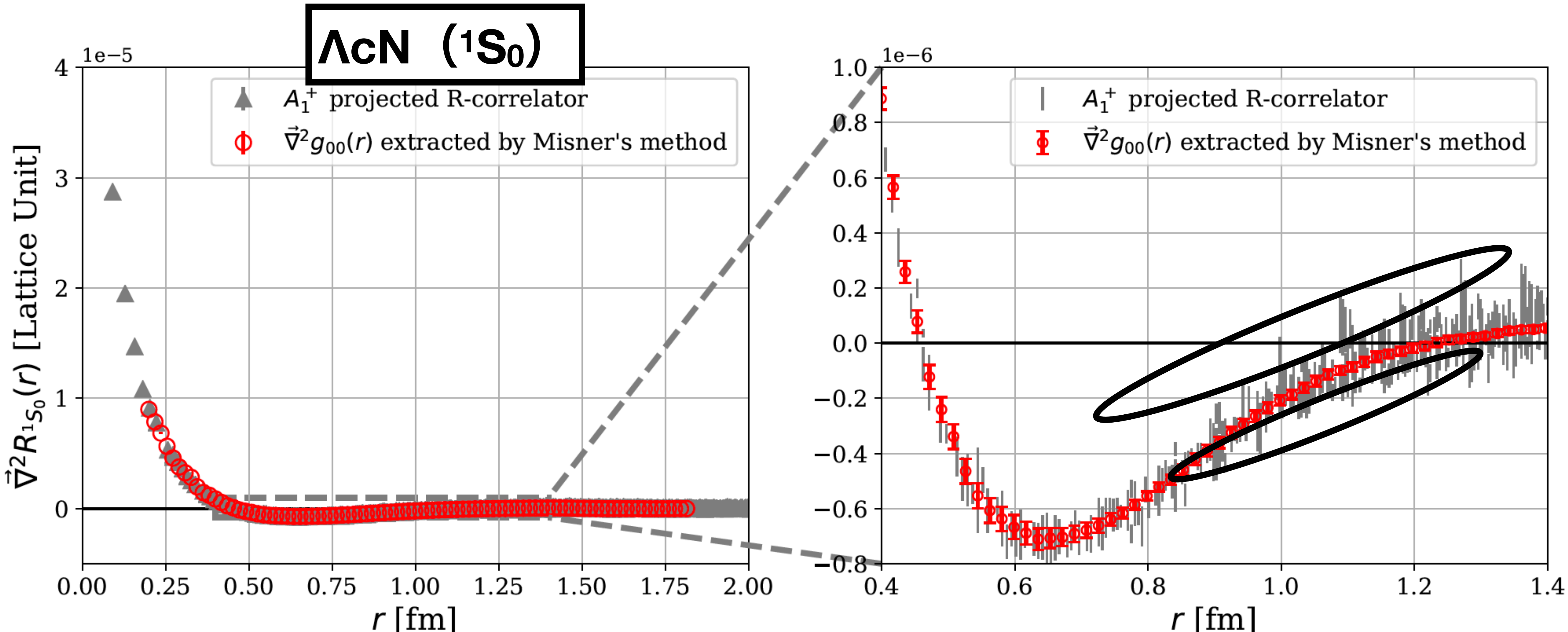
this small difference may come from less #pts for smaller r

Application to NBS wave functions



- l≧4 contributions for A_1^+ projected R-correlator can be found
- The comb-like behavior is removed by Misner's method

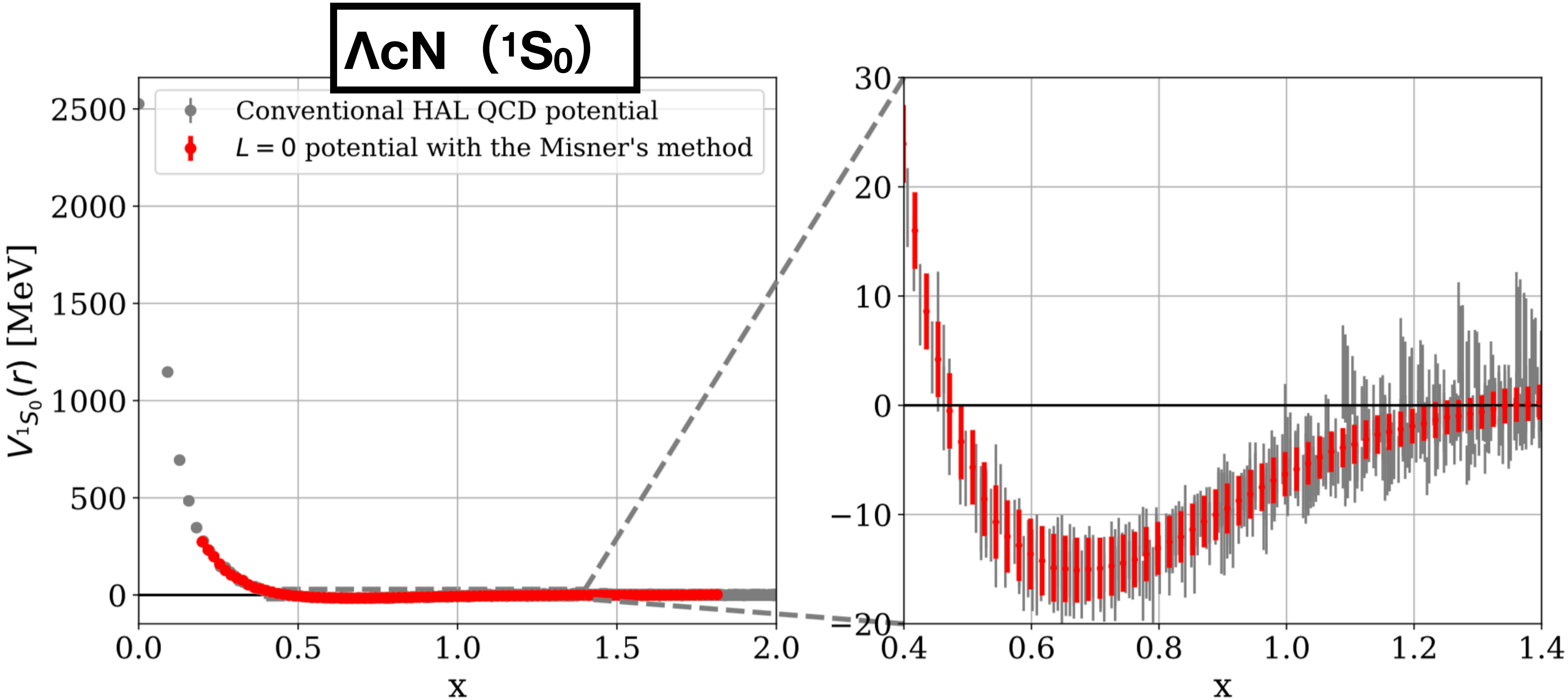
Application to Laplacian term



- **Conventional method:**
  Laplacian=> a finite second-order difference
  => comb-like fluctuation due to l≧4 is enhanced
- **Misner method:**
  Laplacian=> analytically calculable after l=0 extraction
  => The fluctuation is removed

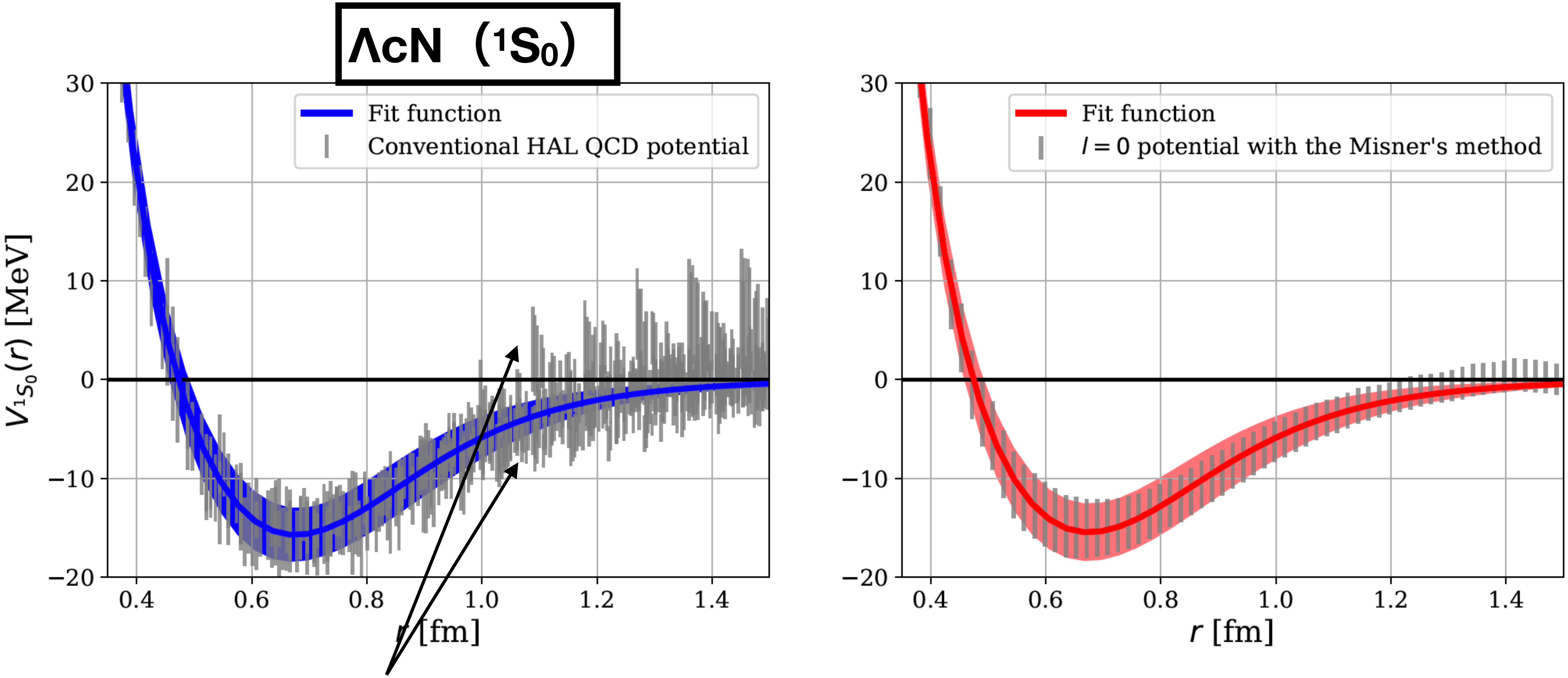
$$\vec{\nabla}^2 g_{lm}(r) = \sum_{n=0}^{n_{\max}} c_{nlm}^{R,\Delta} \frac{1}{r} \frac{\partial^2}{\partial r^2} \left[r G_n^{R,\Delta}(r) \right]$$

Application to HAL potential



- **Conventional method:**
  Enhancement of the fluctuation of laplacian due to l≧4 contributions
  => Potential has large fluctuation (comb-like behavior)
- **Misner method:**
  The fluctuation is removed because of l=0 extraction.

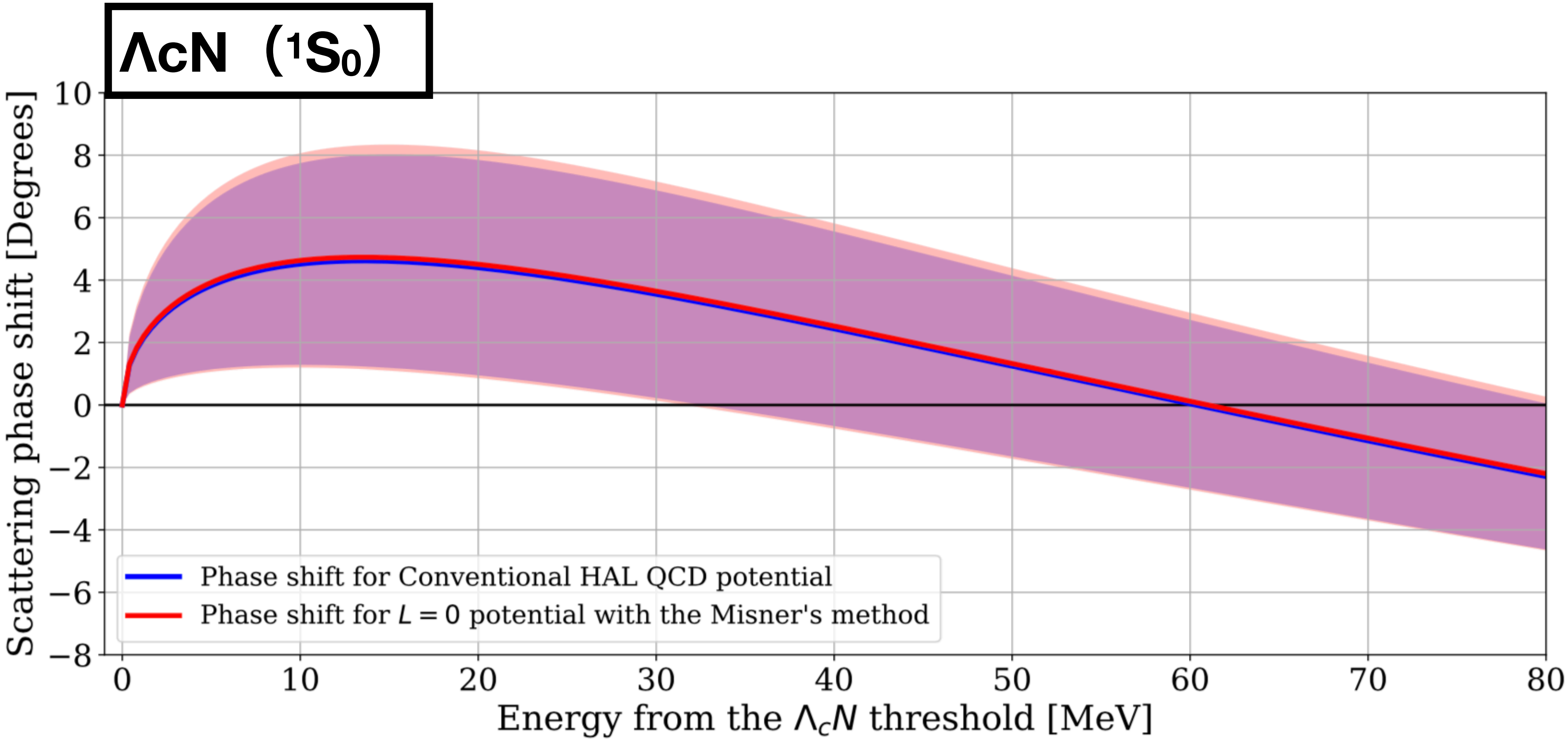
Fit results



The fluctuation is not affected the fit results largely.

Fit to pot. from A1 proj. ≒ Fit to pot. from Misner method

Fit results



The phase shifts are identical with each other.

Summary

We have succeeded in
L=0 extraction

Future works

Use Misner method to extract
higher partial waves

Many systems couple to higher partial waves