Inclusive processes from lattice QCD (?)

with the help of dispersion relation

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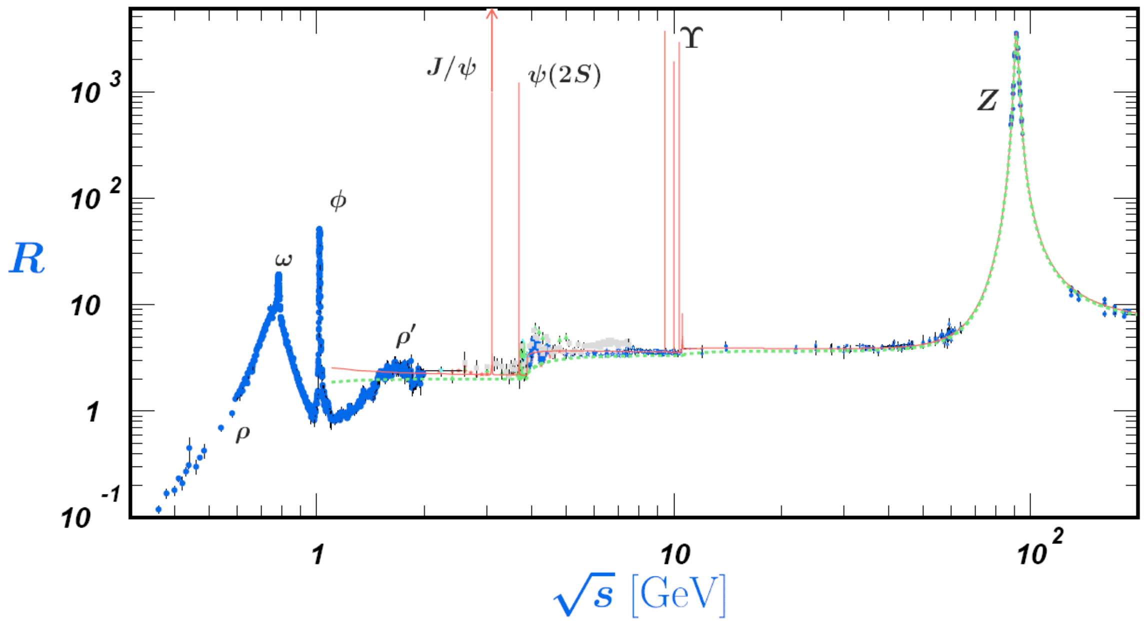
Inclusive processes
= sum over final states

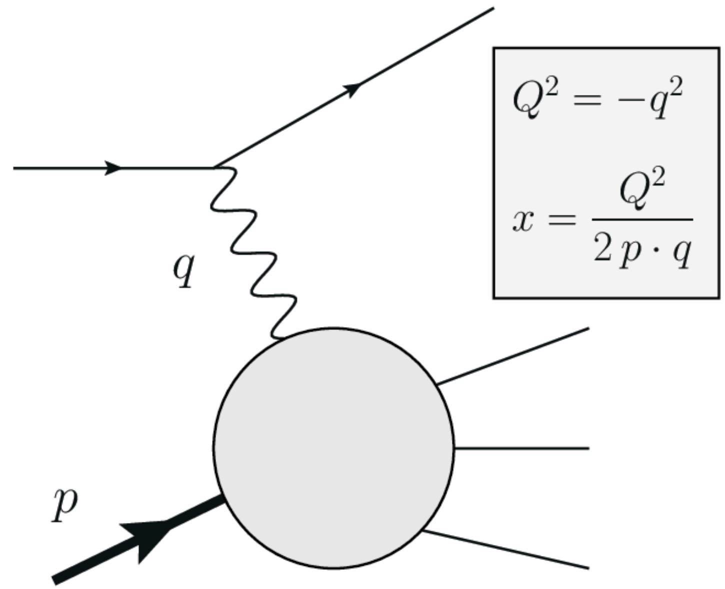
$$e^+e^- \to q\bar{q}$$



- Perturbation theory is applicable (with OPE).
- Resonance region is often ignored (too difficult).

(Deep) (In)elastic scattering

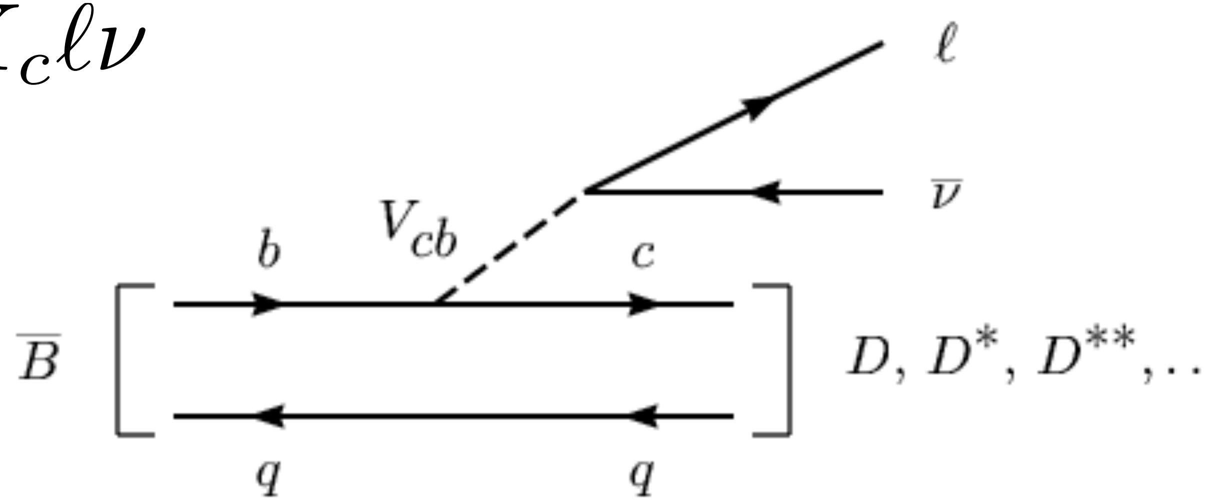
$$e^- N \to e^- X$$



- Bjorken scaling → parton model → PDF
- Resonance region is often ignored (too difficult).

Inclusive B meson decays

$$B \to X_c \ell \nu$$



- Perturbation theory is used with heavy quark expansion.
- Resonance region is often ignored (too difficult).

Lattice calculation

- Standard approach = OPE, then matrix element

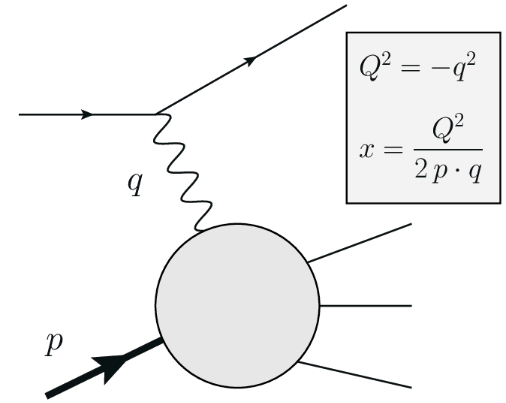
$$\frac{m\langle\bar{q}q\rangle}{Q^4} \qquad \langle x\rangle \qquad \frac{\langle B|D_\perp^2|B\rangle}{m_B^2}$$

- (more) direct calculation??

DIS as an example:

see also, QCDSF, PRL 118, 242001 (2017)

(Deep) (In)elastic scattering



see also, QCDSF, PRL 118, 242001 (2017)

<u>structure function</u>

$$W_{\mu\nu} = \frac{1}{2}\sum_{\mathrm{pol}}\sum_{X}\langle N(p)|J_\mu|X(p_X)\rangle\langle X(p_X)|J_\nu|N(p)\rangle$$
$$\times\,(2\pi)^3\delta^{(4)}(p-p_X+q)$$

optical theorem: $\boxed{W_{\mu\nu} = \frac{1}{\pi}\mathrm{Im}T_{\mu\nu}}$

<u>forward-scattering matrix element</u>

$$T_{\mu\nu} = \frac{1}{2}\sum_{\mathrm{pol}}\int d^4x\, e^{iqx}\langle N(p)|T\{J_\mu(x)J_\nu(0)\}|N(p)\rangle$$

Can we calculate this on the lattice?

$$T_{\mu\nu} = \frac{1}{2}\sum_{\mathrm{pol}}\int d^4x\, e^{iqx}\langle N(p)|T\{J_\mu(x)J_\nu(0)\}|N(p)\rangle$$

- with a fixed 4-momentum *q*

On the Euclidean lattice, one may start from

$$M_{\mu\nu}(t) \equiv \int d^3\mathbf{x}\, e^{-i\mathbf{q}\cdot\mathbf{x}}\langle N(\mathbf{p})|J_\mu(\mathbf{x},t)J_\nu(\mathbf{0},0)|N(\mathbf{p})\rangle$$

to obtain

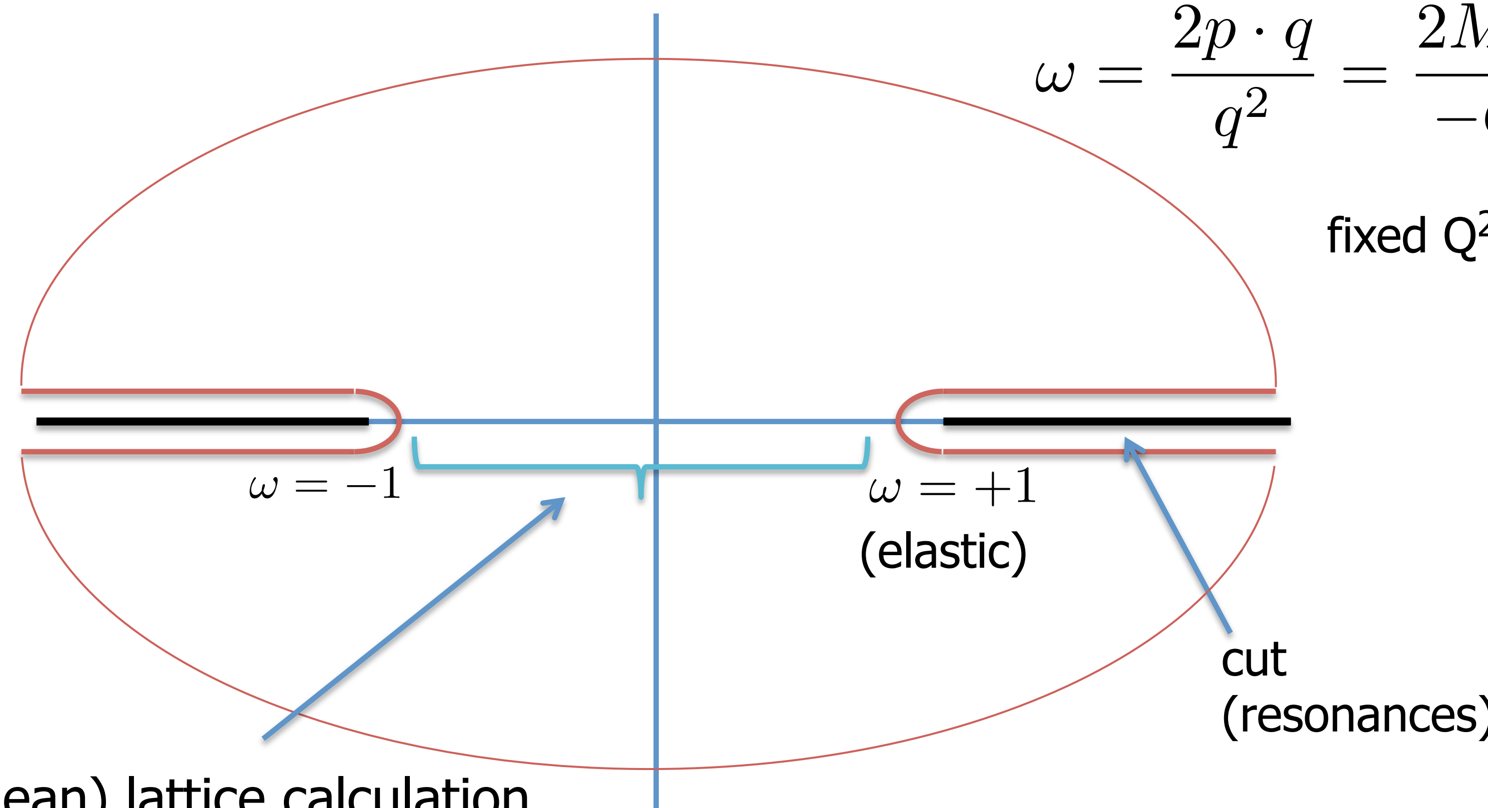
$$T_{\mu\nu}(p,q) = \int_0^\infty dt\, e^{q^0 t} M_{\mu\nu}(t)$$

Analytic continuation is allowed until one encounters any singularity.

$$T_{\mu\nu}(p,q)$$

$$\omega = \frac{2p \cdot q}{q^2} = \frac{2Mq^0}{-Q^2}$$

fixed Q^2



(Euclidean) lattice calculation
only in the region $|\omega|<1$.
(no singularity)

Contour integral (schematically…)

$$T(\omega) = \int_{-\infty}^{-1} d\omega' \frac{W(\omega')}{\omega' - \omega} + \int_{1}^{\infty} d\omega' \frac{W(\omega')}{\omega' - \omega}$$

$$= 2\omega \int_{1}^{\infty} d\omega' \frac{W(\omega')}{\omega'^2 - \omega^2}$$

$\omega' = 1/x$

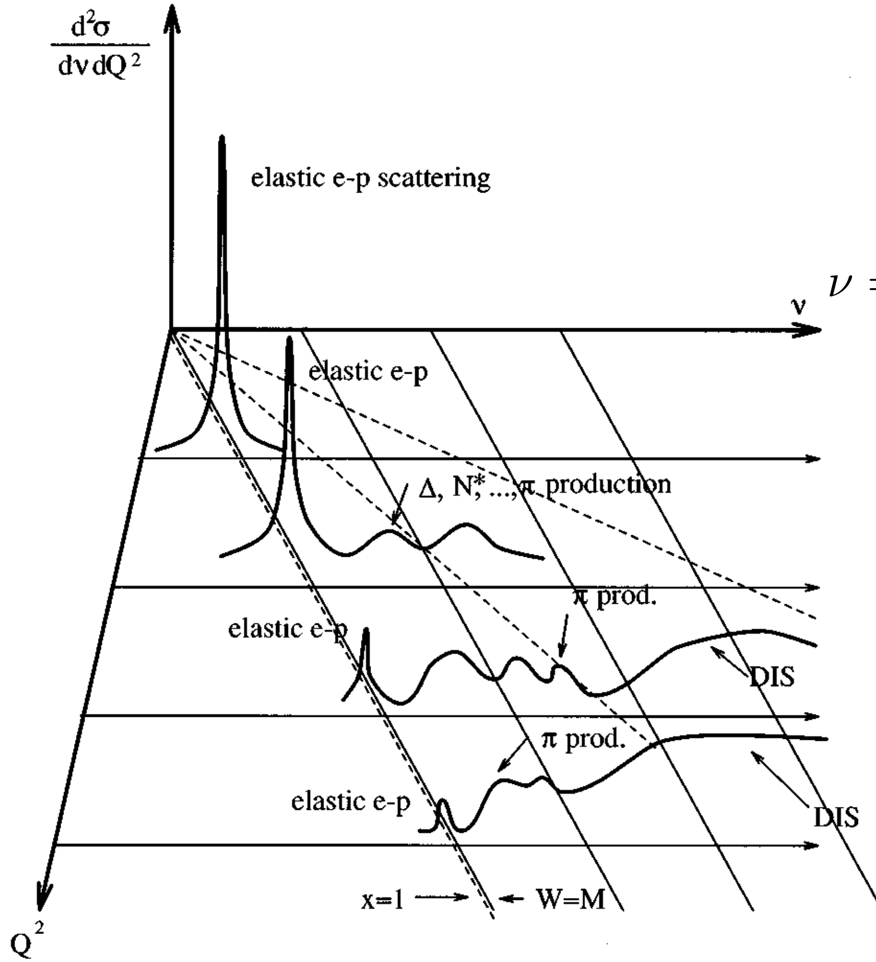
$$= 2\omega \int_{0}^{1} dx \frac{W(x)}{1 - (\omega x)^2}$$

$$= 2 \sum_{n=0}^{\infty} \omega^{2n+1} \int_{0}^{1} dx\, x^{2n} W(x)$$

moments of PDF

Calculation of T(w) at w=0 (and its derivatives) gives arbitrary moments of structure functions.

Badelek, Kwiecinski, RMP 68, 445 (1996)



$$\nu = \frac{p \cdot q}{M} = \frac{Q^2}{2M}\omega$$

Advantages:

- Theoretically clean (only analyticity is used).
- Can obtain the maximum set of info (arbitrary moments) at once…, in principle.
- Can apply to low Q^2 structure functions, for which PDF is no longer a valid description.

- Also related to the Cottingham sum rule, for (M_p-M_n).
- Also related to the two-photon exchange contrib to the Lamb shift.

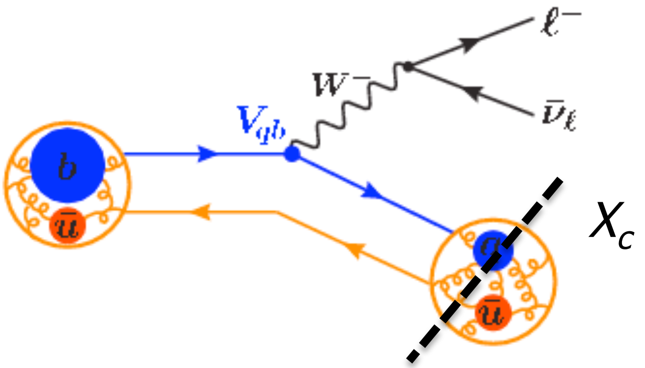
Disadvantages:

- Four-point function is computationally challenging (many contractions), work in progress (H. Ohki and H. Fukaya)
- S/N would be bad due to nucleon prop on both ends.

B meson semileptonic decays

- Determination of $|V_{xb}|$
  - $|V_{cb}|$, in particular
  - BR ~ 10% (e) + 10% (μ)

  - exclusive = decays to individual final states (D, D*, …)
  - inclusive = sum over all possible final states including charm



Formulation is very similar to DIS.

Partial decay rate:

$$d\Gamma \sim |V_{cb}|^2 l^{\mu\nu} W_{\mu\nu}$$

Structure function:

$$W_{\mu\nu} = \sum_X (2\pi)^3 \delta^4(p_B - q - p_X) \frac{1}{2M_B} \langle B(p_B)|J_\mu^\dagger(0)|X\rangle\langle X|J_\nu(0)|B(p_B)\rangle$$

sum over all final states

Optical theorem: $\boxed{-\frac{1}{\pi}\mathrm{Im}T_i = W_i}$

analytic function of q^2 and $v.q$

Forward scattering matrix element:

$$T_{\mu\nu} = i\int d^4x e^{-iqx} \frac{1}{2M_B} \langle B|T\{J_\mu^\dagger(x)J_\nu(0)\}|B\rangle$$

$$T_{\mu\nu} = i\int d^4x e^{-iqx}\frac{1}{2M_B}\langle B|T\{J_\mu^\dagger(x)J_\nu(0)\}|B\rangle$$

analytic function of q^2 and $v.q$

Analytic structure:

$$\int_{-\infty}^{(v\cdot q)_{\max}} \frac{d(v\cdot q')}{\pi}\frac{\mathrm{Im}T(v\cdot q')}{v\cdot q' - v\cdot q} = T(v\cdot q)$$

$\boxed{v\cdot q}$ (with fixed q^2)

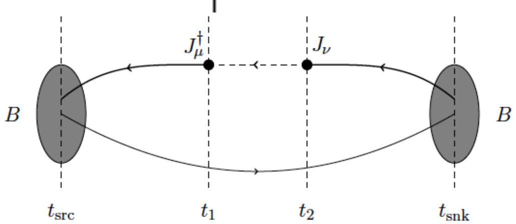
$= m_B - \omega$

unphysical cut $b \to bb\bar{c}$

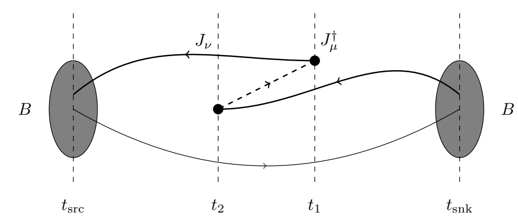
physical cut $b \to c$

$\frac{1}{2M_B}(M_B^2 + q^2 - m_X^2)$

$\frac{1}{2M_B}((2M_B + M_X)^2 - q^2 - M_B^2)$

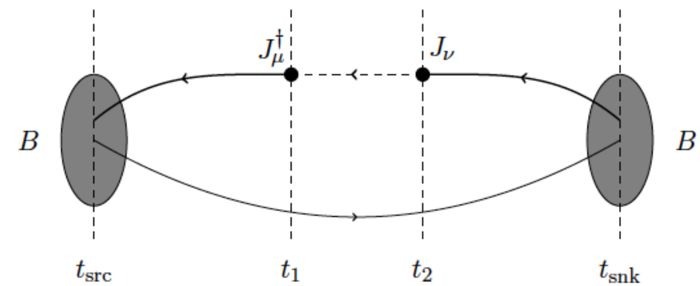


"Euclidean region":
calculable on the lattice in
unphysical kinematical region



Forward-scattering ME on the lattice

1. Calculate the four-point function:



2. Extract the matrix element by taking a ratio to two-points

$$C_{\mu\nu}^{JJ}(t;\mathbf{q}) = \int d^3\mathbf{x}\, e^{i\mathbf{q}\cdot\mathbf{x}} \frac{1}{2M_B} \langle B(\mathbf{0})|J_\mu^\dagger(\mathbf{x},t) J_\nu(0)|B(\mathbf{0})\rangle$$

3. "Fourier transform"

$$\boxed{T_{\mu\nu}^{JJ}(\omega,\mathbf{q}) = \int_0^\infty dt\, e^{\omega t} C_{\mu\nu}^{JJ}(t;\mathbf{q})}$$

→ $T_{\mu\nu}(v\cdot q, q^2)$ at $p_X=(\omega,-\boldsymbol{q})$, $q=(m_B-\omega,\boldsymbol{q})$

JLQCD lattice ensemble

- Mobius domain-wall fermion (2012~)
  - 2+1-flavor (uds)
  - chiral symmetry
    - residual mass < O(1 MeV)
  - lattice spacing : 1/a = 2.4, 3.6, 4.5 GeV
  - volume : L = 2.7 fm（32^3, 48^3, 64^3 lattices）
  - ud quark masses : m_π = 230, 300, 400, 500 MeV
  - statistics : 50-400 measurements
- Valence quarks
  - charm/bottom (MDW) + strange (MDW)
  - bottom is lighter than physical, $m_b = (1.25)^4\, m_c$
  - on Oakforest-PACS with IROIRO++

(3.4 GeV B_s meson)

β = 4.17, 1/a ~ 2.4 GeV, 32^3x64 (x12)

m_{ud}	m_π [MeV]	MD time
m_s = 0.030		
0.007	310	10,000
0.012	410	10,000
0.019	510	10,000
m_s = 0.040		
0.0035	230	10,000
0.0035 (48^3x96)	230	10,000
0.007	320	10,000
0.012	410	10,000
0.019	510	10,000

β = 4.35, 1/a ~ 3.6 GeV, 48^3x96 (x8)

m_{ud}	m_π [MeV]	MD time
m_s = 0.018		
0.0042	300	10,000
0.0080	410	10,000
0.0120	500	10,000
m_s = 0.025		
0.0042	300	10,000
0.080	410	10,000
0.0120	510	10,000

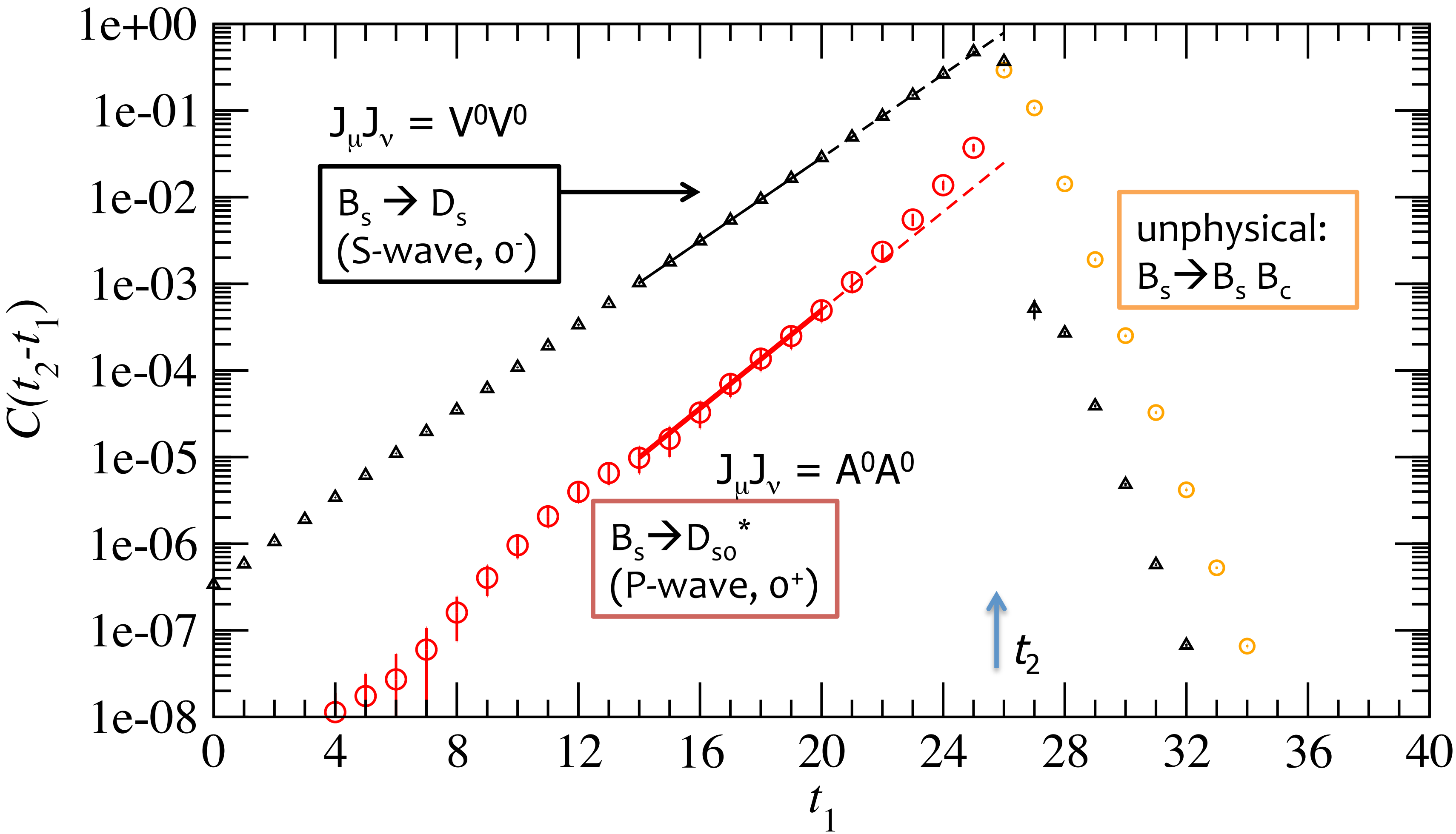
β = 4.47, 1/a ~ 4.6 GeV, 64^3x128 (x8)

0.0030	~ 300	10,000

$$C^{JJ}_{\mu\nu}(t;\mathbf{q}) = \int d^3\mathbf{x}\, e^{i\mathbf{q}\cdot\mathbf{x}} \frac{1}{2M_B} \langle B(\mathbf{0})|J^\dagger_\mu(\mathbf{x},t)J_\nu(0)|B(\mathbf{0})\rangle$$

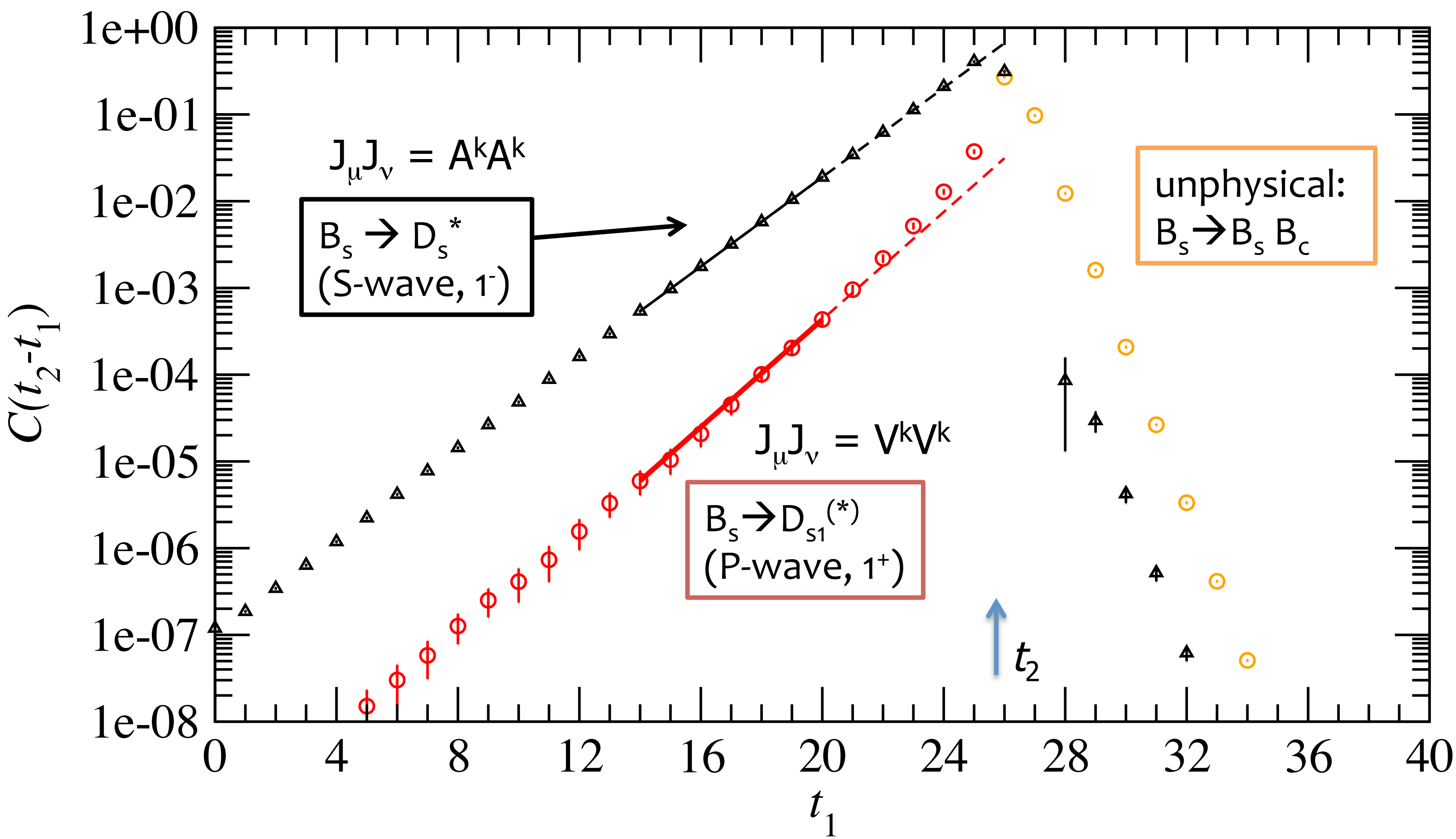
t_1 t_2

$m_b = 1.25^4 m_c$, zero recoil (**q**=**0**) 1/a = 3.6 GeV



$$C^{JJ}_{\mu\nu}(t;\mathbf{q}) = \int d^3\mathbf{x}\, e^{i\mathbf{q}\cdot\mathbf{x}} \frac{1}{2M_B} \langle B(\mathbf{0})| J^\dagger_\mu(\mathbf{x}, \underbrace{t}_{t_1}) J_\nu(\underbrace{0}_{t_2}) |B(\mathbf{0})\rangle$$

$m_b = 1.25^4 m_c$, zero recoil ($\mathbf{q}=\mathbf{0}$)   1/a = 3.6 GeV



“Fourier transform”

- Time direction should be analytically continued to go time-like, i.e. energy ω:

$$e^{ip_0 t} \rightarrow e^{\omega t} \qquad : \qquad T^{JJ}_{\mu\nu}(\omega, \mathrm{q}) = \int_0^\infty dt\, e^{\omega t} C^{JJ}_{\mu\nu}(t; \mathrm{q})$$

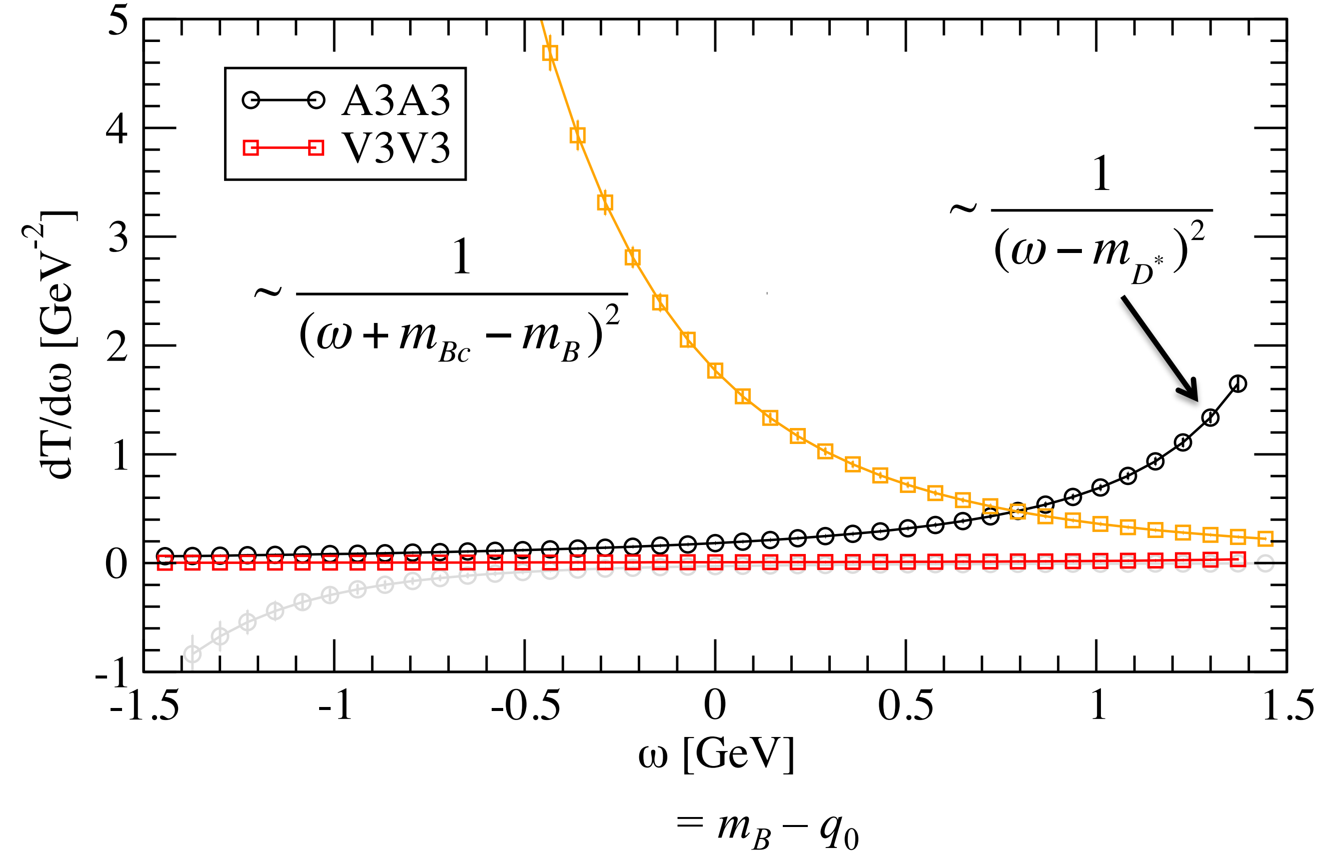
  - Can be understood by a Taylor expansion in p_0, and then reconstruct with $ip_0=\omega$.
  - Only below any singularity: pole, cut, …
  - Obviously,

$$e^{-mt} \xrightarrow{F.T.} \frac{1}{\omega - m}$$

$$T^{JJ}_{\mu\nu}(\omega, \mathbf{q}) = \int_0^\infty dt\, e^{\omega t} C^{JJ}_{\mu\nu}(t; \mathbf{q})$$

m_b=1.25$^4 m_c$, zero recoil (**q**=**0**) $1/a$ = 3.6 GeV

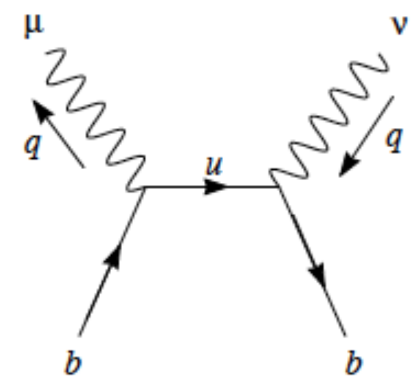
derivative to avoid divergence (contact term)



Comparison with Continuum

(Based on discussions with P. Gambino)

- Heavy Quark Expansion (tree-level formulae):
  Blok, Koyrakh, Shifman, Vainshtein, PRD49, 3356 (1994).
  Manohar, Wise, PRD49, 1310 (1993).
  Falk, Ligeti, Neubert, Nir, PLB326, 145 (1994)
  Balk, Korner, Pirjol, Schilcher, ZP C64, 37 (1994).

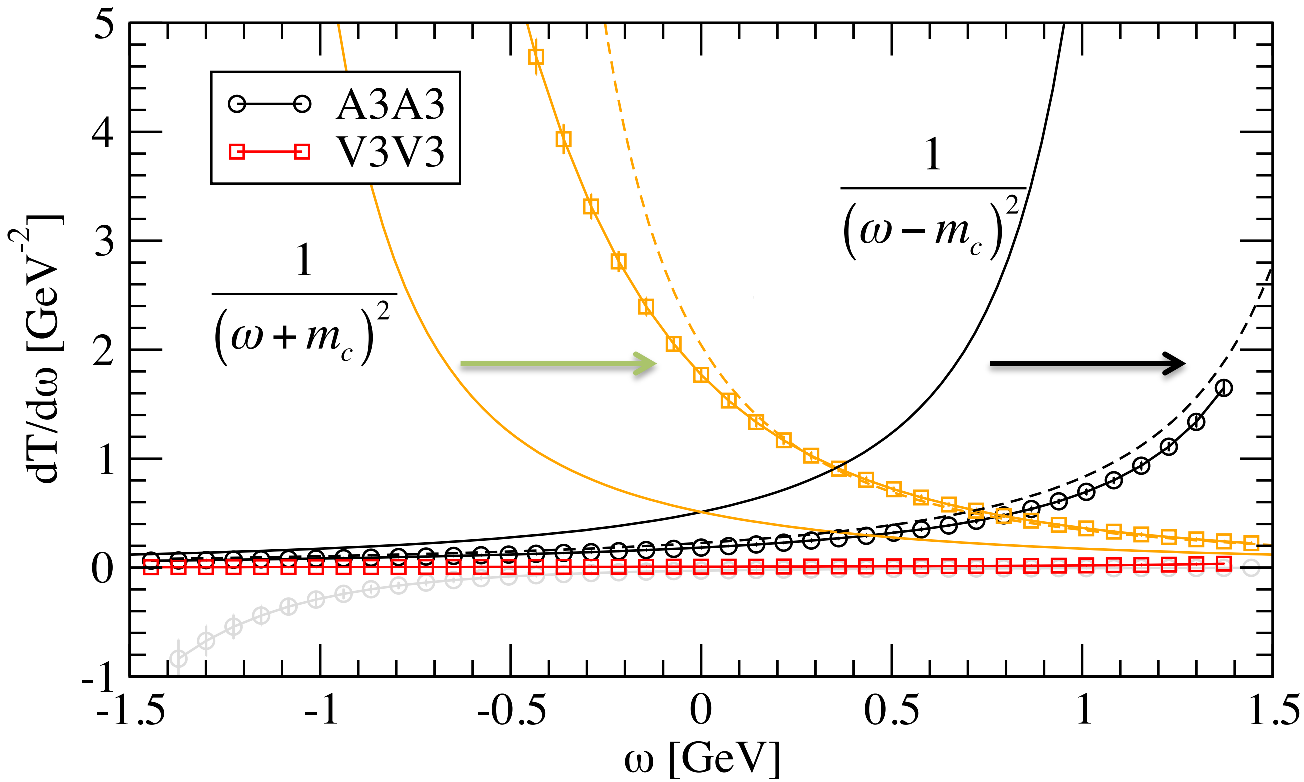


- Expand $\dfrac{1}{m_b \not{v} - \not{q} + \not{k} - m_c}$ in small *k*.

- Zero-recoil limit ($V_k V_K$ or $A_k A_k$ channel, leading order)

$$T_1^{VV} = -\frac{\omega - m_c}{\omega^2 - m_c^2}, \quad T_1^{AA} = -\frac{\omega + m_c}{\omega^2 - m_c^2} \qquad (\omega = m_B - q_0)$$

… pole at $\omega = -m_c$ ($V_k V_k$) or $\omega = m_c$ ($A_k A_k$)



Significant shift due to $m_c \rightarrow m_D$

or to $m_c \rightarrow m_{Bc}\text{-}m_B$

Comparison to Experiment?

- Need the Cauchy's integral, which corresponds to the moments

$$\left\langle \frac{1}{(m_B - E_X)^n} \right\rangle$$

- Previously, only $\langle E_X^n \rangle$ are analyzed.
- Need to supplement the region $m_B - E_X < 0$ using perturbation theory.

No conclusions

- No new results…, sorry!

- Yet, new opportunities for lattice.
  - Learn from sum rules, the golden age of particle physics
  - Apply with new technologies!