Lattice QCD analysis of charmed tetraquark candidates

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from Lattice QCI

HAL QCD (Hadrons to Atomic nuclei from Lattice QCD)

S. Aoki, T. Aoyama, Y. Akahoshi, K. Sasaki, T. Miyamoto (YITP, Kyoto Univ.) T. Doi, T. M. Doi, S. Gongyo, T. Hatsuda, T. Iritani (RIKEN) Y. Ikeda, N. Ishii, K. Murano, H. Nemura (RCNP, Osaka Univ.) T. Inoue (Nihon Univ.)

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Single hadron spectroscopy from LQCD

★ Low-lying hadrons on physical point (physical m_q)





- \checkmark a few % accuracy already achieved for single hadrons
- ✓ LQCD now can predict undiscovered charm hadrons ($\Xi^{(*)}_{cc}$, Ω_{ccc} ,...)

Next challenge in spectroscopy : hadron resonances

Hadron resonances

Particle data group

http://www-pdg.lbl.gov/

p	1/2+		A(1232)	3/2+		2+	1/2+		20	1/2+		Λ.	1/2+	
11	1/2+		A(1600)	3/2+	***	Σ^{0}	1/2+		2-	1/2+		A-(2595)+	1/2"	
N(1440)	1/2+		A(1620)	1/2-	****	Σ-	1/2+		E(1530)	3/2+		A-(2625)+	3/2-	***
N(1520)	3/2-	****	A(1700)	3/2-	****	Σ(1385)	3/2+	****	E(1620)			A, (2765)+		
N(1535)	1/2-	****	A(1750)	$1/2^{+}$	•	$\Sigma(1480)$		•	E(1690)		***	A,(2880)+	5/2+	
N(1650)	1/2-	****	A(1900)	1/2	**	Σ(1560)		**	Ξ(1820)	3/2-	***	A _c (2940) ⁺		***
N(1675)	5/2-		A(1905)	5/2+		Σ(1580)	3/2-	•	IT(1950)		•••	Σ.(2455)	$1/2^+$	****
N(1680)	5/2+	****	A(1910)	$1/2^+$	****	$\Sigma(1620)$	1/2-	•	E(2030)	$\geq \frac{5}{2}^7$	•••	$\Sigma_{c}(2520)$	3/2+	***
N(1700)	3/2-	•••	$\Delta(1920)$	3/2+	***	$\Sigma(1660)$	$1/2^+$	•••	Ξ(2120)			Σ_(2800)		***
N(1710)	$1/2^+$		∆ (1930)	5/2-	***	$\Sigma(1670)$	3/2-	****	E(2250)		**	Ξ.	$1/2^{+}$	***
N(1720)	3/2+	****	∆(1940)	3/2-	**	$\Sigma(1690)$		**	Ξ(2370)		••	10	$1/2^+$	***
N(1860)	5/2+	••	$\Delta(1950)$	7/2+	****	$\Sigma(1730)$	3/2+	•	E[2500]		•	II'+	$1/2^{+}$	***
N(1875)	3/2-	***	A(2000)	5/2+	**	$\Sigma(1750)$	1/2-	***				29	1/2+	***
N(1880)	$1/2^+$	••	$\Delta(2150)$	1/2-	•	$\Sigma(1770)$	$1/2^+$	•	Ω-	3/2+		三(2645)	3/2+	***
N(1895)	1/2-		$\Delta(2200)$	7/2-	•	$\Sigma(1775)$	5/2-		£2250)~		•••	E(2790)	1/2-	
N(1900)	3/2+	•••	∆ (2300)	9/2+	••	$\Sigma(1040)$	3/2+	•	£2380)~		••	E(2815)	3/2-	•••
N(1990)	7/2+	••	∆(2350)	5/2-	•	$\Sigma(1880)$	$1/2^+$	••	£2(2470)-		••	E(2930)	-	
N(2000)	5/2+	••	Δ(2390)	7/2+	•	Σ(1900)	1/2-	•				三(2978)		
N(2040)	3/2+	•	∆(2400)	9/2-	••	Σ(1915)	5/2+					三(3055)		***
N(2060)	5/2-		∆(2420)	$11/2^{+}$		Σ(1940)	3/2+	•				E_(3080)		***
N(2100)	$1/2^+$	•	$\Delta(2750)$	13/2-		$\Sigma(1940)$	3/2-					E(3123)		
N(2120)	3/2-		∆ (2950)	15/2*		L (2000)	1/2-	•				Ω_c^0	$1/2^+$	•••
N(2190)	7/2-					Σ(2030)	7/2+					$\Omega_{c}(2770)^{0}$	3/2+	***
N[2220]	9/2+		1	1/2*		Σ(2070)	5/2+							
N(2250)	9/2		/(1405)	1/2		Σ(2080)	3/2+					Ξ_{co}^{+}		•
N(2300)	1/2*		A(1520)	3/2		2(2100)	1/2							
N[2570]	5/2		A(1670)	1/2		2 (2250)						Λ_{ϕ}^{0}	$1/2^+$	
N(2600)	11/2		A[1670]	3/2		£ (2455)						$\Lambda_{p}(5912)^{0}$	1/2~	
N[2700]	13/2		A(1710)	3/2		2 (2620)						$\Lambda_{2}(5920)^{0}$	3/2~	•••
			A(1800)	1/2-		2 (3000)						Σo	$1/2^+$	
			A(1810)	1/2+		2(31/0)		<u> </u>				I.	3/2*	
			A(1820)	5/2+								Ξ°, Ξ°	1/2+	
			A(1830)	5/2-								Z ₂ (5935)-	$1/2^+$	••••
			A(1890)	3/2+				_				$\Xi_0(5945)^0$	3/2+	•••
			A[2000]	21/2								E*(5955)-	3/2+	
			A(2020)	7/2+								Ω_{3}^{-}	$1/2^{+}$	•••
			A(2050)	3/2-								-		121
			A[2100]	7/2-								$P_c[4380]^+$		
			A(2110)	5/2+	***							Pc[4450]*		·
			A(2325)	3/2-										
			A(2350)	9/2+				_						
			A[2585]		••									

	LIGHT UN	FLAVORED		STRAN	IGE	CHARMED, STRANGE		60 FL FS	
	ELPO .	- 8 - 9	61.80	(S = 31. C	10-10	(t = 5 =	10		10.1
	r(7-)		r(r)		471		47)	• $\eta_c(15)$	0+(0-+)
• 2"	1-(0-)	 p₃(1690) 	1+(3)	• K ²	1/2[0-)	• D_5	0(0")	 J/ψ(15) 	0-(1)
• = "	1-[0-+]	 <i>ρ</i>(1700) 	1+(1)	• K*	1/2[0"]	• D'=	0(?*)	 χ_{c0}(1P) 	0-(0)
• 1	0.[0.1]	a ₂ (1700)	$1^{-}(2^{+})$	• K3	1/2[0"]	 D[*]₅₈(2317)[±] 	0(0+)	• Xet[1P]	0-[1]
 f₀(500) 	0-[0]	• f _b (1710)	0-[0++]	• K2	1/2(0")	 D₅₀(2460)[±] 	0(1+)	• h _c (1P)	11111
 ρ(770) 	1-(1)	η(1760)	0-[0-+]	A. ² (000)	$1/2[0^+)$	 D₅₁(2536)[±] 	0(1+)	 X₁₂[1P] 	0.15.11
• u(782)	0-(1)	• r(1800)	1-(0-+)	 K*(892) 	$1/2(1^{-})$	 D₁₂(2573) 	0(2+)	• 72(23)	0.10 .1
 i) (958) i) (958) 	0-(0-1)	P_(1810)	0 (2 + +)	 K:[1270] 	$1/2(1^+)$	 D[*]₈₁(2700)[±] 	0(1-)	• 0(25)	0 [4
 #(ass) 	1-(0++)	X(1835)	7/(777)	• K;[1400]	$1/2(1^+)$	$D_{s1}^{*}(2060)^{\pm}$	0(1-)	• ((3873)	7/10
 A)[980] A)[980] 	1 [0]	X(1940)	1-0++)	• K*(1410)	$1/2(1^{-})$	$D_{e1}^{*}(2860)^{\pm}$	0(3_)	• \$(3623)	10 + + +
• 0(10.0)	0 (1)	3(1420)	0-0	 K[*]_g(1430) 	1/2[0+)	D _x ,(3040)+	0(?*)	• X(3972)	1+01+-1
• n(1236)	1+0+-1	• (1870)	0 (3)	 K[*]₂(1430) 	1/2(2+)	BOTT	14 ·	• X(3915)	0*10/2*
• 0(120)	1 11 1	1.14701	0.12	P11.4640	1 200-1			• ~ (28)	0+12++1
- 6112	\sim	1	+-		1		1.10(0-1)	X(2940)	27(277)
• 6112		лг	Ta	rae			1/2(0)	• X(4020)	1025
• (12)			LU	90		200 ADA	avalo j	•	0-0
· (17)						(10) (10) (A BORNE	X(4050)2	21721
(11			~ ~			MIXTUR	E	X(4055)±	7(77)
- 6413		7 (20	$\cap \cap \cap$		and Web	CKM Ma-	• X(4140)	0+(77+)
A(1)			53	OOI		Elements		•	0-(1)
		-01	•••				1/2(1-)	X(4160)	2017
						(6721)*	$1/2(1^+)$	X(4200)+	7(1+)
. 6 (1420)	0 11 1	m [2100]	1-12-+1	witereet		B ₁ (5721) ^a	1/2(1*)	X(4230)	201
• (1420)	0-(1)	6(2100)	0+10++1	 K[*]₄(2045) 	1/2[4+]	B'j(5732)	1(1.)	X(4240)*	77(0-1
6(1430)	0+(2++)	6(2150)	0+(2++)	K ₂ (2250)	$1/2(2^{-})$	 B)(\$747)* 	$1/2(2^+)$	X(4250)*	11721
· - (1450)	1-00++1	(2350)	1+(1)	K ₃ (2320)	$1/2(3^+)$	• B5(5747)*	1/2[2+)	• X(4250)	701
• c(1450)	1+01	• ((2170)	0-(1)	K [*] ₅ (2380)	1/2[5"]	B ₂ (5840)+	1/2(?*)	X(4350)	0+(77+)
· = (1475)	0+10-+1	6(2200)	0+10++1	K ₄ (2500)	$1/2(4^{-})$	B ₂ (5840) ³	$1/2(7^{\circ})$	 X(4360) 	72(1)
• 6(1500)	$0^{+}(0^{+})$	6(2220)	0+12++	K(3100)	r(n)	 B₁(59/0)⁺ 	1/2(?)	 ψ(4415) 	0-(1)
6(1510)	0+(1++)		or 4 + + 1	CHARM	#FD	 B'(abo), 	1/2(71)	 X(4430)[±] 	7(1+)
 P₂(1525) 	$0^{+}(2^{+})$	*(2225)	0+10-+1	(C = :	:1)	BOTTOM, S	TRANGE	 X(4550) 	7?(1)
6(1565)	0+(2++)	P1(2250)	1+(3)	• D [±]	1/2(071)	(Ø = ±1, 5	= = 1		
e(1570)	1+(11	• 6(2300)	0+(2++)	• D ²	1/2(0-1	• 8 ⁰	0(0")	b	b
Pt (1595)	0-(1+-)	6(2300)	$0^{+}(4^{+})$	 D*(2007)⁰ 	1/2(1-)		0(1-1	 <i>n</i>₀(15) 	0+(0-+)
• T_(1600)	1-(1-+)	6(2330)	$0^{+}(0^{+})$	· D*(2010)+	1/2(1-1)	· 8. (5830)*	00.40	 T(15) 	0-(1)
J1(1640)	$1^{-}(1^{++})$	 6(2340) 	$0^{+}(2^{+})$	· Dt(2400)0	1/2(0+1	· #* (5340) ²	0(2+)	 χ_{b0}(1P) 	0+(0++)
6(1640)	$0^{+}(2^{++})$	Ps(2350)	1+(5)	D0(2400)*	1/2(0+1	A* (5850)	7(27)	 χ_{B1}(1P) 	0*(1 + +)
• mg(1645)	0+(2-+)	a ₂ (2450)	$1^{-}(6^{++})$	• /b.(2420) ²	1/2(1+1	D _s j(sase)	.de 1	 b₀(1P) 	7(1+-)
 ω(1650) 	0-(1)	6(2510)	0+(6++)	D-(2420)1	1/2071	BOTTOM, C	HARMED	 	0*(2 * *)
• wg(1670)	0-(3)	071071	I LITLET	D-(2430) ²	1/201*1	(B = C =	· ±1)	$\eta_{b}(25)$	0+(0-+)
 x2(2670) 	$1^{-}(2^{-+})$	OTHER	CLIGHT	· D0(2460)0	1/2(2+1)	• B ⁺ .	0(0-)	 T[25] 	0-(1)
 φ(1680) 	$0^{-}(1^{-})$	Further St	ates	· D\$(2460)=	1/2(2+1)	B _c (25) ¹⁰	0(0")	 T(1D) 	0-(2)
				D(2550) ⁸	1/207			 χ₃₀(2P) 	0+[0++]
		1		D*(2500)	1/2020			 χ₂₁(2P) 	0-[1++]
		1		D*(2640)*	1/2(77)			$b_b(2P)$	7(1)
		1		D(2740) ⁸	1/2071			 XB2(2P) 	0-[2++]
				D(2750)	1/2[3-1	1		• 7(35)	0 (1
				D(3000)8	1/2(73)			• X5((3P)	0-(1)
						1		· /(45)	1 + (1 +)
								• X(10610)*	1+(1+)
								• X[10610]*	22122
								A[10650]*	0-0
								• T(11030)	0-0
								 (erread) 	A (4)

- Most hadrons are consistent with qqq / qq^{bar} quantum number (non-trivial)
- Only 10% is stable, others are unstable (resonances) and some can be fake..
- Understanding hadron resonances from QCD is important issue in hadron physics

Tetraquark candidate Z_c(3900)



- peak in $\pi^{+/-}J/\psi$ invariant mass (minimal quark content cc^{bar} ud^{bar} <--> tetraquark?)
- M ~ 3900, Γ ~ 60 MeV (Breit-Wigner, Flatte) --> just above D^{bar}D* threshold
- J^{PC}=1⁺⁻ is most probable <--> couple to s-wave meson-meson states

Tetraquark candidate Z_c(3900)

\star structure of Z_c(3900) studied by models





conclusion not achievedpoor information on interactions

 \star LQCD simulations for Z_c(3900)



Z_c(3900) on the lattice

Conventional approach: temporal correlation
 identify all relevant Wn(L) (n=0,1,2,3,...)



$$\langle 0|[car{c}uar{d}](t)[car{c}uar{d}]^{\dagger}(0)|0
angle = \sum A_n e^{-W_n t}$$

variational method



✓ No positive evidence for $Z_c(3900)$ in $J^{PC}=1^{+-}$

(observed spectrum consistent with scat. states)

S. Prelovsek et al., PLB 727 (2013), PRD91 (2015). S.-H. Lee et al., PoS Lattice2014 (2014).

* Why is the peak observed in expt.?

(broad) resonance? threshold effect?

★ How can we find resonance in LQCD data?

Strategy for studies of resonances from LQCD

Solution Solution Solution

<u>Conventional approach</u> $\langle 0 | \Phi(x) \Phi^{\dagger}(0) | 0 \rangle = A_1 e^{-W_1 \tau} + A_2 e^{-W_2 \tau} + \cdots$ (W₁, W₂, ... are eigen-energies)

e.g., 4-quark operator

 $\Phi(x) = \bar{q}(x)\bar{q}(x)q(x)q(x)$





hadron resonances

★ Resonance energy does NOT correspond to eigen-energy

- ★ Resonances are embedded into coupled-channel scattering states
- Resonance energy is determined from pole of coupled-channel S-matrix

Strategy for studies of resonances from LQCD

contents

- hadron interactions & HAL QCD method
- strategy to find resonance pole
- coupled-channel scattering
- LQCD results about Z_c(3900)
- summary

hadron resonances

Hadronic interactions from LQCD

hadronic correlation function

$$egin{aligned} egin{aligned} C_{(2)}(ec{r},t) &\equiv \langle 0 | \phi_1(ec{r},t) \phi_2(ec{0},t) \mathcal{J}^\dagger(t=0) | 0
angle \ &= \sum_n A_n \psi_n(ec{r}) e^{-W_n t} \end{aligned}$$



• NBS (Nambu-Bethe-Salpeter) wave function $\psi_n(r)$

(outside interactions) [$\psi_n(r) \rightarrow \sin(k_nr + \delta(k_n)) / k_nr$]

C.D. Lee et al., NPB619 (2001).



Lüscher's finite volume formula

$$egin{aligned} & m{k_n} \cot \delta(m{k_n}) = rac{4\pi}{L^3} \sum_{m \in \mathbb{Z}^3} rac{1}{ec{p}_m^2 - m{k_n}^2} \end{aligned}$$

 $W_n = \sqrt{m_1^2 + k_n^2} + \sqrt{m_2^2 + k_n^2}$



 $ec{x}$

Hadronic interactions from LQCD

hadronic correlation function

$$egin{aligned} egin{aligned} C_{(2)}(ec{r},t) &\equiv \langle 0 | \phi_1(ec{r},t) \phi_2(ec{0},t) \mathcal{J}^\dagger(t=0) | 0
angle \ &= \sum_n A_n \psi_n(ec{r}) e^{-W_n t} \end{aligned}$$



• Energy eigenvalue Wn(L)

• NBS (Nambu-Bethe-Salpeter) wave function $\psi_n(r)$

(outside interactions) [$\psi_n(r) \rightarrow \sin(k_nr + \delta(k_n)) / k_nr$]



difficult with coupled-channel problems

HAL QCD Method

 $\blacktriangleright \psi_n(r) \longrightarrow 2PI \text{ kernel } (\psi = \phi + G_0 U \psi)$

--> phase shift, binding energy, ...

Ishii, Aoki, Hatsuda, PRL 99, 022001 (2007). Ishii et al. [HAL QCD], PLB 712, 437 (2012).

Challenge in hadron scatterings

 \star Excited scattering states become noise when determining W₀ even in single-channel scatterings

$$C_{(2)}(t) = b_0 e^{-W_0 t} + b_1 e^{-W_1 t} + \cdots \longrightarrow b_0 e^{-W_0 t} \quad (t > t^*)$$



★ Sophisticated methods is necessary! talk by T. Doi (Thu.) see for BB systems, Iritani, Doi et al. [HAL QCD], JHEP10 (2016) 101. (single-channel) HAL QCD method -- potential as a representation of S-matrix --

• The scattering states do exist, and we should tame the scattering states

time-dependent HAL QCD method

Ishii [HAL QCD], PLB 712 (2012).

 \checkmark define energy-independent potential U(r,r')

$$egin{aligned} dec{r}' oldsymbol{U}(ec{r},ec{r}')\psi_n(ec{r}') &= (E_n-H_0)\,\psi_n(ec{r})\ U(ec{r},ec{r}') &\equiv \sum_{n \, < n} \,\,(E_n-H_0)\,\psi_n(ec{r}) ec{\psi}_n(ec{r}')ec{\psi}_n(ec{r}')ec{r}') \end{aligned}$$

 \rightarrow All elastic states share the same potential U(r,r')

$$U \psi_{0} = (E_{0}-H_{0}) \psi_{0}$$
$$U \psi_{1} = (E_{1}-H_{0}) \psi_{1}$$

nelastic nelastic ch2 elastic ch1

✓ derive U(r,r') from time-dependent Schrödinger-type eq.

$$\int dec{r}' m{U}(ec{r},ec{r}') R(ec{r}',t) = \left(-rac{\partial}{\partial t} + rac{1}{4m}rac{\partial^2}{\partial t^2} - H_0
ight) R(ec{r},t)$$

$$\begin{aligned} R(\vec{r},t) &= C_{(2)}(\vec{r},t) / \left(C_{(1)}(t) \right)^2 \\ &= b_0 \psi_0(\vec{r}) e^{-(W_0 - 2m)t} + b_1 \psi_1(\vec{r}) e^{-(W_1 - 2m)t} + \cdots \end{aligned}$$

(single-channel) HAL QCD method -- potential as a representation of S-matrix --

• The scattering states do exist, and we should tame the scattering states

time-dependent HAL QCD method

Ishii [HAL QCD], PLB 712 (2012).

 \checkmark define energy-independent potential U(r,r')

$$egin{aligned} dec{r}' U(ec{r},ec{r}')\psi_n(ec{r}') &= (E_n-H_0)\,\psi_n(ec{r})\ U(ec{r},ec{r}') &\equiv \sum_{n < n < u}\,\,(E_n-H_0)\,\psi_n(ec{r})ec{\psi}_n(ec{r}')\,ec{r}') \end{aligned}$$

→ All elastic states share the same potential U(r,r')

$$U \psi_{0} = (E_{0}-H_{0}) \psi_{0}$$
$$U \psi_{1} = (E_{1}-H_{0}) \psi_{1}$$



✓ derive U(r,r') from time-dependent Schrödinger-type eq.

$$\int dec{r}' oldsymbol{U}(ec{r},ec{r}') R(ec{r}',t) = \left(-rac{\partial}{\partial t} + rac{1}{4m}rac{\partial^2}{\partial t^2} - H_0
ight) R(ec{r},t)$$

$$R(\vec{r},t) = b_0 \psi_0(\vec{r}) e^{-(W_0 - 2m)t} + b_1 \psi_1(\vec{r}) e^{-(W_1 - 2m)t} + \cdots$$

 \Rightarrow Scat. states are no more contamination than signal ($t^* \sim (E_{ch2} - E_{ch1})^{-1}$)

How can we find resonances?

resonance

2nd shee

If we have complete set of expt. data,





- pole position --> resonance energy
- residue --> coupling to scat. state, partial decay

Resonance pole from lattice QCD



Coupled-channel HAL QCD method

measure relevant NBS wave function --> channel is defined

$$\langle 0|\phi_1^a(ec{x}+ec{r},t)\phi_2^a(ec{0},t)\mathcal{J}^\dagger(0)|0
angle = \sqrt{Z_1^aZ_2^a}\sum_n A_n oldsymbol{\psi}_n^a(ec{r})e^{-oldsymbol{W}_n t}
ight)$$

see for full details, Aoki et al. (HAL QCD), PRD87 (2013); Proc. Jpn. Acad., Ser. B, 87 (2011).

\star define coupled-channel potential using $\psi^{a}(r)$

$$\left(
abla^2 + (ec{k}_n^a)^2
ight)\psi_n^a(ec{r}) = 2\mu^a\sum_b\int dec{r}' U^{ab}(ec{r},ec{r}')\psi_n^b(ec{r}')$$

★ coupled-channel potential U^{ab}(r,r'):

- U^{ab}(r,r') is faithful to **coupled-channel S-matrix**
- U^{ab}(r,r') is **energy independent** (until new threshold opens)
- Non-relativistic approximation is not necessary
- U^{ab}(r,r') contains all 2PI contributions



$Z_c(3900)$ in $I^G(J^{PC})=1^+(1^{+-})$ -- $\pi J/\psi$ - $\rho\eta_c$ - $D^{bar}D^*$ coupled-channel --

Y. Ikeda et al., [HAL QCD], PRL117, 242001 (2016).



♦ Nf=2+1 full QCD

- Iwasaki gauge
- clover Wilson quark
- 32³ x 64 lattice



Tsukuba-type Relativistic Heavy Quark (charm)

• remove leading cutoff errors $O((m_c a)^n)$, $O(\Lambda_{QCD} a)$, ...

→ We are left with O(($a\Lambda_{QCD}$)²) syst. error (~ a few %)

 $\frac{\text{light meson mass (MeV)}}{m_{\pi}=411(1), 572(1), 701(1)}$ $m_{\rho}=896(8), 1000(5), 1097(4)$

 $\begin{array}{l} \underline{charm\ meson\ mass\ (MeV)} \\ m_{\eta c} = 2988(1),\ 3005(1),\ 3024(1) \\ m_{J/\psi} = \ 3097(1),\ 3118(1),\ 3143(1) \\ m_D = \ 1903(1),\ 1947(1),\ 2000(1) \\ m_{D^*} = \ 2056(3),\ 2101(2),\ 2159(2) \end{array}$

Lattice QCD setup : thresholds

Thresholds in I^GJ^P=1+1+ channel

Physical thresholds

D^{bar}D* = 3872 $\pi \psi' = 3821$ $\pi \pi \eta_c = 3256$ $\pi J/\psi = 3232$

LQCD simulation

D^{bar}D* = 3959, 4048, 4159



• $M_{\pi\psi}$ > $M_D^{bar}_{D^*}$ due to heavy m_{π}

 $\pi J/\psi = 3508, 3688, 3844$

• ρ --> $\pi\pi$ decay not allowed w/ L~3fm

S-wave πJ/ψ - ρη_c - D^{bar}D* coupled-channel analysis

D^{bar}D* potential (single-channel calc.)



- Time-slice dependence indicates
- \Rightarrow coupling to lower channels (large contribution from πJ/ψ and/or $\rho\eta_c$)
- single channel D^{bar}D* potential NOT reliable (huge non-locality)
- coupled-channel analysis is necessary

3x3 potential matrix ($\pi J/\psi$ - $\rho \eta_c$ - $D^{bar}D^*$)



3x3 potential matrix ($\pi J/\psi$ - $\rho \eta_c$ - $D^{bar}D^*$)



3x3 potential matrix ($\pi J/\psi - \rho \eta_c - D^{bar}D^*$)



t-dependence on potential matrix





t=11 12

13

14 15

2.5

2.5

2.0

t=11

2.0

12

13 14 15

1.5

1.5

r [fm]

r [fm]



D^{bar}D* single channel simulation



Structure of $Z_c(3900)$ studied by the most ideal scattering process

- S-wave πJ/ψ ρη_c D^{bar}D* coupled-channel scattering
- \Rightarrow Z_c(3900) is observed in π J/ ψ --> 2-body scattering is the most ideal reaction



1. invariant mass spectrum of 2-body scattering

of scat. particles proportional to imaginary part of amplitude

 $N_{
m sc} \propto ({
m flux}) \cdot \sigma(W) \propto {
m Im} f(W)$

2. pole position of S-matrix

- analytic continuation of c.c. S-matrix onto complex energy plane
- understand nature of $Z_c(3900)$
- Results w/ m_{π} =410MeV are shown. (weak quark mass dependence observed)

Mass spectrum of $\pi J/\psi$ (2-body scattering)



Enhancement just above D^{bar}D* threshold

= effect of strong $V^{\pi J/\psi}$, DbarD* (black --> $V^{\pi J/\psi}$, DbarD*=0)

- branching fraction consistent with expt. analysis
- Ine shape not Breit-Wigner

 \checkmark Is Z_c(3900) a conventional resonance? --> pole of S-matrix

Pole of S-matrix on complex energy plane



Pole of S-matrix ($\pi J/\psi$:2nd, $\rho\eta_c$:2nd, $D^{bar}D^*$:2nd)



- Pole corresponding to "virtual state"
- Pole contribution to scat. observable is small (far from scat. axis)
- Z_c(3900) is not a resonance but "threshold cusp" induced by strong V^{πJ/ψ,DbarD*}

Comparison with expt. data: -- spectrum of Y(4260) 3-body decay --



$Y(4260) --> \pi \pi J/\psi \& \pi D^{bar}D^*$

 $d\Gamma_{Y \to \pi + f} = (2\pi)^4 \delta(W_3 - E_{\pi}(\vec{p}_{\pi}) - E_f(\vec{q}_f)) d^3 p_{\pi} d^3 q_f |T_{Y \to \pi + f}(\vec{p}_{\pi}, \vec{q}_f; W_3)|^2$

√ 3-body T-matrix: $T_{Y->π+f}(W_3=4260MeV)$

$$T_{Y \to \pi + f}(\vec{p}_{\pi}, \vec{q}_{f}; W_{3}) = \sum_{n = \pi J/\psi, \bar{D}D^{*}} C^{Y \to \pi + n} \left[\delta_{nf} + \int d^{3}q' \frac{t_{nf}(\vec{q'}, \vec{q}_{f}, \vec{p}_{\pi}; W_{3})}{W_{3} - E_{\pi}(\vec{p}_{\pi}) - E_{n}(\vec{q'}, \vec{p}_{\pi}) + i\epsilon} \right]$$



employ physical hadron masses to compare w/ expt. data $\checkmark V^{LQCD}(r)$ is taken into account --> calculate t-matrix for subsystem

c.f., 10+ parameters needed in models

Invariant mass of 3-body decay



Ikeda [HAL QCD], J. Phys. G45, 024002 (2018).

• Expt. data reproduced well by 2 parameters





• Without off-diagonal $V^{\pi J/\psi, \ DbarD^*}$ (dashed curves), peak structures are not reproduced.

conclusion: $Z_c(3900)$ is threshold cusp caused by strong $V^{\pi J/\psi, DbarD^*}$

HAL QCD method

- NBS wave function $\psi(\mathbf{r}) \rightarrow 2PI \text{ kernel} (\psi = \phi + G_0 U \psi)$
- Crucial for multi-hadrons & coupled-channel scatterings

Aoki, Hatsuda, Ishii, PTP123, 89 (2010). Ishii et al. [HAL QCD], PLB 712, 437 (2012). Aoki et al. (HAL QCD), PRD87, 034512 (2013).

Tetraquark candidate Z_c(3900)

- $Z_c(3900)$ is threshold cusp induced by strong V^{DbarD*, $\pi J/\psi$}
 - pole position very far from scat. axis
 - expt. data of Y(4260) decay well reproduced
 - no peak structure w/o $V^{DbarD^*, \pi J/\psi}$

Ikeda et al. [HAL QCD], PRL117, 242001 (2016).

Reviewed in Ikeda [HAL QCD], J. Phys. G45, 024002 (2018).

Future: many hadron resonances & nuclear structures at physical point