Lattice QCD analysis of charmed tetraquark candidates

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HAL QCD (Hadrons to Atomic nuclei from Lattice QCD)

S. Aoki, T. Aoyama, Y. Akahoshi, K. Sasaki, T. Miyamoto (YITP, Kyoto Univ.)
T. Doi, T. M. Doi, S. Gongyo, T. Hatsuda, T. Iritani (RIKEN)
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T. Inoue (Nihon Univ.)

International Molecule-type Workshop "Frontiers in Lattice QCD and related topics"
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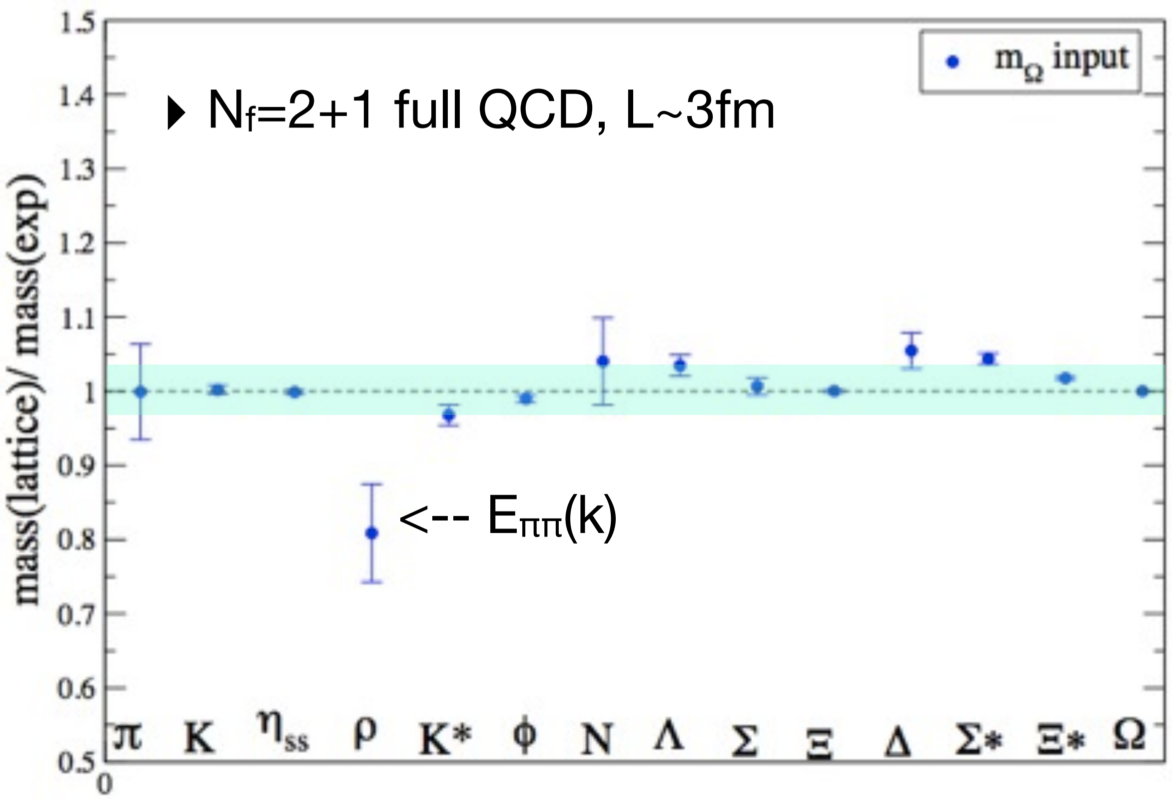
Single hadron spectroscopy from LQCD

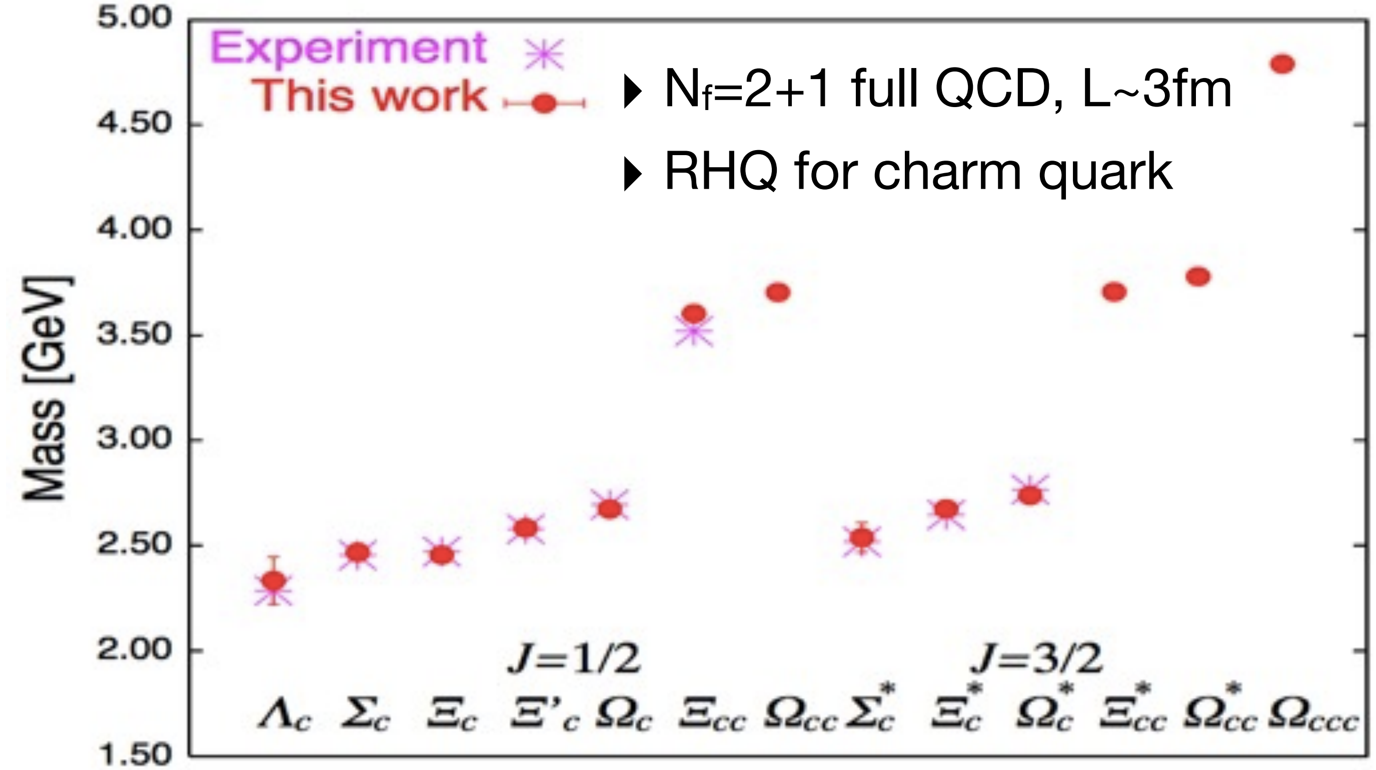
★ **Low-lying hadrons on physical point (physical m_q)**

light-quark sector

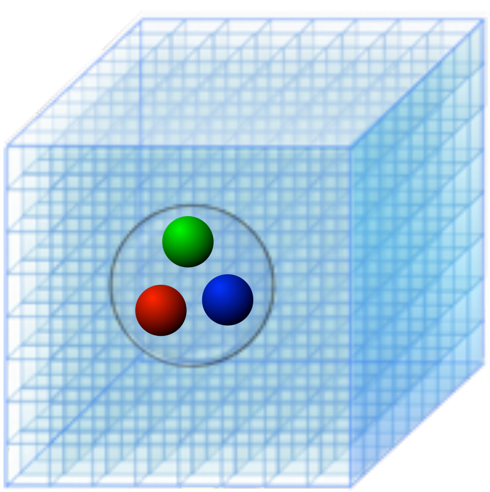


Aoki et al. (PACS-CS), PRD81 (2010).

charm baryons



Namekawa et al. (PACS-CS), PRD84 (2011); PRD87 (2013).



✓ a few % accuracy already achieved for single hadrons

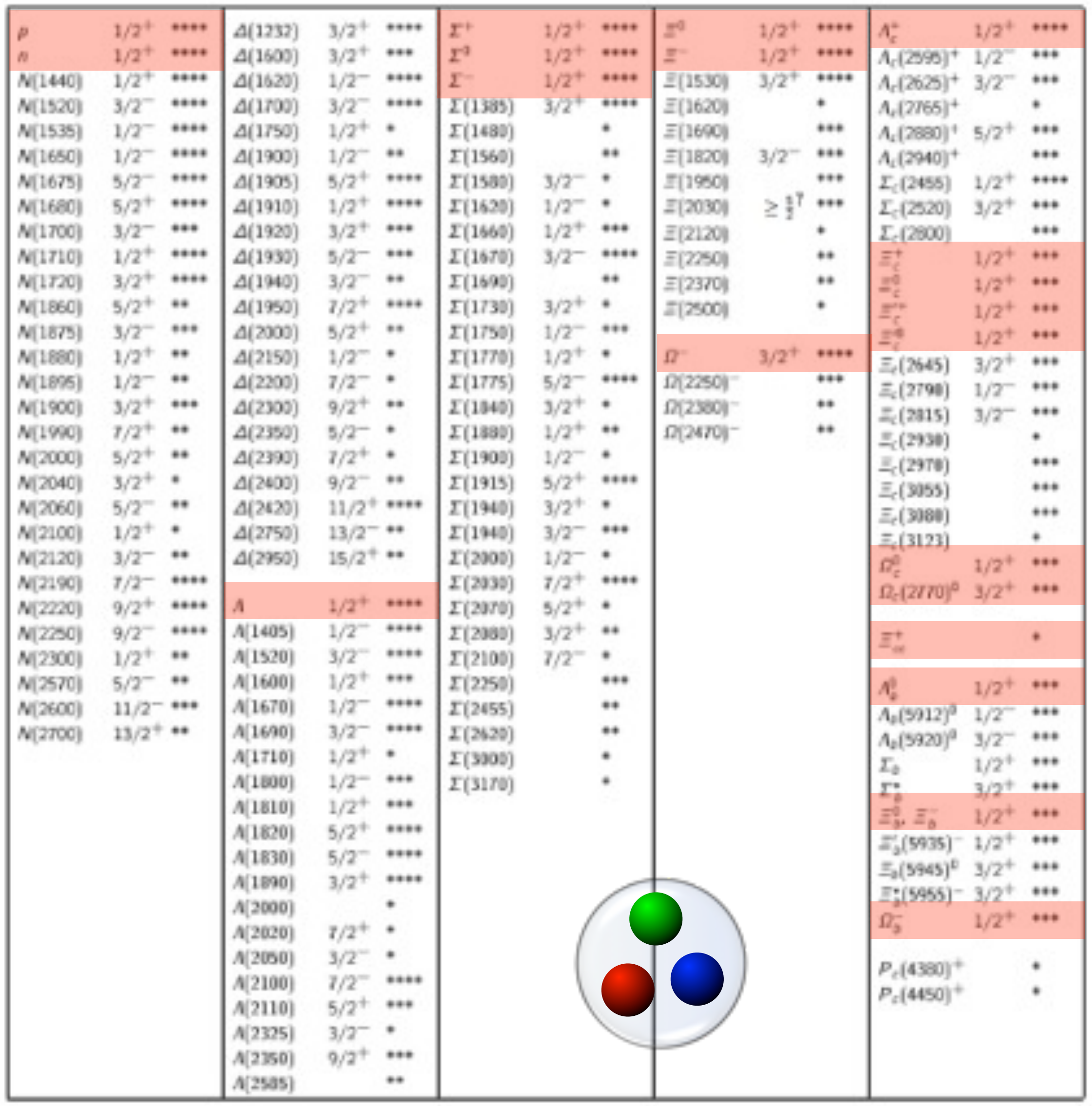
✓ LQCD now can predict undiscovered charm hadrons ($\Xi^{(*)}_{cc}$, Ω_{ccc},...)

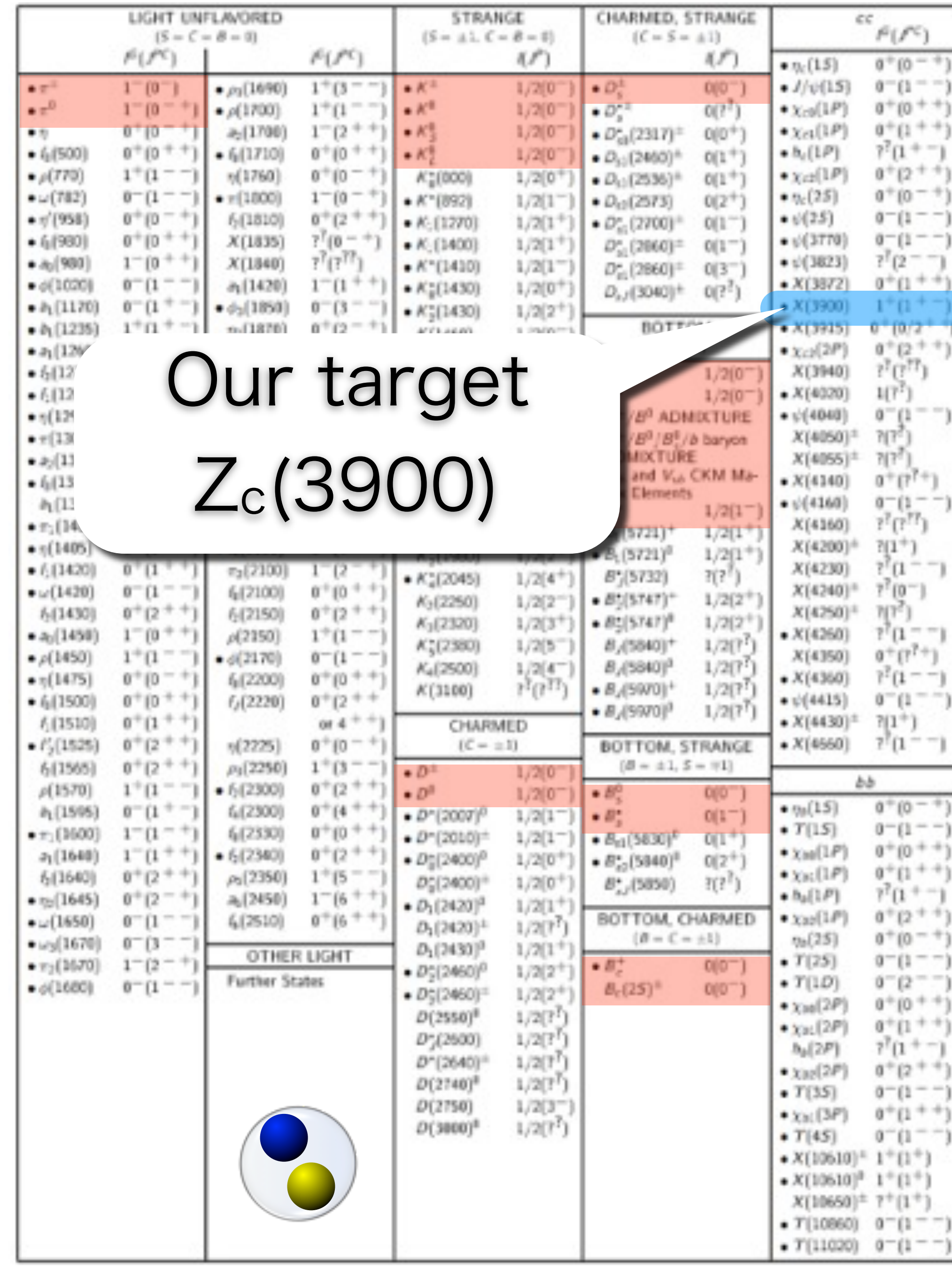
➡ **Next challenge in spectroscopy : hadron resonances**

Hadron resonances

- **Particle data group**

  http://www-pdg.lbl.gov/



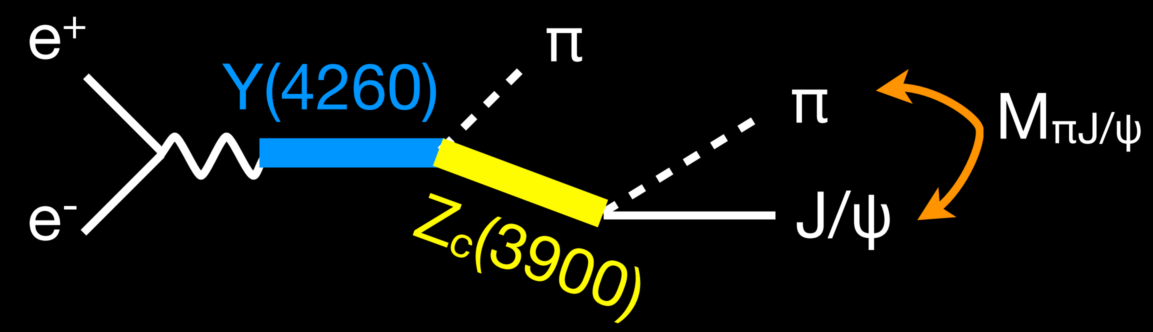


- Most hadrons are consistent with qqq / qq^{bar} quantum number (non-trivial)
- Only 10% is stable, others are unstable (resonances) and some can be fake..
- Understanding hadron resonances from QCD is important issue in hadron physics

Tetraquark candidate $Z_c(3900)$

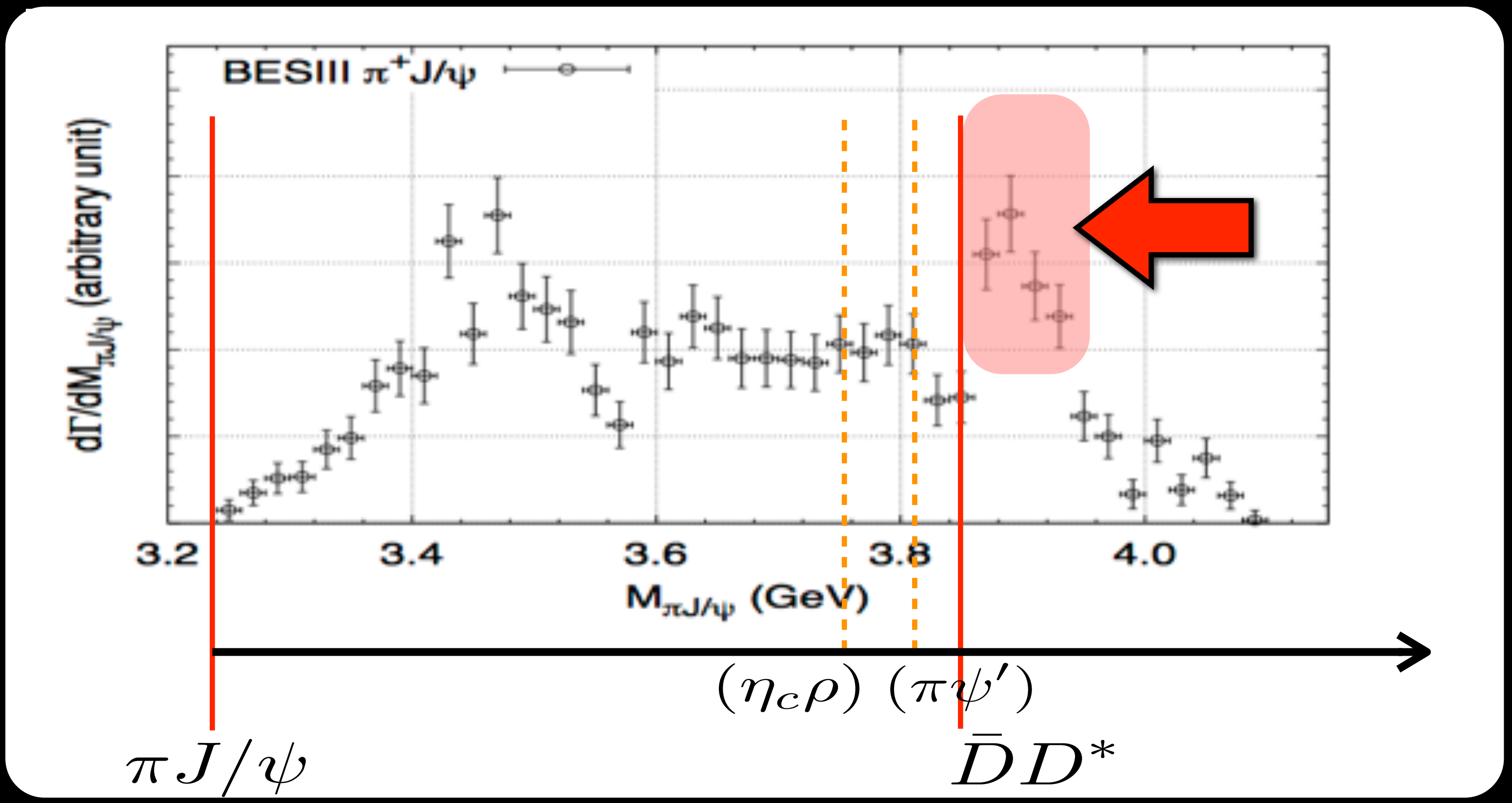
- **Expt. observations**

  Y(4260) 3-body decay



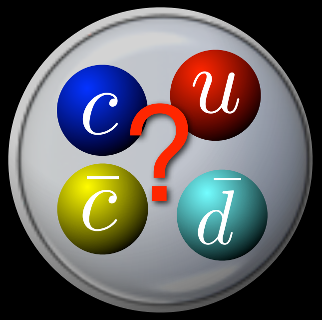
‣ $e^+ + e^-$ --> Y(4260) --> π + $\underline{Z_c(3900)}$

➡ $\pi^{+/-}$ + J/ψ



BESIII Coll., PRL110 (2013).

see also Belle Coll., PRL110 (2013).



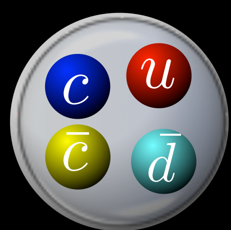
- peak in $\pi^{+/-}J/\psi$ invariant mass (minimal quark content cc^{bar} ud^{bar} <--> tetraquark?)
- M ~ 3900, Γ ~ 60 MeV (Breit-Wigner, Flatte) --> just above $D^{bar}D^*$ threshold
- $J^{PC}=1^{+-}$ is most probable <--> couple to s-wave meson-meson states

Tetraquark candidate $Z_c(3900)$

★ **structure of $Z_c(3900)$ studied by models**

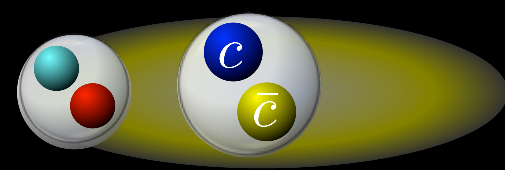
genuine resonance

tetraquark



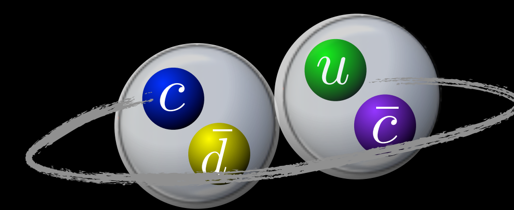
Maiani et al.('13)

J/ψ + π atom



Voloshin('08)

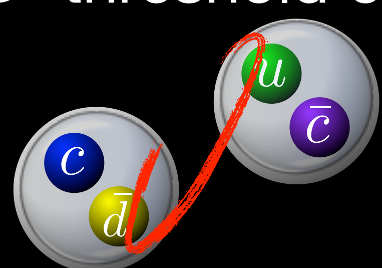
$D^{bar}D^*$ molecule



Nieves et al.('11) + many others

kinematical origin

$D^{bar}D^*$ threshold effect

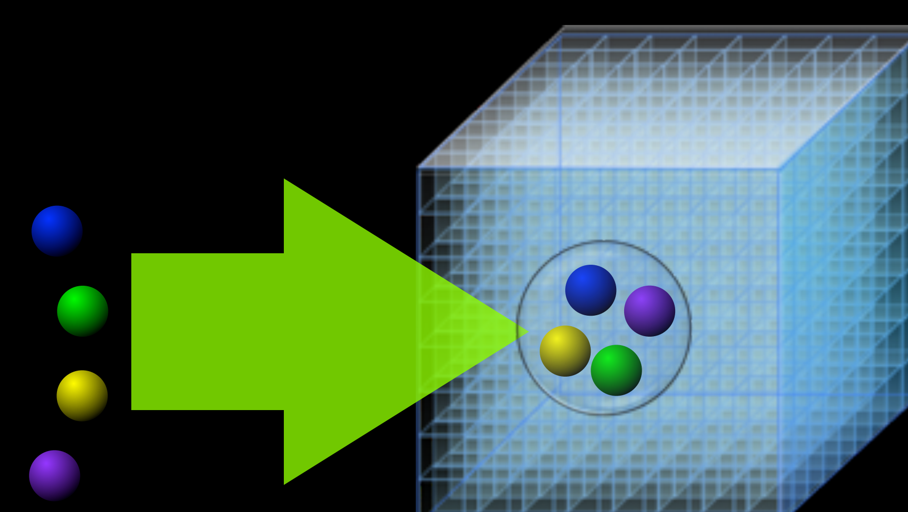


Chen et al.('14), Swanson('15)

conclusion not achieved

➡ **poor information on interactions**

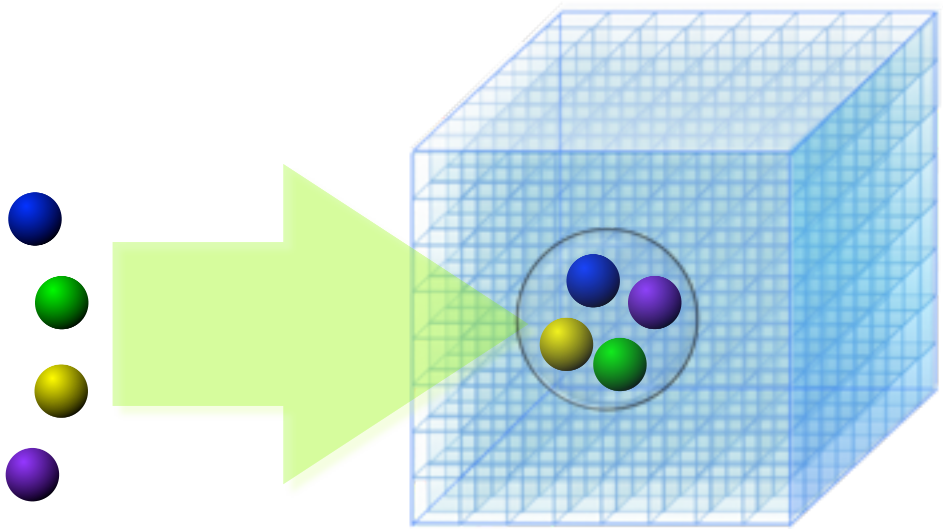
★ LQCD simulations for $Z_c(3900)$



$Z_c(3900)$ on the lattice

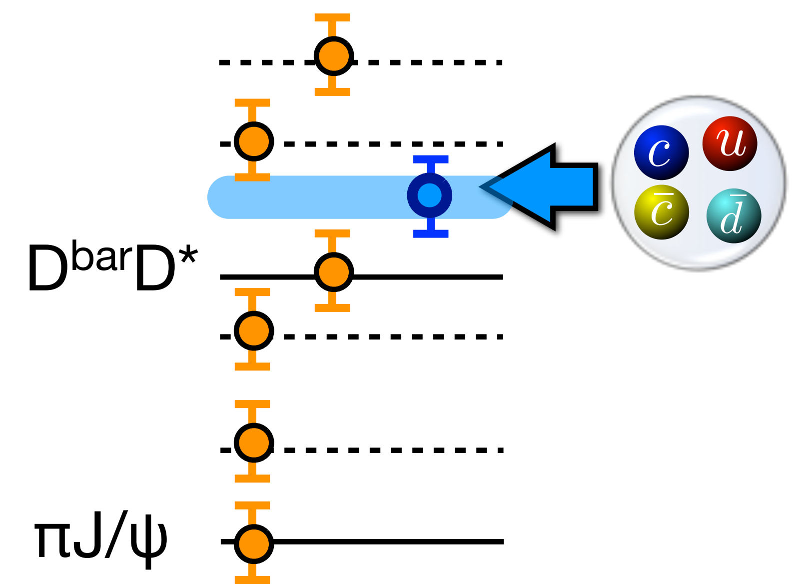
✦ Conventional approach: temporal correlation

➡ identify all relevant $W_n(L)$ (n=0,1,2,3,...)



$$\langle 0|[c\bar{c}u\bar{d}](t)[c\bar{c}u\bar{d}]^\dagger(0)|0\rangle = \sum A_n e^{-W_n t}$$

variational method



✓ No positive evidence for $Z_c(3900)$ in $J^{PC}=1^{+-}$

(observed spectrum consistent with scat. states)

S. Prelovsek et al., PLB 727 (2013), PRD91 (2015).
S.-H. Lee et al., PoS Lattice2014 (2014).

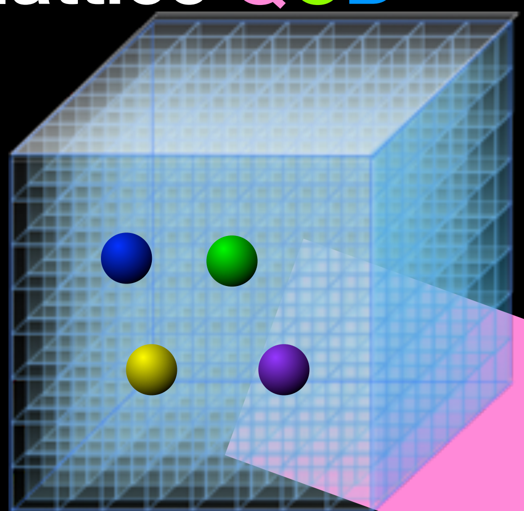
★ Why is the peak observed in expt.?

- (broad) resonance? threshold effect?

★ How can we find resonance in LQCD data?

Strategy for studies of resonances from LQCD

lattice QCD



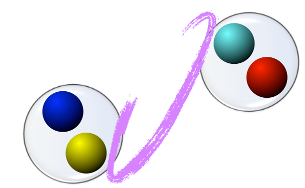
Conventional approach

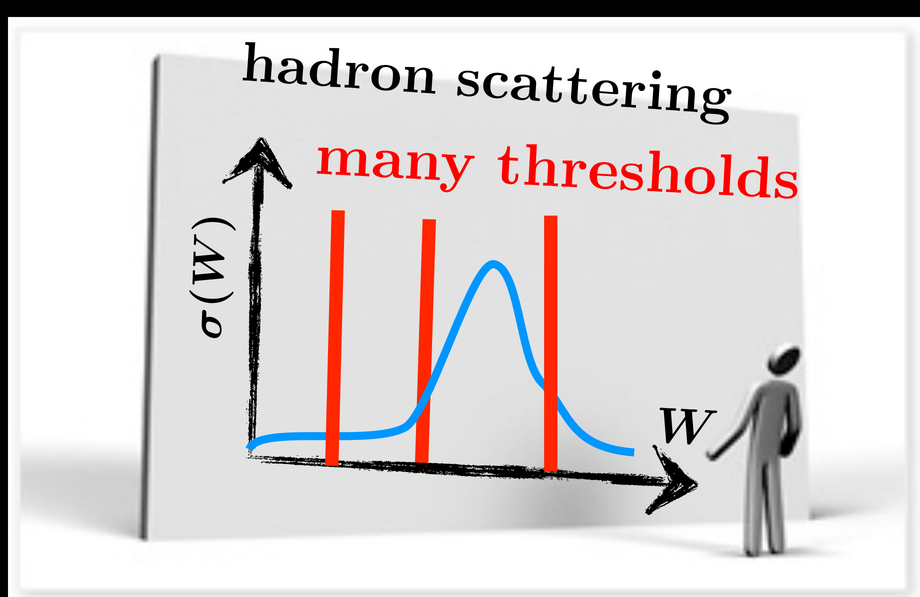
$$\langle 0|\Phi(x)\Phi^\dagger(0)|0\rangle = A_1 e^{-W_1\tau} + A_2 e^{-W_2\tau} + \cdots$$

(W_1, W_2, ... are eigen-energies)

e.g., 4-quark operator

$$\Phi(x) = \bar{q}(x)\bar{q}(x)q(x)q(x)$$





hadron resonances

★ Resonance energy does NOT correspond to eigen-energy

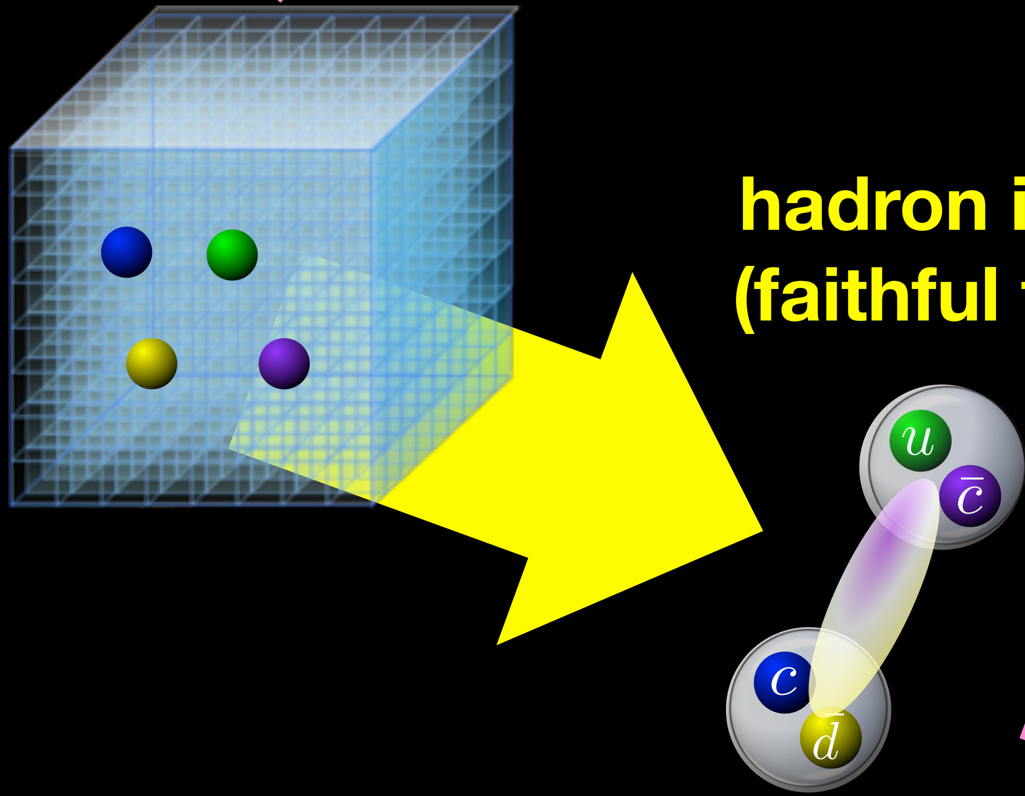
★ Resonances are embedded into coupled-channel scattering states

➡ Resonance energy is determined from pole of coupled-channel S-matrix

Strategy for studies of resonances from LQCD

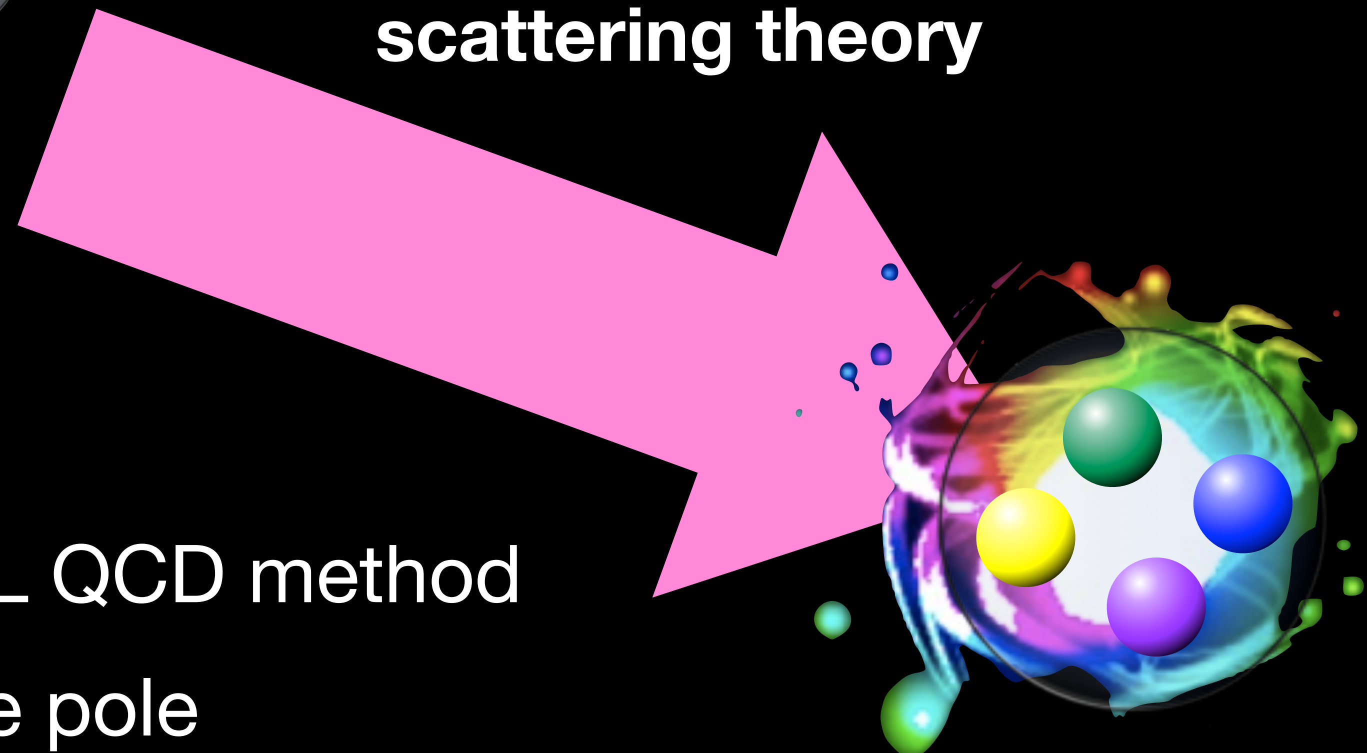
(Resonance search through scattering observable)

lattice QCD



hadron interactions
(faithful to S-matrix)

scattering theory

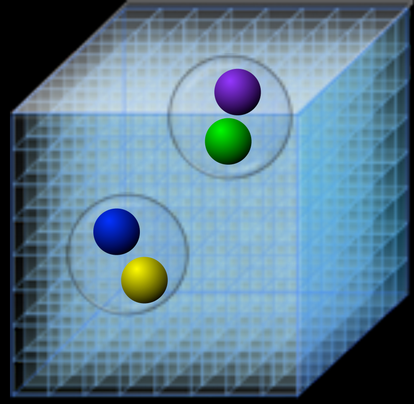


hadron resonances

contents

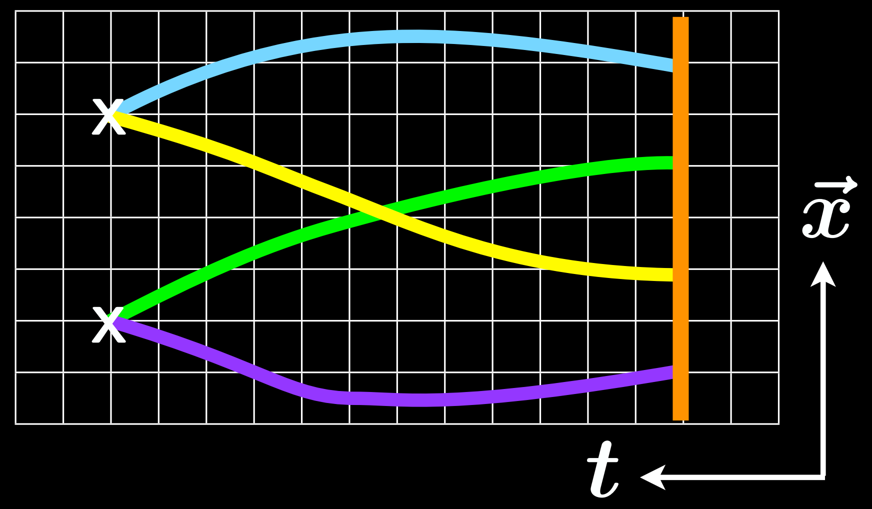
- hadron interactions & HAL QCD method
- strategy to find resonance pole
- coupled-channel scattering
- LQCD results about $Z_c(3900)$
- summary

Hadronic interactions from LQCD



hadronic correlation function

$$C_{(2)}(\vec{r},t) \equiv \langle 0|\phi_1(\vec{r},t)\phi_2(\vec{0},t)\mathcal{J}^\dagger(t=0)|0\rangle$$
$$= \sum_n A_n \psi_n(\vec{r}) e^{-W_n t}$$



- Energy eigenvalue $W_n(L)$
- NBS (Nambu-Bethe-Salpeter) wave function $\psi_n(r)$

(outside interactions)
[$\psi_n(r)$ --> $\sin(k_n r + \delta(k_n)) / k_n r$]

C.D. Lee et al., NPB619 (2001).

Finite Volume Method

- $W_n(L)$ --------> phase shift

Lüscher's formula

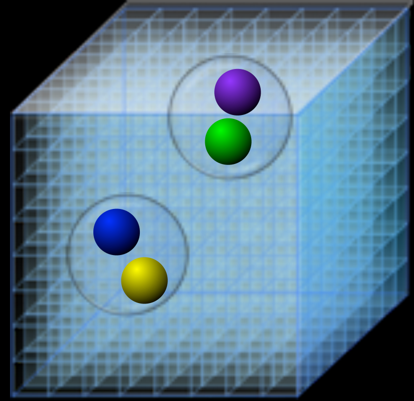
Lüscher, Nucl. Phys. B354, 531 (1991).

Lüscher's finite volume formula

$$k_n \cot\delta(k_n) = \frac{4\pi}{L^3}\sum_{m\in\mathbb{Z}^3}\frac{1}{\vec{p}_m^{\,2} - {k_n}^2}$$

$$W_n = \sqrt{m_1^2 + k_n^2} + \sqrt{m_2^2 + k_n^2}$$

Hadronic interactions from LQCD



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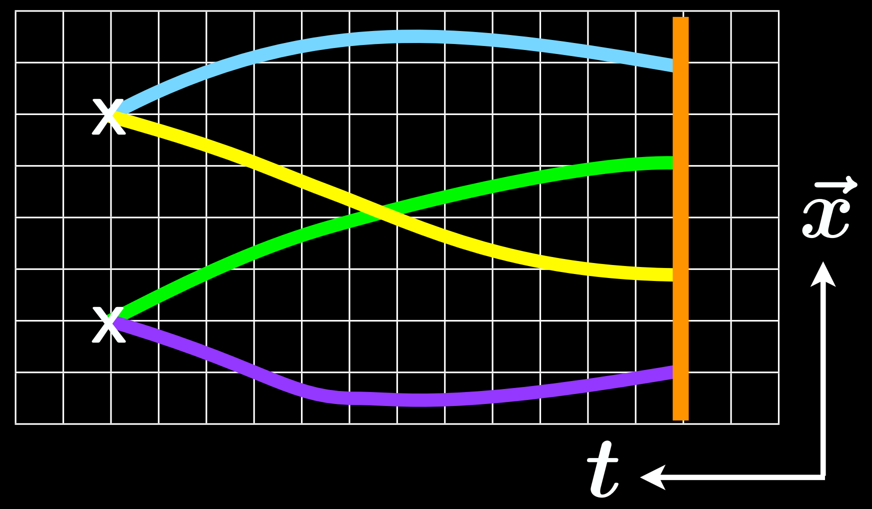

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Finite Volume Method

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↑
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➡ difficult with coupled-channel problems

HAL QCD Method

- $\psi_n(r)$ --> **2PI kernel** ($\psi = \phi + G_0 U \psi$)

--> phase shift, binding energy, ...

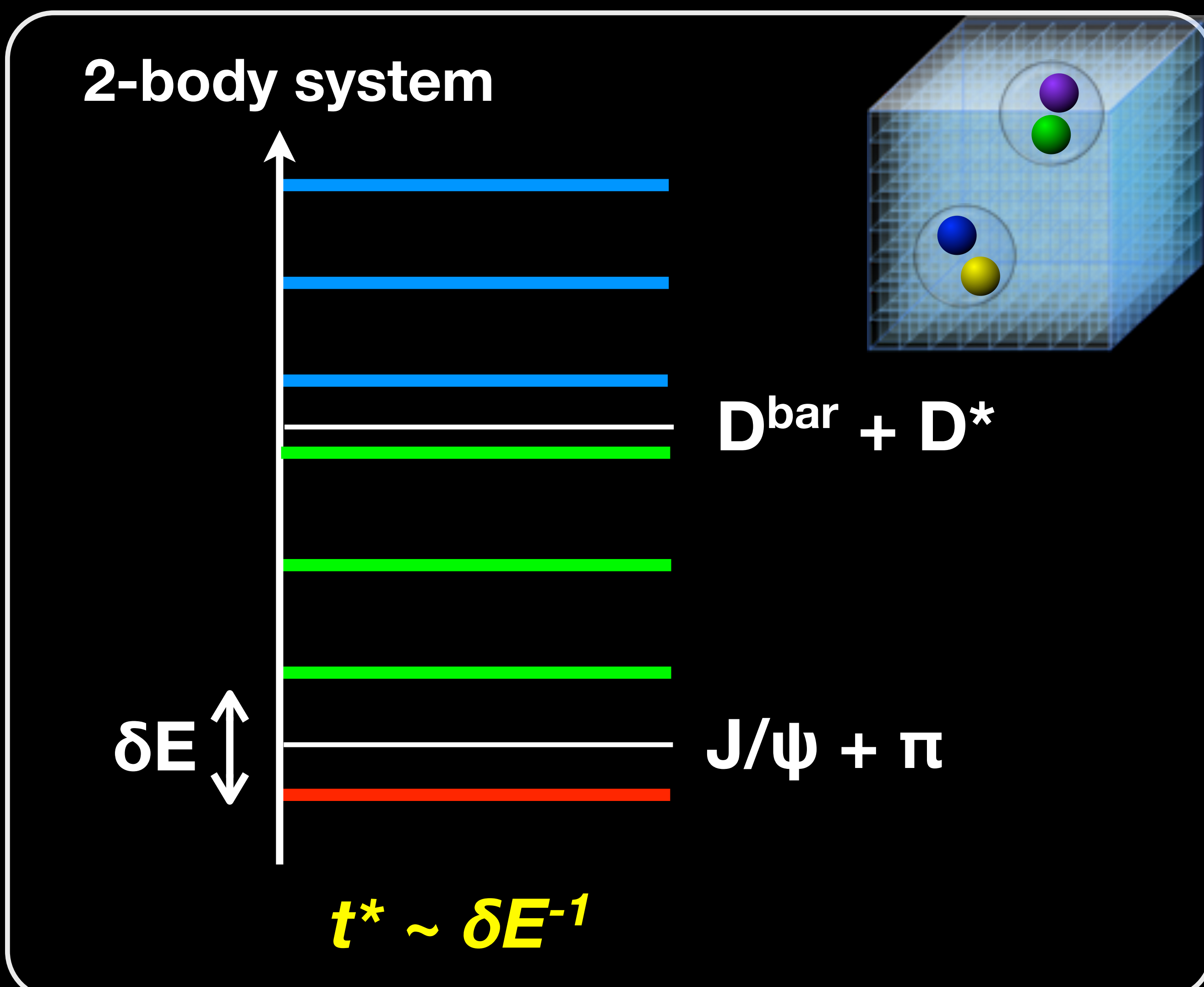
Ishii, Aoki, Hatsuda, PRL 99, 022001 (2007).

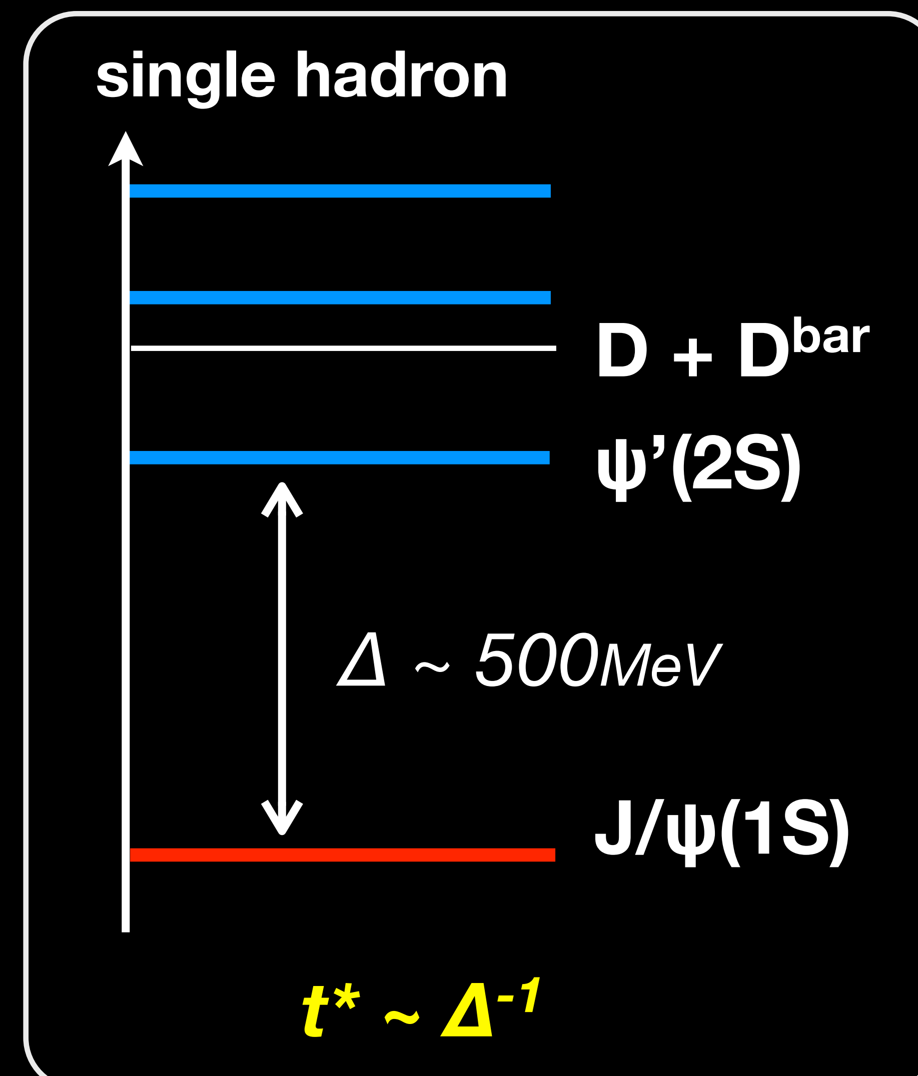
Ishii et al. [HAL QCD], PLB 712, 437 (2012).

Challenge in hadron scatterings

★ Excited scattering states become noise when determining W_0 even in single-channel scatterings

$$C_{(2)}(t) = b_0 e^{-W_0 t} + b_1 e^{-W_1 t} + \cdots \longrightarrow b_0 e^{-W_0 t} \quad (t > t^*)$$





★ Sophisticated methods is necessary! talk by T. Doi (Thu.)

see for BB systems, Iritani, Doi et al. [HAL QCD], JHEP10 (2016) 101.

(single-channel) HAL QCD method -- potential as a representation of S-matrix --

- The scattering states do exist, and we should tame the scattering states

➡ **time-dependent HAL QCD method**

Ishii [HAL QCD], PLB 712 (2012).

✓ define energy-independent potential *U(r,r')*

$$\int d\vec{r}'\, U(\vec{r},\vec{r}')\psi_n(\vec{r}') = (E_n - H_0)\,\psi_n(\vec{r})$$

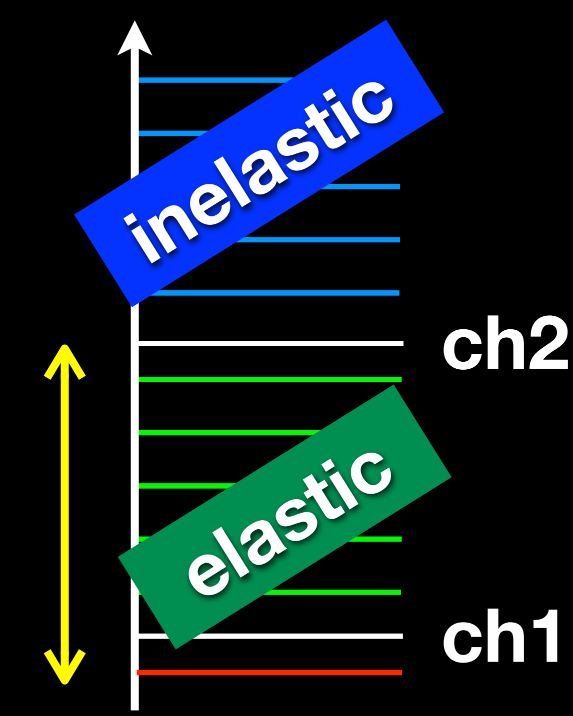
$$U(\vec{r},\vec{r}') \equiv \sum_{n<n_{\rm th}} (E_n - H_0)\,\psi_n(\vec{r})\bar{\psi}_n(\vec{r}')$$

➡ All elastic states share the same potential U(r,r')

$U\ \psi_0 = (E_0 - H_0)\ \psi_0$

$U\ \psi_1 = (E_1 - H_0)\ \psi_1$

⋮



✓ derive *U(r,r')* from time-dependent Schrödinger-type eq.

$$\int d\vec{r}'\, U(\vec{r},\vec{r}') R(\vec{r}',t) = \left(-\frac{\partial}{\partial t} + \frac{1}{4m}\frac{\partial^2}{\partial t^2} - H_0\right) R(\vec{r},t)$$

$$\begin{aligned} R(\vec{r},t) &= C_{(2)}(\vec{r},t)/\left(C_{(1)}(t)\right)^2 \\ &= b_0\psi_0(\vec{r})e^{-(W_0-2m)t} + b_1\psi_1(\vec{r})e^{-(W_1-2m)t} + \cdots \end{aligned}$$

(single-channel) HAL QCD method -- potential as a representation of S-matrix --

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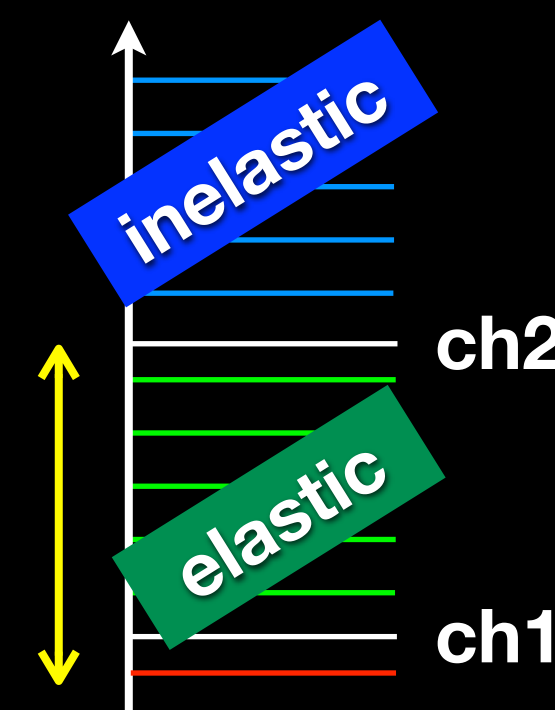

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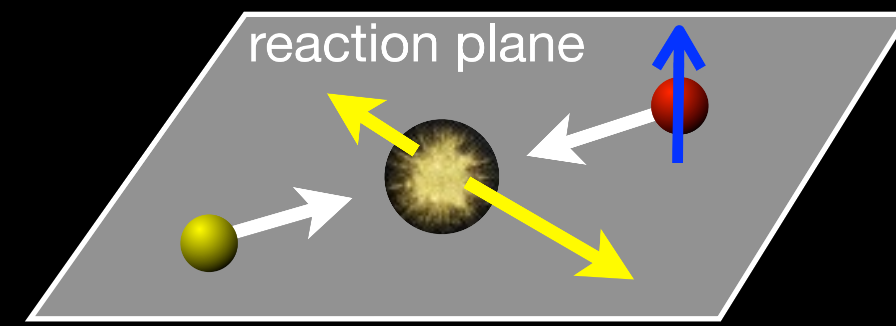
$$R(\vec{r},t) = b_0\psi_0(\vec{r})e^{-(W_0-2m)t} + b_1\psi_1(\vec{r})e^{-(W_1-2m)t} + \cdots$$

➡ **Scat. states are no more contamination than signal** ($t^* \sim (E_{ch2} - E_{ch1})^{-1}$)

How can we find resonances?

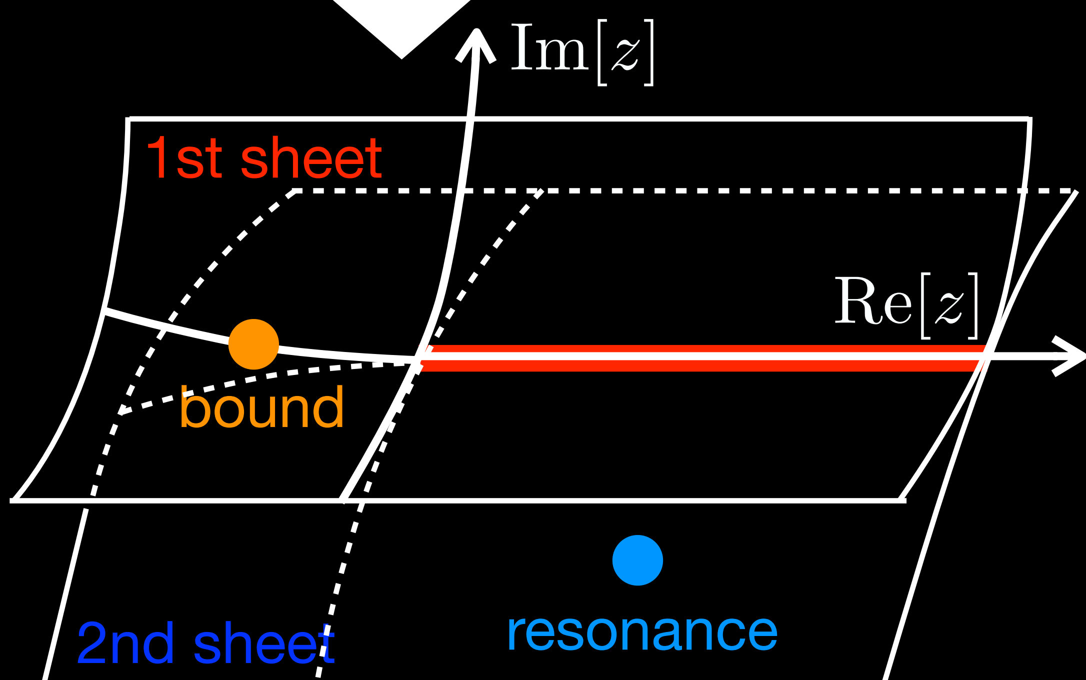


If we have complete set of expt. data,

$S^{(\ell)}(W)$ ⟵ **partial wave analysis**

- cross sections (dσ/dΩ)
- spin polarization observables
- etc.

Pole of S-matrix is **uniquely** determined ⟵ **identity theorem + analyticity of S-matrix**



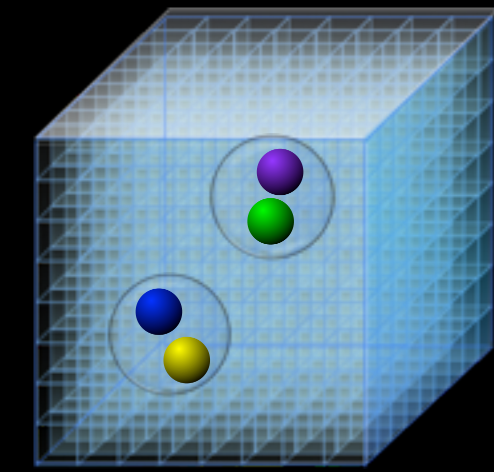
bound state (1st sheet)

- pole position --> binding energy
- residue --> coupling to scattering state

resonance (2nd sheet)

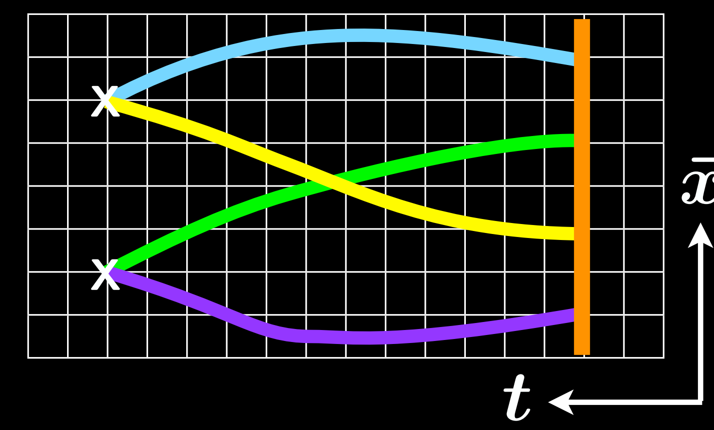
- analytic continuation onto 2nd sheet
- pole position --> resonance energy
- residue --> coupling to scat. state, partial decay

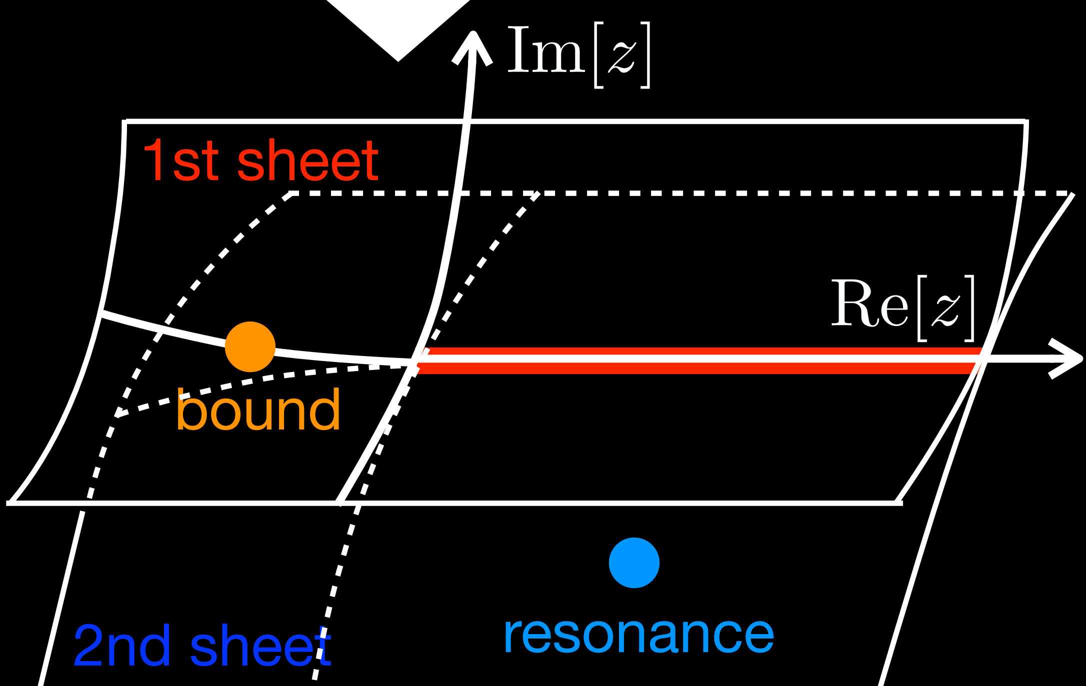
Strategy to search for complex poles on the lattice



Resonance pole from lattice QCD

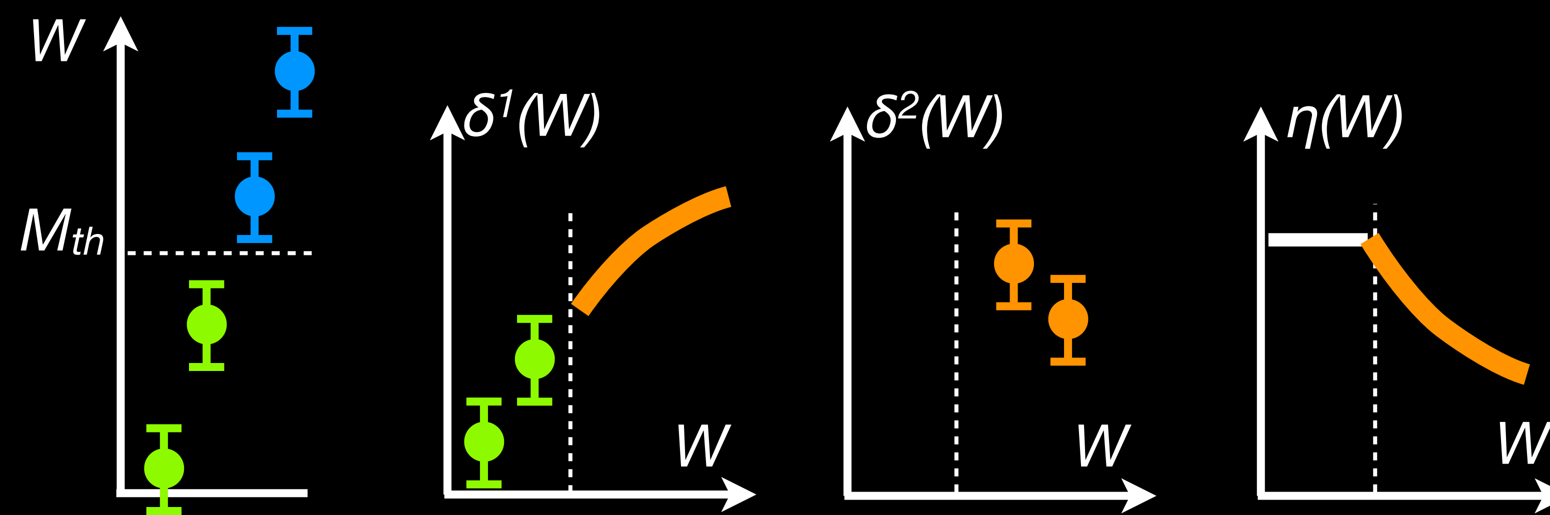
$$S^{(\ell)}(W) \Leftarrow \langle 0|\phi_1(\vec{r},t)\phi_2(\vec{0},t)\mathcal{J}^\dagger(t=0)|0\rangle = \sum_n A_n \psi_n(\vec{r}) e^{-W_n t}$$





- **coupled-channel Lüscher's formula**

➡ $W_n(L)$ --> $\delta^1(W_n)$, $\delta^2(W_n)$, $\eta(W_n)$

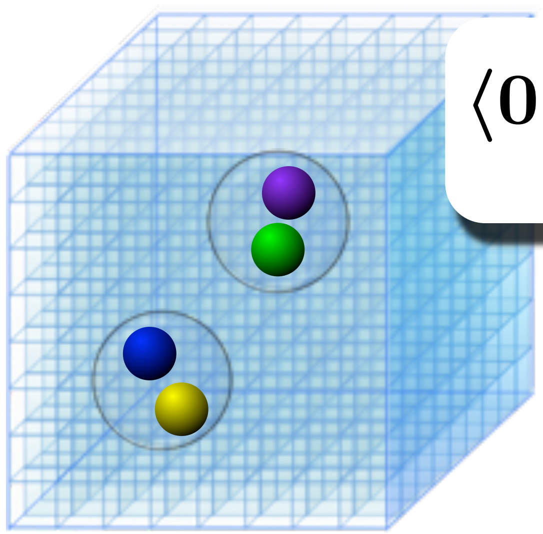


(**coupled-channel scattering difficult**)

- $\delta^1(W_n)$, $\delta^2(W_n)$, $\eta(W_n)$ <-- $W_n(L_1) = W_n(L_2) = W_n(L_3)$

Coupled-channel HAL QCD method

✦ **measure relevant NBS wave function --> channel is defined**



$$\langle 0|\phi_1^a(\vec{x}+\vec{r},t)\phi_2^a(\vec{0},t)\mathcal{J}^\dagger(0)|0\rangle = \sqrt{Z_1^a Z_2^a}\sum_n A_n \psi_n^a(\vec{r}) e^{-W_n t}$$

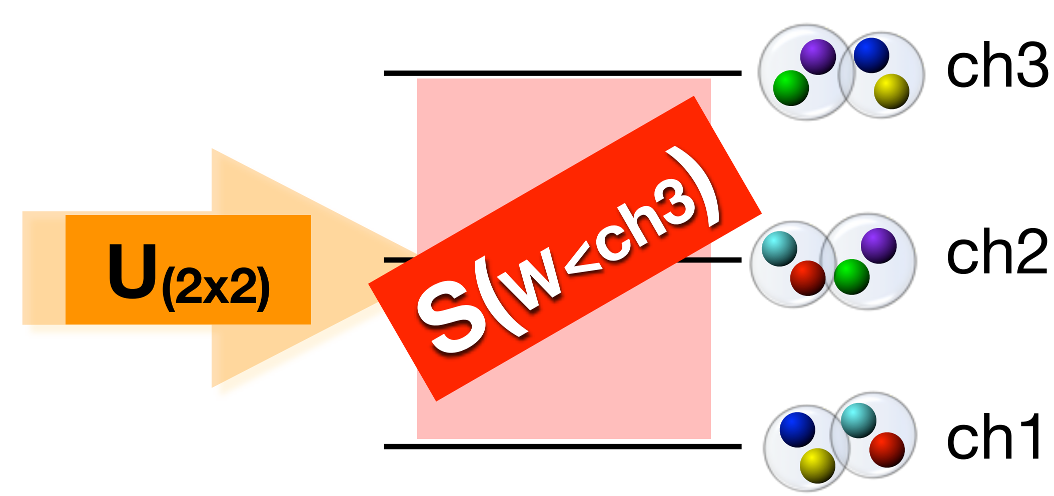
see for full details,
Aoki et al. (HAL QCD), PRD87 (2013); Proc. Jpn. Acad., Ser. B, 87 (2011).

★ **define coupled-channel potential using $\psi^a(r)$**

$$\left(\nabla^2 + (\vec{k}_n^a)^2\right)\psi_n^a(\vec{r}) = 2\mu^a \sum_b \int d\vec{r}' U^{ab}(\vec{r},\vec{r}')\psi_n^b(\vec{r}')$$

★ **coupled-channel potential $U^{ab}(r,r')$:**

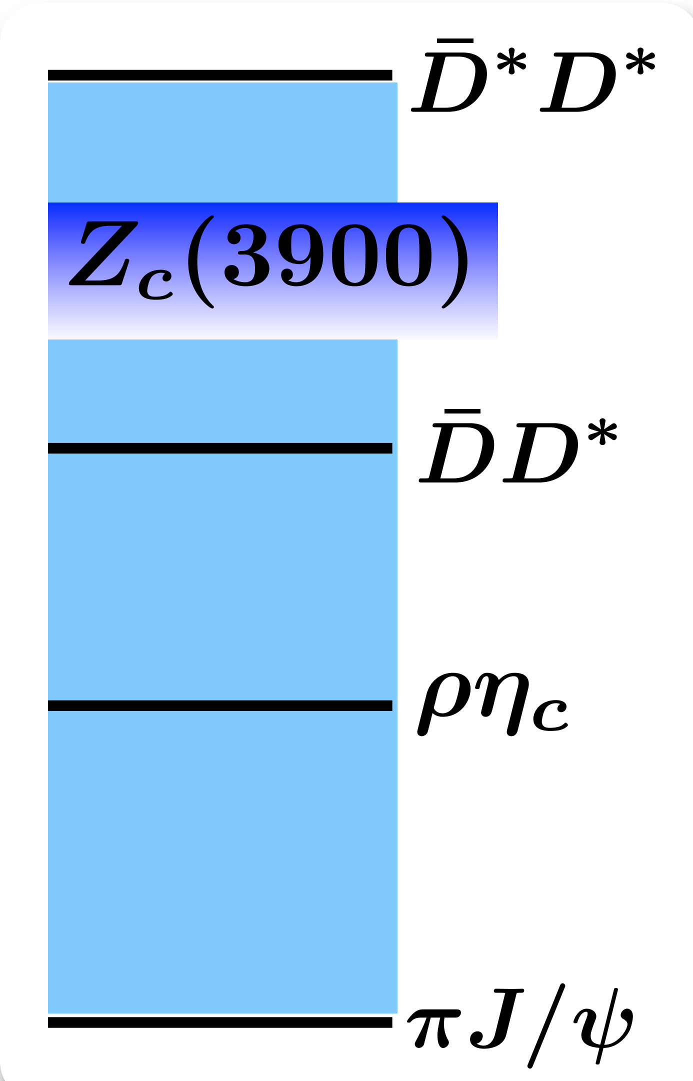
- $U^{ab}(r,r')$ is faithful to **coupled-channel S-matrix**
- $U^{ab}(r,r')$ is **energy independent** (until new threshold opens)
- Non-relativistic approximation is not necessary
- $U^{ab}(r,r')$ contains all 2PI contributions



$Z_c(3900)$ in $I^G(J^{PC})=1^+(1^{+-})$

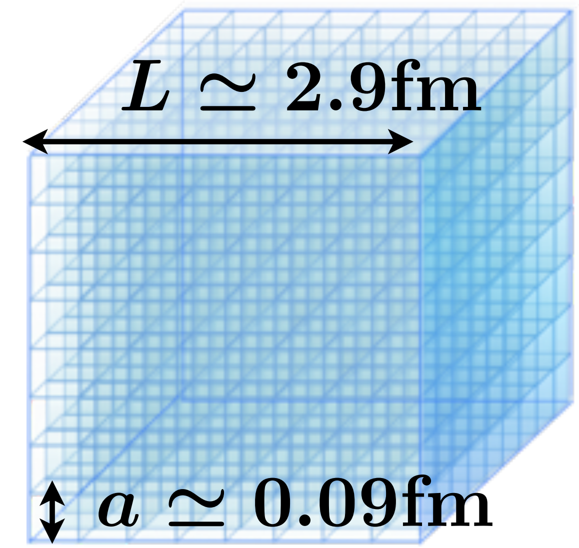
-- $\pi J/\psi$ - $\rho\eta_c$ - $D^{bar}D^*$ coupled-channel --

Y. Ikeda et al., [HAL QCD], PRL117, 242001 (2016).



❖ N_f=2+1 full QCD

- Iwasaki gauge
- clover Wilson quark
- 32^3 x 64 lattice



❖ Tsukuba-type Relativistic Heavy Quark (charm)

- remove leading cutoff errors $O((m_c a)^n)$, $O(\Lambda_{QCD}\, a)$, ...

➡ We are left with $O((a\Lambda_{QCD})^2)$ syst. error (~ a few %)

light meson mass (MeV)

m_π= 411(1), 572(1), 701(1)
m_ρ= 896(8), 1000(5), 1097(4)

charm meson mass (MeV)

$m_{\eta c}$= 2988(1), 3005(1), 3024(1)
$m_{J/\psi}$= 3097(1), 3118(1), 3143(1)
m_D= 1903(1), 1947(1), 2000(1)
m_{D^*}= 2056(3), 2101(2), 2159(2)

Lattice QCD setup : thresholds

✦ Thresholds in $I^G J^P = 1^+ 1^+$ channel

Physical thresholds

- $D^{bar}D^*$ = 3872
- $\pi\psi'$ = 3821
- $\pi\pi\eta_c$ = 3256
- $\pi J/\psi$ = 3232

LQCD simulation

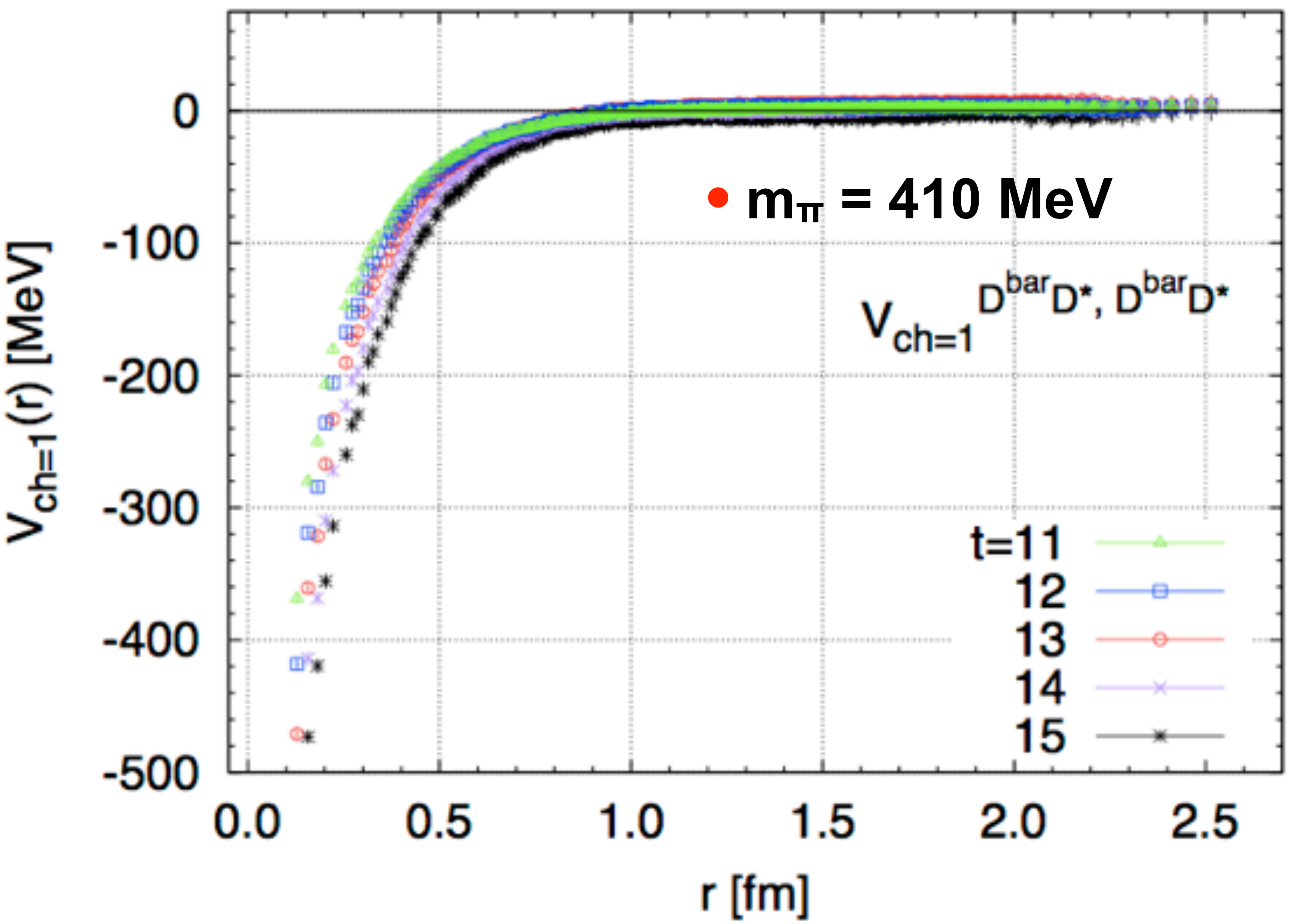
- $D^{bar}D^*$ = 3959, 4048, 4159
- $\rho\eta_c$ = 3884, 4005, 4121
- $\pi J/\psi$ = 3508, 3688, 3844

- $M_{\pi\psi'} > M_{D^{bar}D^*}$ due to heavy m_π
- ρ-->ππ decay not allowed w/ L~3fm

S-wave $\pi J/\psi$ - $\rho\eta_c$ - $D^{bar}D^*$ coupled-channel analysis

$D^{bar}D^{*}$ potential (single-channel calc.)

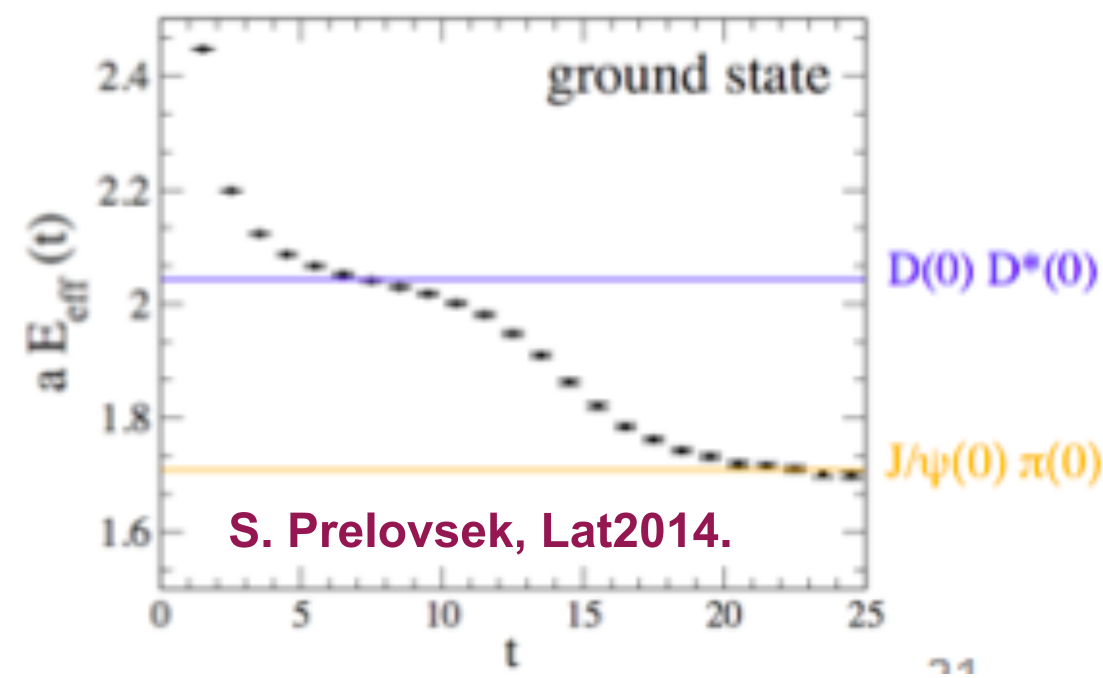
• $D^{bar}D^{*}$: inelastic channel



$D^{bar}D^{*}$ = 3959

Δ = 451

$\pi J/\Psi$ = 3508



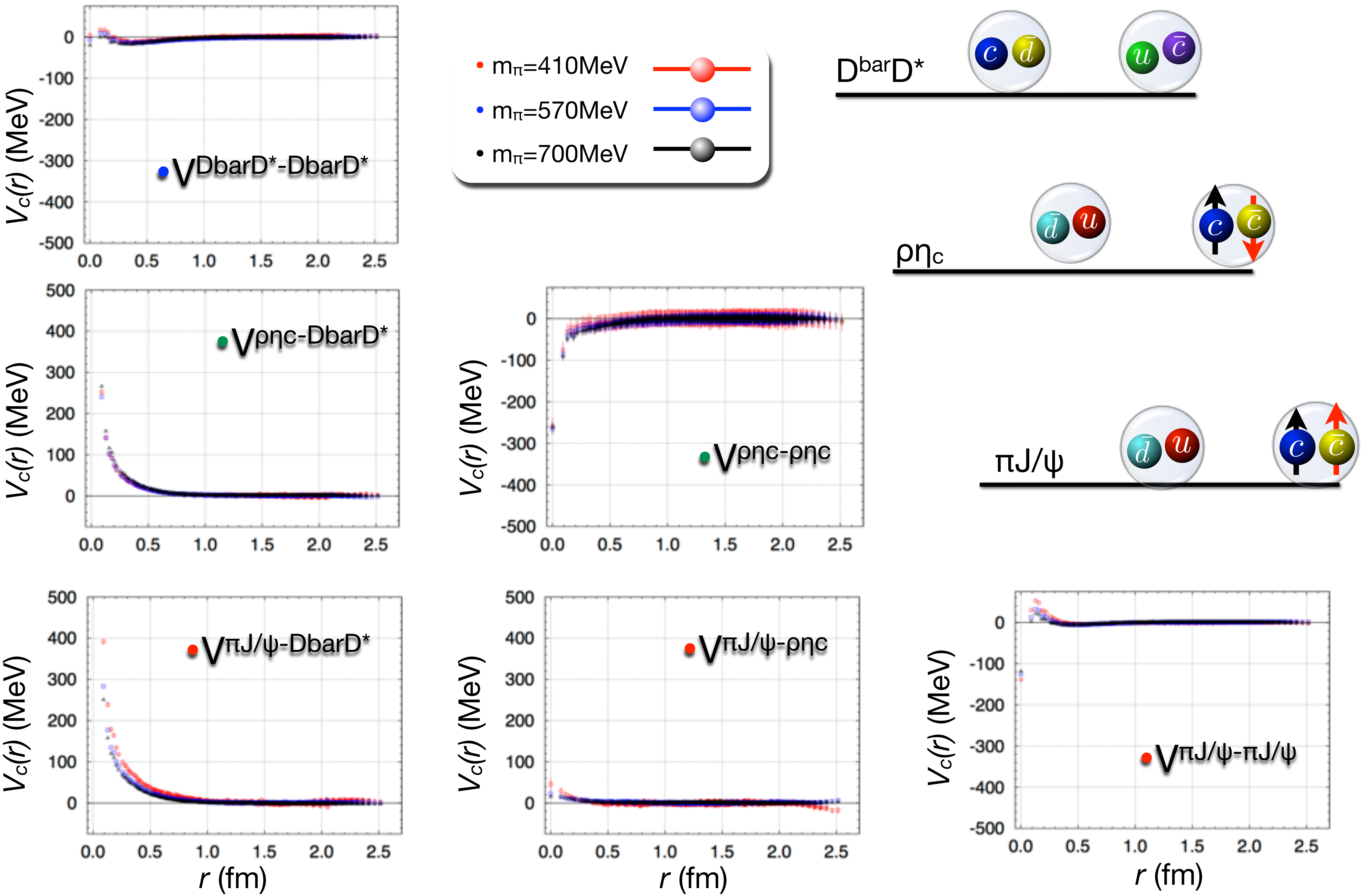
• Time-slice dependence indicates

➡ coupling to lower channels (large contribution from $\pi J/\psi$ and/or $\rho\eta_c$)

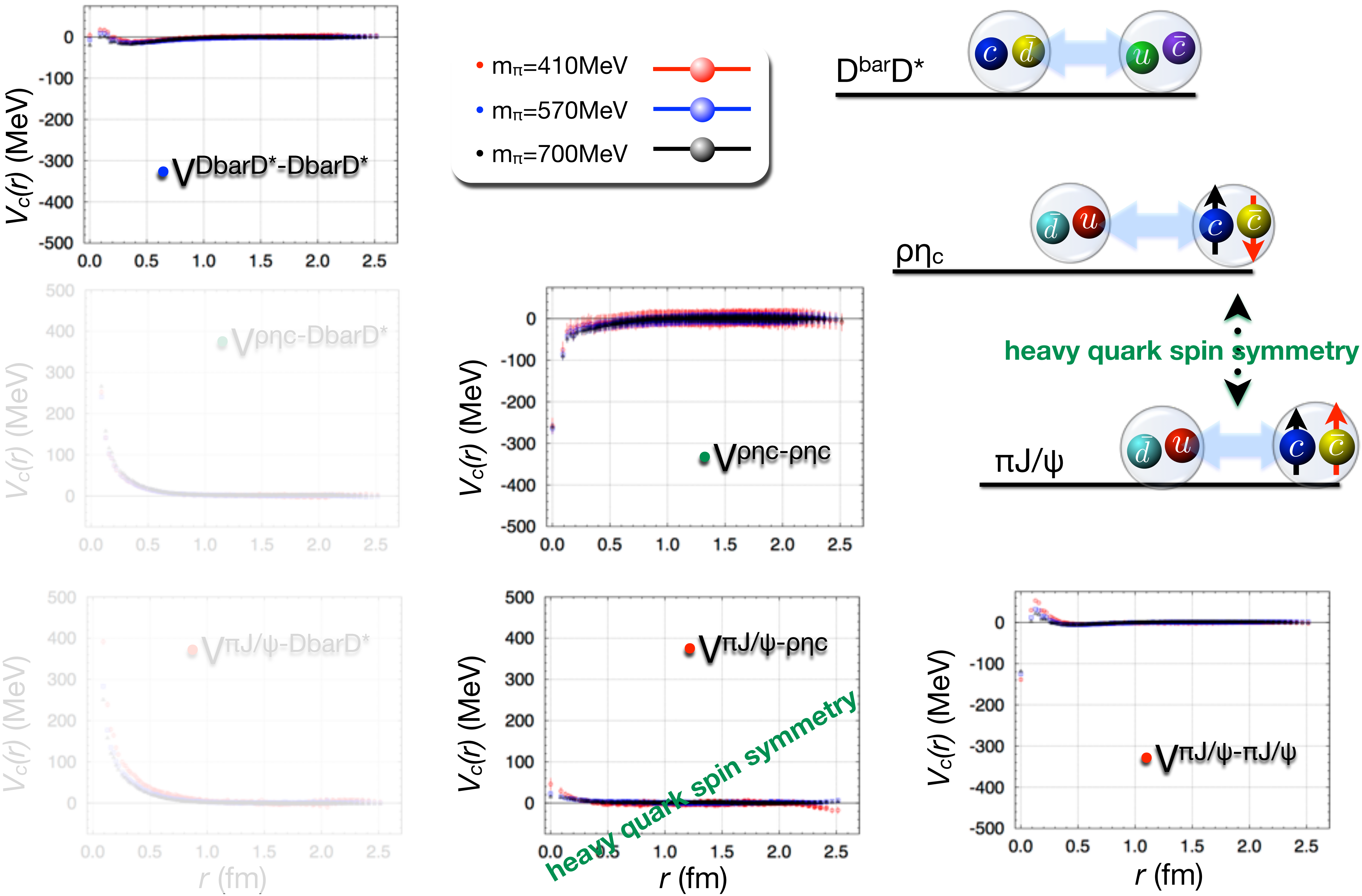
➡ single channel $D^{bar}D^{*}$ potential NOT reliable (huge non-locality)

➡ **coupled-channel analysis is necessary**

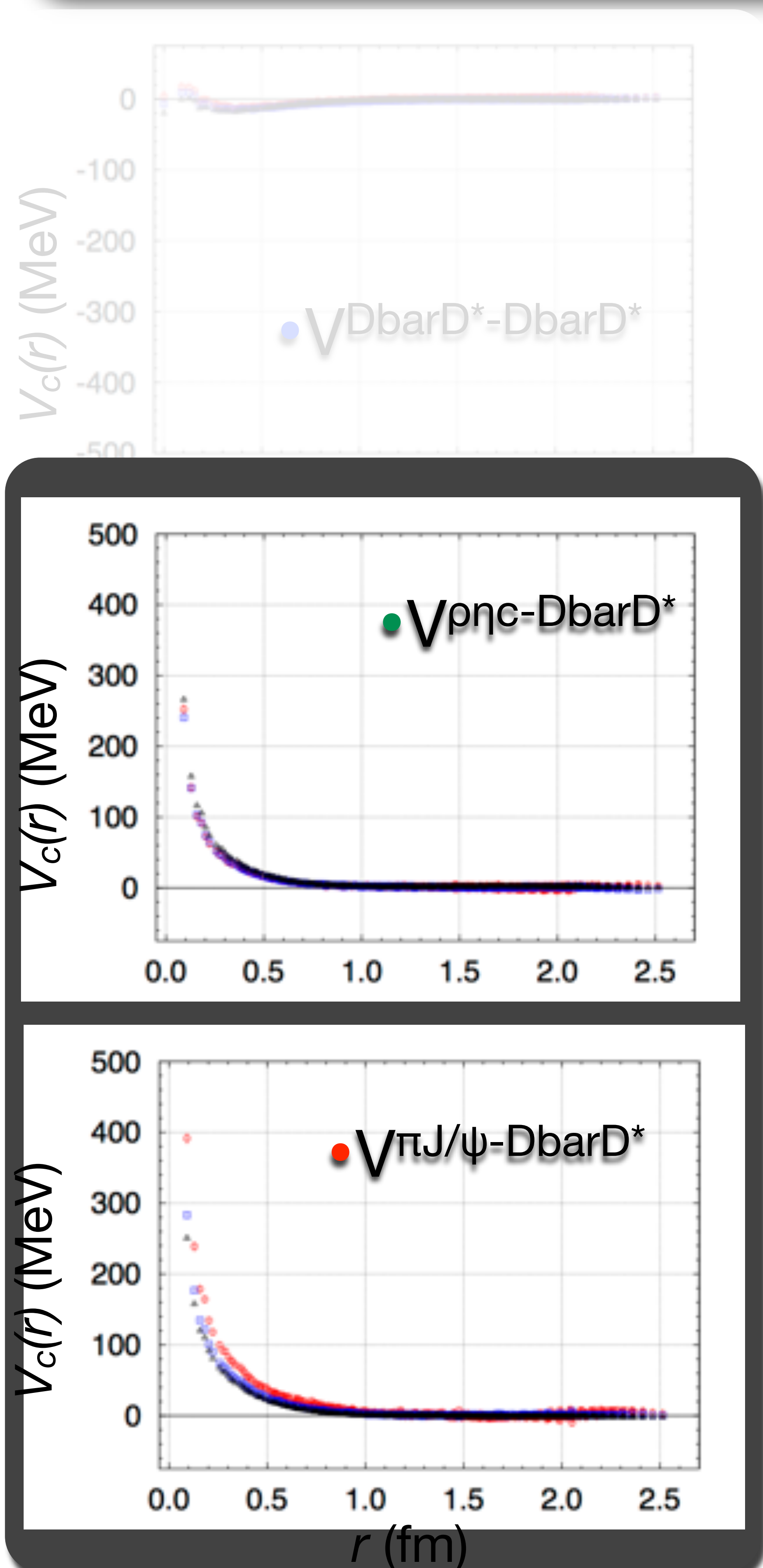
3x3 potential matrix (πJ/ψ - ρηc - DbarD*)

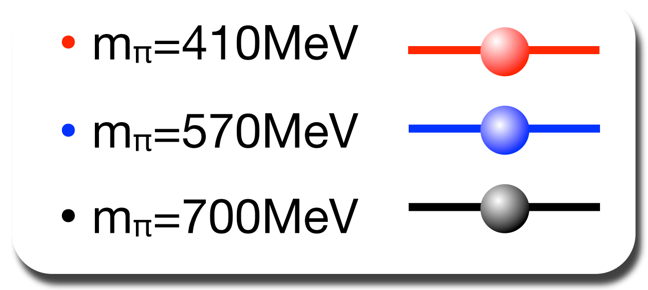


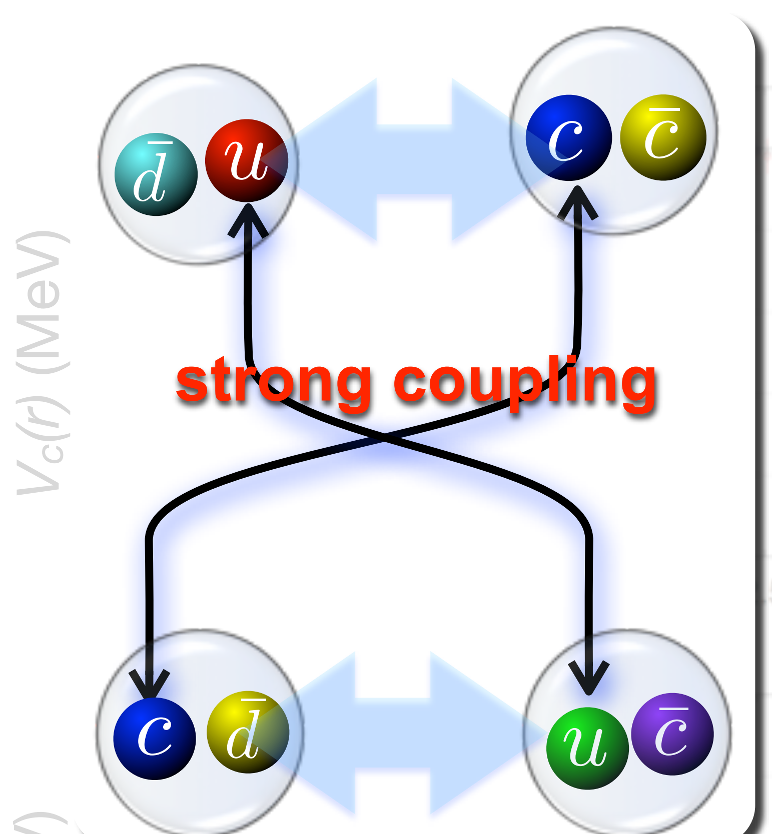
3x3 potential matrix (πJ/ψ - $\rho\eta_c$ - $D^{bar}D^*$)

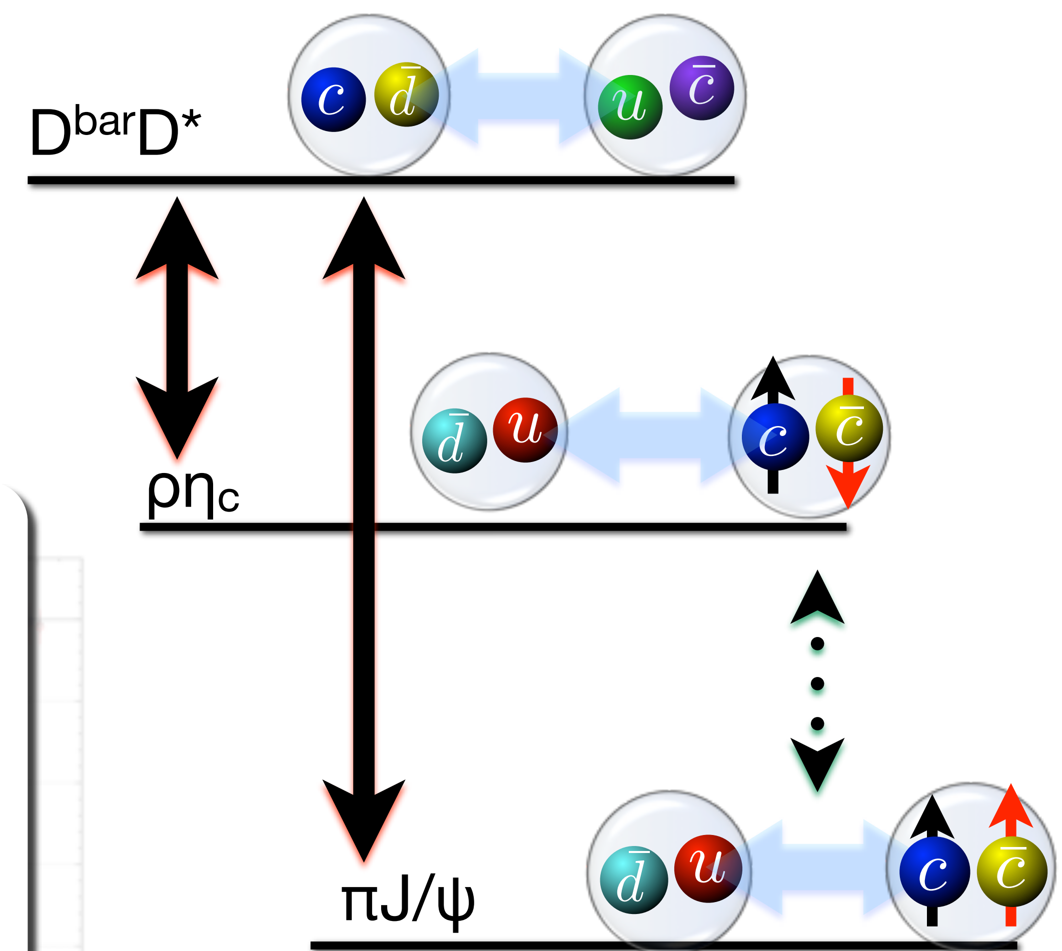


3x3 potential matrix ($\pi J/\psi$ - $\rho\eta_c$ - $D^{bar}D^*$)







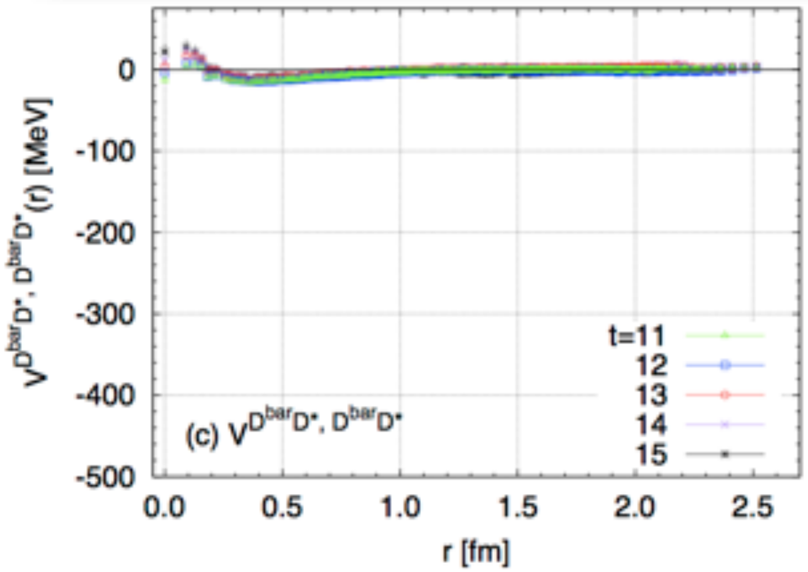


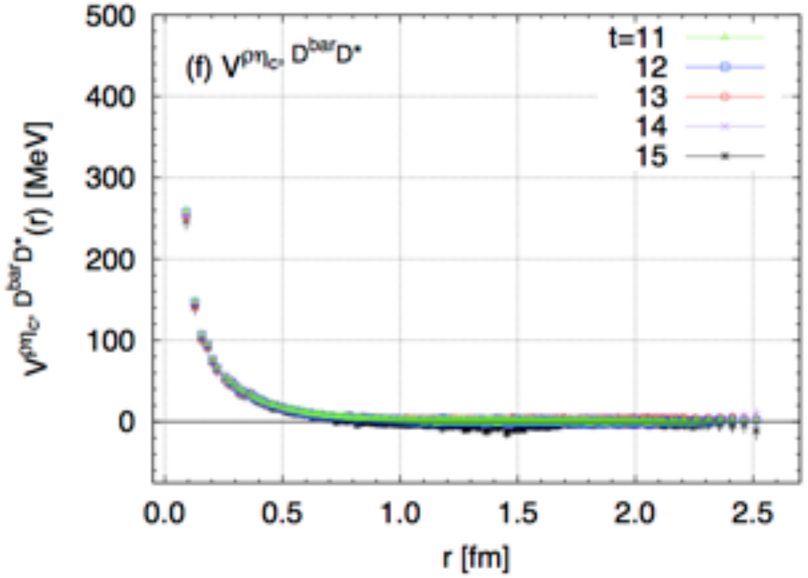
• strong $V^{\pi J/\psi, DbarD^*}$ & $V^{\rho\eta c, DbarD^*}$

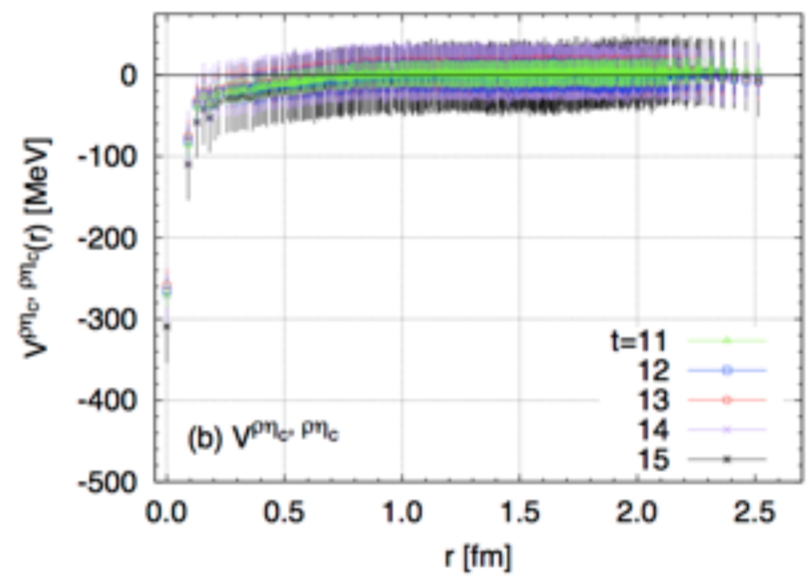
➡ charm quark exchange process

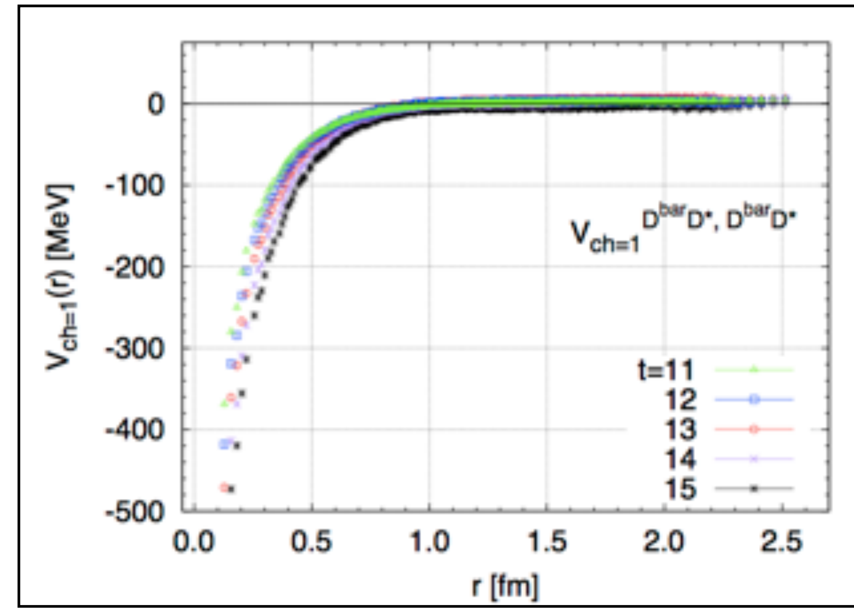
t-dependence on potential matrix

- all $V^{(t=11\text{--}15)}$ are consistent within stat. errors
- coupled-channel simulation works well

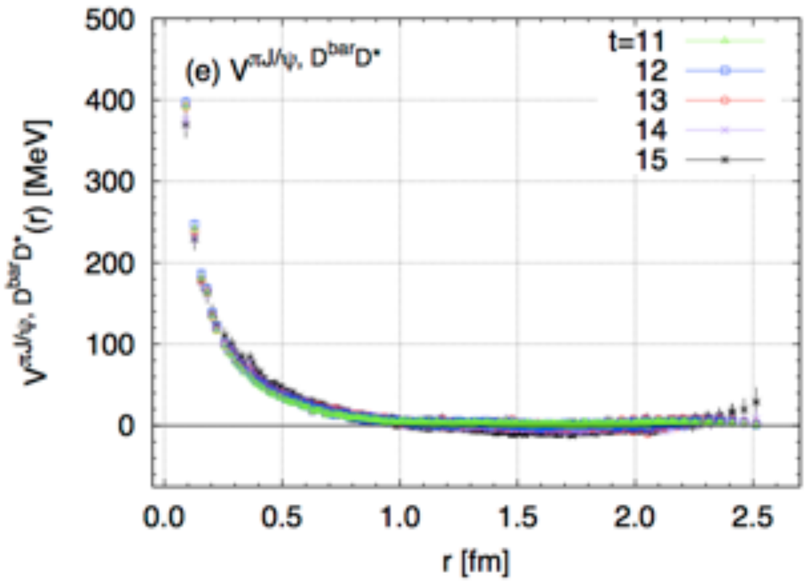


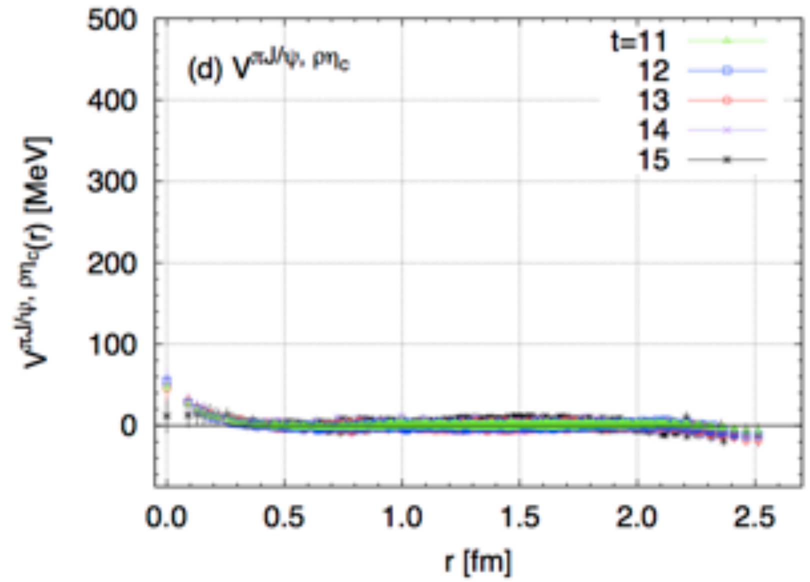


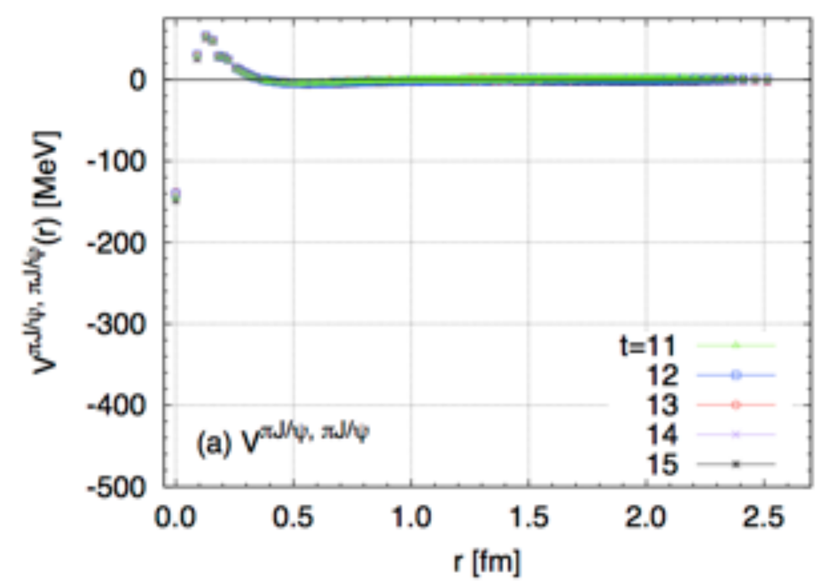




$D^{bar}D^*$ single channel simulation





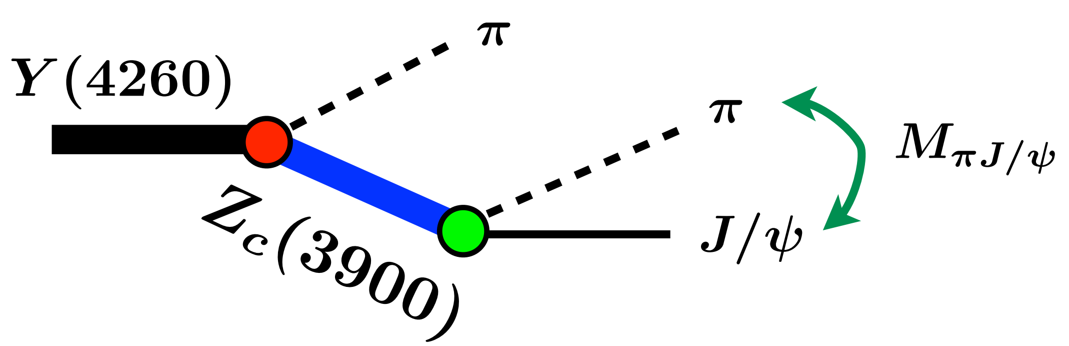


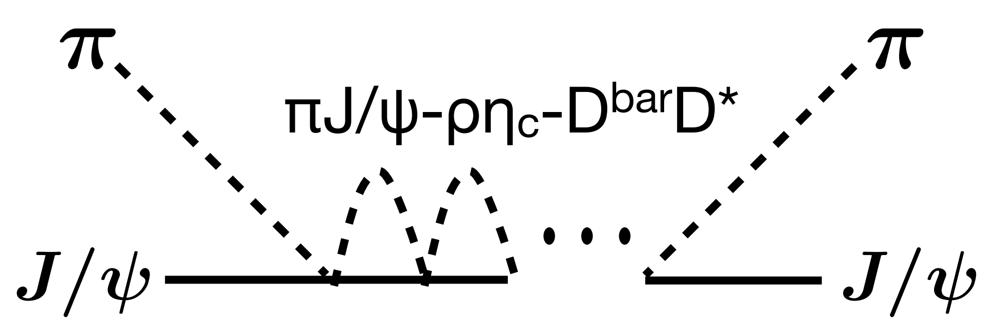
Structure of Z_c(3900)

studied by the most ideal scattering process

- S-wave $\pi J/\psi$ - $\rho\eta_c$ - $D^{bar}D^*$ coupled-channel scattering

➡ **Z_c(3900) is observed in $\pi J/\psi$ --> 2-body scattering is the most ideal reaction**





1. invariant mass spectrum of 2-body scattering

\# of scat. particles proportional to imaginary part of amplitude

$$N_{\rm sc} \propto ({\rm flux}) \cdot \sigma(W) \propto {\rm Im} f(W)$$

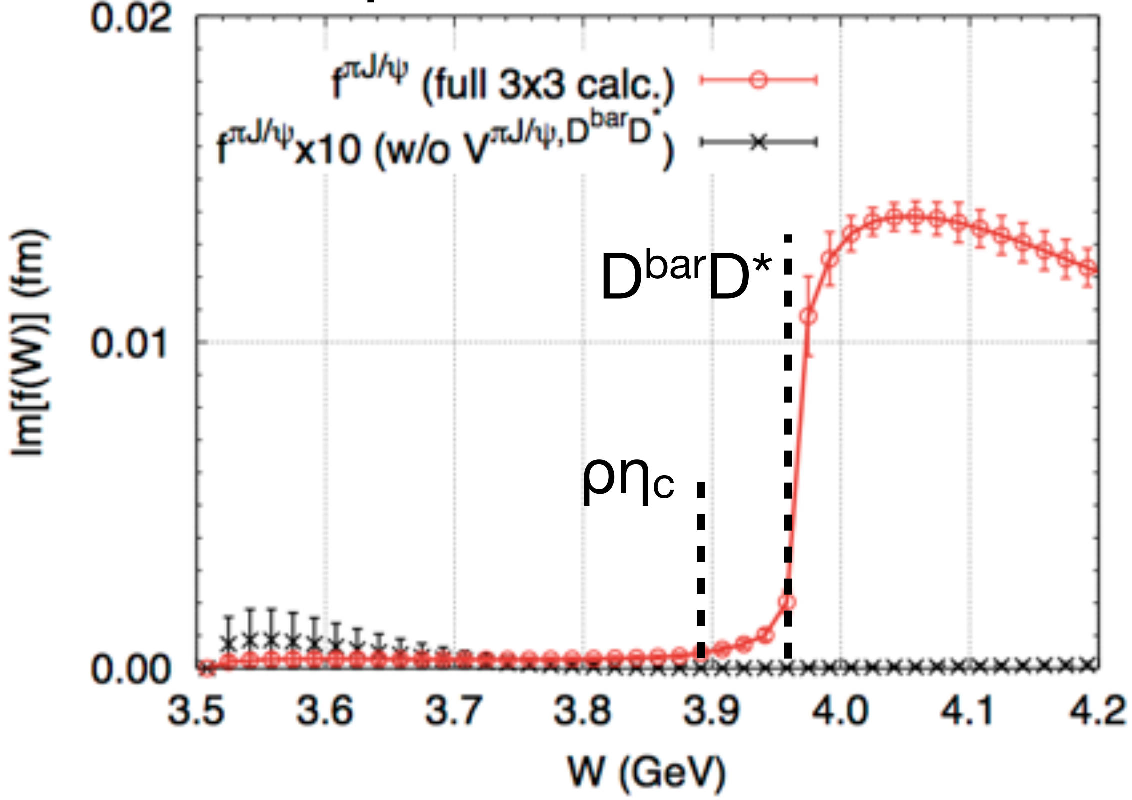
2. pole position of S-matrix

- analytic continuation of c.c. S-matrix onto complex energy plane
- understand nature of *Z_c(3900)*

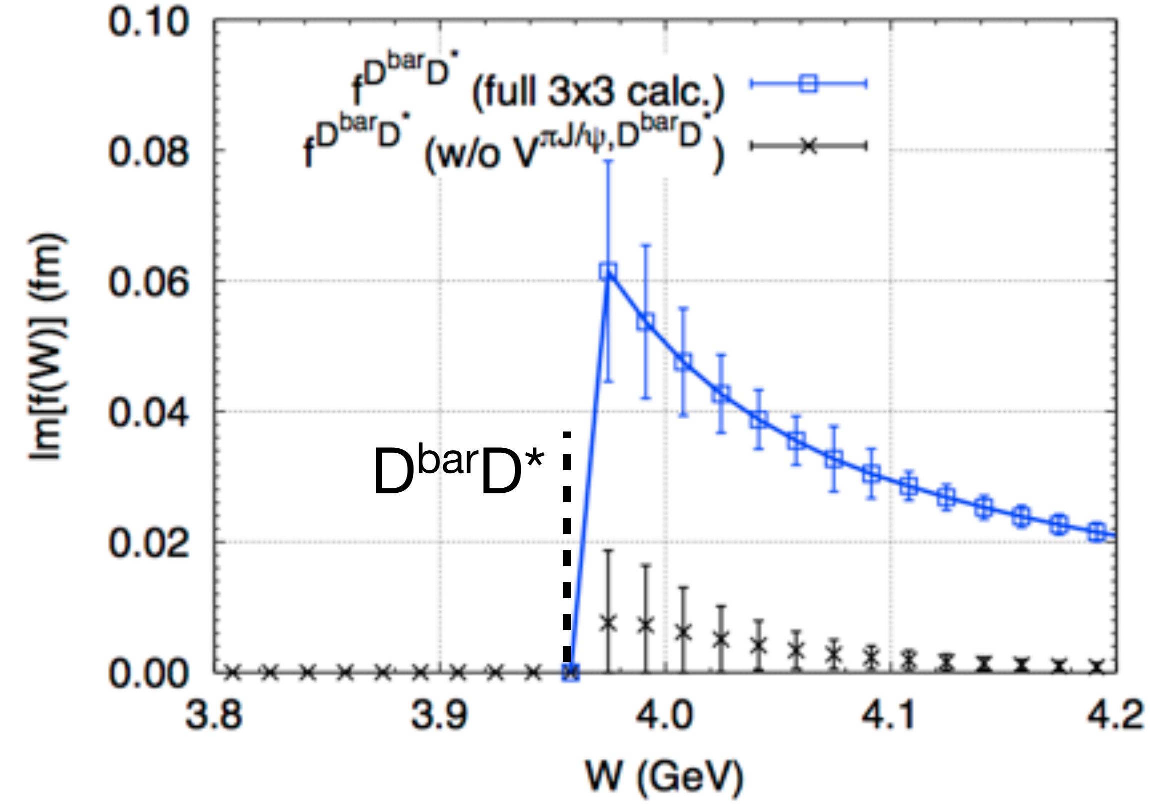
- Results w/ m_π=410MeV are shown. (**weak quark mass dependence observed**)

Mass spectrum of $\pi J/\psi$ (2-body scattering)

- $\pi J/\psi$ invariant mass



- $D^{bar}D^*$ invariant mass



$$\frac{\Gamma(Zc(3900) \to \bar{D}D^*)}{\Gamma(Zc(3900) \to \pi J/\psi)} = 6.2(1.1)(2.7)$$

BESIII Coll., PRL112 (2014).

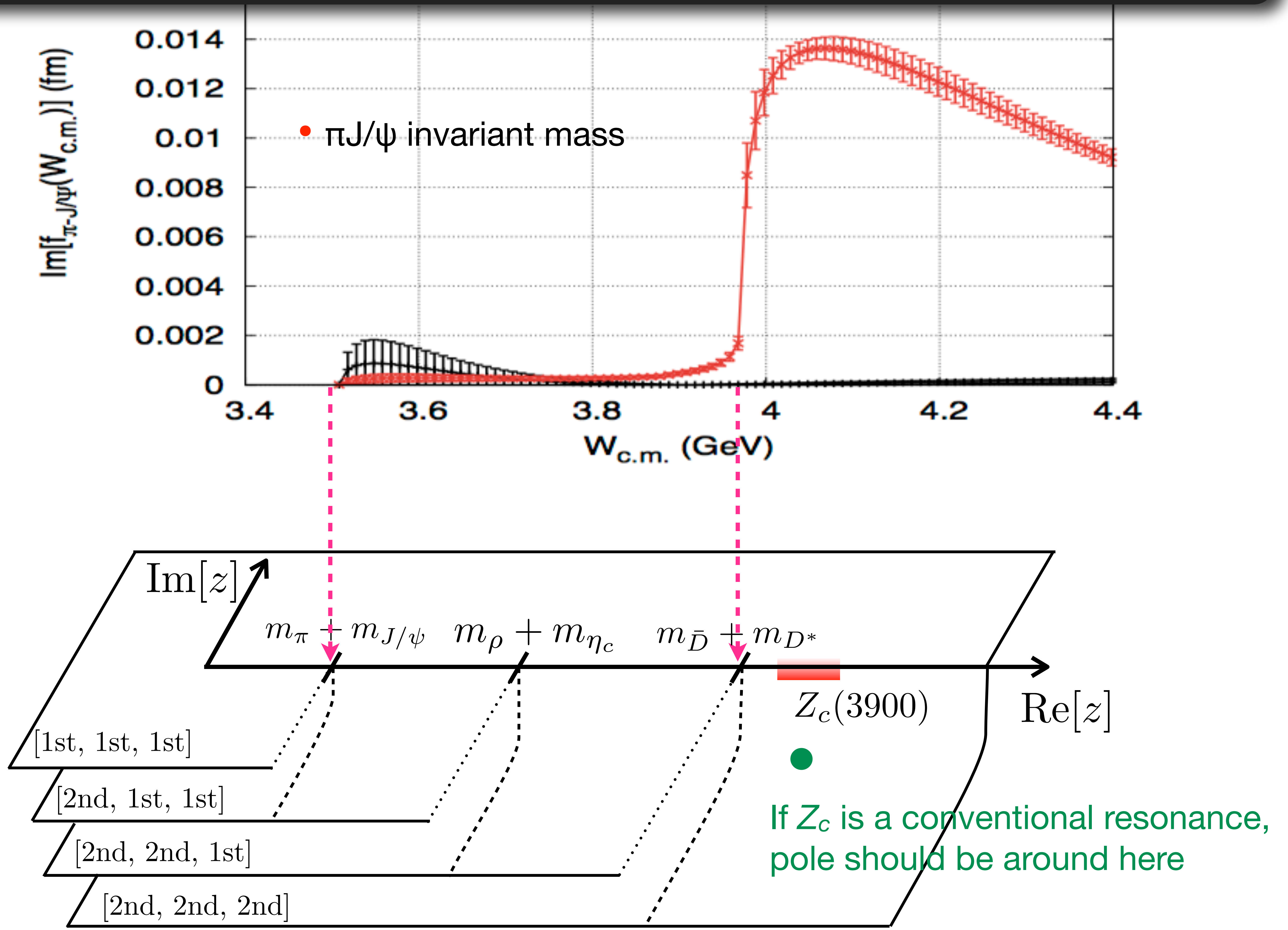
✓ **Enhancement just above $D^{bar}D^*$ threshold**

➡ **effect of strong $V^{\pi J/\psi, DbarD^*}$** (black --> $V^{\pi J/\psi, DbarD^*}$=0)

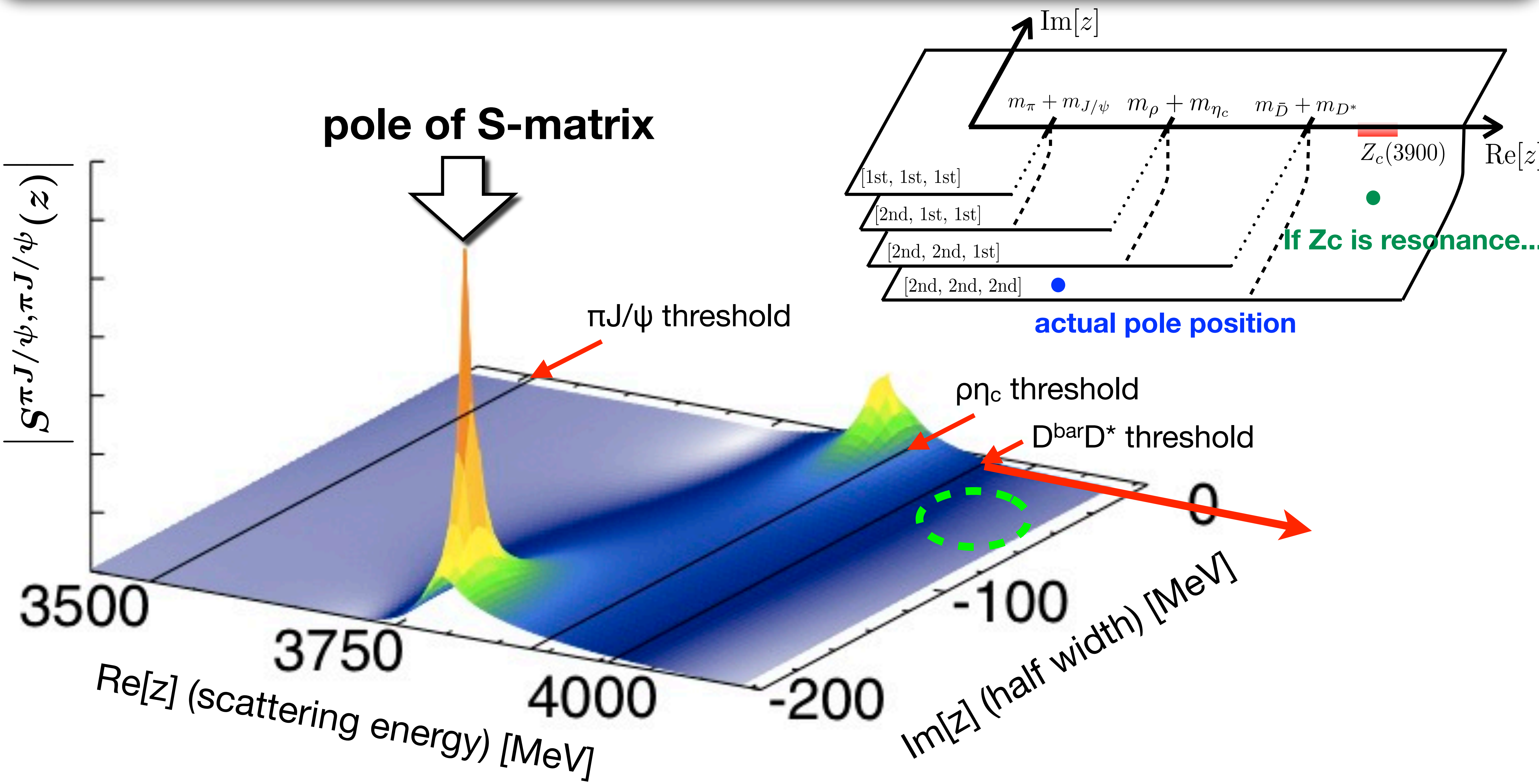
- branching fraction consistent with expt. analysis
- line shape not Breit-Wigner

✓ **Is $Z_c(3900)$ a conventional resonance? --> pole of S-matrix**

Pole of S-matrix on complex energy plane

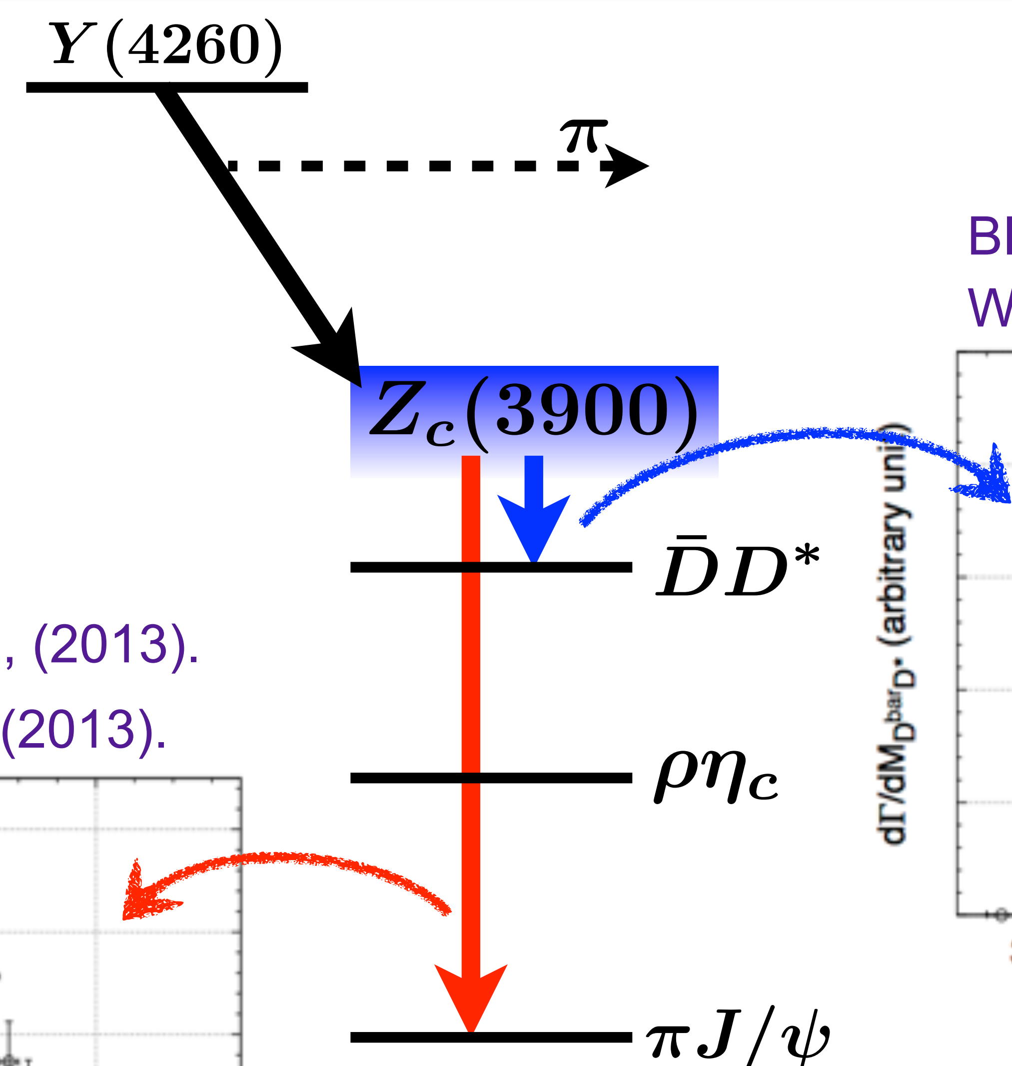


Pole of S-matrix ($\pi J/\psi$:2nd, $\rho\eta_c$:2nd, $D^{bar}D^*$:2nd)

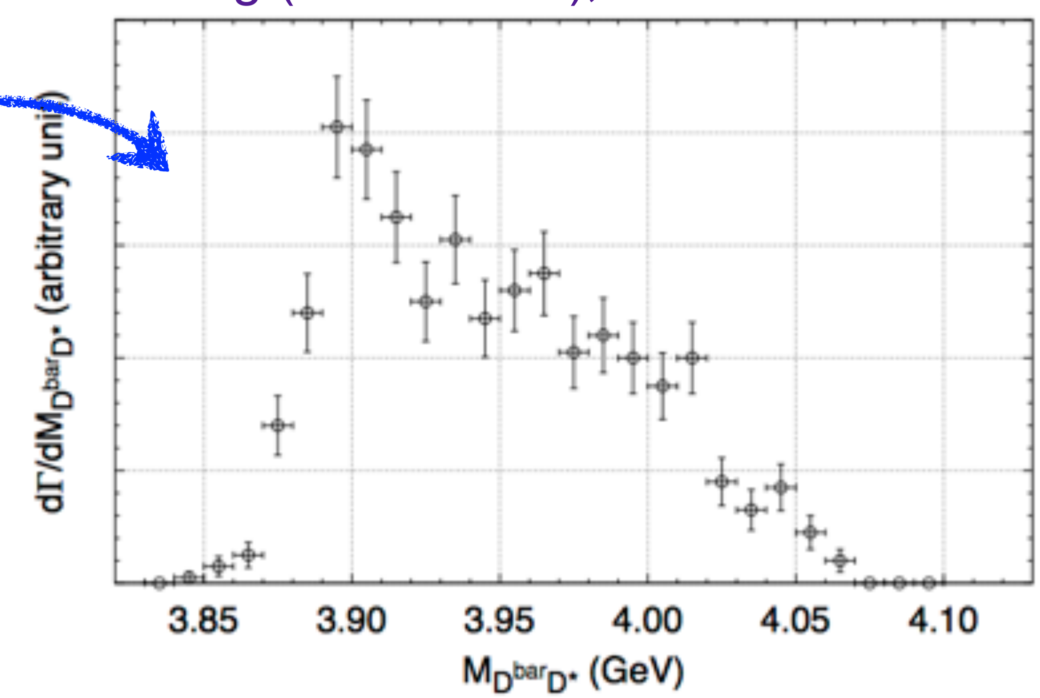


- Pole corresponding to “virtual state”
- **Pole contribution to scat. observable is small** (far from scat. axis)
- **Z_c(3900) is not a resonance but “threshold cusp” induced by strong $V^{\pi J/\psi, DbarD^*}$**

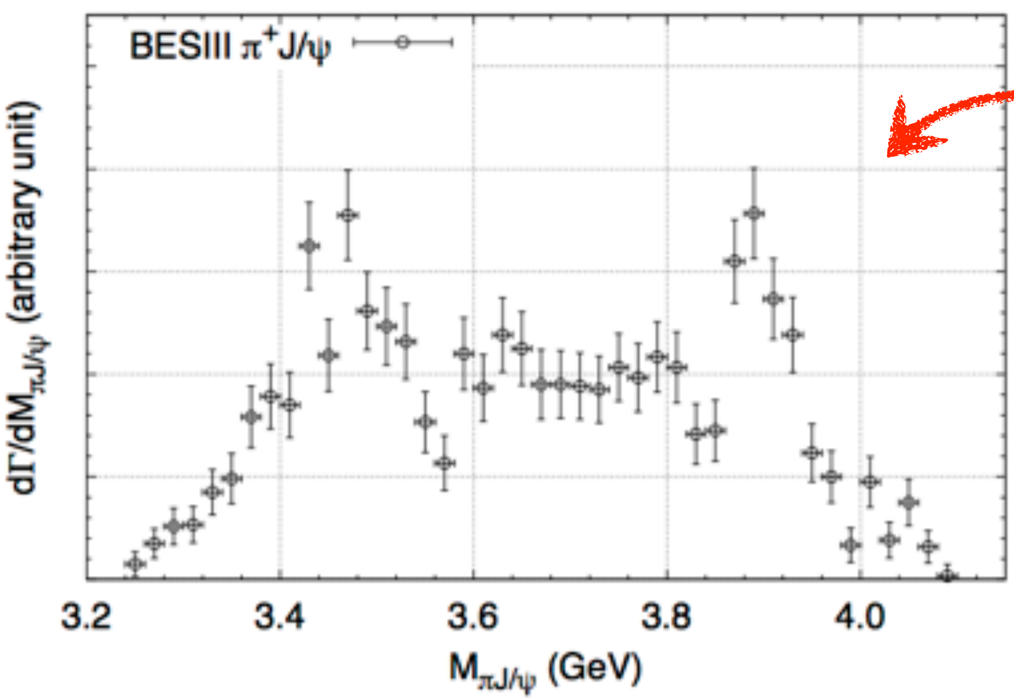
Comparison with expt. data:
-- spectrum of Y(4260) 3-body decay --

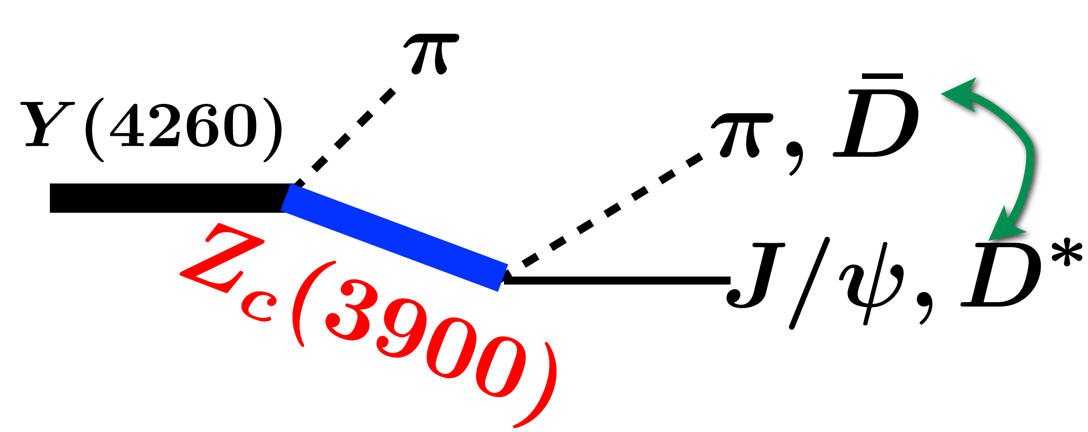


BESIII Coll., PRL112, 022001, (2014).
Wang (BESIII Coll.), MENU2016 talk



BESIII Coll., PRL110, 252001, (2013).
Belle Coll., PRL110, 252002, (2013).



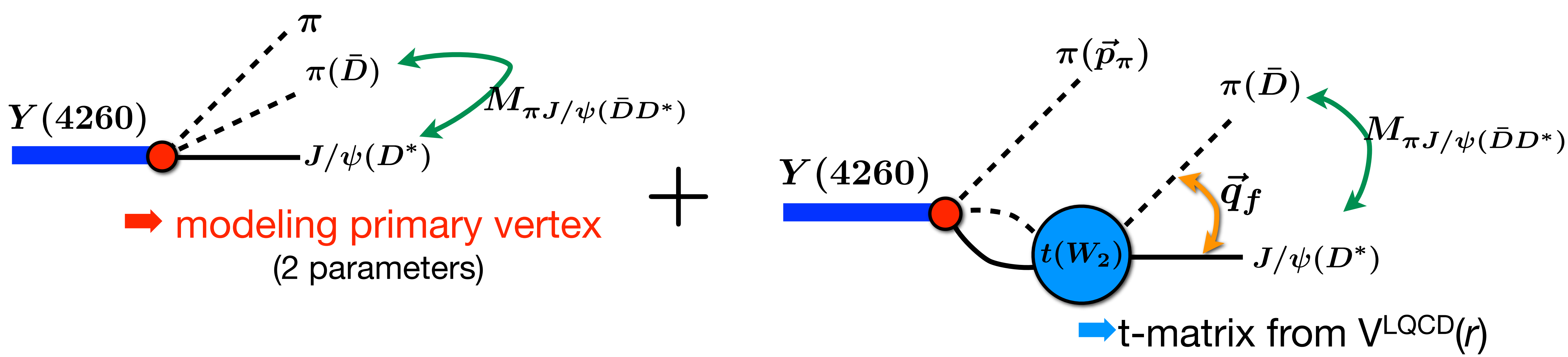


Y(4260) --> ππJ/ψ & πD^{bar}D*

$$d\Gamma_{Y\to\pi+f} = (2\pi)^4\delta(W_3 - E_\pi(\vec{p}_\pi) - E_f(\vec{q}_f))d^3p_\pi d^3q_f|T_{Y\to\pi+f}(\vec{p}_\pi,\vec{q}_f;W_3)|^2$$

✓ 3-body T-matrix: $T_{Y\text{->}\pi+f}(W_3\text{=}4260\text{MeV})$

$$T_{Y\to\pi+f}(\vec{p}_\pi,\vec{q}_f;W_3) = \sum_{n=\pi J/\psi,\bar{D}D^*} C^{Y\to\pi+n}\left[\delta_{nf} + \int d^3q' \frac{t_{nf}(\vec{q}',\vec{q}_f,\vec{p}_\pi;W_3)}{W_3 - E_\pi(\vec{p}_\pi) - E_n(\vec{q}',\vec{p}_\pi) + i\epsilon}\right]$$



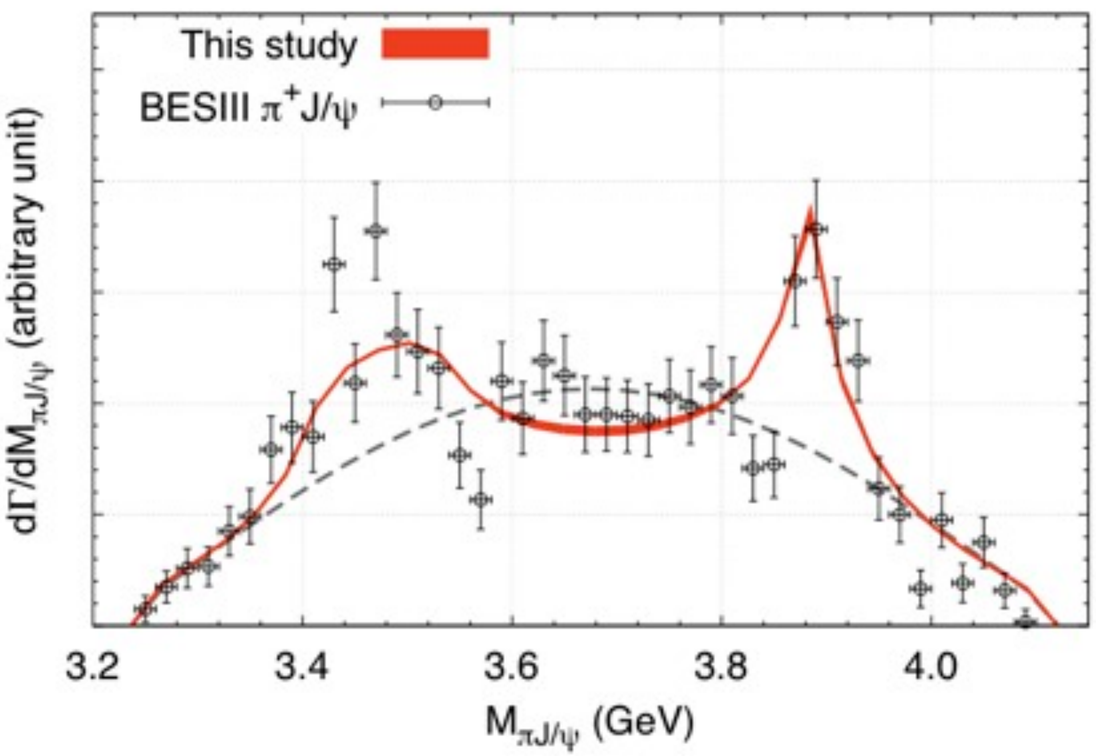
employ physical hadron masses to compare w/ expt. data

✓ $V^{LQCD}(r)$ is taken into account --> calculate t-matrix for subsystem

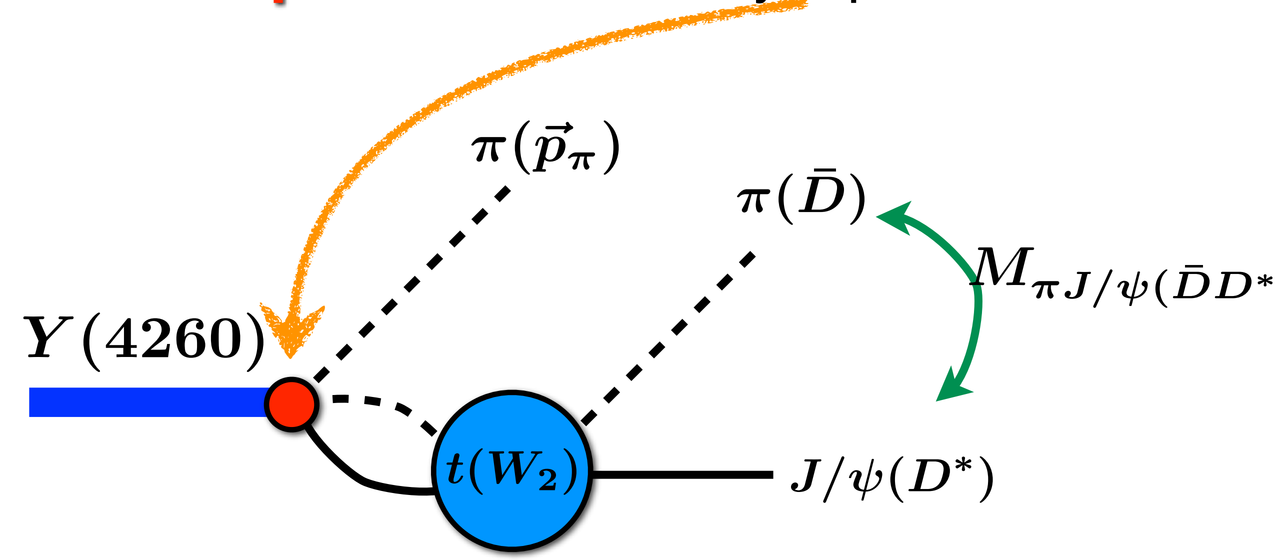
c.f., 10+ parameters needed in models

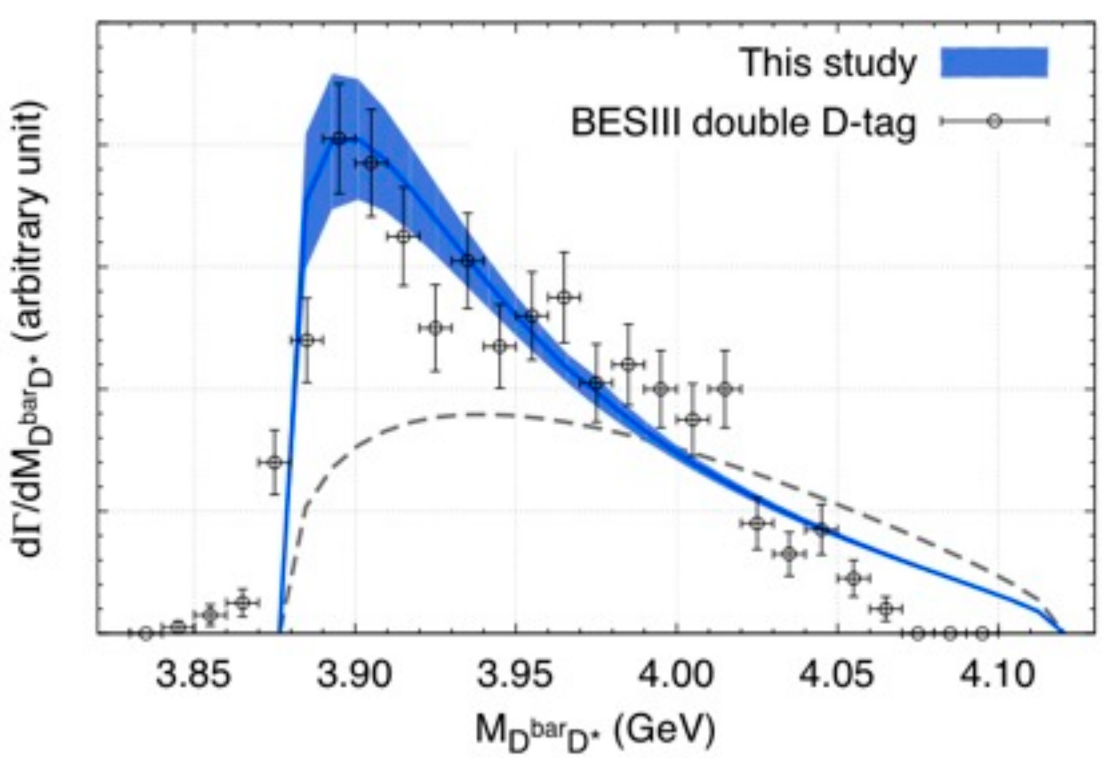
Invariant mass of 3-body decay

Ikeda [HAL QCD], J. Phys. G45, 024002 (2018).



- **Expt. data reproduced well** by 2 parameters





- Without off-diagonal $V^{\pi J/\psi,\ DbarD^*}$ (dashed curves), peak structures are not reproduced.

conclusion: $Z_c(3900)$ is threshold cusp caused by strong $V^{\pi J/\psi,\ DbarD^*}$

Summary

✤ HAL QCD method

- NBS wave function $\psi(r)$ --> 2PI kernel ($\psi = \phi + G_0 U \psi$)
- Crucial for multi-hadrons & coupled-channel scatterings

Aoki, Hatsuda, Ishii, PTP123, 89 (2010).
Ishii et al. [HAL QCD], PLB 712, 437 (2012).
Aoki et al. (HAL QCD), PRD87, 034512 (2013).

✤ Tetraquark candidate $Z_c(3900)$

- $Z_c(3900)$ is threshold cusp induced by strong $V^{DbarD^*, \pi J/\psi}$
  - pole position very far from scat. axis
  - expt. data of Y(4260) decay well reproduced
  - no peak structure w/o $V^{DbarD^*, \pi J/\psi}$

Ikeda et al. [HAL QCD], PRL117, 242001 (2016).
Reviewed in Ikeda [HAL QCD], J. Phys. G45, 024002 (2018).

✤ Future: many hadron resonances & nuclear structures at physical point