Hadronic contributions to muon g-2

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Contents & References

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  Phys. Rev. Lett. 121 (2018) 022003

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  Phys.Rev.Lett. 121 (2018) 202003

- g-2 Hadronic Light-by-Light (HLbL)
  Phys. Rev. D96 (2017) 034515
  Phys. Rev. Lett. 118 (2017) 022005

Collaborators / Machines

g-2 DWF
HVP & HLbL

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James Harrison (Southampton)
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Christoph Lehner (BNL)
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Chulwoo Jung (BNL)
Andreas Jüttner (Southampton)
Luchang Jin (BNL)
Antonin Portelli (Edinburgh)

tau input for
g-2 HVP &
HVP GEVP

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Christoph Lehner (BNL & Regensburg)
Taku Izubuchi (BNL & RBRC)

tau decay

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Kim Maltman (York)
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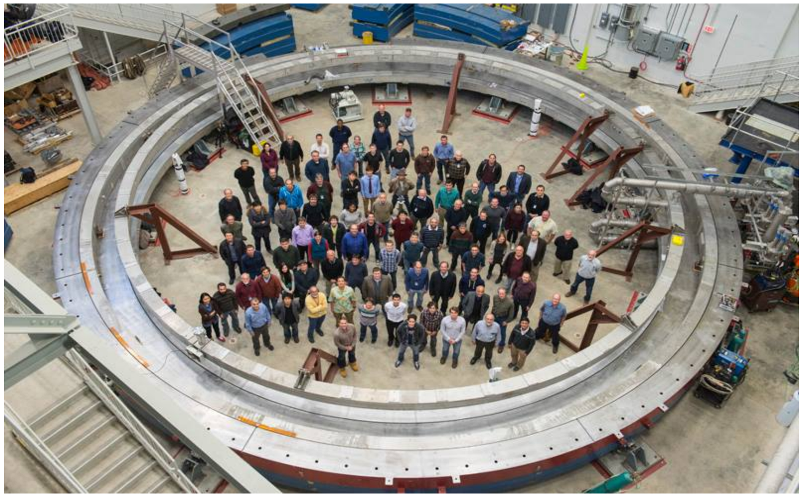
Hiroshi Ohki (RBRC/Nara Women)
Matthew Spraggs (Edinburgh)

Part of related calculation are done by resources from
USQCD (DOE), XSEDE, ANL BG/Q Mira (DOE, ALCC), Edinburgh BG/Q,
BNL BG/Q, RIKEN BG/Q and Cluster (RICC, HOKUSAI)

Support from RIKEN, JSPS, US DOE, and BNL

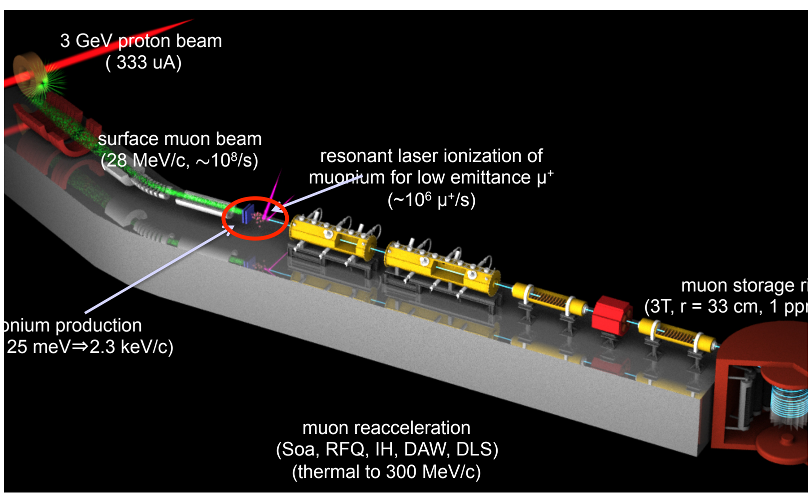
muon anomalous magnetic moment



BNL g-2 till 2004 : ~ 3.7 σ larger than SM prediction

Contribution	Value $\times 10^{10}$	Uncertainty $\times 10^{10}$
QED (5 loops)	11 658 471.895	0.008
EW	15.4	0.1
HVP LO	692.3	**4.2**
HVP NLO	-9.84	0.06
HVP NNLO	1.24	0.01
Hadronic light-by-light	10.5	**2.6**
Total SM prediction	11 659 181.5	4.9
BNL E821 result	11 659 209.1	6.3
FNAL E989/J-PARC E34 goal		$\approx$ **1.6**

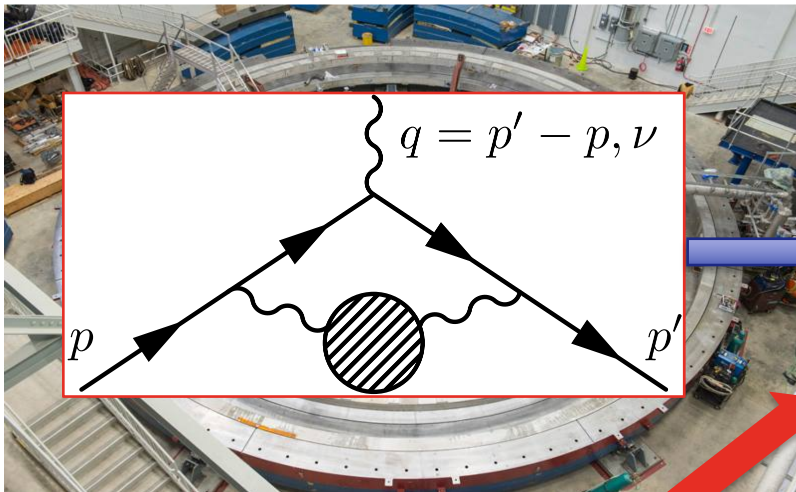
$$a_\mu^{\rm EXP} - a_\mu^{\rm SM} = 27.4\,\underbrace{(2.7)}_{\rm HVP}\,\underbrace{(2.6)}_{\rm HLbL}\,\underbrace{(0.1)}_{\rm other}\,\underbrace{(6.3)}_{\rm EXP} \times 10^{-10}$$



FNAL E989 (began 2017-)
- move storage ring from BNL
- x4 more precise results, 0.14ppm

J-PARC E34
- ultra-cold muon beam
- 0.37 ppm then 0.1 ppm, also EDM

muon anomalous magnetic moment



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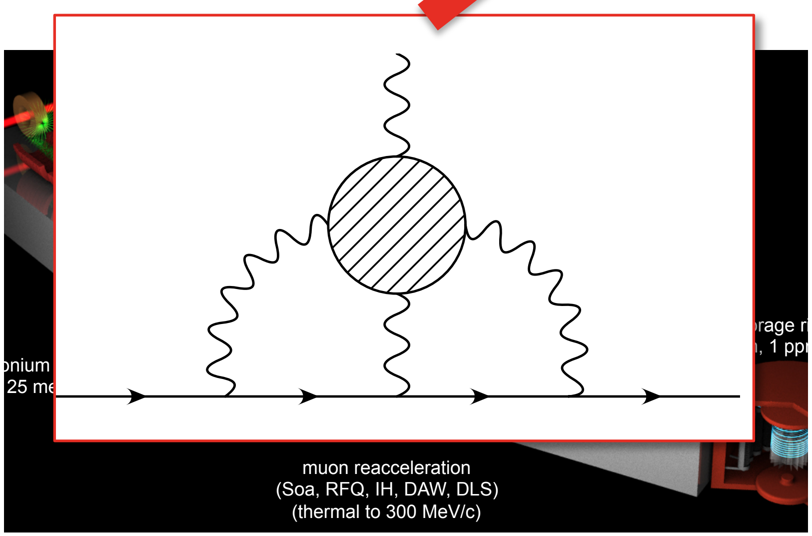

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move storage ring from BNL
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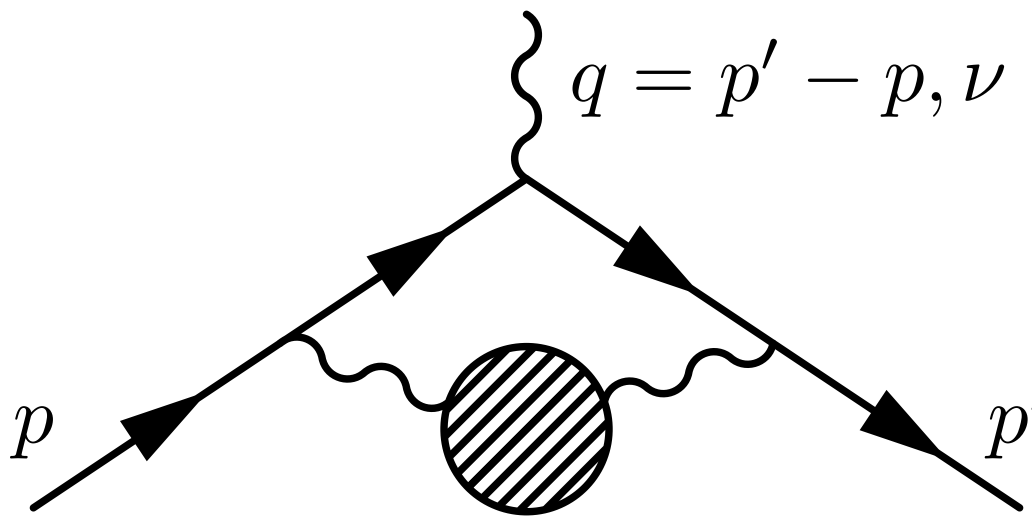
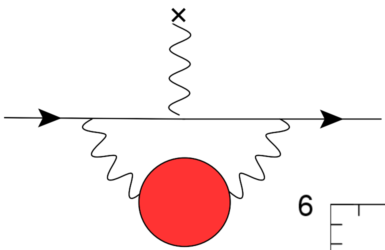
ultra-cold muon beam
0.37 ppm then 0.1 ppm, also EDM

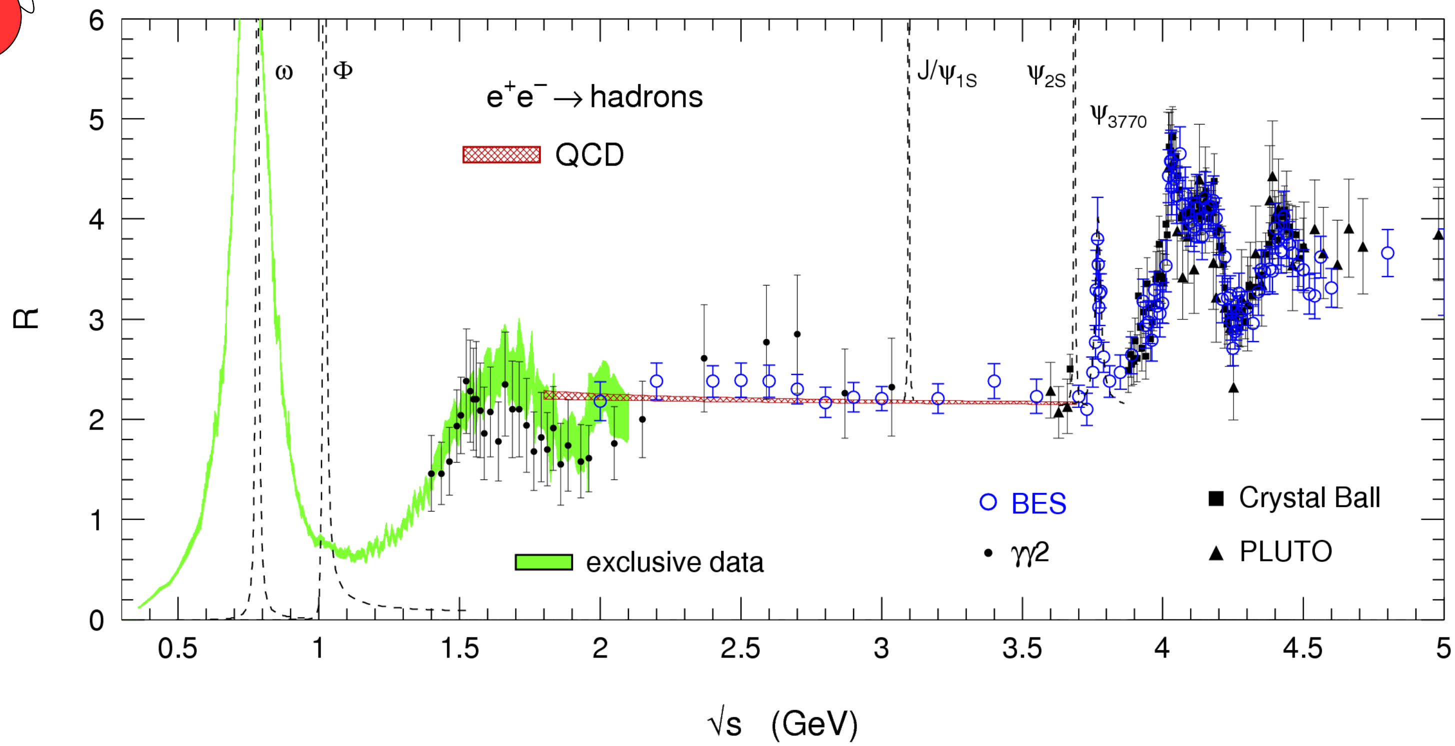
Hadronic Vacuum Polarization (HVP) contribution to g-2



Quark & anti-quark contribution

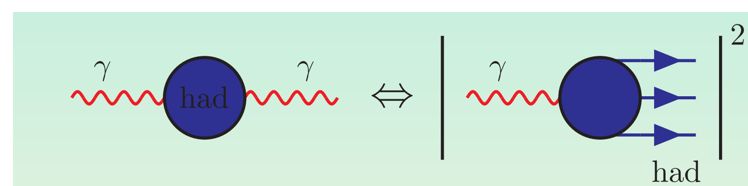


g-2 from R-ratio

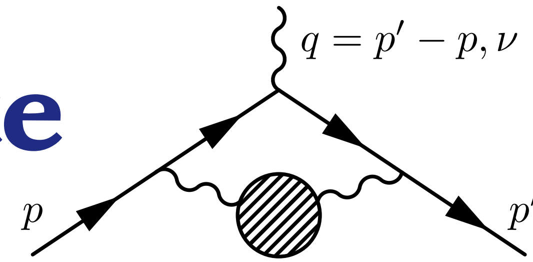


- From experimental e+ e- inclusive hadron decay cross section $\sigma_{\mathrm{total}}(s)$ in time-like $s = q^2 > 0$, and dispersion relation, optical theorem

$$a_\mu^{\mathrm{HVP}} = \frac{1}{4\pi^2}\int_{s_{th}}^{\infty} ds K(s)\sigma_{\mathrm{total}}(s)$$



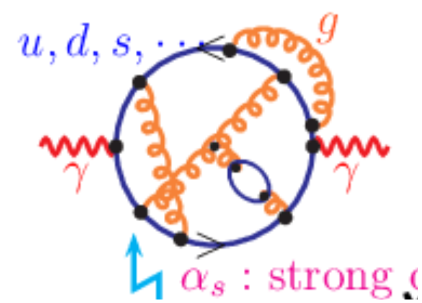
g-2 HVP from Lattice



[Bernecker Meyer 2011, Feng et al. 2013]

In Euclidean space-time, project verctor 2 pt to zero spacial momentum, $\vec{p} = 0$:

$$C(t) = \frac{1}{3} \sum_{x,i} \langle j_i(x) j_i(0) \rangle$$



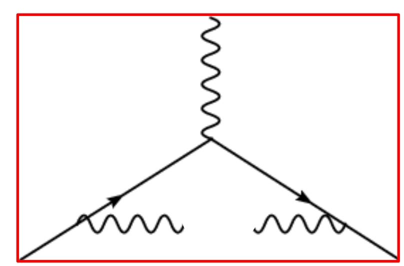
g-2 HVP contribution is

w(t) ~ t^4

$$a_\mu^{HVP} = \sum_t w(t) C(t)$$

$$w(t) = 2 \int_0^\infty \frac{d\omega}{\omega} f_{\text{QED}}(\omega^2) \left[\frac{\cos \omega t - 1}{\omega^2} + \frac{t^2}{2} \right]$$

$f_{QED}(\omega^2)$



- Subtraction $\Pi(0)$ is performed.

  Noise/Signal $\sim e^{(E_{\pi\pi} - m_\pi)t}$, is improved [Lehner et al. 2015] .

Euclidean time correlation from e^+e^- $R(s)$ data

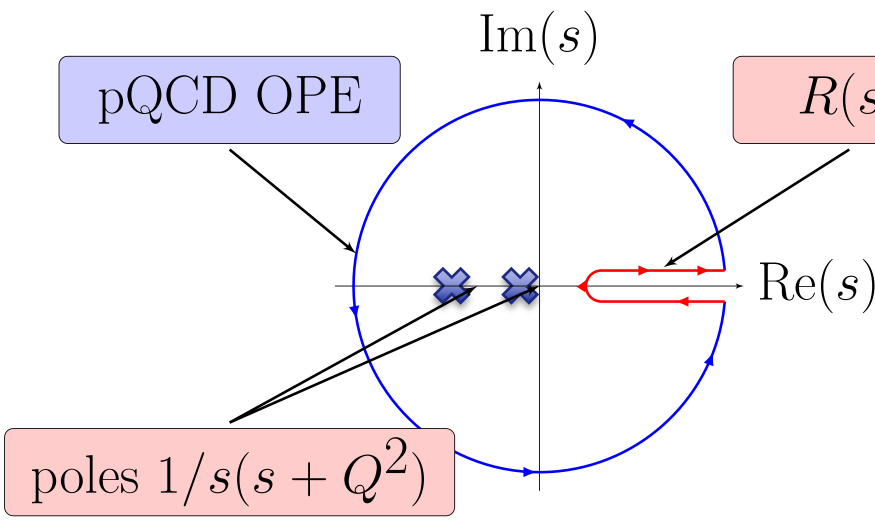
From e^+e^- $R(s)$ ratio, using disparsive relation, zero-spacial momentum projected Euclidean correlation function $C(t)$ is obtained

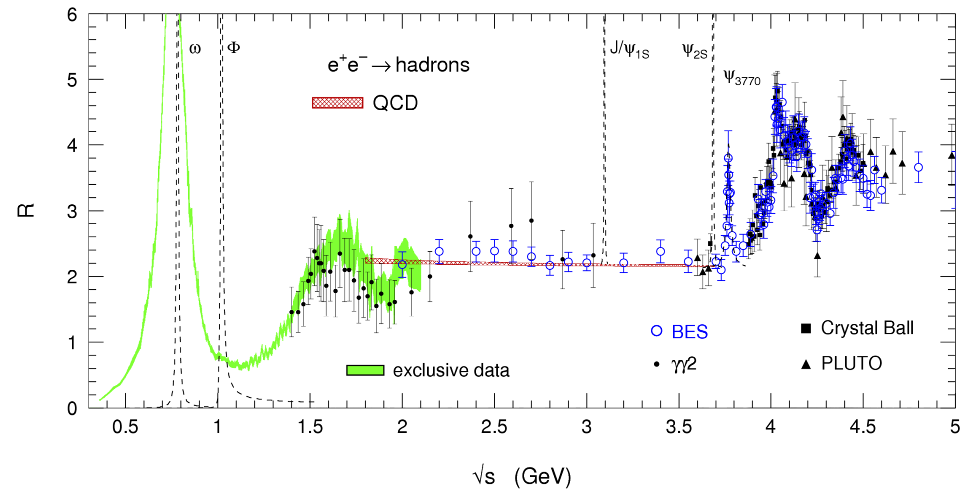
$$\hat{\Pi}(Q^2) = Q^2 \int_0^\infty ds \frac{R(s)}{s(s+Q^2)}$$

Lattice can compute Integral of Inclusive cross sections accurately

$$C^{\text{R-ratio}}(t) = \frac{1}{12\pi^2} \int_0^\infty \frac{d\omega}{2\pi} \hat{\Pi}(\omega^2) e^{i\omega t} = \frac{1}{12\pi^2} \int_0^\infty ds \sqrt{s} R(s) e^{-\sqrt{s}t}$$

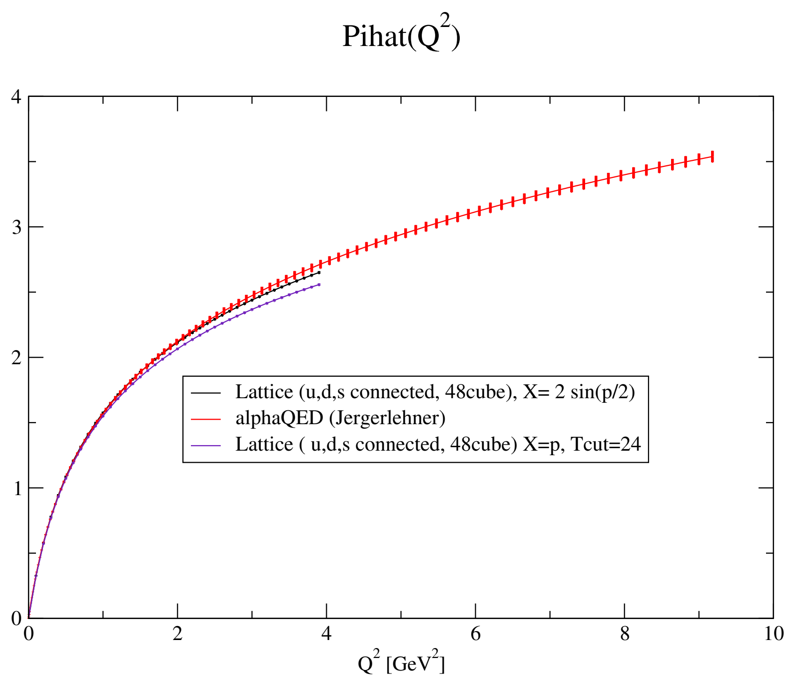
- $C(t)$ or $w(t)C(t)$ are directly comparable to Lattice results with the proper limits ($m_q \to m_q^{\text{phys}}, a \to 0, V \to \infty$, QED ...)
- Lattice: long distance has large statistical noise, (short distance: discretization error, removed by $a \to 0$ and/or pQCD)
- R-ratio : short distance has larger error

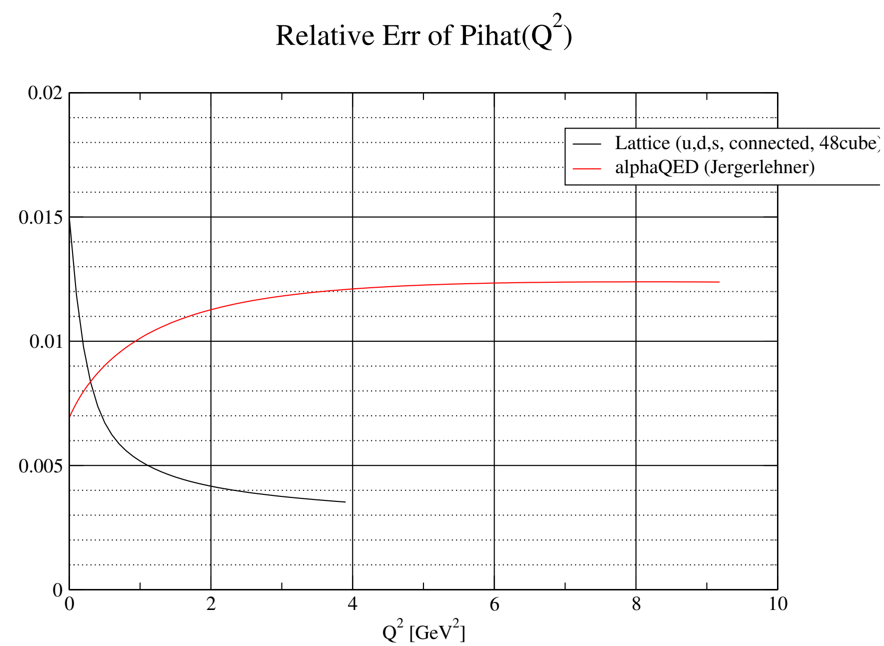




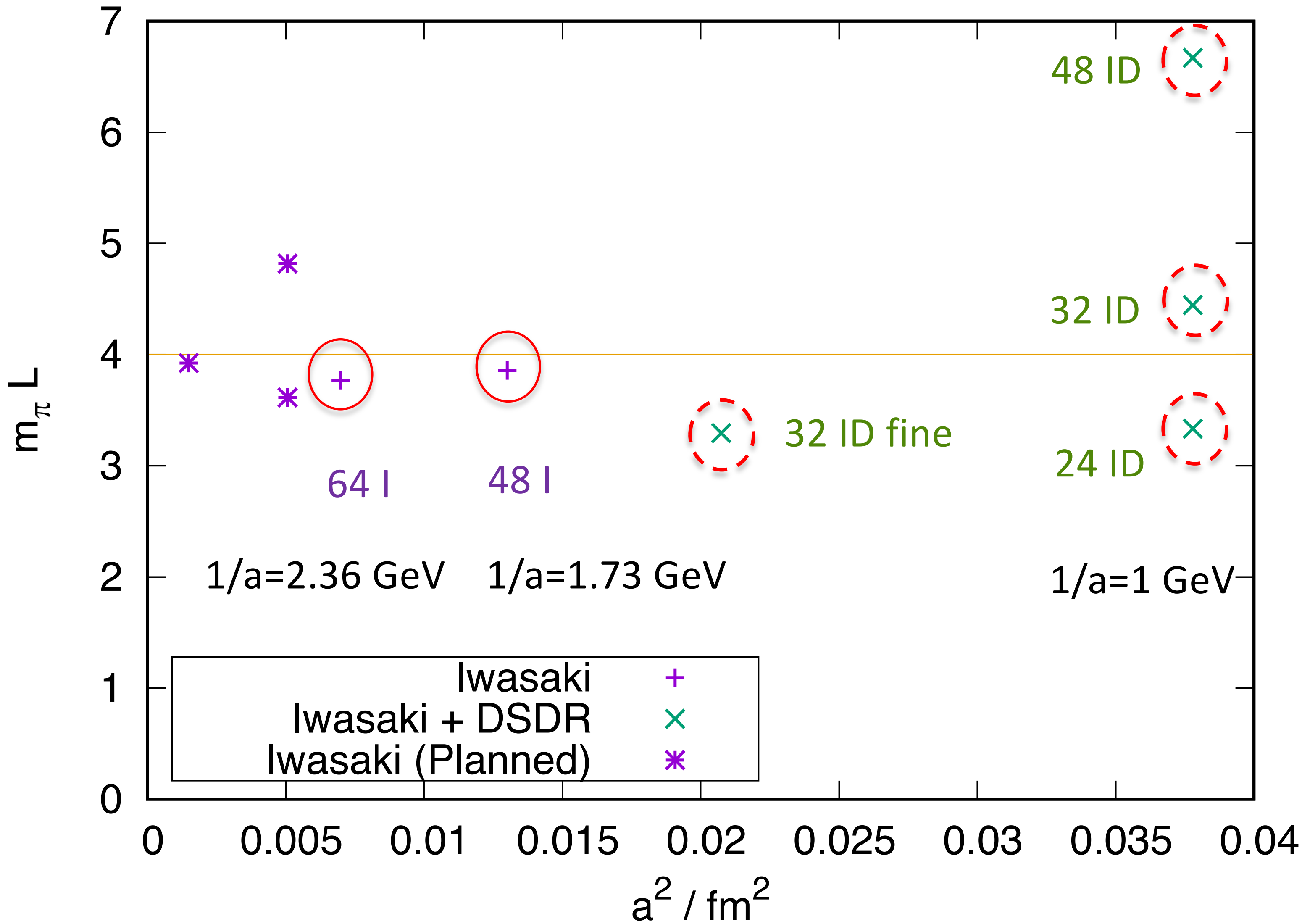
$$\hat{\Pi}(Q^2) = Q^2 \int_0^\infty ds \frac{R(s)}{s(s+Q^2)}$$

$(1/a = 1.78$ **GeV,** **Relative statistical error)**

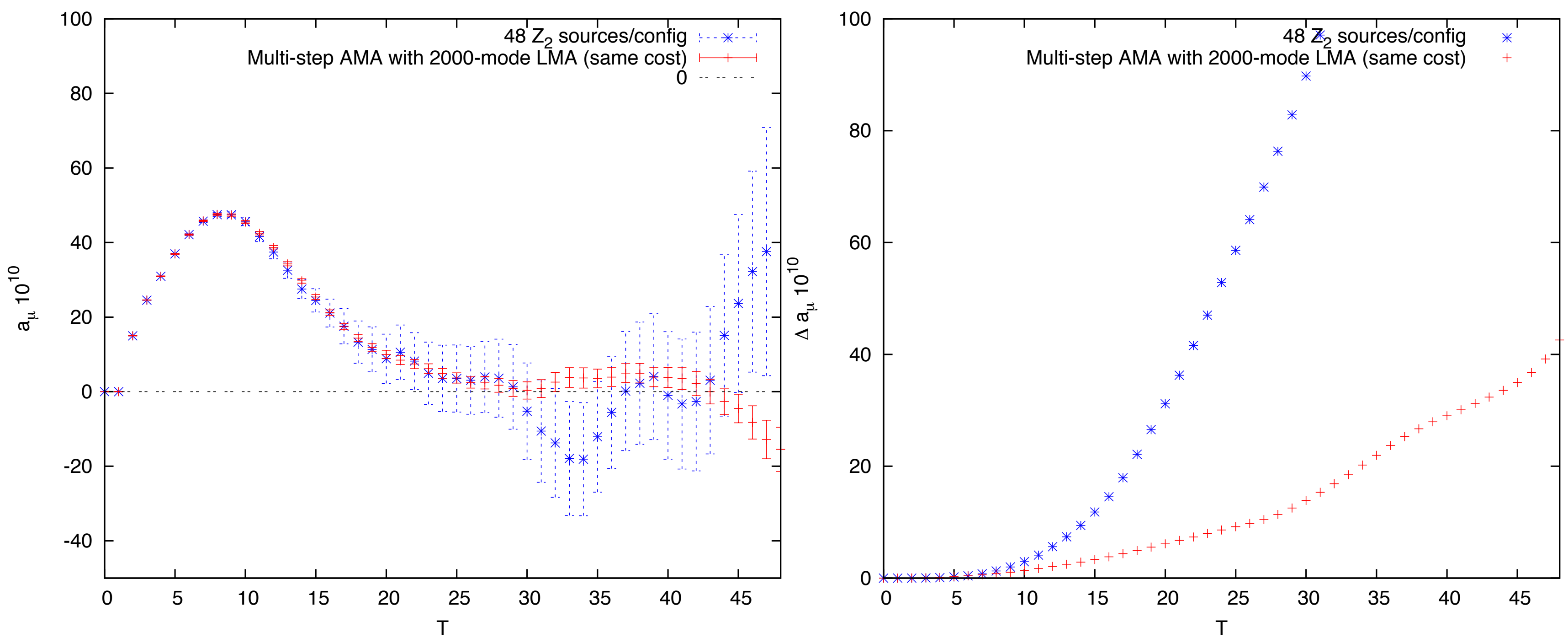




Nf=2+1 DWF QCD ensemble at physical quark mass



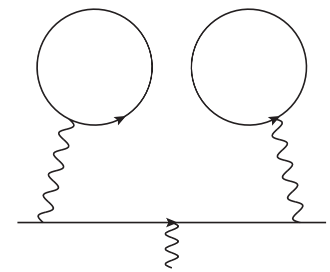
DWF light HVP
[2016 Christoph Lehner]



120 conf (a=0.11fm), 80 conf (a=0.086fm)   physical point Nf=2+1 Mobius DWF
4D full volume LMA with 2,000 eigen vector (of e/o preconditioned zMobius D^+D)
EV compression (1/10 memory) using local coherence [C. Lehner Lat2017 Poster]
In addition, 50 sloppy / conf via multi-level AMA
more than x 1,000 speed up compared to simple CG

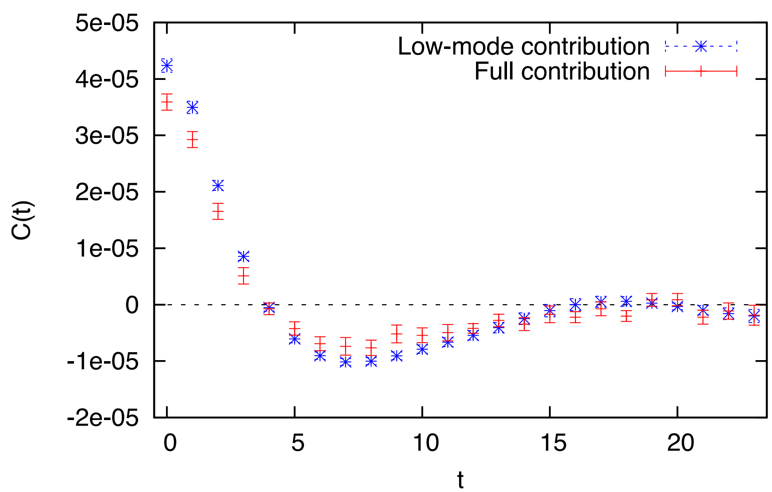
disconnected quark loop contribution

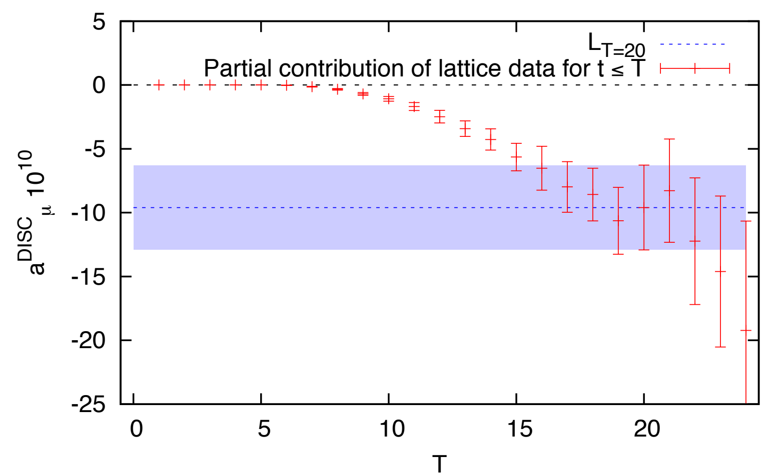
- [C. Lehner et al. (RBC/UKQCD 2015, arXiv:1512.09054, PRL)]
- Very challenging calculation due to statistical noise
- Small contribution, vanishes in SU(3) limit,
  Qu+Qd+Qs = 0
- Use low mode of quark propagator, treat it exactly
  (all-to-all propagator with sparse random source)
- First non-zero signal



$$a_\mu^{\mathrm{HVP\ (LO)\ DISC}} = -9.6(3.3)_{\mathrm{stat}}(2.3)_{\mathrm{sys}} \times 10^{-10}$$

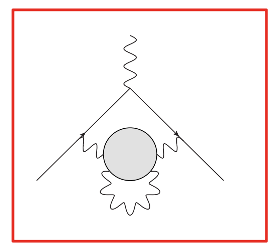
Sensitive to m_π
crucial to compute at physical mass

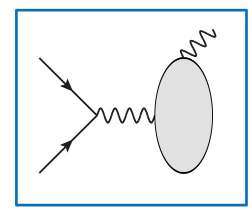


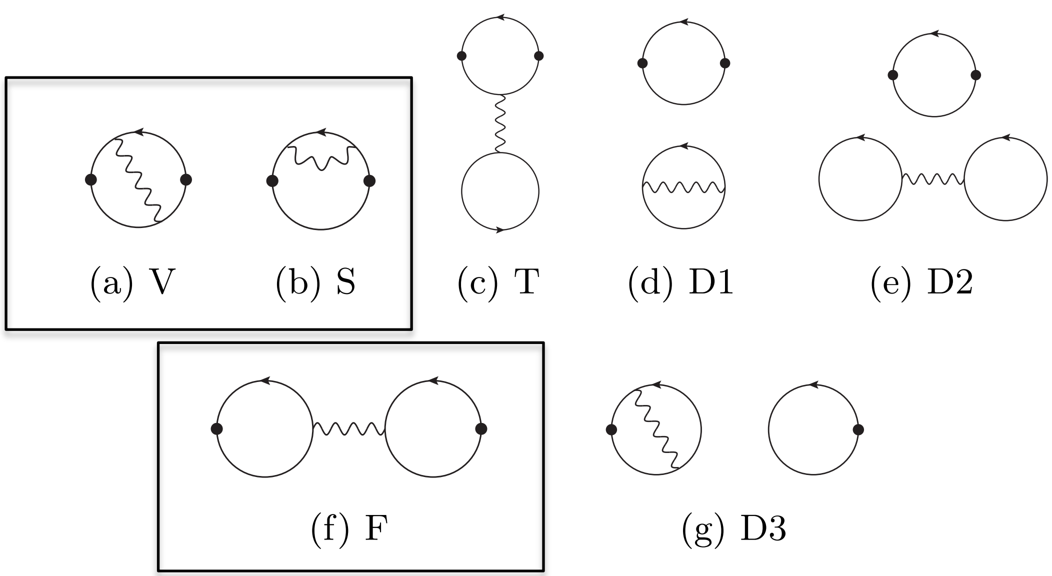


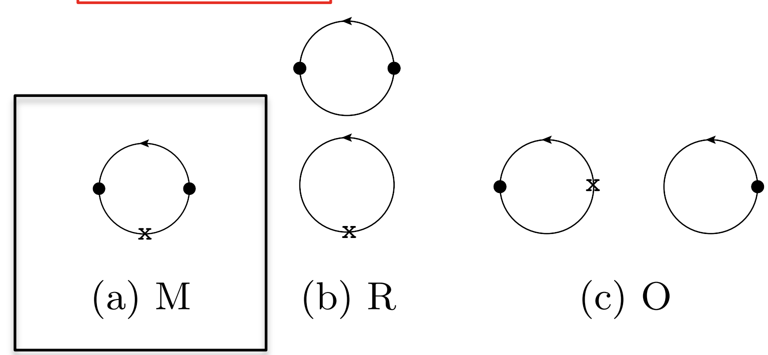
HVP QED+ strong IB corrections

- HVP is computed so far at Iso-symmetric quark mass, needs to compute isospin breaking corrections : Qu, Qd, mu-md ≠0
- u,d,s quark mass and lattice spacing are re-tuned using {charge,neutral} x{pion,kaon} and (Omega baryon masses)
- For now, V, S, F, M are computed : assumes EM and IB of sea quark and also shift to lattice spacing is small (correction to disconnected diagram)
- Point-source method : stochastically sample pair of 2 EM vertices a la important sampling with exact photon



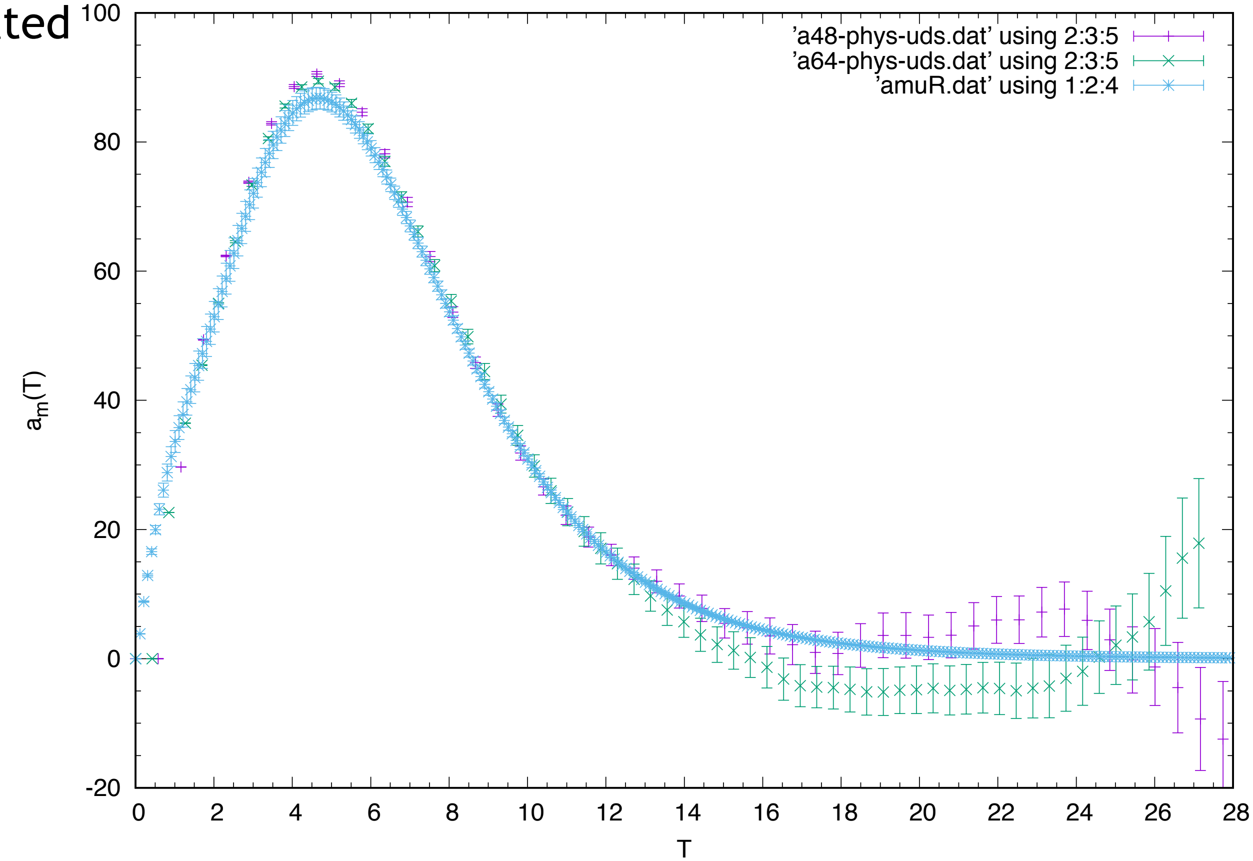






Comparison of R-ratio and Lattice [F. Jegerlehner alphaQED 2016]

- Covariance matrix among energy bin in R-ratio is not available, assumes 100% correlated



$$a_\mu^{HVP} = \sum_t w(t)C(t)$$

$$w(t) = 2\int_0^\infty \frac{d\omega}{\omega} f_{\text{QED}}(\omega^2)\left[\frac{\cos\omega t - 1}{\omega^2} + \frac{t^2}{2}\right]$$

$$C(t) = \frac{1}{3}\sum_{x,i}\langle j_i(x) j_i(0)\rangle$$

Combine R-ratio and Lattice
[Christoph Lehner et al PRL18]

- Use short and long distance from R-ratio using smearing function, and mid-distance from lattice

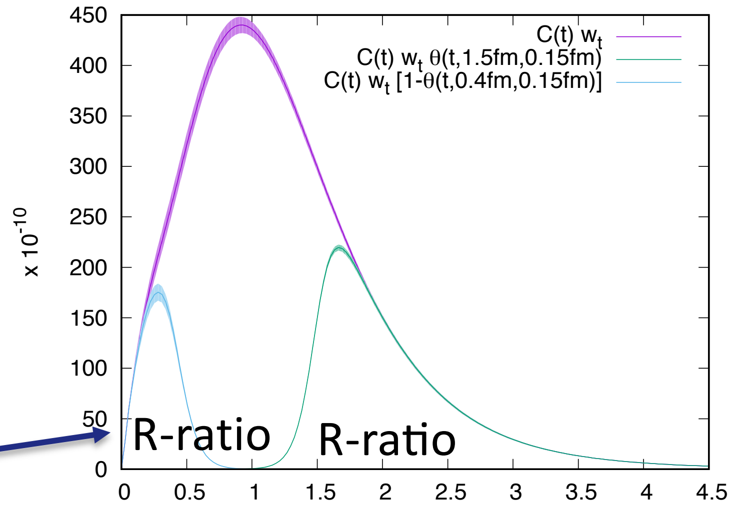
$$\Theta(t,\mu,\sigma) \equiv [1 + \tanh[(t-\mu)/\sigma]]/2$$

$$a_\mu = \sum_t w_t C(t) \equiv a_\mu^{\mathrm{SD}} + a_\mu^{\mathrm{W}} + a_\mu^{\mathrm{LD}}$$

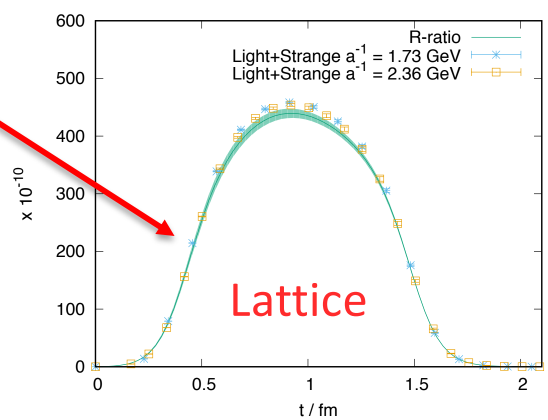
$$a_\mu^{\mathrm{SD}} = \sum_t C(t) w_t [1 - \Theta(t, t_0, \Delta)]\,,$$

$$a_\mu^{\mathrm{W}} = \sum_t C(t) w_t [\Theta(t, t_0, \Delta) - \Theta(t, t_1, \Delta)]\,,$$

$$a_\mu^{\mathrm{LD}} = \sum_t C(t) w_t \Theta(t, t_1, \Delta)$$

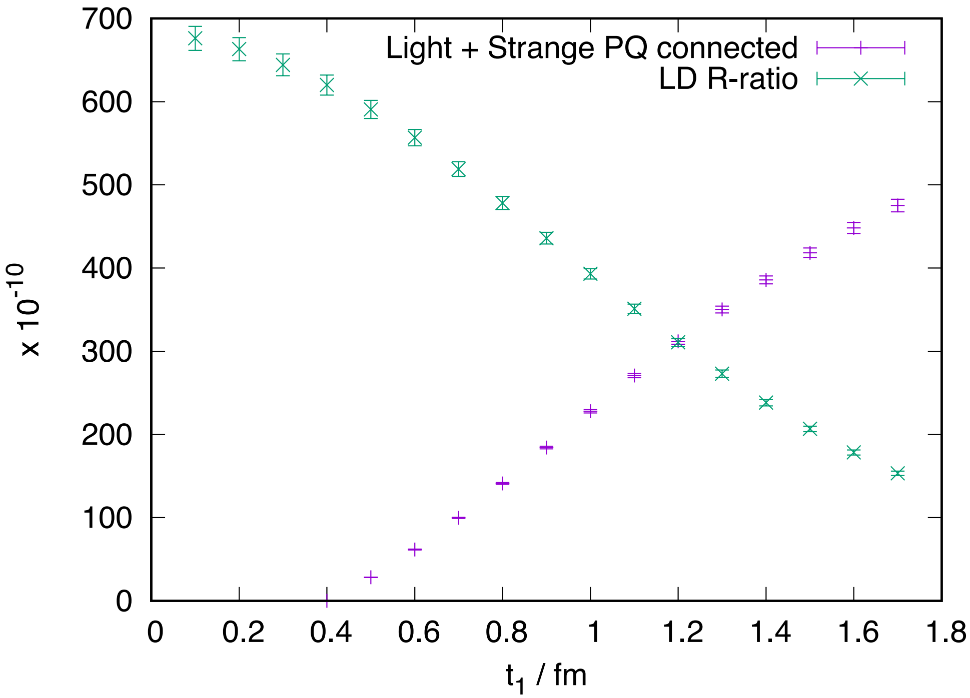


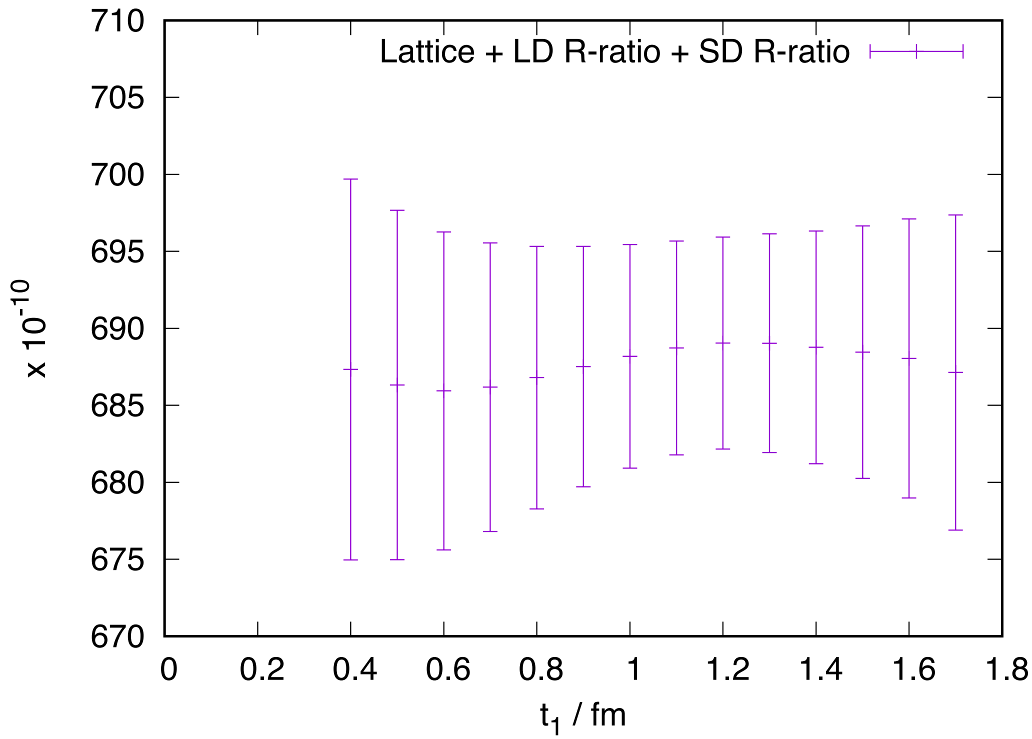
+



R-ratio + Lattice

t0 = 0.4 fm

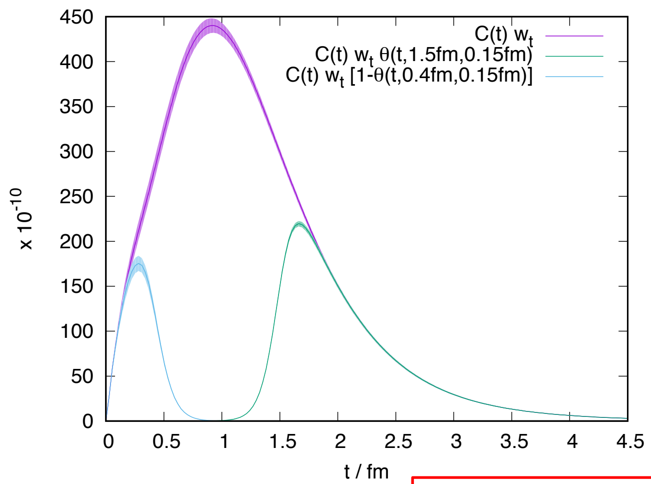




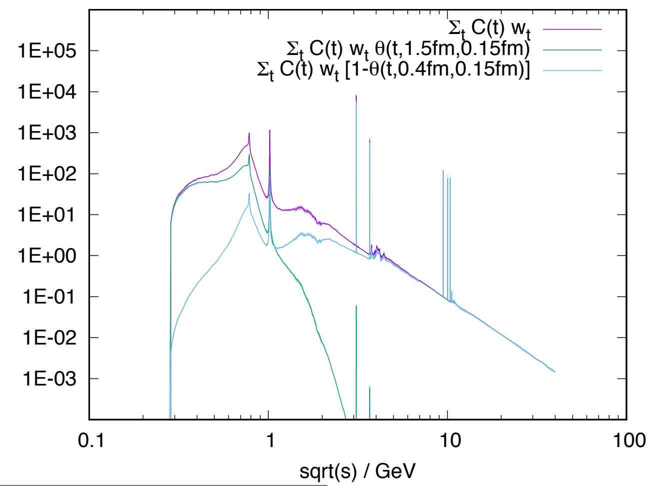
t1 dependence is flat => a consistency between R-ratio and Lattice
t1 = 1.2 fm, R-ratio : Lattice = 50:50
t1=1.2 fm current error (note 100% correlation in R-ratio) is minimum

How does this translate to the time-like region?

Euclidean from Lattice

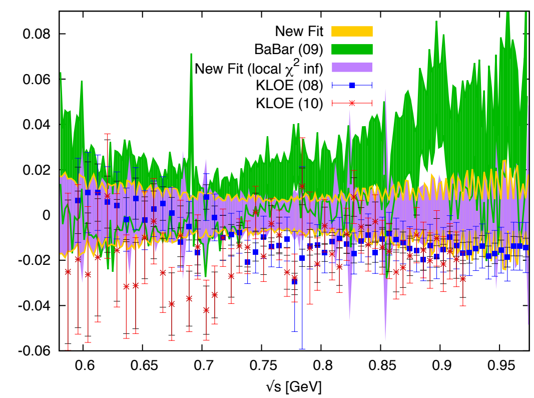


Time-like from e+e-



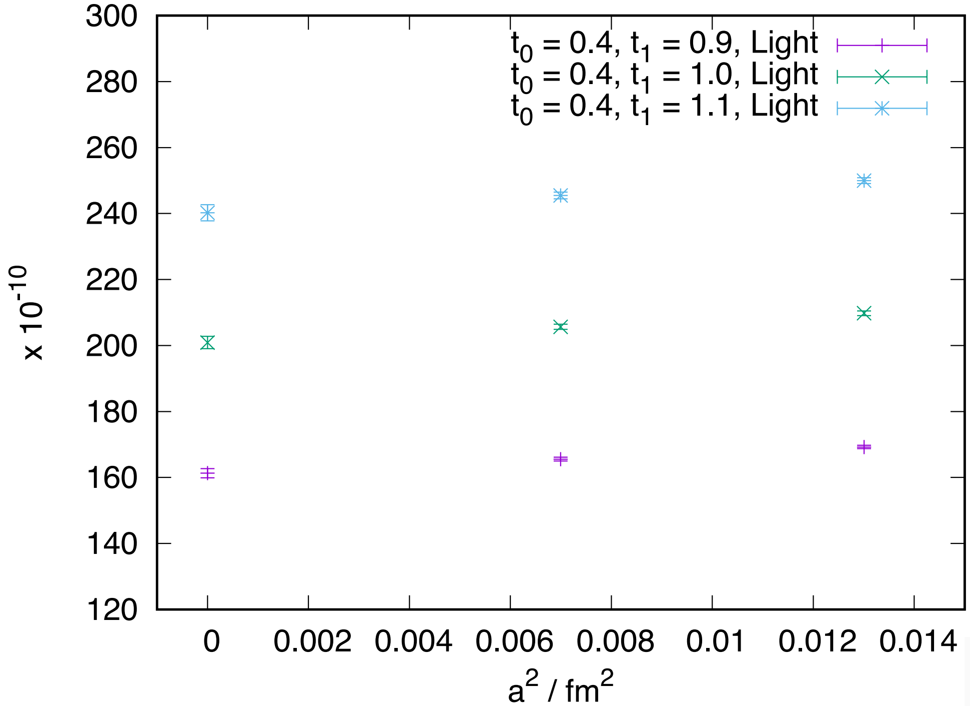
Blue : low-pass window
Green: high-pass window
Purple : total

Most of $\pi\pi$ peak is captured by window from $t_0 = 0.4$ fm to $t_1 = 1.5$ fm, so replacing this region with lattice data reduces the dependence on BaBar versus KLOE data sets.



Continuum limit of a^W

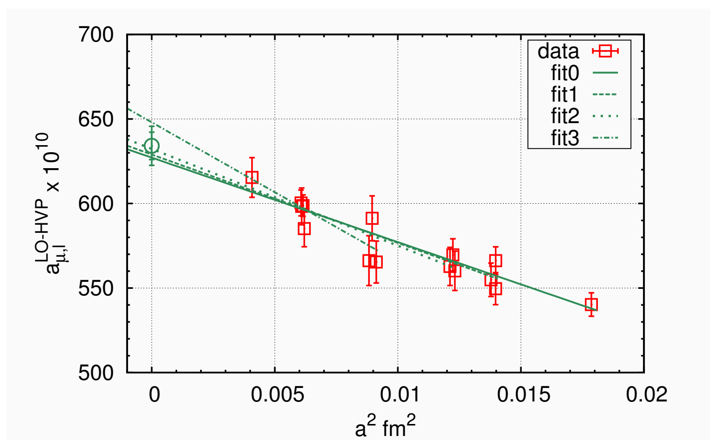
Continuum limit of a_μ^{W} from our lattice data; below $t_0 = 0.4$ fm and $\Delta = 0.15$ fm



RBC/UKQCD [C. Lehner Lat17]

Continuum extrapolation is mild

c.f BMWc [K. Miura Lat17]



Reconstruction of HVP from multi-channel Greens function

- Using N operators O_n, n=0,1,…, N-1

- Point vector
- 2 Π operator
- 4 Π operator

$$\mathcal{O}_0 = \sum_x \bar{\psi}(x)\gamma_\mu\psi(x),\ \mu \in \{1,2,3\}$$

$$\mathcal{O}_n = \left|\sum_{xyz} \bar{\psi}(x)f(x-z)e^{-i\vec{p}_\pi\cdot\vec{z}}\gamma_5 f(z-y)\psi(y)\right|^2$$

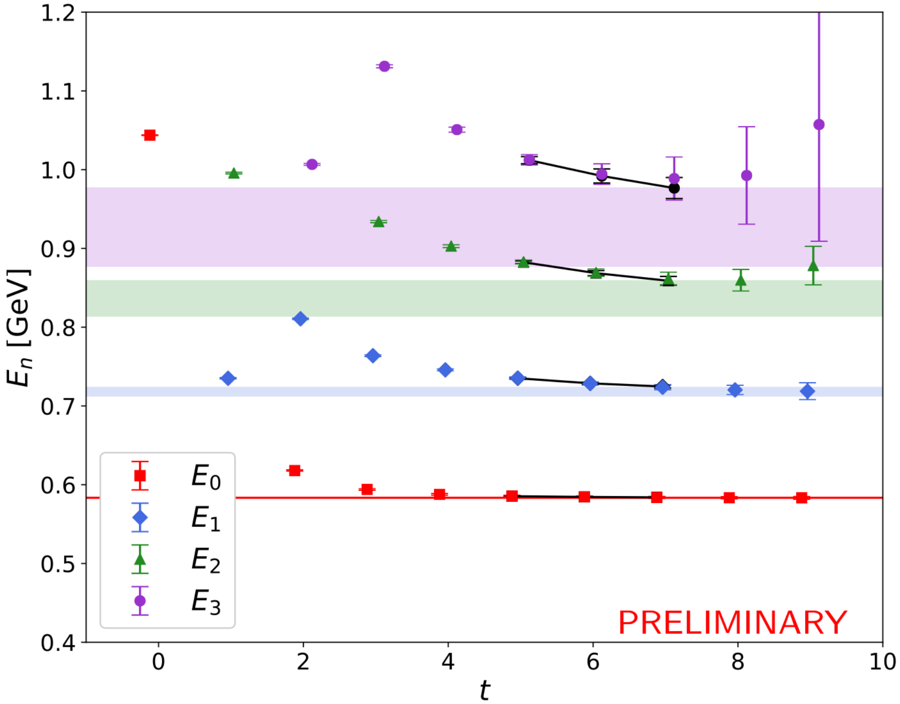
< O_i(t) O_j(0) > (using distillation)

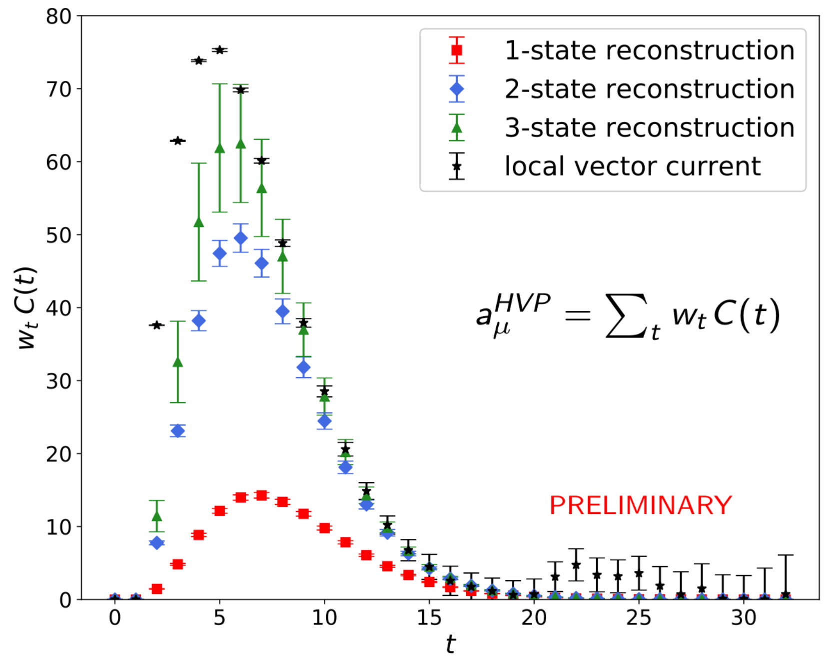
- Solve NxN spectrum E_n of eigenstates |E_n> and Overwrap factors <E_n|O_0|0> (GEVP)

- Reconstruct V-V correlator (or bound)

$$C^{\text{latt.}}(t) = \sum_n |\langle\Omega|\,\mathcal{O}\,|n\rangle|^2 e^{-E_n t}$$

GEVP & Reconstruct I=1 VV





$$C(t)\; V = C(t+\delta t)\; V\, \Lambda(\delta t)\,, \qquad V_{im} \propto \langle \Omega |\, \mathcal{O}_i\, | m \rangle$$

$$C^{\mathrm{latt.}}(t) = \sum_n |\, \langle \Omega |\, \mathcal{O}\, | n \rangle\, |^2 e^{-E_n t}$$

Bounds for a_μ

- Upper & lower bounds from unitarity

$$\widetilde{C}(t; t_{\rm max}, E) = \begin{cases} C(t) & t < t_{\rm max} \\ C(t_{\rm max}) e^{-E(t - t_{\rm max})} & t \geq t_{\rm max} \end{cases}$$

Upper bound: $E = E_0$, lowest state in spectrum

Lower bound: $E = \log[\frac{C(t_{\rm max})}{C(t_{\rm max}+1)}]$

- Also bounds for the n in [N+1, ∞] states contribution

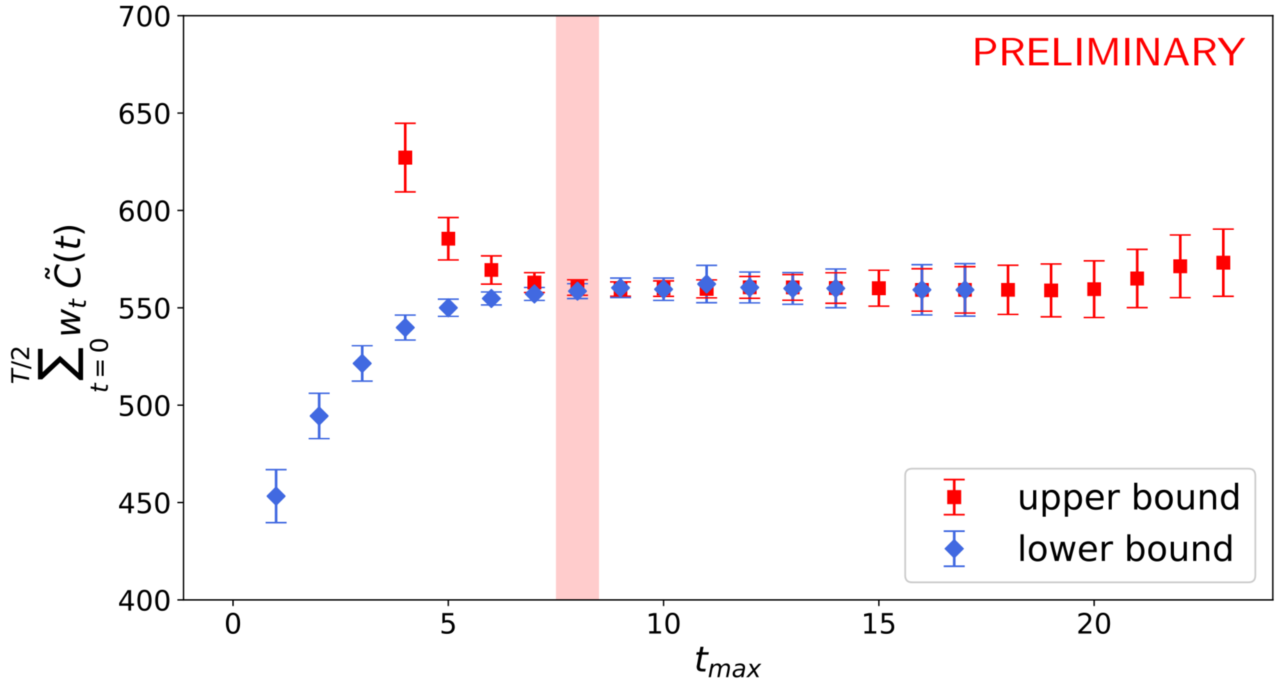
Replace $C(t) \to C(t) - \sum_n^N |c_n|^2 e^{-E_n t}$

$\Longrightarrow$ Long distance convergence now $\propto e^{-E_{N+1} t}$

$\Longrightarrow$ Smaller overall contribution from neglected states

test of GEVP+Bounding method
[A. Meyer]

Improved Bounding Method



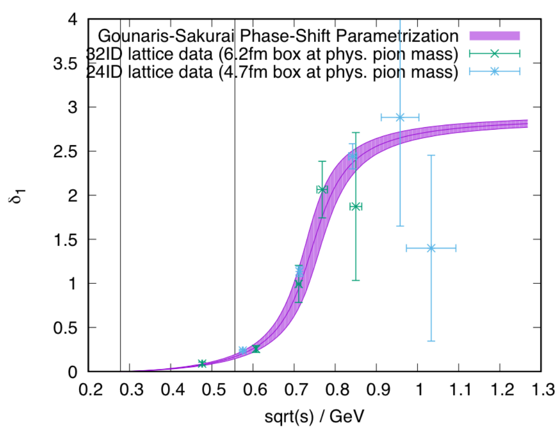
No bounding method:	$a_\mu^{HVP} = 577(31) \times 10^{-10}$
Bounding method $t_{\max} = 2.3$ fm, no improvement:	$a_\mu^{HVP} = 564.0(9.1) \times 10^{-10}$
Bounding method $t_{\max} = 1.7$ fm, 1 state improvement:	$a_\mu^{HVP} = 561.5(4.5) \times 10^{-10}$
Bounding method $t_{\max} = 1.6$ fm, 2 state improvement:	$a_\mu^{HVP} = 559.5(3.8) \times 10^{-10}$

Very large lattice spacing: $a^{-1} = 1.015$ GeV, finite volume effects
Could expect $10 - 20\%$ systematic errors on HVP

Finite Volume correction estimates

- scalar QED
- 24cube vs 32cube
- Using pion form factor (Gounaris-Sakurai parametrization) & Luscher's FV formula

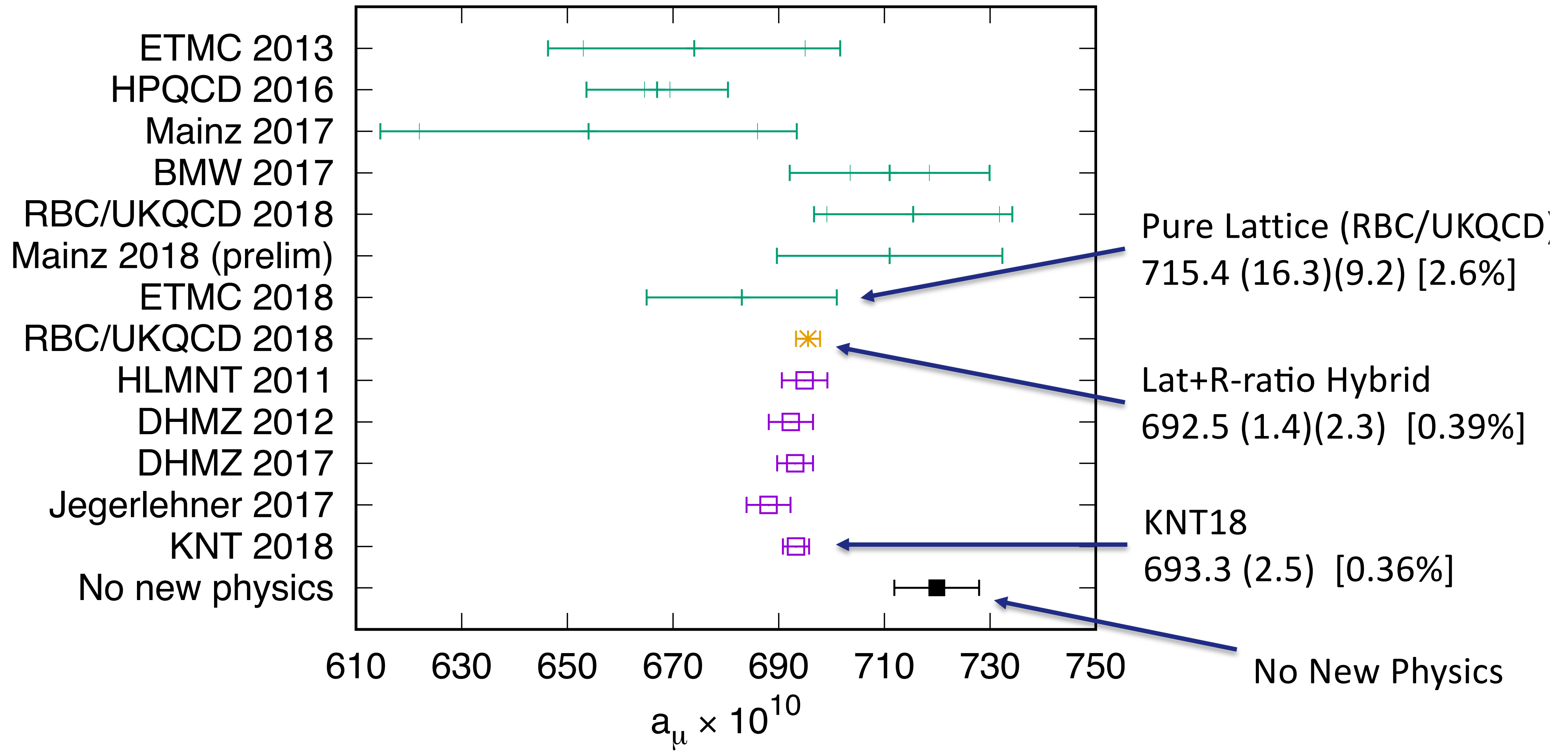
$$a_\mu^{HVP}(L = 6.22 \text{ fm}) - a_\mu^{HVP}(L = 4.66 \text{ fm}) = \begin{cases} 12.2 \times 10^{-10} & \text{sQED} \\ 21.6(6.3) \times 10^{-10} & \text{LQCD} \\ 20(3) \times 10^{-10} & \text{GSL} \end{cases}$$

- 

- Revised FV estimation :

$$a_\mu^{HVP}(L = \infty) - a_\mu^{HVP}(L = 5.47 \text{ fm}) = 22(1) \times 10^{-10}$$

HVP results
[Christoph Lehner et al PRL18]



- Significant improvements is in progress for statistical error using 2π and 4π (!) states in addition to EM current (GEVP, GS-parametrization)
- Checking finite volume and discretization error as well as Isospin V effects

Example error budget from RBC/UKQCD 2018 (**Fred's alphaQED17 results used for window result)**

	Window t=[0.4, 1 fm]	Pure Lattice
$a_\mu^{\rm ud,\ conn,\ isospin}$	$202.9(1.4)_{\rm S}(0.2)_{\rm C}(0.1)_{\rm V}(0.2)_{\rm A}(0.2)_{\rm Z}$	$649.7(14.2)_{\rm S}(2.8)_{\rm C}(3.7)_{\rm V}(1.5)_{\rm A}(0.4)_{\rm Z}(0.1)_{\rm E48}(0.1)_{\rm E64}$
$a_\mu^{\rm s,\ conn,\ isospin}$	$27.0(0.2)_{\rm S}(0.0)_{\rm C}(0.1)_{\rm A}(0.0)_{\rm Z}$	$53.2(0.4)_{\rm S}(0.0)_{\rm C}(0.3)_{\rm A}(0.0)_{\rm Z}$
$a_\mu^{\rm c,\ conn,\ isospin}$	$3.0(0.0)_{\rm S}(0.1)_{\rm C}(0.0)_{\rm Z}(0.0)_{\rm M}$	$14.3(0.0)_{\rm S}(0.7)_{\rm C}(0.1)_{\rm Z}(0.0)_{\rm M}$
$a_\mu^{\rm uds,\ disc,\ isospin}$	$-1.0(0.1)_{\rm S}(0.0)_{\rm C}(0.0)_{\rm V}(0.0)_{\rm A}(0.0)_{\rm Z}$	$-11.2(3.3)_{\rm S}(0.4)_{\rm V}(2.3)_{\rm L}$
$a_\mu^{\rm QED,\ conn}$	$0.2(0.2)_{\rm S}(0.0)_{\rm C}(0.0)_{\rm V}(0.0)_{\rm A}(0.0)_{\rm Z}(0.0)_{\rm E}$	$5.9(5.7)_{\rm S}(0.3)_{\rm C}(1.2)_{\rm V}(0.0)_{\rm A}(0.0)_{\rm Z}(1.1)_{\rm E}$
$a_\mu^{\rm QED,\ disc}$	$-0.2(0.1)_{\rm S}(0.0)_{\rm C}(0.0)_{\rm V}(0.0)_{\rm A}(0.0)_{\rm Z}(0.0)_{\rm E}$	$-6.9(2.1)_{\rm S}(0.4)_{\rm C}(1.4)_{\rm V}(0.0)_{\rm A}(0.0)_{\rm Z}(1.3)_{\rm E}$
$a_\mu^{\rm SIB}$	$0.1(0.2)_{\rm S}(0.0)_{\rm C}(0.2)_{\rm V}(0.0)_{\rm A}(0.0)_{\rm Z}(0.0)_{\rm E48}$	$10.6(4.3)_{\rm S}(0.6)_{\rm C}(6.6)_{\rm V}(0.1)_{\rm A}(0.0)_{\rm Z}(1.3)_{\rm E48}$
$a_\mu^{\rm udsc,\ isospin}$	$231.9(1.4)_{\rm S}(0.2)_{\rm C}(0.1)_{\rm V}(0.3)_{\rm A}(0.2)_{\rm Z}(0.0)_{\rm M}$	$705.9(14.6)_{\rm S}(2.9)_{\rm C}(3.7)_{\rm V}(1.8)_{\rm A}(0.4)_{\rm Z}(2.3)_{\rm L}(0.1)_{\rm E48}(0.1)_{\rm E64}(0.0)_{\rm M}$
$a_\mu^{\rm QED,\ SIB}$	$0.1(0.3)_{\rm S}(0.0)_{\rm C}(0.2)_{\rm V}(0.0)_{\rm A}(0.0)_{\rm Z}(0.0)_{\rm E}(0.0)_{\rm E48}$	$9.5(7.4)_{\rm S}(0.7)_{\rm C}(6.9)_{\rm V}(0.1)_{\rm A}(0.0)_{\rm Z}(1.7)_{\rm E}(1.3)_{\rm E48}$
$a_\mu^{\rm R-ratio}$	$460.4(0.7)_{\rm RST}(2.1)_{\rm RSY}$	
a_μ	$692.5(1.4)_{\rm S}(0.2)_{\rm C}(0.2)_{\rm V}(0.3)_{\rm A}(0.2)_{\rm Z}(0.0)_{\rm E}(0.0)_{\rm E48}(0.0)_{\rm b}(0.1)_{\rm c}(0.0)_{\overline{\rm S}}(0.0)_{\overline{\rm Q}}(0.0)_{\rm M}(0.7)_{\rm RST}(2.1)_{\rm RSY}$	$715.4(16.3)_{\rm S}(3.0)_{\rm C}(7.8)_{\rm V}(1.9)_{\rm A}(0.4)_{\rm Z}(1.7)_{\rm E}(2.3)_{\rm L}(1.5)_{\rm E48}(0.1)_{\rm E64}(0.3)_{\rm b}(0.2)_{\rm c}(1.1)_{\overline{\rm S}}(0.3)_{\overline{\rm Q}}(0.0)_{\rm M}$

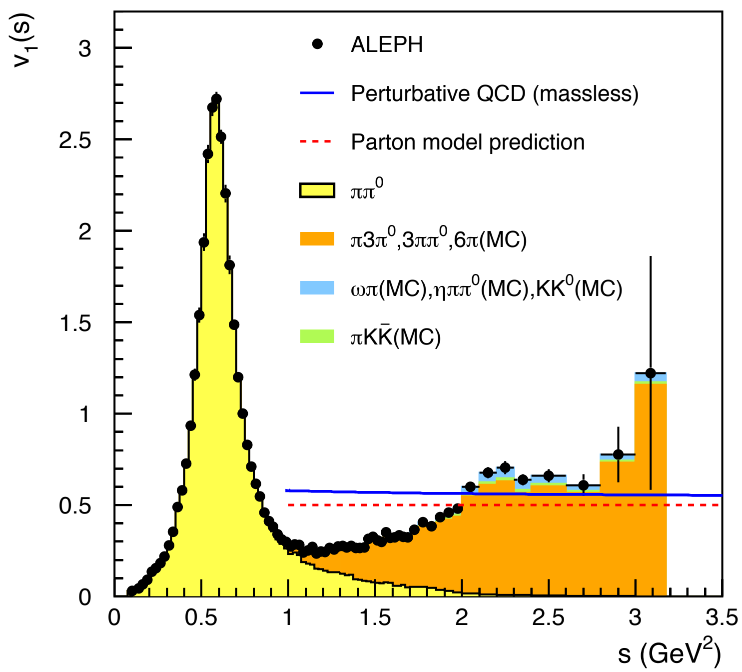
TABLE I. Individual and summed contributions to a_μ multiplied by 10^{10}. The left column lists results for the window method with $t_0 = 0.4$ fm and $t_1 = 1$ fm. The right column shows results for the pure first-principles lattice calculation. The respective uncertainties are defined in the main text.

For the pure lattice number the dominant errors are (S) statistics, (V) finite-volume errors, and (C) the continuum limit extrapolation uncertainty.

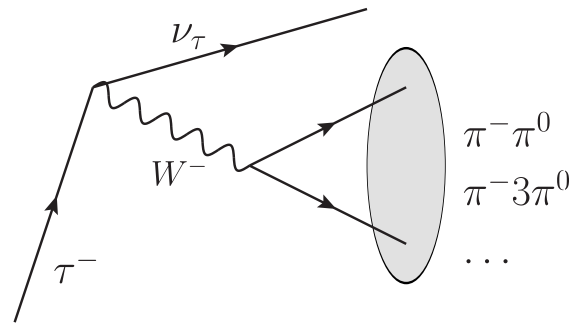
For the window method there are additional R-ratio systematic (RSY) and R-ratio statistical (RST) errors.

[M. Bruno's slide]

Tau input for g-2 HVP

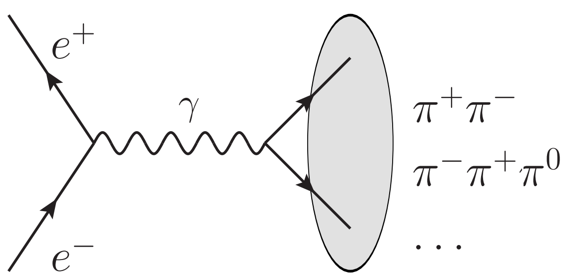


[M. Bruno et al, arXiv:1811.00508]



$V - A$ current

Final states $I = 1$ charged



EM current

Final states $I = 0, 1$ neutral

τ data can improve $a_\mu[\pi\pi]$
$\rightarrow 72\%$ of total Hadronic LO

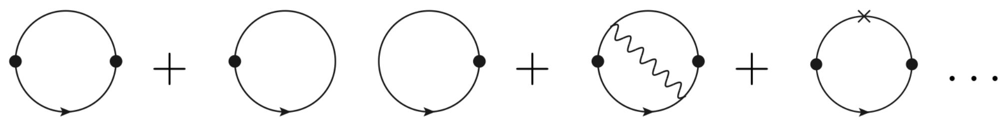
or $a_\mu^{ee} \neq a^\tau \rightarrow$ NP [Cirigliano et al '18]

amu & isospin components

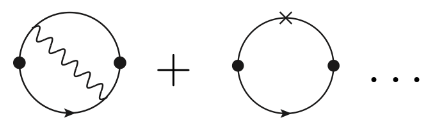
Isospin decomposition of u, d current

$$j_\mu^\gamma = \frac{i}{6}\left(\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d\right) + \frac{i}{2}\left(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d\right) = j_\mu^{(0)} + j_\mu^{(1)}$$

$$G_{00}^\gamma \leftarrow \langle j_k^{(0)}(x) j_k^{(0)}(0)\rangle =$$



$$G_{01}^\gamma \leftarrow \langle j_k^{(0)}(x) j_k^{(1)}(0)\rangle =$$

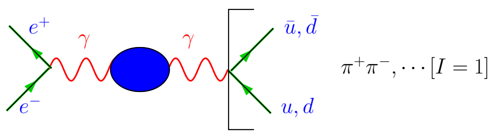


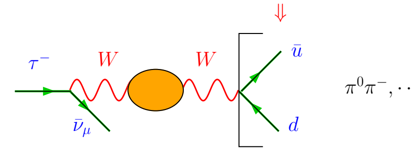
$$G_{11}^\gamma \leftarrow \langle j_k^{(1)}(x) j_k^{(1)}(0)\rangle =$$

Decompose $a_\mu = a_\mu^{(0,0)} + a_\mu^{(0,1)} + a_\mu^{(1,1)}$

difference b/w tau decay and e+e-

[M. Bruno's slide]



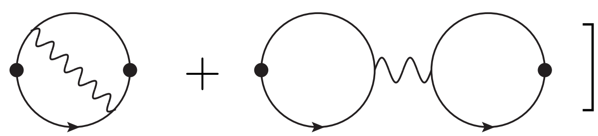


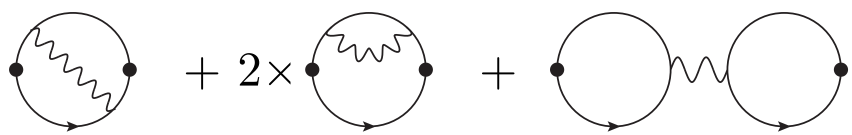
$$\frac{i}{2}\left(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d\right), \begin{bmatrix} I=1 \\ I_3=0 \end{bmatrix} \quad \rightarrow \quad j_\mu^{(1,-)} = \frac{i}{\sqrt{2}}\left(\bar{u}\gamma_\mu d\right), \begin{bmatrix} I=1 \\ I_3=-1 \end{bmatrix}$$

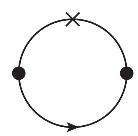
Isospin 1 charged correlator $G_{11}^W = \frac{1}{3}\sum_k \int d\vec{x}\, \langle j_k^{(1,+)}(x) j_k^{(1,-)}(0)\rangle$

$$\Delta a_\mu[\pi\pi,\tau] = 4\alpha^2 \sum_t w_t \times \left[G_{01}^\gamma(t) + G_{11}^\gamma(t) - G_{11}^W(t)\right]$$

$$\delta G^{(1,1)} \equiv G_{11}^\gamma - G_{11}^W$$

$$= Z_V^4 (4\pi\alpha)\frac{(Q_\mathrm{u}-Q_\mathrm{d})^4}{4}\Big[$$  $$\Big]$$

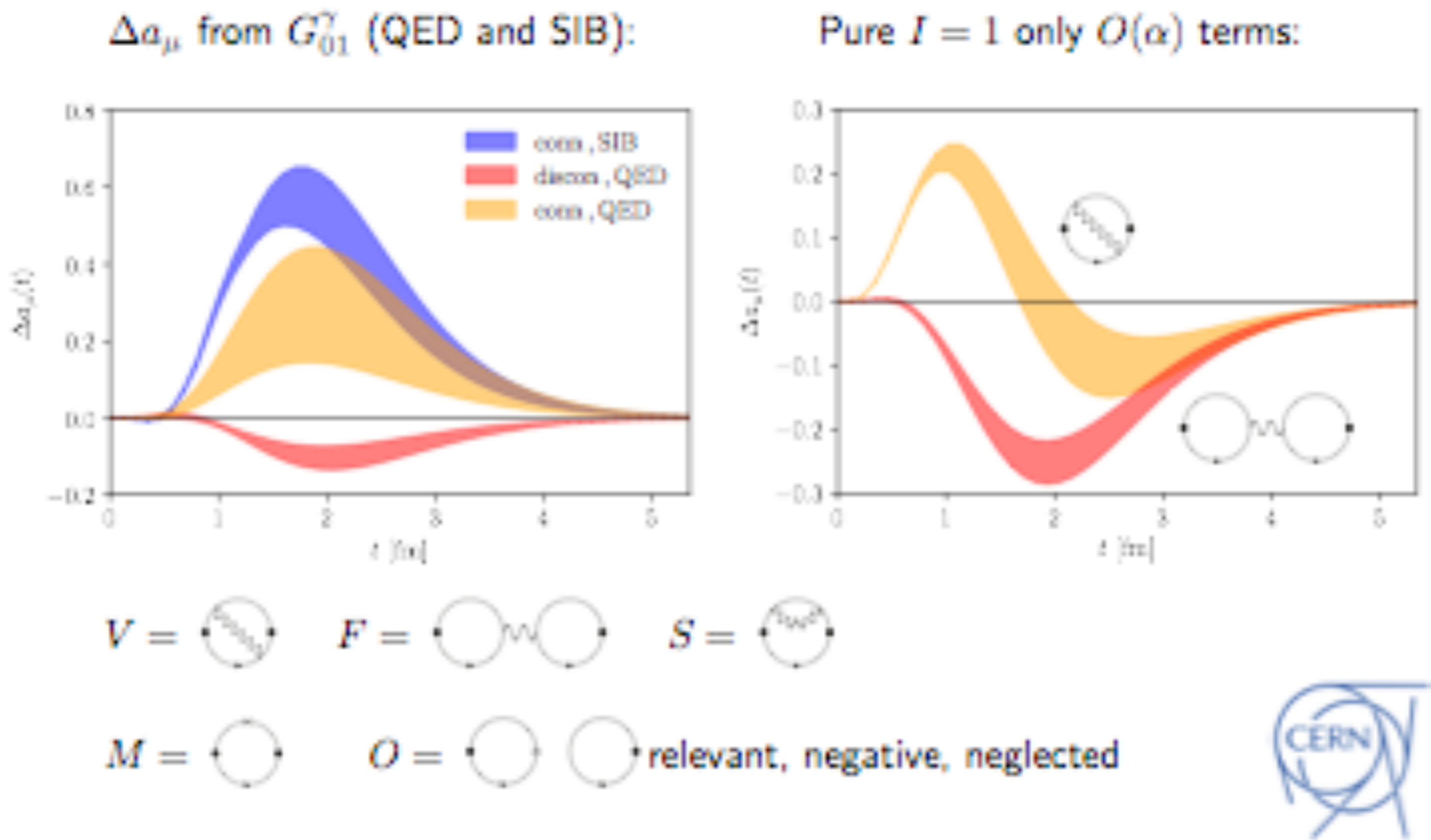
$$G_{01}^\gamma = Z_V^4 \frac{(Q_u^2-Q_d^2)^2}{2}(4\pi\alpha)\Big[$$  $$+\dots\Big]$$

$$+Z_V^2\frac{Q_u^2-Q_d^2}{2}(m_u-m_d)\Big[2\times$$  $$+\dots\Big]$$

$\dots$ = subleading diagrams currently not included

[M. Bruno’s slide]

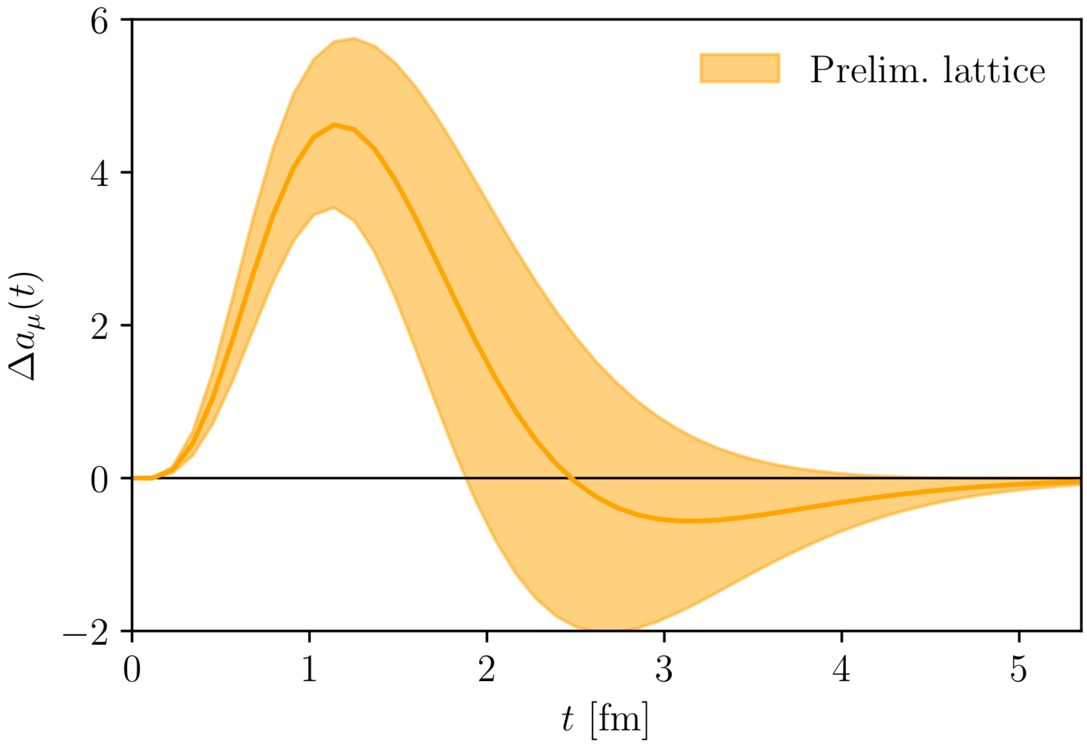
Δa_μ (Preliminary)



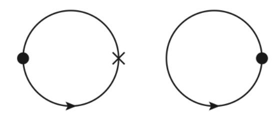
Tau spectral function (vector, Strange=0) is very welcome !

$$\Delta a_\mu[\pi\pi,\tau] = 4\alpha^2 \sum_t w_t \times \left[G^\gamma_{01}(t) + G^\gamma_{11}(t) - G^W_{11}(t)\right]$$

Preliminary lattice (full) calculation: $G^\gamma_{01} + \delta G$



Not included:

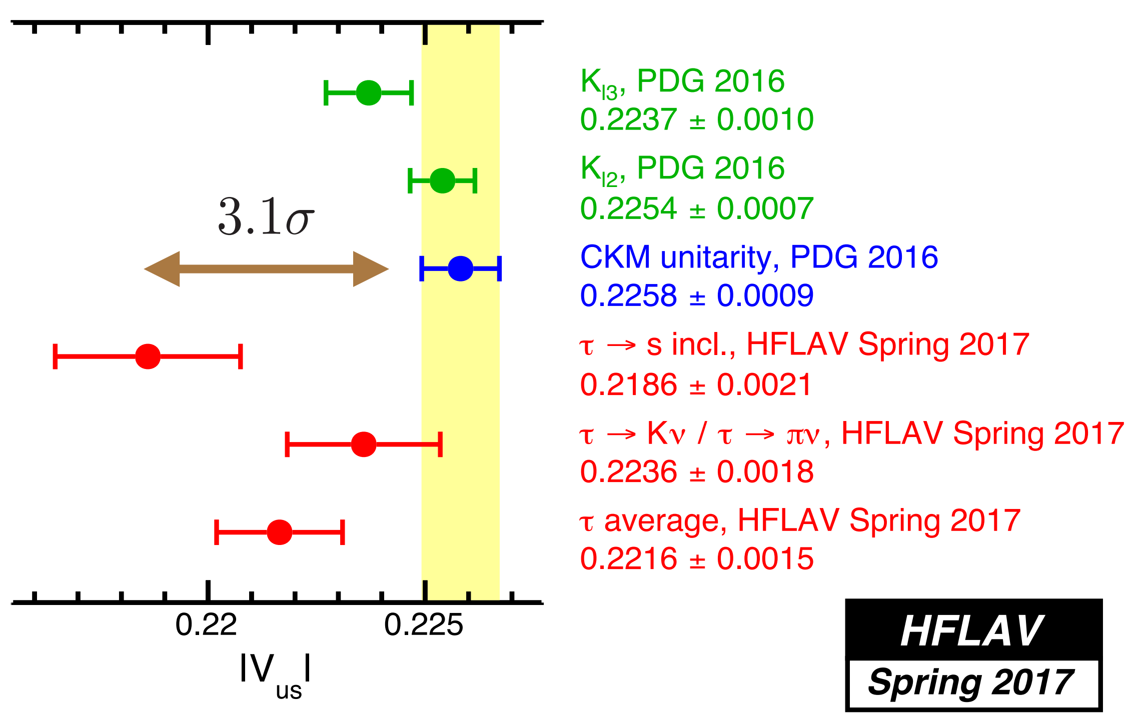
1.  relevant
2. sub-leading $1/N_c$, $1/N_f$
3. finite-volume errors
4. discretization errors

CKM V_{us} from Inclusive tau decay

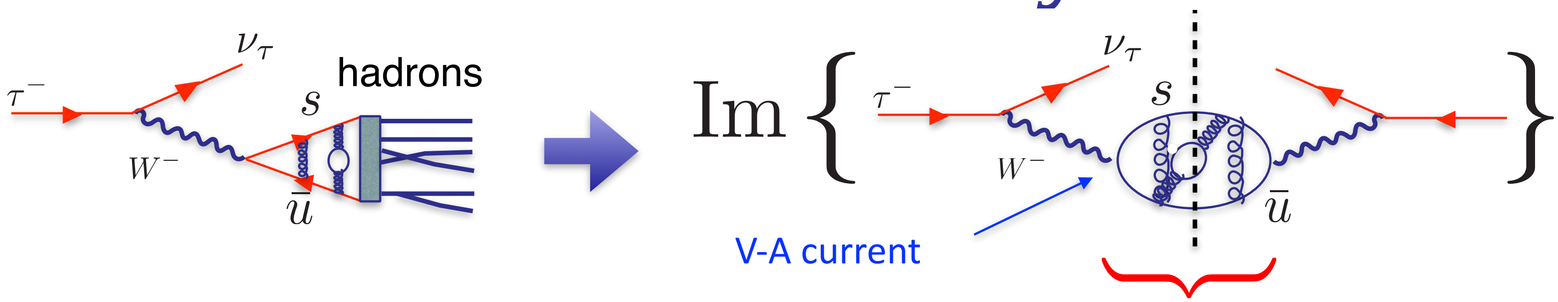
Yet another by-product of muon g-2 HVP

Phys.Rev.Lett. 121 (2018) 202003

[Hiroshi Ohki et al.]



Tau decay



(Hadronic) vacuum polarization function $\Pi(Q^2)$

- Experiment side : $\tau \to \nu + had$ through V-A vertex. EW correction S_{EW}

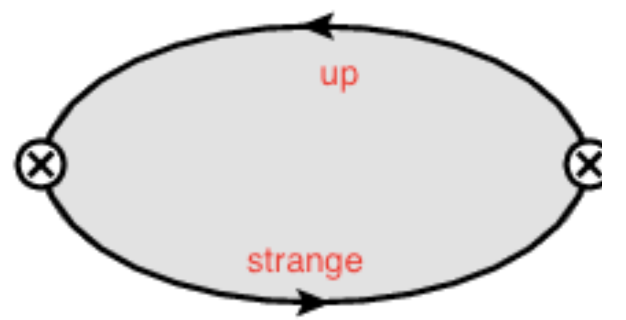
$$R_{ij} = \frac{\Gamma(\tau^- \to \text{hadrons}_{ij}\ \nu_\tau)}{\Gamma(\tau^- \to e^- \bar{\nu}_e \nu_\tau)}$$

$$= \frac{12\pi |V_{ij}|^2 S_{EW}}{m_\tau^2} \int_0^{m_\tau^2} \left(1 - \frac{s}{m_\tau^2}\right) \underbrace{\left[\left(1 + 2\frac{s}{m_\tau^2}\right) \mathrm{Im}\Pi^{(1)}(s) + \mathrm{Im}\Pi^{(0)}(s)\right]}_{\equiv\ \mathrm{Im}\,\Pi(s)}$$

- Lattice side : The Spin=0 and 1, vacuum polarization, Vector(V) or Axial (A) current-current two point

$$\Pi^{\mu\nu}_{ij;V/A}(q^2) = i \int d^4x e^{iqx} \langle 0 | T J^{\mu}_{ij;V/A}(x) J^{\dagger\mu}_{ij;V/A}(0) | 0 \rangle$$

$$= (q^\mu q^\nu - q^2 g^{\mu\nu}) \Pi^{(1)}_{ij;V/A}(q^2) + q^\mu q^\nu \Pi^{(0)}_{ij;V/A}$$



Finite Energy Sum Rule (FESR)

[Shifman, Vainstein, Zakharov 1979]

- FESR = Optical theorem (Unitarity) + Dispersion relation (Analyticity)
- Optical theorem relate S=-1 spectral function $\rho^{0/1}_{V/A,ij}(s)$ and HVP $\Pi^{0/1}_{V/A,ij}(s)$ for given quantum number: flavor (us or ud), spin (0 or 1), parity (V or A)

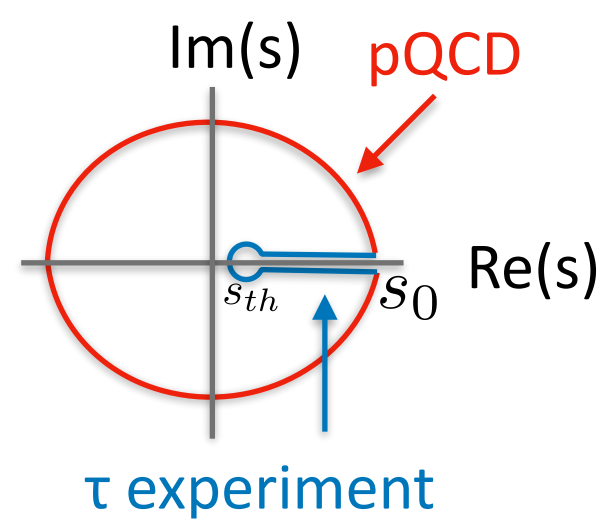
$$\frac{1}{\pi}\text{Im}\Pi(s) = \rho(s)$$

- Do *finite* radius contour integral for arbitrary regular weight function $w(s)$

$$\int_{s_{th}}^{s_0} ds\rho(s)w(s) = +\frac{i}{2\pi}\oint_{|s|=s_0} ds\Pi(s)w(s)$$

- Real axis integral is extracted from experimental decay energy distribution dR_τ/ds

$$\frac{dR_{ij;V/A}}{ds} = \frac{12\pi^2|V_{ij}|^2 S_{EW}}{m_\tau^2}\omega_\tau(s)\rho(s)$$



$|V_{us}|$ determination from FESR

[E. Gamiz, *et al.*, 2003, 2005, Maltman et al 2006]

- Inclusive differencial τ decay rate with wieght $w(s)$

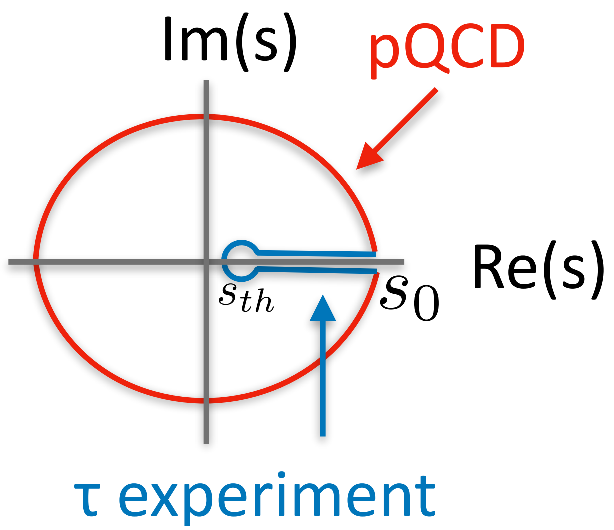
$$R^{\omega}{}_{ij}(s_0) \equiv \int_{s_{th}}^{s_0} ds \frac{dR_{ij}}{ds} \frac{\omega(s/s_0)}{\omega_\tau(s/m_\tau^2)}$$

- Take difference between up-down and up-strange channel

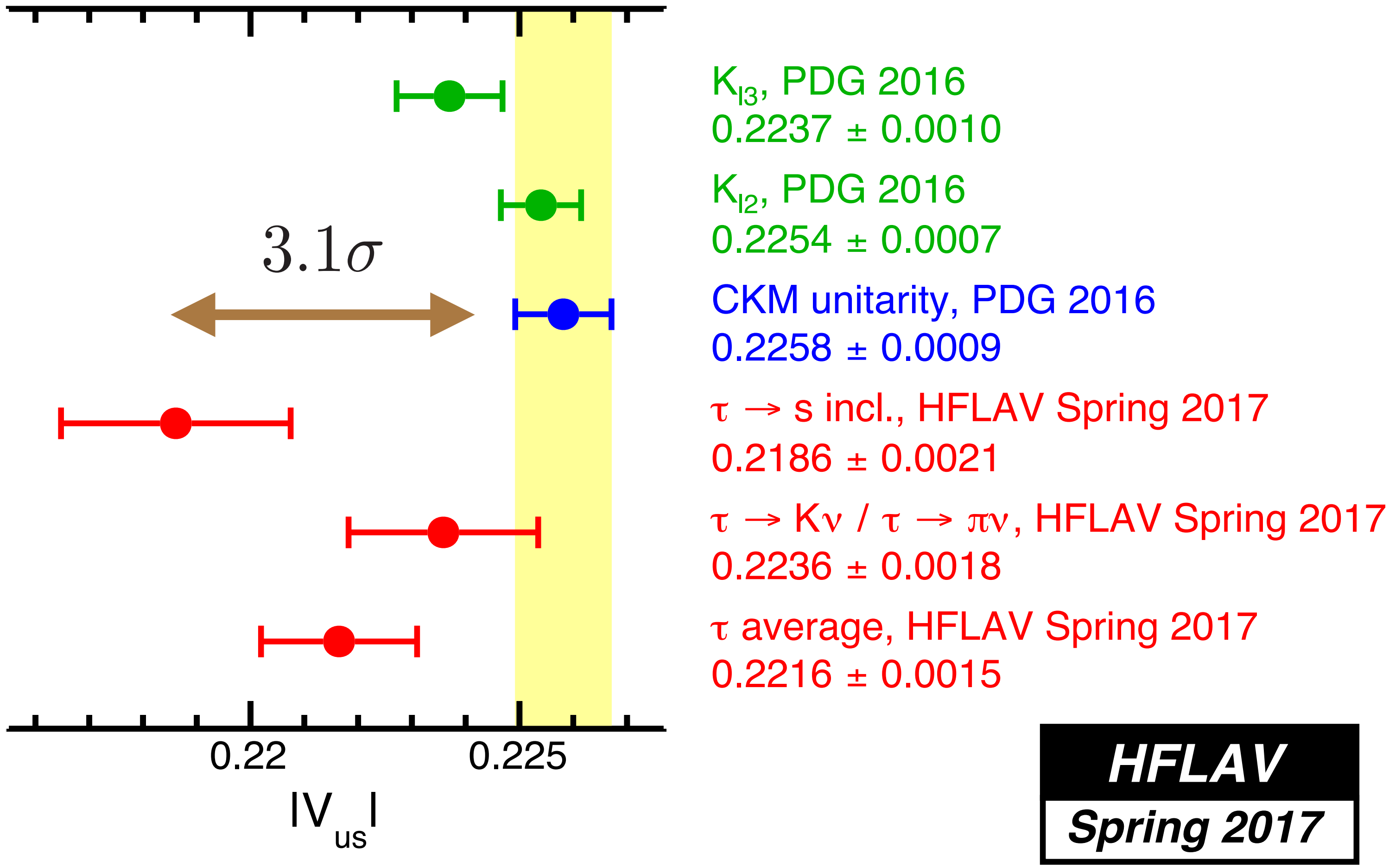
$$\Delta R^{\omega} = \frac{R^{\omega}_{ud}}{|V_{ud}|^2} - \frac{R^{\omega}_{us}}{|V_{us}|^2}$$

- $|V_{ud}|$ and m_s as input, selecting $s_0 = m_\tau^2, \omega = \omega_\tau(s/s_0)$

$$|V_{us}| = \sqrt{\frac{R^{\omega}_{us}(s_0)}{\frac{R^{\omega}_{ud}(s_0)}{|V_{ud}|^2} - [\Delta R^{\omega}(s_0)]^{\text{pQCD}}}}$$



- For $s > s_0$, fixed-order or contour-improved pQCD is used. OPE condensations at dim=4,6 ... are input/assumed. (a source of unaccounted uncertainties)



- τ result v.s. non-τ result : more than 3 σ deviation : |Vus| puzzle
- new physics effect?
- incl. analysis uses Finite energy sum rule (FESR)
- pQCD and higher order OPE for FESR:

  underestimation of truncation error and/or non-perturbative effects ?

  (c.f. alternative FESR approach, R. Hudspith et. al arXiv:1702.01767)

Our new method : Combining FESR and Lattice

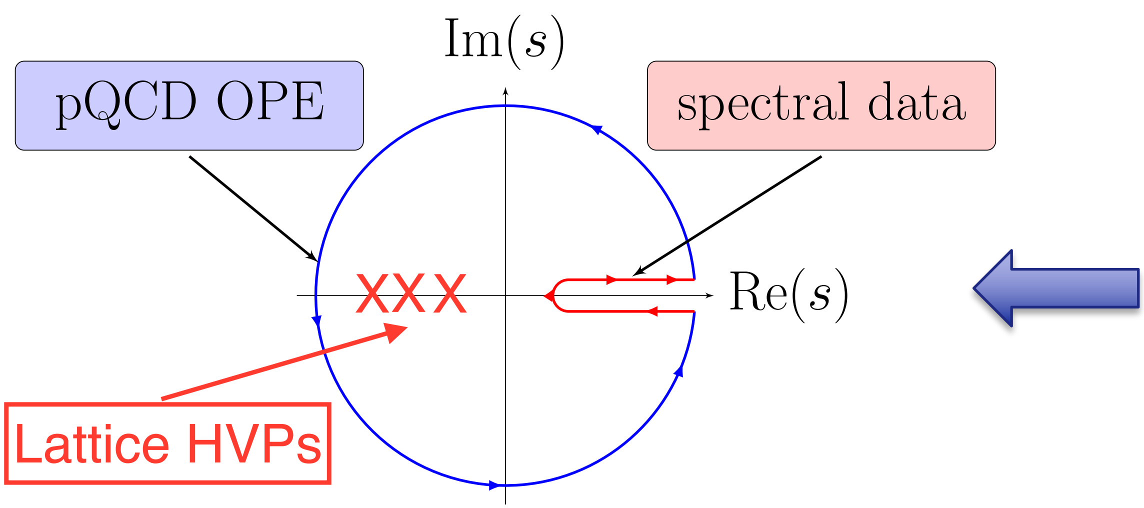
- If we have a reliable estimate for $\Pi(s)$ in Euclidean (space-like) points, $s = -Q_k^2 < 0$, we could extend the FESR with weight function $w(s)$ to have poles there,

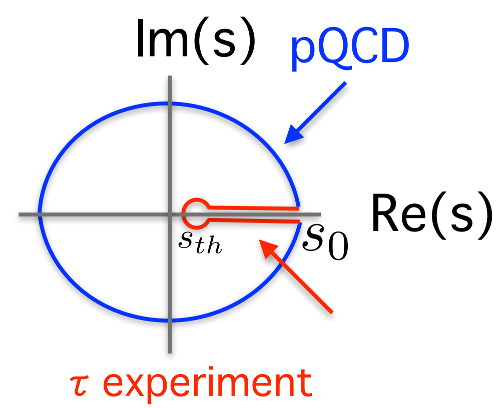
$$\int_{s_{th}}^{\infty} w(s)\mathrm{Im}\Pi(s) = \pi \sum_{k}^{N_p} \mathrm{Res}_k[w(s)\Pi(s)]_{s=-Q_k^2}$$

$$\Pi(s) = \left(1 + 2\frac{s}{m_\tau^2}\right) \mathrm{Im}\Pi^{(1)}(s) + \mathrm{Im}\Pi^{(0)}(s) \propto s \quad (|s| \to \infty)$$

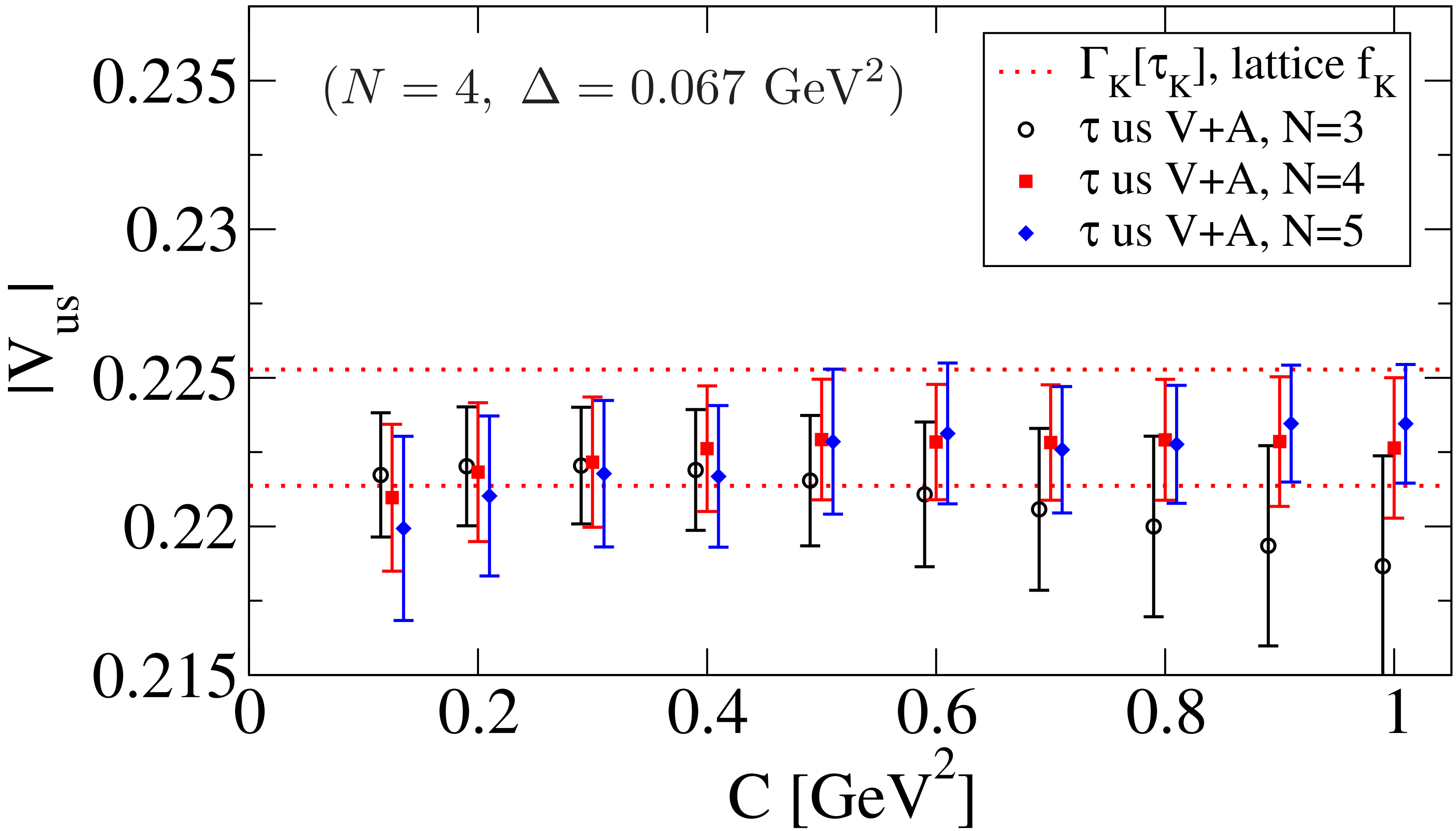
- For $N_p \geq 3$, the $|s| \to \infty$ circle integral vanishes.

$$w(s) = \prod_{k}^{N_p} \frac{1}{(s + Q_k^2)}$$





Lattice Inclusive $|V_{us}|$ determinations

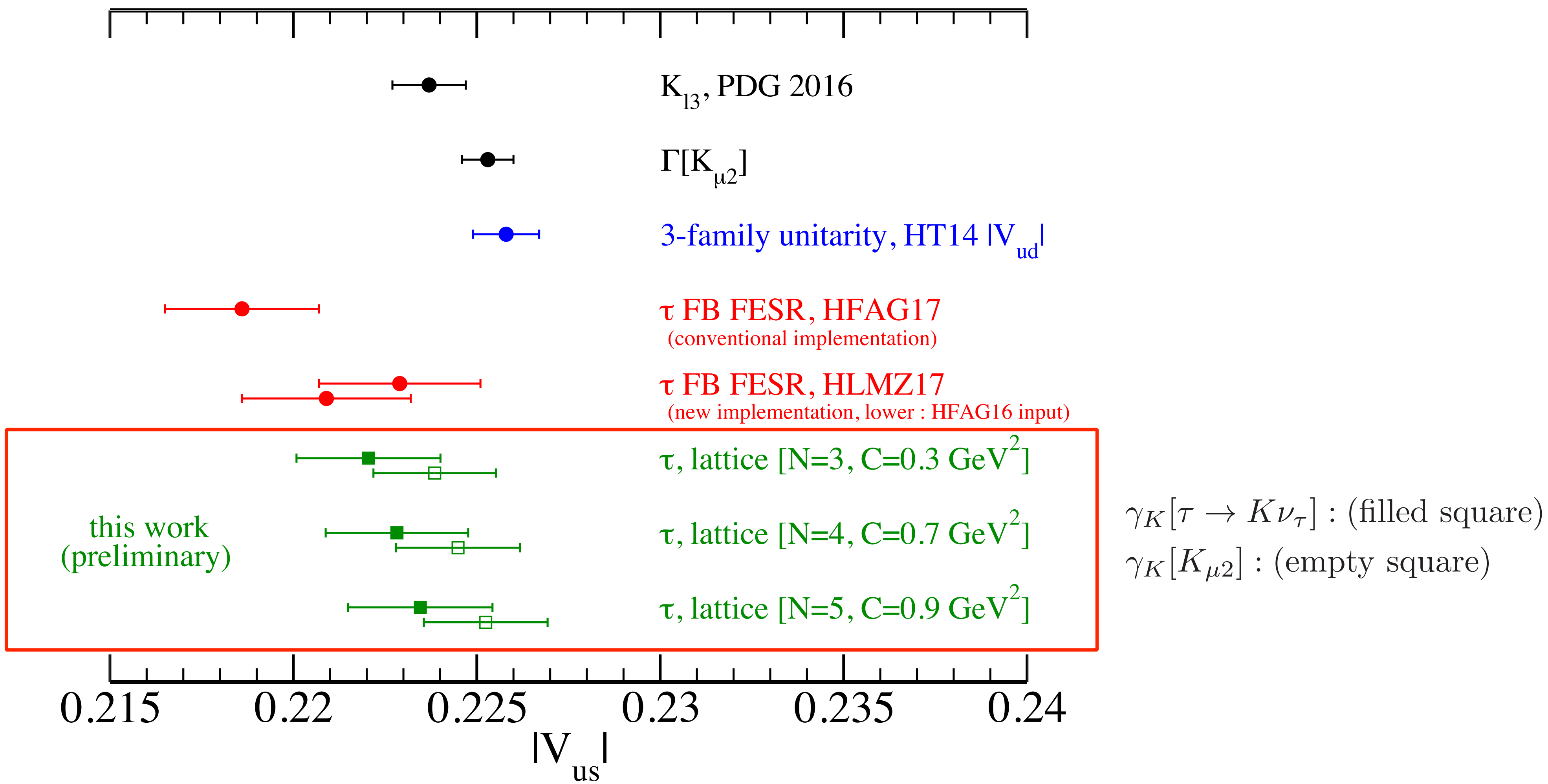


Theory and experimental errors are included.

The result is stable against changes of C and N.

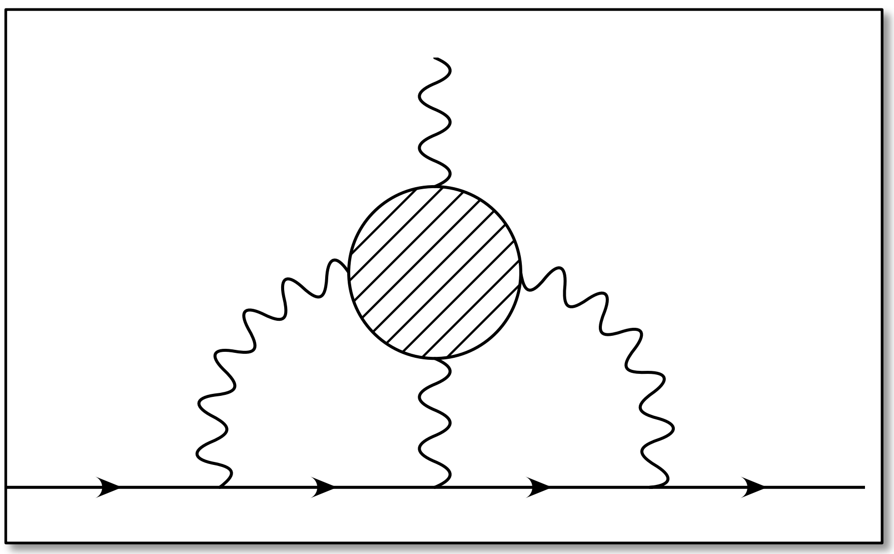
$N = 4, C = 0.7[\text{GeV}^2] : |V_{us}| = 0.2228(15)_{exp}(13)_{th}$ (0.87% total error)

Comparison to $|V_{us}|$ from others



Tau spectral function (vector/axial, Strange=-1) is very welcome !

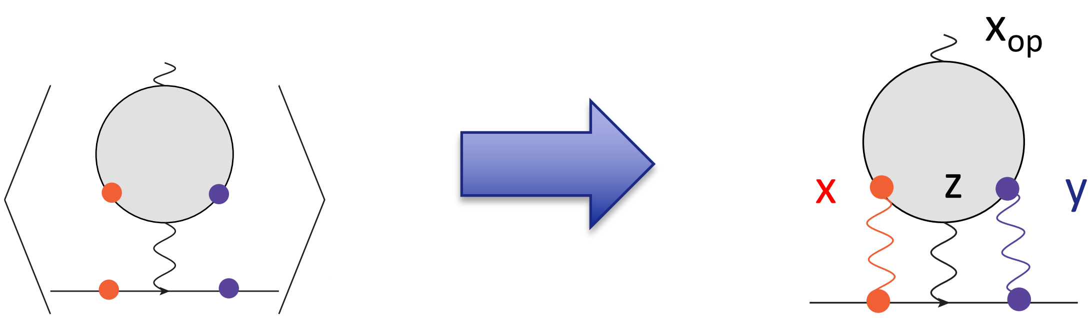
Hadronic Light-by-Light (HLbL) contributions

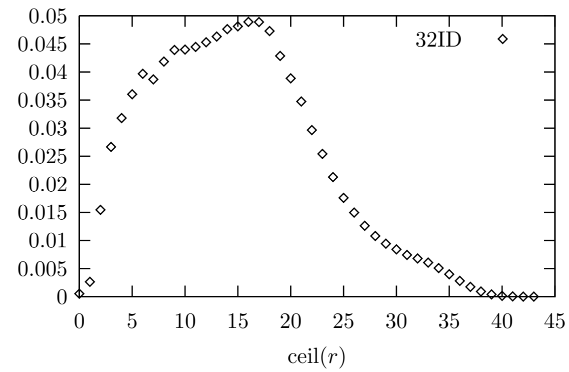


Coordinate space Point photon method

[Luchang Jin et al. , PRD93, 014503 (2016)]

- Treat all 3 photon propagators exactly (3 analytical photons) , which makes the quark loop and the lepton line connected :
  disconnected problem in Lattice QED+QCD -> connected problem with analytic photon
- QED 2-loop in coordinate space. Stochastically sample, two of quark-photon vertex location x,y, z and x_{op} is summed over space-time exactly





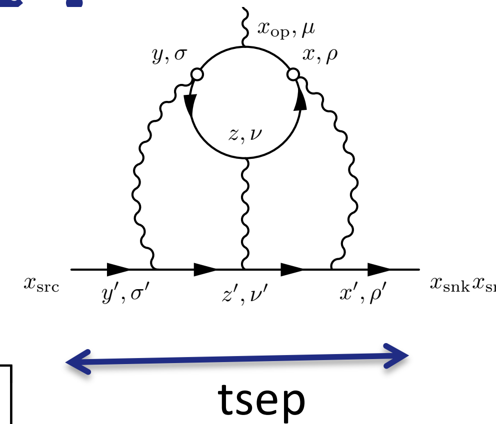
- Short separations, Min[|x-z|,|y-z|,|x-y|] < R ~ O(0.5) fm, which has a large contribution due to confinement, are summed for all pairs
- longer separations, Min[|x-z|,|y-z|,|x-y|] >= R, are done stochastically with a probability shown above (Adaptive Monte Carlo sampling)

Dramatic Improvement !

Luchang Jin

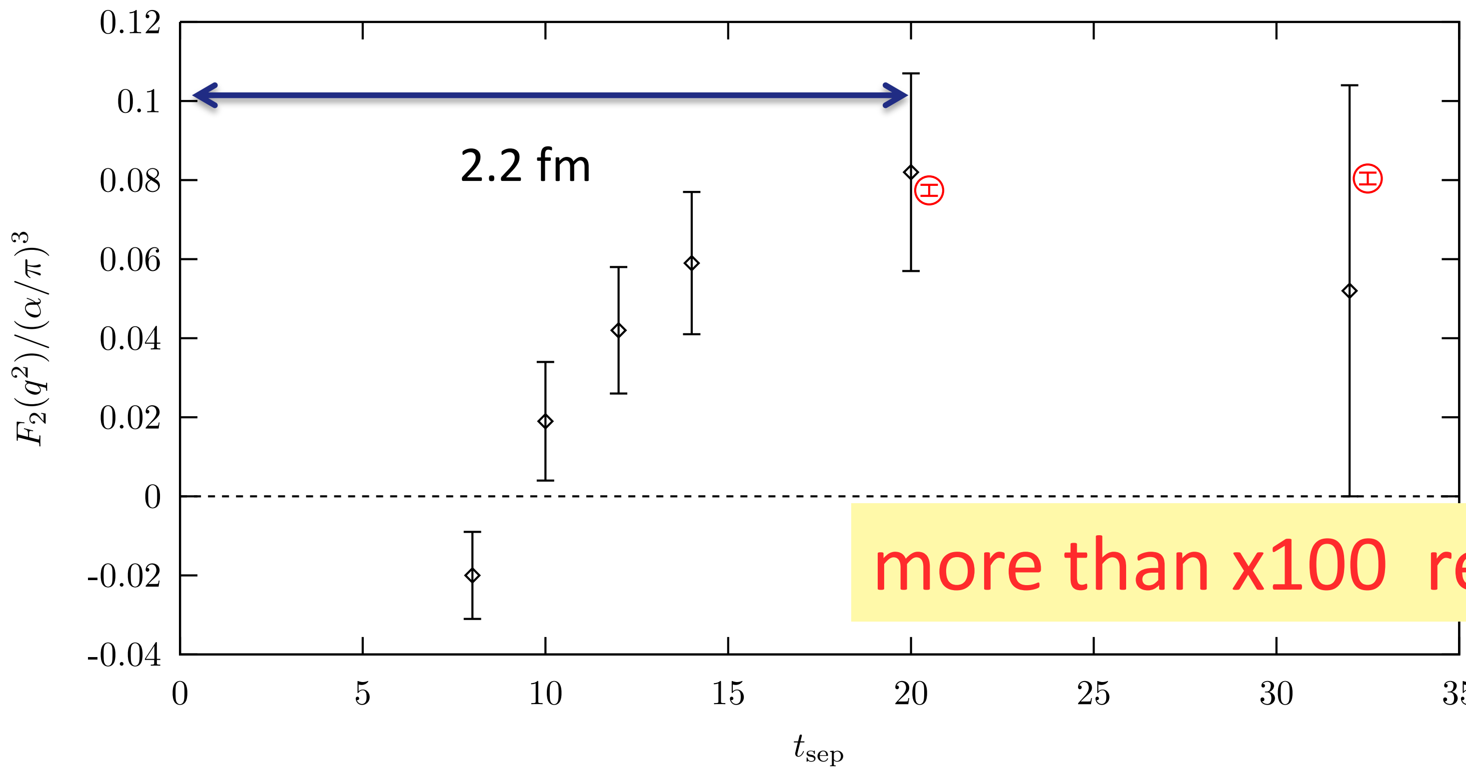
a=0.11 fm, 24^3x64 (2.7 fm)^3,

m_π = 329 MeV, m_μ =~ 190 MeV, e=1



$q = 2\pi/L$ $N_{\rm prop} = 81000$

$q = 0$ $N_{\rm prop} = 26568$

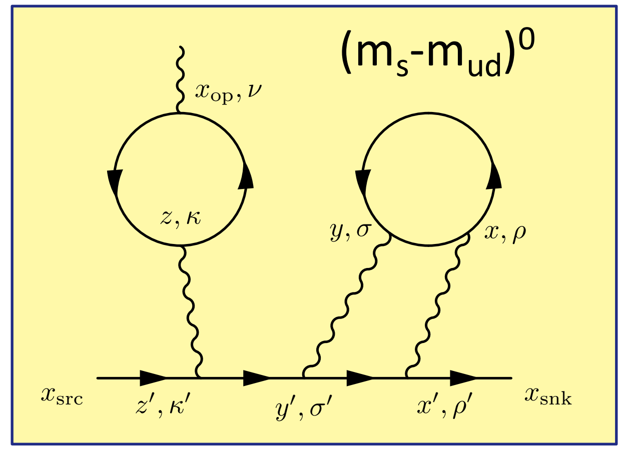


more than x100 reduced cost !

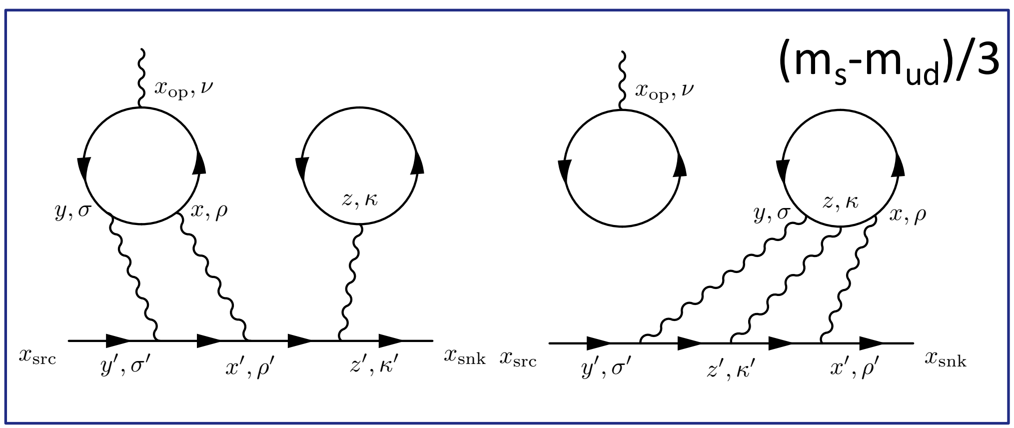
Method	$F_2/(\alpha/\pi)^3$	$N_{\rm conf}$	$N_{\rm prop}$	$\sqrt{\rm Var}$
Conserved	0.0825(32)	12	$(118 + 128) \times 2 \times 7$	0.65
Mom.	0.0804(15)	18	$(118 + 128) \times 2 \times 3$	0.24

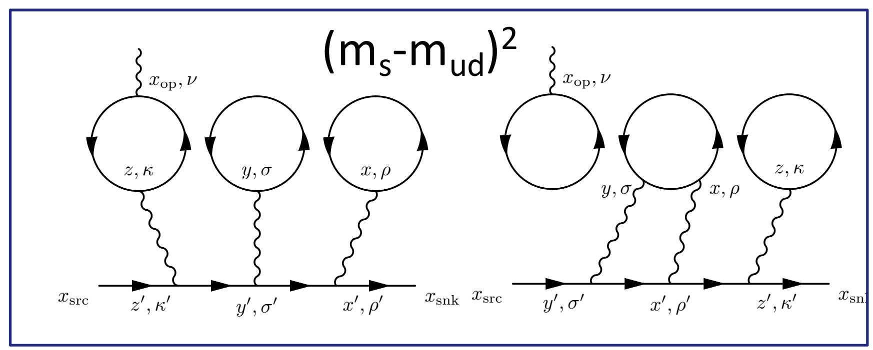
SU(3) hierarchies for d-HLbL

- At m_s=m_{ud} limit, following type of disconnected HLbL diagrams survive $Q_u + Q_d + Q_s = 0$
- Physical point run using similar techniques to c-HLbL.



- other diagrams suppressed by
  $O(m_s-m_{ud})$ /3 and $O((m_s-m_{ud})^2)$

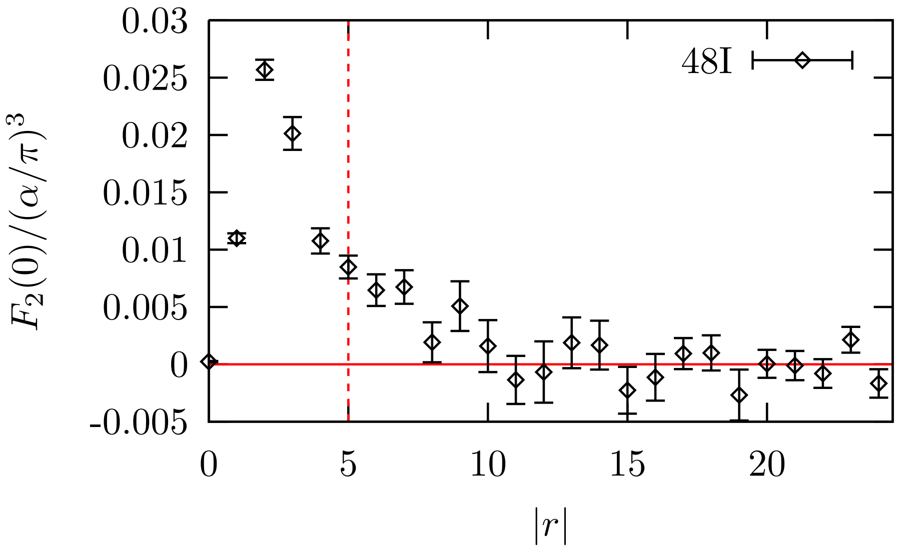


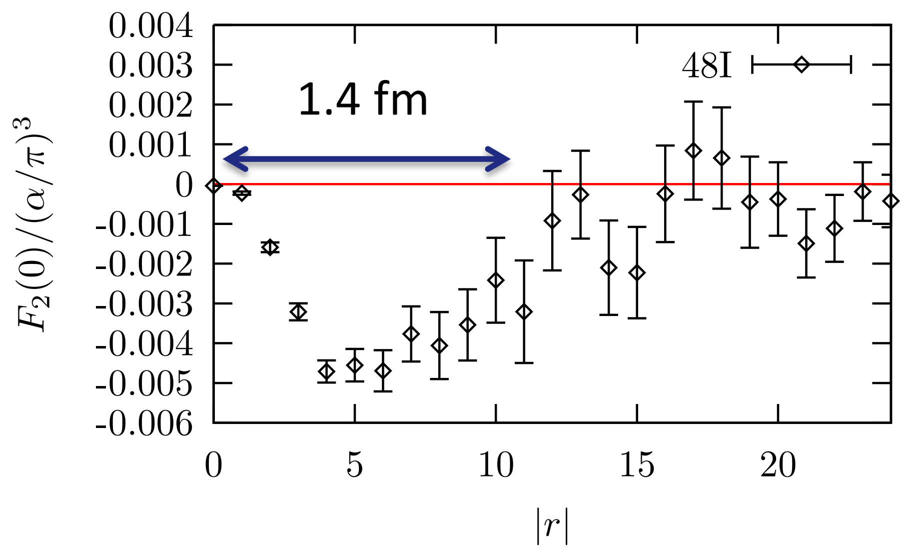


140 MeV Pion, connected and disconnected LbL results

[Luchang Jin et al. , Phys.Rev.Lett. 118 (2017) 022005]

- left: connected, right : leading disconnected

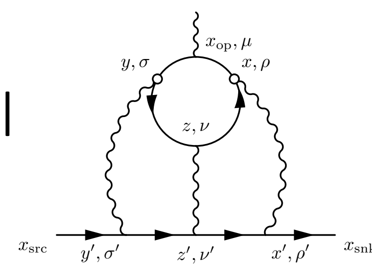




- Using AMA with 2,000 zMobius low modes, AMA

(statistical error only)

r = |x-y|



$$\left.\frac{g_\mu - 2}{2}\right|_{\rm cHLbL} = (0.0926 \pm 0.0077) \times \left(\frac{\alpha}{\pi}\right)^3 = (11.60 \pm 0.96) \times 10^{-10}$$

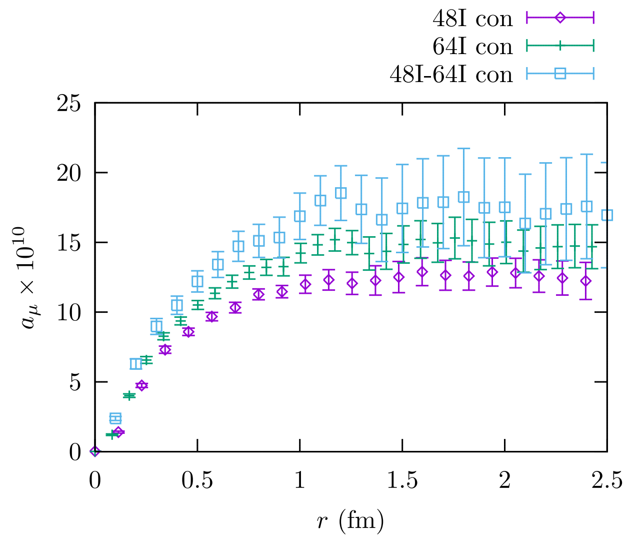
$$\left.\frac{g_\mu - 2}{2}\right|_{\rm dHLbL} = (-0.0498 \pm 0.0064) \times \left(\frac{\alpha}{\pi}\right)^3 = (-6.25 \pm 0.80) \times 10^{-10}$$

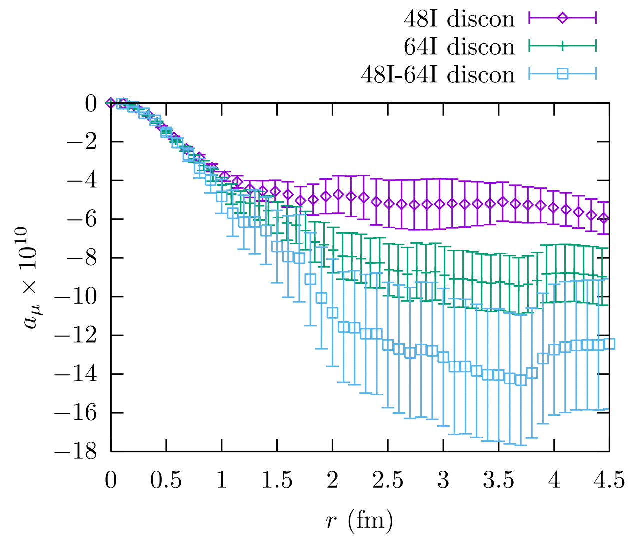
$$\left.\frac{g_\mu - 2}{2}\right|_{\rm HLbL} = (0.0427 \pm 0.0108) \times \left(\frac{\alpha}{\pi}\right)^3 = (5.35 \pm 1.35) \times 10^{-10}$$

Continuum / infinite volume extrapolation

- Discretization error 1/a = 2.7, 1.4 GeV at physical quark mass

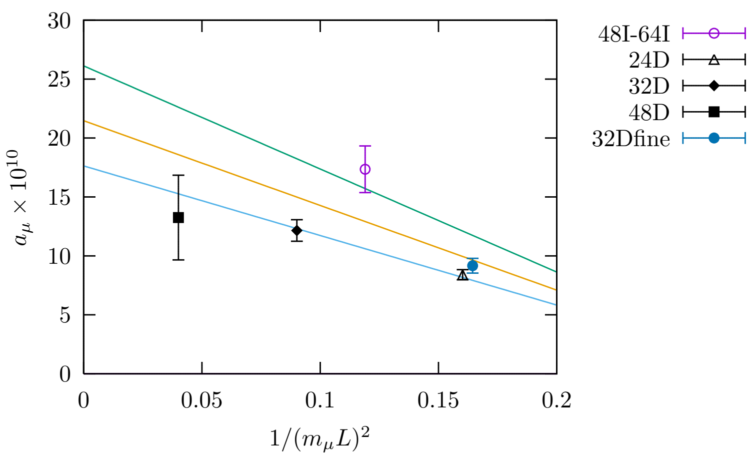
connected



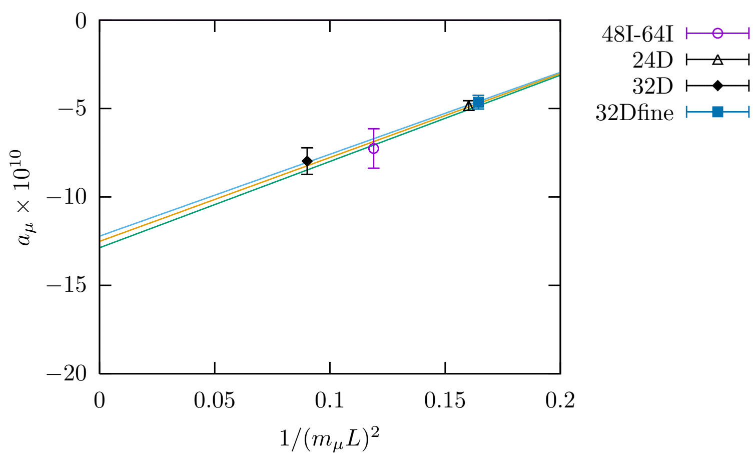


disconnected

- Finite volume L = 4.8, 6.4, 9.6 fm at 1/a=1 GeV at physical mass



Connected diagrams



Disconnected diagrams

HLbL
(connected + leading disconnected)

- The connected + leading disconnected contribution
- continuum, infinite volume limit

$$F_2(a,L) \;=\; F_2\left(1-\frac{c_1}{(m_\mu L)^2}\right)(1-c_2\,a^2)$$

- preliminary results [Luchang Jin, Schwinger Fest 2018-12-04]

$$a_\mu^{\mathrm{cHLbL}} \;=\; (26.12 \pm 3.87_{\mathrm{stat}} \pm 0.28_{\mathrm{sys},a^2}) \times 10^{-10}$$
$$a_\mu^{\mathrm{dHLbL}} \;=\; (-12.87 \pm 2.27_{\mathrm{stat}} \pm 1.63_{\mathrm{sys},a^2}) \times 10^{-10}$$

$$a_\mu^{\mathrm{HLbL}} \;=\; (13.24 \pm 4.40_{\mathrm{stat}} \pm 1.91_{\mathrm{sys},a^2}) \times 10^{-10}$$

F. Jegerlehner , x 10^{11}

Contribution	BPP	HKS	KN	MV	PdRV	N/JN
π^0, η, η'	85±13	82.7±6.4	83±12	114±10	114±13	99±16
π, K loops	−19±13	−4.5±8.1	–	0±10	−19±19	−19±13
axial vectors	2.5±1.0	1.7±1.7	–	22± 5	15±10	22± 5
scalars	−6.8±2.0	–	–	–	−7± 7	−7± 2
quark loops	21± 3	9.7±11.1	–	–	2.3	21± 3
total	83±32	89.6±15.4	80±40	136±25	105±26	116±39

Summary & Perspectives

- **HVP**
  - New methods using low mode for connected at physical quark mass,
  - disconnected quark loop at physical quark mass, QED and IB studies are included
  - Combining with R-ratio experiment data for cross-check and improvement => 0.4 % error
  - Eventually the window will be enlarged for a pure LQCD prediction (currently 2.6 % error)
  - Significant improvements is in progress for statistical error using 2π and 4π (!) states in addition to EM current (GEVP, GS-parametrization)
  - Checking finite volume and discretization error as well as Isospin V effects

- τ input for g-2 HVP

  We could compute Inclusive hadron cross sections at Euclidean q^2 from the first principle Lattice QCD with Isospin breaking effects

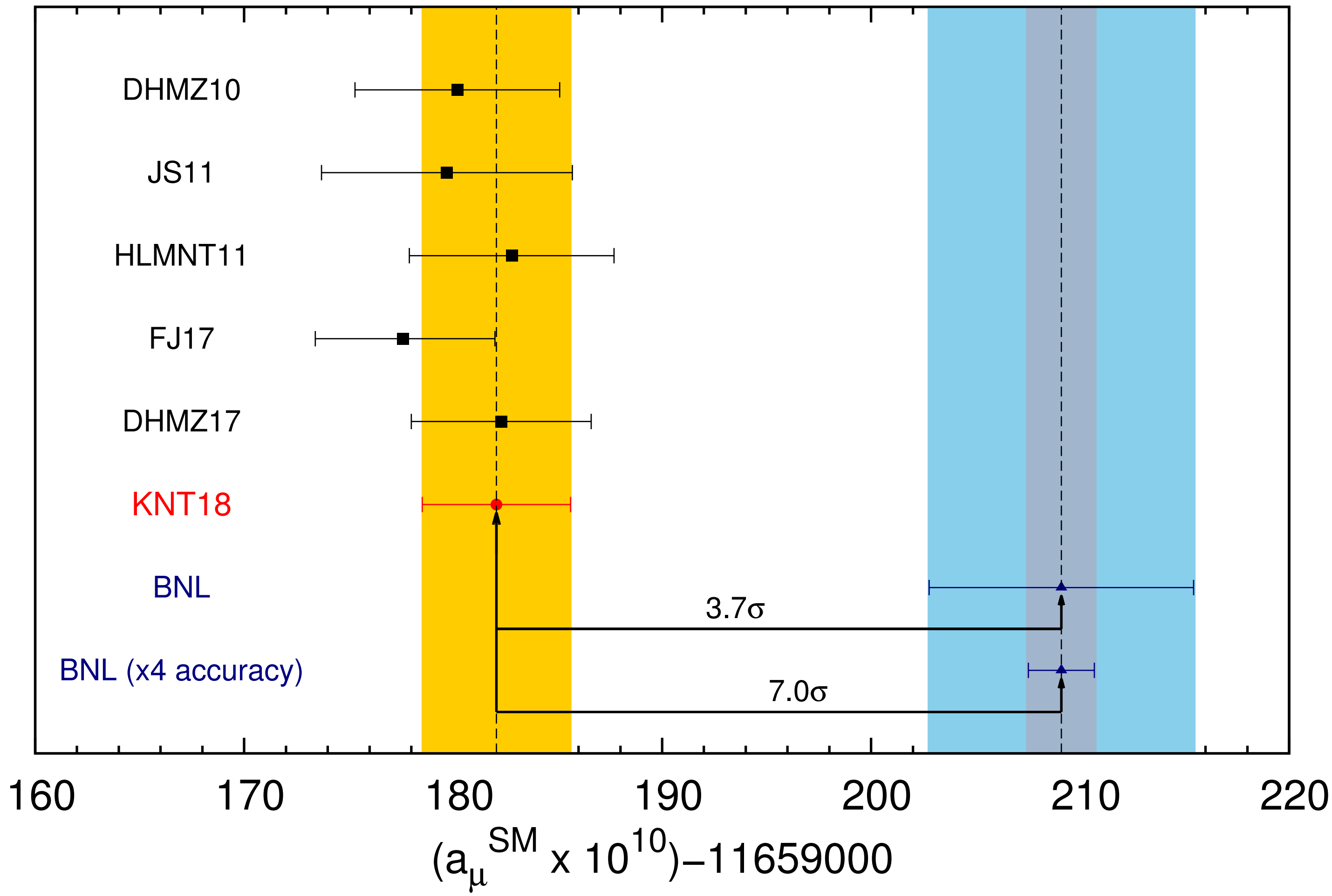
- τ inclusive analysis and Vus

  |Vus| from tau inclusive could be competitive to Kaon determination

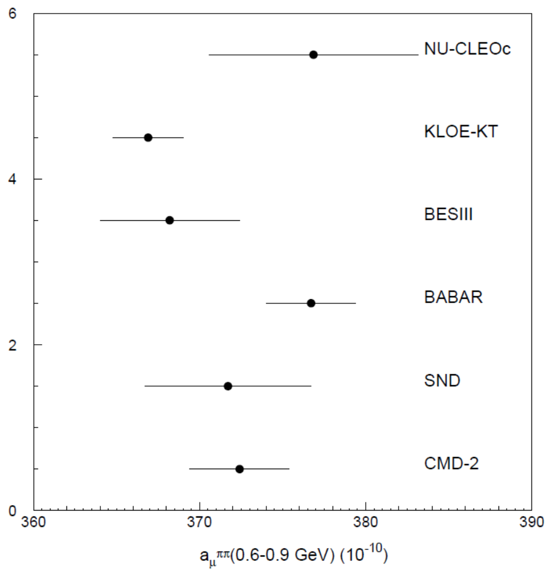
  Improved
  τ spectral functions
  will be very valuable

- **HLbL**
  - computing connected and leading disconnected diagrams, take continue & infinite volume limits
  - preliminary result not very different from the model results (Glasgow consensus)
  - pi0 pole contribution & higher order disconnected diagrams are in progress

KNT18 a_μ^{SM} update



The BABAR/KLOE discrepancy for $\pi\pi\gamma(\gamma)$



- BABAR and KLOE measurements most precise to date, but in poor agreement
- Others are in between, but not precise enough to decide
- No progress achieved in understanding the reason(s) of the discrepancy
- consequence: accuracy of combined results degraded
- imperative to improve accuracy of prediction (forthcoming g-2 results at FNAL, J-PARC)
- Other efforts at VEPP-2000 underway
- Design a new independent BABAR analysis

M. Davier ISR BABAR g-2 g-2 HVP Workshop, KEK 13/02/2018 25

Cross check, combine, and improve by LQCD data

[Luchang Jin's analogy]

Precession of Mercury and GR

Amount (arc-sec/century)	Cause
5025.6	Coordinate (due to precession of equinoxes)
531.4	Gravitational tugs of the other planets
0.0254	Oblateness of the sun (quadrupole moment)
42.98±0.04	General relativity
5600.0	Total
5599.7	Observed

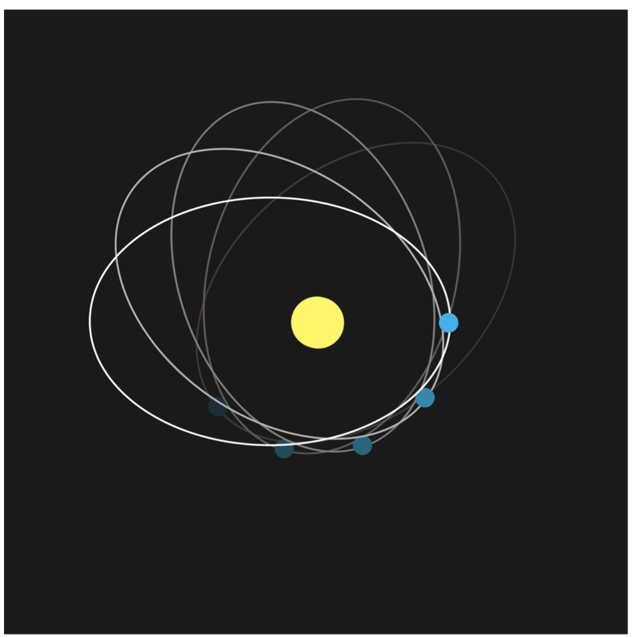
discrepancy recognized since 1859

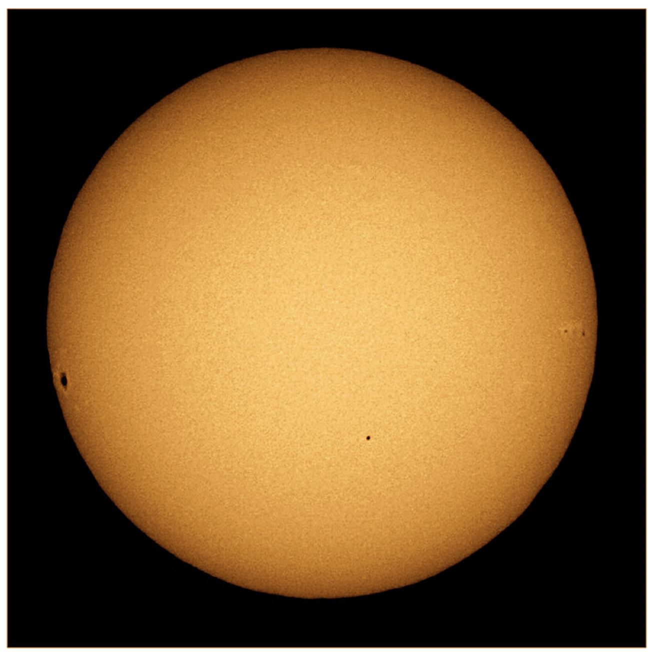
Known physics

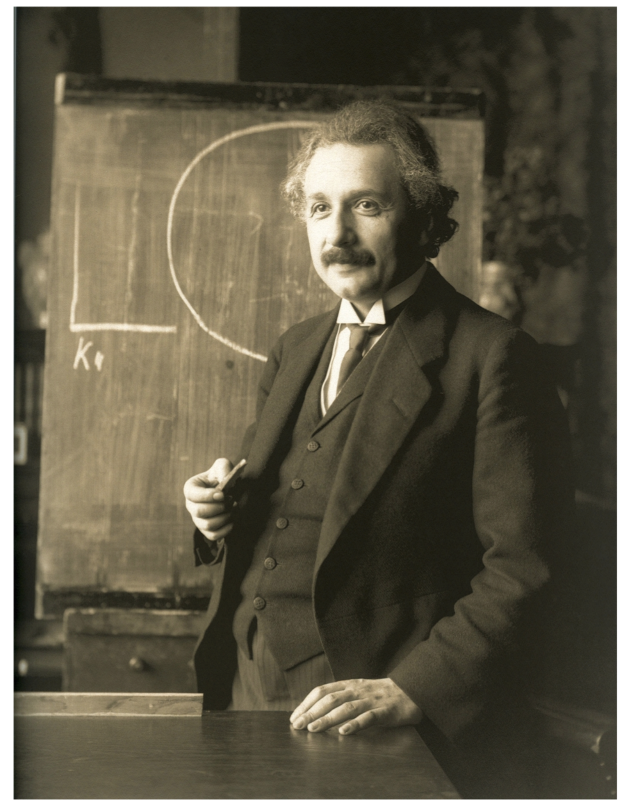
1915 by-then New physics
GR revolution

http://worldnpa.org/abstracts/abstracts_6066.pdf

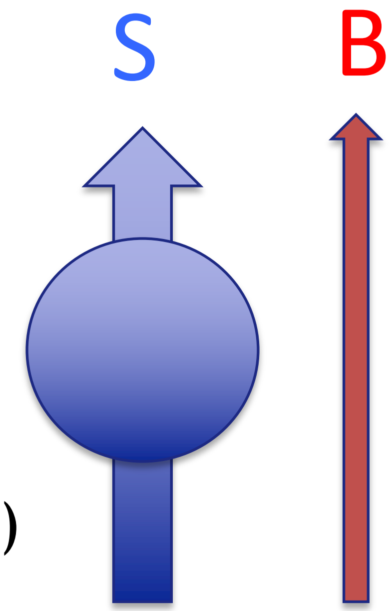
precession of perihelion







Anomalous magnetic moment

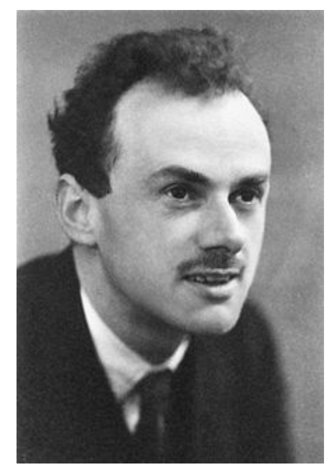


- Fermion's energy in the external magnetic field:

$$V(x) = -\vec{\mu}_l \cdot \vec{B} \qquad \vec{\mu}_l = g_l \frac{e}{2m_l} \vec{S}_l$$

- Magnetic moment Lande g-factor tree level value 2
- 1928 P.A.M. Dirac "Quantum Theory of Electron"
  Dirac equation (relativity, minimal gauge interaction)

$$i[\partial_\mu - ieA_\mu(x)]\gamma^\mu \psi(x) = m\psi(x)$$



- Non-relativistic and weak constant magnetic field
  limits of the Dirac equation :

$$-i\hbar \frac{\partial \psi}{\partial t} = \left[\frac{\nabla^2}{2m} + \frac{e}{2m} \left(\vec{L} + 2\vec{S} \right) \cdot \vec{B} \right] \psi$$

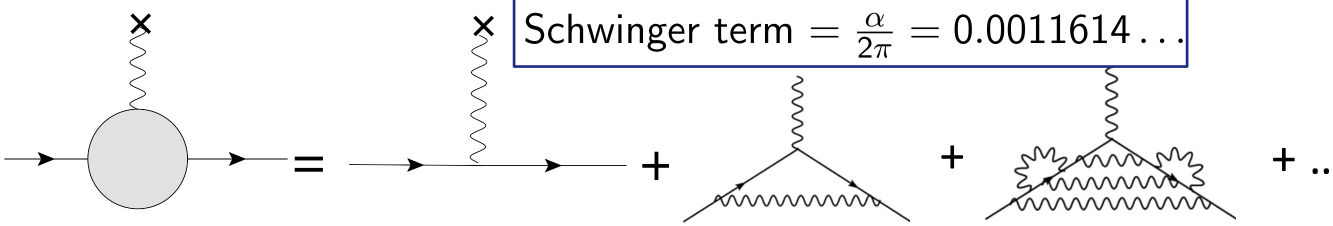
$$g_l = 2$$ (for Dirac Fermion l = e, μ, τ, ….)

SM Theory

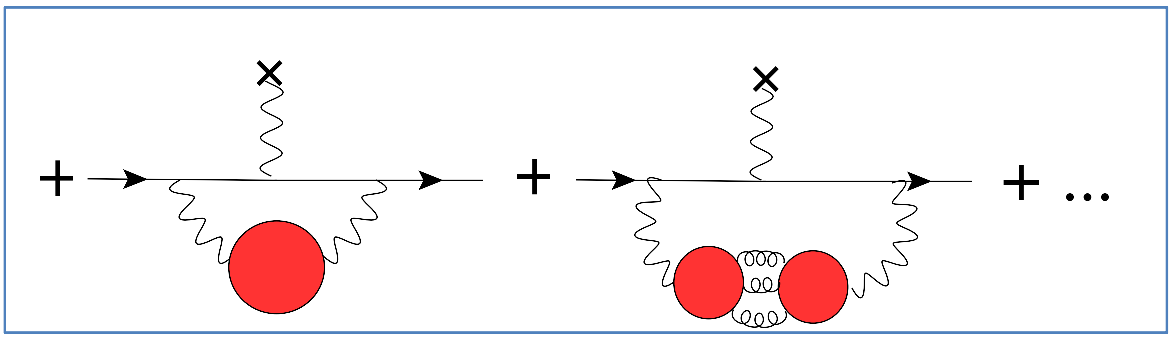
$$\gamma^\mu \rightarrow \Gamma^\mu(q) = \left(\gamma^\mu F_1(q^2) + \frac{i\,\sigma^{\mu\nu}\,q_\nu}{2m} F_2(q^2)\right)$$

- QED, hadronic, EW contributions

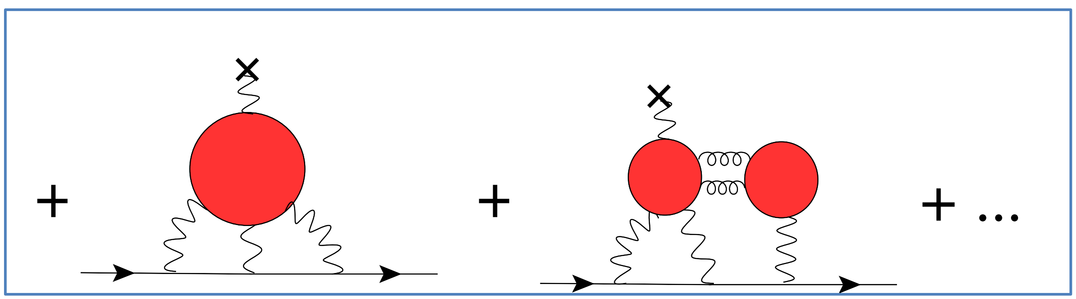
Schwinger term $= \frac{\alpha}{2\pi} = 0.0011614\ldots$



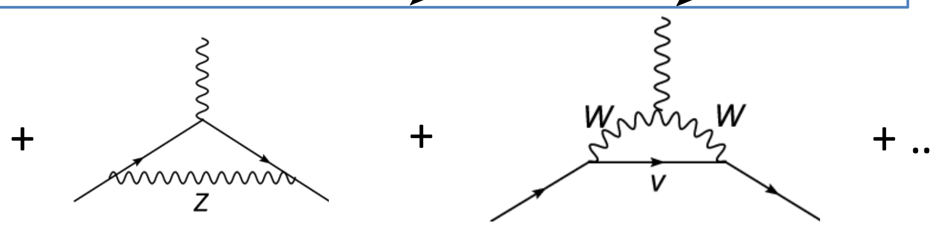
QED (5-loop)
Aoyama et al.
PRL109,111808 (2012)



Hadronic vacuum polarization (HVP)



Hadronic light-by-light (Hlbl)



Electroweak (EW)
Knecht et al 02
Czarnecki et al. 02