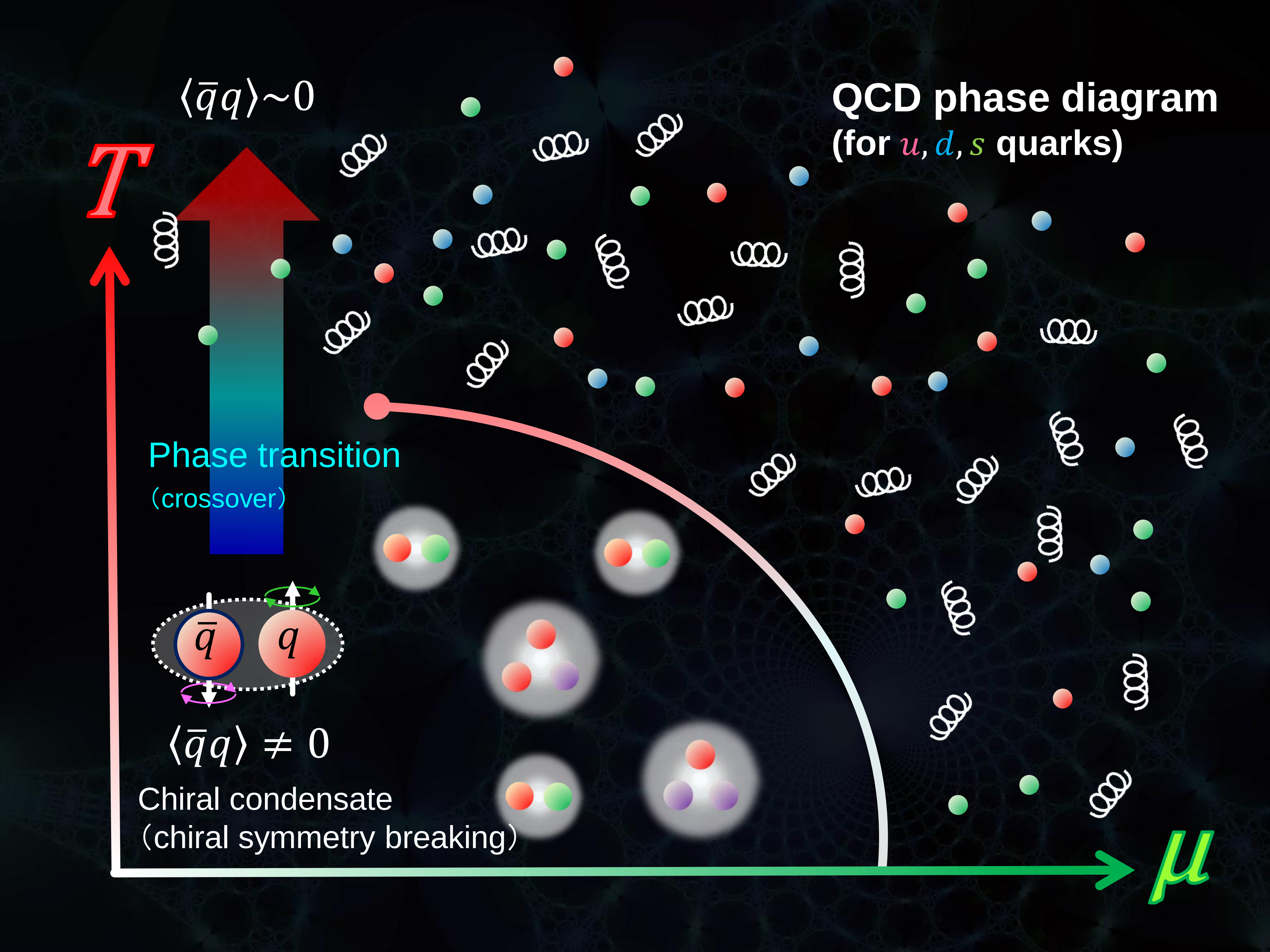
Frontiers in Lattice QCD and related topics

Axial U(1) symmetry in high temperature phase of two-flavor QCD

Kei Suzuki (JAEA)

from JLQCD Collaboration:
Sinya Aoki (YITP), Yasumichi Aoki (RIKEN R-CCS), Guido Cossu (Edinburgh), Hidenori Fukaya (Osaka U.), Shoji Hashimoto (KEK)

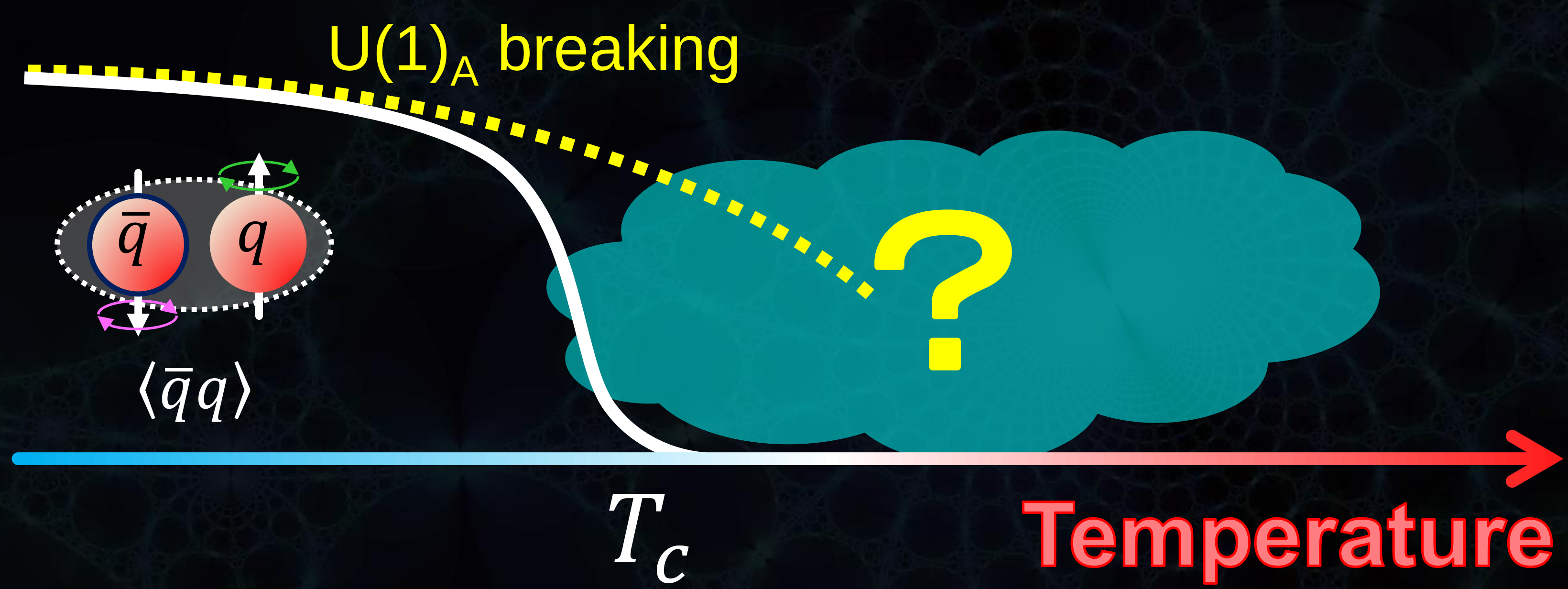


QCD phase diagram
(for u, d, s quarks)
$\langle \bar{q}q \rangle \sim 0$
T
Phase transition
（crossover）
$\bar{q}$ q
$\langle \bar{q}q \rangle \neq 0$
Chiral condensate
（chiral symmetry breaking）
μ

$U(1)_A$ symmetry (in vacuum, broken by anomaly) is restored above Tc？

- Above Tc, chiral symmetry breaking by $\langle \bar{q}q \rangle$ is restored
  ⇒How about $U(1)_A$ symmetry?

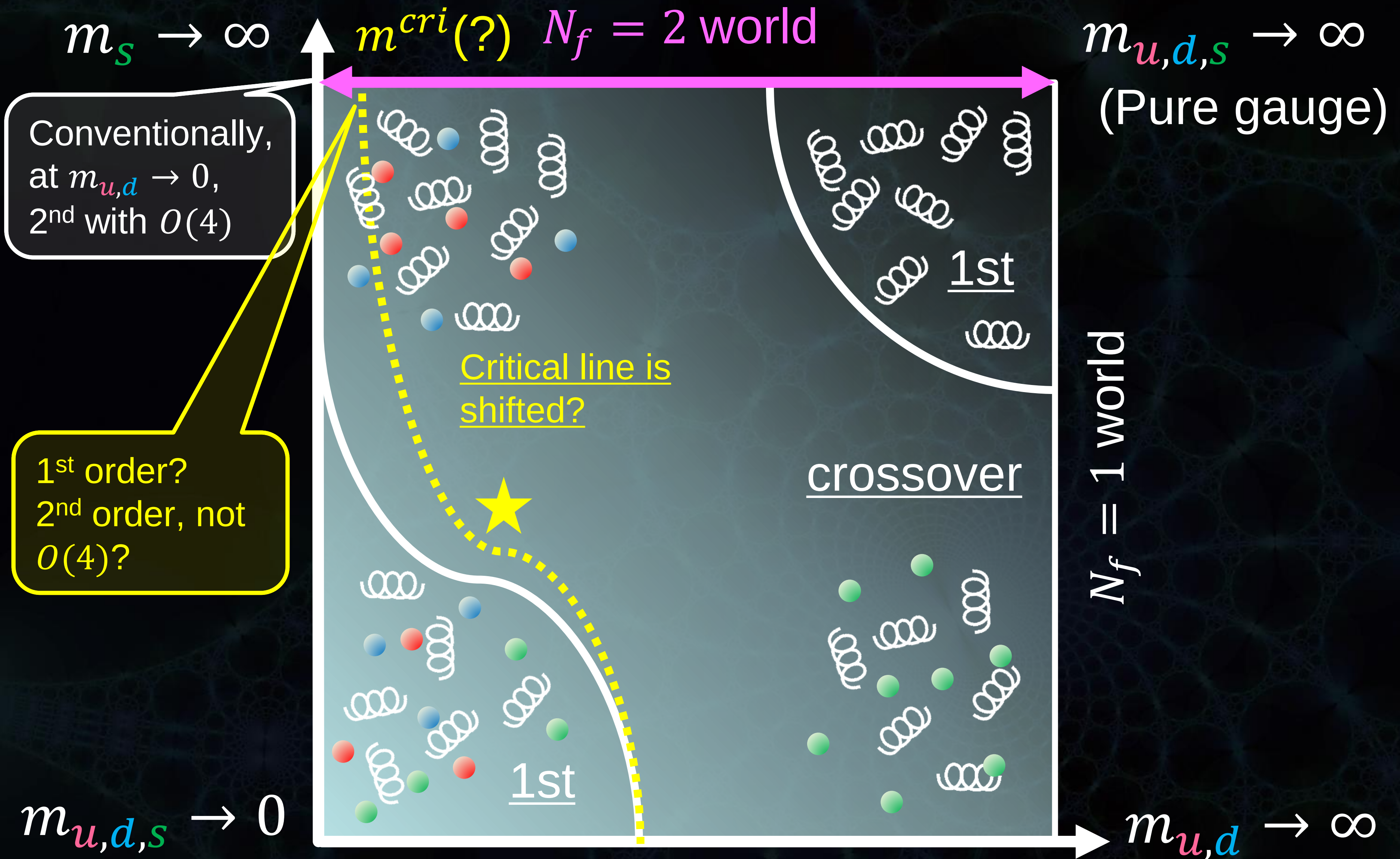
$$\Delta_{\pi-\delta} = \int_0^\infty d^4x\,[\pi^a(x)\pi^a(x) - \delta^a(x)\delta^a(x)]$$



Cf.) S. Aoki, H. Fukaya, and Y. Taniguchi, PRD86

If $U(1)_A$ is restored…

Colombia plot is modified?



U(1)$_A$ symmetry above Tc ⇒Long-standing problem in QCD

- Gross-Pisarski-Yaffe (Dilute instanton gas model, 1981) restored at enough high T
- Cohen (1996) w/o zero mode (or instanton)⇒restored
- Aoki-Fukaya-Taniguchi (theory, 2012) zero mode suppressed, restored in chiral limit at $N_f = 2$
- HotQCD (DW, 2012) broken
- JLQCD (topology fixed overlap, 2013) restored
- TWQCD (optimal DW, 2013) restored
- LLNL/RBC (DW, 2014) broken (restored at higher T?)
- Dick et al. (overlap on HISQ, 2015) broken
- Sharma et al. (overlap on DW, 2015,2016,2018) broken
- Brandt et al. (Wilson, 2016) restored
- Ishikawa et al. (Wilson, 2017) restored
- JLQCD (reweighted overlap on DW, 2016) restored
- Gomez Nicola-Ruiz de Elvira (theory, 2017) restored
- Rohrhofer et al. (DW, 2017) restored

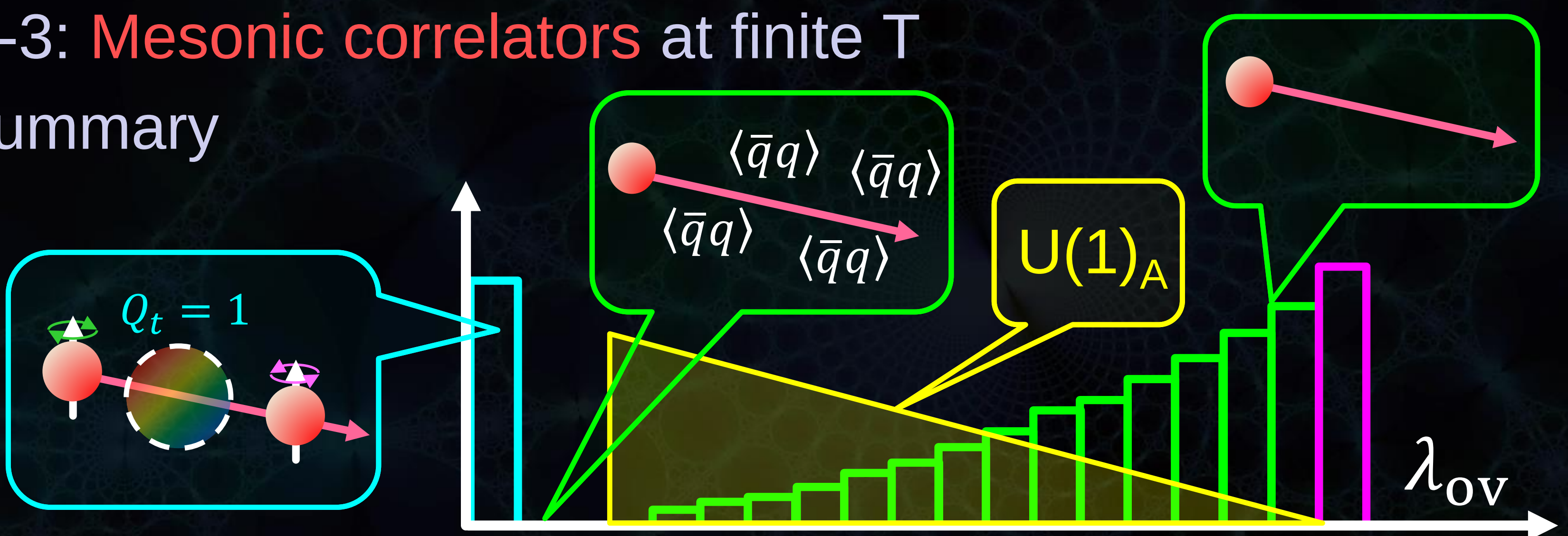
⇒ Many suggestions from lattice QCD (and models)…

$U(1)_A$ symmetry restoration by JLQCD Collaboration
⇒overlap fermion (exact chiral symmetry on the lattice)

	valence/sea quark	Setup
G. Cossu et al. PRD87 (2013)	OV on OV (Topology fixed sector)	
A. Tomiya et al. PRD96 (2017)	DW on DW OV on DW OV on (reweighted) OV	1/a=1.7GeV (a=0.11fm)
In progress	OV on DW OV on (reweighted) OV	1/a=2.6GeV (a=0.076fm) (Finer lattice)

Outline

1. Introduction
2. $U(1)_A$ and topology from Dirac spectra
3. Results
   - 3-1: $U(1)_A$ susceptibility at finite T
   - 3-2: Topological susceptibility at finite T
   - 3-3: Mesonic correlators at finite T
4. Summary

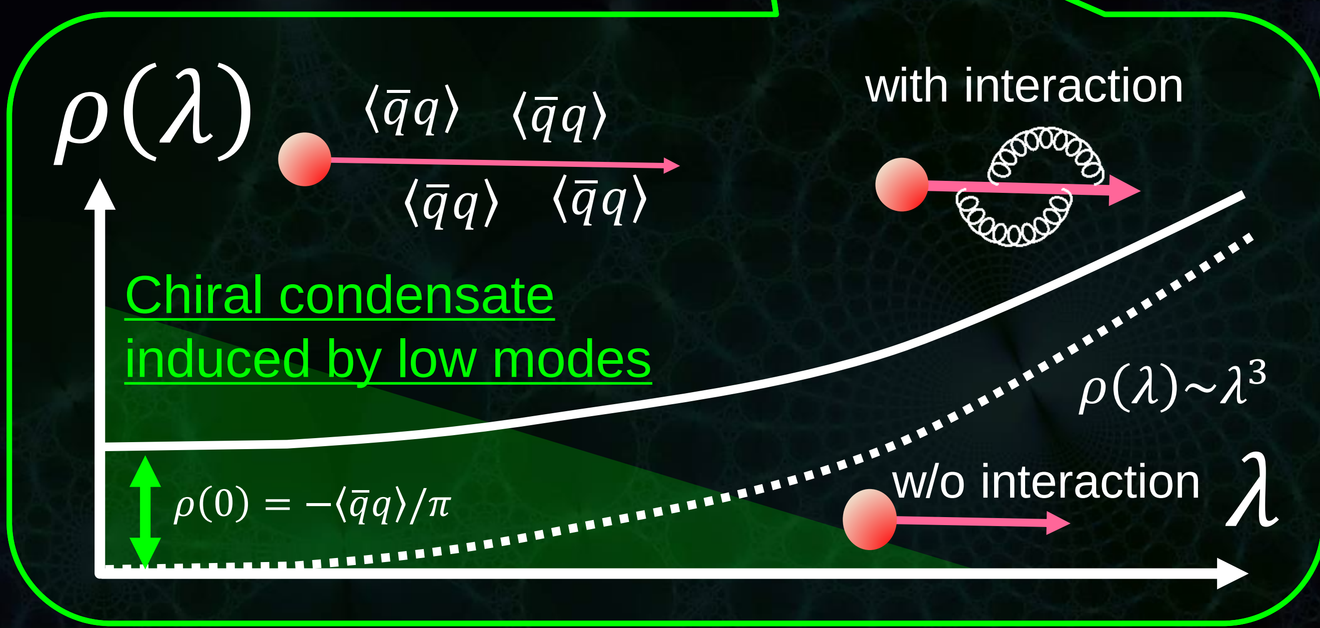


Chiral condensate and Dirac spectra

Banks-Casher relation:

$$\langle \bar{q}q \rangle = \lim_{m\to 0} \int_0^\infty d\lambda\, \rho(\lambda)\, \frac{2m}{\lambda^2 + m^2}$$

$$\rho(\lambda) \equiv \lim_{V\to\infty} \frac{1}{V} \Sigma_{\lambda'} < \delta(\lambda - \lambda') >$$



G. Cossu et al. (JLQCD) PRD87 (2013), 114514

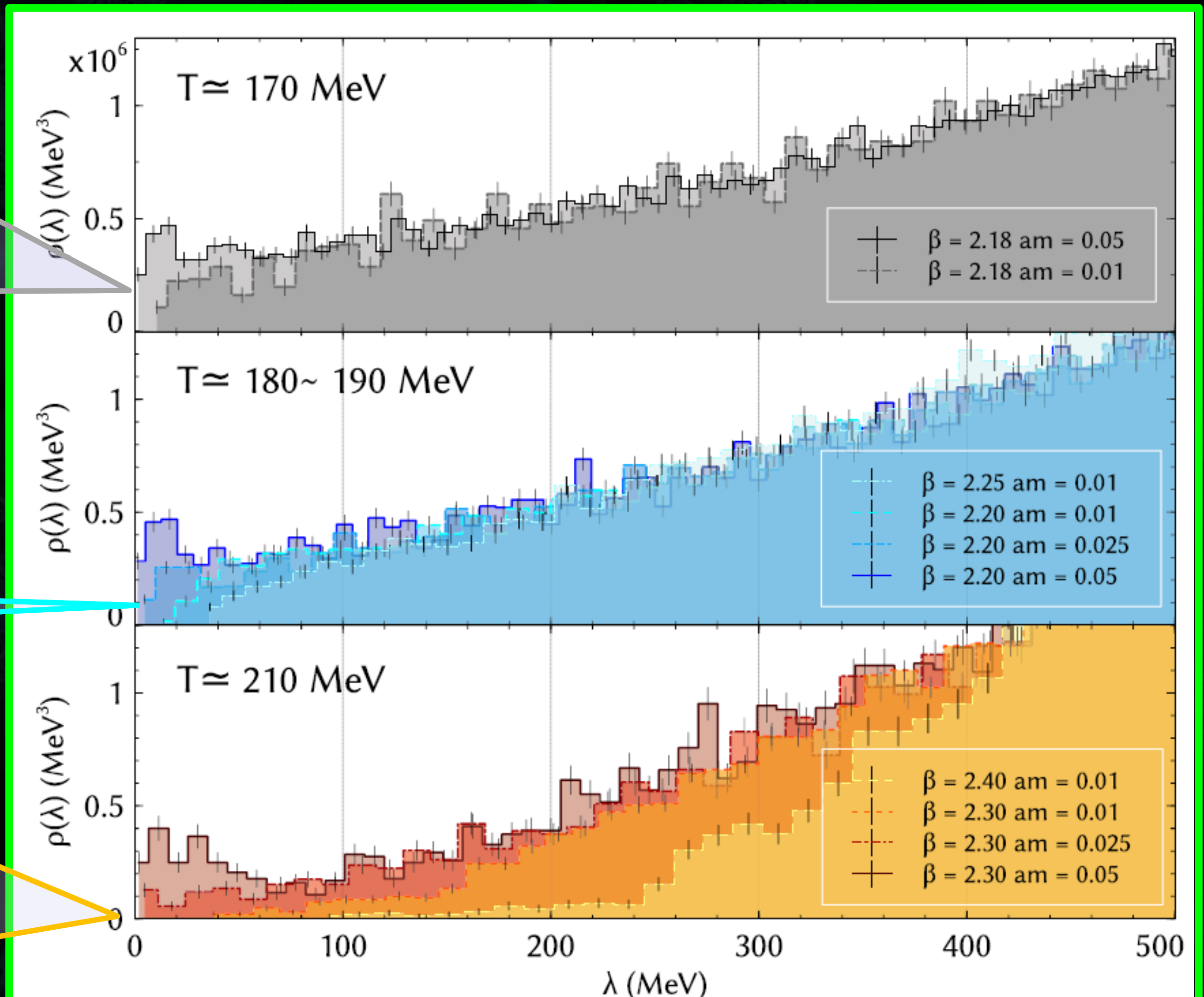
T-dependence of Dirac spectra

Low T :
ρ(0)≠0
⇒Spontaneous chiral symmetry breaking

$\langle \bar{q}q \rangle$ $\langle \bar{q}q \rangle$ $\langle \bar{q}q \rangle$ $\langle \bar{q}q \rangle$

Critical Temp.

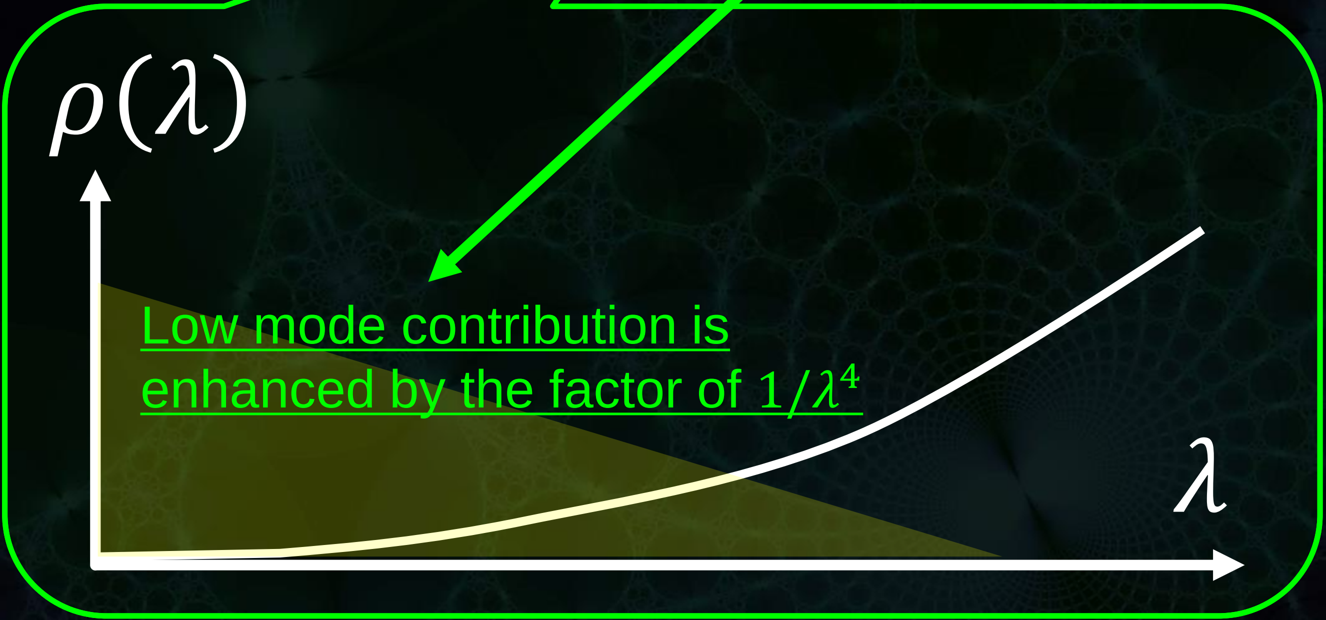
High T :
ρ(0)=0
⇒Chiral symmetry restoration



Low energy → High energy

U(1)$_A$ susceptibility and low modes of Dirac spectra

$$\Delta_{\pi-\delta} = \int_0^\infty d\lambda\, \rho(\lambda)\, \frac{2m^2}{(\lambda^2+m^2)^2}$$



Cf.) Banks-Casher relation: $\langle \bar{q}q \rangle = \lim_{m\to 0} \int_0^\infty d\lambda\, \rho(\lambda)\, \frac{2m}{\lambda^2+m^2}$

S. Aoki, H. Fukaya, and Y. Taniguchi PRD86 (2012), 114512
A. Tomiya et al. (JLQCD) PRD96 (2017), 034509

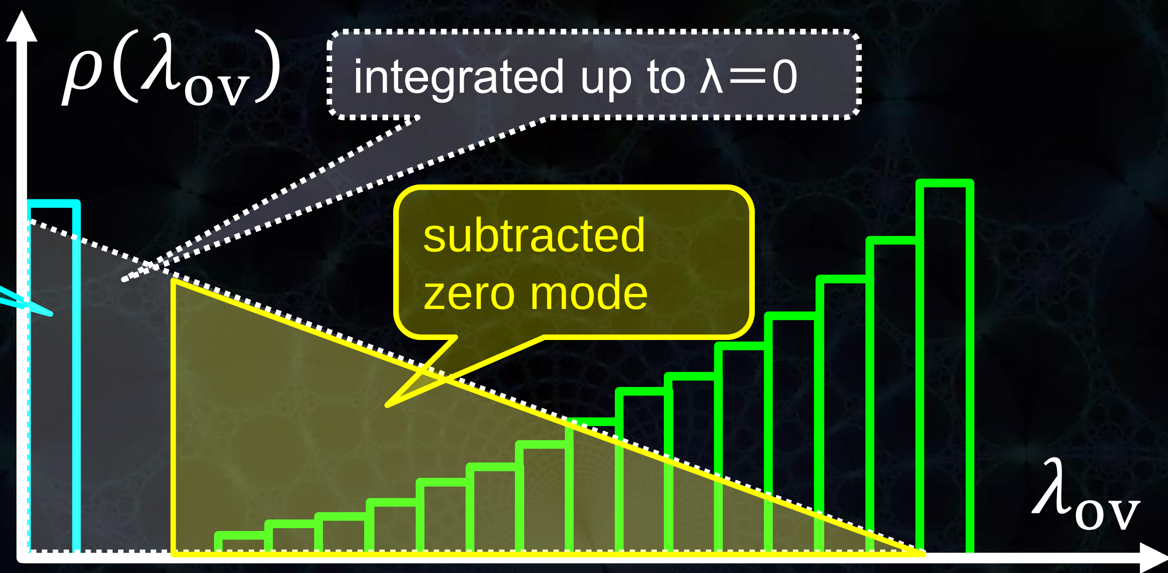
Note:

$U(1)_A$ susc. = Low modes + ~~Zero~~ mode ?

$$\Delta_{\pi-\delta} = \int_0^\infty d\lambda\, \rho(\lambda) \frac{2m^2}{(\lambda^2+m^2)^2}$$

$$\Delta^{\mathrm{ov}}_{\pi-\delta} \equiv \frac{1}{V(1-m^2)^2} \sum_i \frac{2m^2(1-\lambda^{(i)2}_{\mathrm{ov}})^2}{\lambda^{(i)4}_{\mathrm{ov}}}$$

The factor of $1/\lambda^4$ enhances zero-mode contribution?



In $V \to \infty$ limit, we know zero-mode contribution is suppressed:

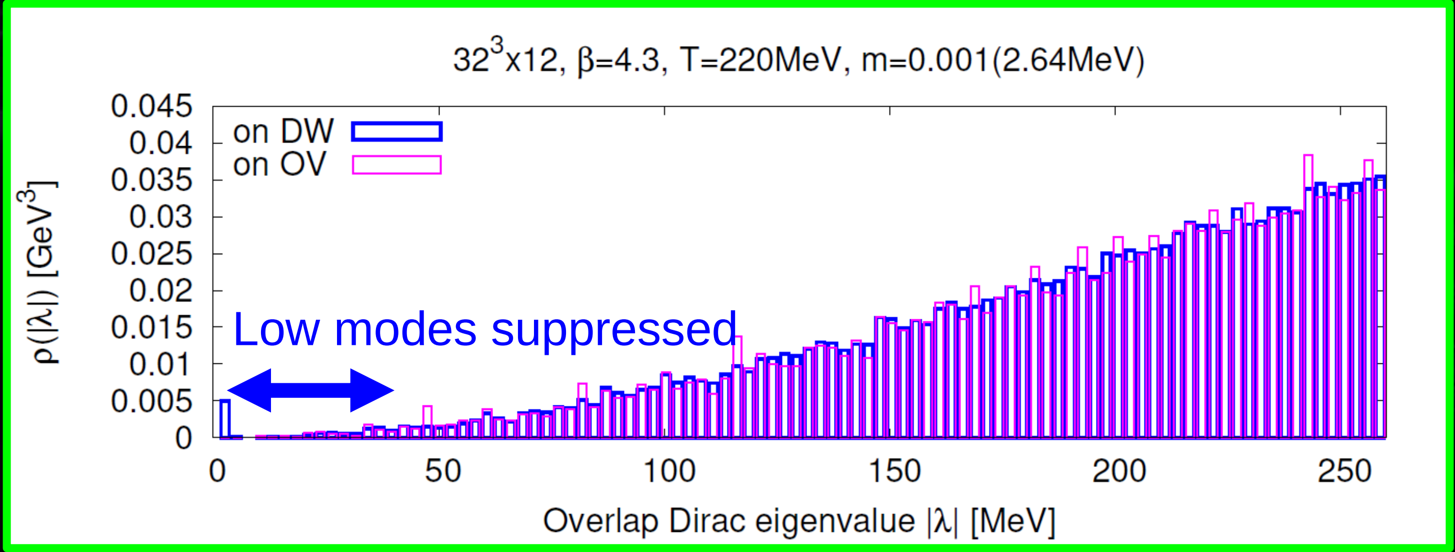
$$\Delta^{\mathrm{ov}}_{0-mode} = \frac{2N_0}{Vm^2} (\propto 1/\sqrt{V})$$

New order parameter:
we subtract zero mode

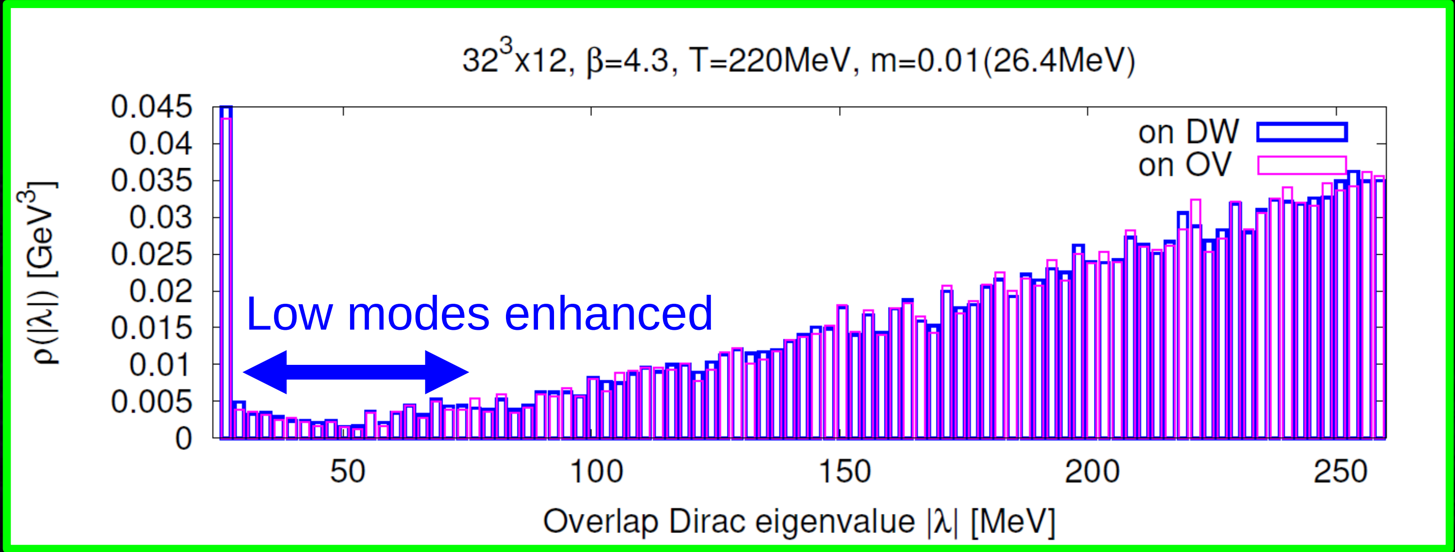
$$\bar{\Delta}^{\mathrm{ov}}_{\pi-\delta} \equiv \Delta^{\mathrm{ov}}_{\pi-\delta} - \frac{2N_0}{Vm^2}$$

Overlap Dirac spectra at T = 220MeV

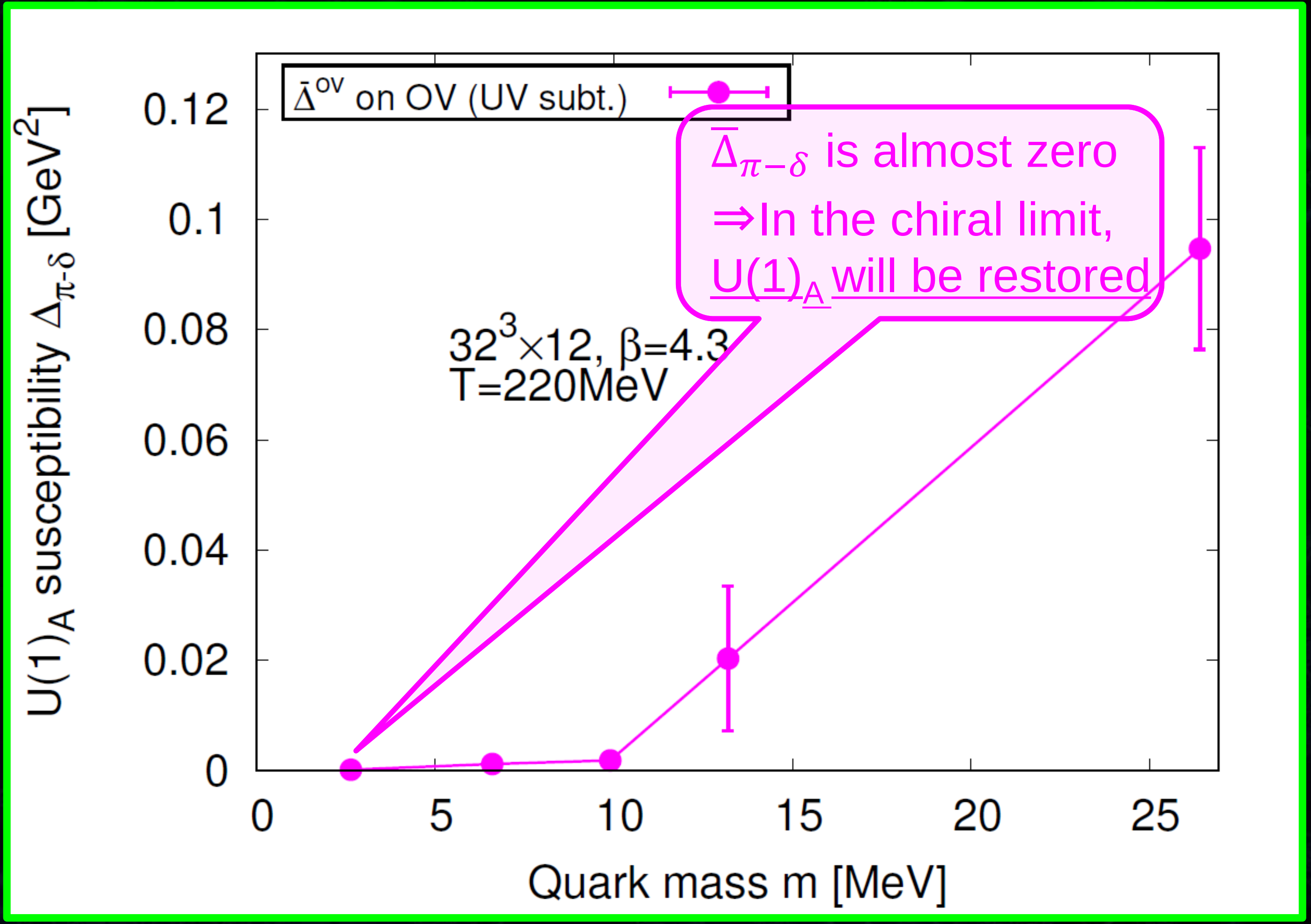
m_q=2.6MeV



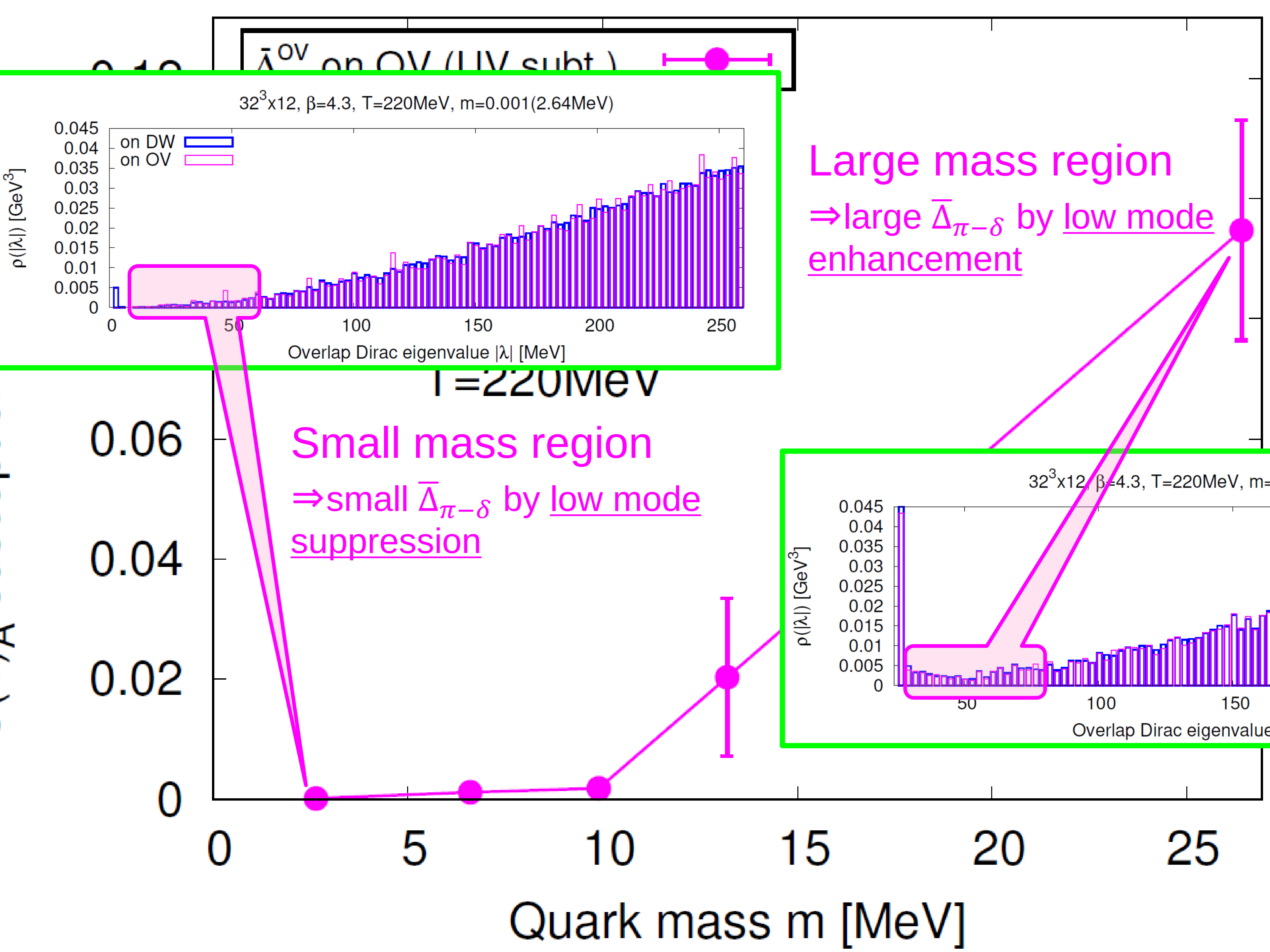
m_q=26MeV



U(1)$_A$ susceptibility at T = 220MeV

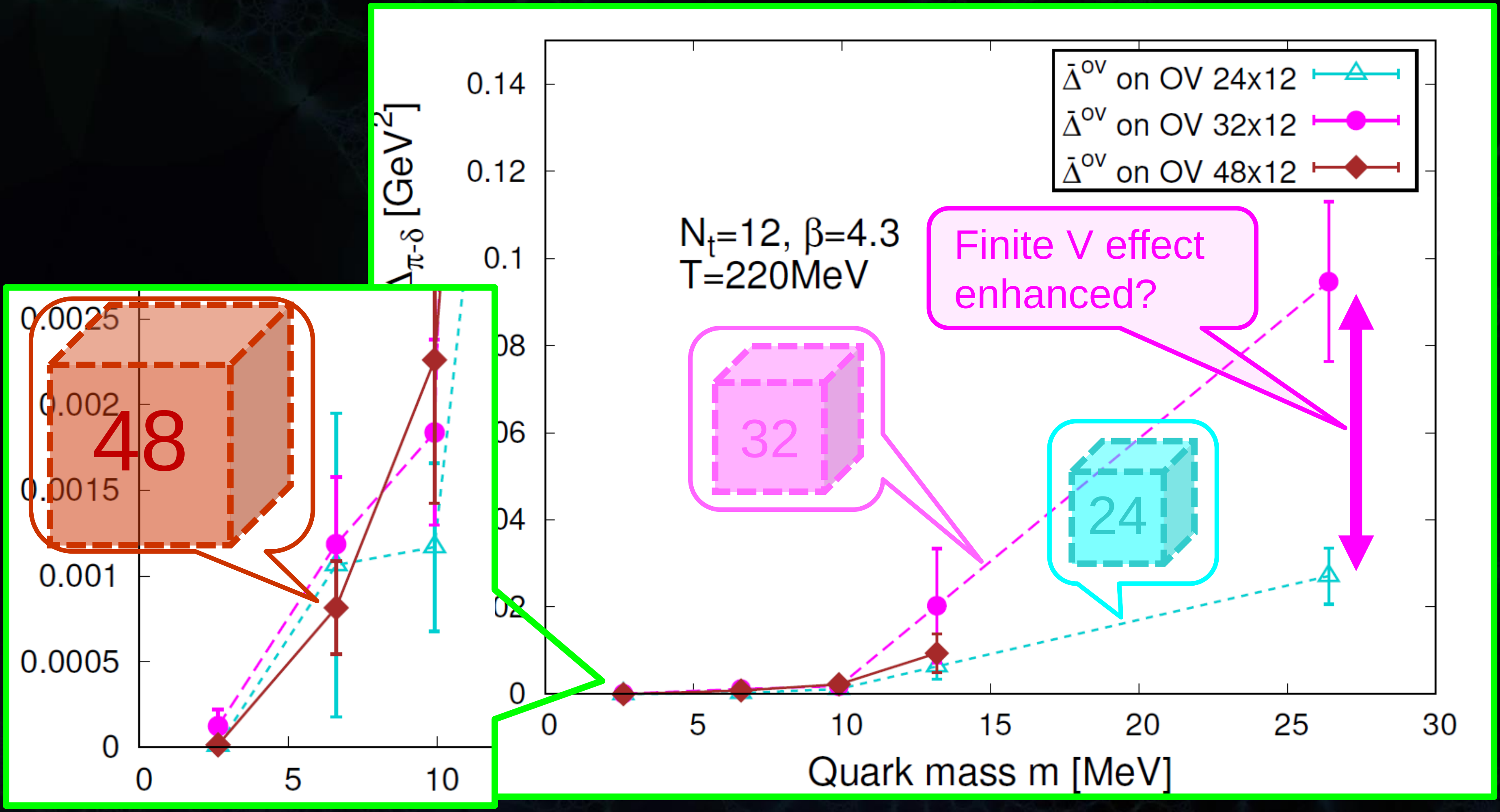


⇒At m_q =2.6MeV, we found suppression of 10^{-4}GeV^2



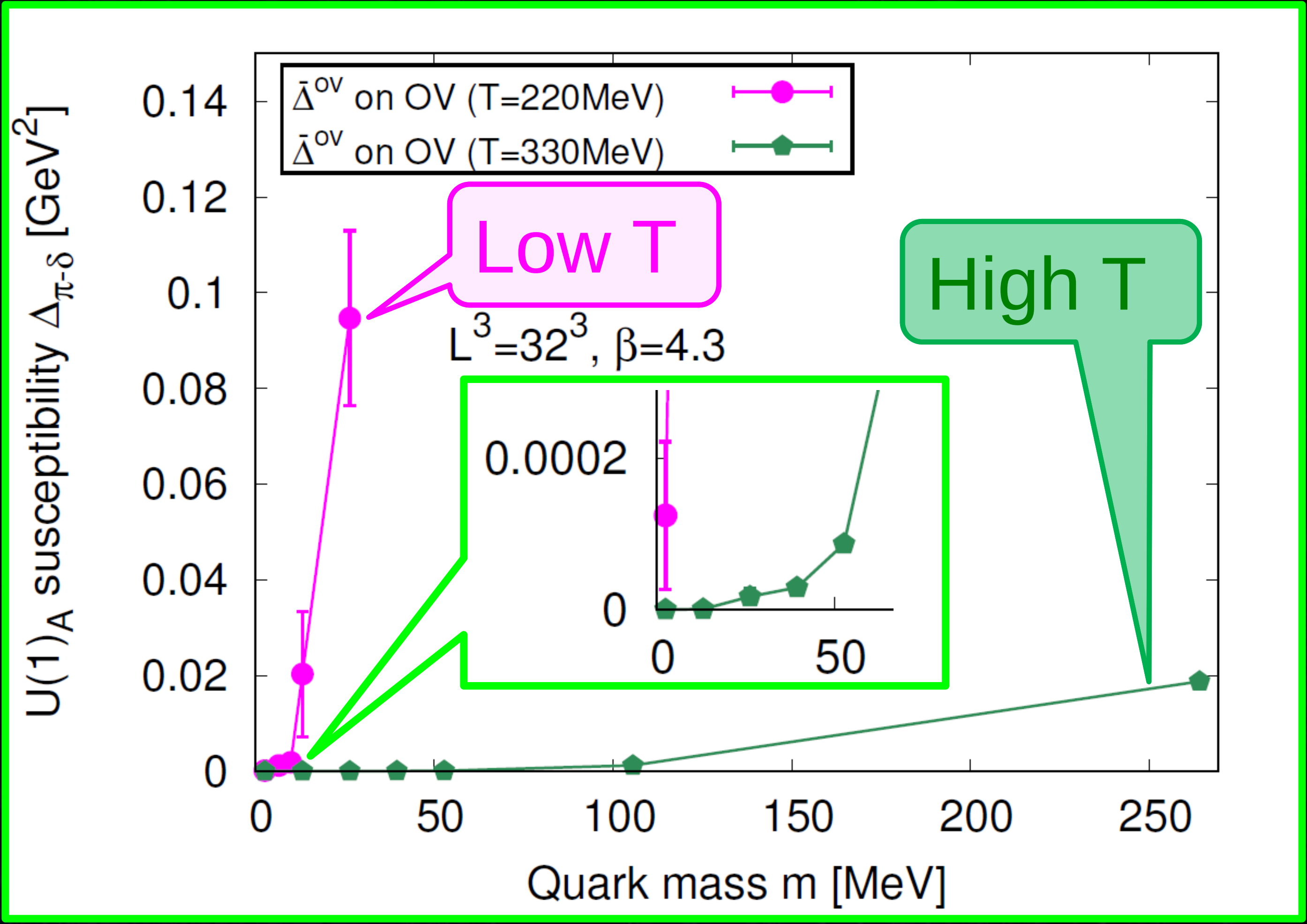
on OV (UV subt.)
T=220MeV
32^3x12, β=4.3, T=220MeV, m=0.001(2.64MeV)
on DW
on OV
ρ(|λ|) [GeV^3]
Overlap Dirac eigenvalue |λ| [MeV]
Large mass region
⇒large $\bar{\Delta}_{\pi-\delta}$ by low mode enhancement
Small mass region
⇒small $\bar{\Delta}_{\pi-\delta}$ by low mode suppression
0.06
0.04
0.02
0
0 5 10 15 20 25
Quark mass m [MeV]

$U(1)_A$ susceptibility (Volume dependence)



⇒For small m_q, V-dependence seems to be small

$U(1)_A$ susceptibility (T=220, 330MeV)



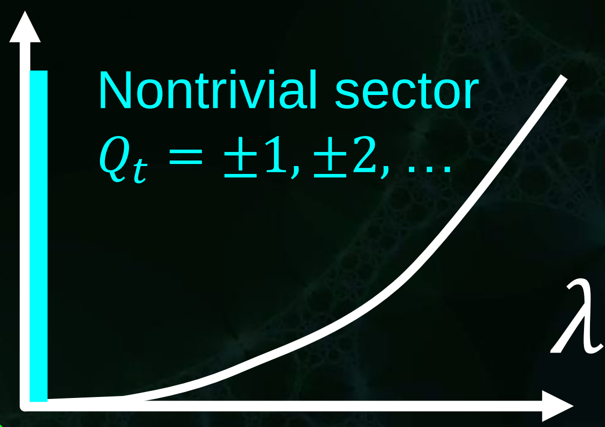
⇒ With increasing T, $\underline{U(1)_A \text{ is more resotored}}$

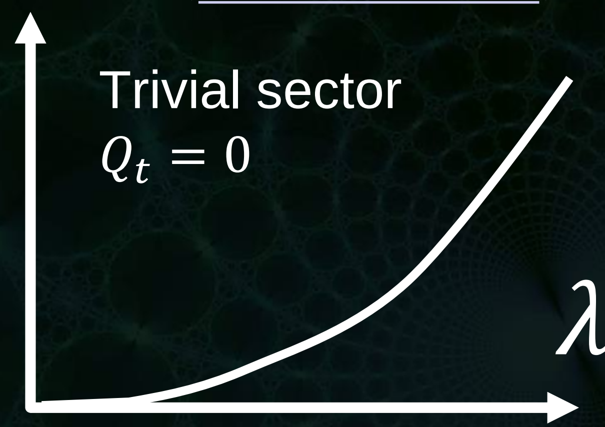
Topological susceptibility and zero mode of Dirac spectra

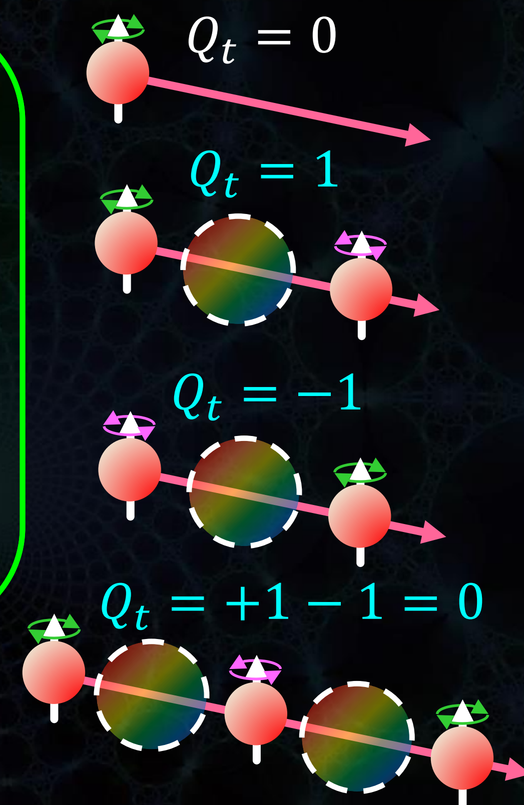
$$\chi_t \equiv \frac{\langle Q_t^2 \rangle}{V}, \qquad Q_t = n_+ - n_-$$

Topological charge Q_t is related to #of Dirac zero mode（Index theorem）

$\rho(\lambda)$

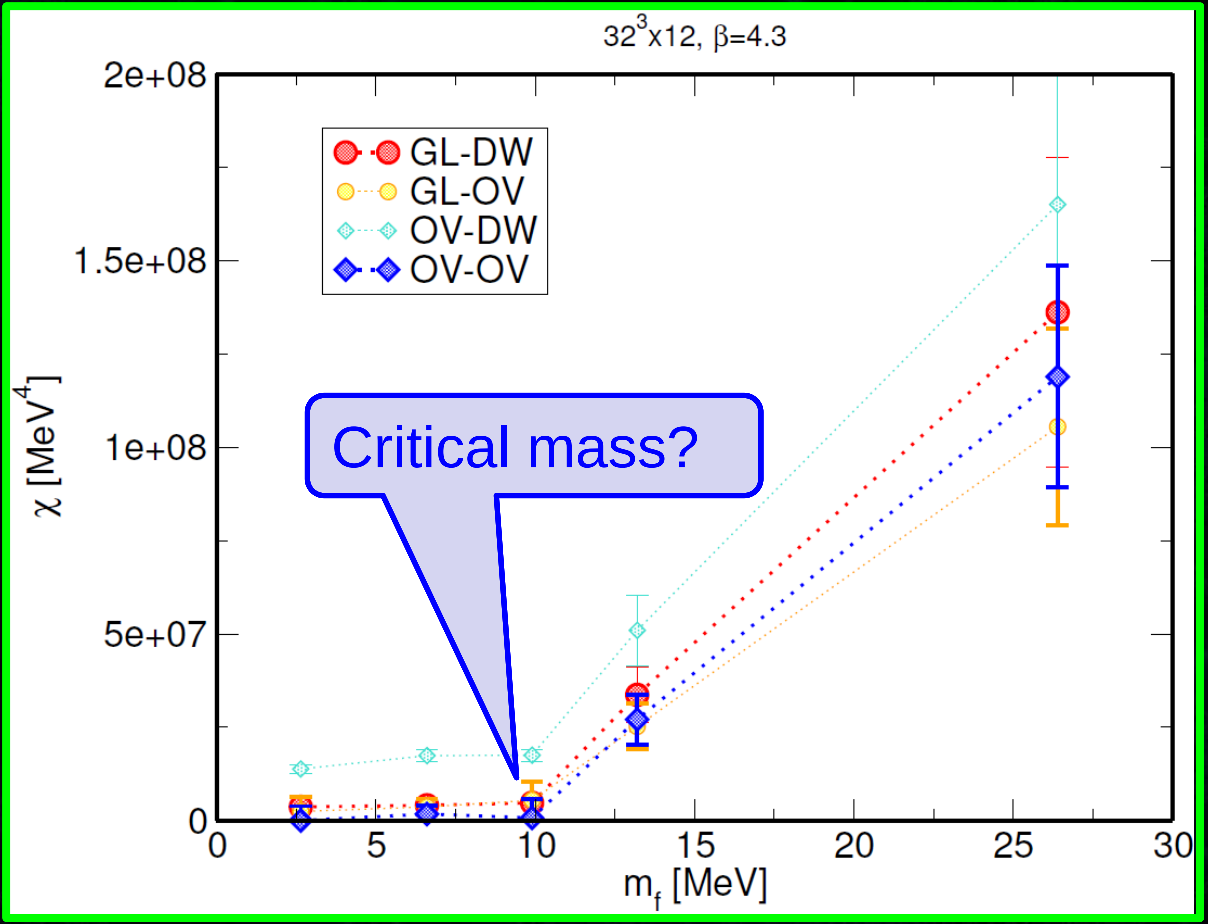






Cf.) Gluonic definition: $Q_t \equiv \frac{g^2}{32\pi^2} \int d^4x\, G^a_{\mu\nu} \tilde{G}^a_{\mu\nu}$

Top. susceptibility at T = 220MeV

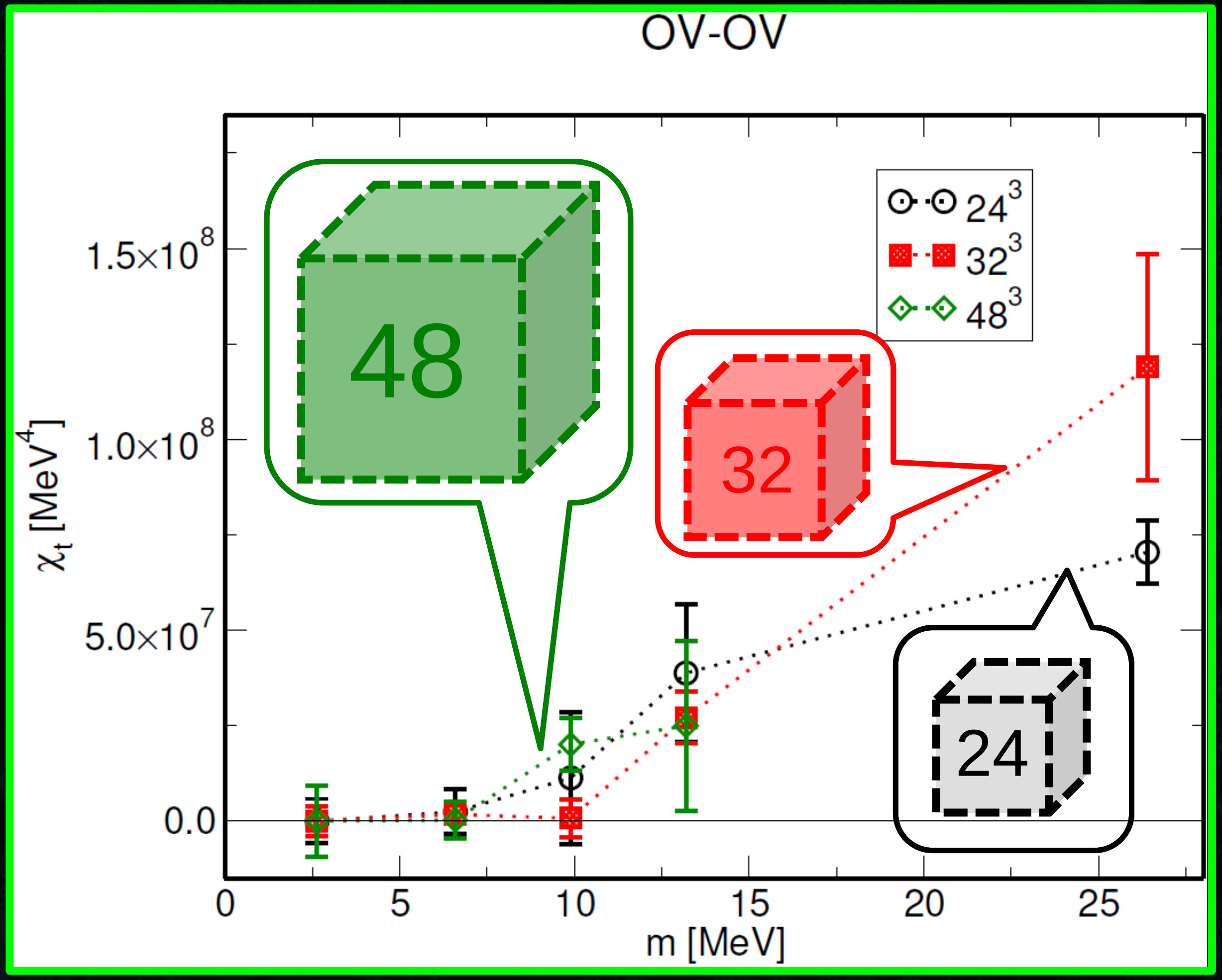


⇒For small m_q, $\chi_t = 0$

⇒Around m_q~10MeV, we found a jump (critical mass?)

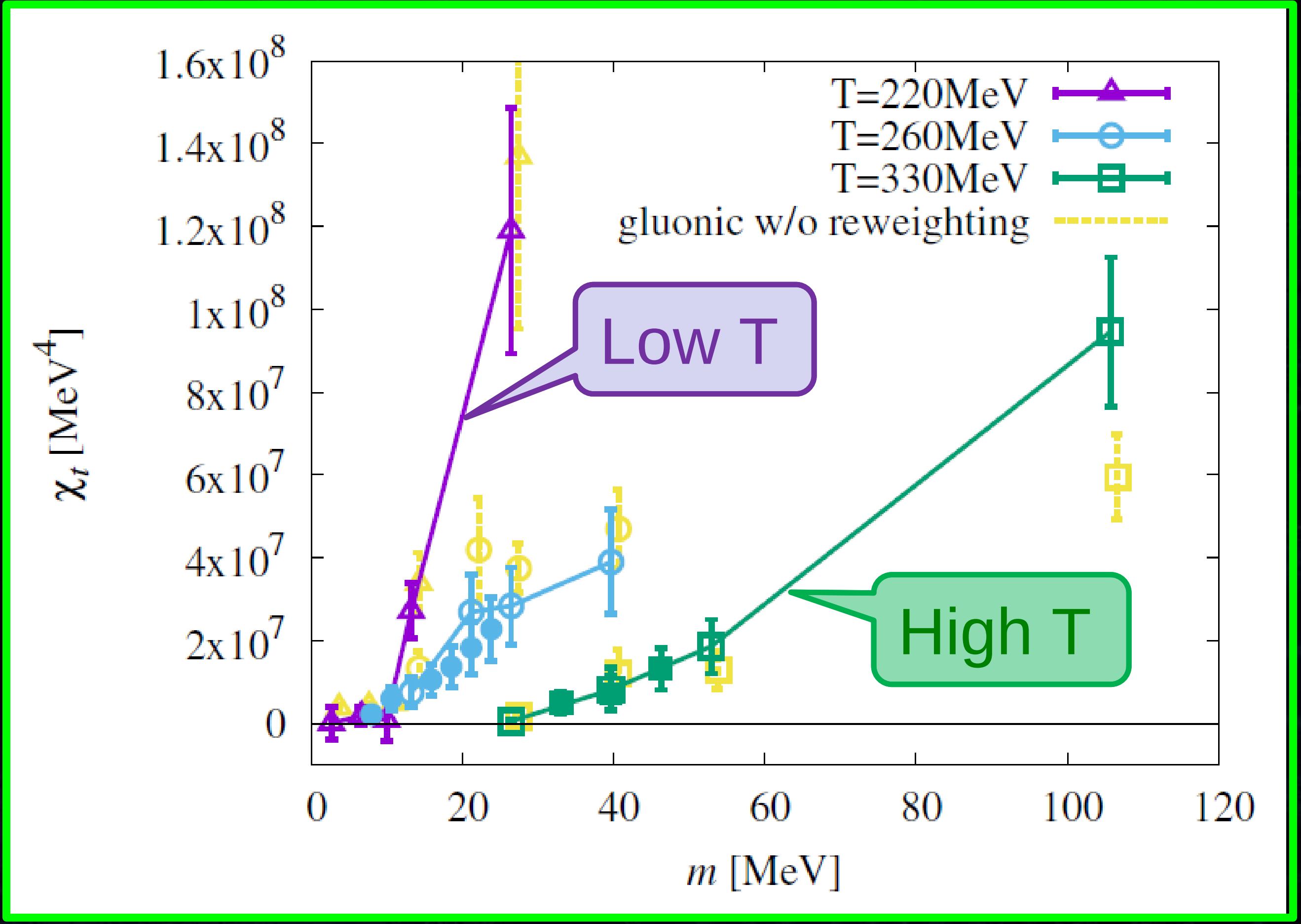
Cf.) S. Aoki, H. Fukaya, and Y. Taniguchi PRD86 (2012), 114512

Top. susceptibility (Volume depend.)



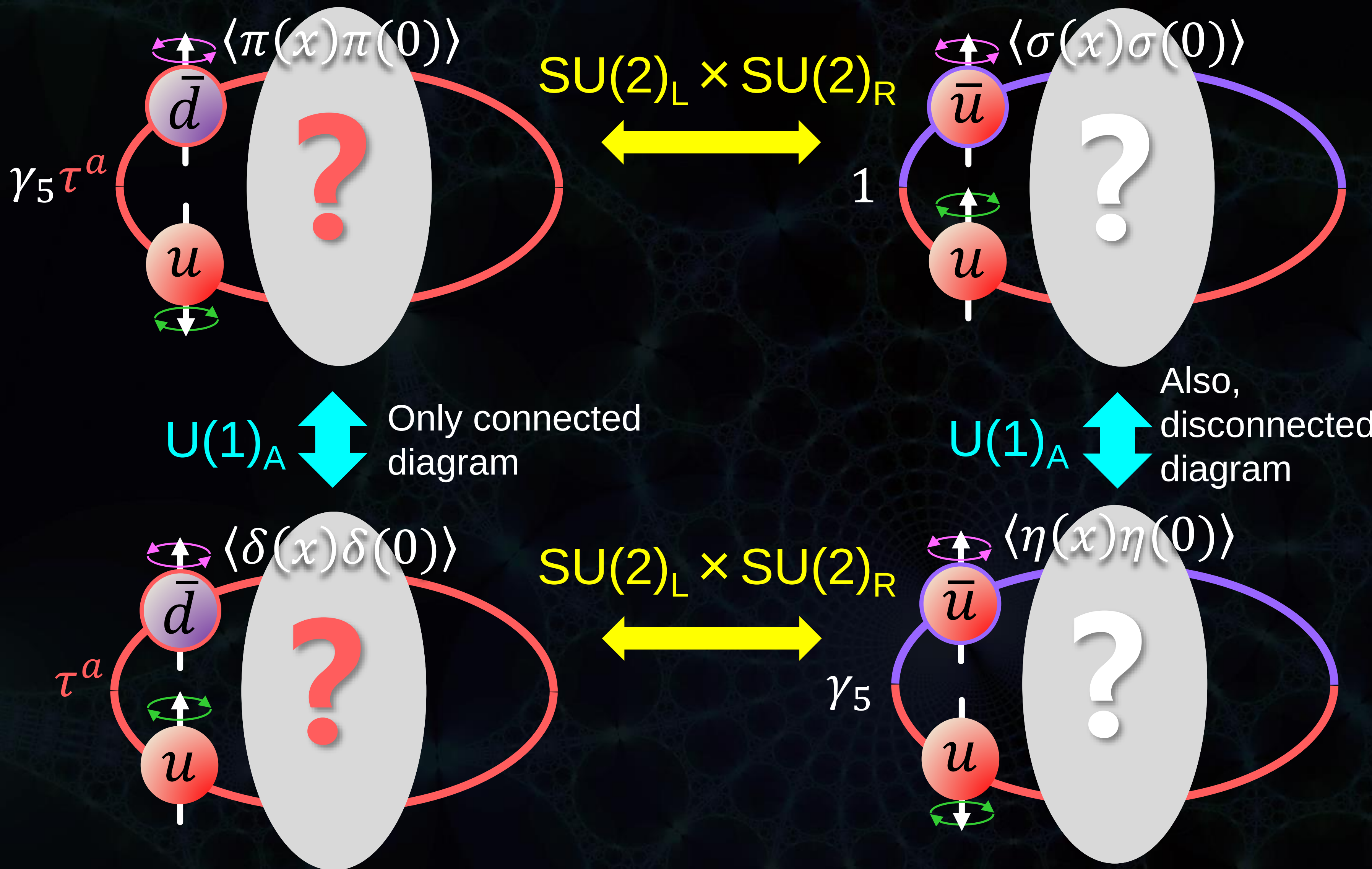
⇒For small m_q, no volume dependence

Top. susceptibility (T=220, 260, 330MeV)

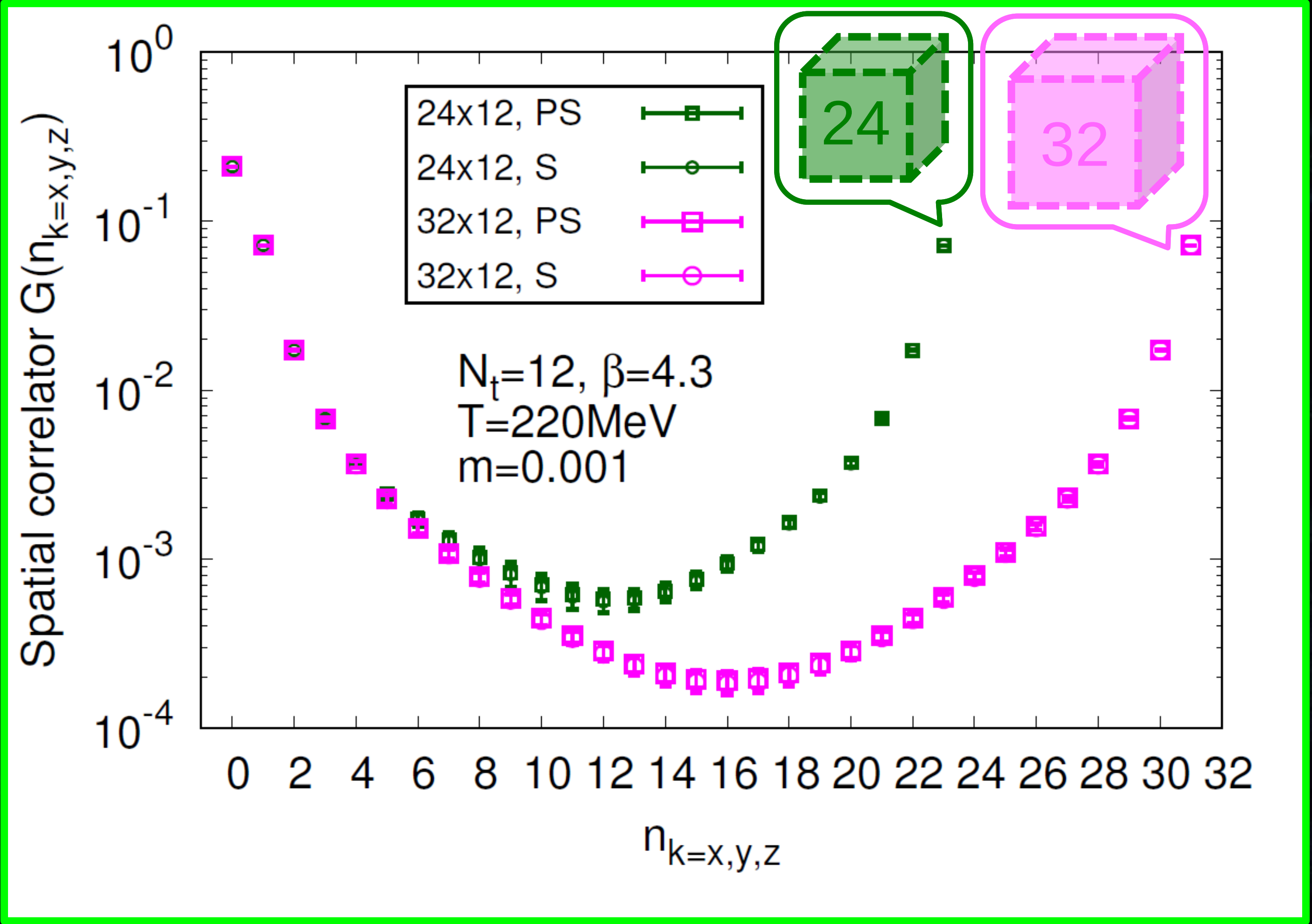


⇒With increasing T, χ_t is more suppressed

Mesonic correlators (PS for $N_f = 2$)

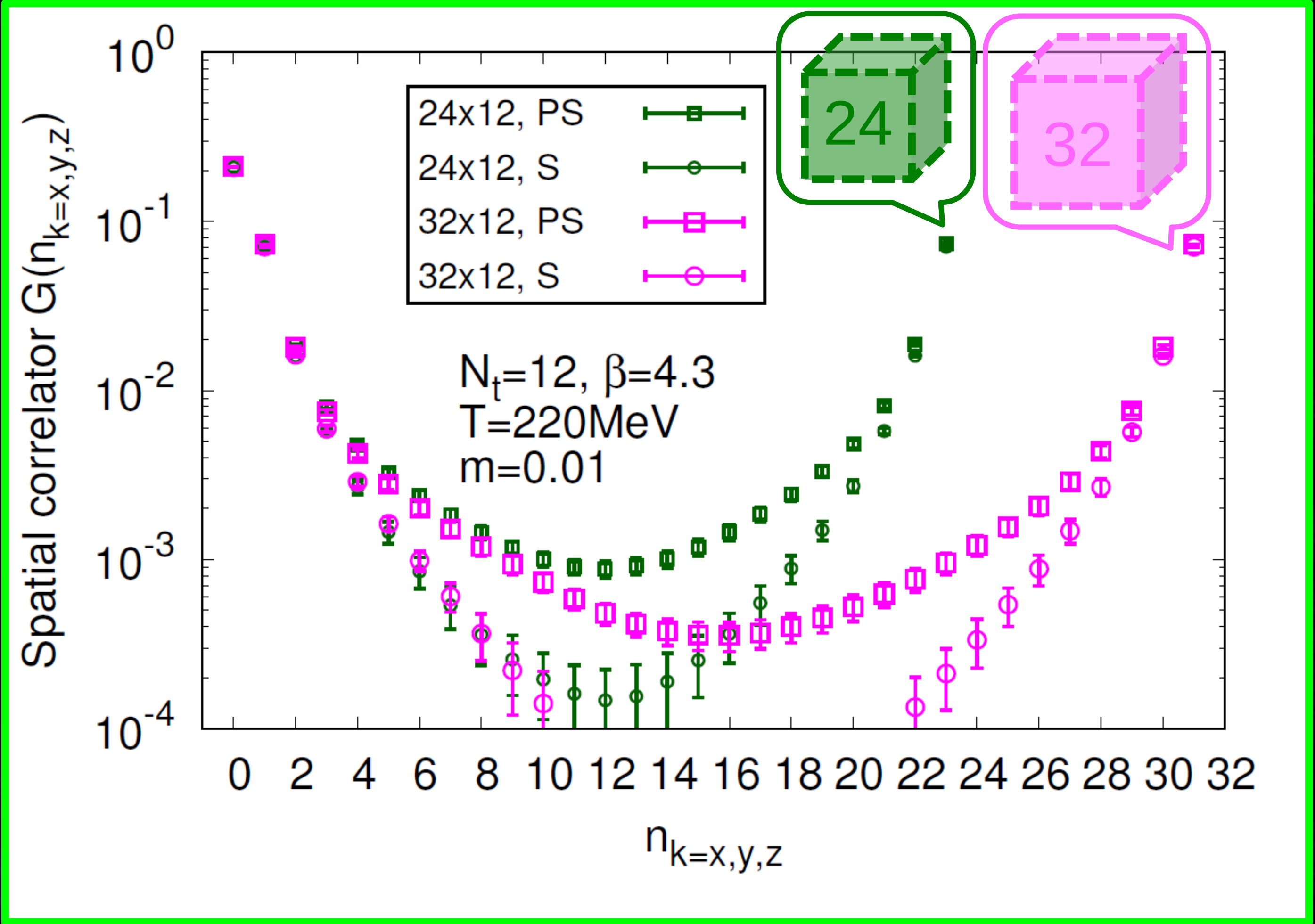


PS-S(Connected) Correlators: $U(1)_A$ partners



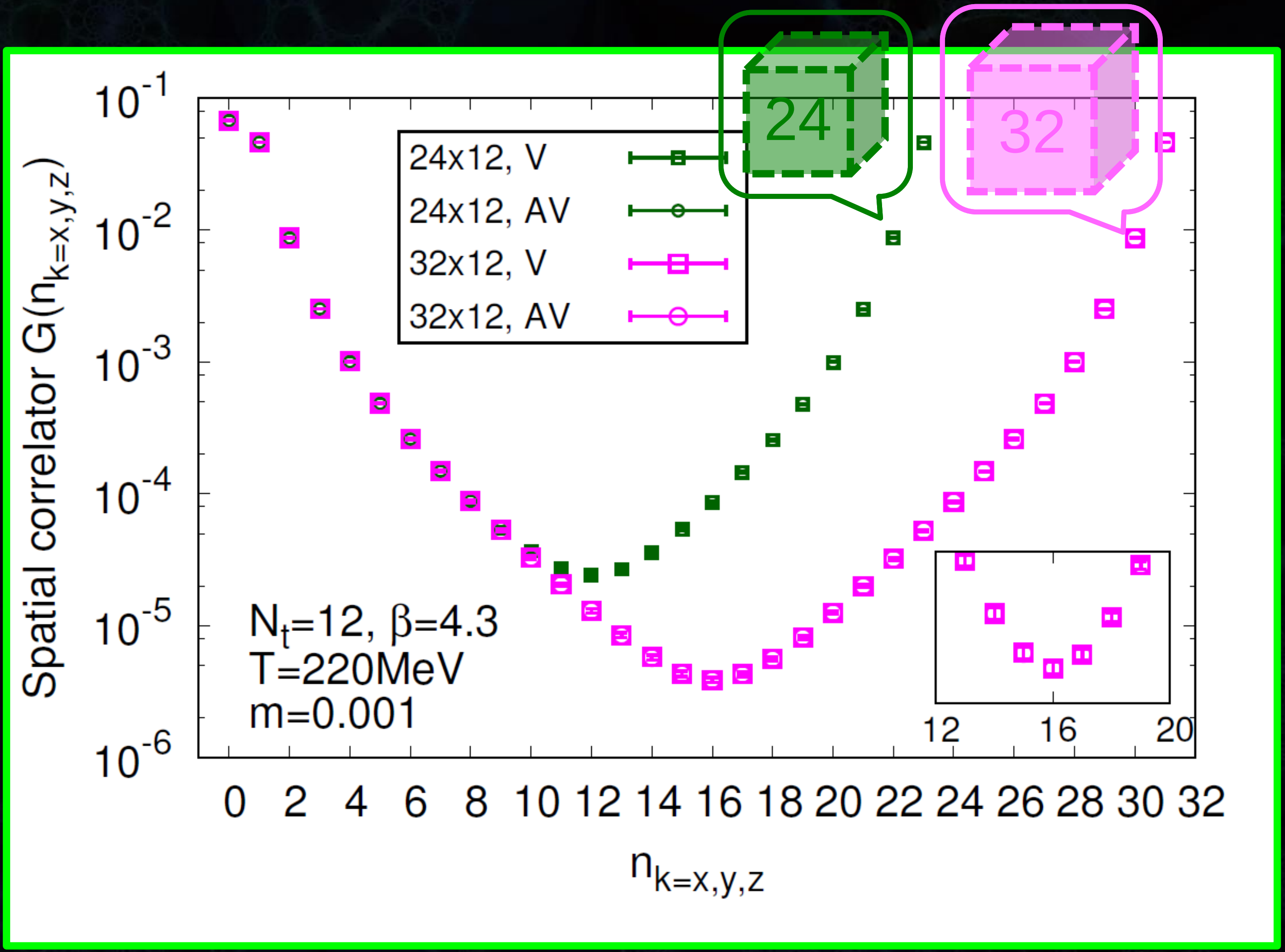
⇒Small m_q : $U(1)_A$ restoration, Large m_q : $U(1)_A$ breaking

PS-S(Connected) Correlators: $U(1)_A$ partners



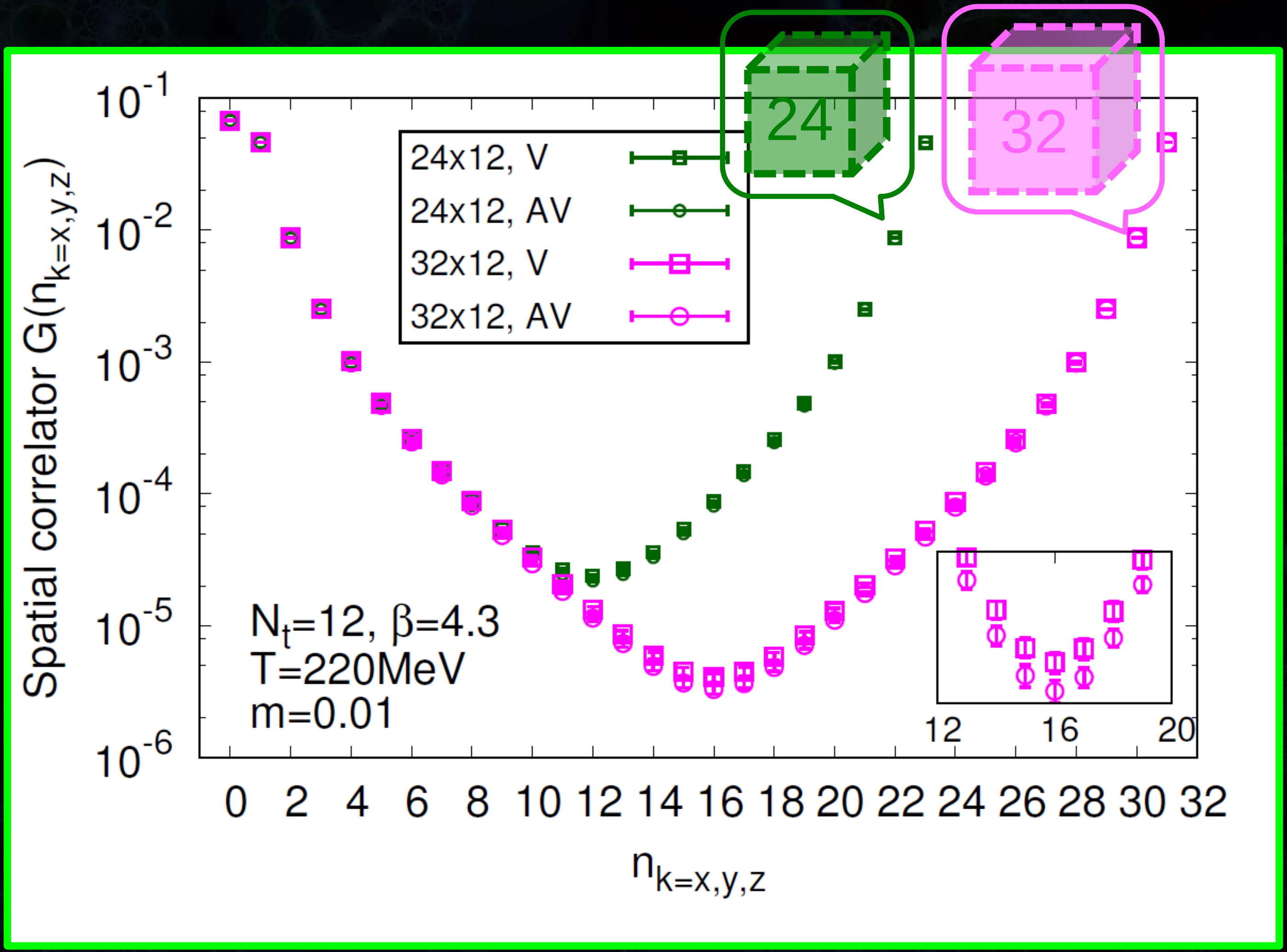
⇒Small m_q : $U(1)_A$ restoration, Large m_q : $U(1)_A$ breaking

V-AV Correlators : Chiral partners



⇒Small m_q : Chiral restoration, Large m_q : Chiral breaking

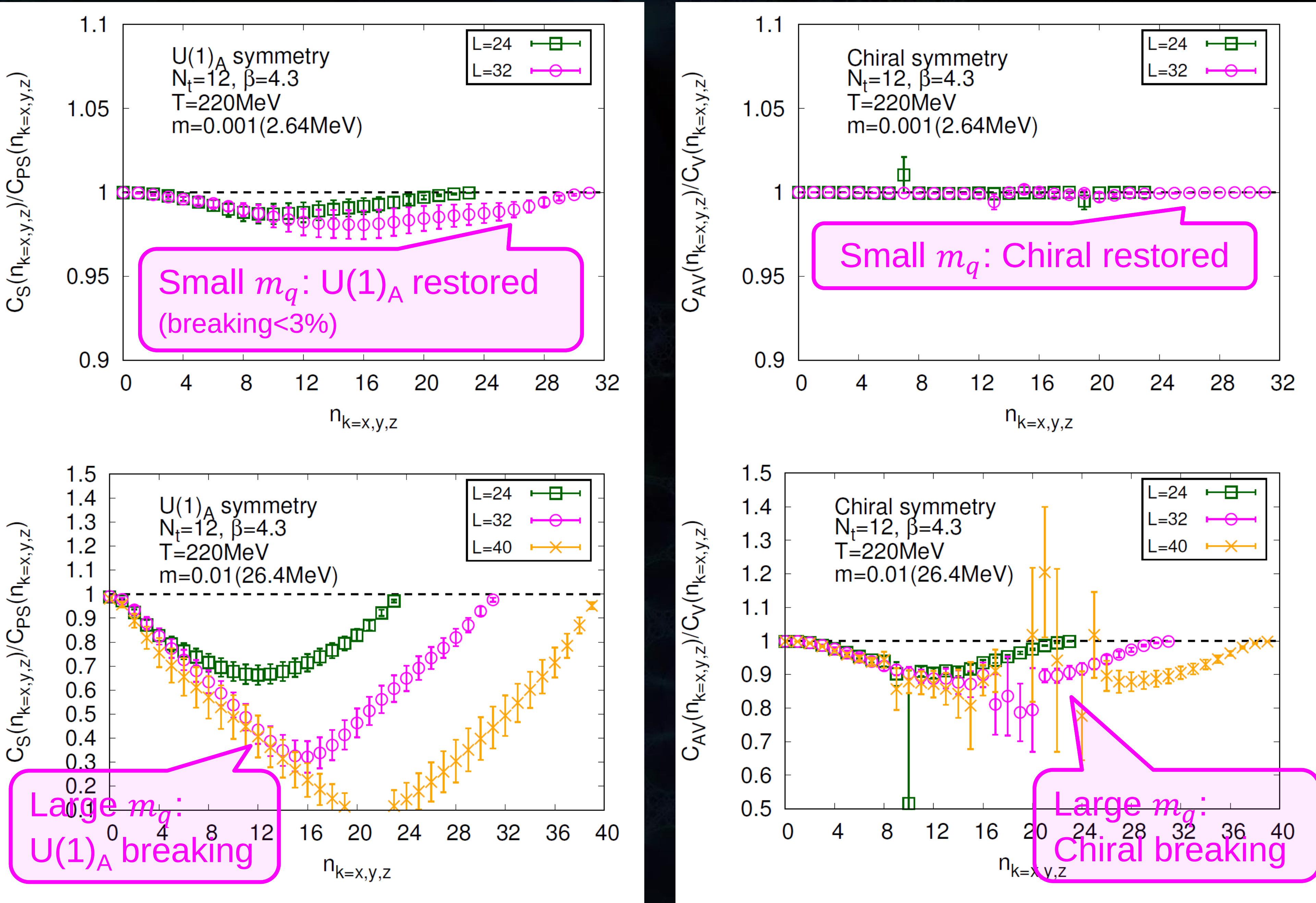
V-AV Correlators : Chiral partners



⇒Small m_q : Chiral restoration, Large m_q : Chiral breaking

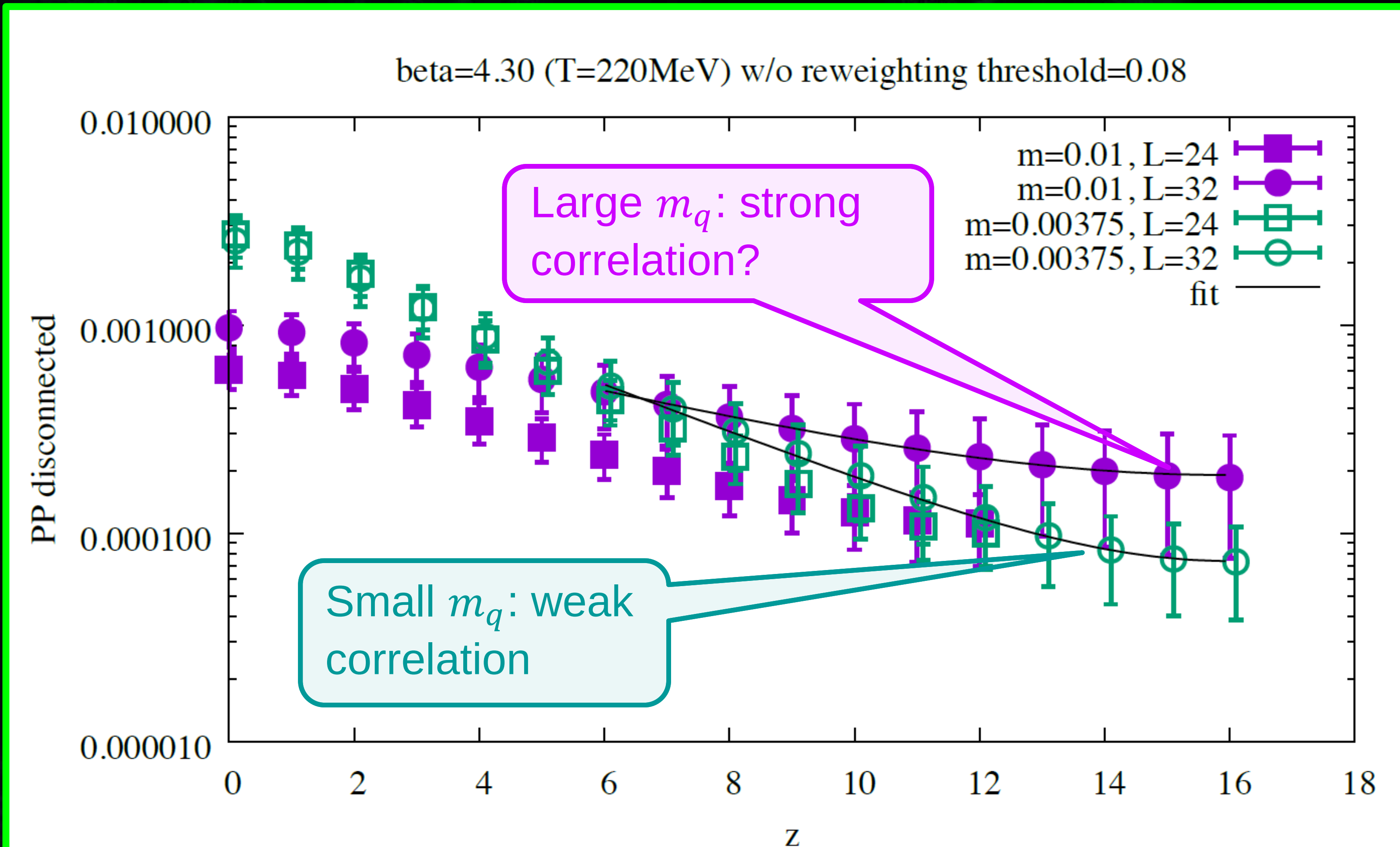
Correlator ratios(C_S/C_{PS}, C_{AV}/C_V)

JLQCD, preliminary (2019)



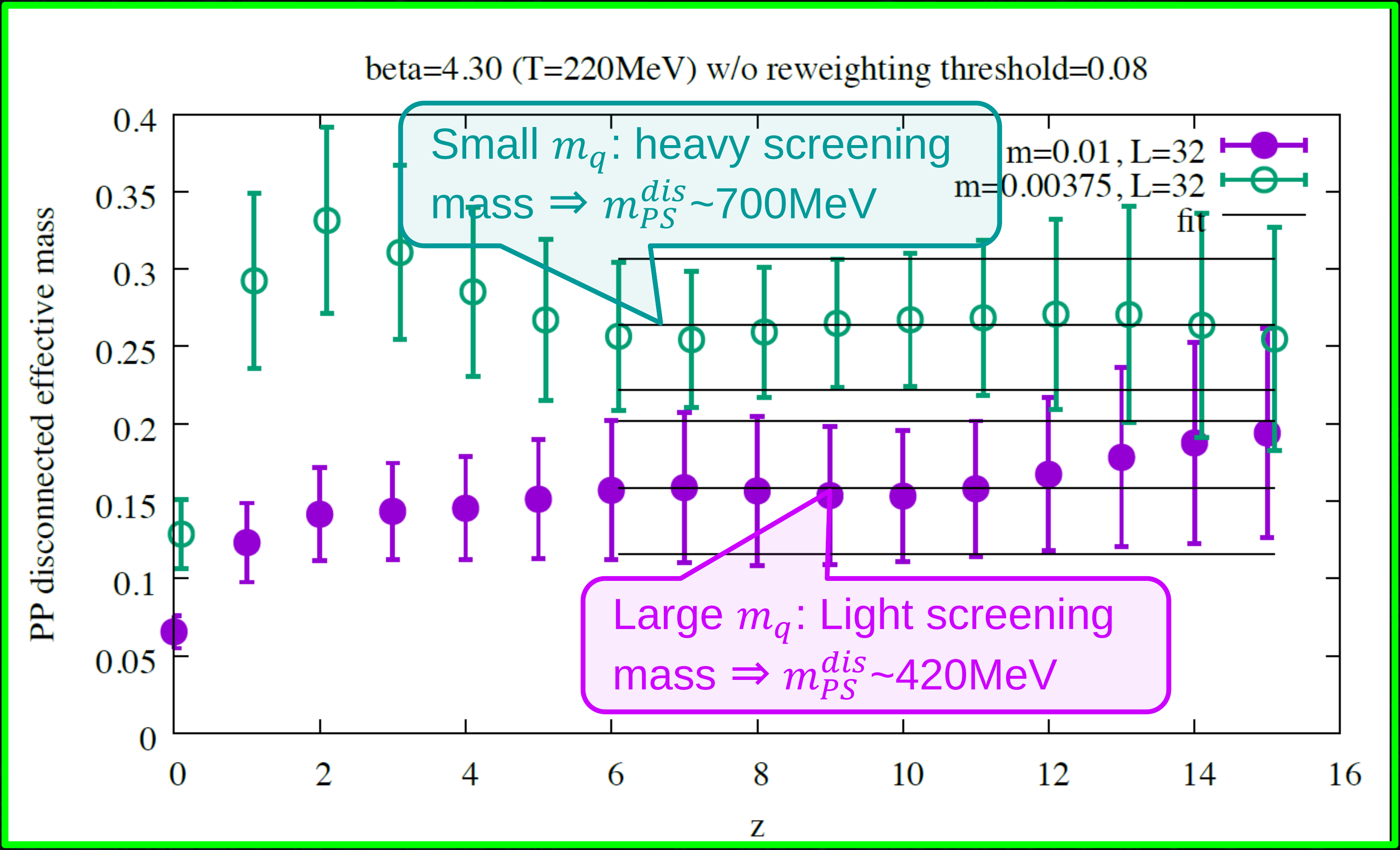
PS(<u>Disconnected</u>) correlator

from Dirac modes



⇒ Large m_q: Correlation becomes strong ⇒ screening masses?

PS(Disconnected) screening mass



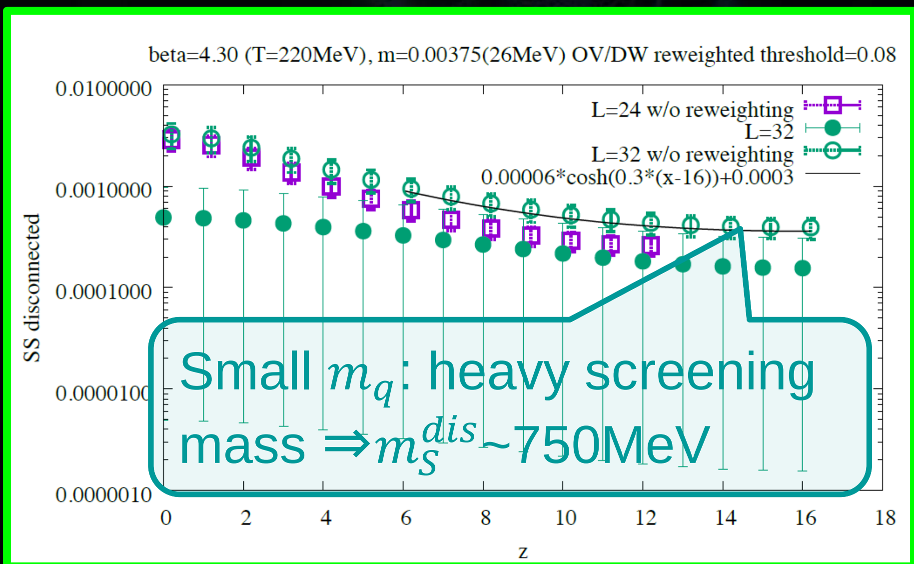
⇒ Small m_q: $m_{PS}^{dis} \sim m_{PS}^{con}$

⇒ Large m_q: m_{PS}^{dis}[~420MeV] $<$ m_{PS}^{con}[~700MeV]

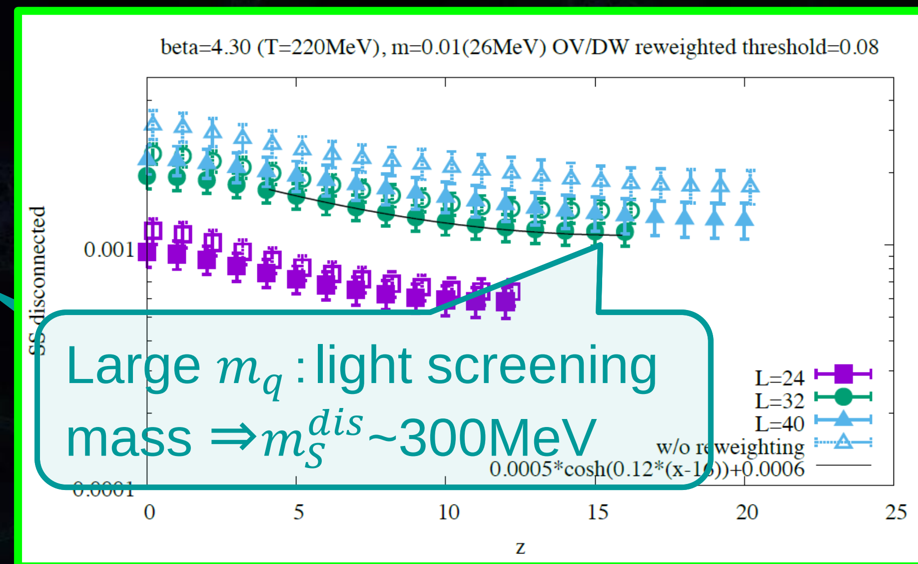
Scalar (Disconnected) correlator

from Dirac modes

Small m_q :



Large m_q :



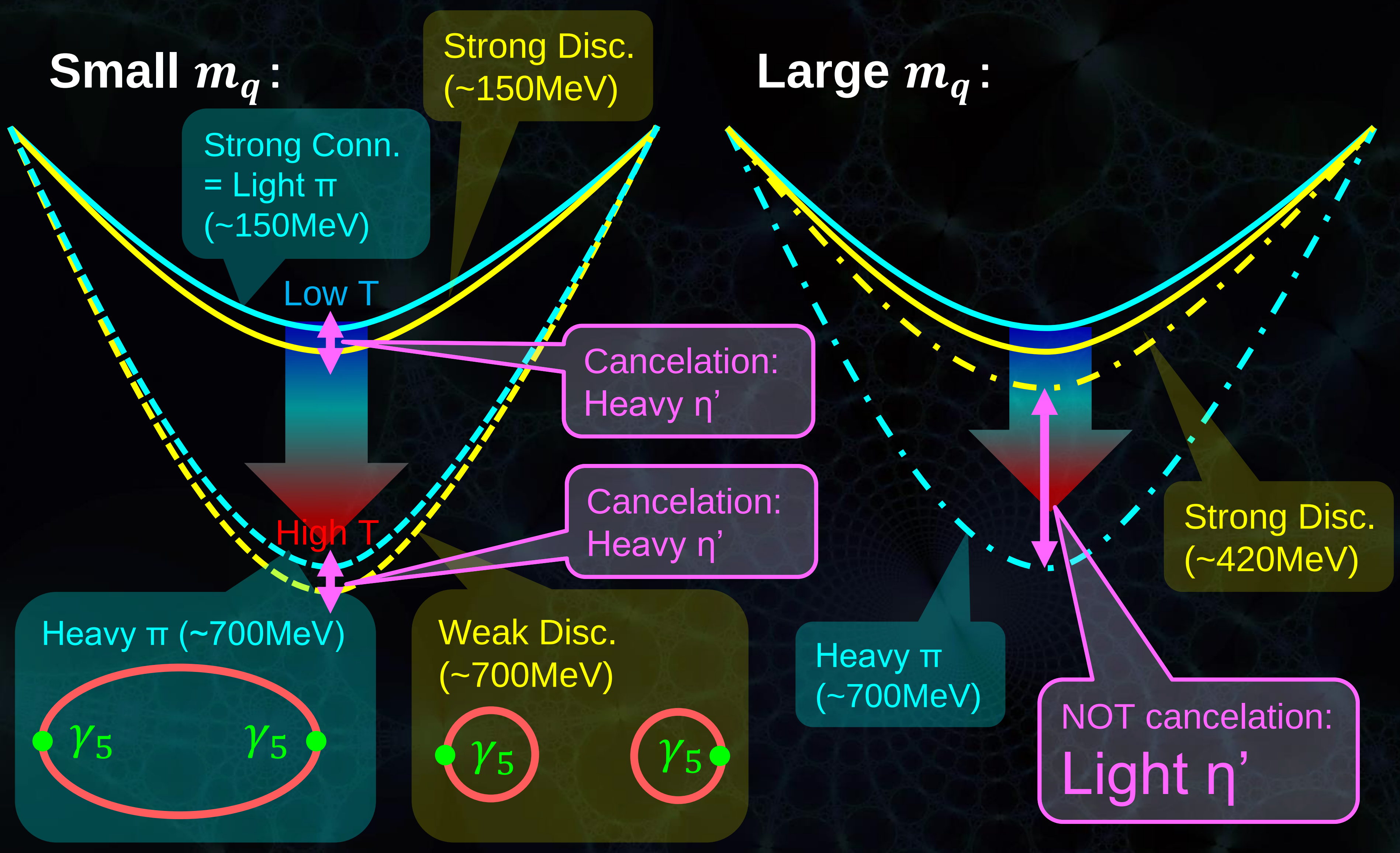
⇒Large m_q:

m_S^{dis}~300MeV < m_{PS}^{dis}~420MeV !?

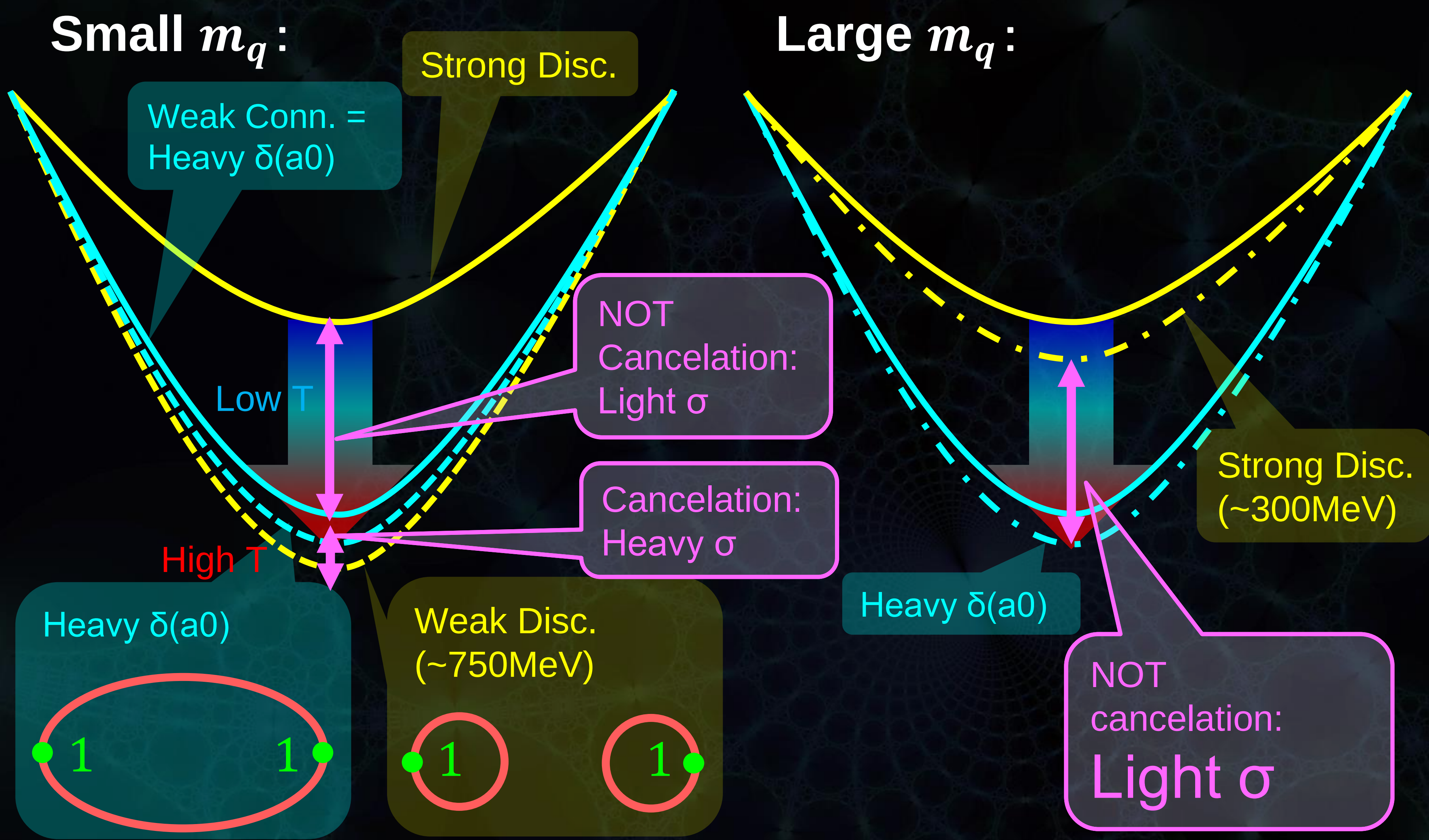
⇒Long-distance correlations by scalar particle?

（Finite volume effect between L=24 [~1.8fm] and L=32 [~2.4fm]?）

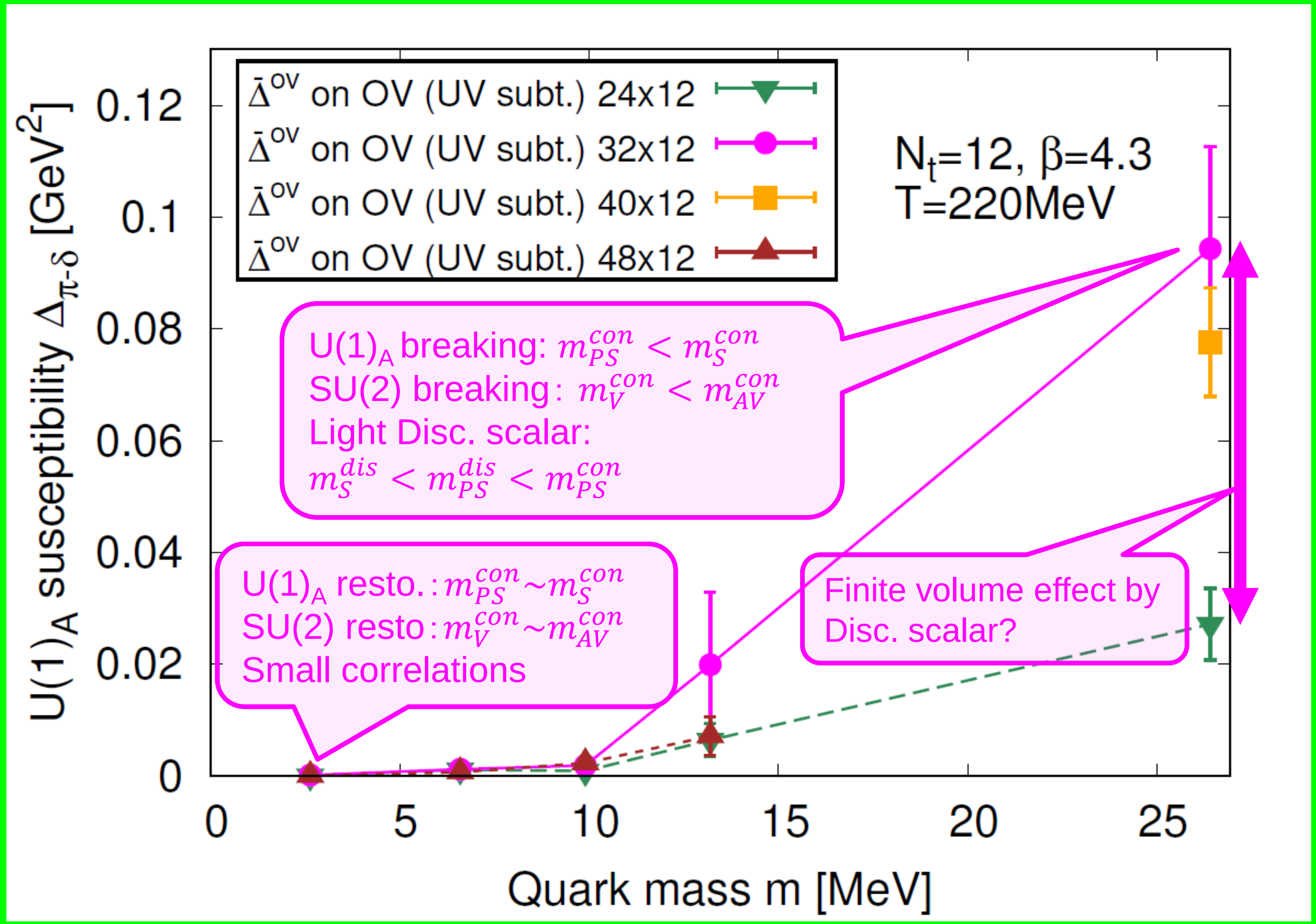
PS Conn. (π) − PS Disc. = η' correlation



S Conn. (δ) − S Disc. = σ correlation



Summary : $U(1)_A$ and correlators



Summary and Outlook

- In high-temperature phase ($T > T_c$) at $N_f = 2$, we found that
- U(1)$_A$ susceptibility is strongly suppressed in the chiral limit (for T=220-330MeV)
- Top. susceptibility shows a critical m_q in a few 10 MeV (for T=220-330MeV)
- Long-distance (disc.) correlations at large m_q

- Another symmetry for mesonic correlators

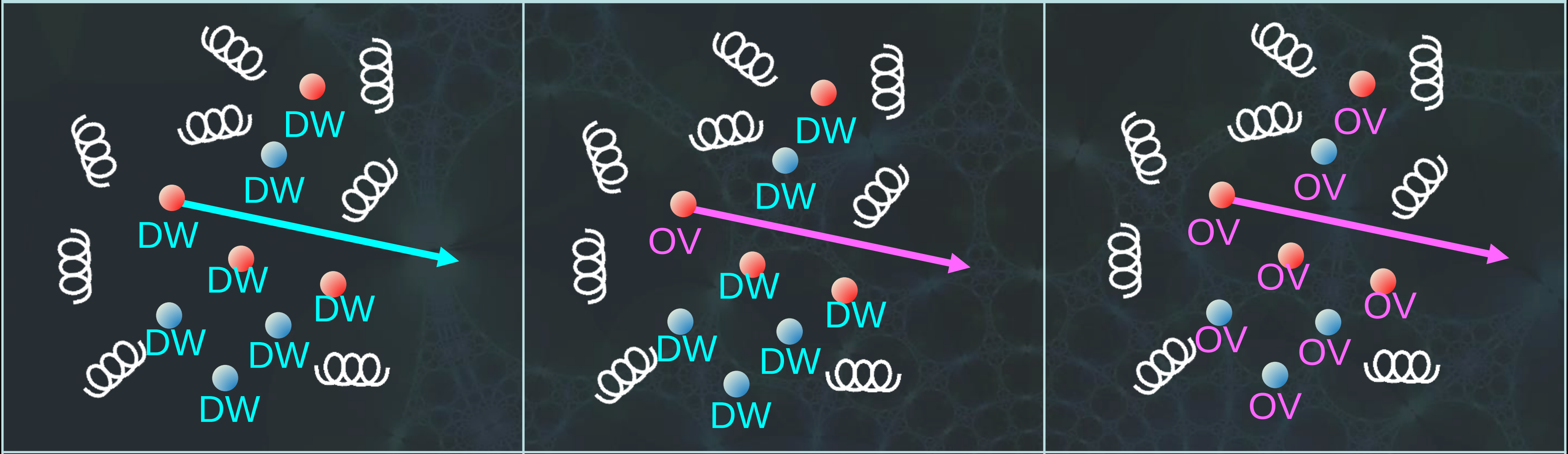
⇒Next talk (by C. Rohrhofer)

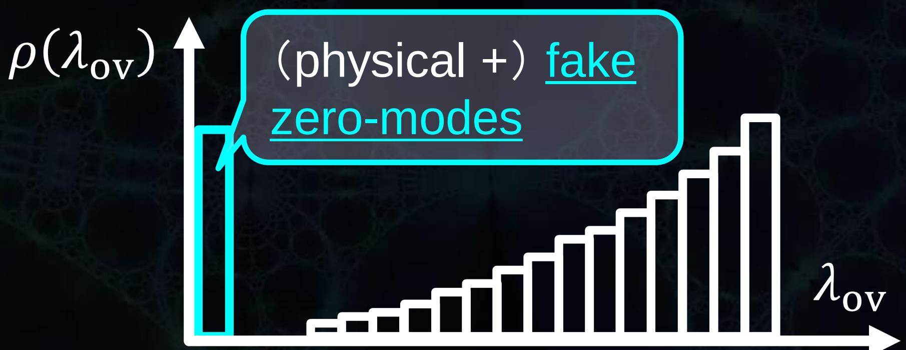
- Near T_c ($N_t = 14$?, chiral transition?)
- $N_f = 2 + 1$ sector

Backup

Valence quark and Sea quark

DW on DW	**OV on DW**	**OV on OV**
Almost good chiral symmetry	Fake zero-mode appears as an artifact	Exact chiral symmetry, but, very high cost





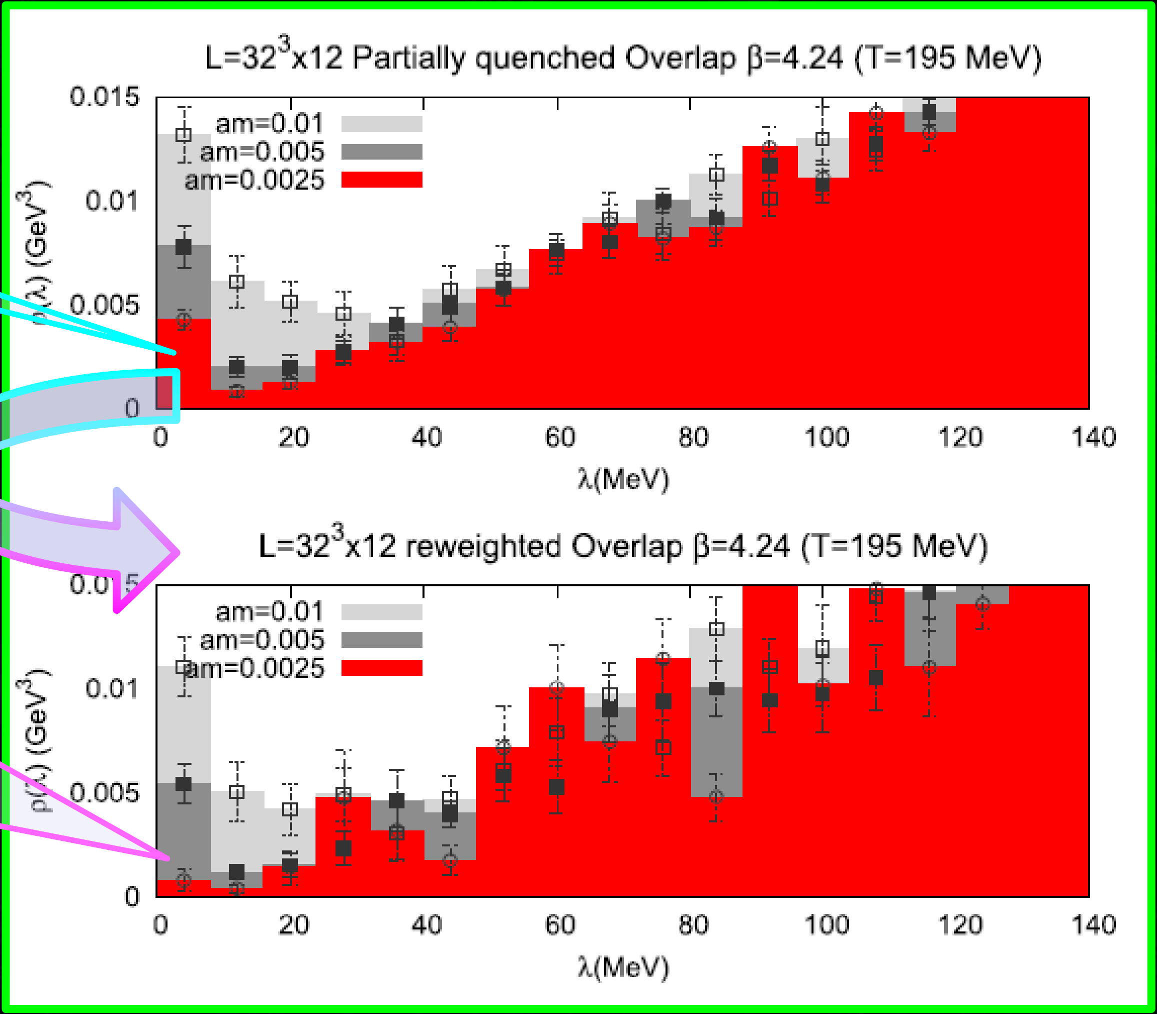
DW / OV reweighting
⇒can remove fake zero mode

A. Tomiya et al. (JLQCD) PRD96 (2017) 034509

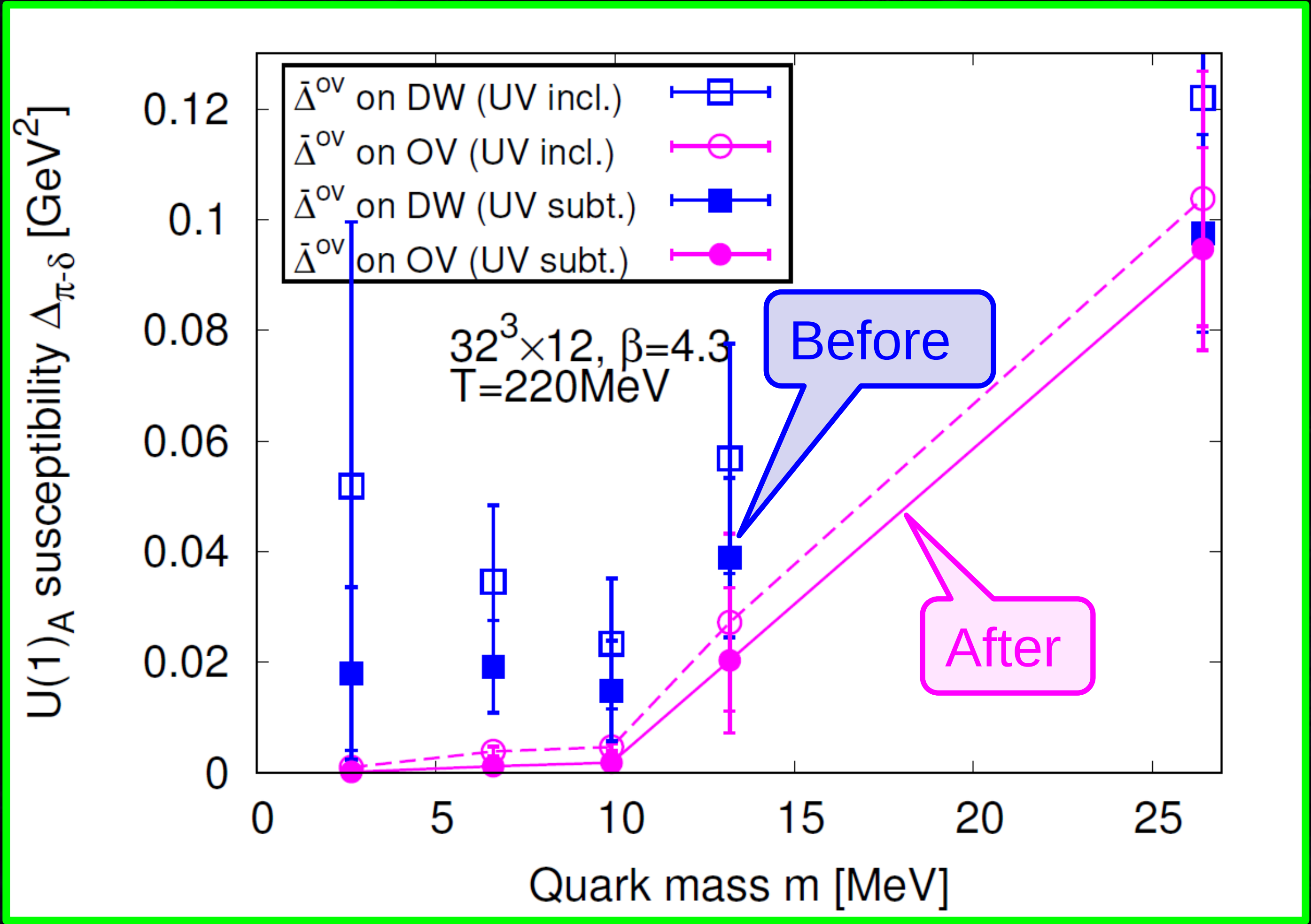
DW/OV reweighting removes fake zero-modes

OV on DW:
Fake zero-modes by partially quenched

OV on OV:
removed fake zero-modes
⇒ Only physical zero-modes survive!



$U(1)_A$ susceptibility (DW/OV reweighting)



⇒DW/OV reweighting is crucial in small m region

S. Aoki, H. Fukaya, and Y. Taniguchi PRD86 (2012), 114512
A. Tomiya et al. (JLQCD) PRD96 (2017), 034509

Note 1 :

$U(1)_A$ susc. = Low modes + ~~Zero~~ mode ?

$$\Delta_{\pi-\delta} \equiv \int_0^\infty d\lambda\, \rho(\lambda) \frac{2m^2}{(\lambda^2+m^2)^2}$$

$$\rho_{0-mode}(\lambda) = \frac{1}{V} \sum_{0-mode} \delta(\lambda)$$

$$\Delta_{\mathrm{zero}} = \int_0^\infty d\lambda \frac{1}{V} \sum_{0-mode} \delta(\lambda) \frac{2m^2}{(\lambda^2+m^2)^2}$$

$$= \frac{1}{V} \sum_{0-mode} \frac{2m^2}{m^4}$$

$$= \frac{1}{V} \sum_{0-mode} \frac{2}{m^2} = \frac{2N_0}{Vm^2}$$

$$\langle N_{L+R}^2 \rangle = \mathcal{O}(V)$$
$$\langle N_{L+R} \rangle = \mathcal{O}(\sqrt{V})$$

$$\lim_{V\to\infty} \Delta_{\mathrm{zero}} = 0$$

Zero mode contributions in $\Delta_{\pi-\delta}$ will be suppressed in $V \to \infty$ limit

Note 2:

$U(1)_A$ susc. = Physics + Ultraviolet divergence?

$$\Delta_{\pi-\delta} = \int_0^\infty d\lambda\, \rho(\lambda) \frac{2m^2}{(\lambda^2+m^2)^2}$$

$\rho(\lambda) \sim \lambda^3$

$\sim 1/\lambda^4$

$$\Delta^{\rm ov}_{\pi-\delta} \propto m^2 \ln\Lambda + \cdots$$

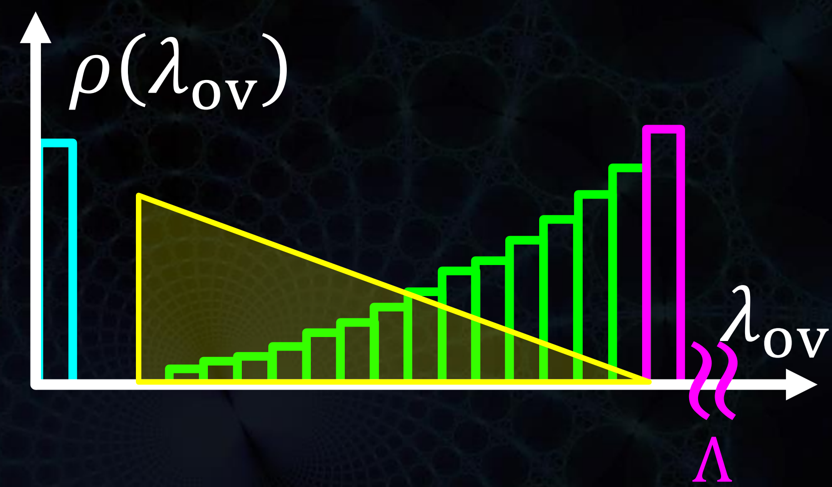
The term depends on cutoff Λ and valence quark mass m

We assume valence quark mass dependence of $\Delta_{\pi-\delta}$ (for small m):

$$\Delta_{\pi-\delta}(m) = \frac{a}{m^2} + b + cm^2 + O(m^4)$$

Zero-mode
(disappears in $V \to \infty$)

$m^2 \ln\Lambda$
(disappears in $m \to 0$)



⇒ From 3 eqs. for $\Delta_{\pi-\delta}(m_1)$, $\Delta_{\pi-\delta}(m_2)$, $\Delta_{\pi-\delta}(m_3)$, a and c are eliminated

⇒ $\Delta_{\pi-\delta} \sim b + O(m^4)$ (, that depends on sea quark mass)