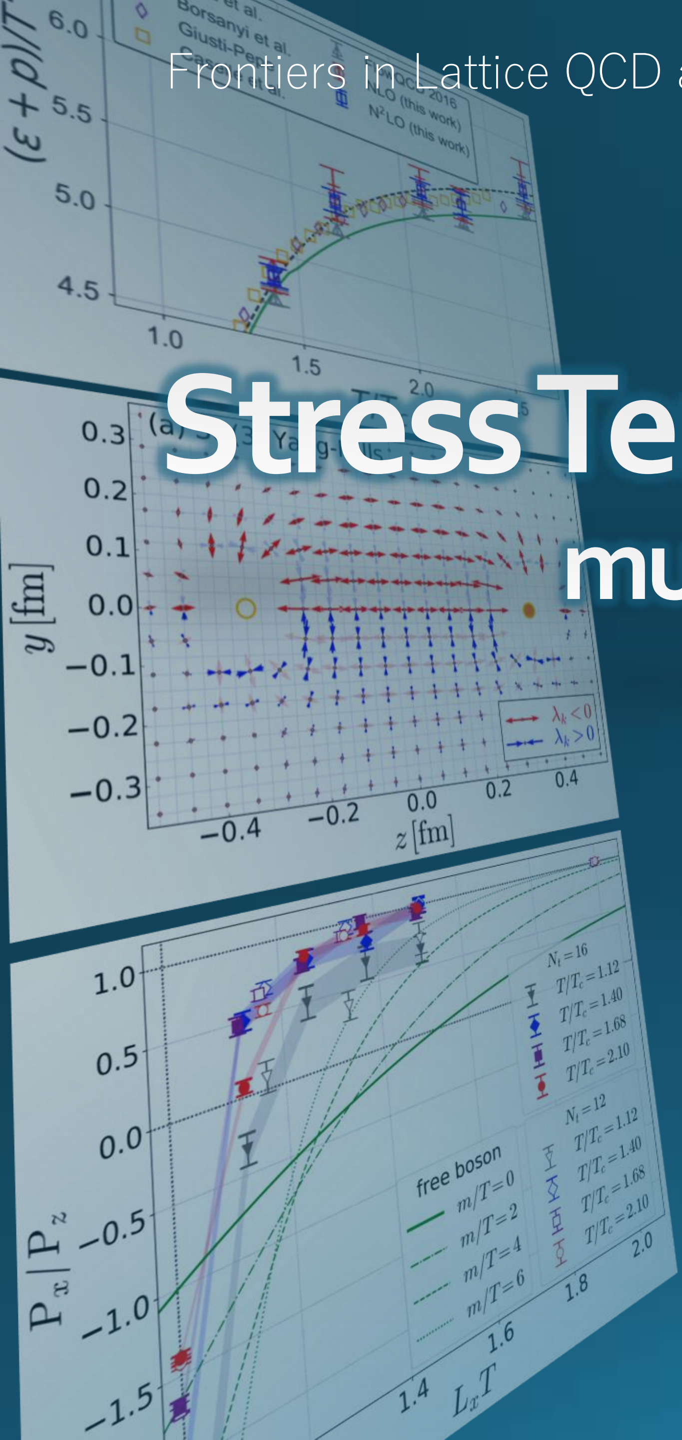
Frontiers in Lattice QCD and related topics, YITP, Kyoto, Japan, 2019/Apr./25



Stress Tensor on the Lattice

multi- & single quark systems, Casimir Effect, and etc.

Masakiyo Kitazawa

(Osaka U.)

Energy-Momentum Tensor

energy momentum

$$T_{\mu\nu} = \begin{bmatrix} T_{00} & T_{01} & T_{02} & T_{03} \\ T_{10} & T_{11} & T_{12} & T_{13} \\ T_{20} & T_{21} & T_{22} & T_{23} \\ T_{30} & T_{31} & T_{32} & T_{33} \end{bmatrix}$$

pressure

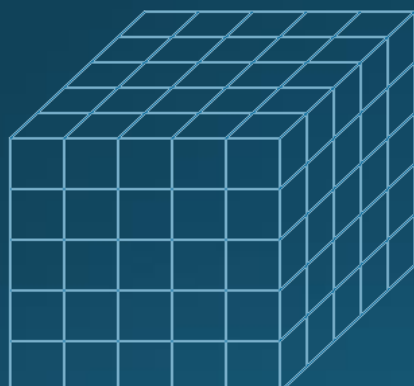
stress

All components are important physical observables!

$T_{\mu\nu}$: nontrivial observable on the lattice

① Definition of the operator is nontrivial

because of the explicit breaking of Lorentz symmetry



ex: $T_{\mu\nu} = F_{\mu\rho}F_{\nu\rho} - \frac{1}{4}\delta_{\mu\nu}FF$

$F_{\mu\nu} =$ 

② Its measurement is extremely noisy

due to high dimensionality and etc.

Contents

1. Constructing EMT
2. Thermodynamics

FlowQCD, PR**D90**,011501 (2014); PR**D94**, 114512 (2016);
WHOT-QCD, PR**D96**, 014509 (2017); Iritani+, PTEP **2019**, 023B02 (2019)

3. Flux Tube

FlowQCD, PLB789, 210 (2019); Yanagihara+, in prep.

4. Static Quark Systems at Nonzero T

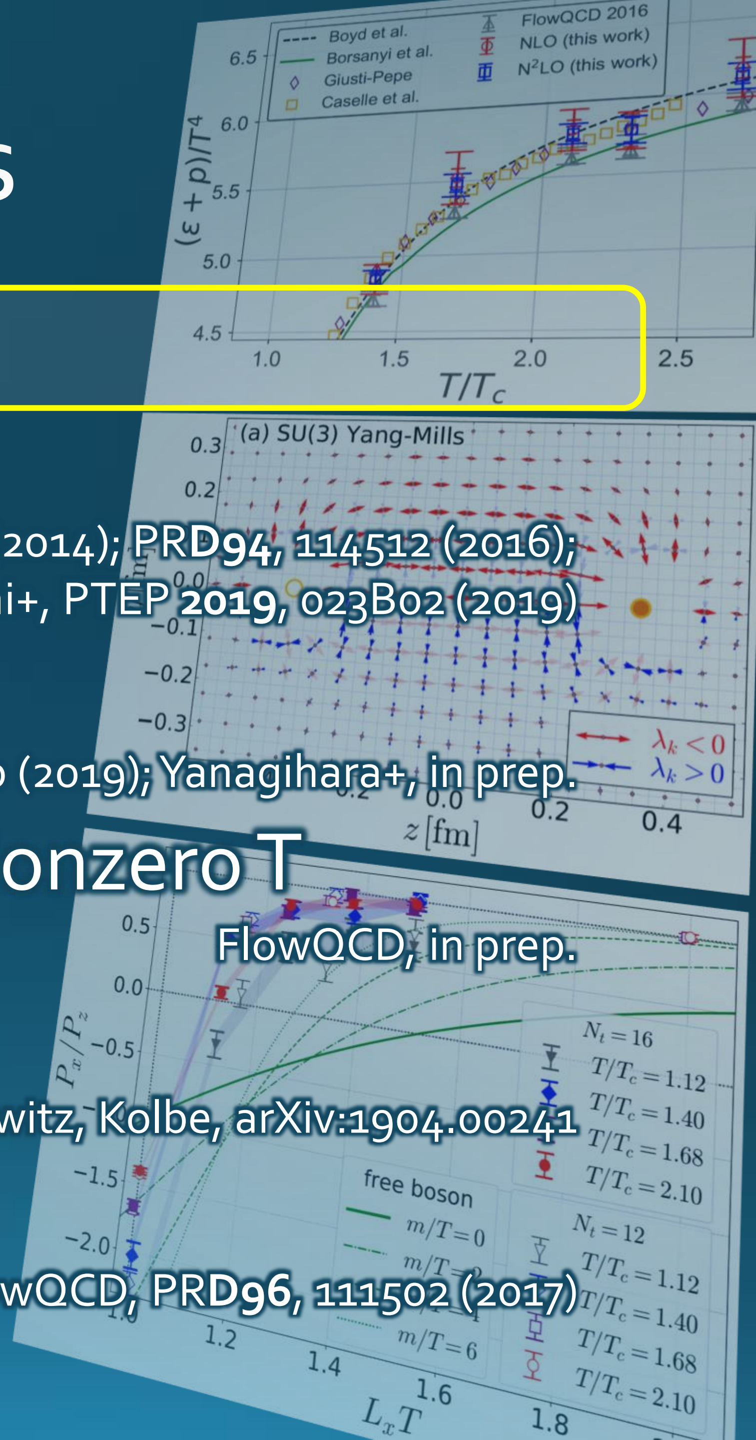
FlowQCD, in prep.

5. Casimir Effect

MK, Mogliacci, Horowitz, Kolbe, arXiv:1904.00241

6. Correlation Function

FlowQCD, PR**D96**, 111502 (2017)



Yang-Mills Gradient Flow

Luscher 2010
Narayanan, Neuberger, 2006
Luscher, Weiss, 2011

$$\frac{\partial}{\partial t} A_\mu(t,x) = -\frac{\partial S_{\mathrm{YM}}}{\partial A_\mu}$$

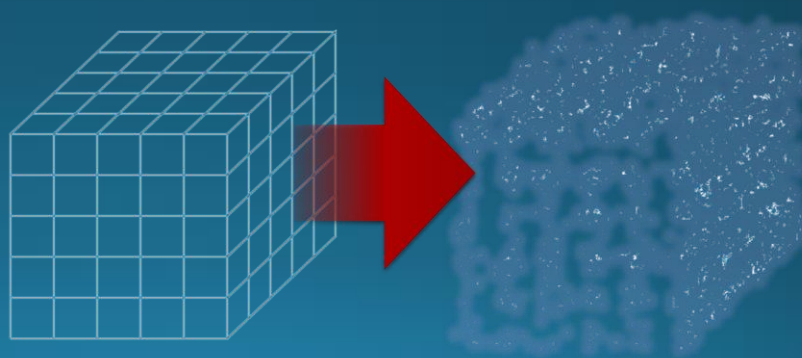
$$A_\mu(0,x) = A_\mu(x)$$

t: "flow time"
dim:[length^2]

leading

$$\partial_t A_\mu = D_\nu G_{\mu\nu} = \partial_\nu \partial_\nu A_\mu + \cdots$$

- diffusion equation in 4-dim space
- diffusion distance $d \sim \sqrt{8t}$
- "continuous" cooling/smearing
- No UV divergence at t>0



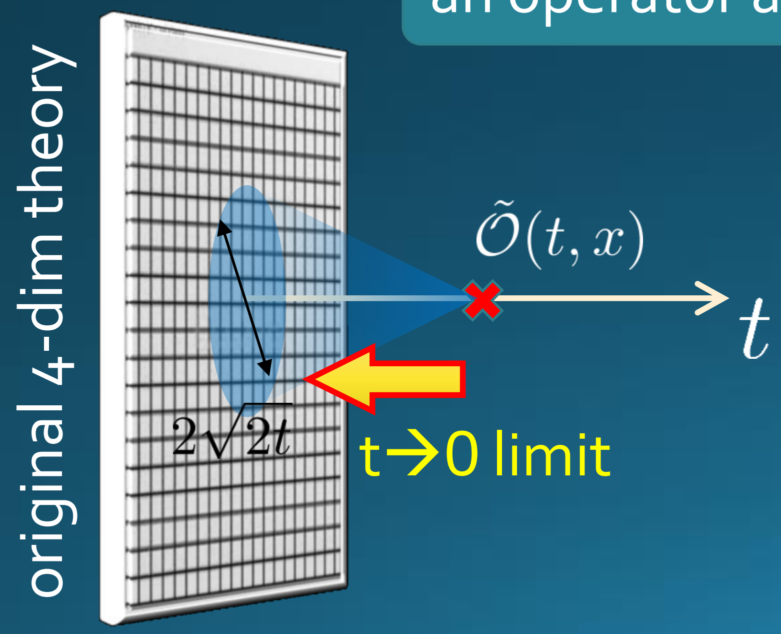
Small Flow-Time Expansion

Luescher, Weisz, 2011
Suzuki, 2013

$$\tilde{\mathcal{O}}(t,x) \xrightarrow[t\to 0]{} \sum_i c_i(t)\mathcal{O}_i^R(x)$$

an operator at t>0

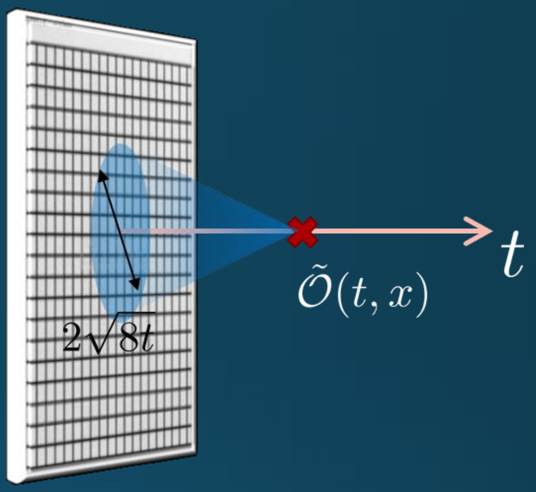
remormalized operators of original theory



Constructing EMT 1

Suzuki, 2013

$$\tilde{\mathcal{O}}(t,x) \underset{t\to 0}{\longrightarrow} \sum_i c_i(t)\mathcal{O}_i^R(x)$$



- **Gauge-invariant dimension 4 operators**

$$\begin{cases} U_{\mu\nu}(t,x) = G_{\mu\rho}(t,x)G_{\nu\rho}(t,x) - \frac{1}{4}\delta_{\mu\nu}G_{\mu\nu}(t,x)G_{\mu\nu}(t,x) \\ E(t,x) = \frac{1}{4}\delta_{\mu\nu}G_{\mu\nu}(t,x)G_{\mu\nu}(t,x) \end{cases}$$

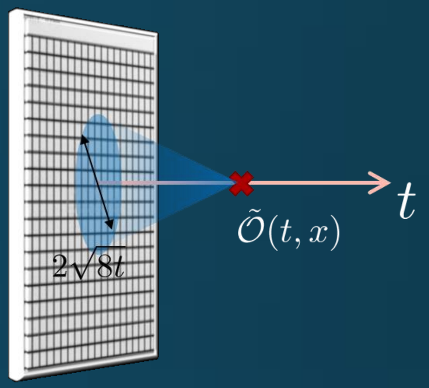
Constructing EMT

Suzuki, 2013

$$U_{\mu\nu}(t,x) = \alpha_U(t)\left[T^R_{\mu\nu}(x) - \frac{1}{4}\delta_{\mu\nu}T^R_{\rho\rho}(x)\right] + \mathcal{O}(t)$$

$$E(t,x) = \langle E(t,x)\rangle + \alpha_E(t)T^R_{\rho\rho}(x) + \mathcal{O}(t)$$

vacuum subtr.



Remormalized EMT

$$T^R_{\mu\nu}(x) = \lim_{t\to 0}\left[c_1(t)U_{\mu\nu}(t,x) + \delta_{\mu\nu}c_2(t)E(t,x)_{\text{subt.}}\right]$$

Perturbative coefficient:
Suzuki (2013); Makino, Suzuki (2014); Harlander+ (2018); Iritani, MK, Suzuki, Takaura (2019)

Perturbative Coefficients

Suzuki, PTEP 2013, 083B03
Harlander+, 1808.09837
Iritani, MK, Suzuki, Takaura, PTEP 2019

$$T_{\mu\nu}(t) = c_1(t) U_{\mu\nu}(t) + \delta_{\mu\nu} c_2(t) E(t)$$

	LO	1-loop	2-loop	3-loop
$c_1(t)$	○	○	○	
$c_2(t)$	× zero	○	○	○

Suzuki (2013) Harlander+(2018) Iritani, MK, Suzuki, Takaura, 2019

☐ Choice of the scale of g²

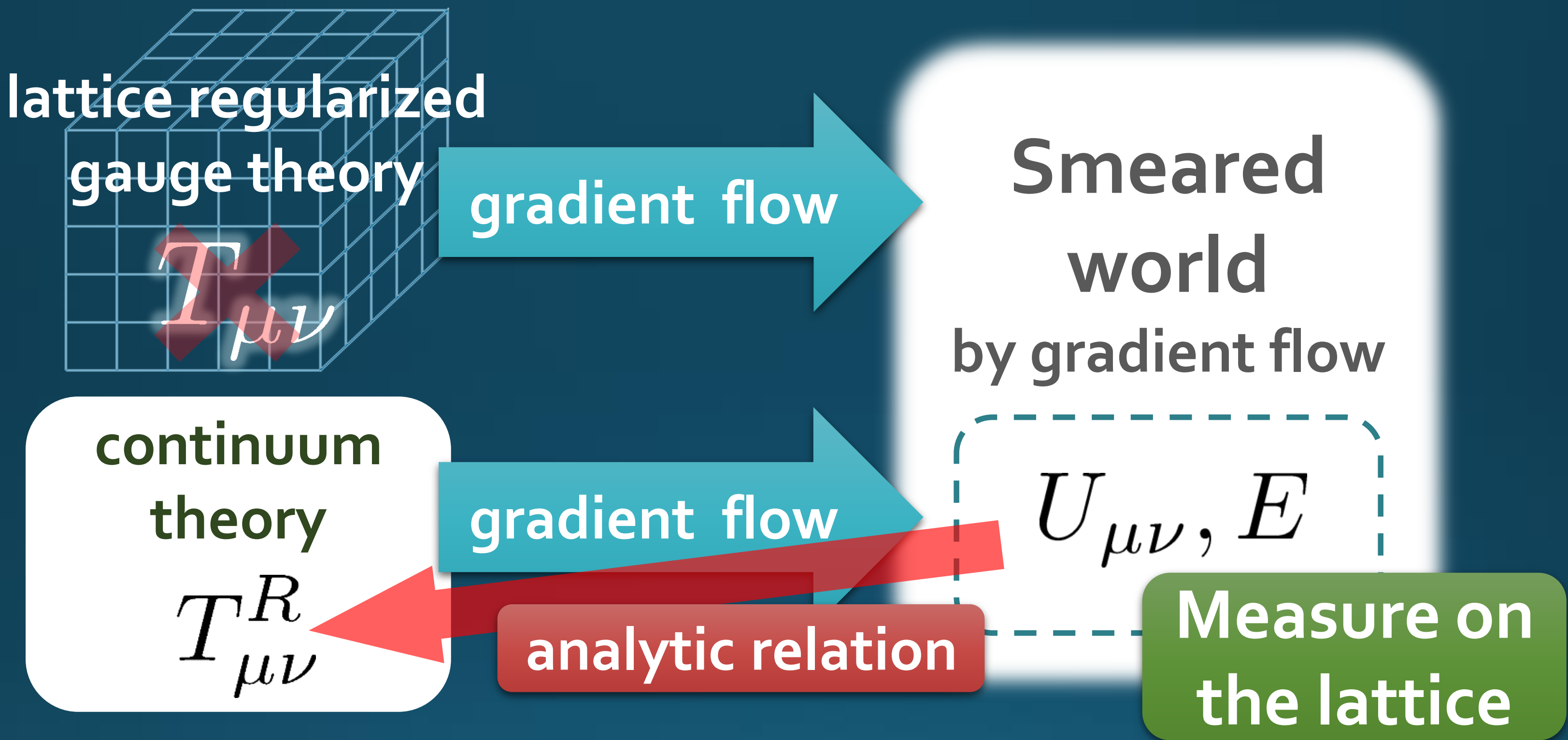
$$c_1(t) = c_1\left(g^2(\mu(t))\right)$$

Previous: $\mu_d(t) = 1/\sqrt{8t}$

Improved: $\mu_0(t) = 1/\sqrt{2e^{\gamma_E} t}$

Harlander+ (2018)

Gradient Flow Method



Take Extrapolation (t,a)→(0,0)

$$\langle T_{\mu\nu}(t)\rangle_{\rm latt} = \langle T_{\mu\nu}(t)\rangle_{\rm phys} + C_{\mu\nu}t + D_{\mu\nu}\frac{a^2}{t} + \cdots$$

O(t) terms in SFTE   lattice discretization

Contents

1. Constructing EMT
2. Thermodynamics

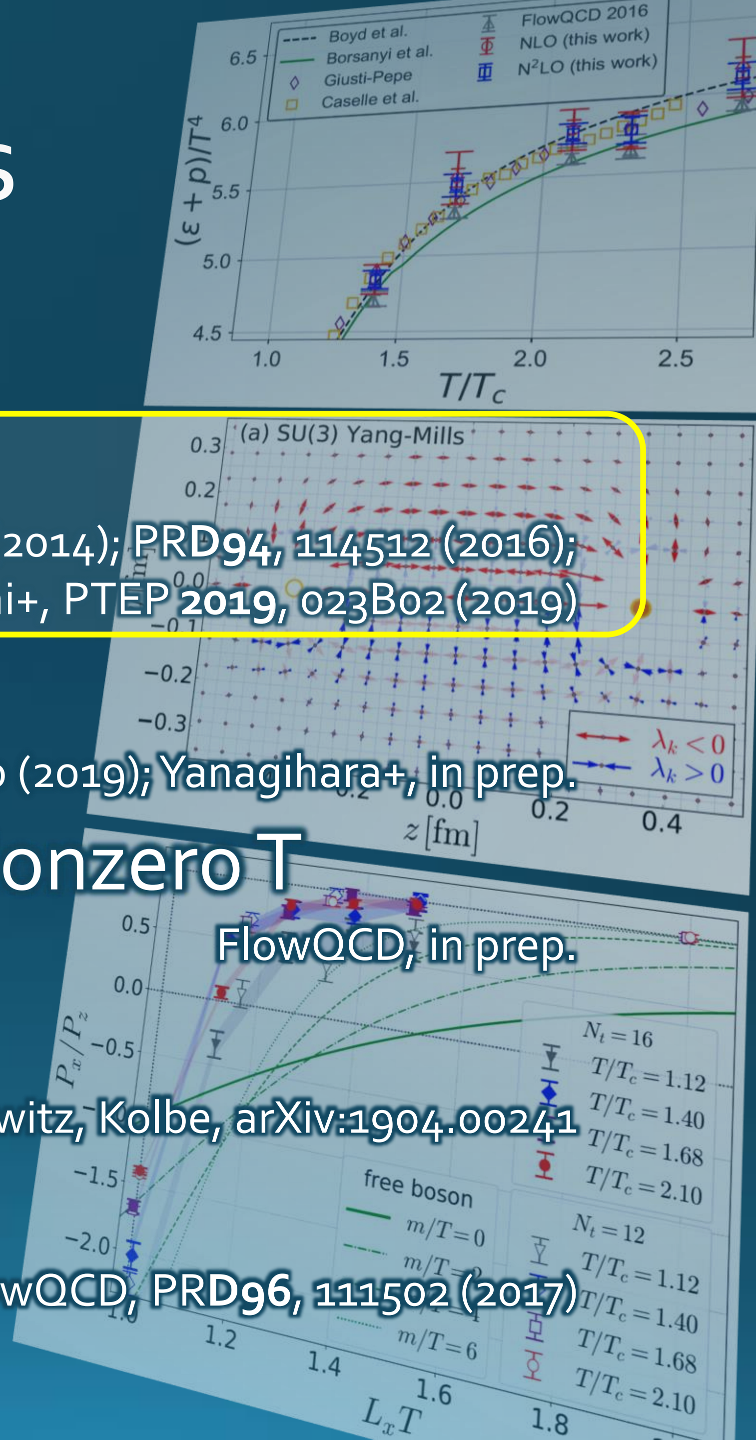
   FlowQCD, PR**D90**,011501 (2014); PR**D94**, 114512 (2016);
   WHOT-QCD, PR**D96**, 014509 (2017); Iritani+, PTEP **2019**, 023B02 (2019)
3. Flux Tube

   FlowQCD, PLB789, 210 (2019); Yanagihara+, in prep.
4. Static Quark Systems at Nonzero T

   FlowQCD, in prep.
5. Casimir Effect

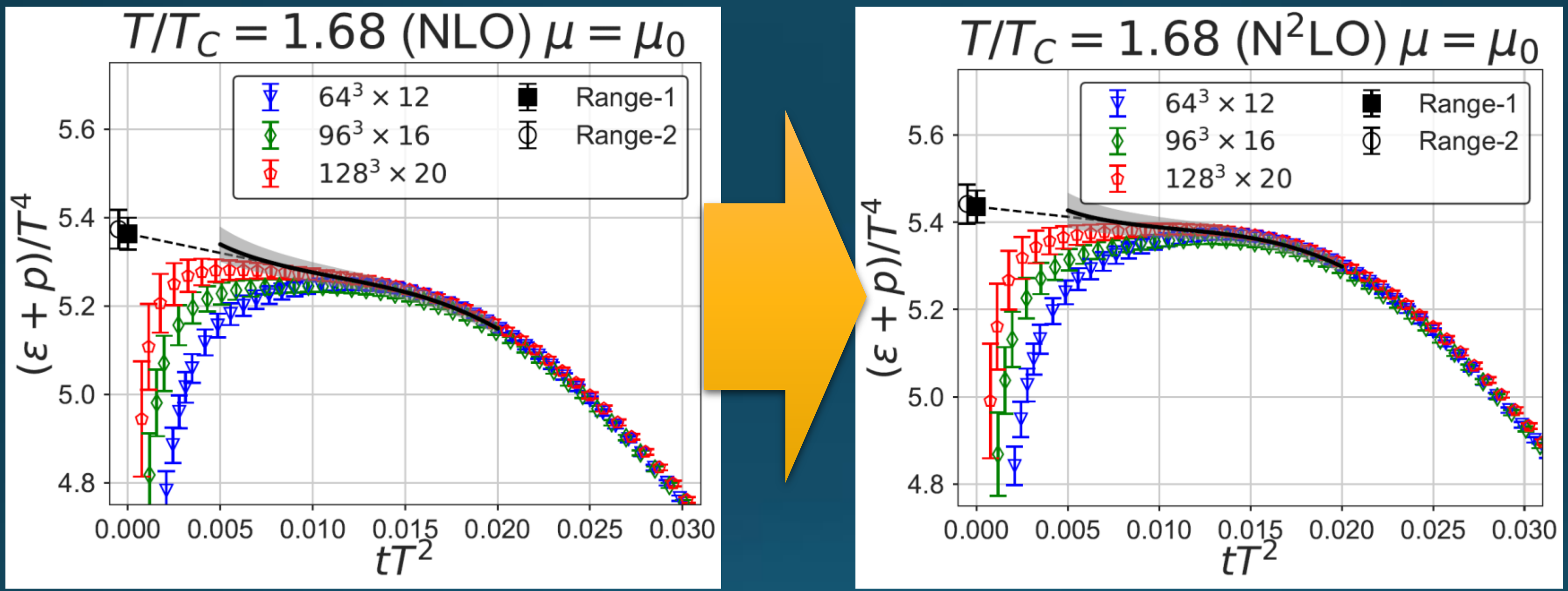
   MK, Mogliacci, Horowitz, Kolbe, arXiv:1904.00241
6. Correlation Function

   FlowQCD, PR**D96**, 111502 (2017)



Higher Order Coefficient: ε+p

NLO (1-loop) **N²LO (2-loop)**



Iritani, MK, Suzuki, Takaura, PTEP 2019

- t dependence becomes milder with higher order coeff.
- Better t→0 extrapolation

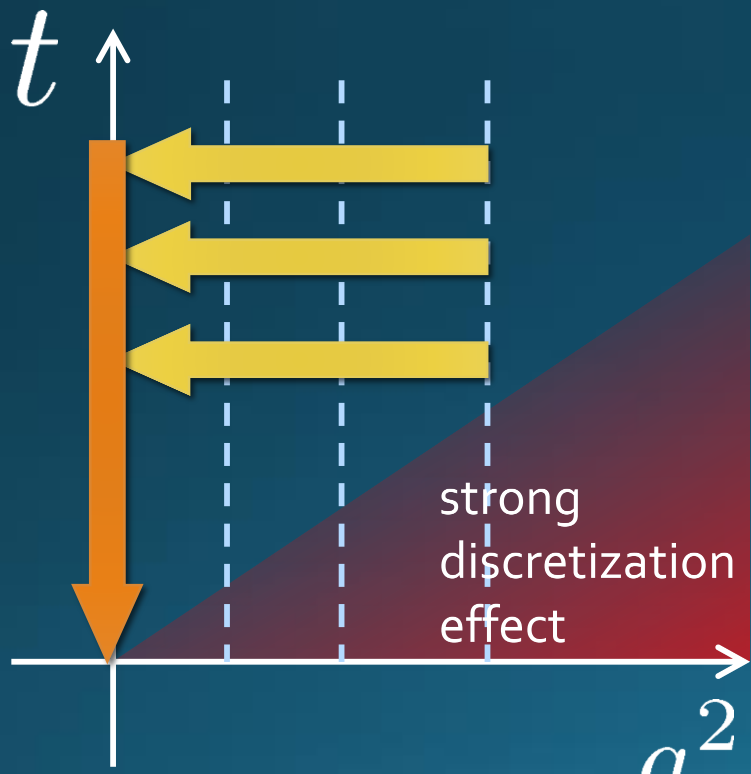
- Systematic error: μ_0 or μ_d, uncertaintyof Λ (±3%), fit range
- Extrapolation func: linear, higher order term in c_1 ($\sim g^6$)

Double Extrapolation

t→0, a→0

$$\langle T_{\mu\nu}(t)\rangle_{\mathrm{latt}} = \langle T_{\mu\nu}(t)\rangle_{\mathrm{phys}} + C_{\mu\nu}t + D_{\mu\nu}(t)\frac{a^2}{t}$$

O(t) terms in SFTE　lattice discretization



Continuum extrapolation

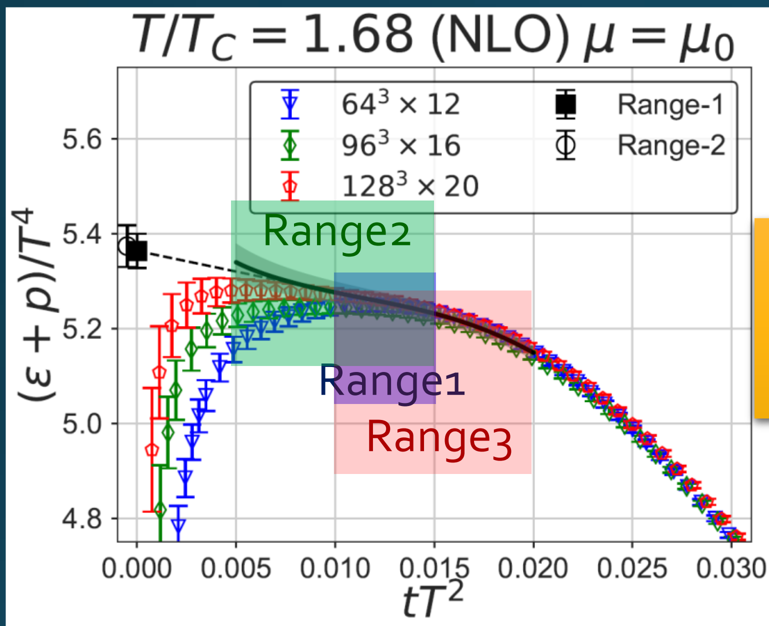
$$\langle T_{\mu\nu}(t)\rangle_{\mathrm{cont}} = \langle T_{\mu\nu}(t)\rangle_{\mathrm{lat}} + C(t)a^2$$

Small t extrapolation

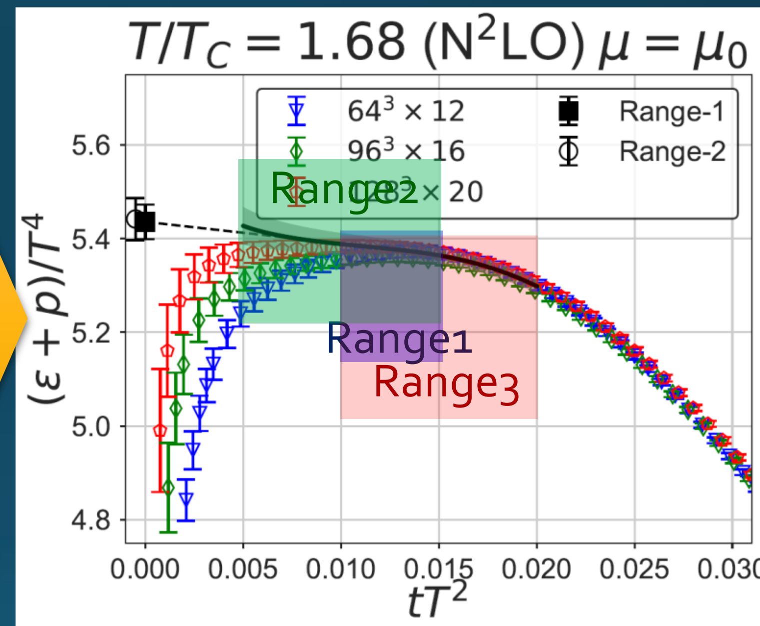
$$\langle T_{\mu\nu}\rangle = \langle T_{\mu\nu}(t)\rangle + C't$$

Higher Order Coefficient: ε+p

NLO (1-loop)



N²LO (2-loop)



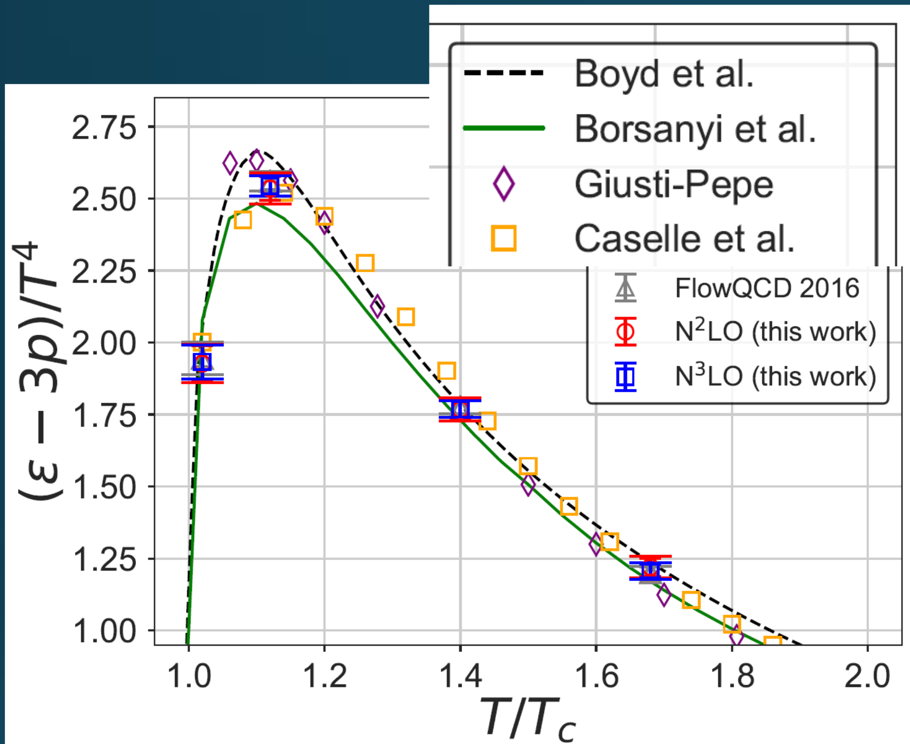
Iritani, MK, Suzuki, Takaura, PTEP 2019

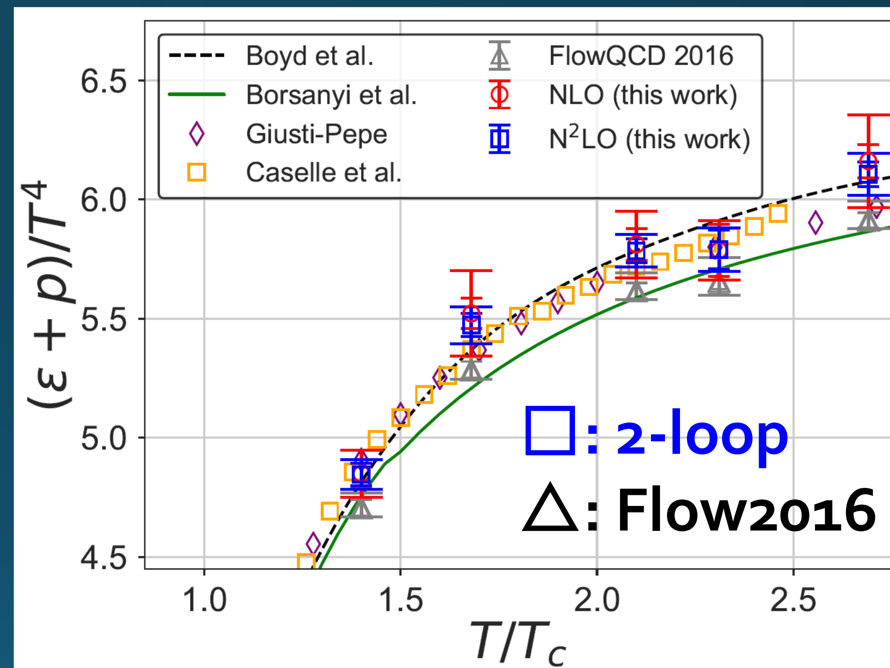
- t dependence becomes milder with higher order coeff.
- Better t→0 extrapolation

- Systematic error: μ_0 or μ_d, uncertaintyof Λ (±3%), fit range
- Extrapolation func: linear, higher order term in c_1 ($\sim g^6$)

Effect of Higher-Order Coeffs.

Iritani, MK, Suzuki, Takaura, 2019





Systematic error: μ_0 or μ_d, Λ, t→0 function, fit range

More stable extrapolation with higher order c_1 & c_2 (pure gauge)

Gradient Flow for Fermions

$$\partial_t \psi(t,x) = D_\mu D_\mu \psi(t,x)$$

$$\partial_t \bar{\psi}(t,x) = \psi(t,x) \overleftarrow{D}_\mu \overleftarrow{D}_\mu$$

$$D_\mu = \partial_\mu + A_\mu(t,x)$$

Luscher, 2013
Makino, Suzuki, 2014
Taniguchi+ (WHOT) 2016; 2017

- Not "gradient" flow but a "diffusion" equation.

- Divergence in field renormalization of fermions.
- All observables are finite at t>0 once Z(t) is fixed.

$$\tilde{\psi}(t,x) = Z(t)\psi(t,x)$$

- Energy-momentum tensor from SFTE Makino, Suzuki, 2014

EMT in QCD

$$T_{\mu\nu}(t,x) = c_1(t)U_{\mu\nu}(t,x) + c_2(t)\delta_{\mu\nu}\big(E(t,x) - \langle E\rangle_0\big)$$
$$+ c_3(t)\big(O_{3\mu\nu}(t,x) - 2O_{4\mu\nu}(t,x) - \mathrm{VEV}\big)$$
$$+ c_4(t)\big(O_{4\mu\nu}(t,x) - \mathrm{VEV}\big) + c_5(t)\big(O_{5\mu\nu}(t,x) - \mathrm{VEV}\big)$$

$$T_{\mu\nu}(x) = \lim_{t\to 0} T_{\mu\mu}(t,x)$$

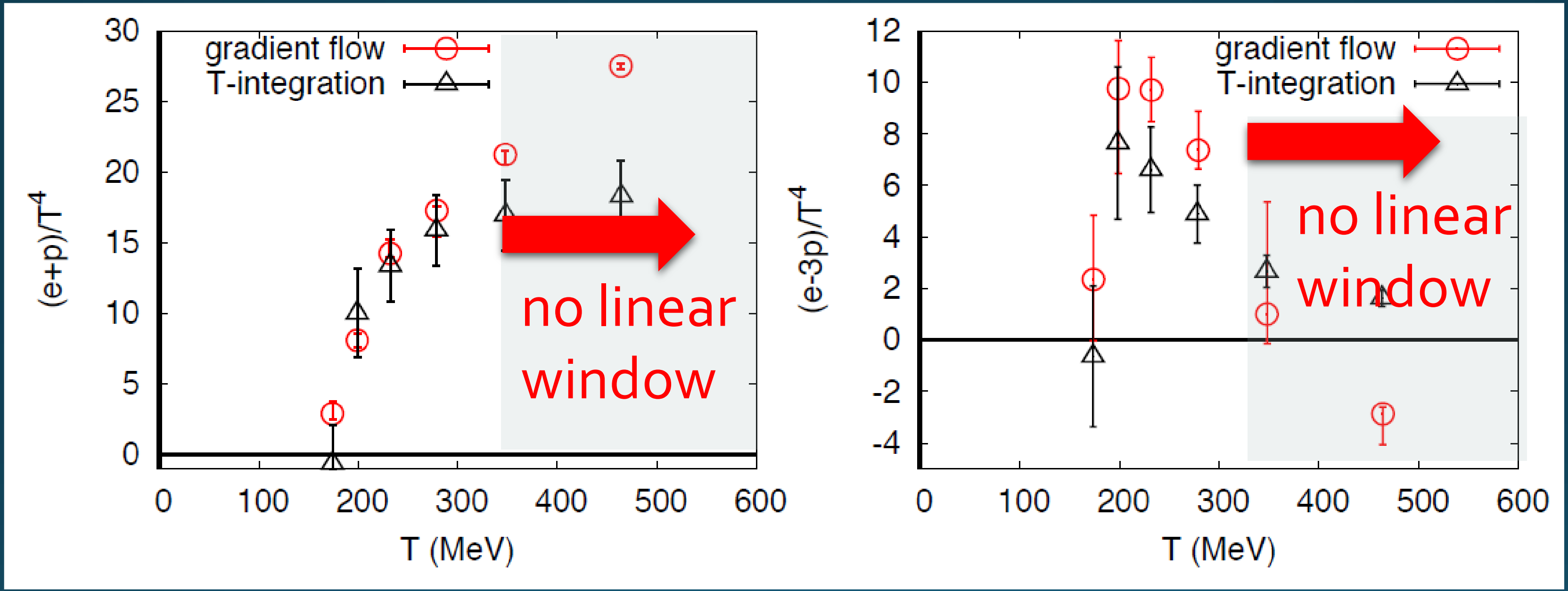
$$\tilde{O}^f_{3\mu\nu}(t,x) \equiv \varphi_f(t)\bar{\chi}_f(t,x)\left(\gamma_\mu \overleftrightarrow{D}_\nu + \gamma_\nu \overleftrightarrow{D}_\mu\right)\chi_f(t,x),$$
$$\tilde{O}^f_{4\mu\nu}(t,x) \equiv \varphi_f(t)\delta_{\mu\nu}\bar{\chi}_f(t,x)\overleftrightarrow{\not{D}}\chi_f(t,x),$$
$$\tilde{O}^f_{5\mu\nu}(t,x) \equiv \varphi_f(t)\delta_{\mu\nu}\bar{\chi}_f(t,x)\chi_f(t,x),$$

$$\varphi_f(t) \equiv \frac{-6}{(4\pi)^2 t^2 \left\langle \bar{\chi}_f(t,x)\overleftrightarrow{\not{D}}\chi_f(t,x)\right\rangle_0}.$$

2+1 QCD EoS from Gradient Flow

Taniguchi+ (WHOT-QCD), PR**D96**, 014509 (2017)

$m_{PS}/m_V \approx 0.63$



- Agreement with integral method except for N_t=4, 6
- N_t=4, 6: No stable extrapolation is possible
- Statistical error is substantially suppressed!

Physical mass: Kanaya+ (WHOT-QCD), 1710.10015

Contents

1. Constructing EMT
2. Thermodynamics

FlowQCD, PRD**90**,011501 (2014); PRD**94**, 114512 (2016);
WHOT-QCD, PRD**96**, 014509 (2017); Iritani+, PTEP **2019**, 023B02 (2019)

3. Flux Tube

FlowQCD, PLB789, 210 (2019); Yanagihara+, in prep.

4. Static Quark Systems at Nonzero T

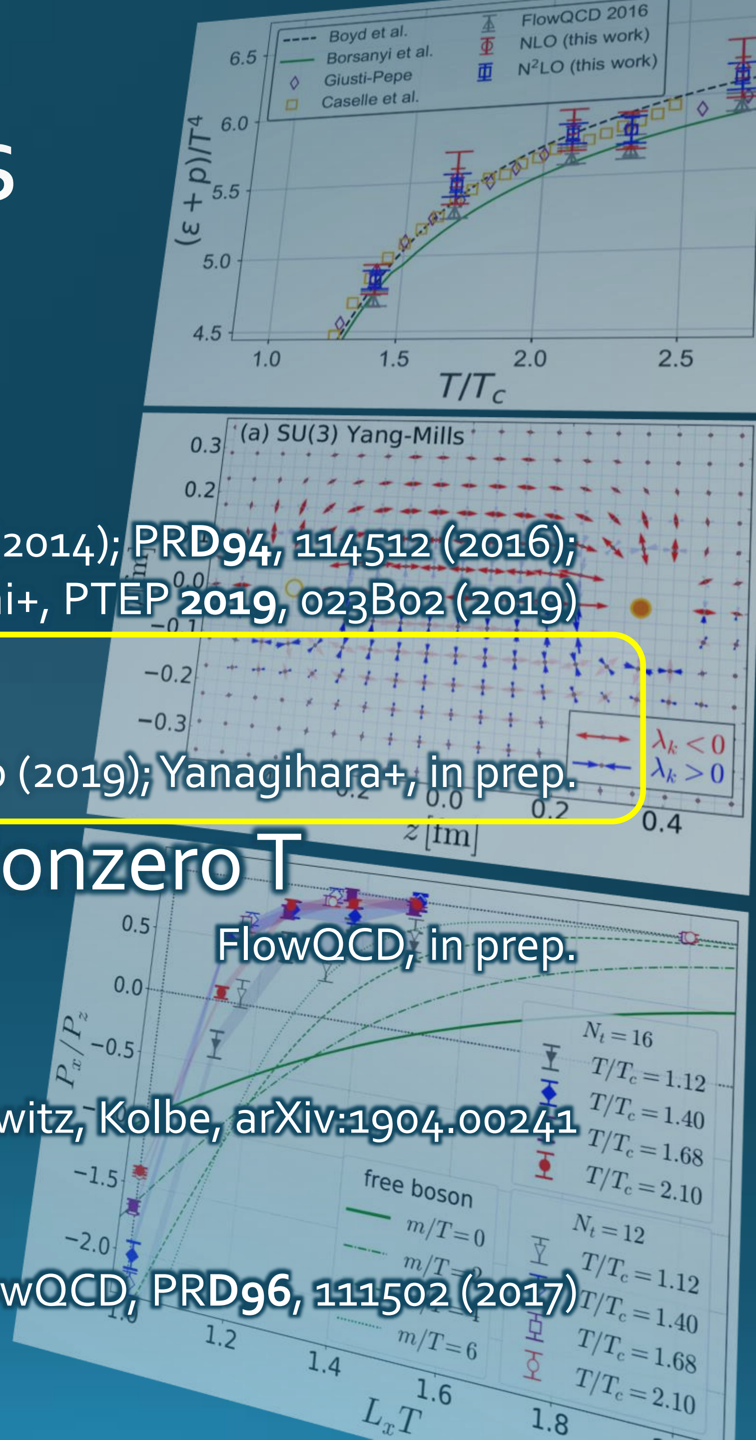
FlowQCD, in prep.

5. Casimir Effect

MK, Mogliacci, Horowitz, Kolbe, arXiv:1904.00241

6. Correlation Function

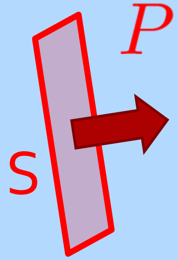
FlowQCD, PRD**96**, 111502 (2017)



Stress = Force per Unit Area

Stress = Force per Unit Area

Pressure

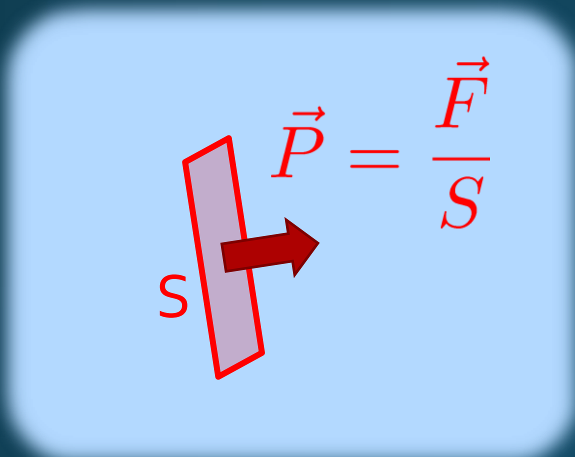


$$\vec{P} = \frac{\vec{F}}{S}$$

$$\vec{P} = P\vec{n}$$

Stress = Force per Unit Area

Pressure

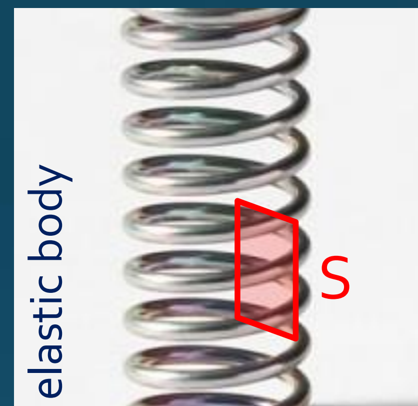


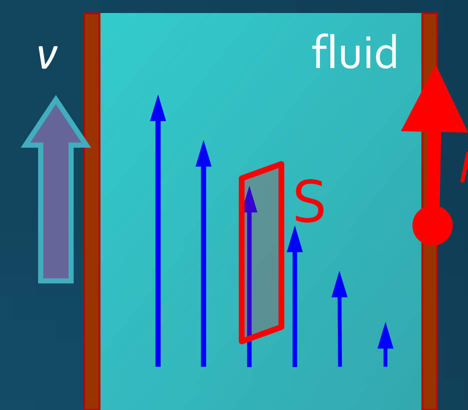
$$\vec{P} = P\vec{n}$$

In thermal medium

$$T_{ij} = P\delta_{ij}$$

Generally, F and n are not parallel





$$\frac{F_i}{S} = \sigma_{ij} n_j$$

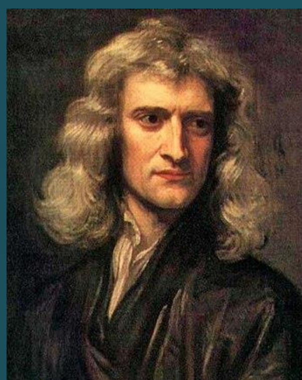
Stress Tensor

$$\sigma_{ij} = -T_{ij}$$

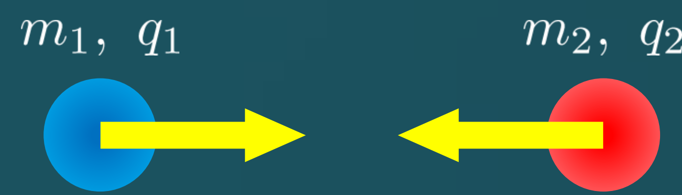
Landau
Lifshitz

Force

Action-at-a-distance



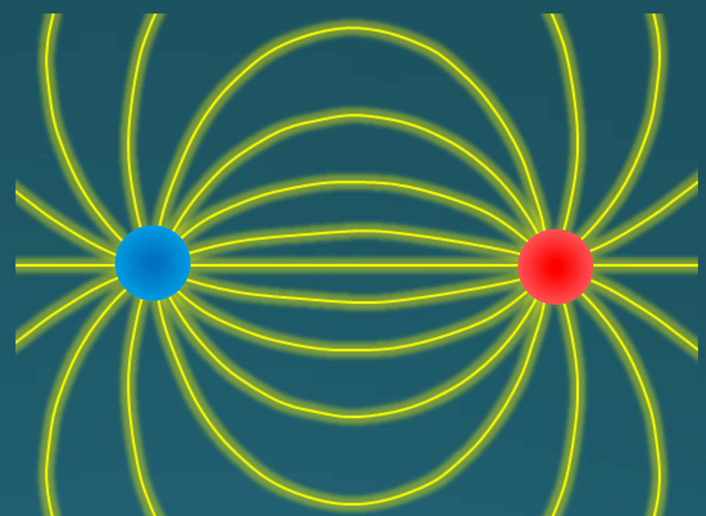
Newton
1687



$$F = -G\frac{m_1 m_2}{r^2} \qquad F = -\frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}$$

Local interaction

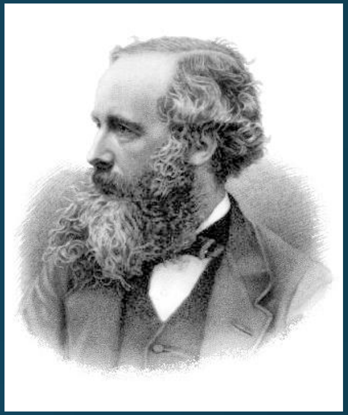
Faraday
1839



Maxwell Stress

(in Maxwell Theory)

$$\sigma_{ij} = \varepsilon_0 E_i E_j + \frac{1}{\mu_0} B_i B_j - \frac{1}{2}\delta_{ij}\left(\varepsilon_0 E^2 + \frac{1}{\mu_0} B^2\right)$$

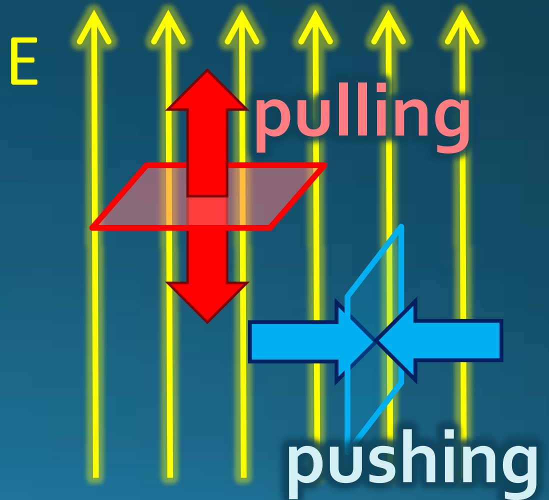


Maxwell

$$\vec{E} = (E, 0, 0)$$

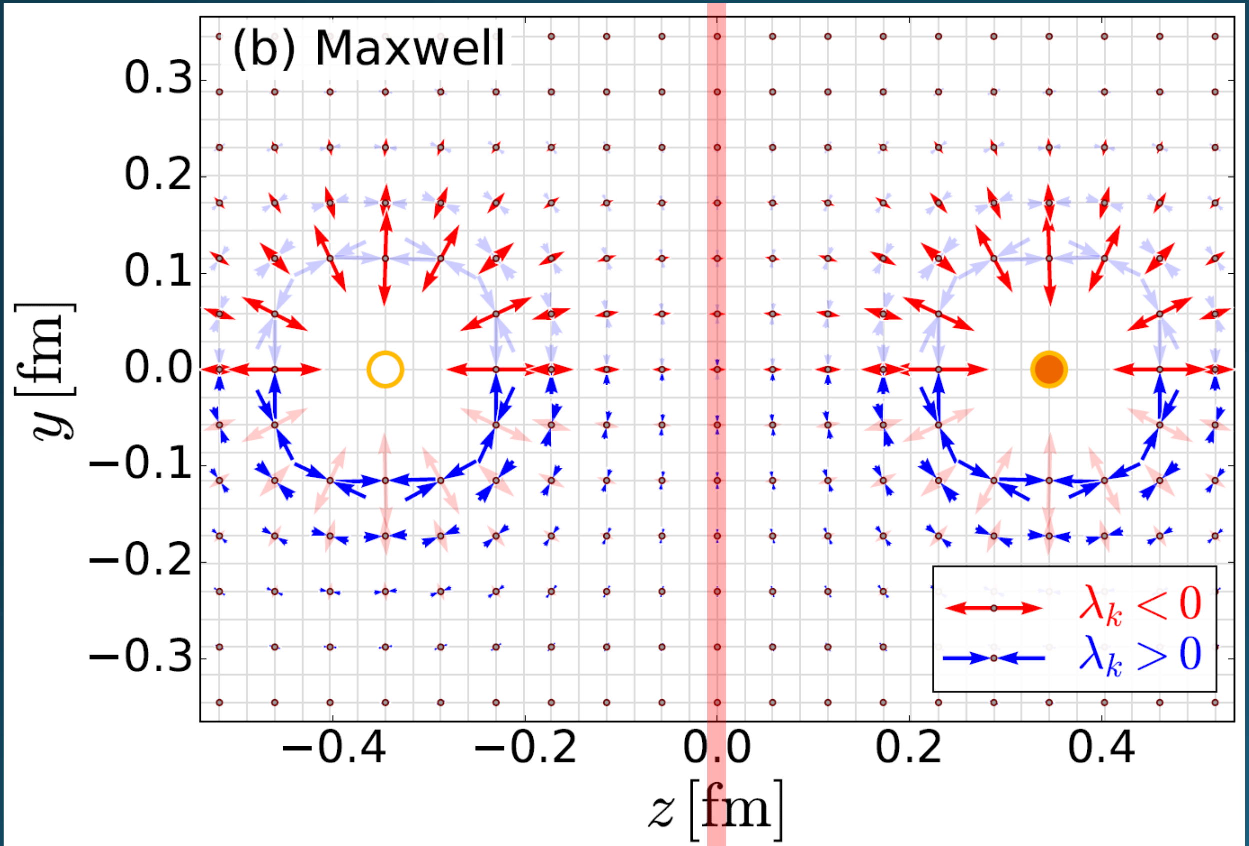
$$T_{ij} = \begin{pmatrix} -E^2 & 0 & 0 \\ 0 & E^2 & 0 \\ 0 & 0 & E^2 \end{pmatrix}$$

- Parallel to field: **Pulling**
- Vertical to field: **Pushing**



Maxwell Stress

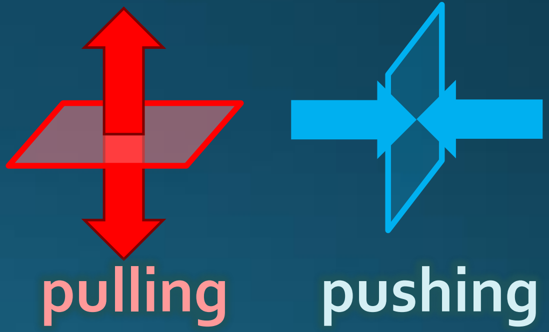
(in Maxwell Theory)



$$T_{ij} v_j^{(k)} = \lambda_k v_i^{(k)}$$

$$(k = 1, 2, 3)$$

length: $\sqrt{|\lambda_k|}$

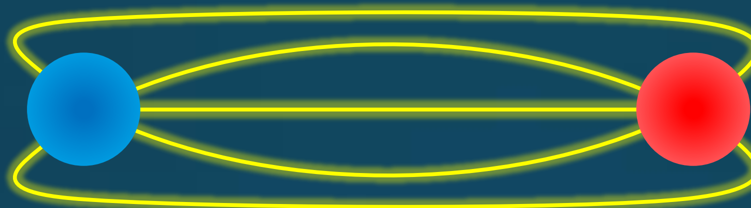


Definite physical meaning

- ☐ Distortion of field, line of the field
- ☐ Propagation of the force as local interaction

Quark-Anti-quark system

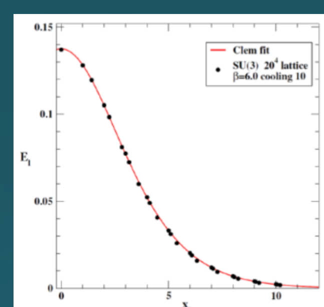
Formation of the flux tube → confinement



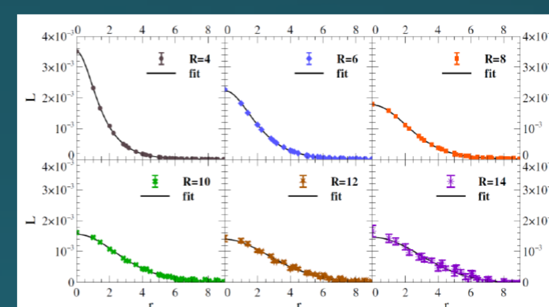
Previous Studies on Flux Tube

- ☐ Potential
- ☐ Action density
- ☐ Color-electric field

so many studies…



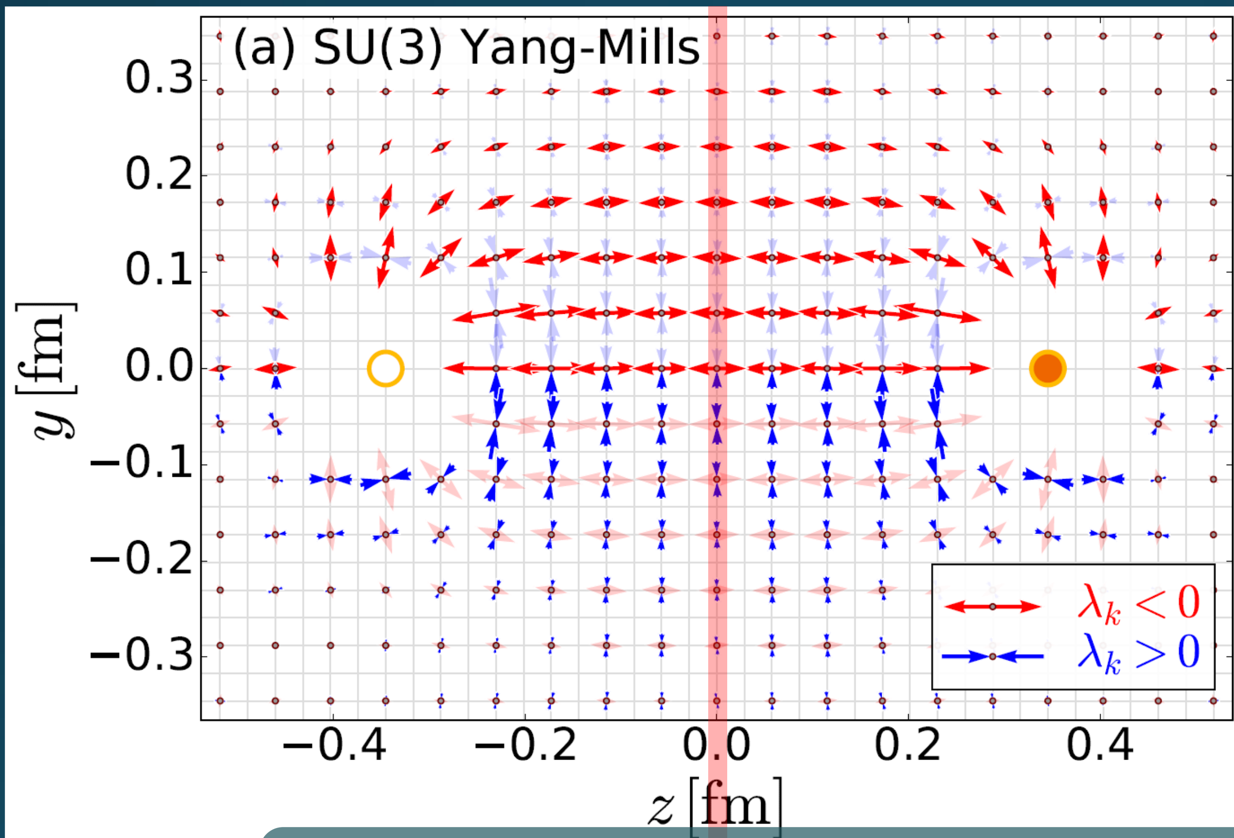
Cea+ (2012)



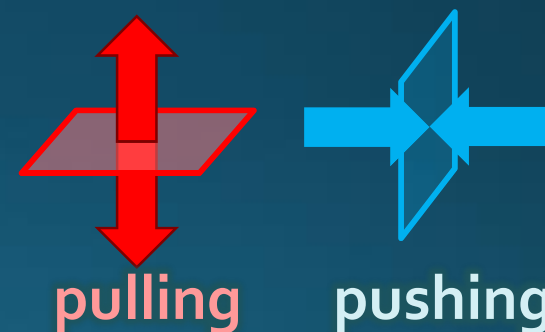
Cardoso+ (2013)

Stress Tensor in $Q\bar{Q}$ System

Yanagihara+, 1803.05656
PLB, in press



Lattice simulation
SU(3) Yang-Mills
a=0.029 fm
R=0.69 fm
t/a^2=2.0

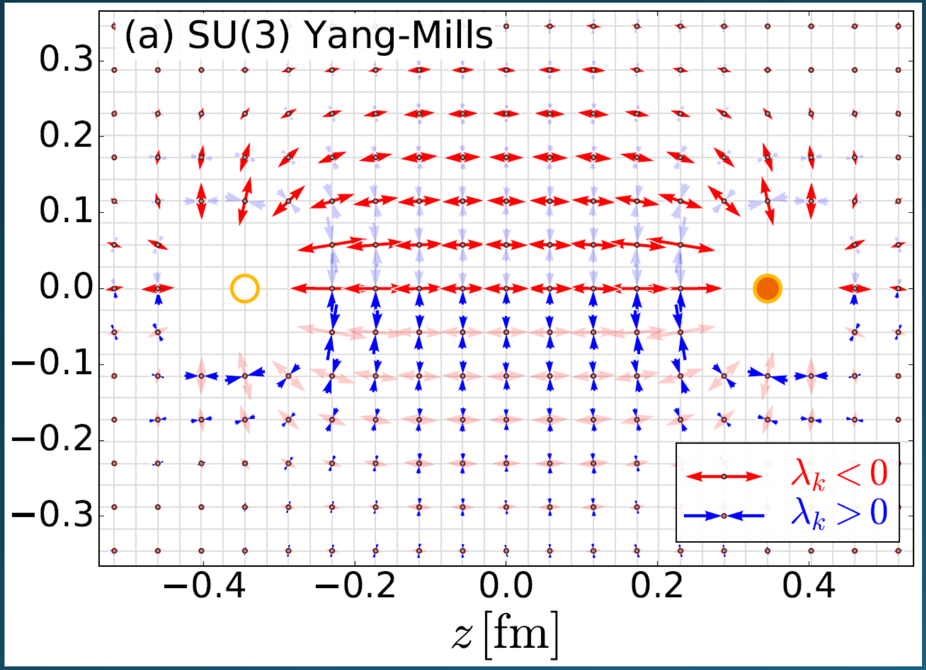


Definite physical meaning

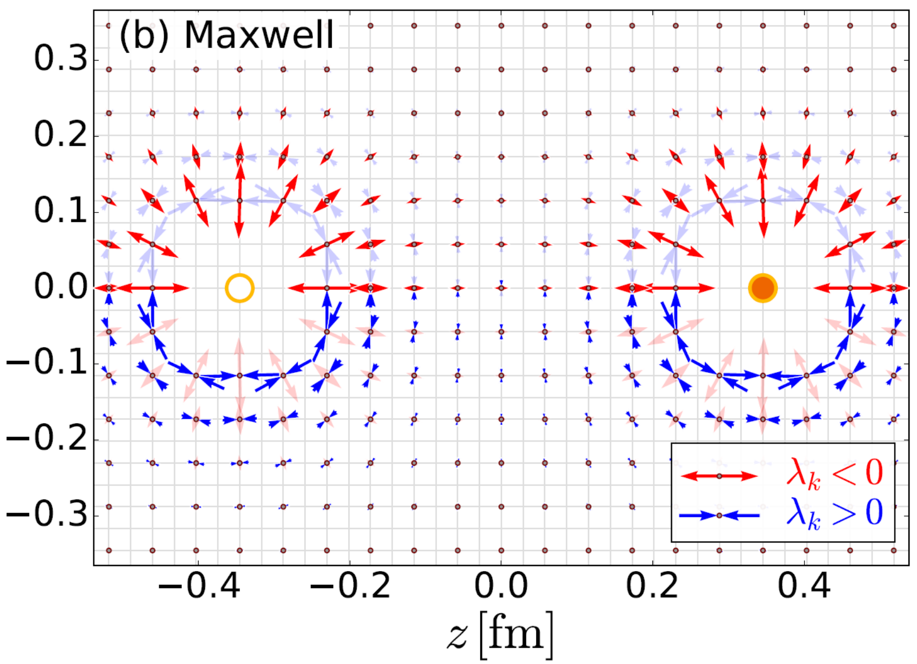
- ☐ Distortion of field, line of the field
- ☐ Propagation of the force as local interaction
- ☐ Manifestly gauge invariant

SU(3) YM vs Maxwell

SU(3) Yang-Mills
(quantum)



Maxwell
(classical)



Propagation of the force is clearly different in YM and Maxwell theories!

Lattice Setup

Yanagihara+, 1803.05656

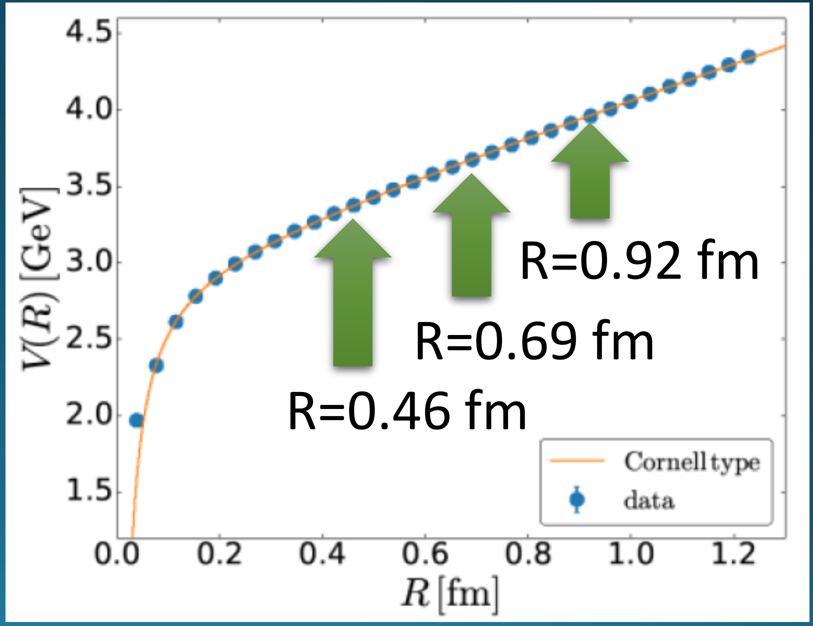
- SU(3) Yang-Mills (Quenched)
- Wilson gauge action
- Clover operator

- APE smearing / multi-hit

- fine lattices (a=0.029-0.06 fm)
- continuum extrapolation

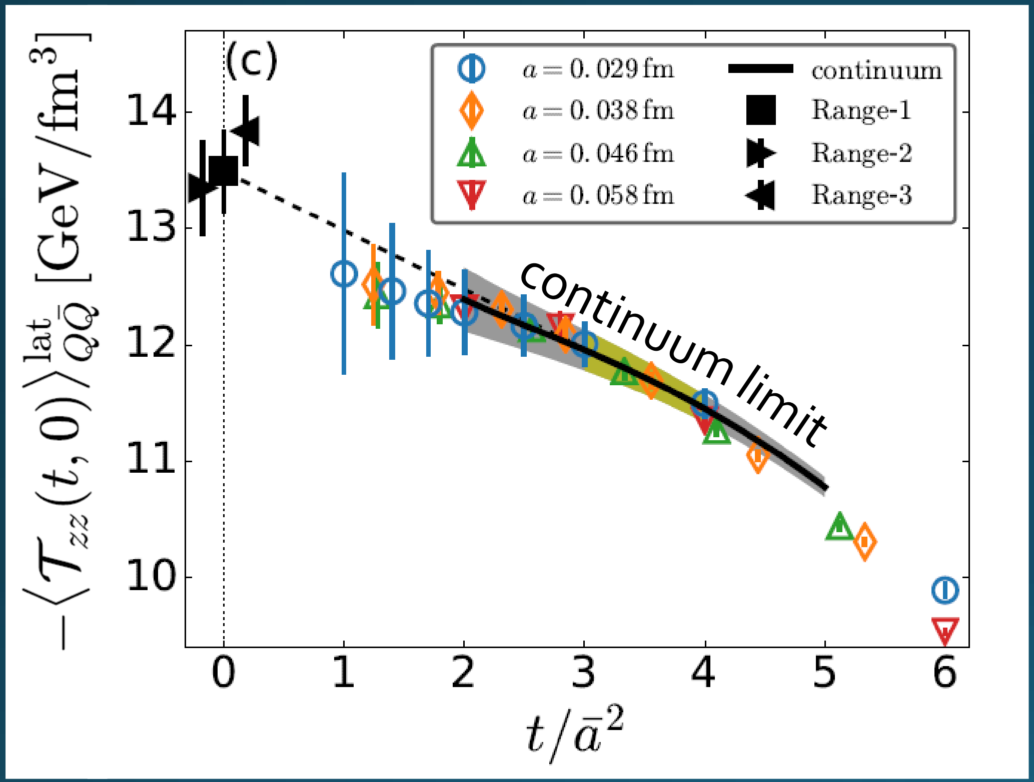
- Simulation: bluegene/Q@KEK

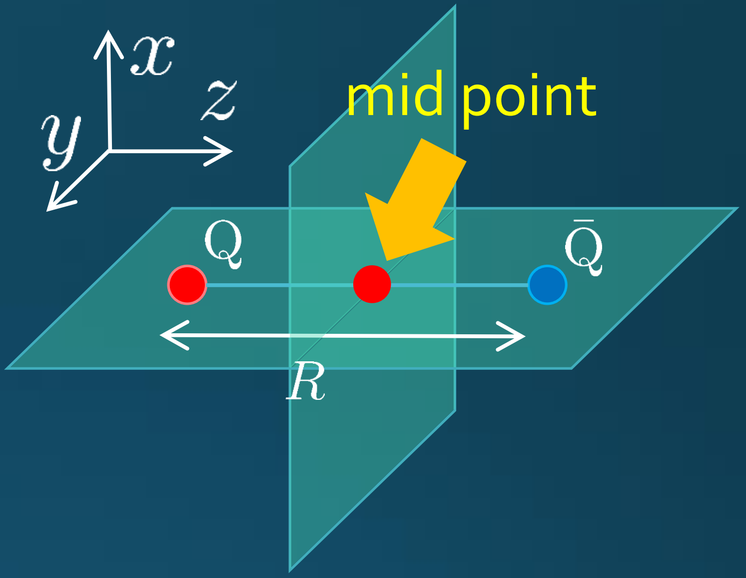
β	a [fm]	$N_{\rm size}^4$	$N_{\rm conf}$	R/a		
6.304	0.058	48^4	140	8	12	16
6.465	0.046	48^4	440	10	–	20
6.513	0.043	48^4	600	–	16	–
6.600	0.038	48^4	1,500	12	18	24
6.819	0.029	64^4	1,000	16	24	32
			R [fm]	0.46	0.69	0.92

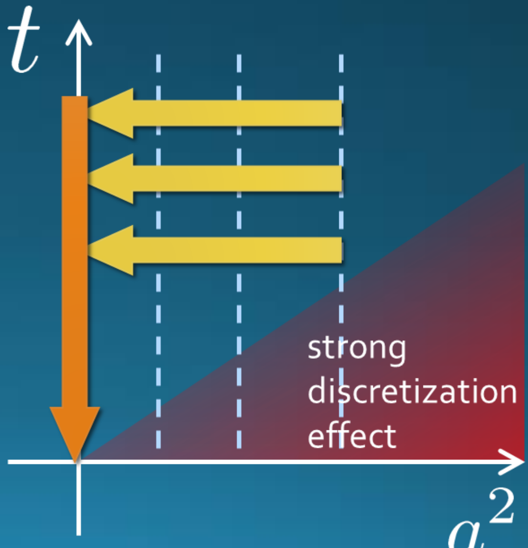


$$\langle O(x)\rangle_{Q\bar{Q}} = \lim_{T\to\infty} \frac{\langle \delta O(x)\delta W(R,T)\rangle}{\langle W(R,T)\rangle}$$

Continuum Extrapolation
at mid-point



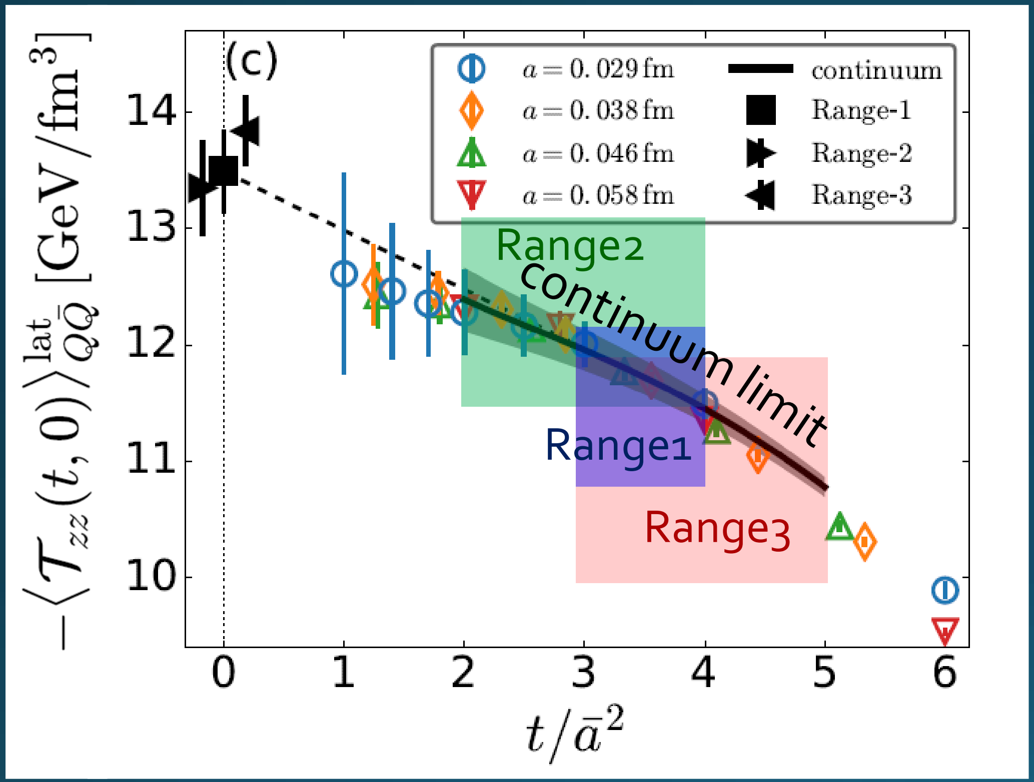


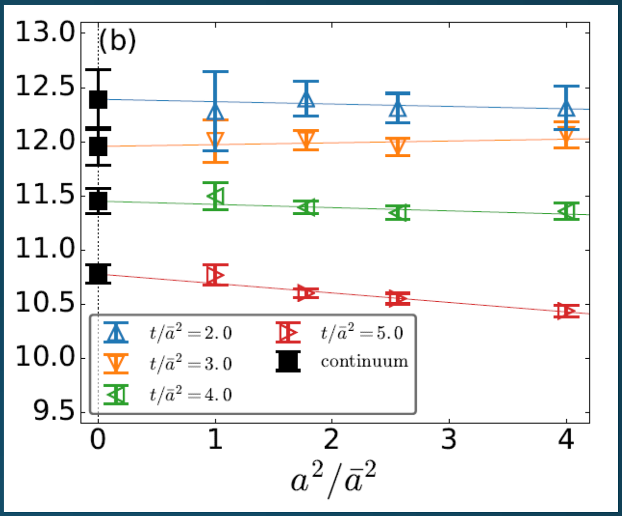


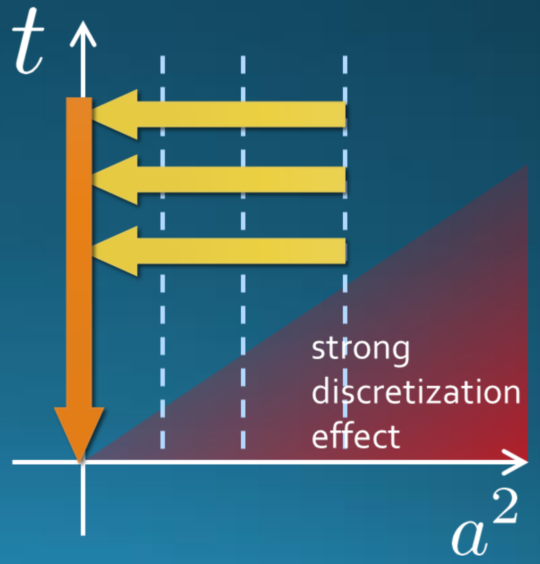
- a→0 extrapolation with fixed t

t→0 Extrapolation

at mid-point







- ☐ a→0 extrapolation with fixed t
- ☐ Then, t→0 with three ranges

Stress Distribution on Mid-Plane

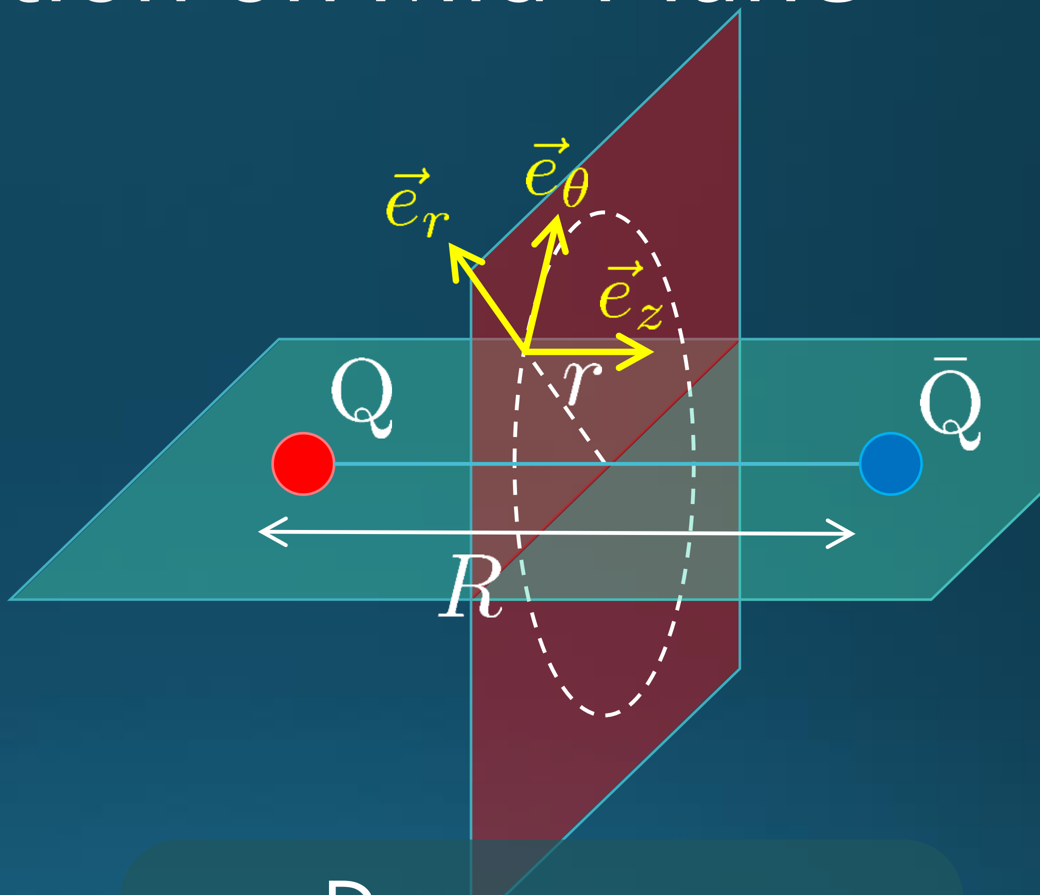
From rotational symm. & parity

EMT is diagonalized
in Cylindrical Coordinates

$$T_{cc'}(r) = \begin{pmatrix} T_{rr} & & & \\ & T_{\theta\theta} & & \\ & & T_{zz} & \\ & & & T_{44} \end{pmatrix}$$

$$T_{rr} = \vec{e}_r^{\,T} T \vec{e}_r$$

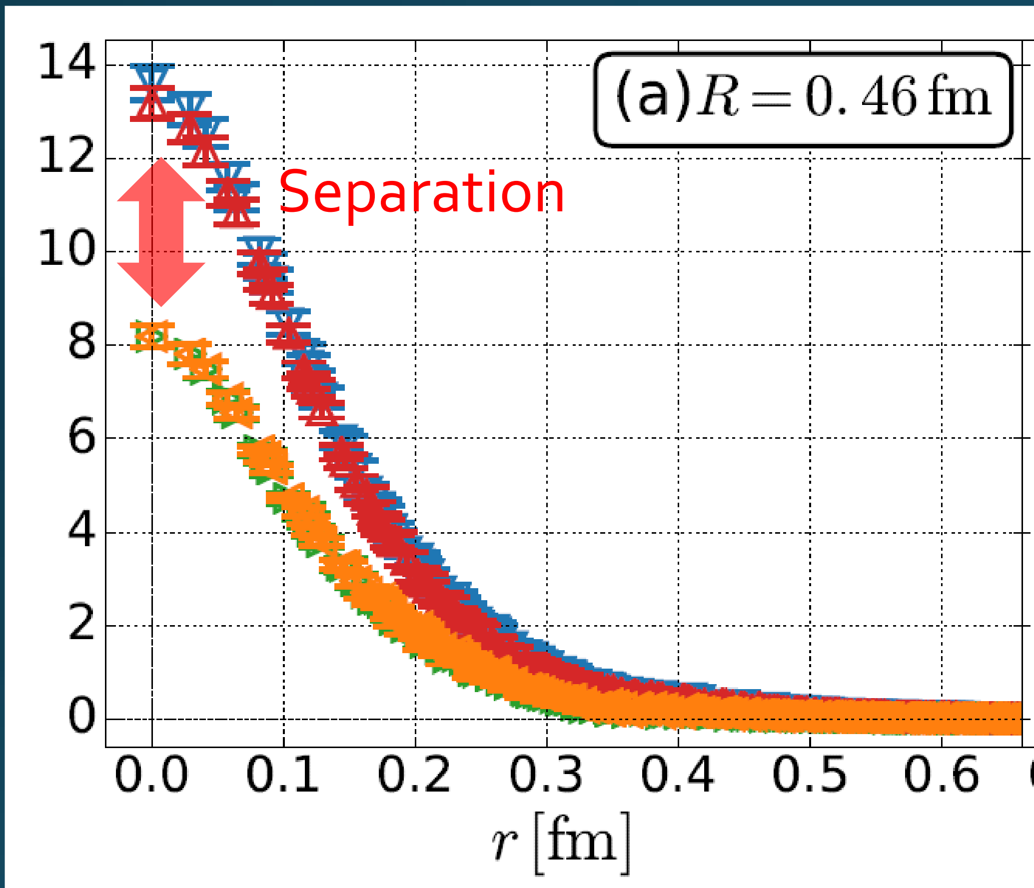
$$T_{\theta\theta} = \vec{e}_\theta^{\,T} T \vec{e}_\theta$$

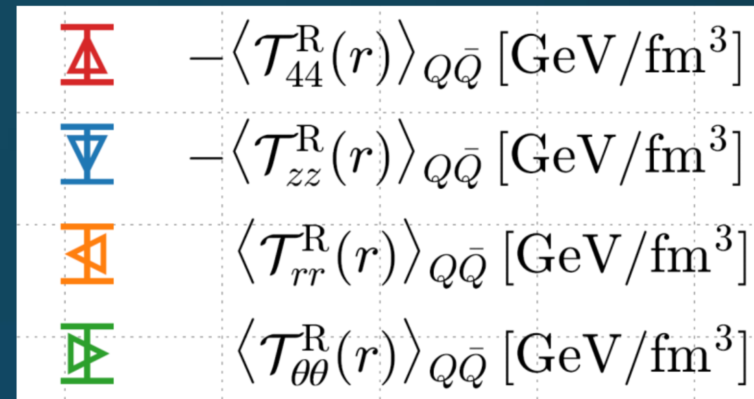


Degeneracy
in Maxwell theory

$$T_{rr} = T_{\theta\theta} = -T_{zz} = -T_{44}$$

Mid-Plane





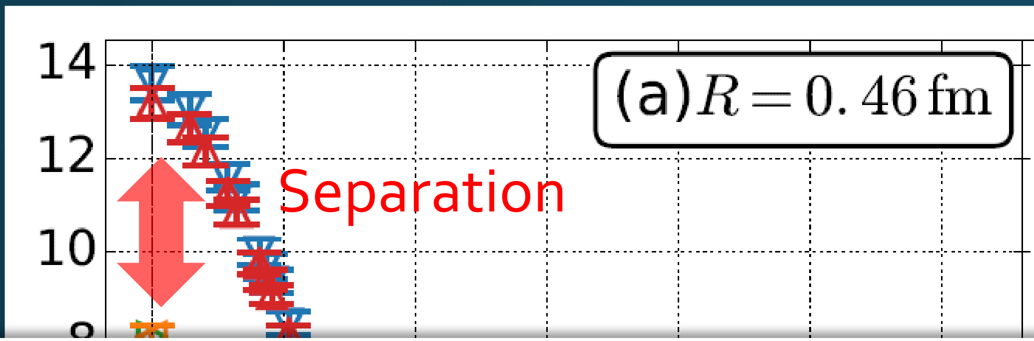
Continuum Extrapolated!

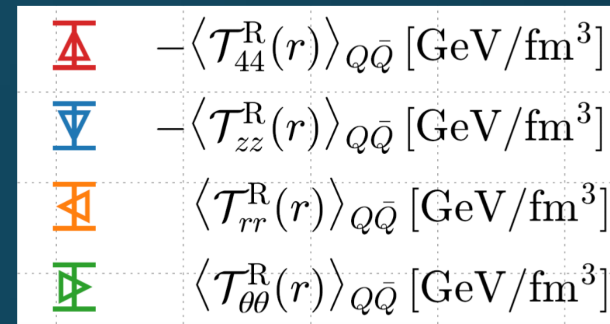
In Maxwell theory

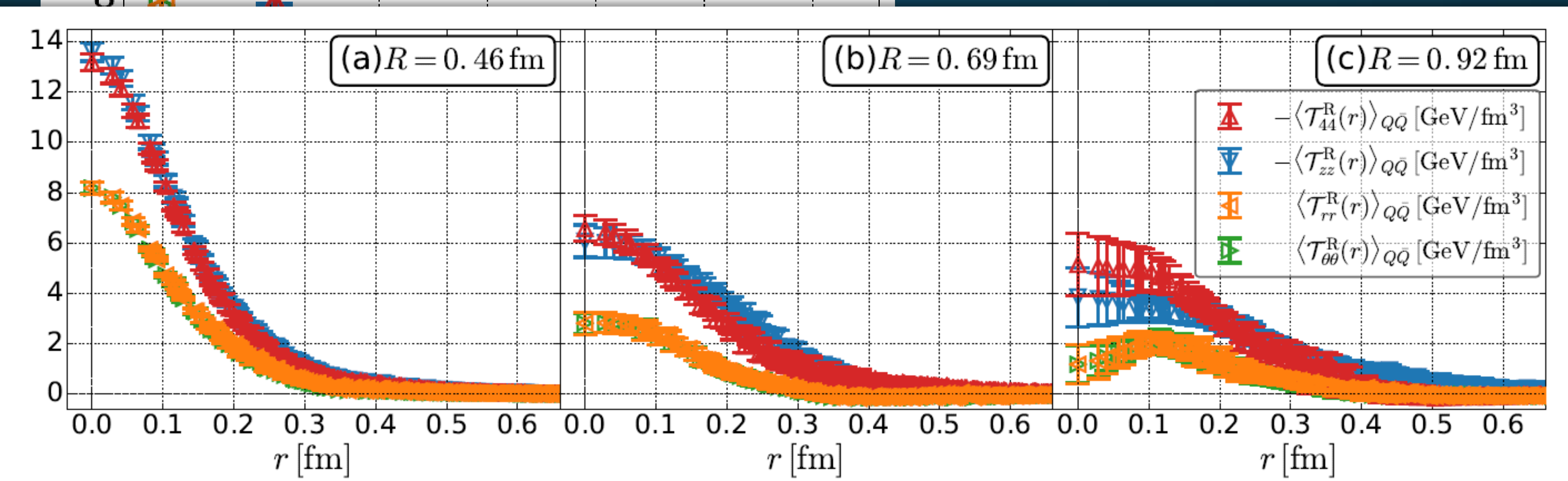
$$T_{rr} = T_{\theta\theta} = -T_{zz} = -T_{44}$$

- ☐ Degeneracy: $T_{44} \simeq T_{zz}, \quad T_{rr} \simeq T_{\theta\theta}$
- ☐ Separation: $T_{zz} \neq T_{rr}$
- ☐ Nonzero trace anomaly $\sum T_{cc} \neq 0$

Mid-Plane







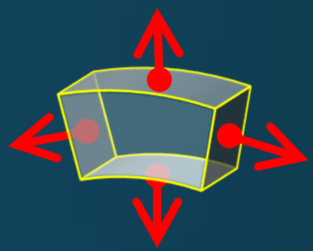
- ☐ Degeneracy: $T_{44} \simeq T_{zz}, \quad T_{rr} \simeq T_{\theta\theta}$
- ☐ Separation: $T_{zz} \neq T_{rr}$
- ☐ Nonzero trace anomaly $\sum T_{cc} \neq 0$

Momentum Conservation

Yanagihara+, in prep.

- In cylindrical coordinats,

$$\partial_i T_{ij} = 0 \Rightarrow \partial_r (r T_{rr}) = T_{\theta\theta} - r \partial_z T_{rz}$$

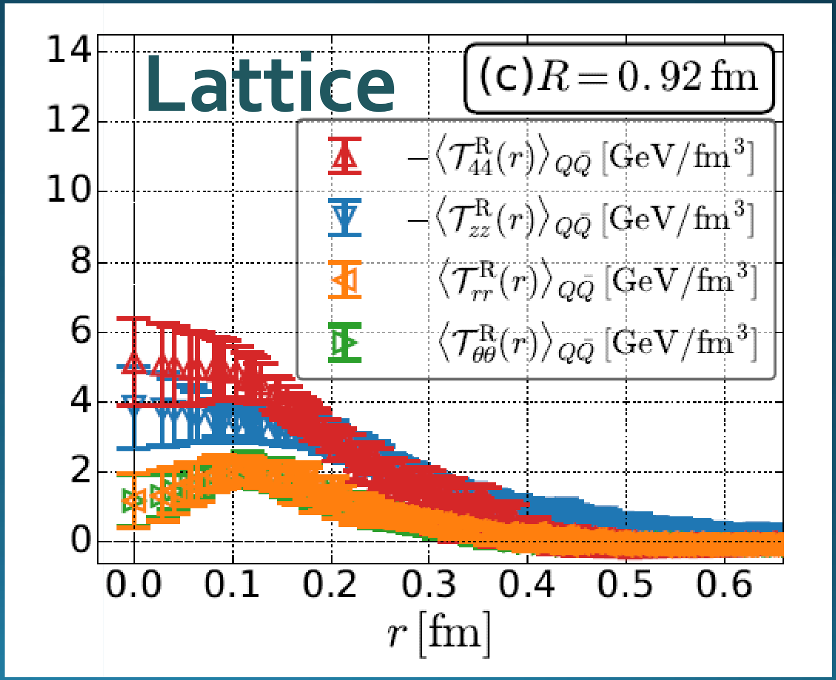


- For infinitely-long flux tube

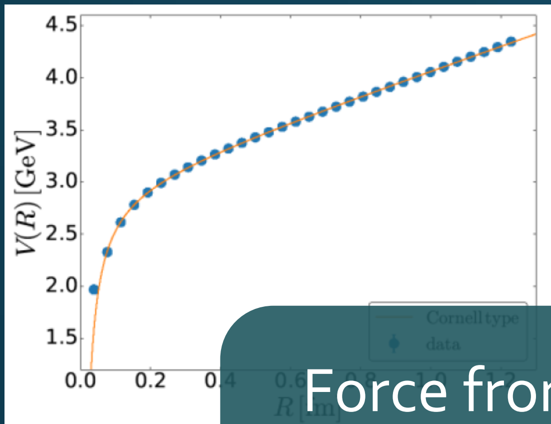
$$\partial_r (r T_{rr}) = T_{\theta\theta}$$

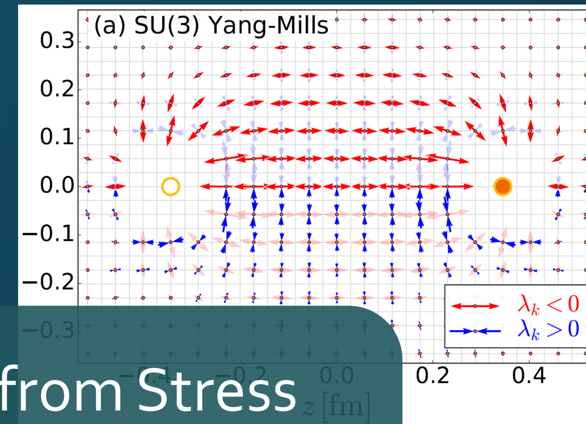
⇒ **T_{rr} and $T_{\theta\theta}$ must separate!**

Effect of boundaries is important for the flux tube at R=0.92fm



Force





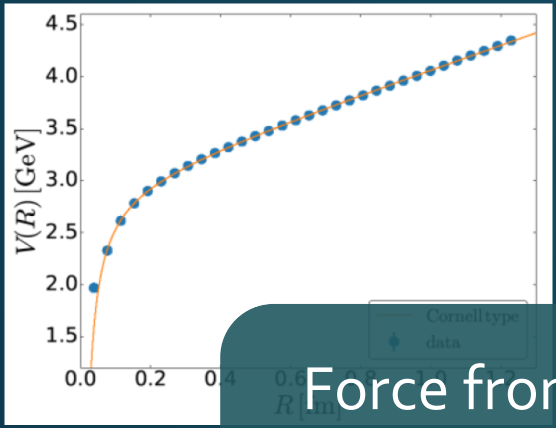
Force from Potential

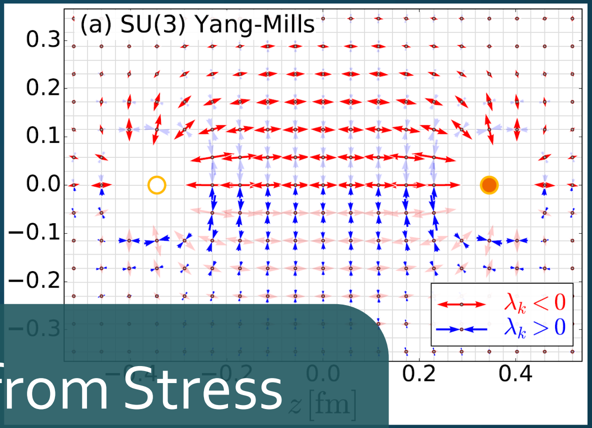
$$F_{\rm pot} = -\frac{dV}{dR}$$

Force from Stress

$$F_{\rm stress} = \int_{\rm mid.} d^2x T_{zz}(x)$$

Force



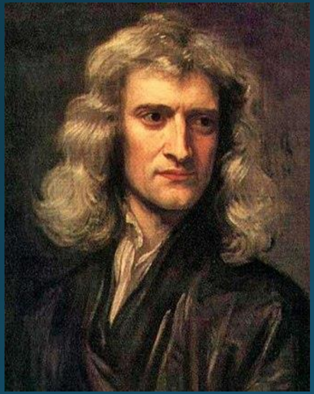


Force from Potential

$$F_{\text{pot}} = -\frac{dV}{dR}$$

Force from Stress

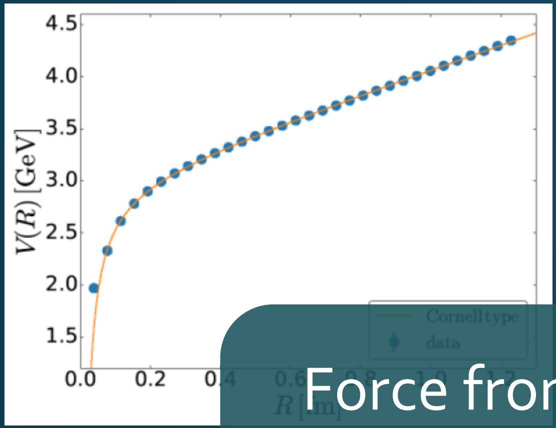
$$F_{\text{stress}} = \int_{\text{mid.}} d^2x T_{zz}(x)$$

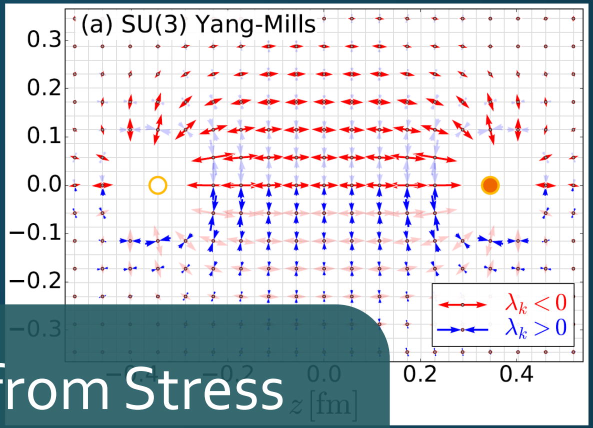


Newton
1687

Faraday
1839

Force



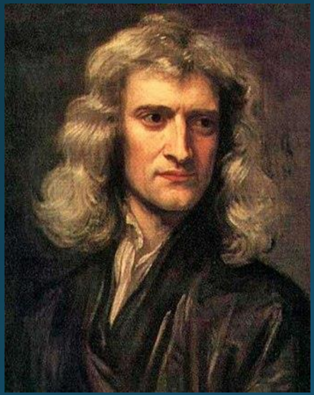


Force from Potential

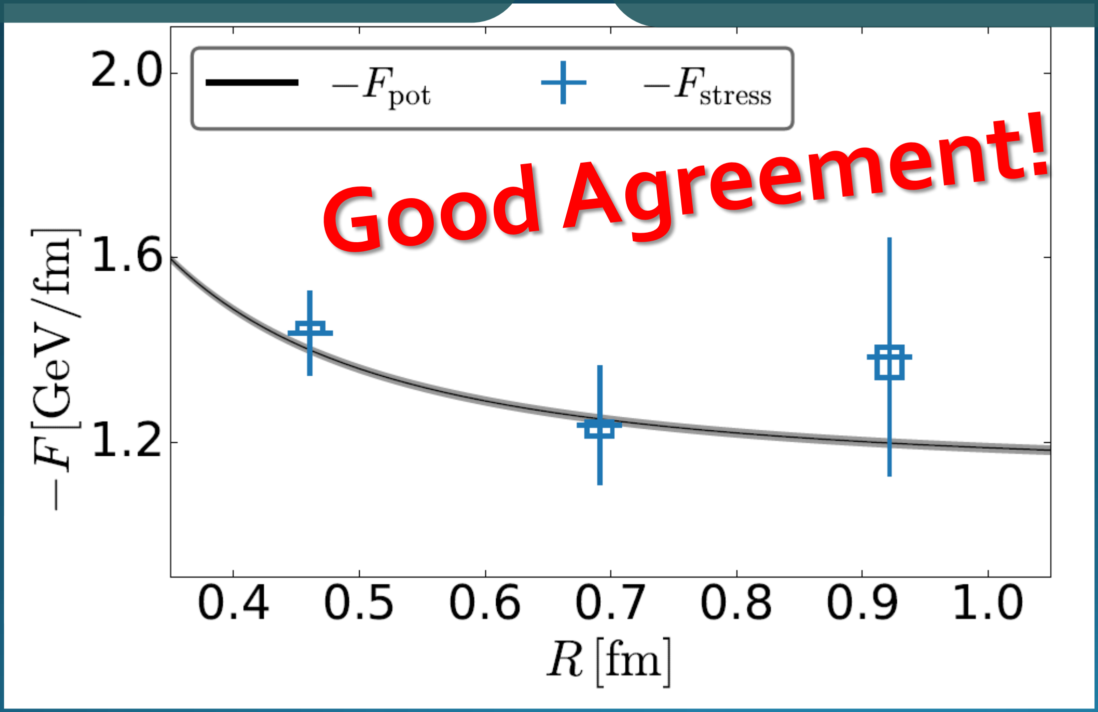
$$F_{\text{pot}} = -\frac{dV}{dR}$$

Force from Stress

$$F_{\text{stress}} = \int_{\text{mid.}} d^2x T_{zz}(x)$$



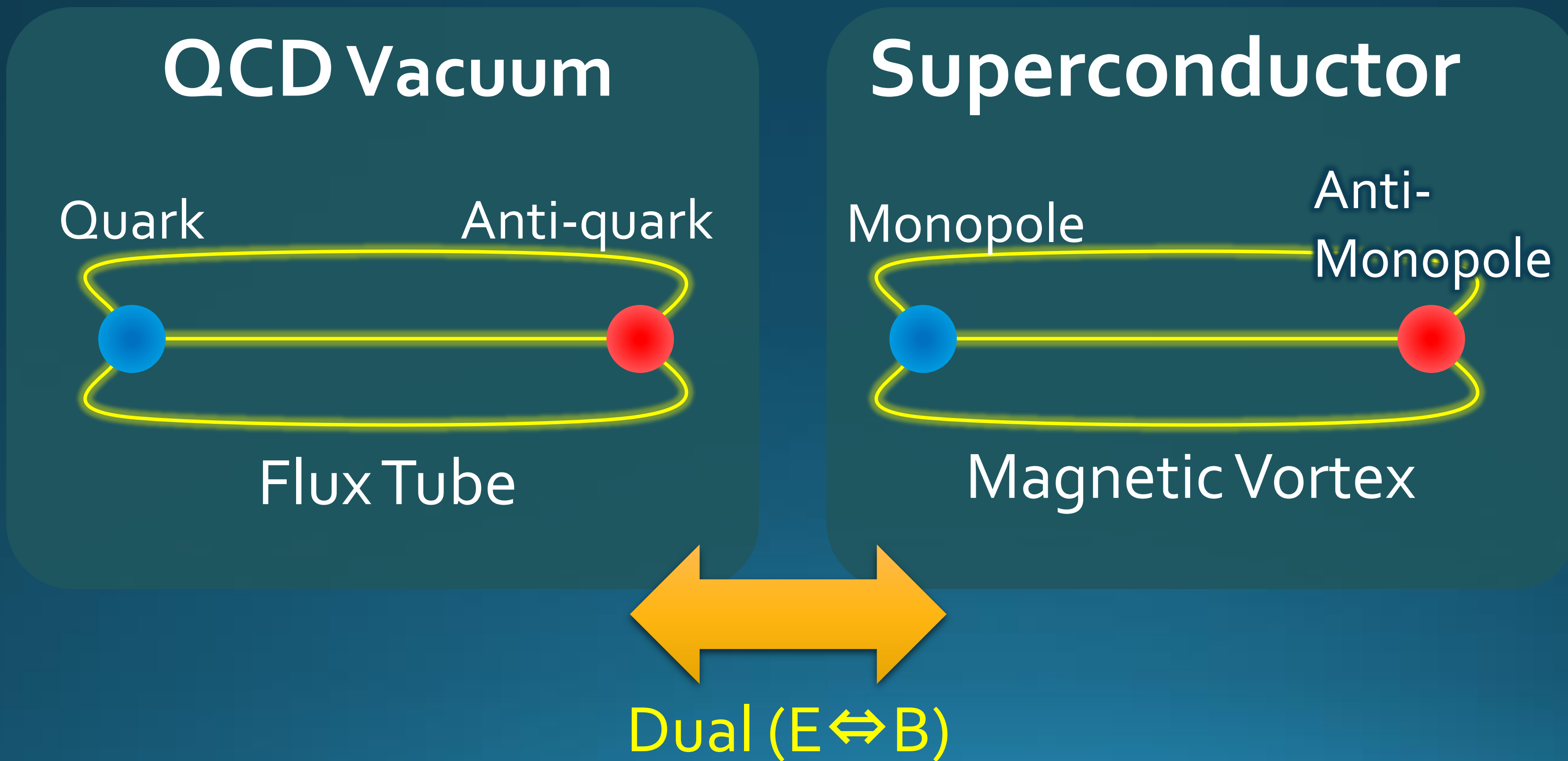
Newton
1687



Faraday
1839

Dual Superconductor Picture

Nambu, 1970
Nielsen, Olesen, 1973
t 'Hooft, 1981
...



Abelian-Higgs Model

Yanagihara, Iritani, MK, in prep.

Abelian-Higgs Model

$$\mathcal{L}_{\rm AH} = -\frac{1}{4}F_{\mu\nu}^2 + |(\partial_\mu + igA_\mu)\phi|^2 - \lambda(\phi^2 - v^2)^2$$

GL parameter: $\kappa = \sqrt{\lambda}/g$

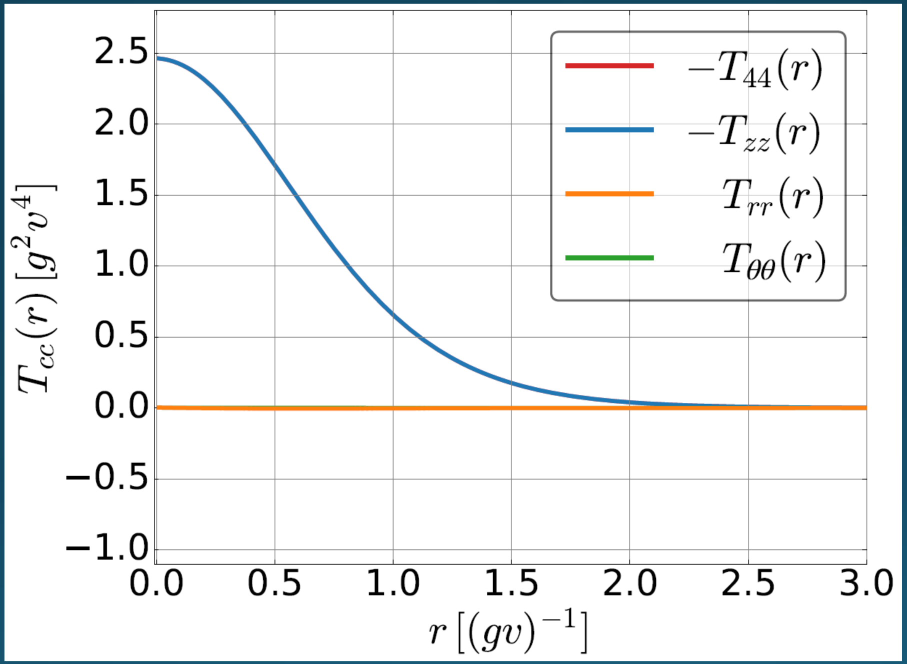
- type-I : $\kappa < 1/\sqrt{2}$
- type-II : $\kappa > 1/\sqrt{2}$
- Bogomol'nyi bound : $\kappa = 1/\sqrt{2}$

Infinitely long tube

- degeneracy
  $T_{zz}(r) = T_{44}(r)$ Luscher, 1981
- momentum conservation
  $$\frac{d}{dr}(rT_{rr}) = T_{\theta\theta}$$

Stress Tensor in AH Model
infinitely-long flux tube

Bogomol'nyi bound : $\kappa = 1/\sqrt{2}$



$$T_{rr} = T_{\theta\theta} = 0$$

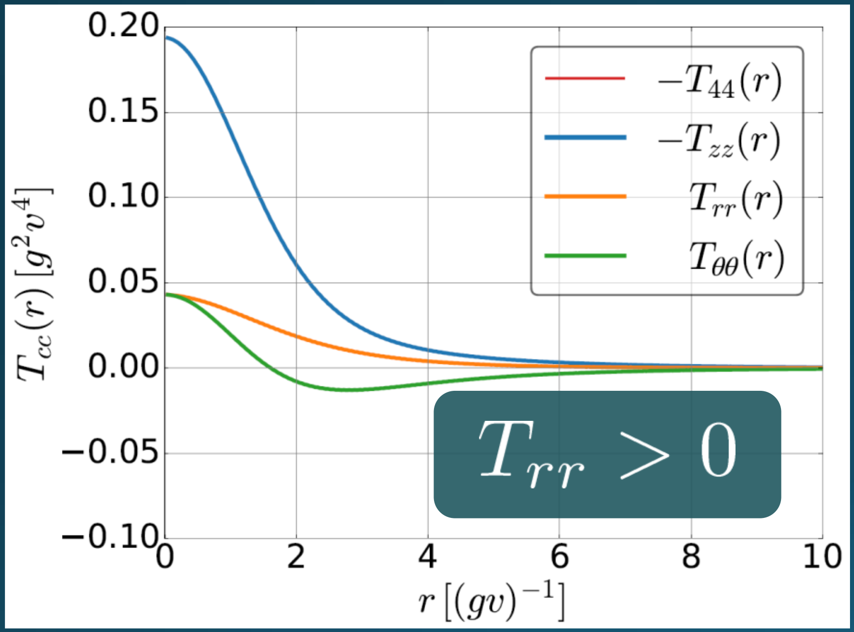
de Vega, Schaposnik, PR**D14**, 1100 (1976).

Stress Tensor in AH Model

infinitely-long flux tube

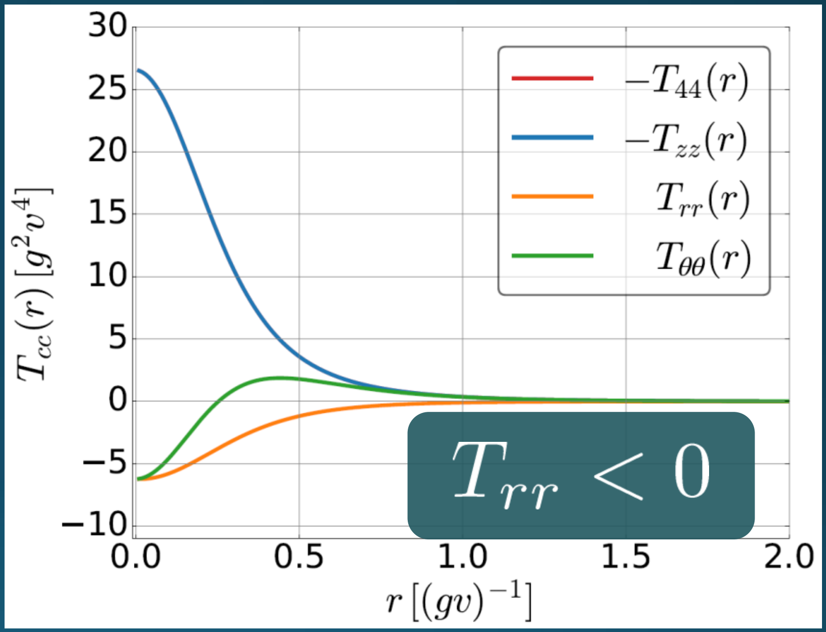
Type-I

$\kappa = 0.1$



Type-II

$\kappa = 3.0$



- ☐ No degeneracy bw T_{rr} & $T_{\theta\theta}$
- ☐ $T_{\theta\theta}$ changes sign

conservation law

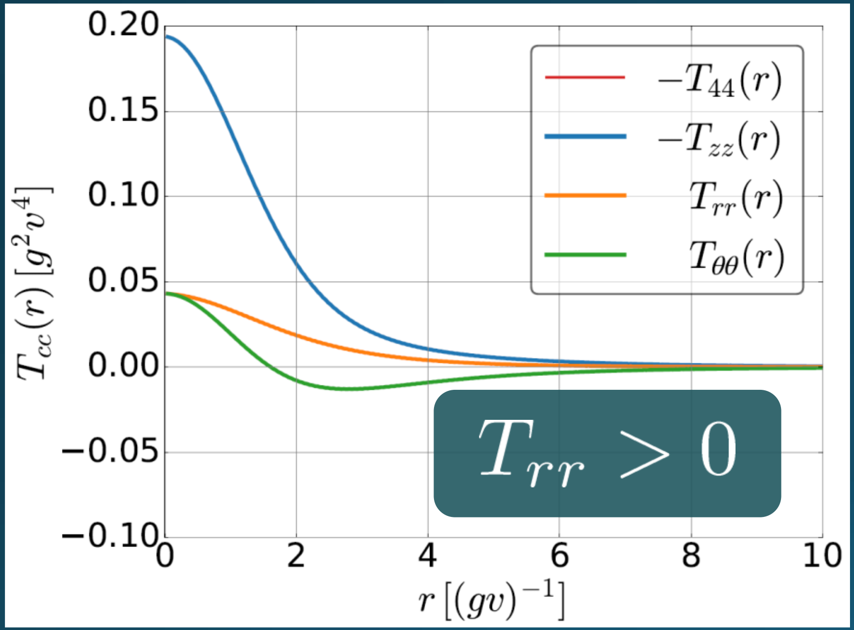
$$\frac{d}{dr}(rT_{rr}) = T_{\theta\theta}$$

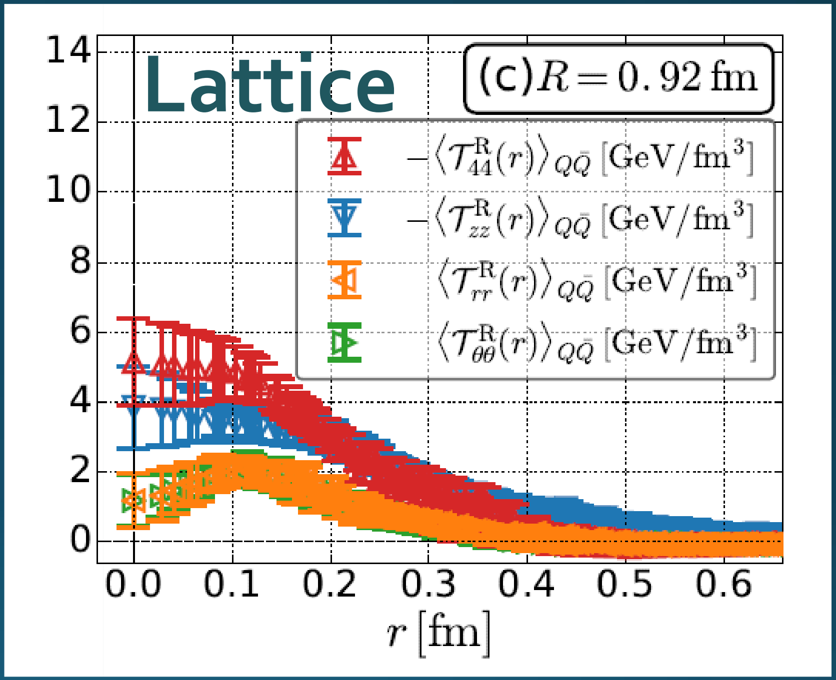
Stress Tensor in AH Model

infinitely-long flux tube

Type-I

$\kappa = 0.1$





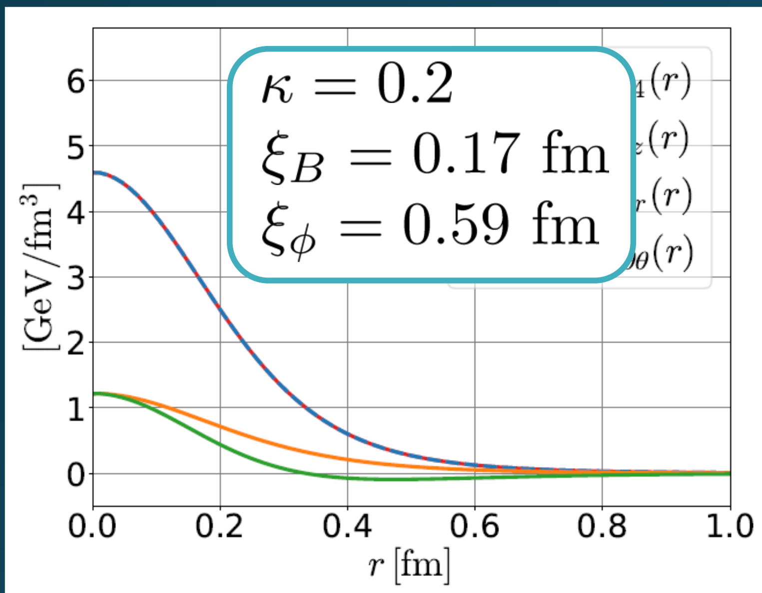
- ☐ No degeneracy bw T_{rr} & $T_{\theta\theta}$
- ☐ $T_{\theta\theta}$ changes sign

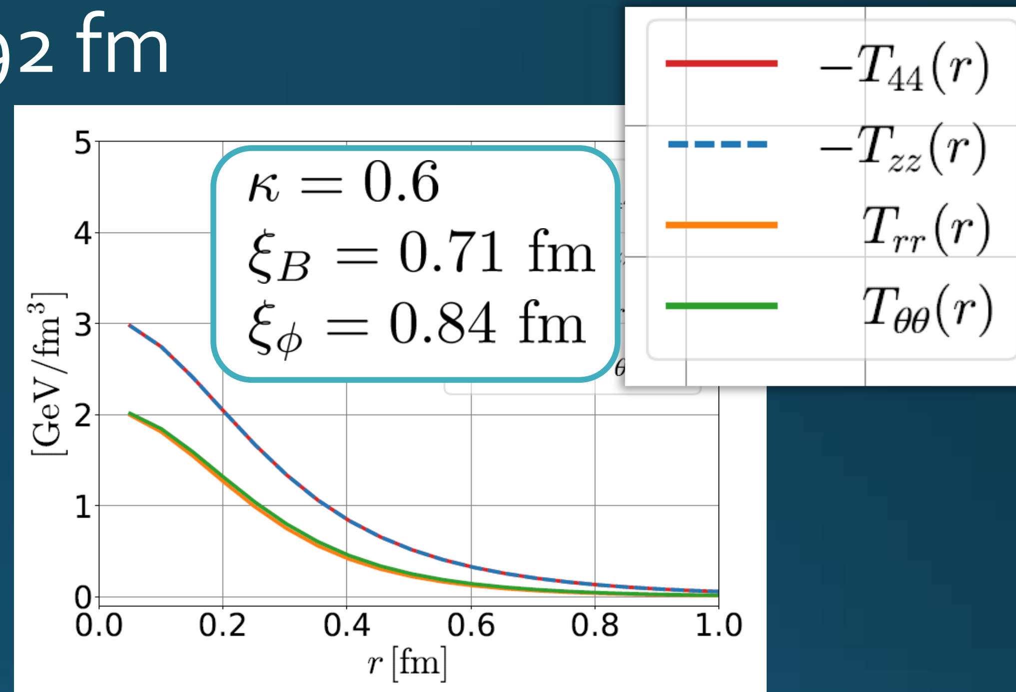
Inconsistent with lattice result

$$T_{rr} \simeq T_{\theta\theta}$$

Flux Tube with Finite Length

R=0.92 fm

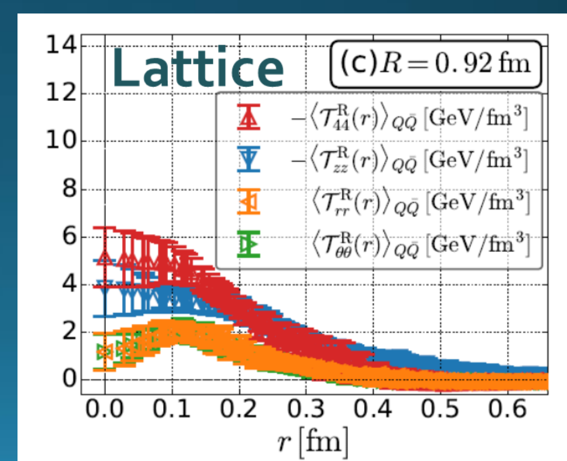




Left: $T_{zz}(0)$, $T_{rr}(0)$ reproduce lattice result

Right: A parameter satisfying $T_{rr} \approx T_{\theta\theta}$

No parameters to reproduce lattice data at R=0.92fm.



Contents

1. Constructing EMT
2. Thermodynamics

FlowQCD, PR**D90**,011501 (2014); PR**D94**, 114512 (2016);
WHOT-QCD, PR**D96**, 014509 (2017); Iritani+, PTEP **2019**, 023B02 (2019)

3. Flux Tube

FlowQCD, PLB789, 210 (2019); Yanagihara+, in prep.

4. Static Quark Systems at Nonzero T

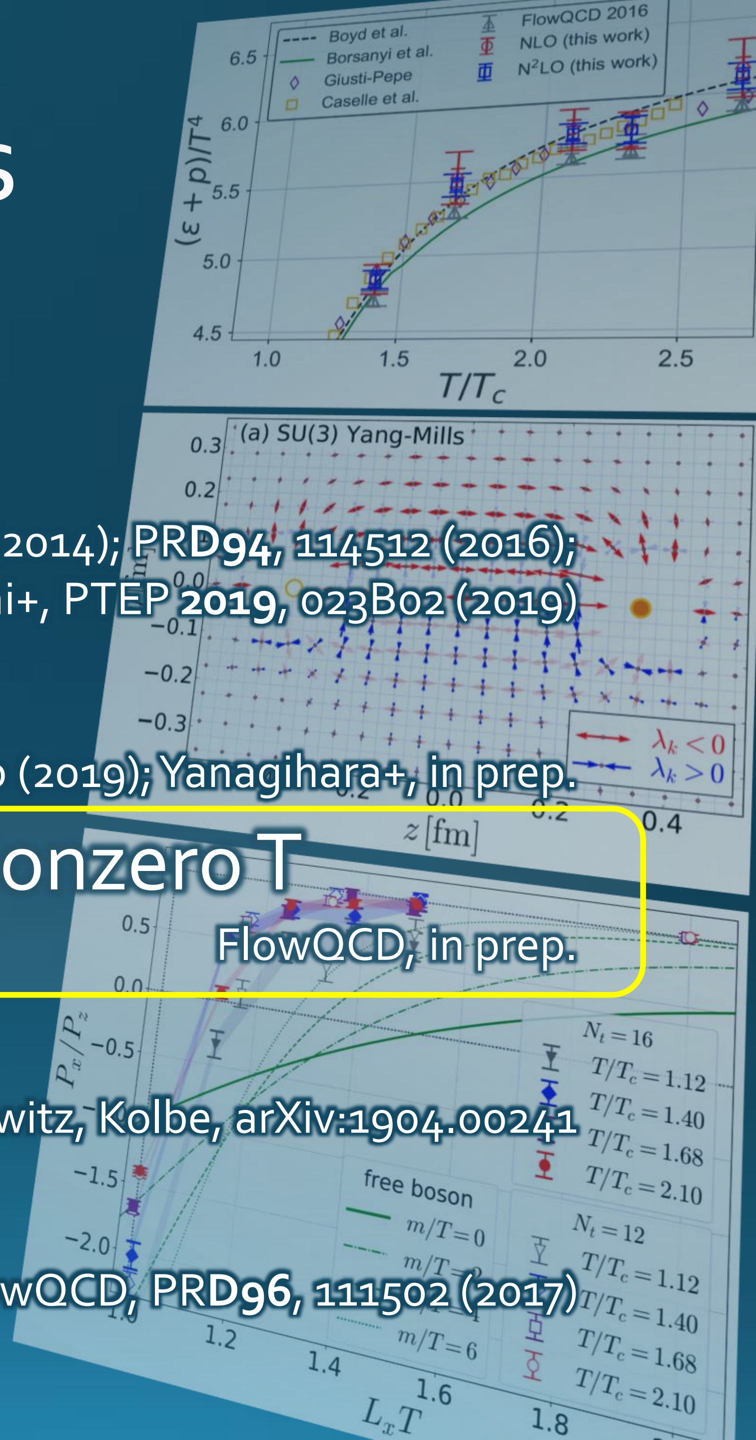
FlowQCD, in prep.

5. Casimir Effect

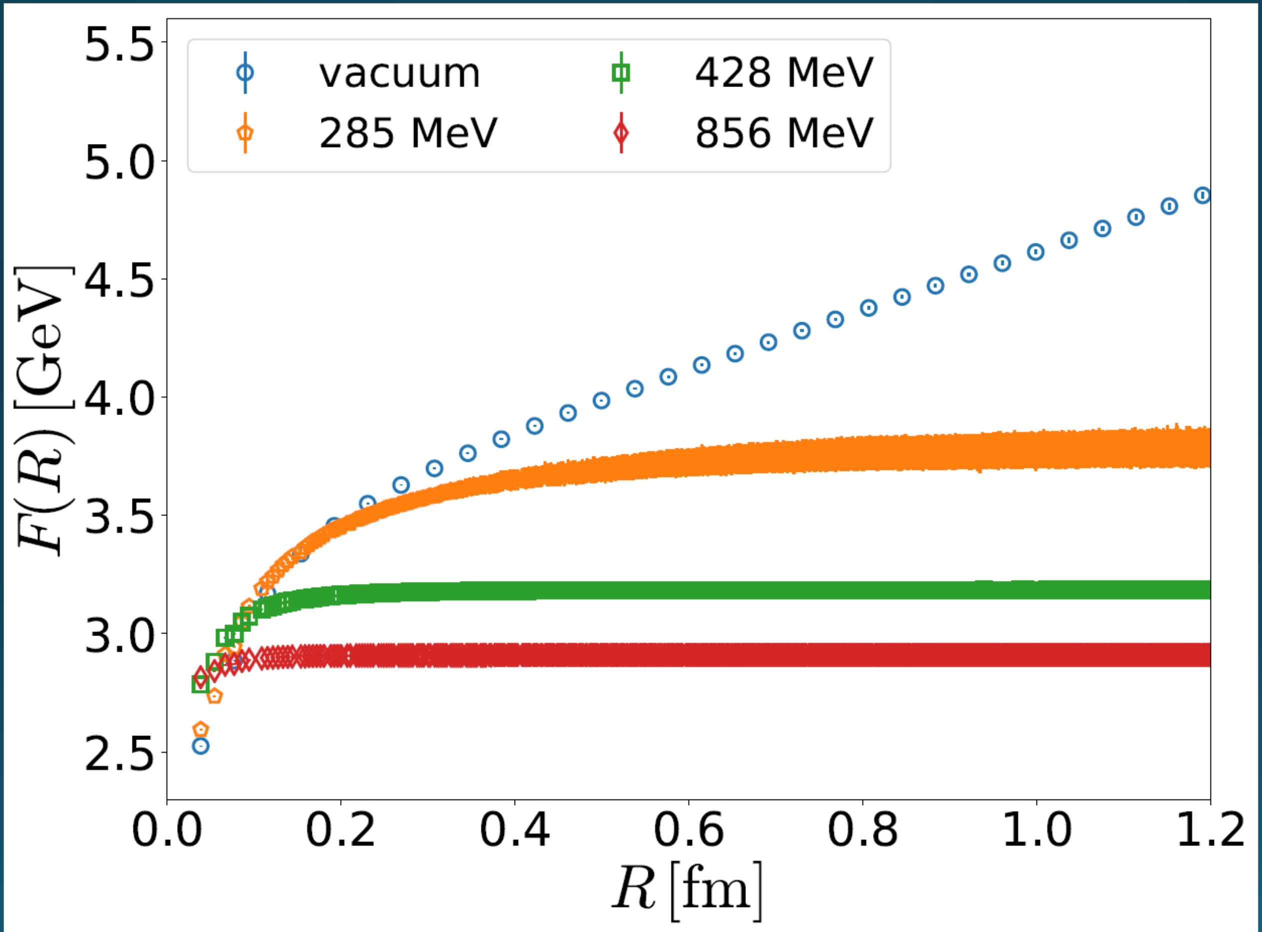
MK, Mogliacci, Horowitz, Kolbe, arXiv:1904.00241

6. Correlation Function

FlowQCD, PR**D96**, 111502 (2017)



Screening of $Q\bar{Q}$ Force above T_c



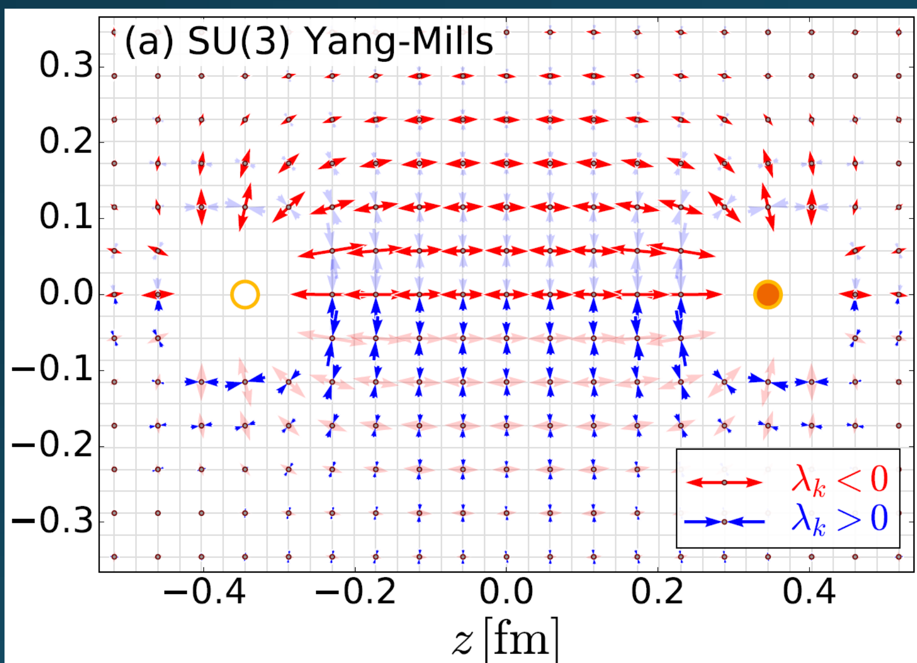
vacuum

high temperature

Q-Qbar force is screened in the deconfined phase.

Temperature Dependence

Vacuum
(Current Universe)



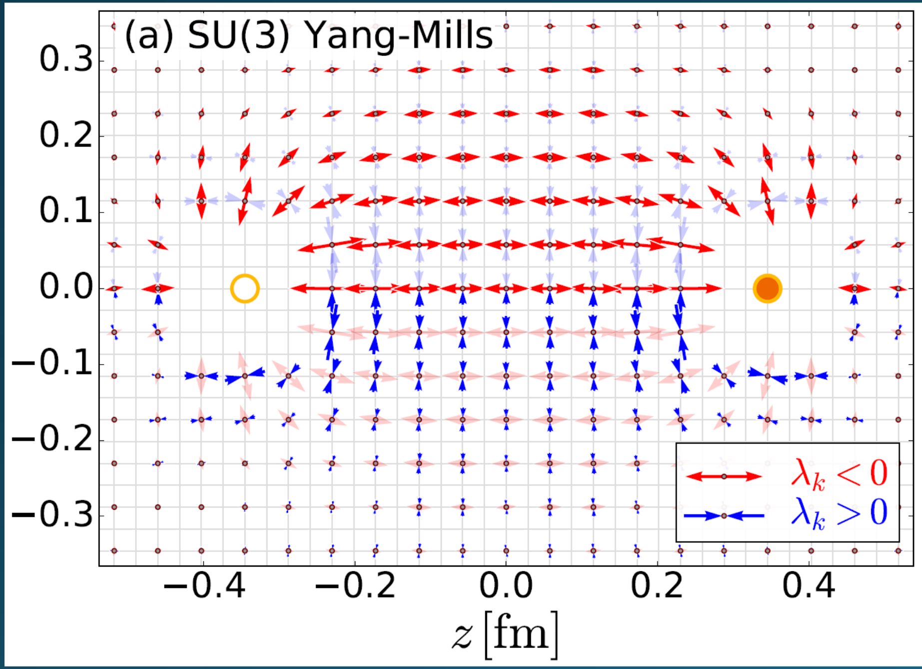
High Temperature
(Early Universe)

?

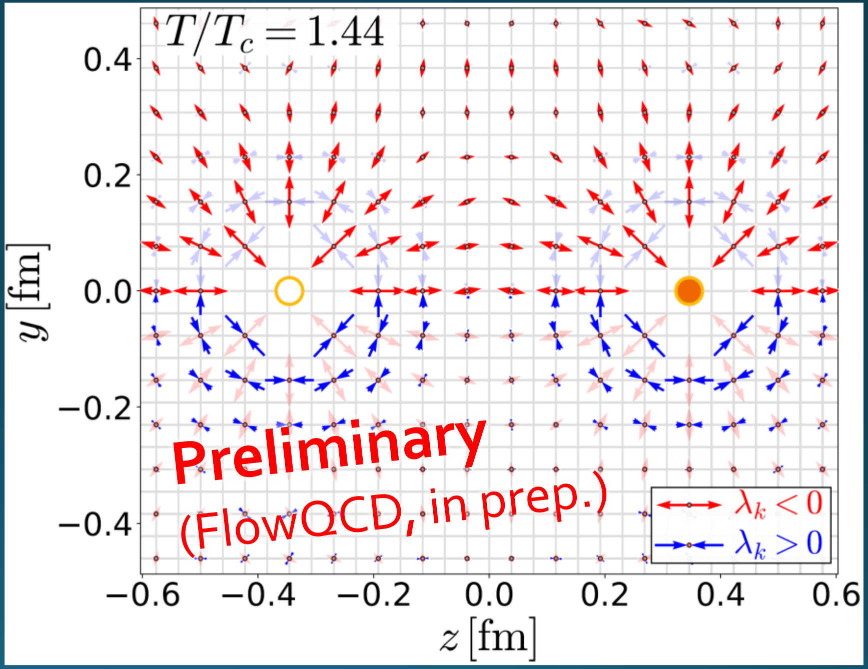
$$\langle T_{\mu\nu}(x)\rangle_{\mathrm{Q\bar{Q}}} = \frac{\langle \delta T_{\mu\nu}(x)\delta\Omega(y)\Omega^{\dagger}(z)\rangle}{\langle \Omega(y)\Omega^{\dagger}(z)\rangle}$$

Temperature Dependence

Vacuum
(Current Universe)



High Temperature
(Early Universe)

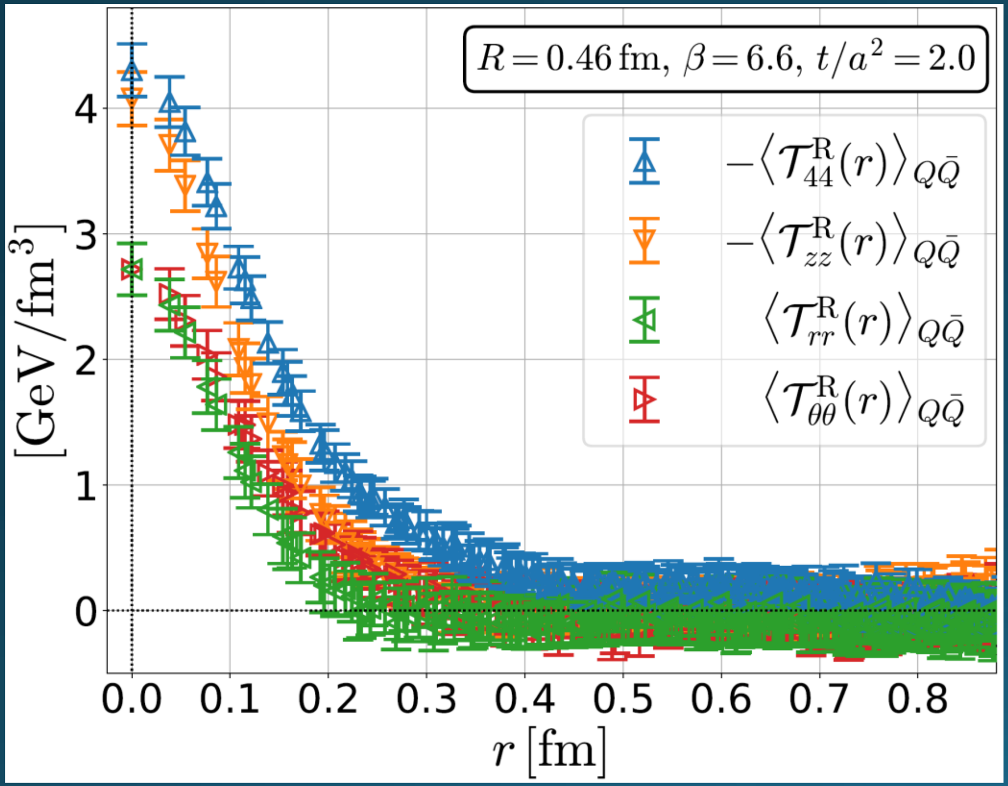


$T = 1.44 T_c$

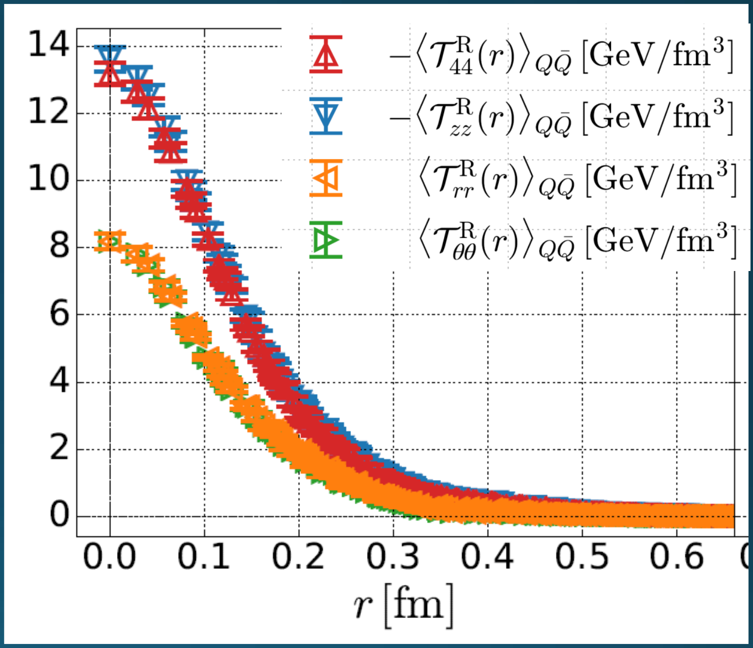
- Singlet projection for $T = 1.44 T_c$
- Flux-tube structure is screened above T_c.

Mid Plane

T=1.44Tc, R=0.46 fm



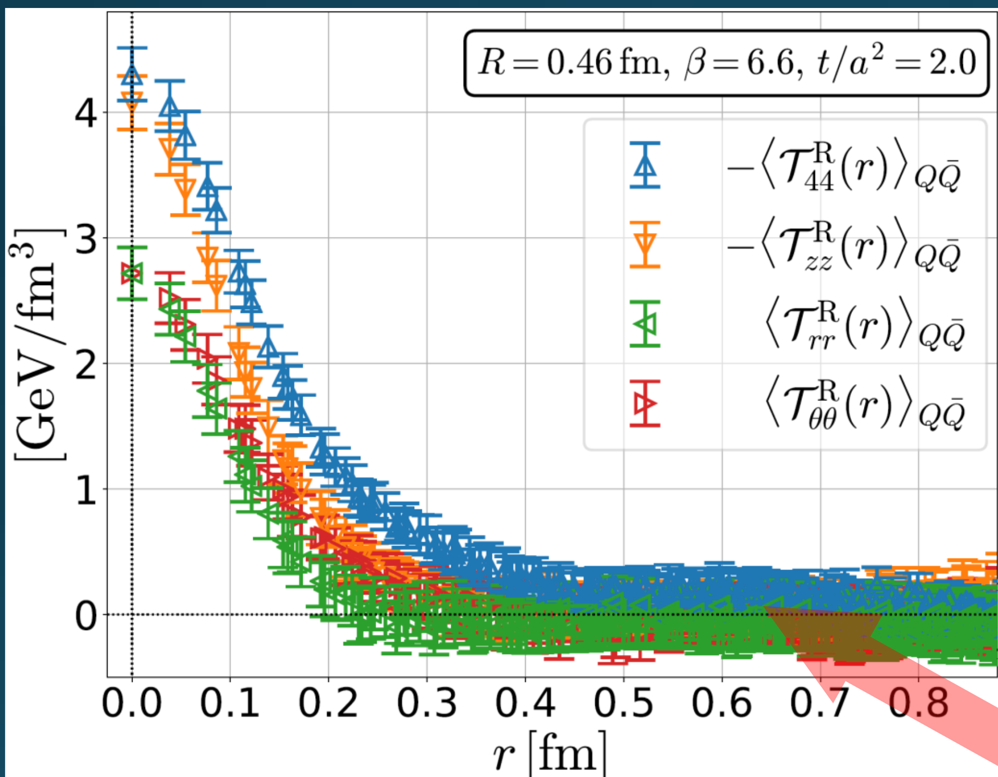
Vacuum, R=0.46 fm



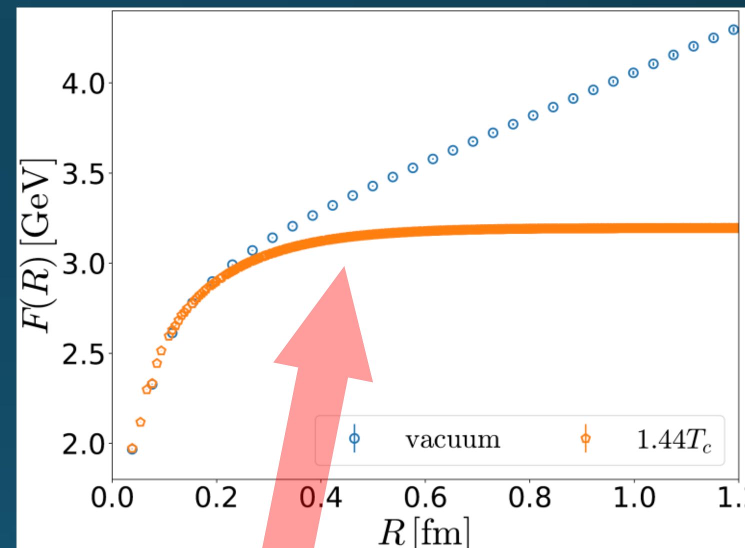
- ☐ Separation b/w T_{44} & T_{zz}?

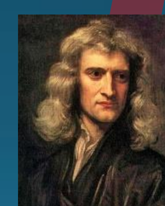
Mid Plane

T=1.44Tc



Free Energy





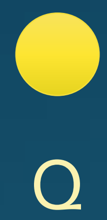
F=0.41 GeV/fm

F=0.30 GeV/fm
Before t->0 limit

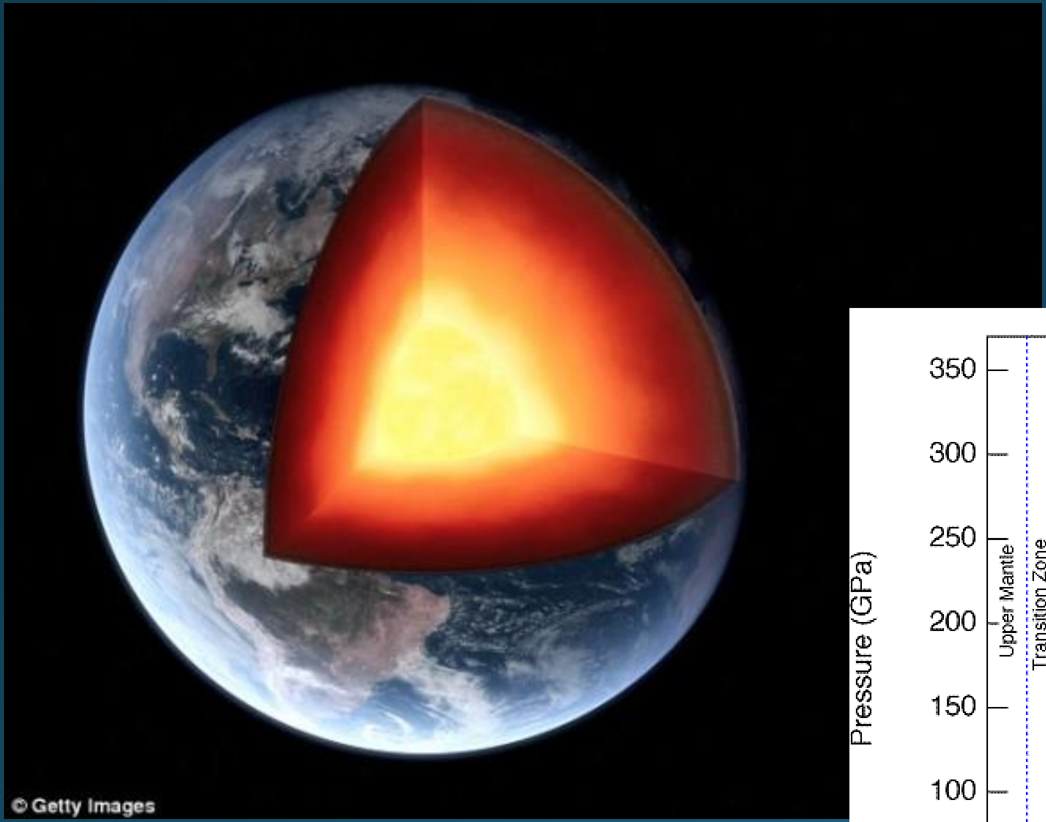
- ☐ Separation b/w T_{44} & T_{zz}?

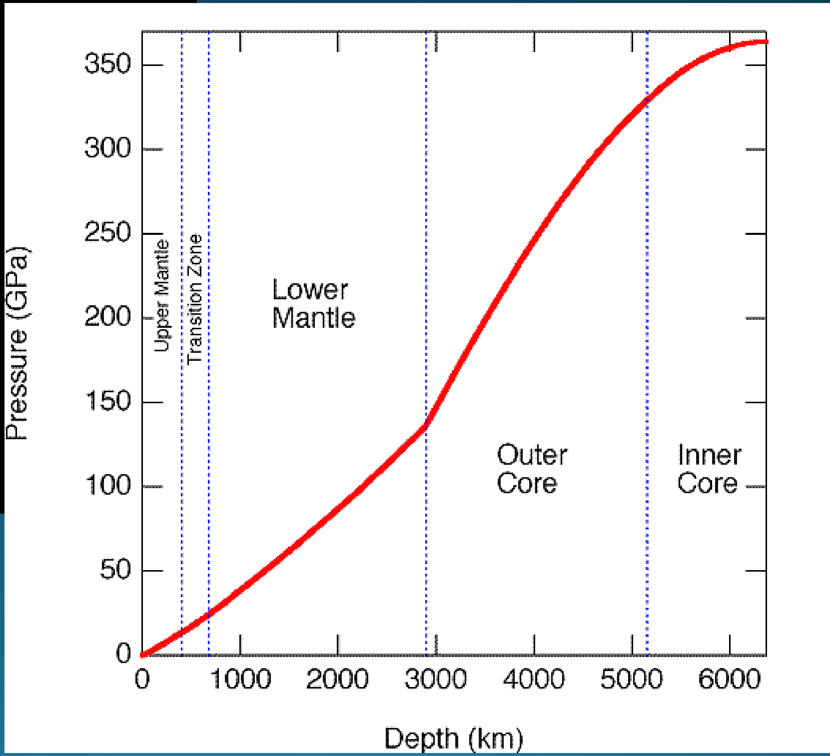
Stress Tensor around A Quark

in a deconfined phase



Pressure inside the Earth

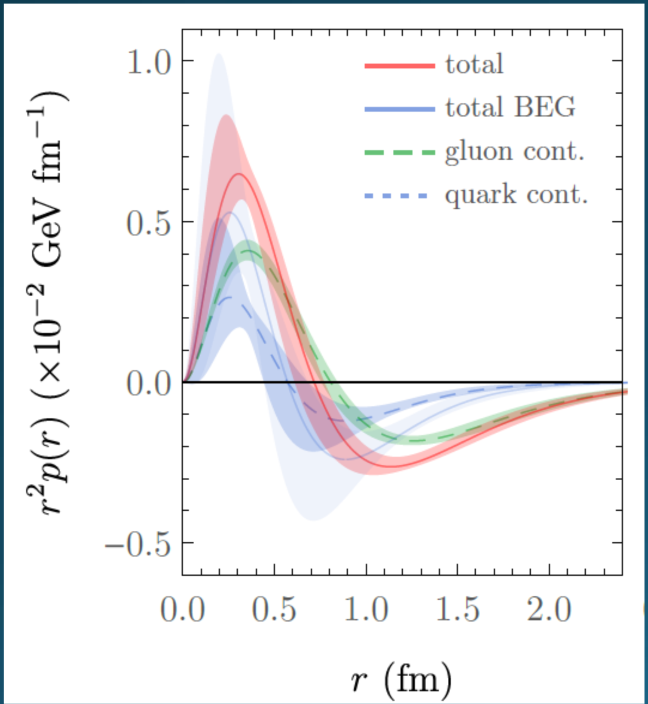




Pressure inside Hadrons

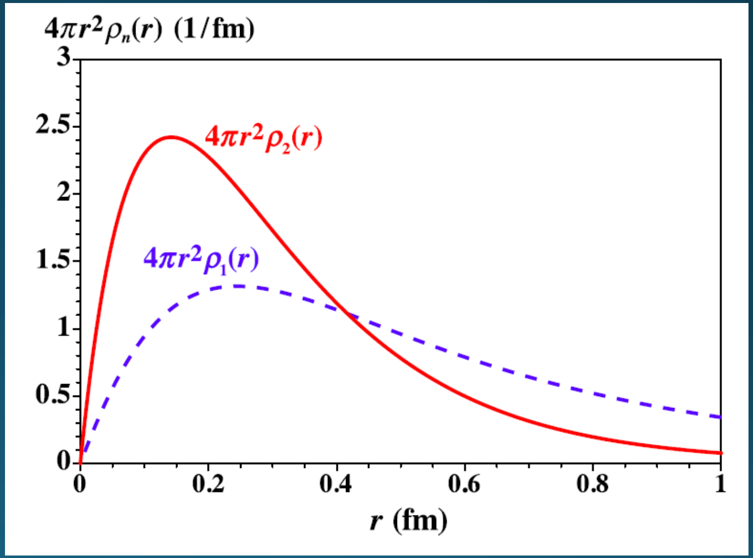
EMT distribution inside hadrons now accessible??

Pressure @ proton



arXiv:1810.07589
Nature, 557, 396 (2018)

EMT distribution @ pion

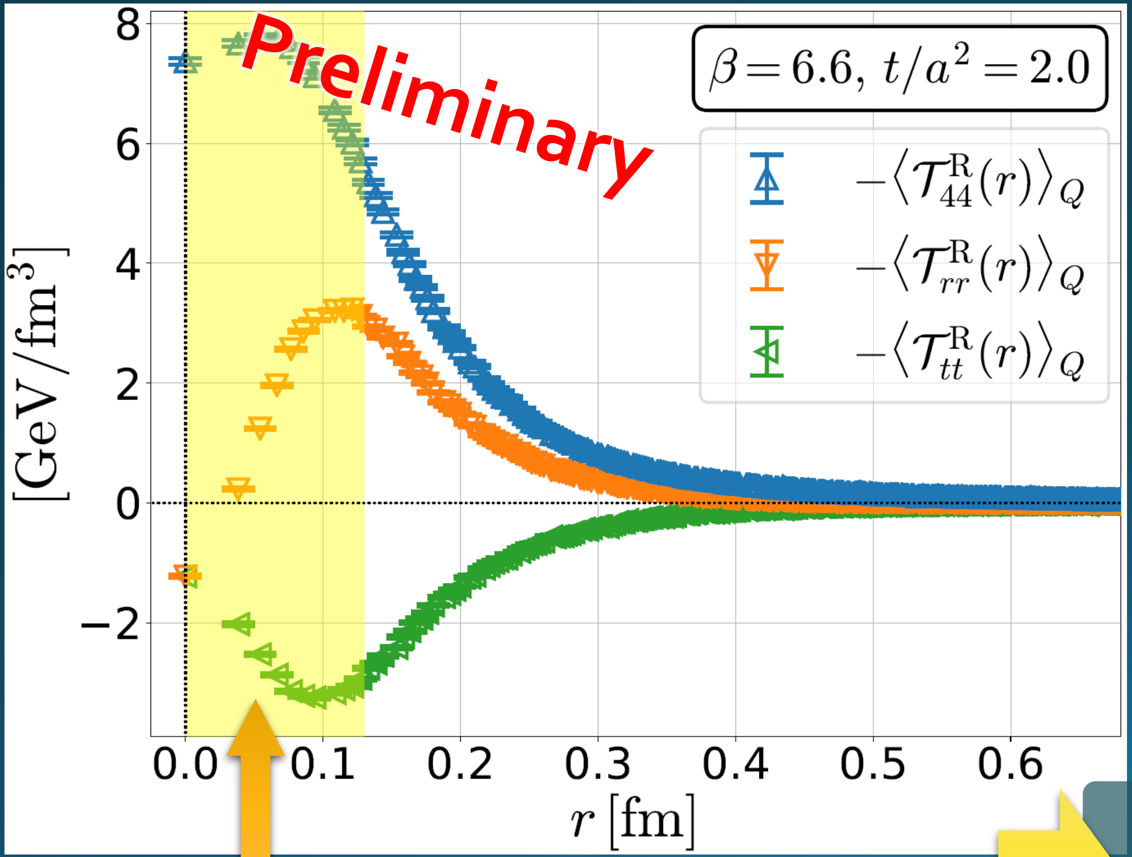


Kumano, Song, Teryaev
Phys. Rev. D 97, 014020 (2018)

Stress Tensor around A Quark

in a deconfined phase

$$\langle T_{\mu\nu}(x)\rangle_{\mathrm{Q}} = \frac{\langle \delta T_{\mu\nu}(x)\delta\Omega(0)\rangle}{\langle\Omega\rangle}$$



Not reliable

Yanagihara+, in prep.
Quenched QCD
48^3x12 ($T \approx 1.4 T_c$)
fixed t , a

Spherical Coordinates

- Energy density
  $-\langle T_{44}\rangle = \varepsilon$
- Longitudinal pressure
  $-\langle T_{rr}\rangle = -p(r)$
- Transverse pressure
  $-\langle T_{tt}\rangle$

☐ Screening mass
☐ Strong coupling const.

Contents

1. Constructing EMT
2. Thermodynamics

FlowQCD, PR**D90**,011501 (2014); PR**D94**, 114512 (2016);
WHOT-QCD, PR**D96**, 014509 (2017); Iritani+, PTEP **2019**, 023B02 (2019)

3. Flux Tube

FlowQCD, PLB789, 210 (2019); Yanagihara+, in prep.

4. Static Quark Systems at Nonzero T

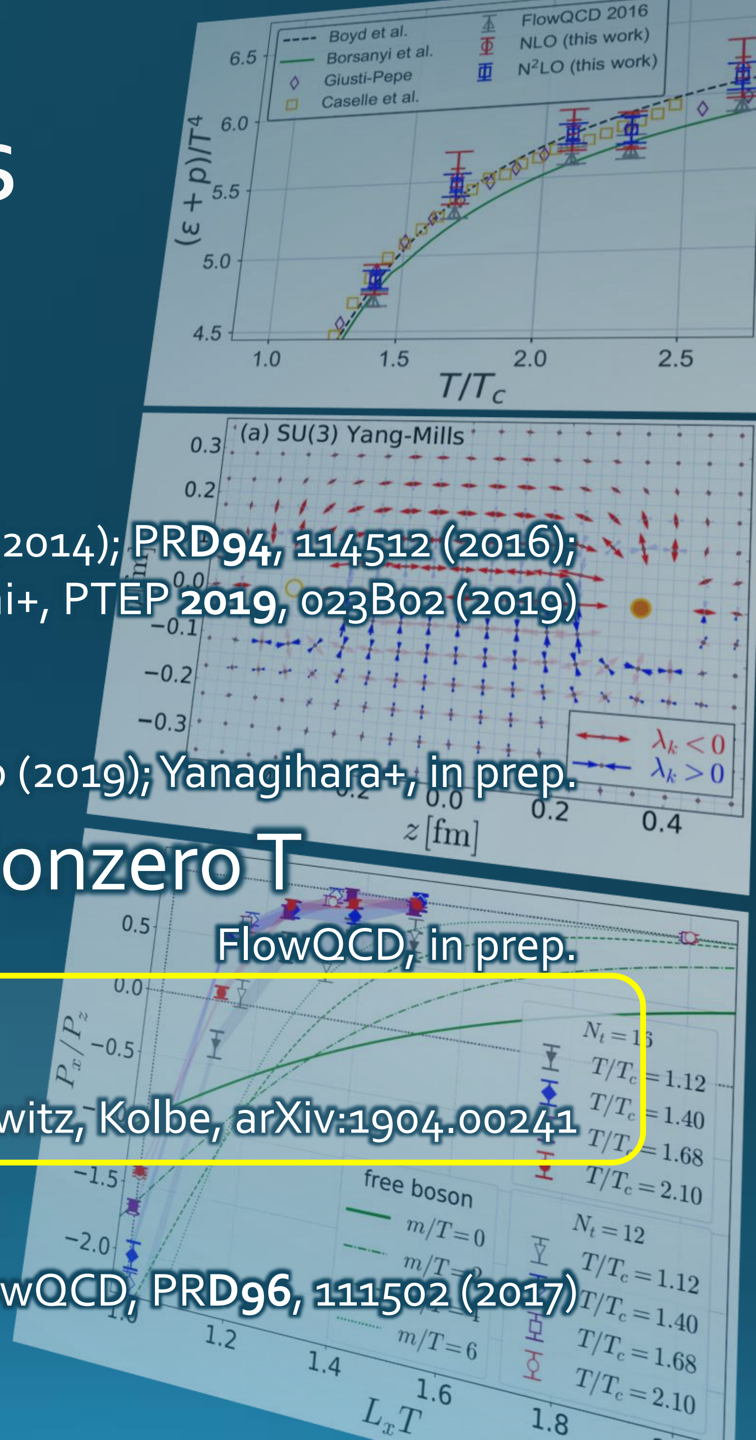
FlowQCD, in prep.

5. Casimir Effect

MK, Mogliacci, Horowitz, Kolbe, arXiv:1904.00241

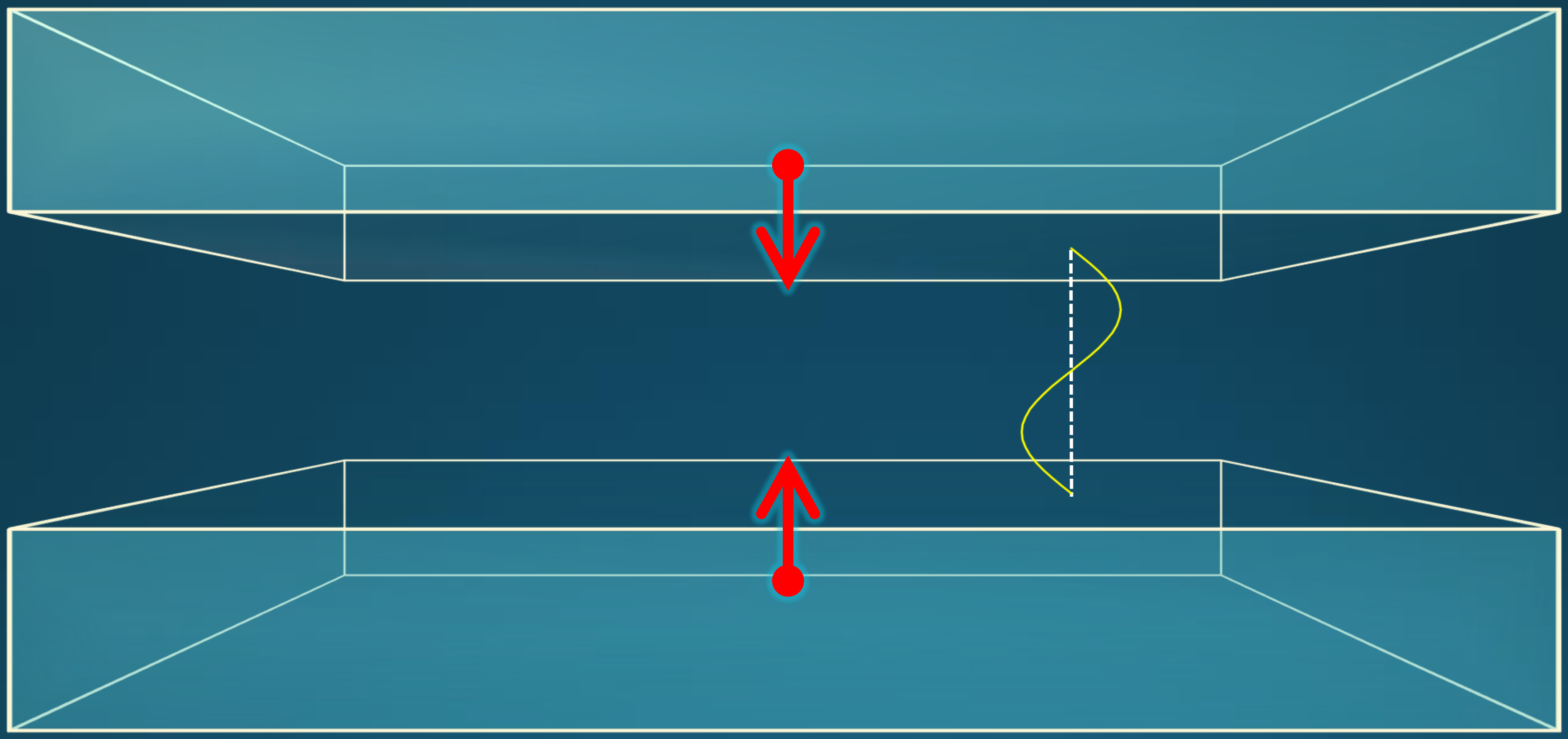
6. Correlation Function

FlowQCD, PR**D96**, 111502 (2017)



Casimir Effect

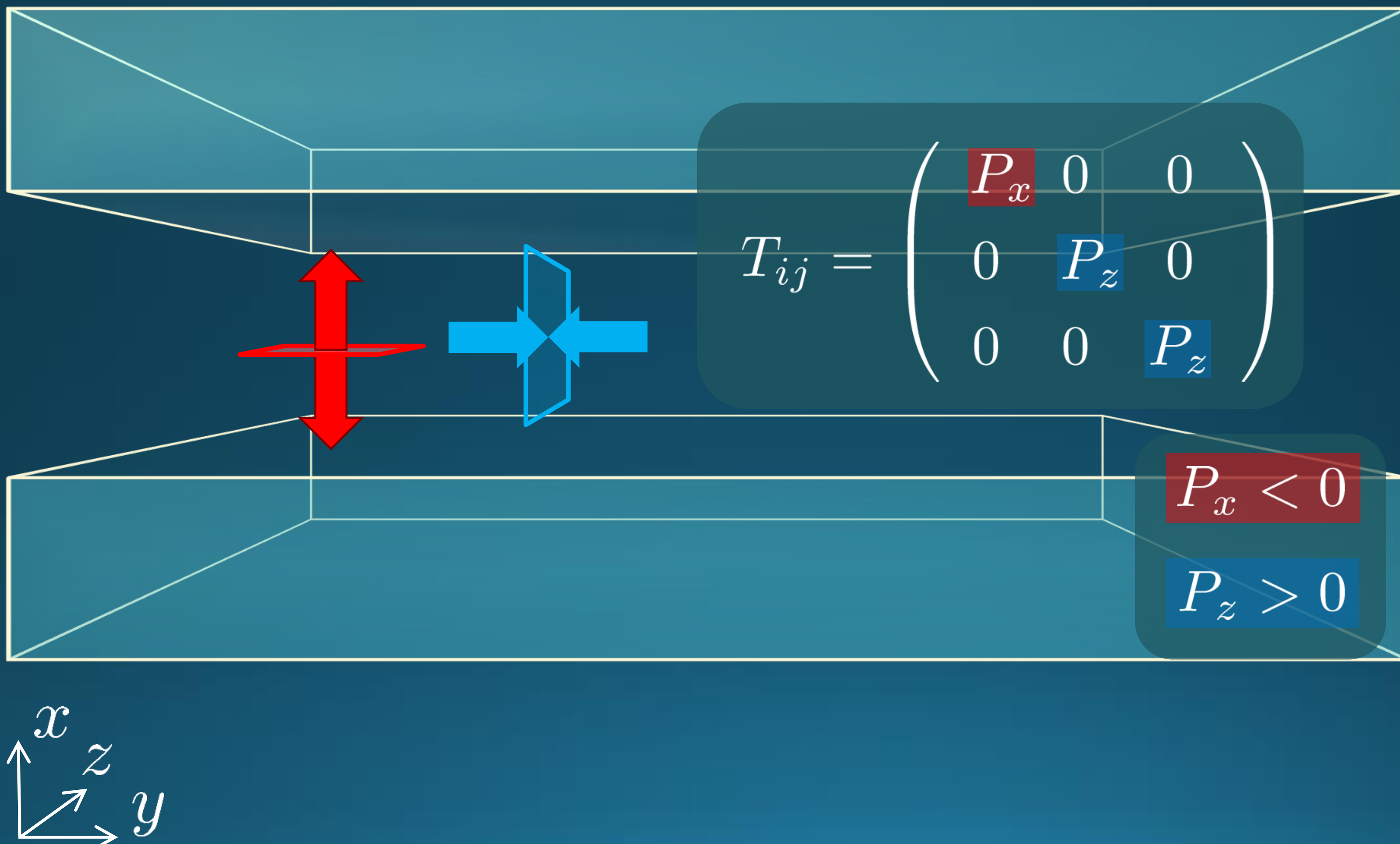
Casimir Effect



attractive force between two conductive plates

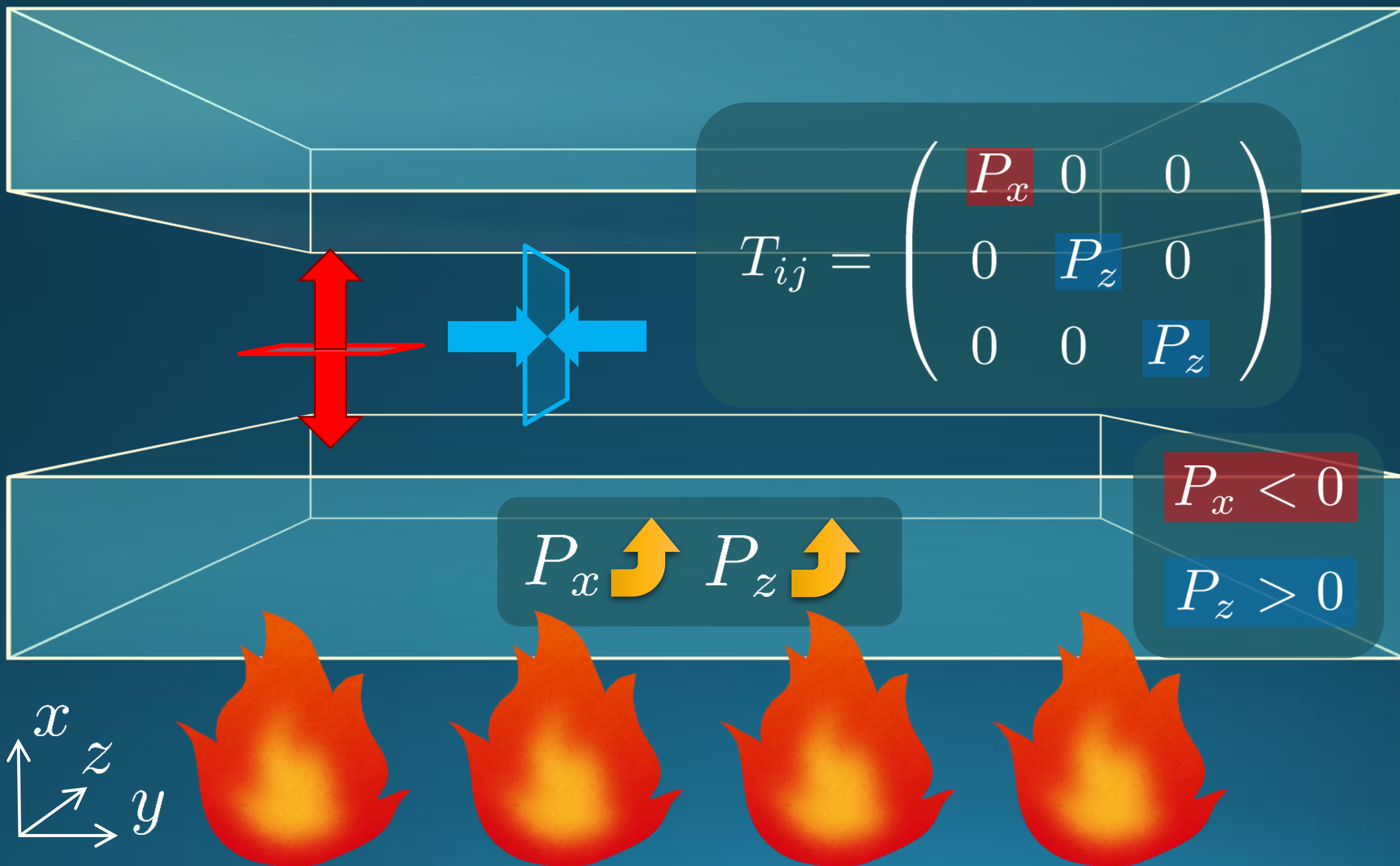
Casimir Effect

Brown, Maclay
1969



Casimir Effect

Brown, Maclay
1969



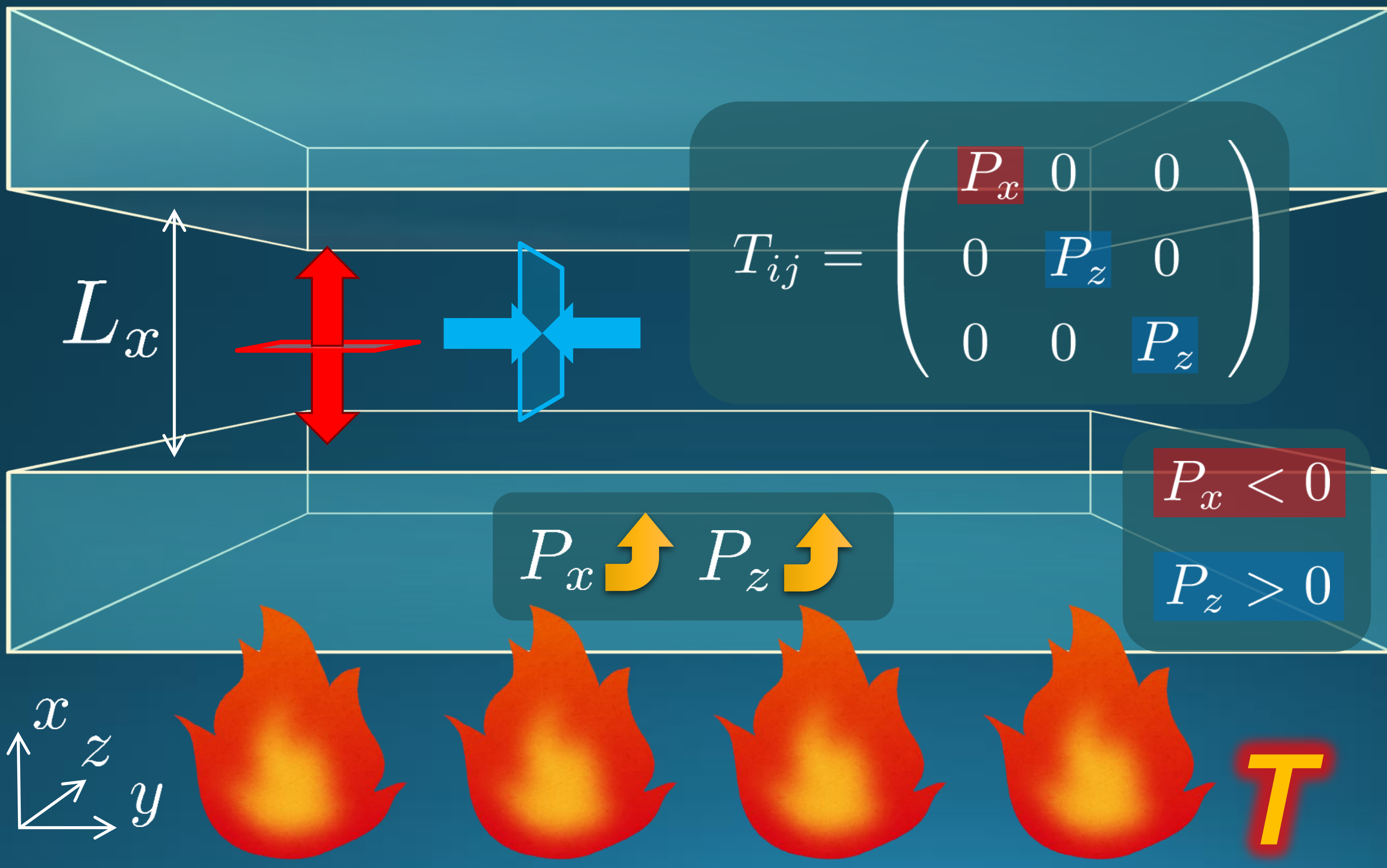
$$T_{ij} = \begin{pmatrix} P_x & 0 & 0 \\ 0 & P_z & 0 \\ 0 & 0 & P_z \end{pmatrix}$$

$P_x < 0$

$P_z > 0$

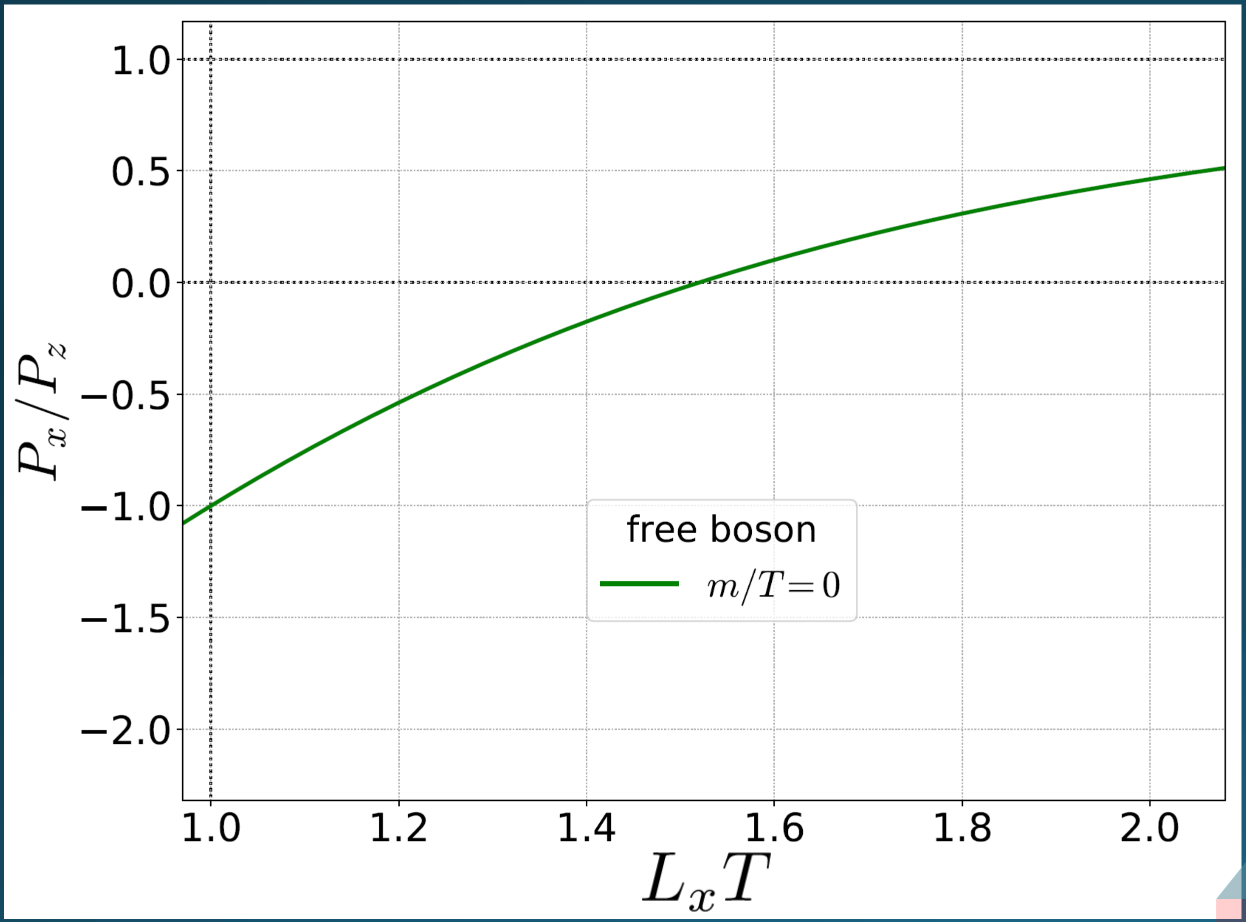
Casimir Effect

Brown, Maclay
1969



Pressure Anisotropy @ T≠o

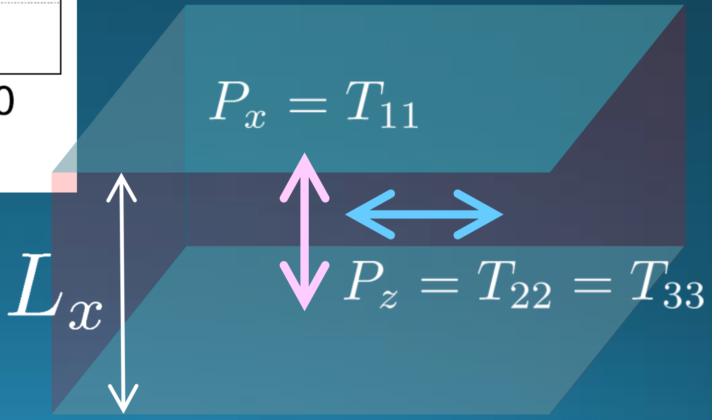
MK, Mogliacci, Kolbe, Horowitz, 1904.00241



Free scalar field

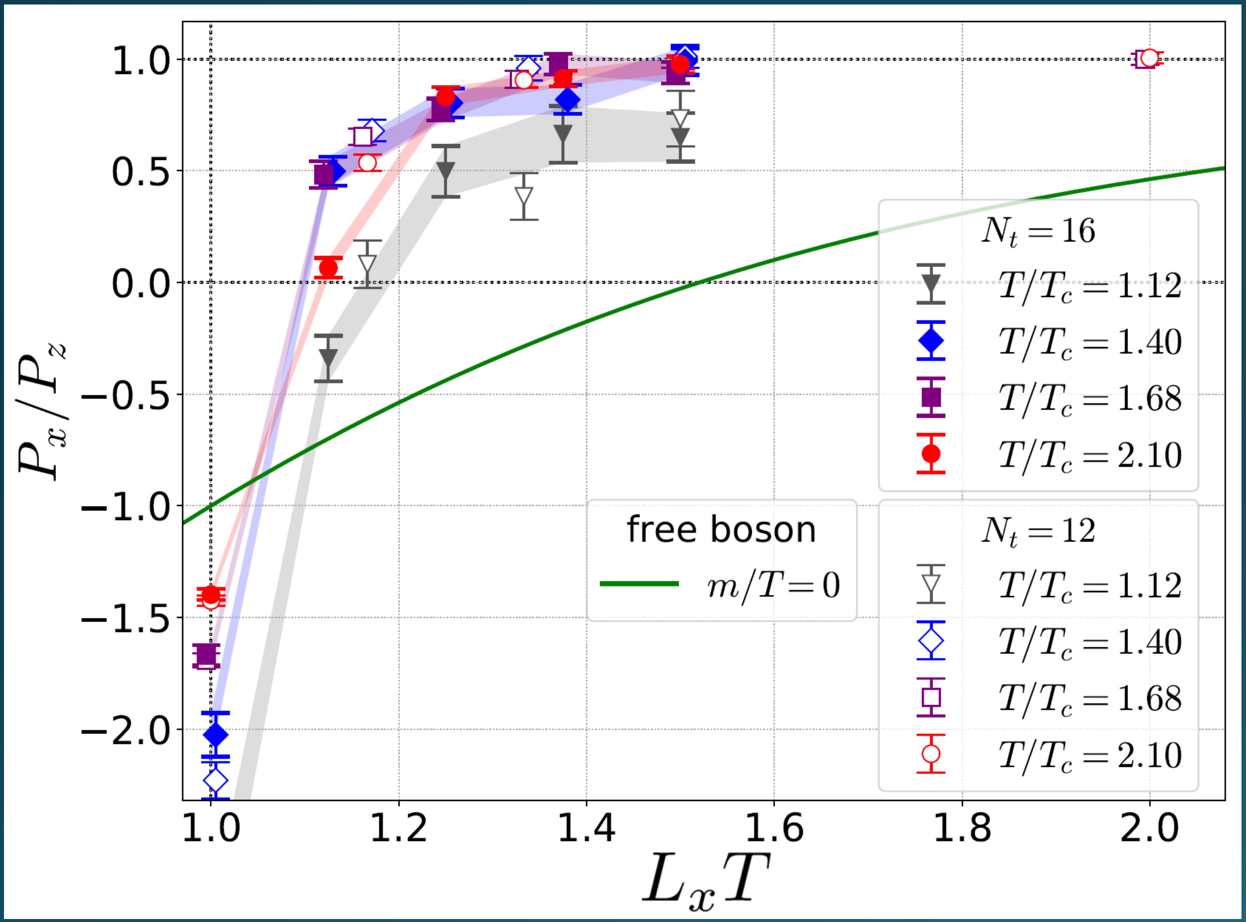
- $L_2 = L_3 = \infty$
- Periodic BC

Mogliacci+, 1807.07871



Pressure Anisotropy @ T≠0

MK, Mogliacci, Kolbe, Horowitz, 1904.00241



Free scalar field

- ☐ $L_2=L_3=\infty$
- ☐ Periodic BC

Mogliacci+, 1807.07871

Lattice result

- ☐ Periodic BC
- ☐ Only t→0 limit
- ☐ Error: stat.+sys.

Medium near T_c is remarkably insensitive to finite size!

Thermodynamics on the Lattice

Various Methods

- **Integral**, differential, moving frame, non-equilibrium, …
- rely on thermodynamic relations valid in V→∞

$$P = \frac{T}{V} \ln Z \qquad sT = \varepsilon + P$$

→ **Not applicable to anisotropic systems**

- We employ **Gradient Flow Method**

$$\varepsilon = \langle T_{00} \rangle \quad P = \langle T_{11} \rangle$$

Components of EMT are directly accessible!

Numerical Setup

- SU(3) YM theory
- Wilson gauge action

- N_t = 16, 12
- N_z/N_t=6
- 2000~4000 confs.
- Even N_x

- No Continuum extrap.

T/T_c	β	N_z	N_τ	N_x	$N_{\rm vac}$
1.12	6.418	72	12	12, 14, 16, 18	64
	6.631	96	16	16, 18, 20, 22, 24	96
1.40	6.582	72	12	12, 14, 16, 18	64
	6.800	96	16	16, 18, 20, 22, 24	128
1.68	6.719	72	12	12, 14, 16, 18, 24	64
	6.719	96	12	14, 18	64
	6.941	96	16	16, 18, 20, 22, 24	96
2.10	6.891	72	12	12, 14, 16, 18, 24	72
	7.117	96	16	16, 18, 20, 22, 24	128
2.69	7.086	72	12	12, 14, 16, 18	-
$\simeq$ 8.1	8.0	72	12	12, 14, 16, 18	-
$\simeq$ 25	9.0	72	12	12, 14, 16, 18	-

- Same Spatial volume
  - $12 \times 72^2 \times 12$ ~ $16 \times 96^2 \times 16$
  - $18 \times 72^2 \times 12$ ~ $24 \times 96^2 \times 16$

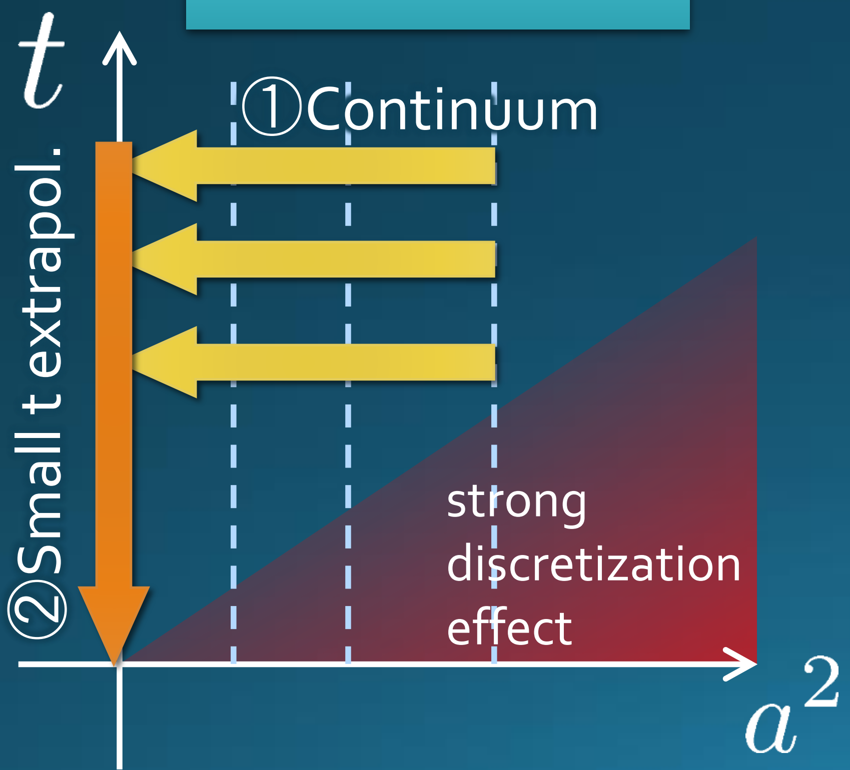
Simulations on
OCTOPUS/Reedbush

Extrapolations t→0, a→0

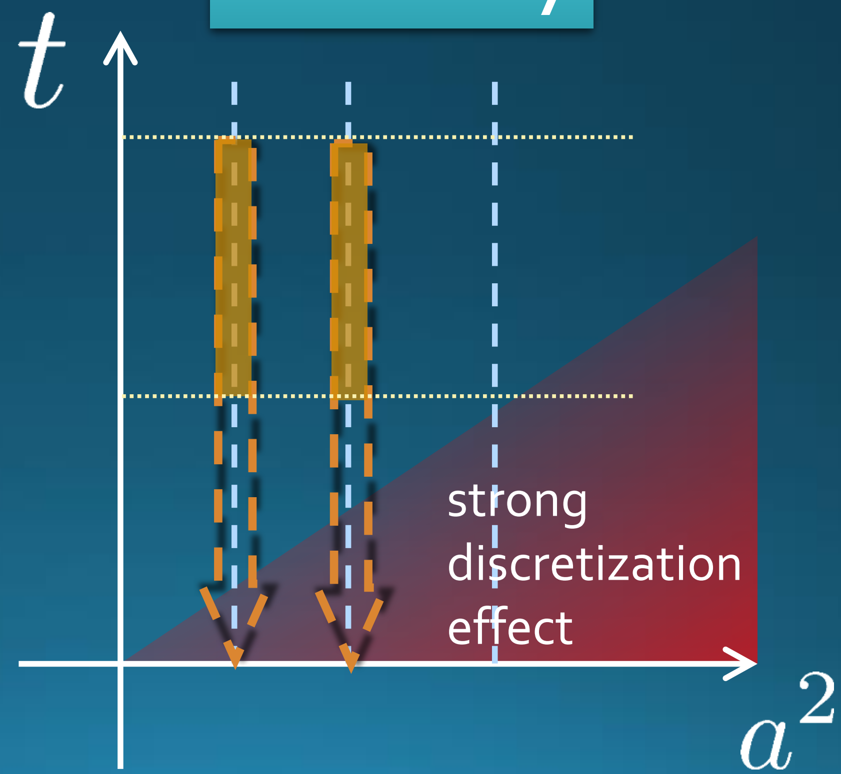
$$\langle T_{\mu\nu}(t)\rangle_{\rm latt} = \langle T_{\mu\nu}(t)\rangle_{\rm phys} + C_{\mu\nu}t + D_{\mu\nu}(t)\frac{a^2}{t}$$

O(t) terms in SFTE | lattice discretization

FlowQCD2016

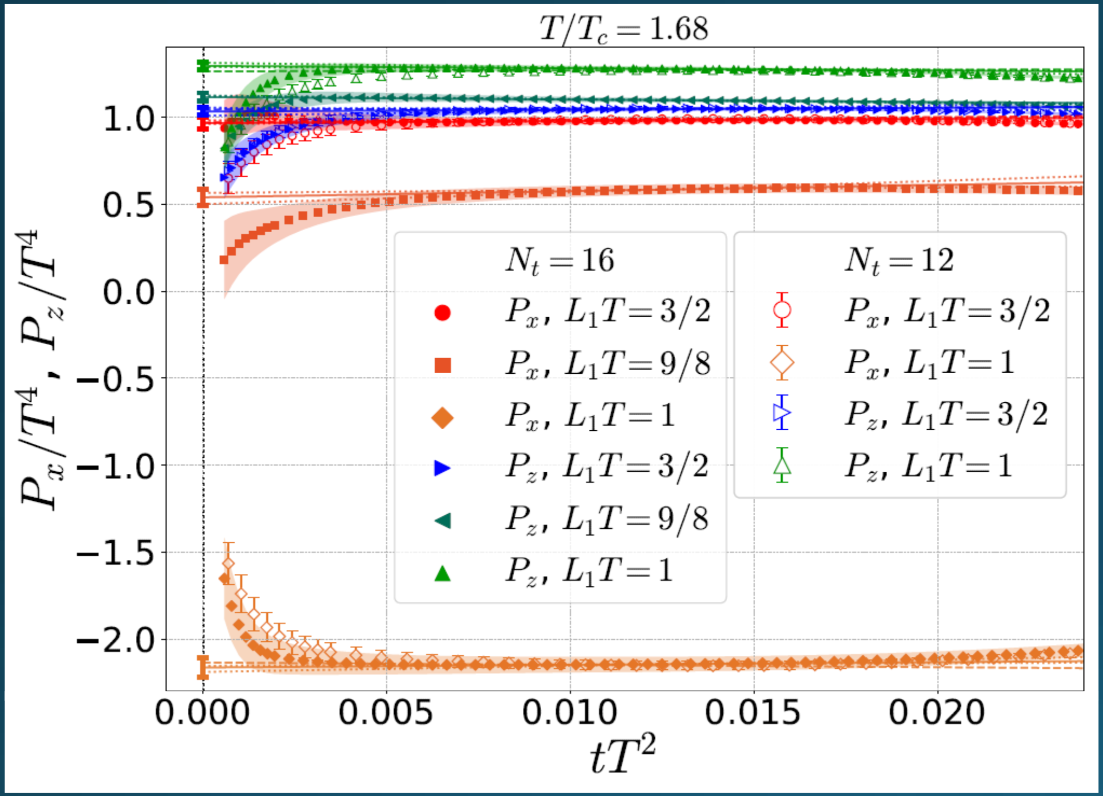


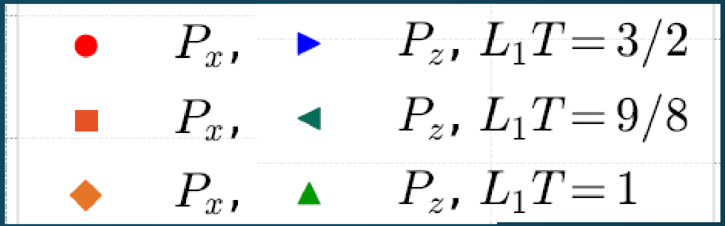
This Study



Small-t Extrapolation

$T/T_c = 1.68$





Filled: N_t=16 / Open: N_t=12

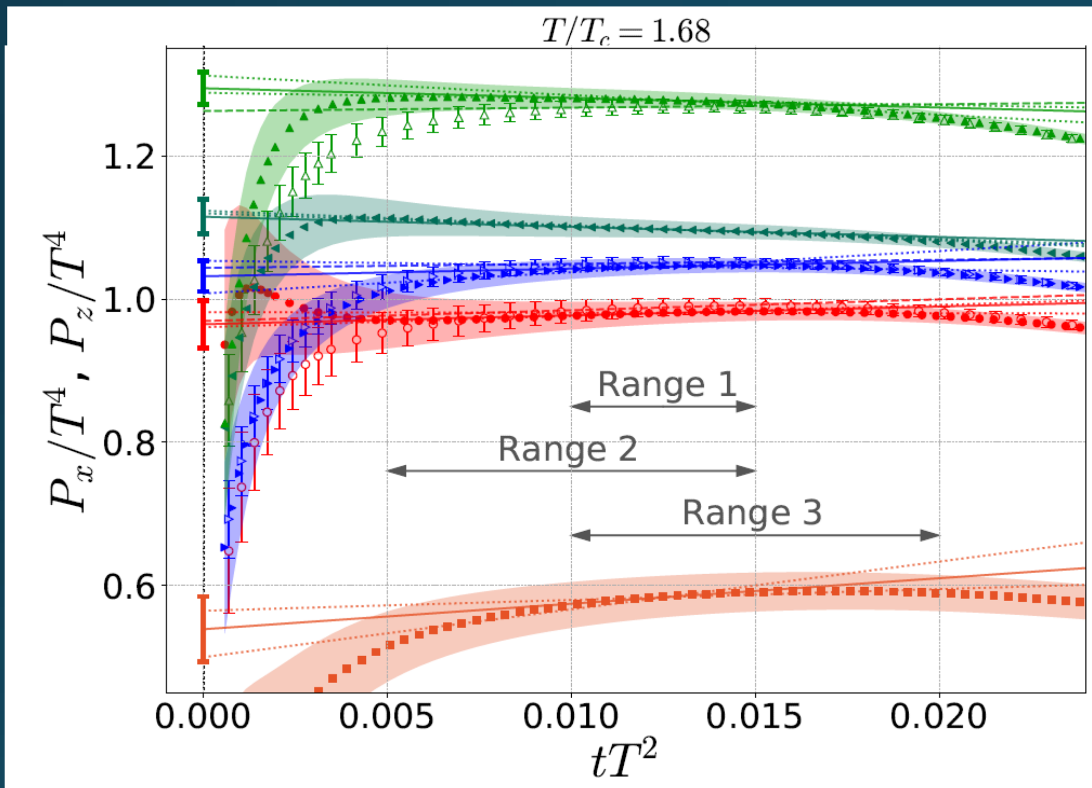
Small-t extrapolation

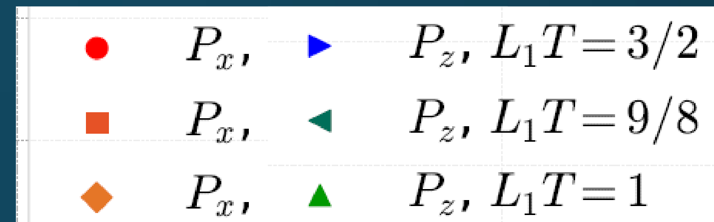
- Solid: N_t=16, Range-1
- Dotted: N_t=16, Range-2,3
- Dashed: N_t=12, Range-1

- ☐ Stable small-t extrapolation
- ☐ No N_t dependence within statistics for L_xT=1, 1.5

Small-t Extrapolation

$T/T_c = 1.68$





Filled: N_t=16 / Open: N_t=12

Small-t extrapolation

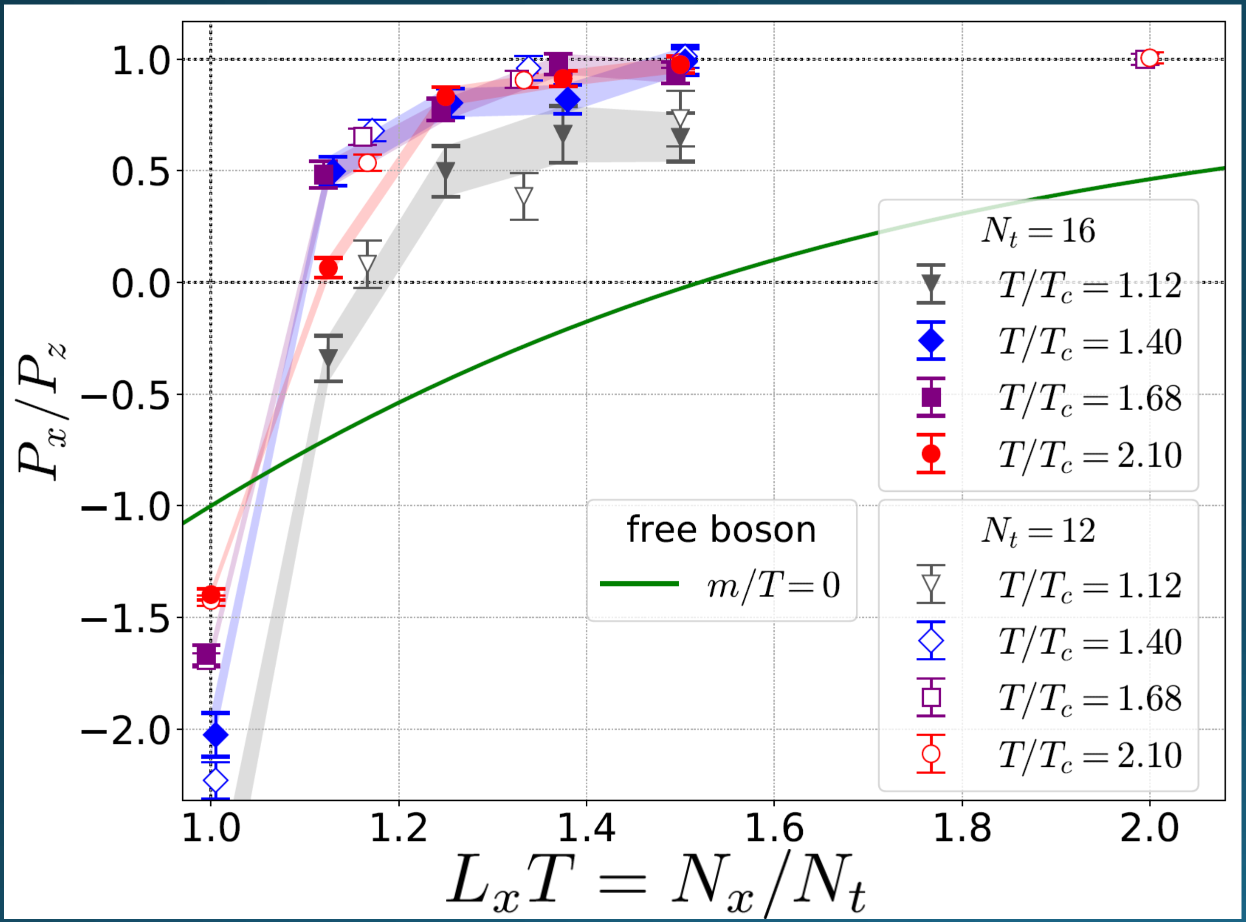
- Solid: N_t=16, Range-1
- Dotted: N_t=16, Range-2,3
- Dashed: N_t=12, Range-1

☐ Stable small-t extrapolation

☐ No N_t dependence within statistics for L_xT=1, 1.5

Pressure Anisotropy @ T≠o

MK, Mogliacci, Kolbe, Horowitz, 1904.00241



Free scalar field

- $L_2=L_3=\infty$
- Periodic BC

Mogliacci+, 1807.07871

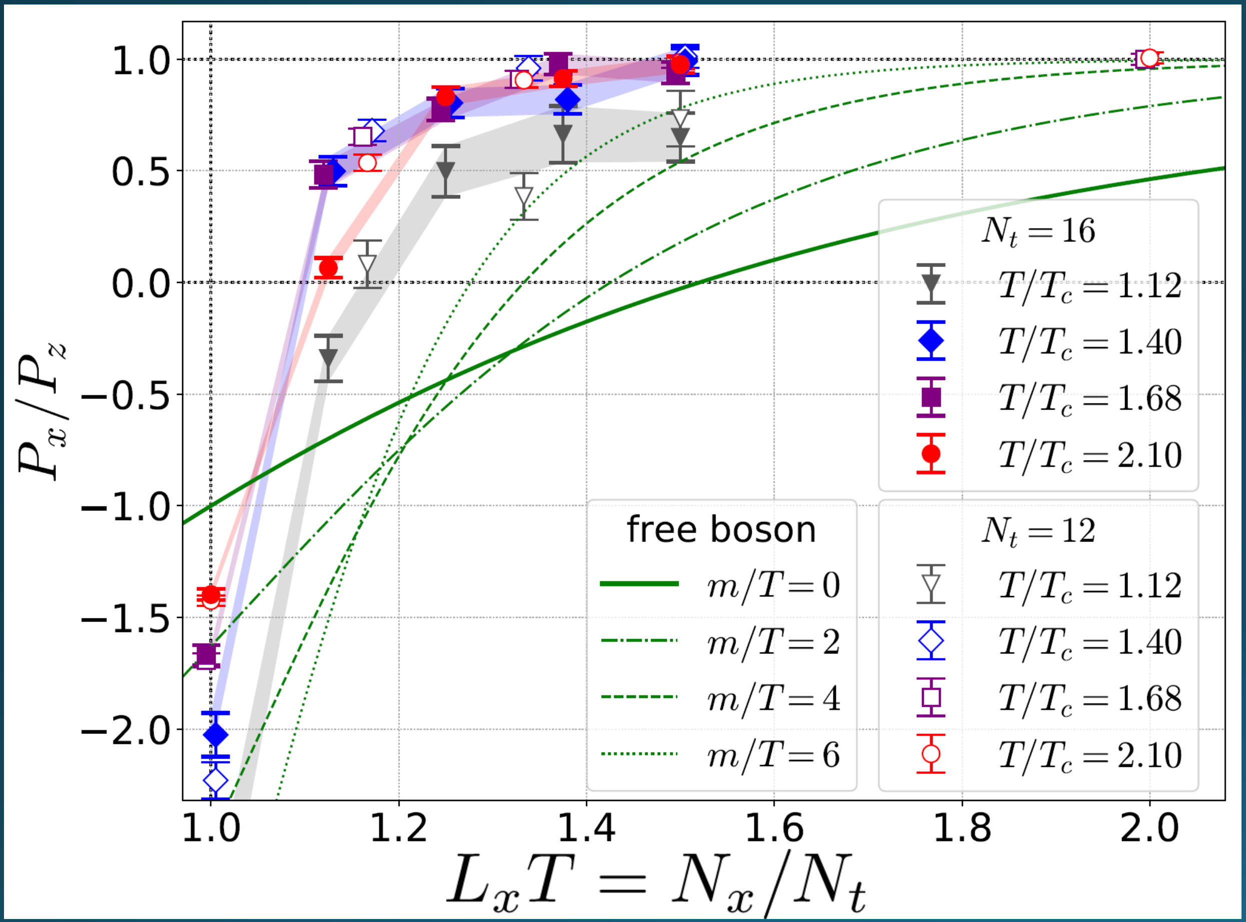
Lattice result

- Periodic BC
- Only t→0 limit
- Error: stat.+sys.

Medium near T_c is remarkably insensitive to finite size!

Pressure Anisotropy @ T≠0

MK, Mogliacci, Kolbe, Horowitz, 1904.00241



Free scalar field

- $L_2=L_3=\infty$
- Periodic BC

Mogliacci+, 1807.07871

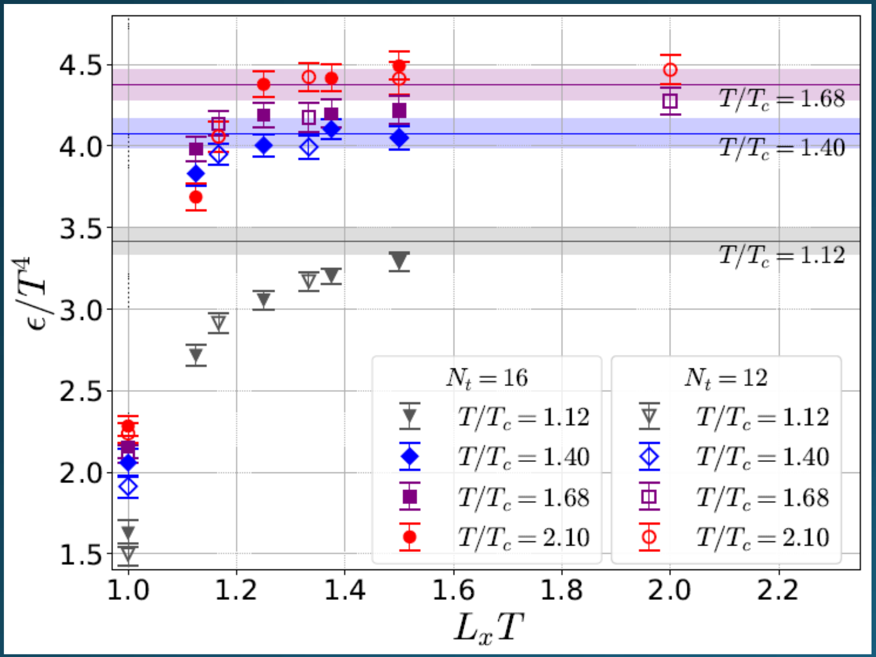
Lattice result

- Periodic BC
- Only t→0 limit
- Error: stat.+sys.

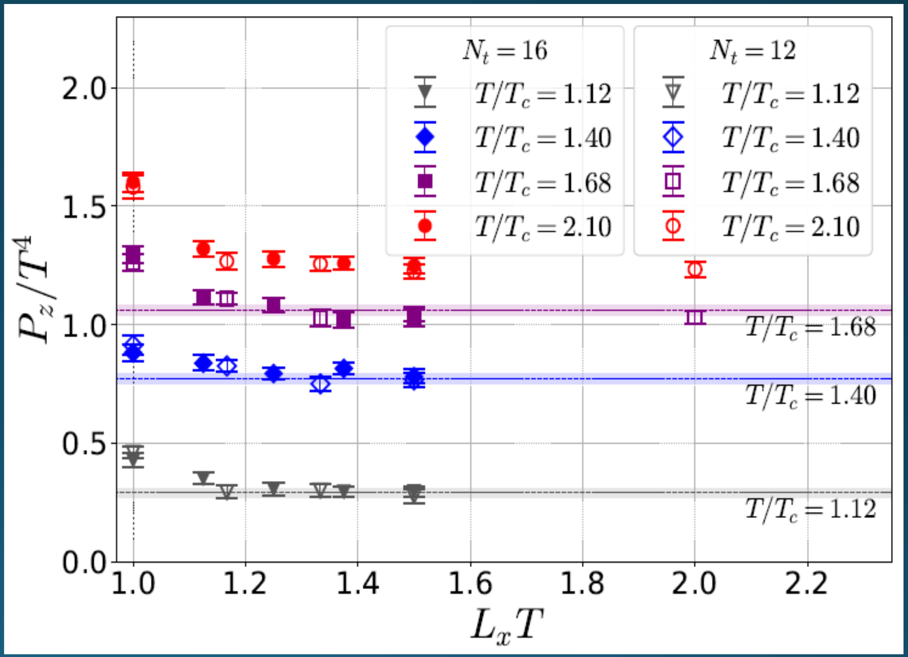
Medium near T_c is remarkably insensitive to finite size!

Energy densty / transverse P

Energy Density



Transverse Pressure P_z



Higher T

High-T limit: massless free gluons

How does the anisotropy approach this limit?

Difficulties

- ☐ Vacuum subtraction requires large-volume simulations.
- ☐ Lattice spacing not available → $c_1(t)$, $c_2(t)$ are not determined.

Higher T

High-T limit: massless free gluons
How does the anisotropy approach this limit?

Difficulties

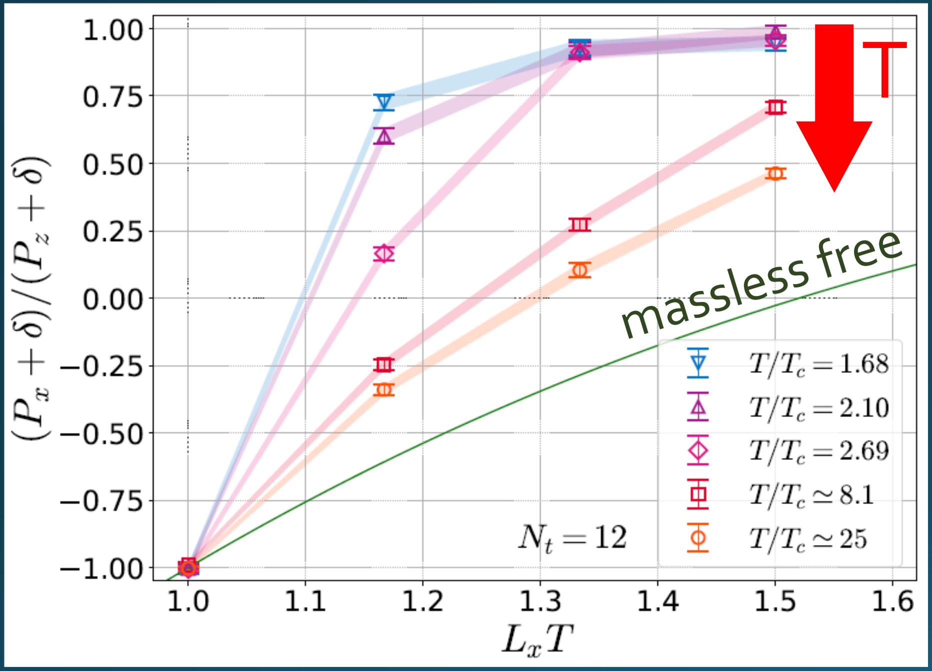
- ☐ Vacuum subtraction requires large-volume simulations.
- ☐ Lattice spacing not available → $c_1(t)$, $c_2(t)$ are not determined.

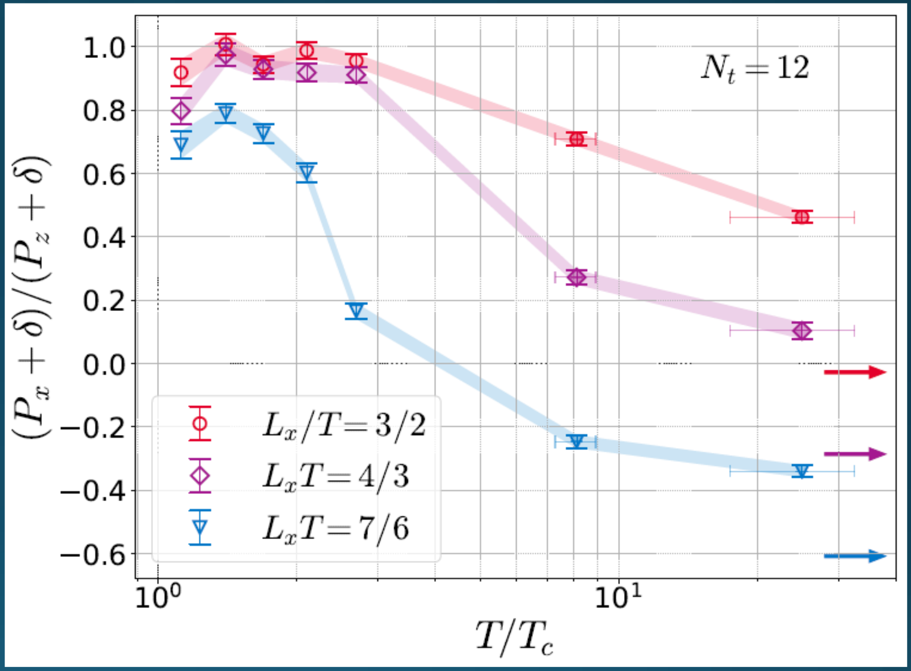
We study

$$\frac{P_x + \delta}{P_z + \delta} \qquad \delta = -\frac{1}{4}\sum_{\mu} T^{\mathrm{E}}_{\mu\mu}$$

No vacuum subtr. nor Suzuki coeffs. necessary!

$$\frac{P_x + \delta}{P_z + \delta}$$





$T/T_c \cong 8.1$ (β=8.0) / $T/T_c \cong 25$ (β=9.0)

- ☐ Ratio approaches the asymptotic value.
- ☐ But, large deviation exists even at $T/T_c \sim 25$.

Contents

1. Constructing EMT
2. Thermodynamics

FlowQCD, PR**D90**,011501 (2014); PR**D94**, 114512 (2016);
WHOT-QCD, PR**D96**, 014509 (2017); Iritani+, PTEP **2019**, 023B02 (2019)

3. Flux Tube

FlowQCD, PLB789, 210 (2019); Yanagihara+, in prep.

4. Static Quark Systems at Nonzero T

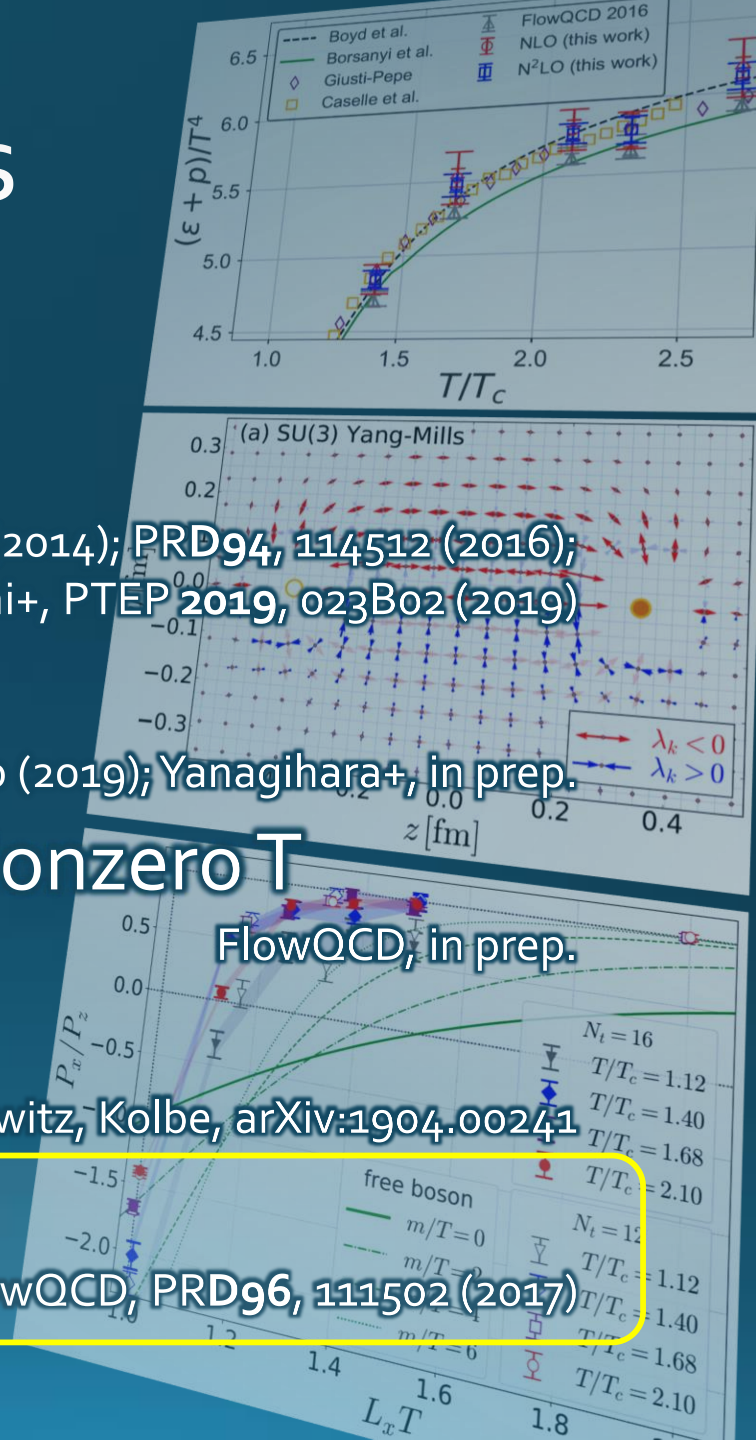
FlowQCD, in prep.

5. Casimir Effect

MK, Mogliacci, Horowitz, Kolbe, arXiv:1904.00241

6. Correlation Function

FlowQCD, PR**D96**, 111502 (2017)



EMT Correlator: Motivation

- Transport Coefficient

Kubo formula → viscosity

$$\eta = \int_0^\infty dt \int_0^{1/T} d\tau \int d^3x \langle T_{12}(x,-i\tau) T_{12}(0,t) \rangle$$

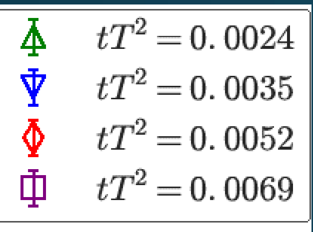
Karsch, Wyld, 1987
Nakamura, Sakai, 2005
Meyer; 2007, 2008
...
Borsanyi+, 2018
Astrakhantsev+, 2018

- Energy/Momentum Conservation

$\langle \bar{T}_{0\mu}(\tau) \bar{T}_{\rho\sigma}(0) \rangle$: τ-independent constant

- Fluctuation-Response Relations

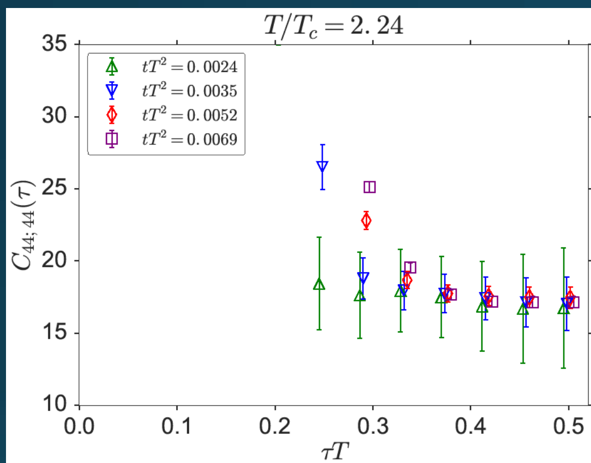
$$c_V = \frac{\langle \delta E^2 \rangle}{VT^2} \qquad E + p = \frac{\langle \bar{T}_{01}^2 \rangle}{VT} = \frac{\langle \bar{T}_{11} \bar{T}_{00} \rangle}{VT}$$

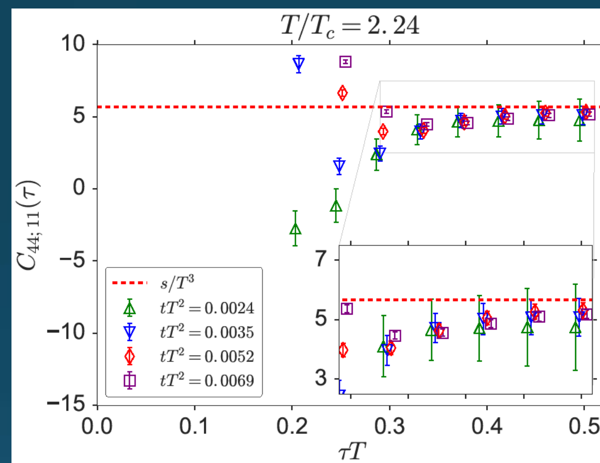


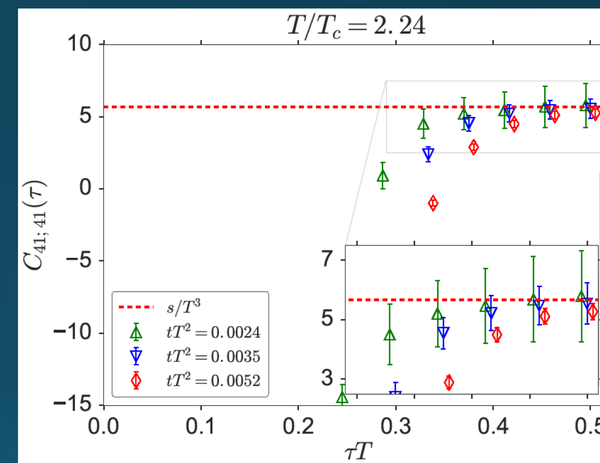
EMT Euclidean Correlator

FlowQCD, PR **D96**, 111502 (2017)

$\langle \bar{T}_{44}(\tau)\bar{T}_{44}(0)\rangle$ $\langle \bar{T}_{44}(\tau)\bar{T}_{11}(0)\rangle$ $\langle \bar{T}_{41}(\tau)\bar{T}_{41}(0)\rangle$





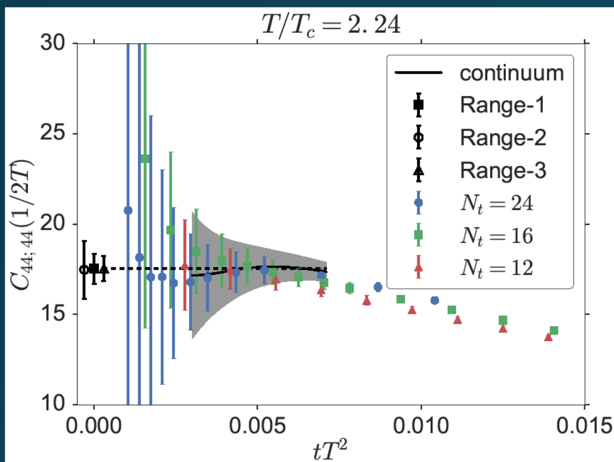


- ☐ τ-independent plateau in all channels → conservation law
- ☐ Confirmation of fluctuation-response relations
- ☐ New method to measure c_V

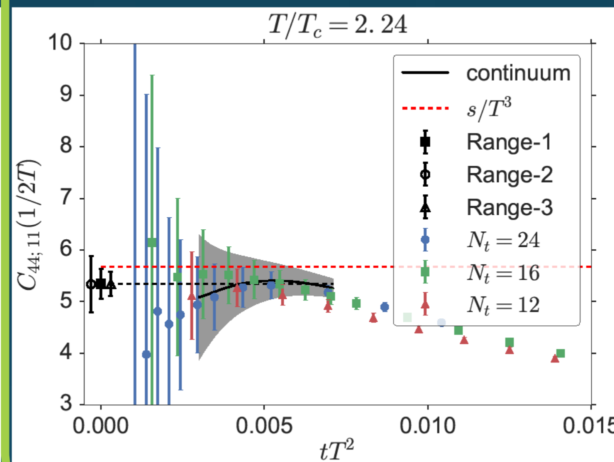
  - ☐ Similar result for (41;41) channel: Borsanyi+, 2018
  - ☐ Perturbative analysis: Eller, Moore, 2018

Fluctuation-Response Relations

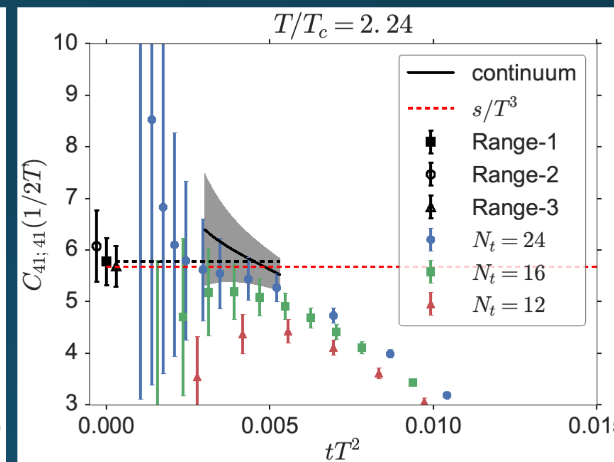
$\langle T_{44}(\tau) T_{44}(0) \rangle$



$\langle T_{44}(\tau) T_{11}(0) \rangle$



$\langle T_{41}(\tau) T_{41}(0) \rangle$



New measurement of c_V

c_V/T^3

T/T_c	$C_{44;44}(\tau_m)$	Ref.[19]	Ref.[11]	ideal gas
1.68	$17.7(8)(^{+2.1}_{-0.4})$	$22.8(7)^*$	17.7	21.06
2.24	$17.5(0.8)(^{+0}_{-0.1})$	$17.9(7)^{**}$	18.2	21.06

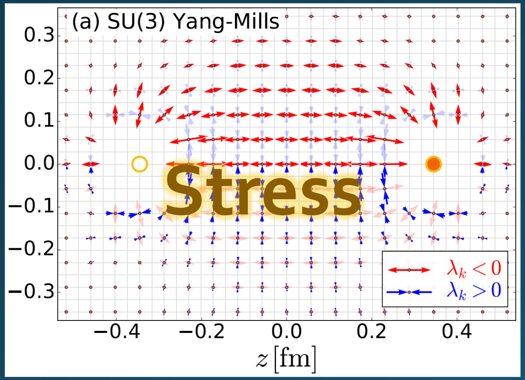
Confirmation of FRR

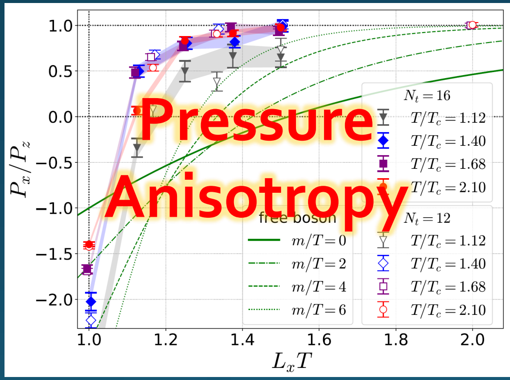
$$E + p = \frac{\langle \bar{T}_{01}^2 \rangle}{VT} = \frac{\langle \bar{T}_{11} \bar{T}_{00} \rangle}{VT}$$

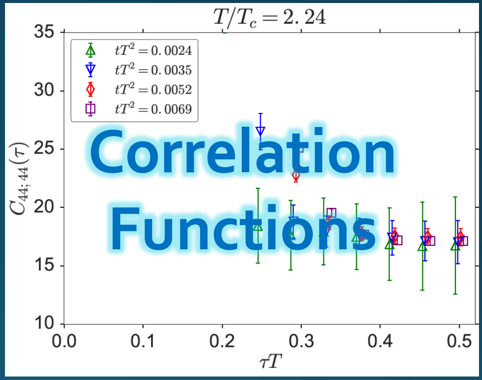
2+1 QCD:
Taniguchi+ (WHOT-QCD), 1711.02262

Summary

- Successful analysis of energy-momentum tensor on the lattice is now available, and various stuides are ongoing!
  - gradient flow method
  - higher-order perturbative coefficients







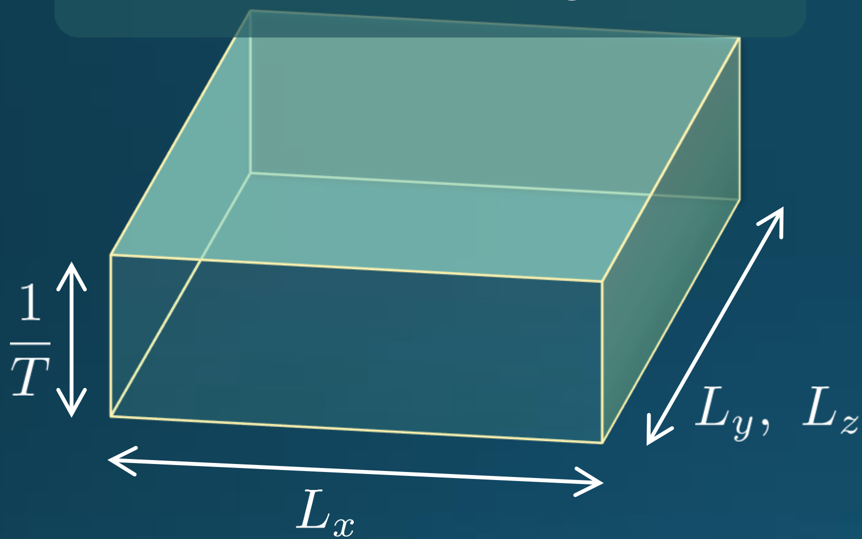
- **So many future studies**
  - Flux tube at **nonzero temperature**
  - EMT distribution **inside hadrons**
  - viscosity / other operators / instantons / **full QCD**

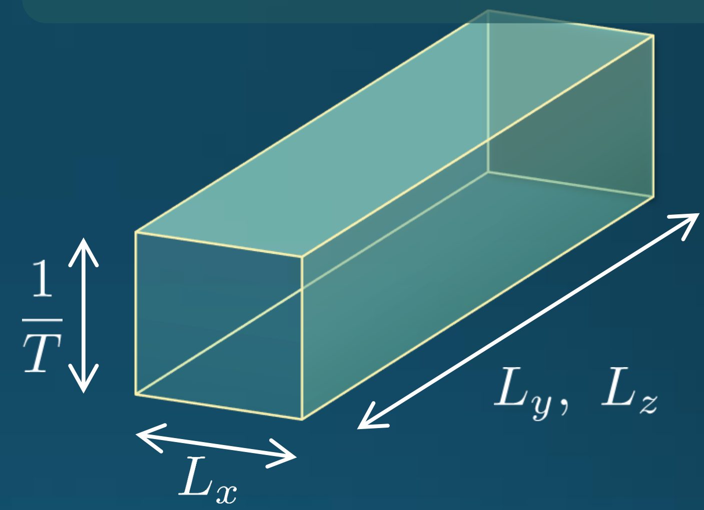
backup

Two Special Cases with PBC

$$1/T \ll L_x = L_y = L_z$$

$$1/T = L_x, \; L_y = L_z$$





$$T_{11} = T_{22} = T_{33}$$

$$T_{44} = T_{11}, \; T_{22} = T_{33}$$

$$\frac{p_1}{p_2} = 1$$

In conformal ($\Sigma_\mu T_{\mu\mu}$=0)

$$\frac{p_1}{p_2} = -1$$

EMT on the Lattice: Conventional

Lattice EMT Operator Caracciolo+, 1990

$$T_{\mu\nu} = Z_6 T^{[6]}_{\mu\nu} + Z_3 T^{[3]}_{\mu\nu} + Z_1 \left(T^{[1]}_{\mu\nu} - \langle T^{[1]}_{\mu\nu} \rangle \right)$$

$$T^{[6]}_{\mu\nu} = (1-\delta_{\mu\nu}) F^a_{\mu\rho} F^a_{\nu\rho}, \quad T^{[3]}_{\mu\nu} = \delta_{\mu\nu} \left(F^a_{\mu\rho} F^a_{\nu\rho} - \frac{1}{4} F^a_{\rho\sigma} F^a_{\rho\sigma} \right), \quad T^{[1]}_{\mu\nu} = \delta_{\mu\nu} F^a_{\rho\sigma} F^a_{\rho\sigma}$$

- Fit to thermodynamics: Z_3, Z_1
- Shifted-boundary method: Z_6, Z_3 — Giusti, Meyer, 2011; 2013; Giusti, Pepe, 2014~; Borsanyi+, 2018

Multi-level algorithm

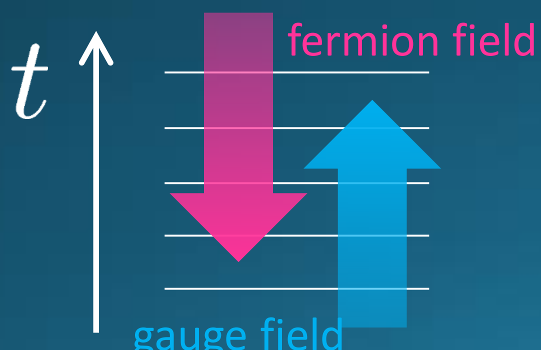
- effective in reducing statistical error of correlator — Meyer, 2007; Borsanyi, 2018; Astrakhantsev+, 2018

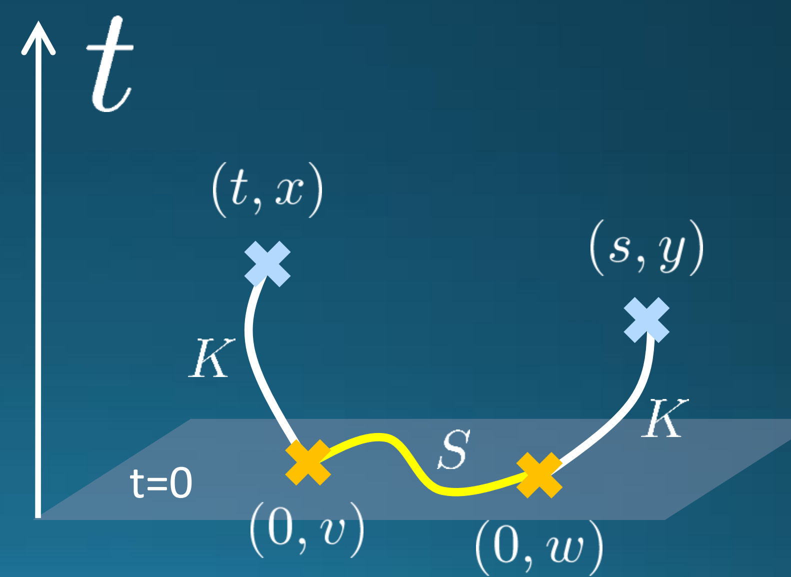
Fermion Propagator

$$S(t,x;s,y) = \langle \chi(t,x)\bar{\chi}(s,y)\rangle$$
$$= \sum_{v,w} K(t,x;0,v)S(v,w)K(s,y;0,w)^{\dagger}$$

$$\Big(\partial_t - D_\mu D_\mu\Big)K(t,x) = 0$$

- propagator of flow equation
- Inverse propagator is needed





N_f=2+1 QCD Thermodynamics

Taniguchi+ (WHOT-QCD),
PR**D96**, 014509 (2017)

- N_f=2+1 QCD, Iwasaki gauge + NP-clover
- $m_{PS}/m_V \approx 0.63$ / almost physical s quark mass

- T=0: CP-PACS+JLQCD (ß=2.05, 28^3x56, a≈0.07fm)
- T>0: 32^3xN_t, N_t = 4, 6, … , 14, 16):
- T≈174-697MeV

- t→0 extrapolation only (No continuum limit)