Critical endpoints of the finite temperature QCD

Yoshifumi Nakamura
RIKEN Center for Computational Science

in collaboration with
Y. Kuramashi, H. Ohno & S. Takeda

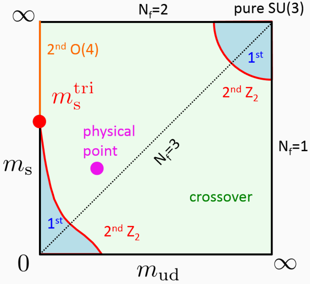
International Molecule-type Workshop
Frontiers in Lattice QCD and related topics
April 15 - April 26 2019
Yukawa Institute for Theoretical Physics, Kyoto University

Contents

- introduction
  - Columbia plot
  - previous studies for critical endpoint
  - previous studies for critical endline
- preliminary results
  - New $\boldsymbol{N_f = 3}$ simulations at $\boldsymbol{N_t = 12}$
  - New $\boldsymbol{N_f = 2+1}$ simulations away from symmetric point
- summary

Columbia plot

nature of finite temperature phase transition of 2+1 flavor QCD at $\mu = 0$ in the plane of $\boldsymbol{m_{u,d}}$ and $\boldsymbol{m_s}$



- 1st order : small $\boldsymbol{m_q}$ region [Pisarski, Wilczek, '84]
- 1st order : heavy $\boldsymbol{m_q}$ region
- crossover : medium $\boldsymbol{m_q}$
- 2nd order ($\boldsymbol{Z_2}$) : boundary between 1st and crossover

At small $\boldsymbol{m_q}$ region

- crossover at the physical point
- critical end point at SU(3) flavor symmetric point, $\boldsymbol{m}^{\rm sym}$, has not been determined yet
- critical end line has not been well determined yet

Previous studies for critical endpoint

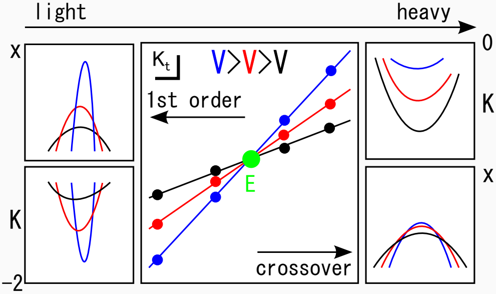
found	N_f	S_G	S_F	N_t	endpoint	ref
y	3	S	W	4	$m_{\mathrm{PS}} > 1$GeV	Iwasaki et al, '96
y	3	S	KS	4	$m_{\mathrm{PS}} \sim 290$ MeV	Karsch et al, '01
y	3	I	p4	4	$m_{\mathrm{PS}} \sim 190$ MeV	Karsch et al, '01
y	3	S	KS	4	$m_{\mathrm{PS}} = 290(20)$ MeV	Karsch et al, '04
y	3	I	p4	4	$m_{\mathrm{PS}} = 67(17)$ MeV	Karsch et al, '04
y	3	S	KS	4	$am_{\mathrm{q}} \approx 0.033$	Karsch et al, '04
y	3	S	KS	4	$am_{\mathrm{q}} = 0.0260(5)$	de Forcrand et al, '07
y	3	S	KS	4	$m_{\mathrm{PS}}/T = 1.680(4)$	de Forcrand et al, '07
y	3	S	KS	6	$m_{\mathrm{PS}}/T = 0954(12)$	de Forcrand et al, '07
n	2+1	I	stout KS	4	$m_{\mathrm{q}}/m^{\mathrm{phy}} \lesssim 0.07$	Endrodi et al, '07
n	2+1	I	stout KS	6	$m_{\mathrm{q}}/m^{\mathrm{phy}} \lesssim 0.12$	Endrodi et al, '07
n	3	I	HISQ	6	$m_{\mathrm{PS}} \lesssim 50$ MeV	Bazavov et al, '17
y	3	I	imp. W	4,6,8	$m_{\mathrm{PS}} \sim 300$ MeV	our, '14
y	3	I	imp. W	8,10	$m_{\mathrm{PS}} \lesssim 170$ MeV	our, '17
n	2+1	I	HISQ	6,8(,12)	$m_\pi \lesssim 80$ MeV	Ding et al, '18, '19

Endrodi et al, '07 : $m_l/m_s = m_l^{phy}/m_s^{phy}$

Ding et al, '18, '19 : $m_s = m_s^{phy}$

Our study for critical endpoints

- Iwasaki gauge + NP O(a) improved Wilson fermions
- chiral condensate (10 - 20 noises for $\mathrm{Tr}\boldsymbol{D}^{-1,-2.-3,-4}$)
- kurtosis intersection method to determine the critical endpoint
- reweighting method to obtain more critical endpoints

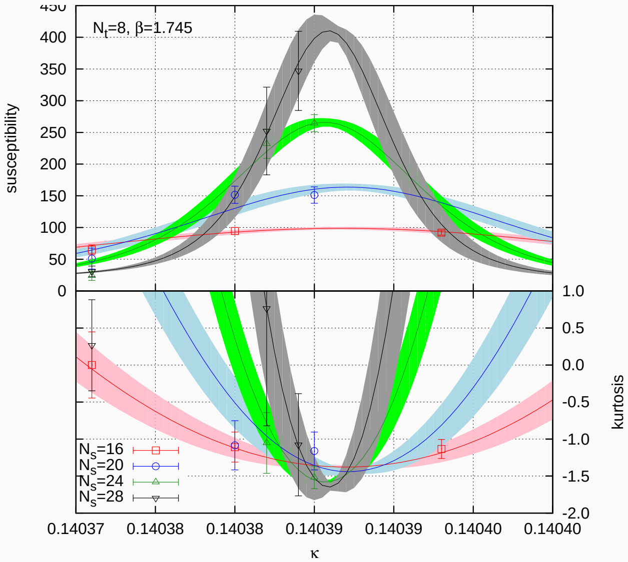


- O(100) zero temperature runs for physical scale setting are covering almost critical endpoints and also transition points of finite temperature simulations

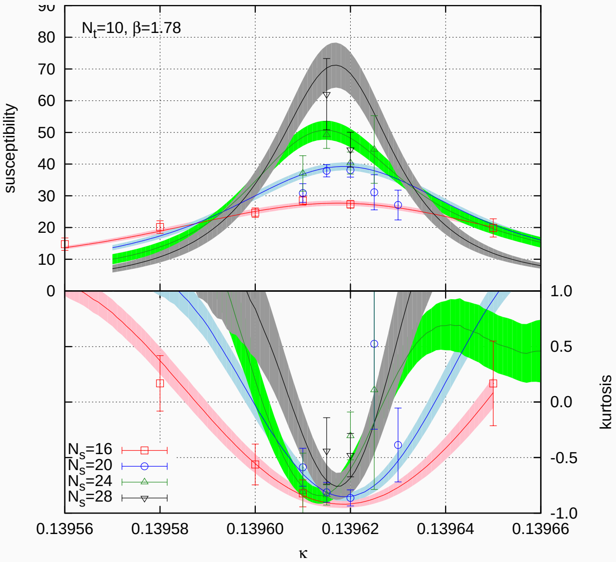
$N_f = 3$ simulation parameters

- $N_t = 4$ ($a \approx 0.23$fm) , $N_l = 6, 8, 10, 12, 16$
    - $\beta = 1.60$, $\kappa = 0.1431 - 0.1439$
    - $\beta = 1.65$, $\kappa = 0.1410 - 0.1415$
    - $\beta = 1.70$, $\kappa = 0.1371 - 0.1390$
- $N_t = 6$ ($a \approx 0.19$fm) , $N_l = 10, 12, 16, 24$
    - $\beta = 1.715$, $\kappa = 0.140900 - 0.141100$
    - $\beta = 1.73$, $\kappa = 0.140420 - 0.140450$
    - $\beta = 1.75$, $\kappa = 0.139620 - 0.139700$
- $N_t = 8$ ($a \approx 0.16$fm) , $N_l = 16, 20, 24, 28$
    - $\beta = 1.745$, $\kappa = 0.140371 - 0.140393$
    - $\beta = 1.74995$, $\kappa = 0.140240$
    - $\beta = 1.76$, $\kappa = 0.139950$
- $N_t = 10$ ($a \approx 0.13$fm), $N_l = 16, 20, 24, 28$
    - $\beta = 1.77$, $\kappa = 0.139800 - 0.139900$
    - $\beta = 1.78$, $\kappa = 0.139550 - 0.139650$
    - $\beta = 1.79$, $\kappa = 0.139300 - 0.139400$

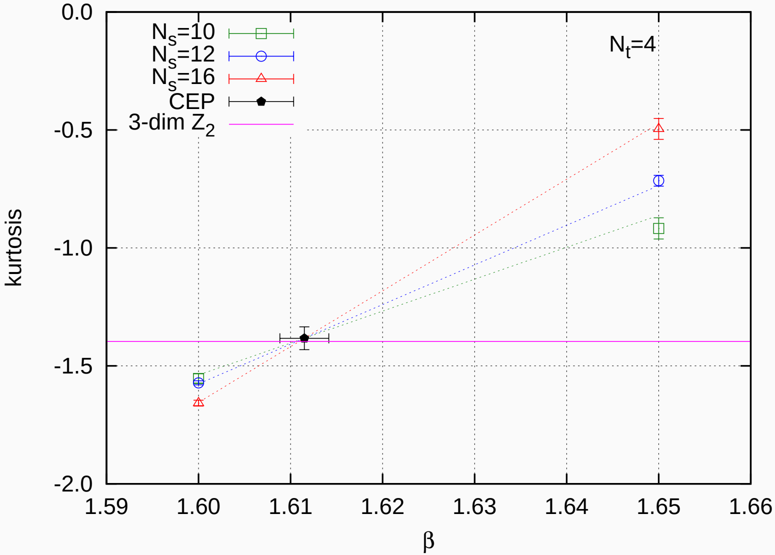
$N_f = 3$ susceptibility and kurtosis (example 1)



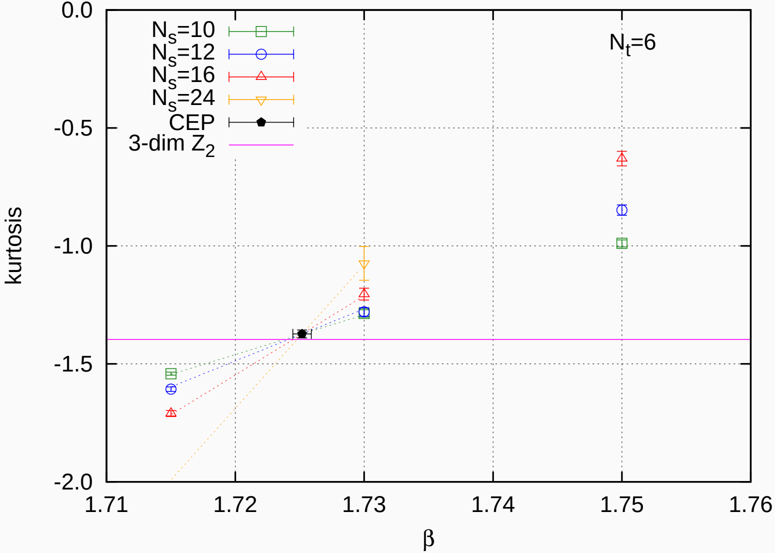
$N_f = 3$ susceptibility and kurtosis (example 2)



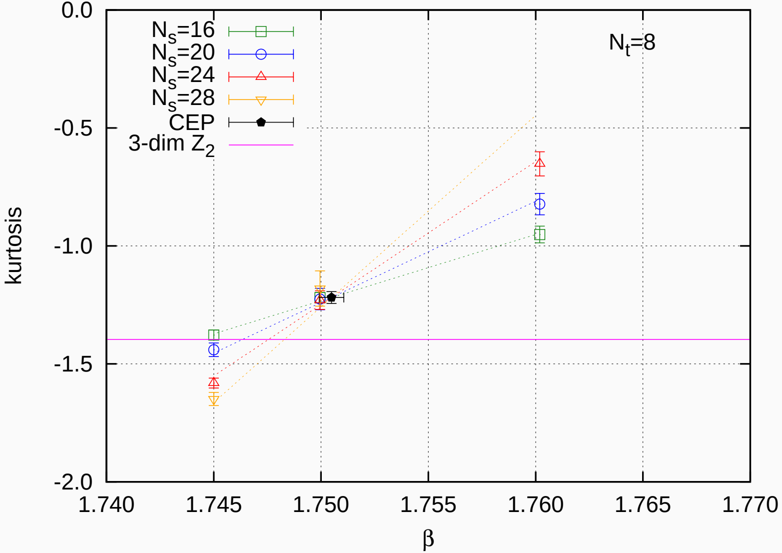
$N_f = 3$ kurtosis intersection ($N_t = 4$)



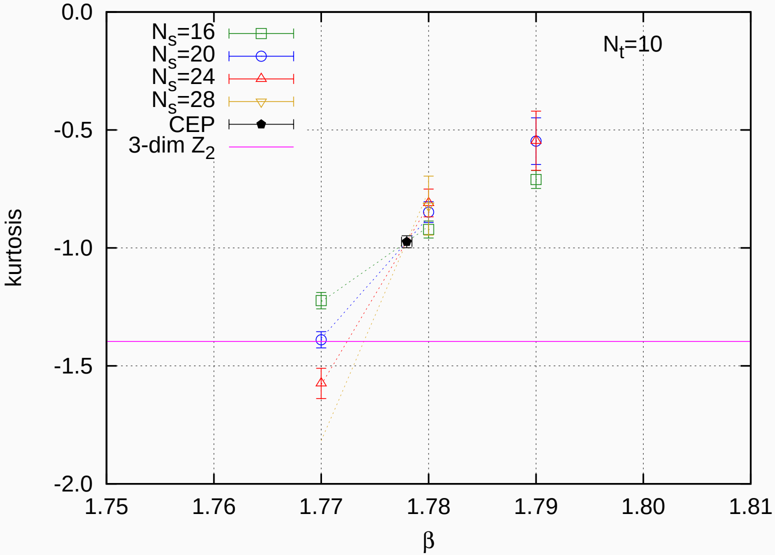
$N_f = 3$ kurtosis intersection ($N_t = 6$)



$N_f = 3$ kurtosis intersection ($N_t = 8$)



$N_f = 3$ kurtosis intersection ($N_t = 10$)



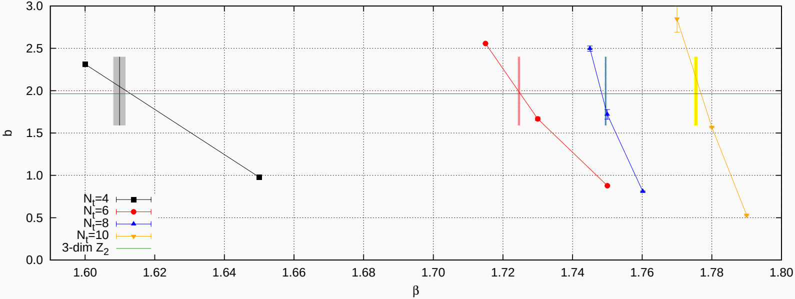
$N_f = 3$ kurtosis intersection fitting

$$K = \left[K_{\rm E} + AN_l^{1/\nu}(\beta - \beta_{\rm E})\right](1 + BN_l^{y_t - y_h})$$

N_t	Fit	$\beta_{\rm E}$	$K_{\rm E}$	ν	A	B	$y_t - y_h$	$\chi^2/\rm dof$
4	**1**	**1.6115(26)**	**−1.383(48)**	**0.84(13)**	**0.88(42)**	×	×	**1.75**
	2	**1.61065(61)**	**−1.396**	**0.63**	**0.313(12)**	×	×	**3.05**
	3	**1.6099(17)**	**−1.396**	**0.63**	**0.311(14)**	**0.10(21)**	**−0.894**	**3.77**
6	**1**	**1.72518(71)**	**−1.373(17)**	**0.683(54)**	**0.58(17)**	×	×	**0.68**
	2	**1.72431(24)**	**−1.396**	**0.63**	**0.418(11)**	×	×	**0.70**
	3	**1.72462(40)**	**−1.396**	**0.63**	**0.422(12)**	**−0.052(52)**	**−0.894**	**0.70**
8	**1**	**1.75049(57)**	**−1.219(25)**	**0.527(55)**	**0.146(88)**	×	×	**0.73**
	2	**1.74721(42)**	**−1.396**	**0.63**	**0.404(36)**	×	×	**5.99**
	3	**1.74953(33)**	**−1.396**	**0.63**	**0.414(13)**	**−1.33(15)**	**−0.894**	**0.73**
10	**1**	**1.77796(48)**	**−0.974(25)**	**0.466(45)**	**0.084(52)**	×	×	**0.22**
	2	**1.7694(16)**	**−1.396**	**0.63**	**0.421(95)**	×	×	**10.03**
	3	**1.77545(53)**	**−1.396**	**0.63**	**0.559(29)**	**−2.97(25)**	**−0.894**	**0.43**

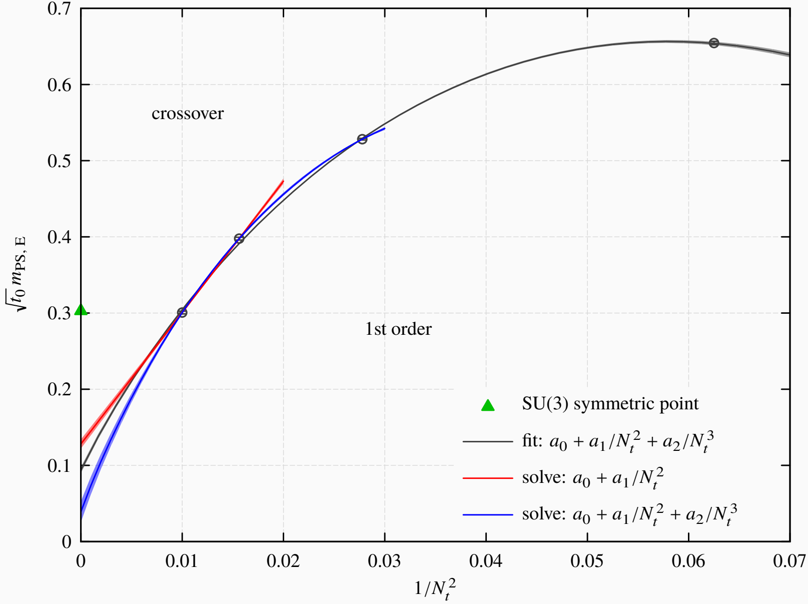
Fit-1: no correction term ($B = y_t - y_h = 0$) and all other parameters are used as fit parameter.
Fit-2: no correction term and assuming the 3D Z_2 universality class for $K_{\rm E}$ and ν.
Fit-3: including correction term and assuming the 3D Z_2 universality class for $K_{\rm E}$, ν and $y_t - y_h$.

$N_f = 3$, $b = \gamma/\nu$ of $\chi_{max} \propto N_l^b$



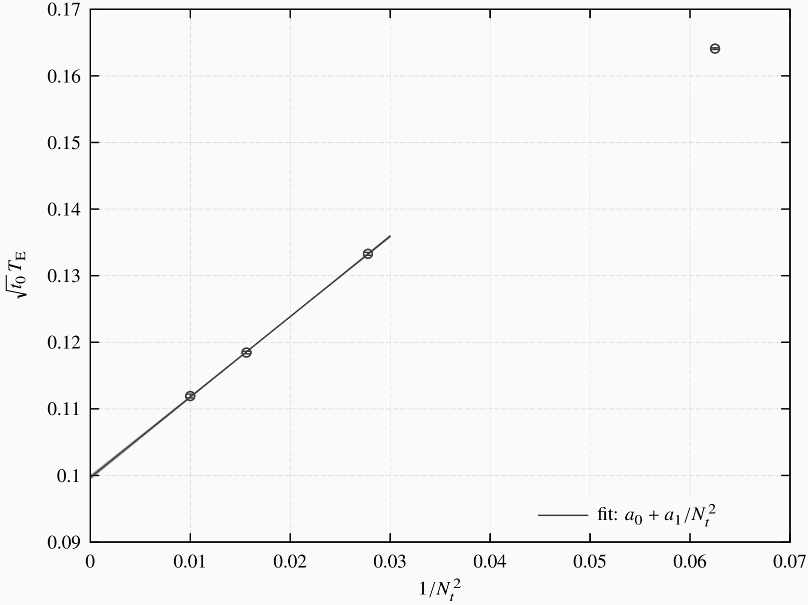
critical endpoints determined by kurtosis intersection and critical exponent are consistent

Critical endpoint (m_{PS})



Continuum extrapolation for $\sqrt{t_0}\, m_{\mathrm{PS,E}}$ is still uncertain

Critical endpoint (*T*)



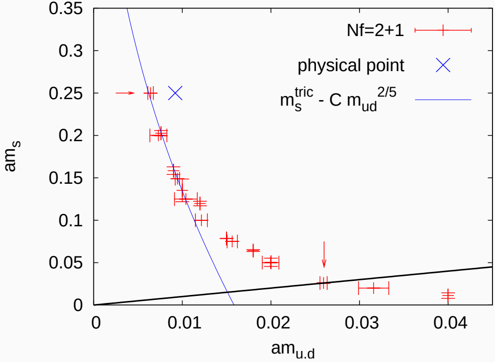
Critical endpoint

In the continuum limit

$$\begin{aligned} m_{\mathrm{PS,E}} &\lesssim 170\mathrm{MeV} \\ T_{\mathrm{E}} &= 134(3)\mathrm{MeV} \\ m_{\mathrm{PS,E}}/T_{\mathrm{E}} &\lesssim 1.3 \end{aligned}$$

We shall extend our study at $N_t = 12$

Previous study for critical end line

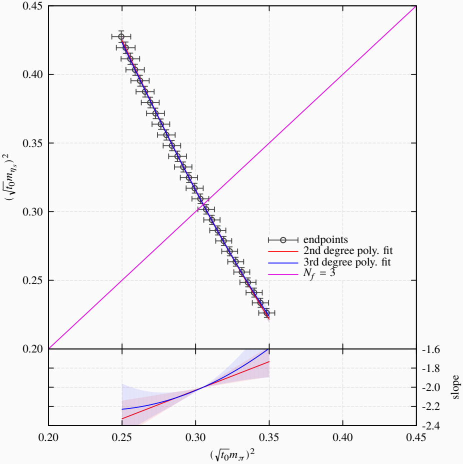


[de Forcrand, Philipsen, ’07]

- staggered fermions
- $\boldsymbol{N_t = 4}$, $\boldsymbol{a} \approx \mathbf{0.3}$ fm
- data exhibits that slope at $\boldsymbol{m}^{\mathrm{sym}}$ is not - 2
- $\boldsymbol{am_s^{crit}} \approx \mathbf{0.7}$ (roughly **5** times larger than $\boldsymbol{m_s^{phy}}$)

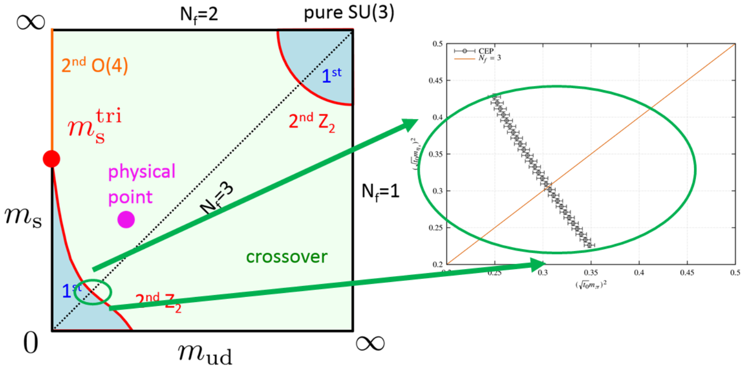
Our study for critical end line(1/2)

around $\boldsymbol{m}^{\mathrm{sym}}$



- Wilson-clover fermion
- $\boldsymbol{N_t = 6}$
- $\boldsymbol{a \approx 0.19}$ fm
- we confirmed that slope at $\boldsymbol{m}^{\mathrm{sym}}$ is - 2

Our study for critical end line(2/2)



We shall extend our study for critical end line away form $\boldsymbol{m}^{\mathrm{sym}}$

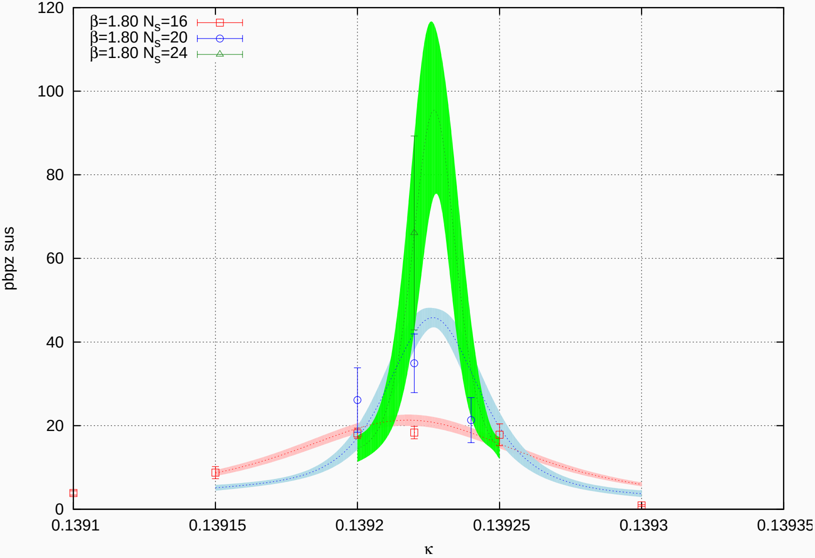
Preliminary results

- New $N_f = 3$ simulation at $N_t = 12$
- New $N_f = 2 + 1$ simulation away from symmetric point

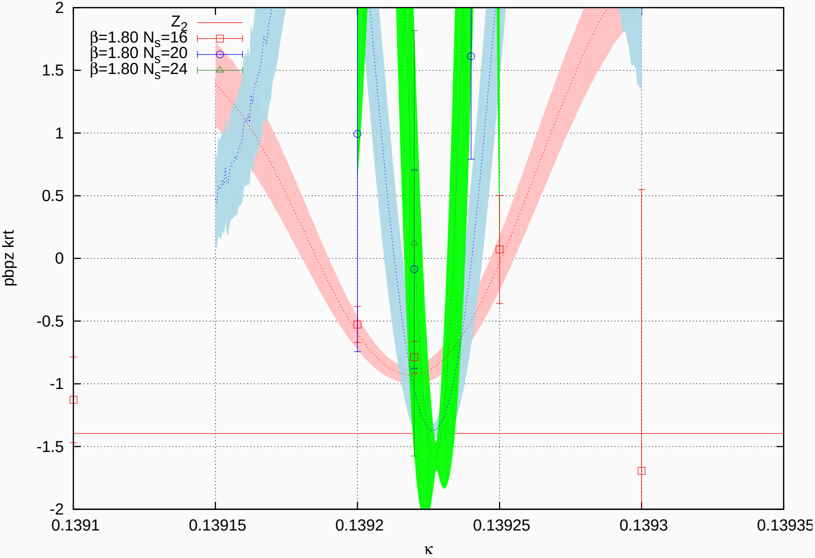
New $N_f = 3$ simulation at $N_t = 12$

- $N_t = 12$ ($a \approx 0.1$fm ?), $N_l = 16, 20, 24, 28, 32$
  - $\beta = 1.78$, $\kappa = 0.1396 - 0.1397$
  - $\beta = 1.79$, $\kappa = 0.1393 - 0.1395$
  - $\beta = 1.80$, $\kappa = 0.1390 - 0.1394$
  - $\beta = 1.81$, $\kappa = 0.13895 - 0.13910$
  - $\beta = 1.82$, $\kappa = 0.13875 - 0.13895$
  - $\beta = 1.83$, $\kappa = 0.13855 - 0.13880$
  - $\beta = 1.84$, $\kappa = 0.13835 - 0.13870$
  - $\beta = 1.85$, $\kappa = 0.13815 - 0.13860$

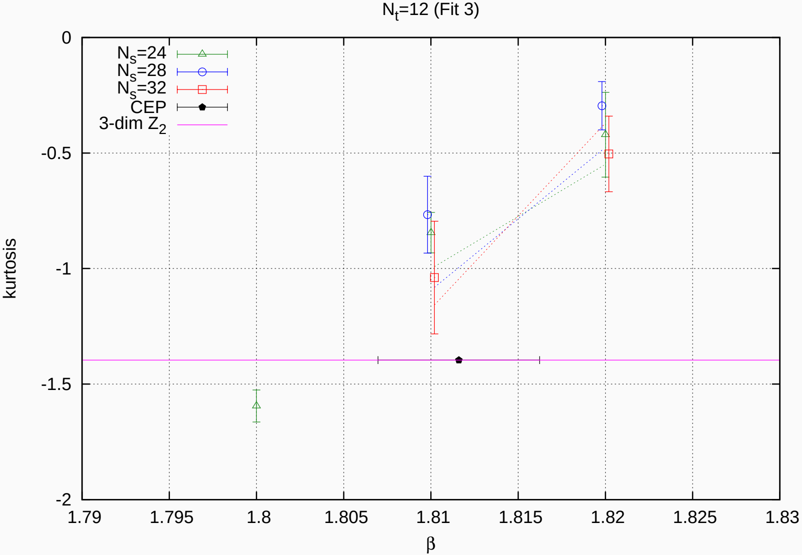
$N_f = 3$ susceptibility ($N_t = 12$)



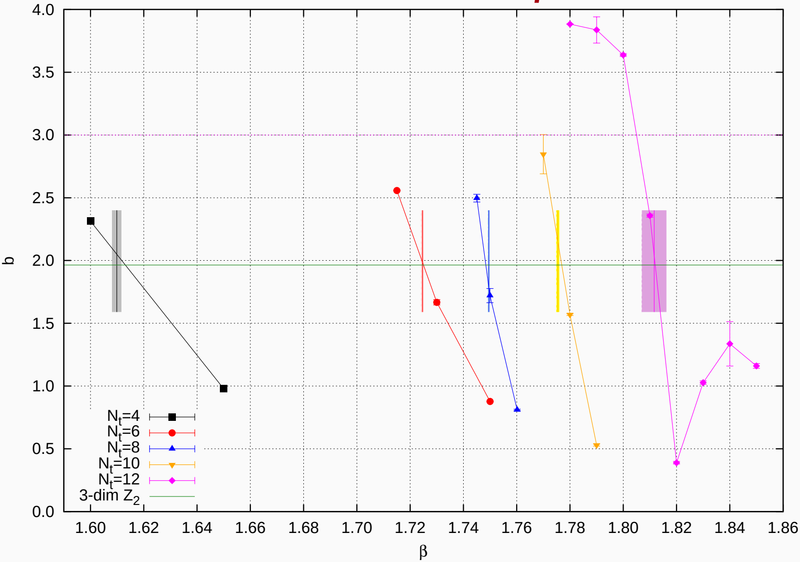
$N_f = 3$ kurtosis ($N_t = 12$)



$N_f = 3$ kurtosis intersection ($N_t = 12$)



$N_f = 3$, $b = \gamma/\nu$ of $\chi_{max} \propto N_l^b$

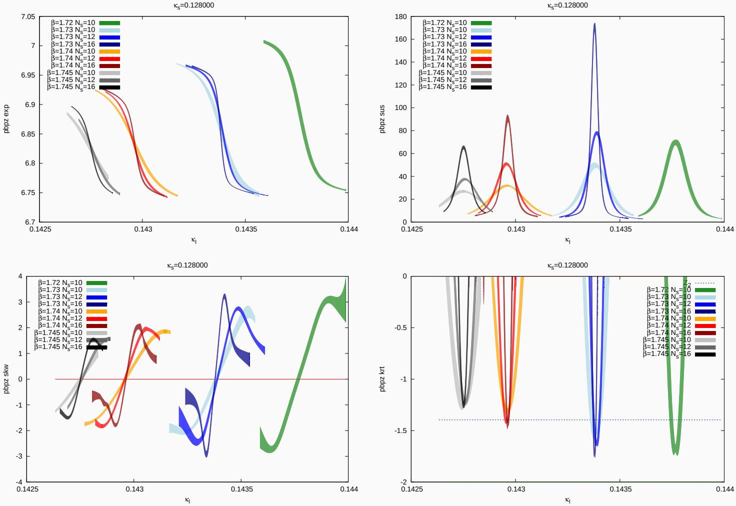


critical endpoints determined by kurtosis intersection and critical exponent are consistent including $N_t = 12$

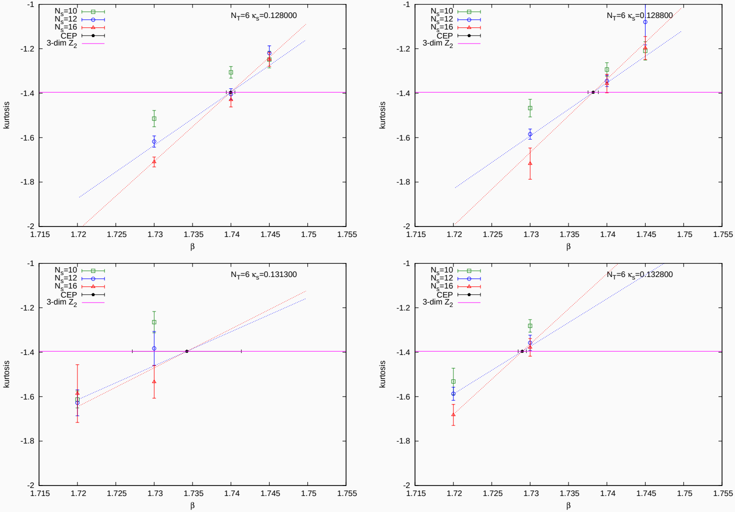
New $N_f = 2 + 1$ simulation away from symmetric point

- $N_t = 6$ ($a \approx 0.19$fm), $N_l = 10, 12, 16, 20, 24$
- symmetric runs
    - $\beta = 1.715$, $\kappa = 0.140900 – 0.141100$
    - $\beta = 1.725$, $\kappa = 0.140600 – 0.140618$
    - $\beta = 1.73$, $\kappa = 0.140420 – 0.140450$
- very heavy m_s runs ($\kappa_s = 0.128$)
    - $\beta = 1.73$, $\kappa_l = 0.143365 – 0.143390$
    - $\beta = 1.74$, $\kappa_l = 0.142970 – 0.143042$
    - $\beta = 1.745$, $\kappa_l = 0.142733 – 0.142790$
- heavy m_s runs ($\kappa_s = 0.1328$)
    - $\beta = 1.72$, $\kappa_l = 0.143160$
    - $\beta = 1.73$, $\kappa_l = 0.142702 – 0.142750$
    - $\beta = 1.735$, $\kappa_l = 0.142508$

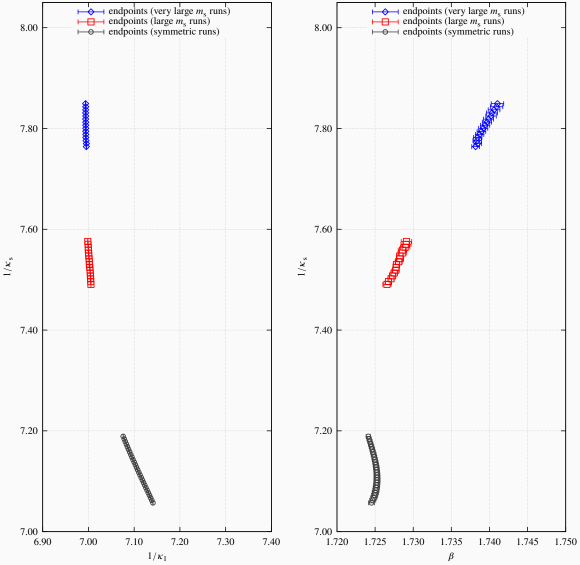
$N_f = 2 + 1$ expectation, susceptibility, skewness and kurtosis (example)



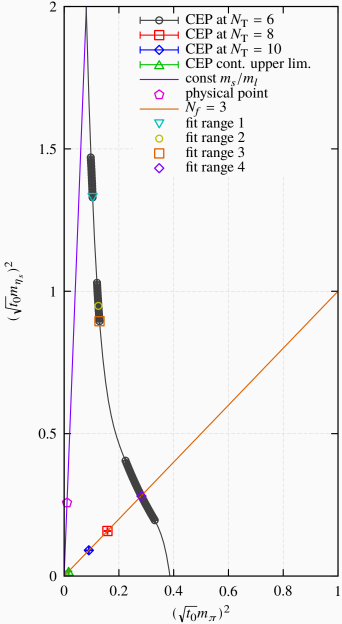
$N_f = 2 + 1$ kurtosis intersection (example)



Critical endpoints in bare parameter plane



Critical endline at $N_t = 6$



$$m_s - m_s^{\text{tric}} \sim m_l^{2/5}$$

[Rajagopal '95]

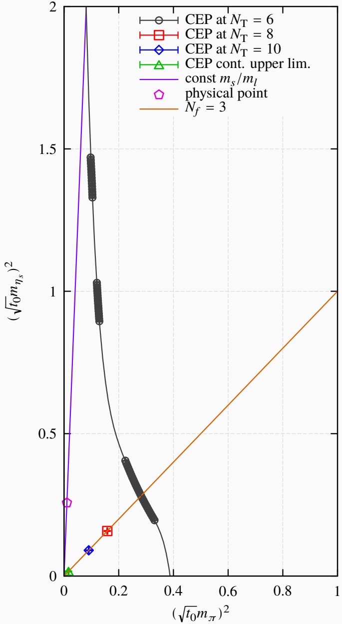
Fitting endpoints for tri-critical point

$$x = (\sqrt{t_0}m_\pi)^2 \propto m_l$$
$$y = (\sqrt{t_0}m_{\eta_s})^2 \propto m_s$$

$$y = b_0 + b_1 x^{2/5}$$

b_0	b_1	χ^2/dof	range (x <)
6.71(8)	-13.3(2)	0.54	f. r. 1
6.44(2)	-12.62(4)	0.97	f. r. 2
6.34(3)	-12.39(6)	3.26	f. r. 3
3.28(9)	-5.1(2)	1655	f. r. 4

Critical endline at $N_t = 6$



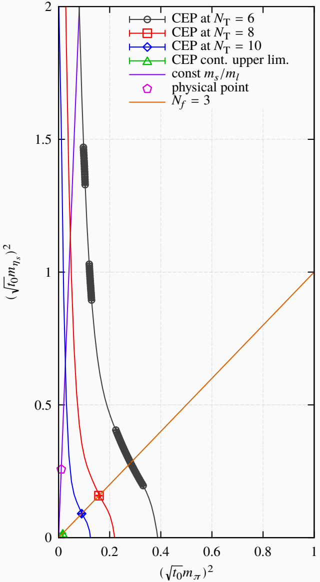
Fiting endpoints for all range with $\boldsymbol{b_0 = 6.71}$

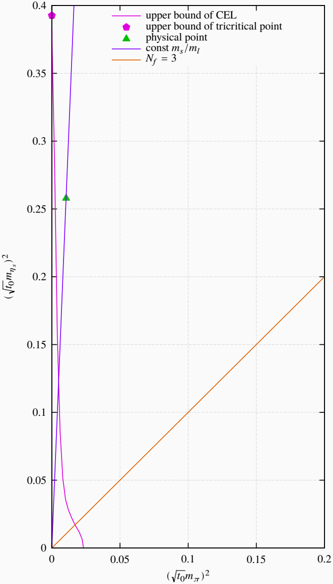
Fit 1 $y = b_0 + a_0 x^{2/5} + a_1 x + a_2 x^2 + a_3 x^3$
χ^2/dof = 158

Fit 2 $y = b_0 + a_0 x^{2/5} + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4$
χ^2/dof = 6.6

Fit 3 $y = b_0 + a_0 x^{2/5} + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5$
χ^2/dof = 1.3

Critical endline at $N_t = 8, 10, \infty$ (estimated upper bound)





assuming same shape

Summary

We are updating the critical endpint in the continuum limit and the critical endline away from the SU(3)-flavor symmetric point at $N_t = 6$ and presented preliminary results for the critical end lines at $N_t = 8, 10$ and in the continuum limit with NP O(a) improved Wilson fermions. We find

- 3 series of multi-ensemble, multi-parameter re-weighting determines well the critical end line
- critical end line at $N_t = 6$ is nice agreement with $m_s - m_s^{tri} \sim m_l^{2/5}$ in small m_l region
- $m_s^{tri} \lesssim 1.5\, m_s^{phy}$ (very preliminary!!)
- tentative tri-critical scaling region : $m_{\eta_s}^2 \gtrsim 5 m_\pi^2$