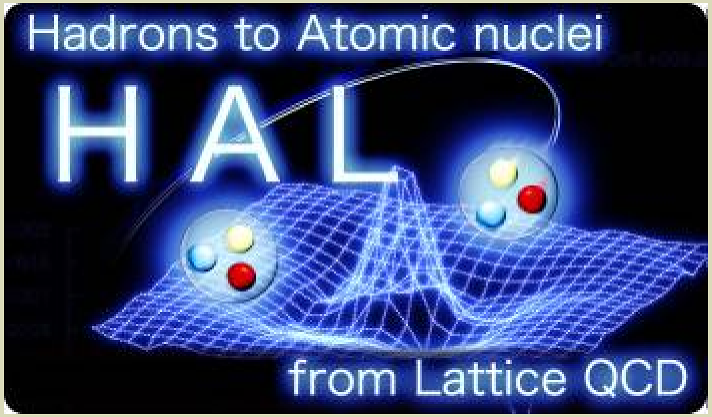
Hyperon-nucleon interaction from lattice quantum chromoDynamics at almost physical masses

H. Nemura[1],
for HAL QCD Collaboration
Y. Akahoshi[2], S. Aoki[2], T. Aoyama[2], T. Doi[3], T. M. Doi[3],
F. Etminan[4], S. Gongyo[3], T. Hatsuda[3], Y. Ikeda[1],
T. Inoue[5], T. Iritani[3], N. Ishii[1], T. Miyamoto[2],
K. Murano[1], and K. Sasaki[2],



[1]*Osaka University,*
[2]*Kyoto University,* [3]*RIKEN,* [4]*University of Birjand,*
[5]*Nihon University*

arXiv:1810.04046 [hep-lat]

Outline

- Introduction
  - Importance of LN-SN tensor force for hypernuclei
  - Brief introduction of HAL QCD method
    - Effective block algorithm for various baryon-baryon channels, CPC**207**,91(2016)[1510.00903]
- Preliminary results of LN-SN potentials at nearly physical point; update from [1702.00734]
  - LN-SN(I=1/2), central and tensor potentials
  - SN(I=3/2), central and tensor potentials
    - Phase shifts of SN(I=3/2) scattering
- Summary

Plan of research

QCD

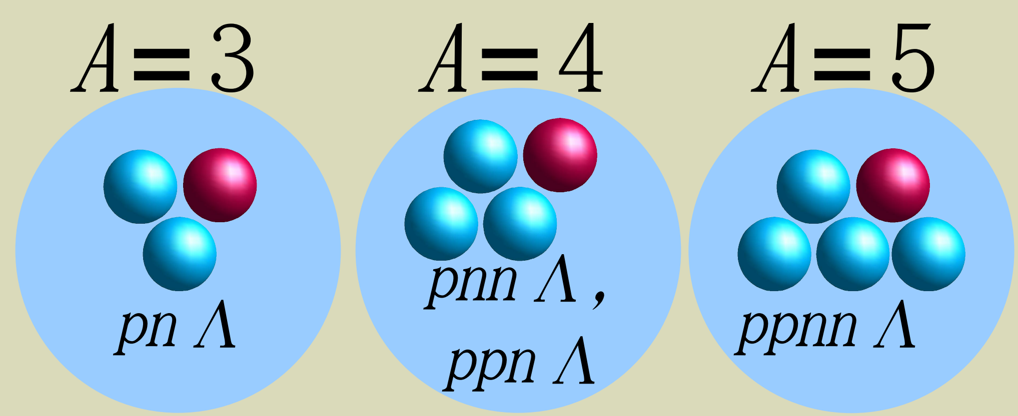
Baryon interaction

J-PARC,
JLab, GSI, MAMI, ...
YN scattering,
hypernuclei

Structure and reaction of (hyper)nuclei

Equation of State (EoS) of nuclear matter

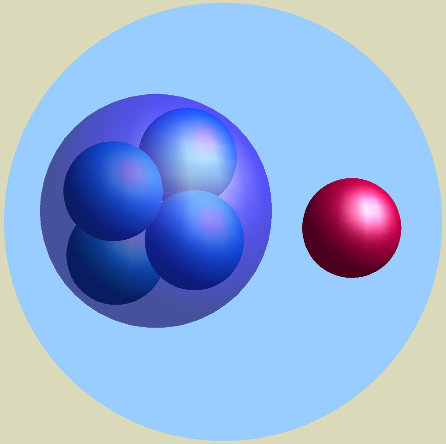
Neutron star and supernova



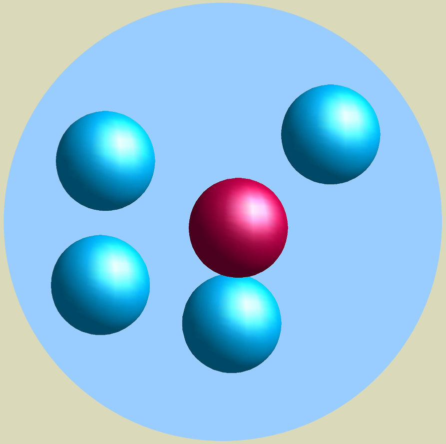
What is realistic picture of hypernuclei?

- $B(\text{total}) = B(^4\text{He}) + B_\Lambda(^5_\Lambda\text{He})$

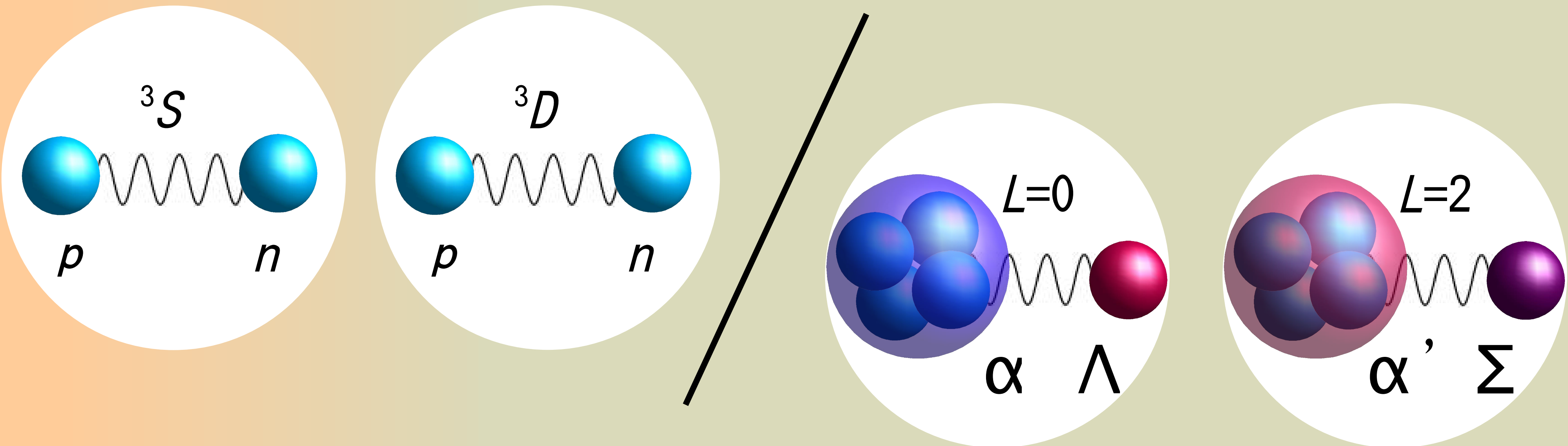
- A conventional picture:
  $B(\text{total})$
  $= B(^4\text{He}) + B_\Lambda(^5_\Lambda\text{He})$
  $=$ 28+3 MeV.



- A (probably realistic) picture:
  $B(\text{total})$
  $= (B(^4\text{He}) - \Delta E_c) + (B_\Lambda(^5_\Lambda\text{He}) + \Delta E_c)$
  $=$??+?? MeV.



Comparison between d=p+n and core+Y



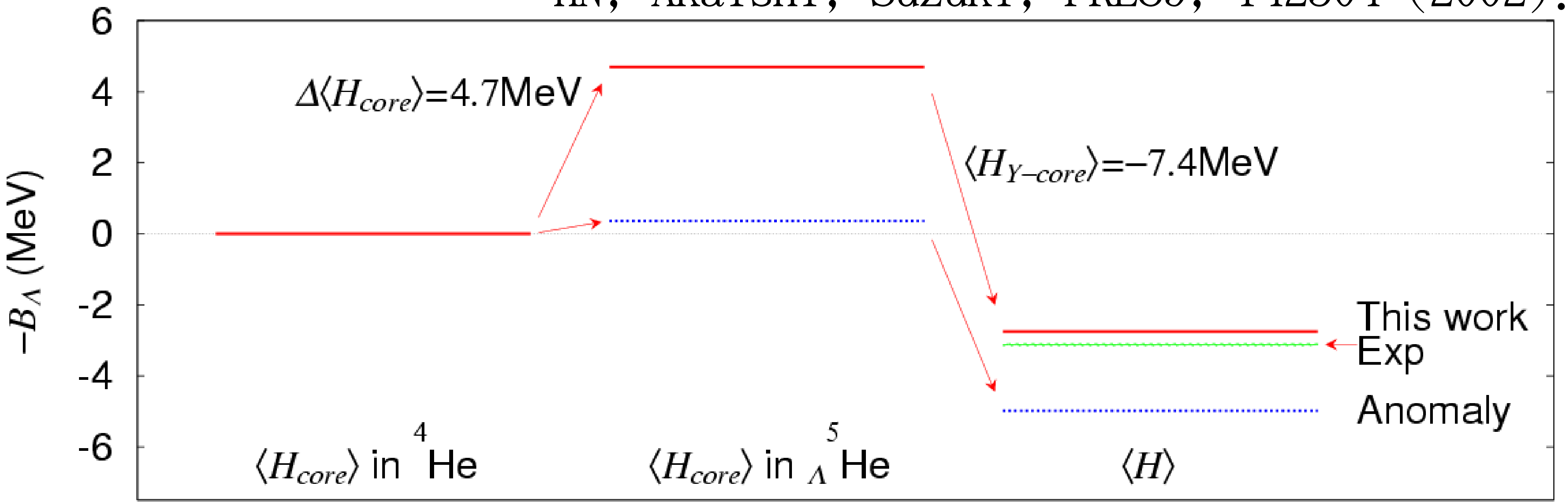
	$\langle T_S \rangle$ (MeV)	$\langle T_D \rangle$ (MeV)	$\langle V_{NN}(\text{central}) \rangle$ (MeV)	$\langle V_{NN}(\text{tensor}) \rangle$ (MeV)	$\langle V_{NN}(\text{LS}) \rangle$ (MeV)
AV8	8.57	11.31	−4.46	−16.64	−1.02
G3RS	10.84	5.64	−7.29	−11.46	0.00

	$\langle T_{Y\text{-}c} \rangle_\Lambda$	$\langle T_{Y\text{-}c} \rangle_\Sigma + \Delta \langle H_C \rangle$	$\langle V_{YN}$(のこり)$\rangle$	$2\langle V_{\Lambda N-\Sigma N}(\text{tensor}) \rangle$
$^5_\Lambda$He	9.11	3.88+4.68	−0.86	−19.51
$^4_\Lambda$H*	5.30	2.43+2.02	0.01	−10.67
$^4_\Lambda$H	7.12	2.94+2.16	−5.05	−9.22

HN, Akaishi, Suzuki, PRL89, 142504 (2002).

Rearrangement effect of ${}^{5}_{\Lambda}$He

HN, Akaishi, Suzuki, PRL89, 142504 (2002).



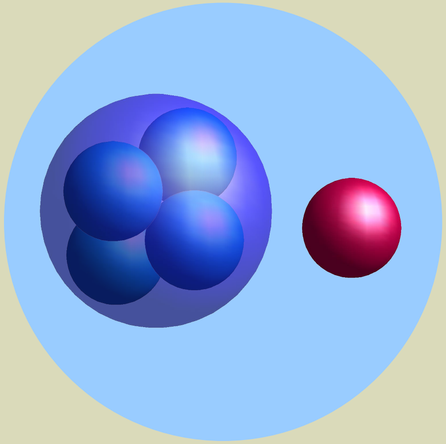
$$H = \sum_{i=1}^{A}\left(m_i c^2 + \frac{\boldsymbol{p}_i^2}{2\,\mathrm{m}_i}\right) - T_{CM} + \sum_{i<j}^{A-1} v_{ij}^{(NN)} + \sum_{i=1}^{A-1} v_{iY}^{(NY)} = H_{core} + H_{Y-core}\ ,$$

$$H_{core} = \sum_{i=1}^{A-1}\frac{\boldsymbol{p}_i^2}{2\,\mathrm{m}_N} - \frac{\left(\sum_{i=1}^{A-1}\boldsymbol{p}_i\right)^2}{2(A-1)m_N} + \sum_{i<j}^{A-1} v_{ij}^{(NN)} = T_{core} + V_{NN}\ .$$

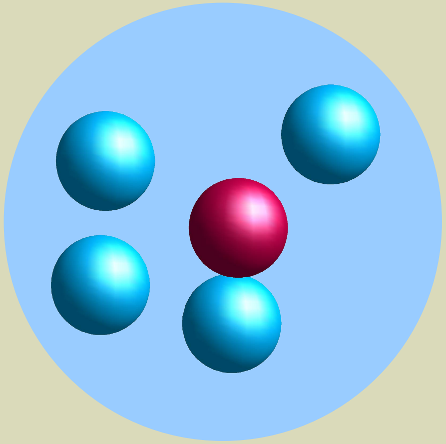
What is realistic picture of hypernuclei?

- $B(\text{total}) = B(^{4}\text{He}) + B_{\Lambda}({}^{5}_{\Lambda}\text{He})$

- A conventional picture:
  $B(\text{total})$
  $= B(^{4}\text{He}) + B_{\Lambda}({}^{5}_{\Lambda}\text{He})$
  $= 28+3$ MeV.



- A (probably realistic) picture:
  $B(\text{total})$
  $= (B(^{4}\text{He}) - \Delta E_{c}) + (B_{\Lambda}({}^{5}_{\Lambda}\text{He}) + \Delta E_{c})$
  $= 24+7$ MeV.



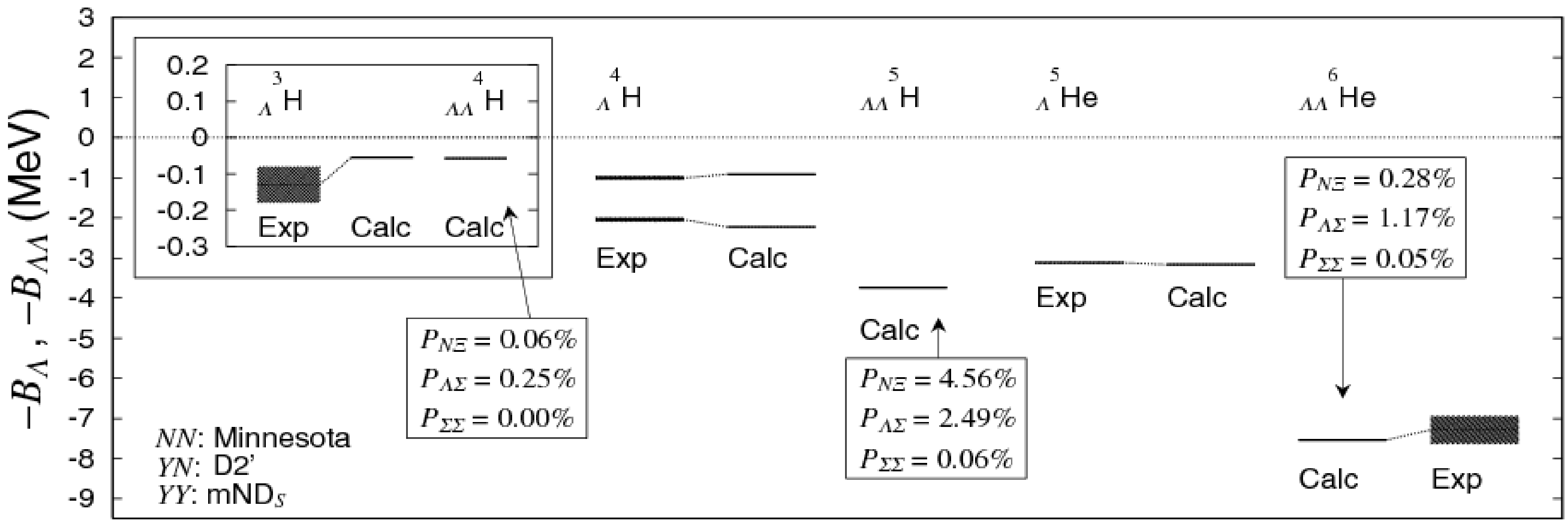


FIG. 1. Λ and $\Lambda\Lambda$ separation energies of $A = 3 - 6$, $S = -1$ and -2 s-shell hypernuclei. The Minnesota NN, D2$'$ YN, and mND$_S$ YY potentials are used. The width of the line for the experimental B_Λ or $B_{\Lambda\Lambda}$ value indicates the experimental error bar. The probabilities of the $N\Xi$, $\Lambda\Sigma$, and $\Sigma\Sigma$ components are also shown for the $\Lambda\Lambda$ hypernuclei.

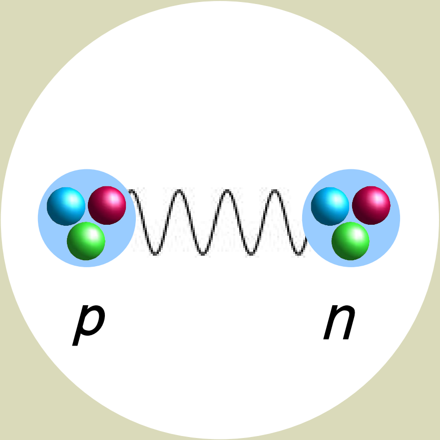
FY calculation with and w/o 3NF

- Three nucleon force does not change the B_Λ so much.
  - A. Nogga, *et al.*, PRL**88**, 172501 (2002).

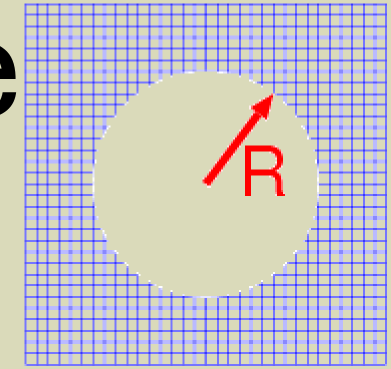
TABLE II. *NN* and 3*N* interaction dependence of the ${}^4_\Lambda$He SE's E^Λ_{sep} and the 0^+-1^+ splitting Δ. We show results for different combinations of *YN*, *NN*, and 3*N* forces (*YNF*, *NNF*, and 3*NF*). All energies are given in MeV.

YNF	*NNF*	3*NF*	$E^\Lambda_{sep}(0^+)$	$E^\Lambda_{sep}(1^+)$	Δ
SC97*e*	Bonn *B*	⋯	1.66	0.80	0.84
SC97*e*	Nijm 93	⋯	1.54	0.72	0.79
SC97*e*	Nijm 93	TM	1.56	0.70	0.82
SC89	Bonn *B*	⋯	2.25	⋯	⋯
SC89	Nijm 93	⋯	2.14	0.02	2.06
SC89	Nijm 93	TM	2.19	⋯	⋯

Lattice QCD calculation



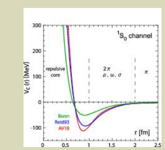
Multi-hadron on lattice



i) basic procedure:

asymptotic region

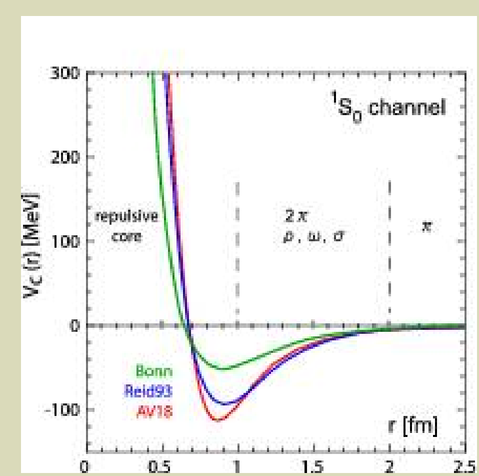
--> phase shift



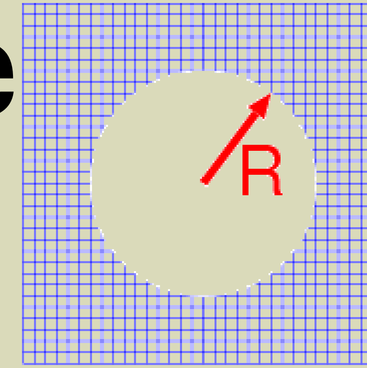
ii) HAL's procedure:

interacting region

--> potential



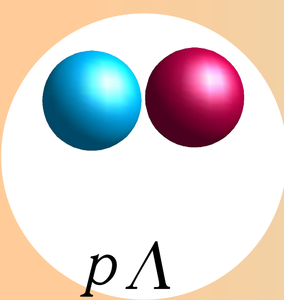
Multi-hadron on lattice

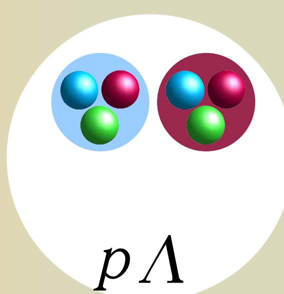


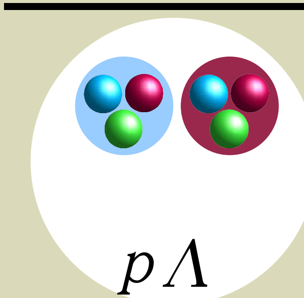
Lattice QCD simulation

$$L = -\frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu} + \bar{q}\gamma^\mu (i\partial_\mu - g t^a A^a_\mu) q - m\bar{q}q$$

$$\begin{aligned} \langle O(\bar{q}, q, U)\rangle &= \int dU\, d\bar{q}\, dq\, \mathrm{e}^{-S(\bar{q}, q, U)} O(\bar{q}, q, U) \\ &= \int dU \det D(U)\, \mathrm{e}^{-S_U(U)} O(D^{-1}(U)) \\ &= \lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} O(D^{-1}(U_i)) \end{aligned}$$



$\rightarrow \langle$ 

$(t)\ \overline{}$ 

$\overline{(t_0)}\ \rangle$

Multi-hadron on lattice

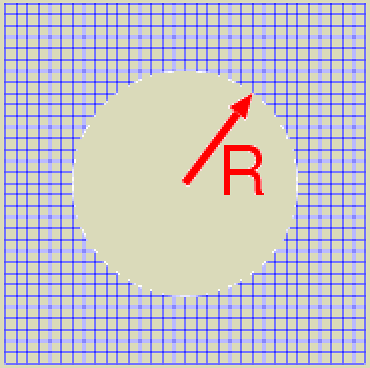
i) basic procedure:

asymptotic region

(or temporal correlation)

--> scattering energy

--> phase shift



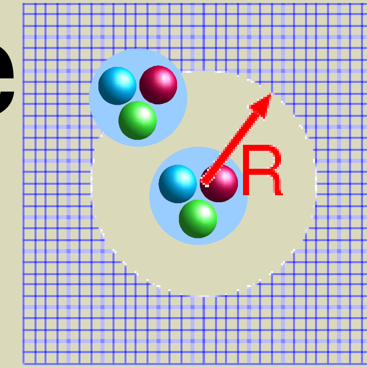
$$E = \frac{k^2}{2\mu}$$

$$k \cot \delta_0(k) = \frac{2}{\sqrt{\pi} L} Z_{00}(1; (kL/(2\pi))^2) = \frac{1}{a_0} + O(k^2)$$

$$Z_{00}(1; q^2) = \frac{1}{\sqrt{4\pi}} \sum_{\vec{n} \in Z^3} \frac{1}{(n^2 - q^2)^s} \qquad \Re\, s > \frac{3}{2}$$

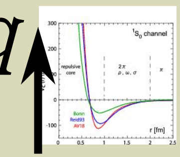
Luscher, NPB354, 531 (1991).
Aoki, et al., PRD71, 094504 (2005).

Multi-hadron on lattice



Lattice QCD simulation

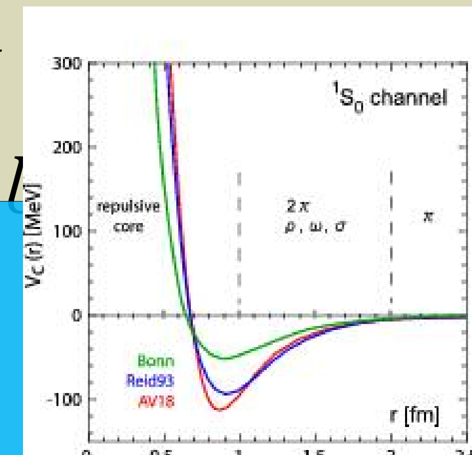
$$L=-\frac{1}{4}G^{a}_{\mu\nu}G^{a\mu\nu}+\bar{q}\gamma^{\mu}(i\partial_{\mu}-g\,t^{a}A^{a}_{\mu})q-m\bar{q}q$$



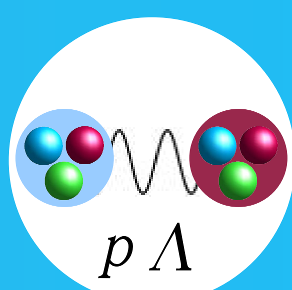
$$\langle O(\bar{q},q,U)\rangle=\int dU\,d\bar{q}\,dq\,e^{-S(\bar{q},q,U)}O(\bar{q},q$$

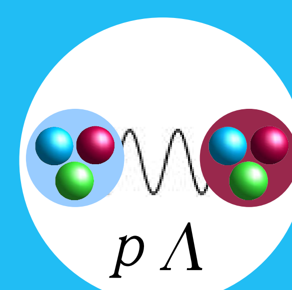
$$=\int dU\det D(U)e^{-S_U(U)}O(D^{-1}(U$$

$$=\lim_{N\to\infty}\frac{1}{N}\sum_{i=1}^{N}O(D^{-1}(U_i))$$



$$F^{(JM)}_{\alpha\beta}(\vec{r},t-t_0)$$

$\rightarrow\langle$ 

$(\vec{r},t)$ 

$(t_0)\rangle$

Calculate the scattering state

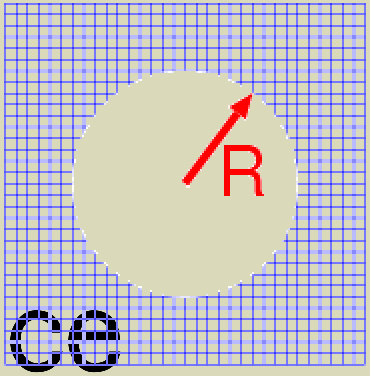
Multi-hadron on lattice

ii) HAL's procedure:

make better use of the lattice output ! (wave function)

interacting region

--> potential



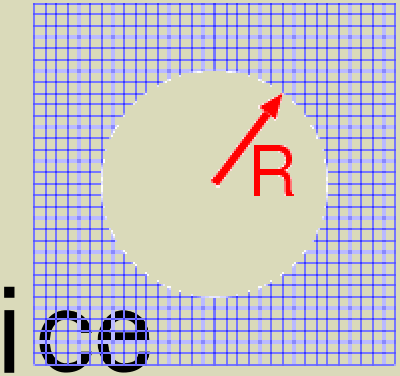
Ishii, Aoki, Hatsuda, PRL99, 022001 (2007); ibid., PTP123, 89 (2010).

NOTE:

> Potential is not a direct experimental observable.
> Potential is a useful tool to give (and to reproduce) the physical quantities. (e.g., phase shift)

....

Multi-hadron on lattice



ii) HAL's procedure:

make better use of the lattice output ! (wave function)

interacting region

--> potential

Ishii, Aoki, Hatsuda,
PRL99, 022001 (2007);
ibid., PTP123, 89 (2010).

=>
- > Phase shift
- > Nuclear many-body problems

In lattice QCD calculations, we compute the normalized four-point correlation function

$$R_{\alpha\beta}^{(J,M)}(\vec{r},t-t_0) = \sum_{\vec{X}} \left\langle 0 \left| B_{1,\alpha}(\vec{X}+\vec{r},t) B_{2,\beta}(\vec{X},t) \overline{\mathcal{J}_{B_3B_4}^{(J,M)}(t_0)} \right| 0 \right\rangle / \exp\{-(m_{B_1}+m_{B_2})(t-t_0)\},$$

$$p = \varepsilon_{abc}\left(u_a C\gamma_5 d_b\right) u_c, \qquad n = -\varepsilon_{abc}\left(u_a C\gamma_5 d_b\right) d_c, \tag{2}$$

$$\Sigma^+ = -\varepsilon_{abc}\left(u_a C\gamma_5 s_b\right) u_c, \qquad \Sigma^- = -\varepsilon_{abc}\left(d_a C\gamma_5 s_b\right) d_c, \tag{3}$$

$$\Sigma^0 = \frac{1}{\sqrt{2}}\left(X_u - X_d\right), \qquad \Lambda = \frac{1}{\sqrt{6}}\left(X_u + X_d - 2X_s\right), \tag{4}$$

$$\Xi^0 = \varepsilon_{abc}\left(u_a C\gamma_5 s_b\right) s_c, \qquad \Xi^- = -\varepsilon_{abc}\left(d_a C\gamma_5 s_b\right) s_c, \tag{5}$$

where

$$X_u = \varepsilon_{abc}\left(d_a C\gamma_5 s_b\right) u_c, \quad X_d = \varepsilon_{abc}\left(s_a C\gamma_5 u_b\right) d_c, \quad X_s = \varepsilon_{abc}\left(u_a C\gamma_5 d_b\right) s_c, \tag{6}$$

The potential is obtained at moderately large imaginary time; no single state saturation is required.

$$R^{(J,M)}_{\alpha\beta}(\vec{r},t-t_0) = \sum_{\vec{X}} \left\langle 0 \left| B_{1,\alpha}(\vec{X}+\vec{r},t) B_{2,\beta}(\vec{X},t) \overline{\mathcal{J}^{(J,M)}_{B_3B_4}(t_0)} \right| 0 \right\rangle / \exp\{-(m_{B_1}+m_{B_2})(t-t_0)\},$$

$$= \sum_n A_n \sum_{\vec{X}} \left\langle 0 \left| B_{1,\alpha}(\vec{X}+\vec{r},0) B_{2,\beta}(\vec{X},0) \right| E_n \right\rangle \mathrm{e}^{-(E_n-m_{B_1}-m_{B_2})(t-t_0)} + O(\mathrm{e}^{-(E_{\rm th}-m_{B_1}-m_{B_2})(t-t_0)}), \quad (4)$$

where E_n ($|E_n\rangle$) is the eigen-energy (eigen-state) of the six-quark system and $A_n = \sum_{\alpha'\beta'} P^{(JM)}_{\alpha'\beta'} \langle E_n | \overline{B}_{4,\beta'} \overline{B}_{3,\alpha'} | 0 \rangle$. At moderately large $t - t_0$ where the inelastic contribution above the pion production $O(\mathrm{e}^{-(E_{\rm th}-2m_N)(t-t_0)}) = O(\mathrm{e}^{-m_\pi(t-t_0)})$ becomes exiguous, we can construct the non-local potential U through $\left(\frac{\nabla^2}{2\mu} - \frac{k^2}{2\mu}\right) R(\vec{r}) = \int d^3r'\, U(\vec{r},\vec{r'}) R(\vec{r'})$. In lattice QCD calculations

[1] The potential is obtained from the NBS wave function at moderately large imaginary time; it would be $t - t_0 \gg 1/m_\pi \sim 1.4$ fm even for the physical pion mass. Furthermore, no single state saturation between the ground state and the first excited states, $t - t_0 \gg (\Delta E)^{-1} = ((2\pi)^2/(2\mu L^2))^{-1}$, is required for the present HAL QCD method[20], which becomes $((2\pi)^2/(2\mu L^2))^{-1} \simeq 4.6$ fm if we consider $L \sim 6$ fm and $m_N \simeq 1$ GeV. In Ref. [14], the validity of the velocity expansion of the NN potential has been examined in quenched lattice QCD simulations at $m_\pi \simeq 530$ MeV and $L \simeq 4.4$ fm.

The potential is obtained at moderately large imaginary time; no single state saturation is required.

$$R^{(J,M)}_{\alpha\beta}(\vec{r}, t-t_0) = \sum_{\vec{X}} \left\langle 0 \left| B_{1,\alpha}(\vec{X}+\vec{r}, t) B_{2,\beta}(\vec{X}, t) \overline{\mathcal{J}^{(J,M)}_{B_3B_4}(t_0)} \right| 0 \right\rangle / \exp\{-(m_{B_1}+m_{B_2})(t-t_0)\},$$

$$= \sum_n A_n \sum_{\vec{X}} \left\langle 0 \left| B_{1,\alpha}(\vec{X}+\vec{r}, 0) B_{2,\beta}(\vec{X}, 0) \right| E_n \right\rangle \mathrm{e}^{-(E_n - m_{B_1} - m_{B_2})(t-t_0)}$$
$$+O(\mathrm{e}^{-(E_{\rm th}-m_{B_1}-m_{B_2})(t-t_0)}), \qquad (4)$$

where E_n ($|E_n\rangle$) is the eigen-energy (eigen-state) of the six-quark system and $A_n = \sum_{\alpha'\beta'} P^{(JM)}_{\alpha'\beta'} \langle E_n | \overline{B}_{4,\beta'} \overline{B}_{3,\alpha'} | 0 \rangle$. At moderately large $t-t_0$ where the inelastic contribution above the pion production $O(\mathrm{e}^{-(E_{\rm th}-2m_N)(t-t_0)}) = O(\mathrm{e}^{-m_\pi (t-t_0)})$ becomes exiguous, we can construct the non-local potential U through $\left(\frac{\nabla^2}{2\mu} - \frac{k^2}{2\mu}\right) R(\vec{r}) = \int d^3r' \, U(\vec{r}, \vec{r'}) R(\vec{r'})$. In lattice QCD calculations

---

[1] The potential is obtained from the NBS wave function at moderately large imaginary time; it would be $t - t_0 \gg 1/m_\pi \sim 1.4$ fm even for the physical pion mass. Furthermore, no single state saturation between the ground state and the first excited states, $t - t_0 \gg (\Delta E)^{-1} = ((2\pi)^2/(2\mu L^2))^{-1}$, is required for the present HAL QCD method[20], which becomes $((2\pi)^2/(2\mu L^2))^{-1} \simeq 4.6$ fm if we consider $L \sim 6$ fm and $m_N \simeq 1$ GeV. In Ref. [14], the validity of the velocity expansion of the NN potential has been examined in quenched lattice QCD simulations at $m_\pi \simeq 530$ MeV and $L \simeq 4.4$ fm.

The potential is obtained at moderately large imaginary time; no single state saturation is required.

$$R^{(J,M)}_{\alpha\beta}(\vec{r},t-t_0) = \sum_{\vec{X}} \left\langle 0 \left| B_{1,\alpha}(\vec{X}+\vec{r},t) B_{2,\beta}(\vec{X},t) \overline{\mathcal{J}^{(J,M)}_{B_3B_4}(t_0)} \right| 0 \right\rangle / \exp\{-(m_{B_1}+m_{B_2})(t-t_0)\},$$

$$= \sum_n A_n \sum_{\vec{X}} \left\langle 0 \left| B_{1,\alpha}(\vec{X}+\vec{r},0) B_{2,\beta}(\vec{X},0) \right| E_n \right\rangle \mathrm{e}^{-(E_n-m_{B_1}-m_{B_2})(t-t_0)} + O(\mathrm{e}^{-(E_{\mathrm{th}}-m_{B_1}-m_{B_2})(t-t_0)}), \quad (4)$$

where E_n ($|E_n\rangle$) is the eigen-energy (eigen-state) of the six-quark system and $A_n = \sum_{\alpha'\beta'} P^{(JM)}_{\alpha'\beta'} \langle E_n | \overline{B}_{4,\beta'} \overline{B}_{3,\alpha'} | 0 \rangle$. At moderately large $t-t_0$ where the inelastic contribution above the pion production $O(\mathrm{e}^{-(E_{\mathrm{th}}-2m_N)(t-t_0)}) = O(\mathrm{e}^{-m_\pi(t-t_0)})$ becomes exiguous, we can construct the non-local potential U through $\left(\frac{\nabla^2}{2\mu} - \frac{k^2}{2\mu}\right) R(\vec{r}) = \int d^3r'\, U(\vec{r},\vec{r}') R(\vec{r}')$. In lattice QCD calculations

1 The potential is obtained from the NBS wave function at moderately large imaginary time; it would be $t-t_0 \gg 1/m_\pi \sim 1.4$ fm even for the physical pion mass. Furthermore, no single state saturation between the ground state and the first excited states, $t-t_0 \gg (\Delta E)^{-1} = ((2\pi)^2/(2\mu L^2))^{-1}$, is required for the present HAL QCD method[20], which becomes $((2\pi)^2/(2\mu L^2))^{-1} \simeq 4.6$ fm if we consider $L \sim 6$ fm and $m_N \simeq 1$ GeV. In Ref. [14], the validity of the velocity expansion of the NN potential has been examined in quenched lattice QCD simulations at $m_\pi \simeq 530$ MeV and $L \simeq 4.4$ fm.

The potential is obtained at moderately large imaginary time; no single state saturation is required.

[1] The potential is obtained from the NBS wave function at moderately large imaginary time; it would be $t - t_0 \gg 1/m_\pi \sim 1.4$ fm. In addition, no single state saturation between the ground state and the excited states with respect to the relative motion, e.g., $t - t_0 \gg (\Delta E)^{-1} = \left((2\pi)^2/(2\mu(La)^2)\right)^{-1} \simeq 8.0$ fm, is required for the HAL QCD method[13].

RECIPE:

Compute the 4pt correlator

$$F_{\alpha\beta,JM}^{\langle B_1B_2\overline{B_3B_4}\rangle}(\vec{r},t-t_0) = \sum_{\vec{X}} \left\langle 0 \left| B_{1,\alpha}(\vec{X}+\vec{r},t) B_{2,\beta}(\vec{X},t) \overline{\mathscr{J}_{B_3B_4}^{(J,M)}(t_0)} \right| 0 \right\rangle, \quad (2.3)$$

Take into account the threshold energy differences for coupled-channel system

$$R_{\alpha\beta,JM}^{\langle B_1B_2\overline{B_3B_4}\rangle}(\vec{r},t-t_0) = \mathrm{e}^{(m_{B_1}+m_{B_2})(t-t_0)} F_{\alpha\beta,JM}^{\langle B_1B_2\overline{B_3B_4}\rangle}(\vec{r},t-t_0)$$
$$= \sum_n A_n \sum_{\vec{X}} \left\langle 0 \left| B_{1,\alpha}(\vec{X}+\vec{r},0) B_{2,\beta}(\vec{X},0) \right| E_n \right\rangle \mathrm{e}^{-(E_n-m_{B_1}-m_{B_2})(t-t_0)} + O(\mathrm{e}^{-(E_{\mathrm{th}}-m_{B_1}-m_{B_2})(t-t_0)}) \quad (2.4)$$

elastic — inelastic

Obtain the potential by using the appropriate equation(s); For spin-singlet,

$$\left(\frac{\nabla^2}{2\mu_\lambda} - \frac{\partial}{\partial t}\right) R_{\lambda\varepsilon}(\vec{r},t) \simeq V_{\lambda\lambda'}^{(\mathrm{LO})}(\vec{r})\theta_{\lambda\lambda'}R_{\lambda'\varepsilon}(\vec{r},t), \text{ with } \theta_{\lambda\lambda'} = \mathrm{e}^{(m_{B_1}+m_{B_2}-m_{B'_1}-m_{B'_2})(t-t_0)}.$$

For spin-triplet, the "tensor force" becomes active

$$\begin{Bmatrix}\mathscr{P}\\ \mathscr{Q}\end{Bmatrix} \times \left\{ V_{\lambda\lambda'}^{(0)}(r) + V_{\lambda\lambda'}^{(\sigma)}(r) + V_{\lambda\lambda'}^{(T)}(r)S_{12} \right\} \theta_{\lambda\lambda'}R_{\lambda'\varepsilon}(\vec{r},t-t_0) = \begin{Bmatrix}\mathscr{P}\\ \mathscr{Q}\end{Bmatrix} \times \left\{ \frac{\nabla^2}{2\mu_\lambda} - \frac{\partial}{\partial t} \right\} R_{\lambda\varepsilon}(\vec{r},t-t_0) \quad (2.7)$$

Where

$$\begin{cases} R(\vec{r};\,^3S_1) = \mathscr{P}R(\vec{r};J=1) \equiv \frac{1}{24}\sum_{\mathscr{R}\in O}\mathscr{R}R(\vec{r};J=1), \\ R(\vec{r};\,^3D_1) = \mathscr{Q}R(\vec{r};J=1) \equiv (1-\mathscr{P})R(\vec{r};J=1). \end{cases} \quad (2.6)$$

In the lowest few orders, we have

$$V(\vec{r},\vec{\nabla}_r) = V^{(0)}(r) + V^{(\sigma)}(r)\vec{\sigma}_1\cdot\vec{\sigma}_2 + V^{(T)}(r)S_{12} + V^{(LS)}_{(ALS)}(r)\vec{L}\cdot(\vec{\sigma}_1 \pm \vec{\sigma}_2) + O(\nabla^2), \quad (2.5)$$

An improved recipe for NY potential:

cf. Ishii (HAL QCD), PLB712 (2012) 437.

- Take account of not only the spatial correlation but also the temporal correlation in terms of the R-correlator:

$$-\frac{1}{2\mu}\nabla^2 R(t,\vec{r}) + \int d^3r' \, U(\vec{r},\vec{r}')R(t,\vec{r}') = -\frac{\partial}{\partial t}R(t,\vec{r})$$

$$\to \frac{k^2}{2\mu}R(t,\vec{r})$$

$$U(\vec{r},\vec{r}') = V_{NY}(\vec{r},\nabla)\delta(\vec{r}-\vec{r}')$$

- A general expression of the potential:

$$V_{NY} = V_0(r) + V_\sigma(r)(\vec{\sigma}_N\cdot\vec{\sigma}_Y) + V_T(r)S_{12} + V_{LS}(r)(\vec{L}\cdot\vec{S}_+) + V_{ALS}(r)(\vec{L}\cdot\vec{S}_-) + O(\nabla^2)$$

Effective block algorithm for various baryon-baryon correlators

HN, CPC207,91(2016), arXiv:1510.00903(hep-lat)

Numerical cost (# of iterative operations) in this algorithm

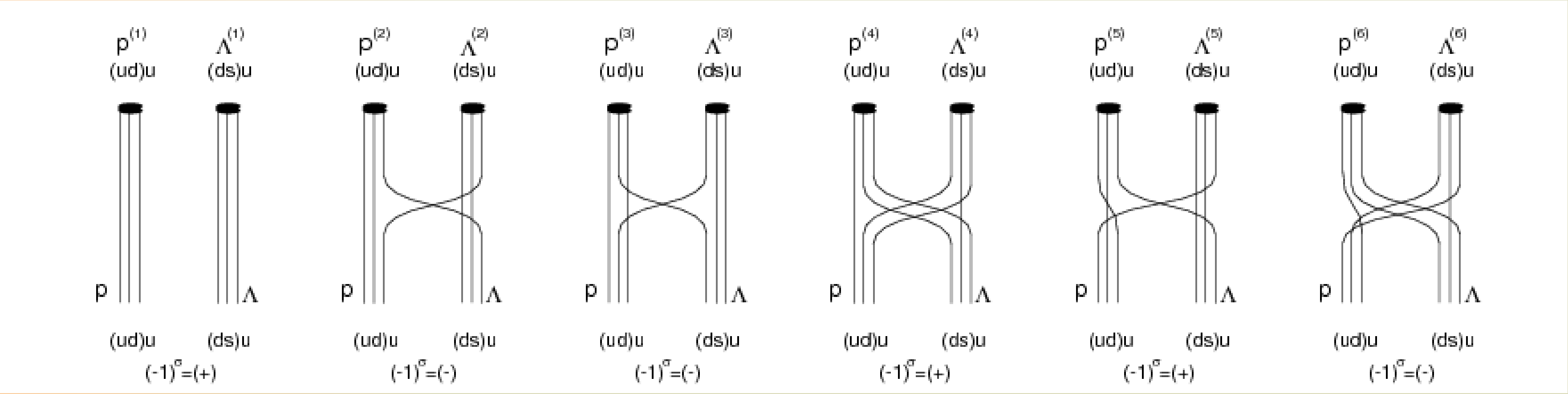
$$1+N_c^{\,2}+N_c^{\,2}N_\alpha^{\,2}+N_c^{\,2}N_\alpha^{\,2}+N_c^{\,2}N_\alpha+N_c^{\,2}N_\alpha = 370$$

In an intermediate step:

$$(N_c!\,N_\alpha)^B\times N_u!\,N_d!\,N_s!\times 2^{N_\Lambda+N_{\Sigma^0}-B} = 3456$$

In a naïve approach:

$$(N_c!\,N_\alpha)^{2B}\times N_u!\,N_d!\,N_s! = 3,981,312$$



Generalization to the various baryon-baryon channels strangeness S=0 to -4 systems

$$\langle pn\overline{pn}\rangle, \tag{4.1}$$

$$\begin{array}{lll}
\langle p\Lambda\overline{p\Lambda}\rangle, & \langle p\Lambda\overline{\Sigma^+ n}\rangle, & \langle p\Lambda\overline{\Sigma^0 p}\rangle, \\
\langle \Sigma^+ n\overline{p\Lambda}\rangle, & \langle \Sigma^+ n\overline{\Sigma^+ n}\rangle, & \langle \Sigma^+ n\overline{\Sigma^0 p}\rangle, \\
\langle \Sigma^0 p\overline{p\Lambda}\rangle, & \langle \Sigma^0 p\overline{\Sigma^+ n}\rangle, & \langle \Sigma^0 p\overline{\Sigma^0 p}\rangle,
\end{array} \tag{4.2}$$

$$\begin{array}{llllll}
\langle \Lambda\Lambda\overline{\Lambda\Lambda}\rangle, & \langle \Lambda\Lambda\overline{p\Xi^-}\rangle, & \langle \Lambda\Lambda\overline{n\Xi^0}\rangle, & \langle \Lambda\Lambda\overline{\Sigma^+\Sigma^-}\rangle, & \langle \Lambda\Lambda\overline{\Sigma^0\Sigma^0}\rangle, & \\
\langle p\Xi^-\overline{\Lambda\Lambda}\rangle, & \langle p\Xi^-\overline{p\Xi^-}\rangle, & \langle p\Xi^-\overline{n\Xi^0}\rangle, & \langle p\Xi^-\overline{\Sigma^+\Sigma^-}\rangle, & \langle p\Xi^-\overline{\Sigma^0\Sigma^0}\rangle, & \langle p\Xi^-\overline{\Sigma^0\Lambda}\rangle, \\
\langle n\Xi^0\overline{\Lambda\Lambda}\rangle, & \langle n\Xi^0\overline{p\Xi^-}\rangle, & \langle n\Xi^0\overline{n\Xi^0}\rangle, & \langle n\Xi^0\overline{\Sigma^+\Sigma^-}\rangle, & \langle n\Xi^0\overline{\Sigma^0\Sigma^0}\rangle, & \langle n\Xi^0\overline{\Sigma^0\Lambda}\rangle, \\
\langle \Sigma^+\Sigma^-\overline{\Lambda\Lambda}\rangle, & \langle \Sigma^+\Sigma^-\overline{p\Xi^-}\rangle, & \langle \Sigma^+\Sigma^-\overline{n\Xi^0}\rangle, & \langle \Sigma^+\Sigma^-\overline{\Sigma^+\Sigma^-}\rangle, & \langle \Sigma^+\Sigma^-\overline{\Sigma^0\Sigma^0}\rangle, & \langle \Sigma^+\Sigma^-\overline{\Sigma^0\Lambda}\rangle, \\
\langle \Sigma^0\Sigma^0\overline{\Lambda\Lambda}\rangle, & \langle \Sigma^0\Sigma^0\overline{p\Xi^-}\rangle, & \langle \Sigma^0\Sigma^0\overline{n\Xi^0}\rangle, & \langle \Sigma^0\Sigma^0\overline{\Sigma^+\Sigma^-}\rangle, & \langle \Sigma^0\Sigma^0\overline{\Sigma^0\Sigma^0}\rangle, & \\
 & \langle \Sigma^0\Lambda\overline{p\Xi^-}\rangle, & \langle \Sigma^0\Lambda\overline{n\Xi^0}\rangle, & \langle \Sigma^0\Lambda\overline{\Sigma^+\Sigma^-}\rangle, & & \langle \Sigma^0\Lambda\overline{\Sigma^0\Lambda}\rangle,
\end{array} \tag{4.3}$$

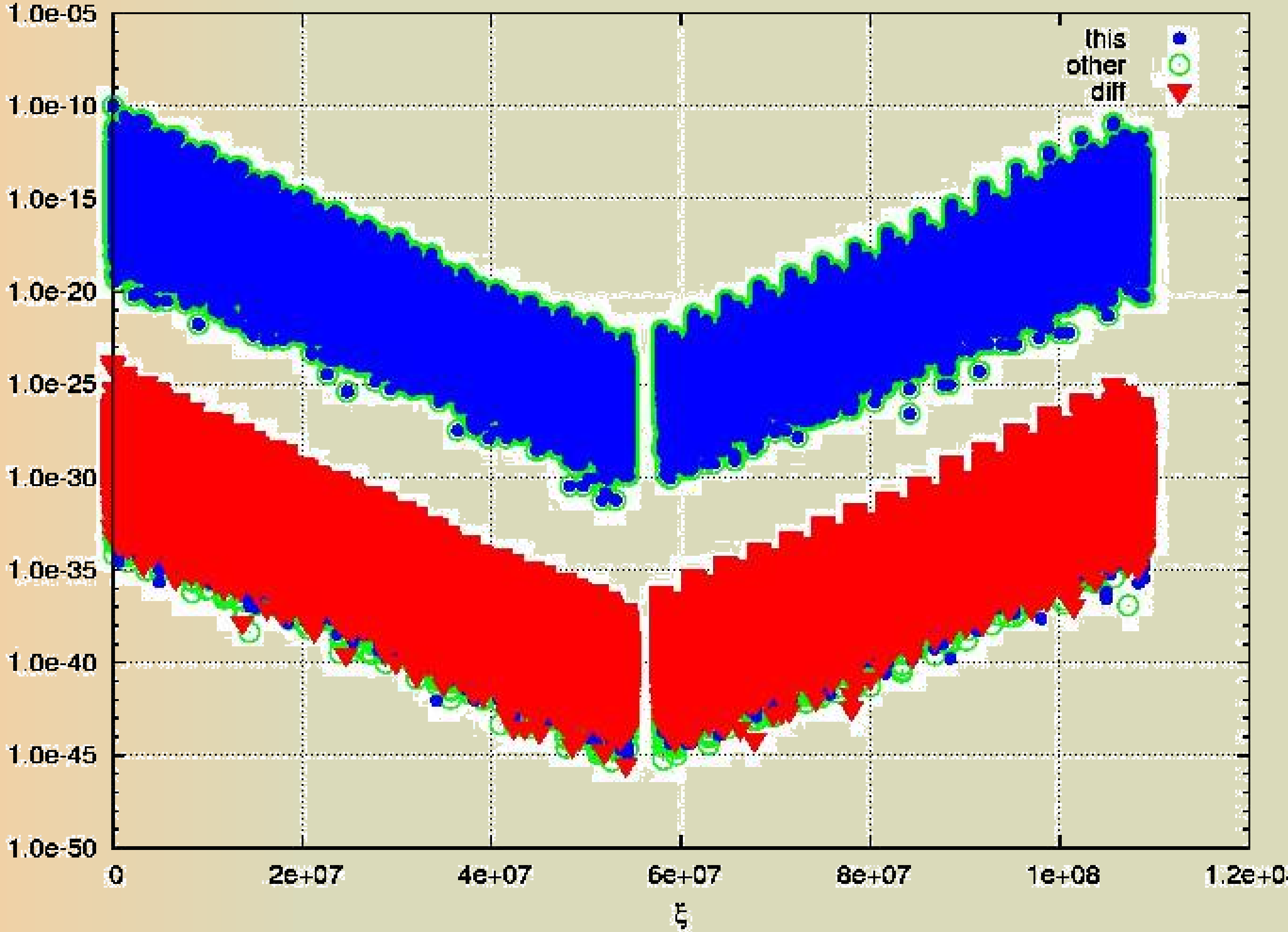
$$\begin{array}{lll}
\langle \Xi^-\Lambda\overline{\Xi^-\Lambda}\rangle, & \langle \Xi^-\Lambda\overline{\Sigma^-\Xi^0}\rangle, & \langle \Xi^-\Lambda\overline{\Sigma^0\Xi^-}\rangle, \\
\langle \Sigma^-\Xi^0\overline{\Xi^-\Lambda}\rangle, & \langle \Sigma^-\Xi^0\overline{\Sigma^-\Xi^0}\rangle, & \langle \Sigma^-\Xi^0\overline{\Sigma^0\Xi^-}\rangle, \\
\langle \Sigma^0\Xi^-\overline{\Xi^-\Lambda}\rangle, & \langle \Sigma^0\Xi^-\overline{\Sigma^-\Xi^0}\rangle, & \langle \Sigma^0\Xi^-\overline{\Sigma^0\Xi^-}\rangle,
\end{array} \tag{4.4}$$

$$\langle \Xi^-\Xi^0\overline{\Xi^-\Xi^0}\rangle. \tag{4.5}$$

Make better use of the computing resources!

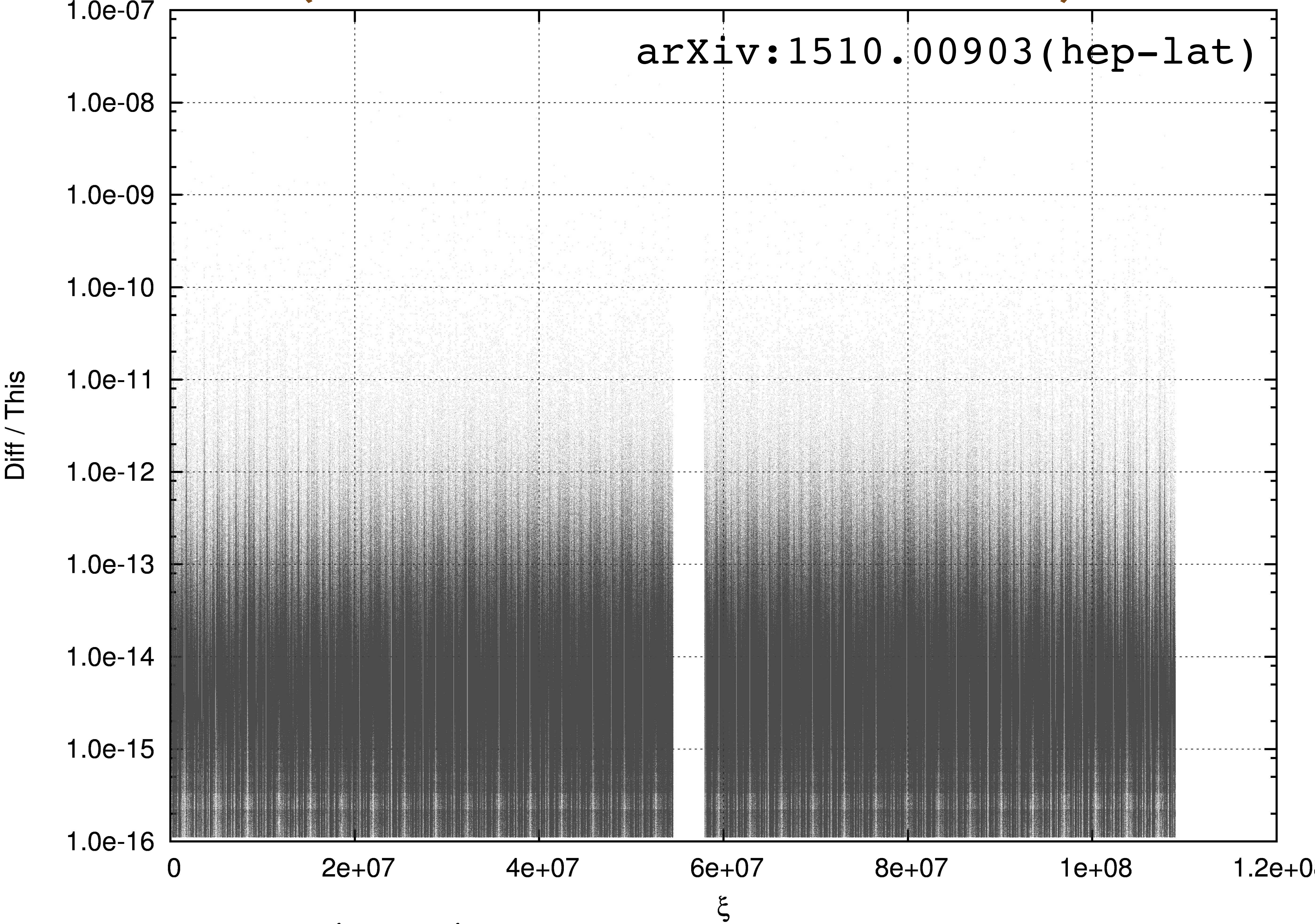
HN, CPC **207**, 91(2016) [arXiv:1510.00903[hep-lat]],
(See also arXiv:1604.08346)

Benchmark (from NN to XiXi channels)



numerical results of the correlators of entire 52 channels from NN to $\Xi\Xi$ systems given in Eqs. (32)−(36), over 31 time-slices, 16^3 points for spatial, and 2^4 points for the spin degrees of freedom, obtained by using this effective block algorithm (dot) and by using the unified contraction algorithm (open circle) as a function of one-dimensionally aligned data point $\xi = \tilde{\alpha} + 2(\tilde{\beta} + 2(\tilde{\alpha}' + 2(\tilde{\beta}' + 2(x + 16(y + 16(z + 16(c + 52((t - t_0 + T) \bmod T)))))))$, where $c = 0, \cdots, 51$ selects one of the 52 channels. The absolute value of their difference is also shown (triangle).

Benchmark (from NN to XiXi channels)



$$\xi = \alpha + 2(\beta + 2(\alpha' + 2(\beta' + 2(x + 16(y + 16(z + 16(c + 52(t - t_0)))))))$$

HA-PACS

- BASE
  - Intel E5-2670 (16core) + NVIDIA M2090 (x4)

    332.8 GFlops — 665 GFlops (x4)

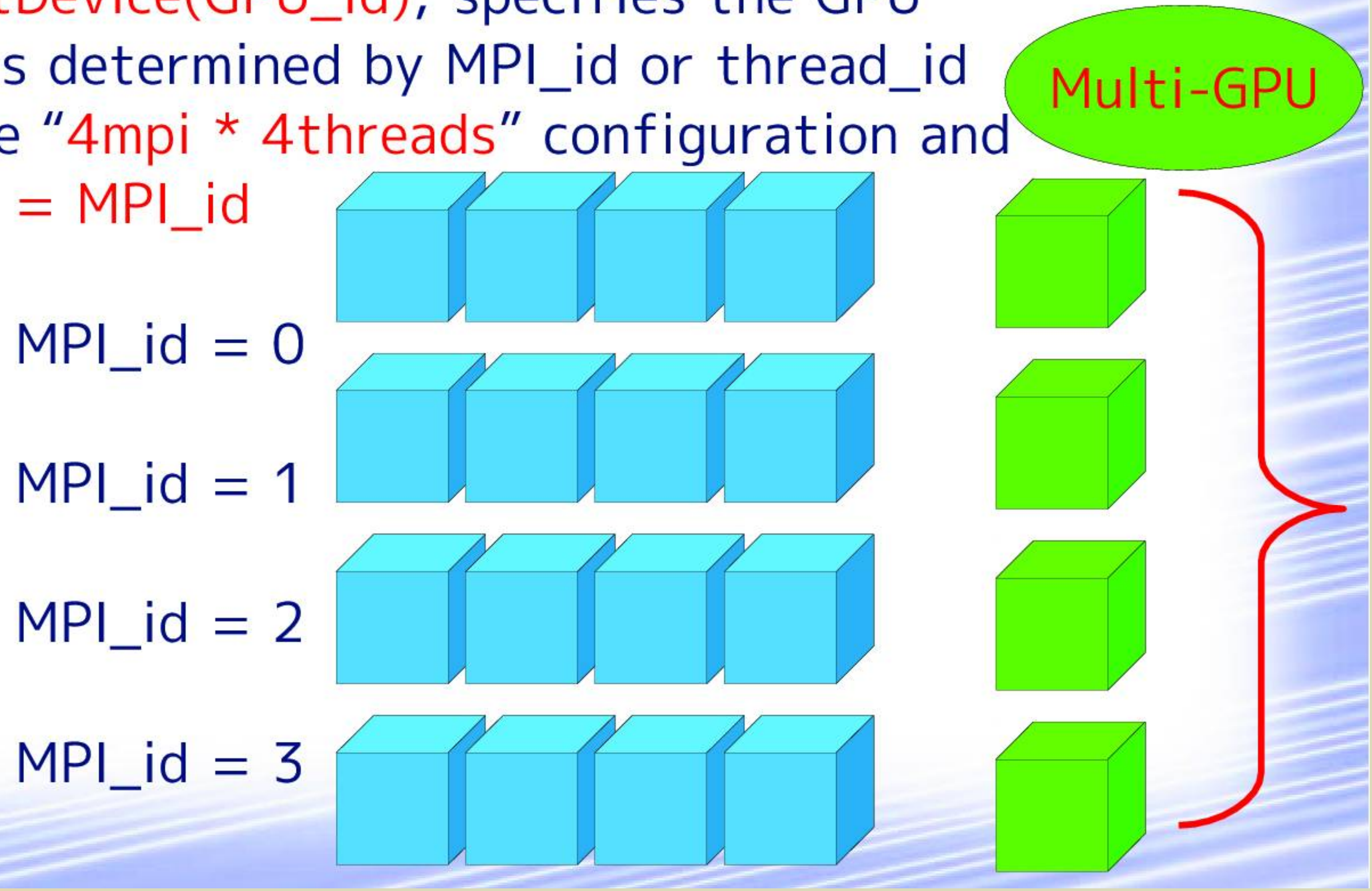
    128 GBytes — 6 GBytes (x4)
- TCA
  - Intel E5-2680v2 (20core) + NVIDIA K20X (x4)

    448 GFlops — 1310 GFlops (x4)

    128 GBytes — 6 GBytes (x4)

MPI+OpenMP + CUDA

- For HA-PACS, 1PE has 16 CPU cores and 4 GPUs:
  - cudaSetDevice(GPU_id); specifies the GPU
  - GPUid is determined by MPI_id or thread_id
  - We take "4mpi * 4threads" configuration and
  - GPU_id = MPI_id



Cygnus (Deneb×48nodes) @Tsukuba

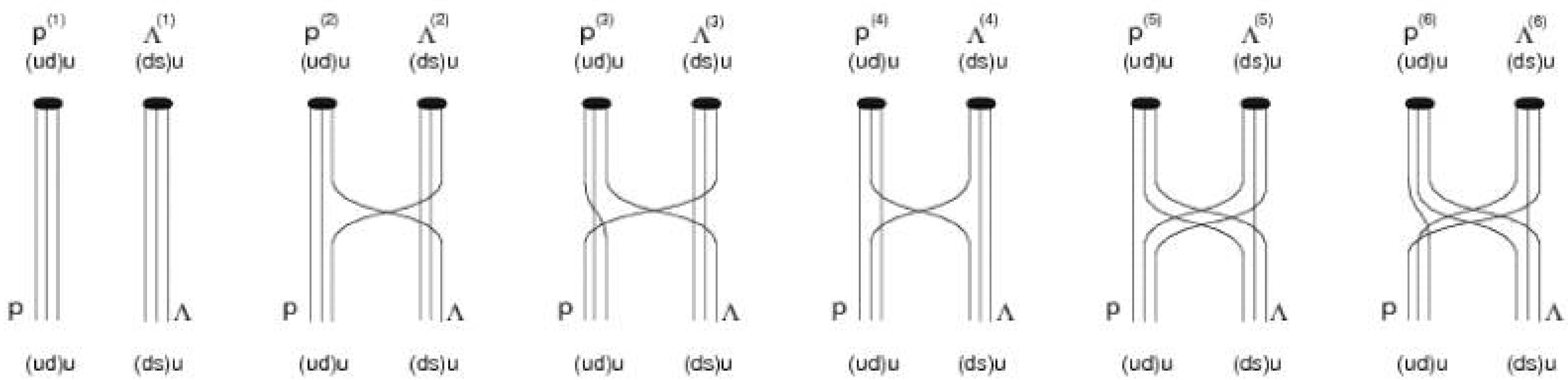
NVIDIA Tesla V100-PCIe (DP: 7 TFLOPS)×4
Memory (GPU): 32 GiB ×4, 900 GB/s ×4

Intel Xeon Gold 6126 2.5 GHz (12core)×2
Memory (CPU): 192 GiB, 255.9 GB/s

4pt correlator through the FFT

- Cuda implementations for various parts:

1 $[B]_\alpha(r,\alpha_1,c_1,\alpha_2,c_2,\alpha_3,c_3)$ $\quad [B]=[N,\Sigma,\Xi,\Lambda(dsu),\Lambda(sud),\Lambda(uds)]$



$$\phi_{\alpha\beta}(r)=\sum_i F^{(i)}\phi^{(i)}_{\alpha\beta}(r) = \sum_i F^{(i)}\sum_x p^{(i)}_\alpha(X+\vec{r})\Lambda^{(i)}_\beta(X)$$
$$=\sum_i F^{(i)}\sum_q \tilde{p}^{(i)}_\alpha(q)\tilde{\Lambda}^{(i)}_\beta(-q)e^{iqr} = \sum_q\left(\sum_i F^{(i)}\tilde{p}^{(i)}_\alpha(q)\tilde{\Lambda}^{(i)}_\beta(-q)\right)e^{iqr}$$

3 $\tilde{p}^{(i)}_\alpha(q)$ $\quad$ 3 $\tilde{\Lambda}^{(i)}_\beta(-q)$ $\quad$ 5 $\phi_{\alpha\beta}(r)$

2 $p^{(i)}_\alpha(X+r)$ $\quad$ 2 $\Lambda^{(i)}_\beta(X)$ $\quad$ 4 $\left(\sum_i F^{(i)}\tilde{p}^{(i)}_\alpha(q)\tilde{\Lambda}^{(i)}_\beta(-q)\right)$

Determination of baryon-baryon potentials at nearly physical point

Almost physical point lattice QCD calculation using N_F=2+1 clover fermion + Iwasaki gauge action

- APE-Stout smearing (ρ=0.1, n_{stout}=6)
- Non-perturbatively O(a) improved Wilson Clover action at β=1.82 on $96^3 \times 96$ lattice
- $1/a$ = 2.3 GeV (a = 0.085 fm)
- Volume: $96^4 \rightarrow (8\text{fm})^4$
- m_π=145MeV, m_K=525MeV
- DDHMC(ud) and UVPHMC(s) with preconditioning
- K.-I.Ishikawa, et al., PoS LAT2015, 075; arXiv:1511.09222 [hep-lat].

- NBS wf is measured using wall quark source with Coulomb gauge fixing, spatial PBD and temporal DBC; #stat=207configs x 4rotation x Nsrc (Nsrc=4 → 20 → 52 → 96 (2015FY+))

ΛN-ΣN potentials at nearly physical point

The methodology for coupled-channel V is based on:
Aoki, et al., Proc.Japan Acad. B87 (2011) 509.
Sasaki, et al., PTEP 2015 (2015) no.11, 113B01.
Ishii, et al., JPS meeting, March (2016).
Nemura, et al., [1702.00734].

#stat: (this/scheduled in FY2015+) 0.2 → 0.54 → 1.00 → 2.00

$t-t_0=5-12$, $t-t_0=5-14$

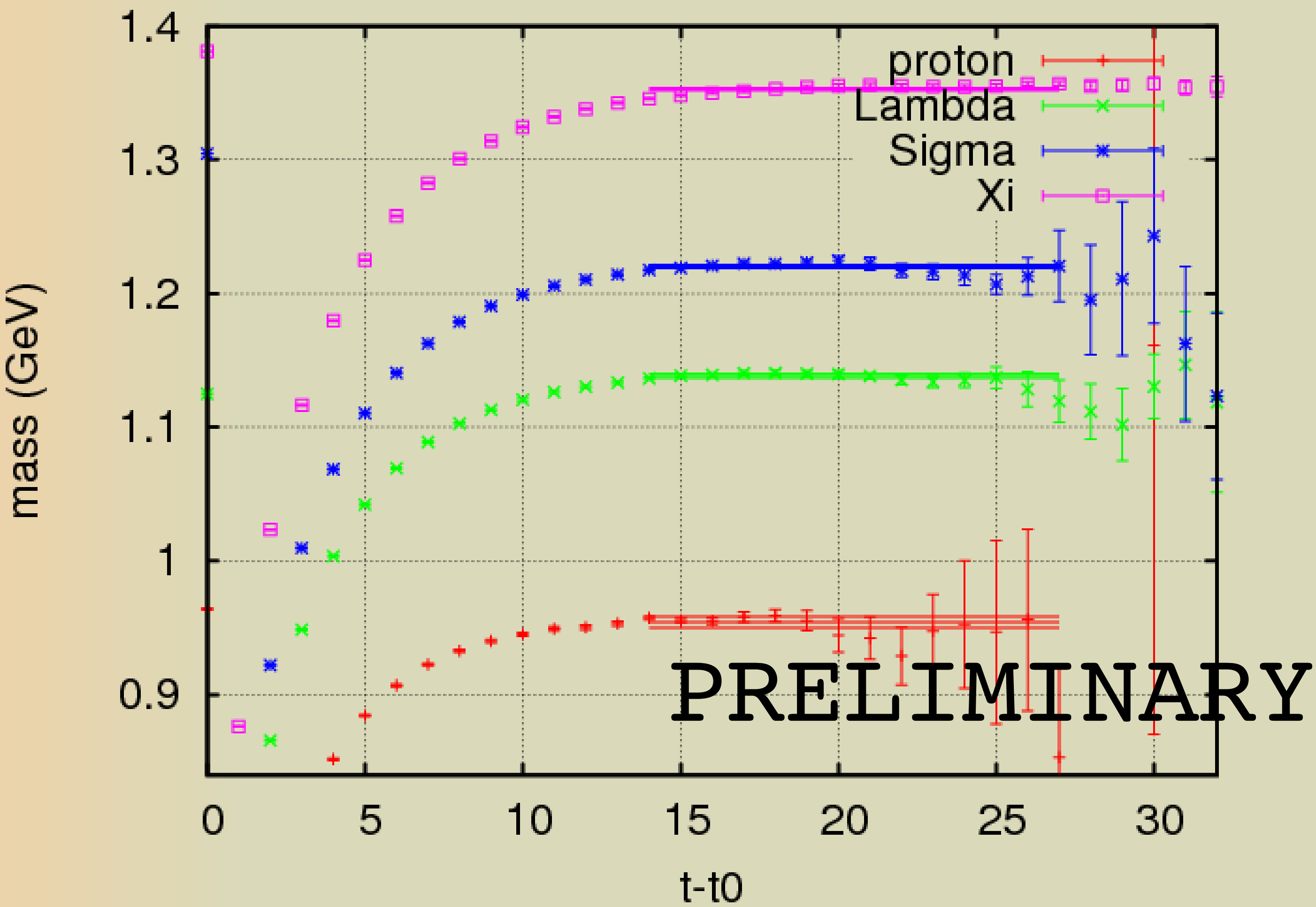
$\Lambda N - \Sigma N \; (I=1/2)$

$V_C(^1S_0)$ $V_C(^3S_1-^3D_1)$ $V_T(^3S_1-^3D_1)$

$\Sigma N \; (I=3/2)$

$V_C(^1S_0)$ $V_C(^3S_1-^3D_1)$ $V_T(^3S_1-^3D_1)$

Effective mass plot of the single baryon's correlation function



Potentials obtained at t-t0 = 5 to 12 are shown;
For the largest statistics, t-t0 = 13 and 14 will also be shown.

TABLE 4

The eigenvalues of the normalization kernel in eq. (3.3) for $S=-1$ two-baryon (BB) system

$S=-1$

I	J	BB	Eigenvalues (uncoupled)	Eigenvalues (coupled)
$\frac{1}{2}$	0	$N\Lambda$	1	0 $\frac{10}{9}$
		$N\Sigma$	$\frac{1}{9}$	
$\frac{1}{2}$	1	$N\Lambda$	1	$\frac{8}{9}$ $\frac{10}{9}$
		$N\Sigma$	1	
$\frac{3}{2}$	0	$N\Sigma$	$\frac{10}{9}$	
$\frac{3}{2}$	1	$N\Sigma$	$\frac{2}{9}$	

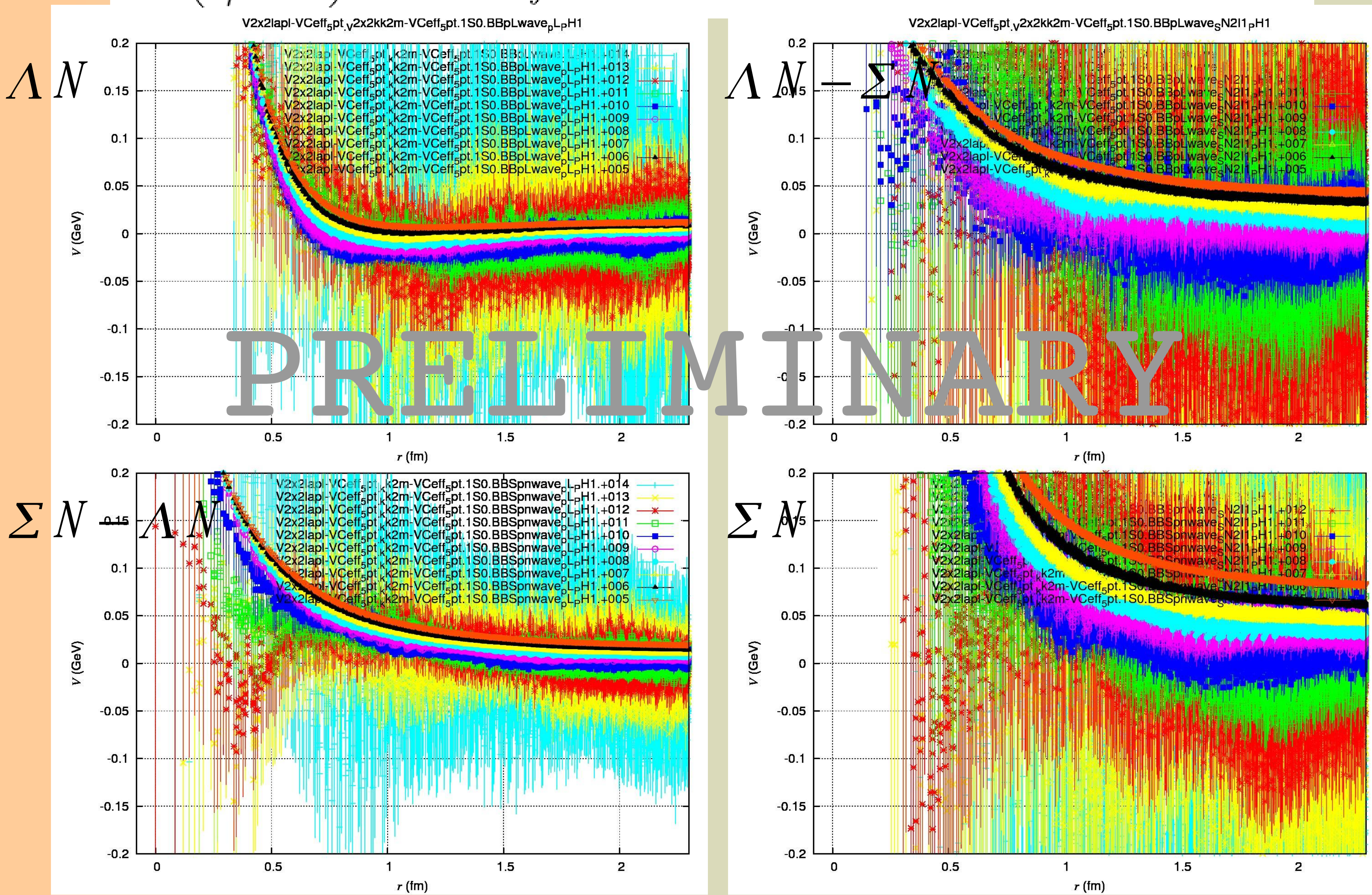
Eigenvalues of single and coupled channels are given.

Oka, Shimizu and Yazaki (1987)

Very preliminary result of LN potential at the physical point

$V_C(^1S_0)$

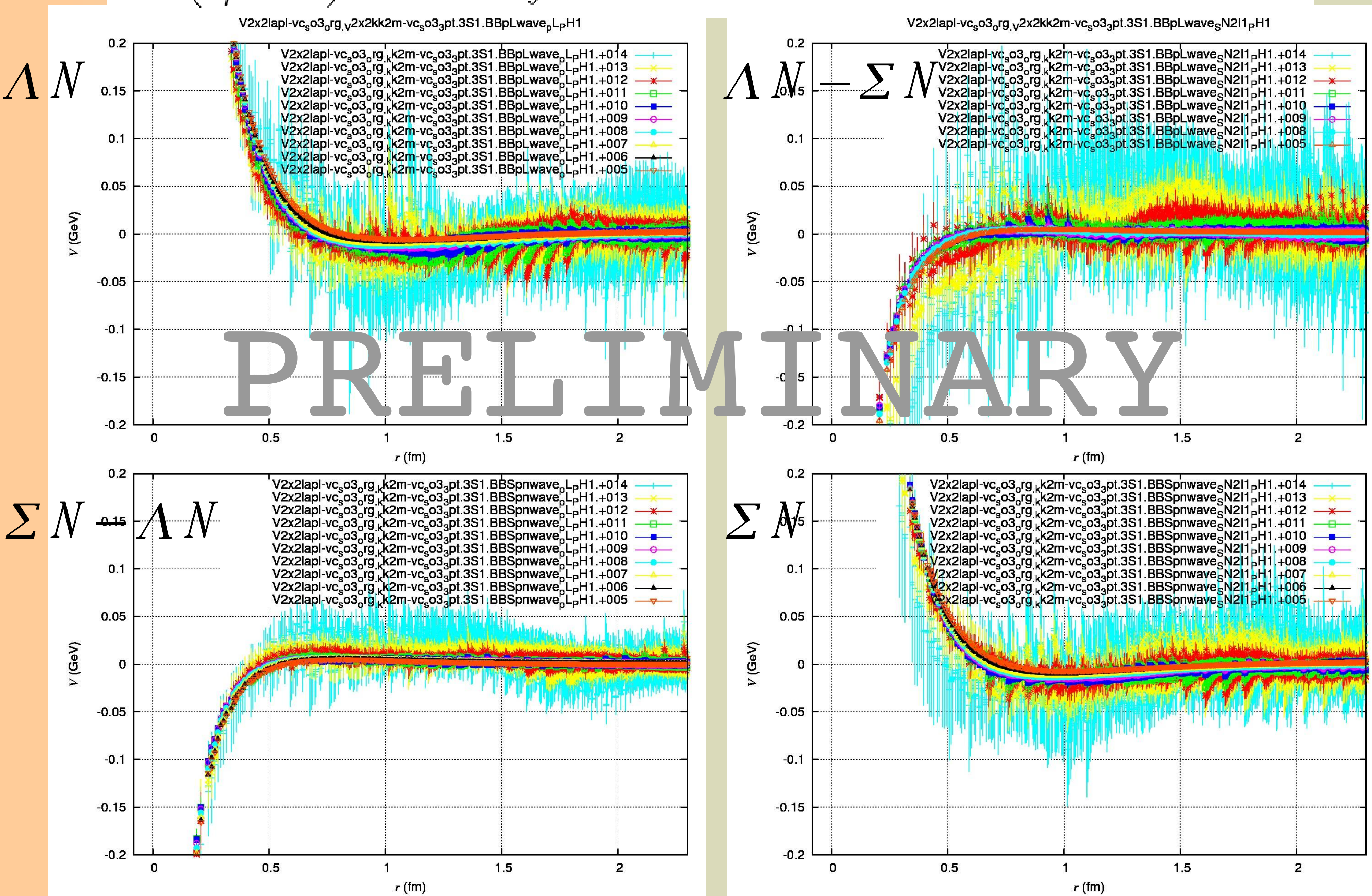
$$\left(\frac{\nabla^2}{2\mu} - \frac{\partial}{\partial t}\right) R(\vec{r},t) = \int d^3r' \, U(\vec{r},\vec{r}')R(\vec{r}',t) + O(k^4) = V_{\rm LO}(\vec{r})R(\vec{r},t) + \cdots (8)$$



Very preliminary result of LN potential at the physical point

$$V_C({}^3S_1 - {}^3D_1)$$

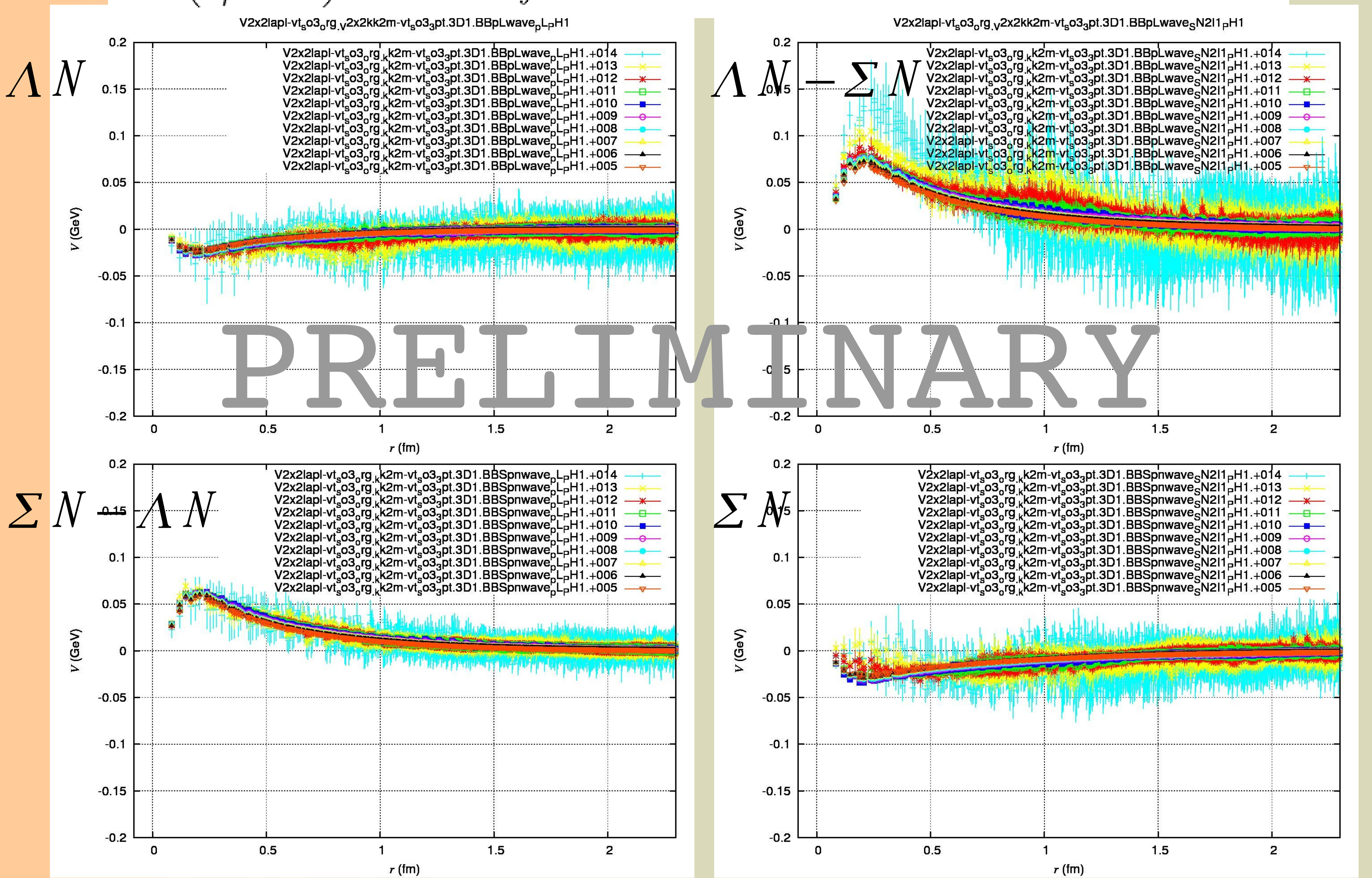
$$\left(\frac{\nabla^2}{2\mu} - \frac{\partial}{\partial t}\right) R(\vec{r},t) = \int d^3r'\, U(\vec{r},\vec{r}')R(\vec{r}',t) + O(k^4) = V_{\rm LO}(\vec{r})R(\vec{r},t) + \cdots (8)$$



Very preliminary result of LN potential at the physical point

$V_T(\,^3S_1 - \,^3D_1)$

$$\left(\frac{\nabla^2}{2\mu} - \frac{\partial}{\partial t}\right) R(\vec{r},t) \;=\; \int d^3r'\, U(\vec{r},\vec{r}')R(\vec{r}',t) + O(k^4) = V_{\rm LO}(\vec{r})R(\vec{r},t) + \cdots (8)$$

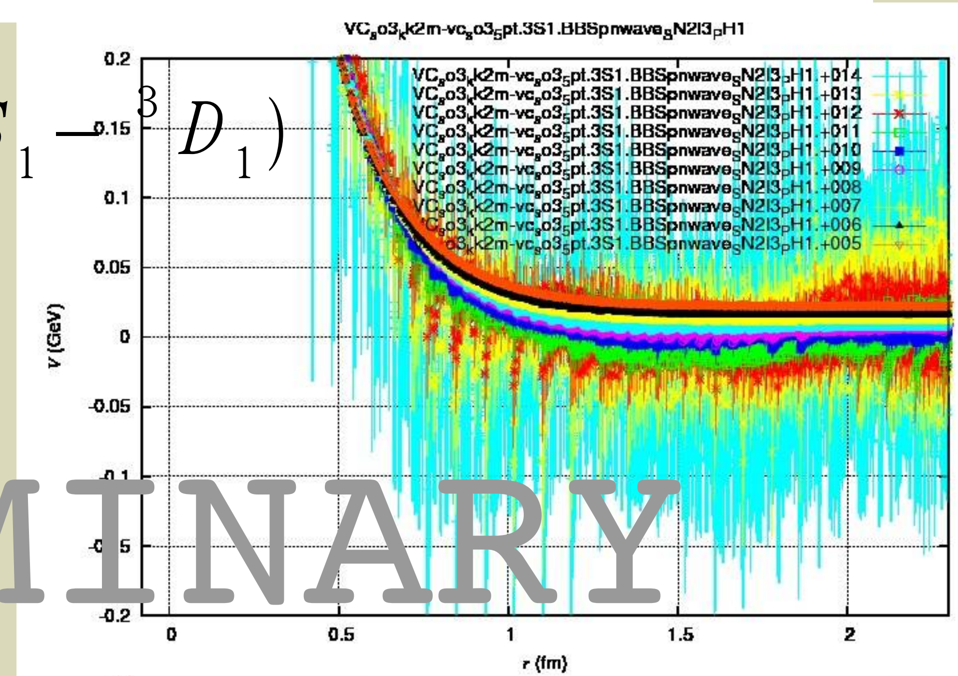


Very preliminary result of LN potential at the physical point

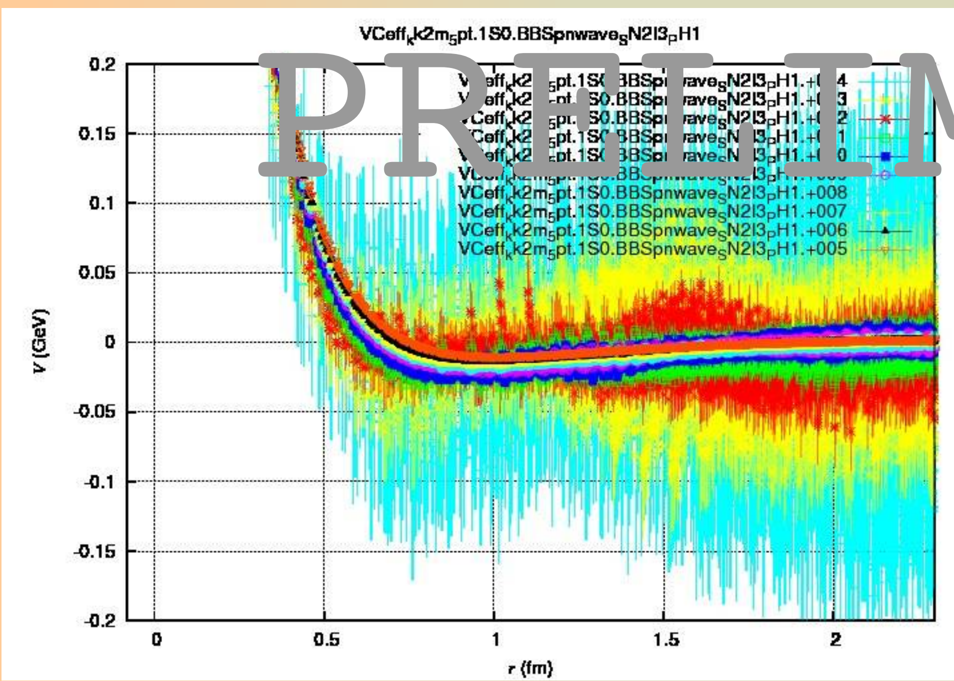
$$\left(\frac{\nabla^2}{2\mu} - \frac{\partial}{\partial t}\right) R(\vec{r},t) = \int d^3r'\, U(\vec{r},\vec{r}')R(\vec{r}',t) + O(k^4) = V_{\rm LO}(\vec{r})R(\vec{r},t) + \cdot (8)$$

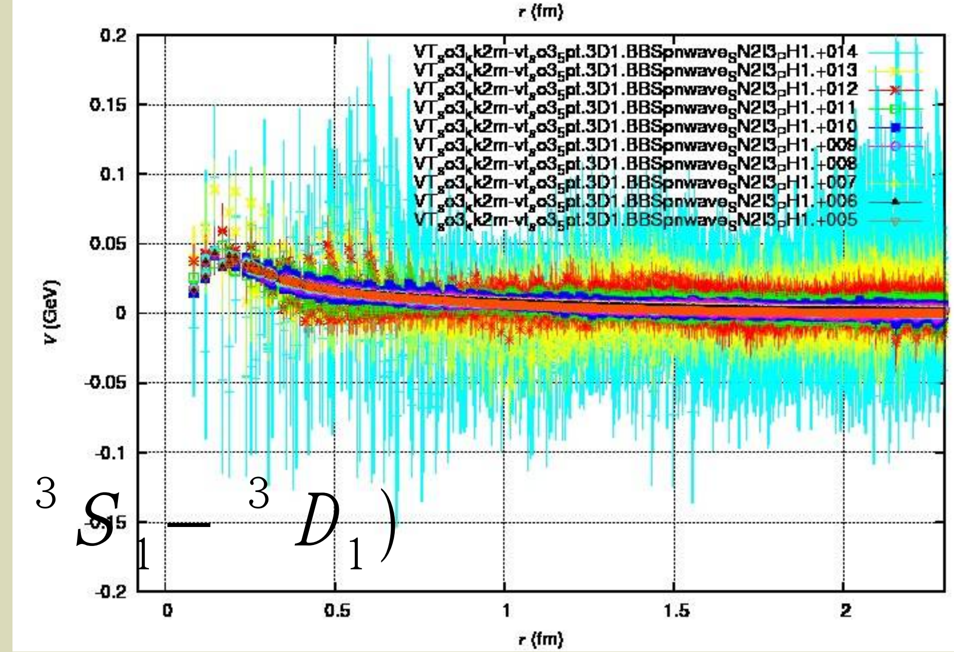
$\Sigma N\,(I=3/2)$

$V_C(\,^3S_1 - \,^3D_1)$



PRELIMINARY



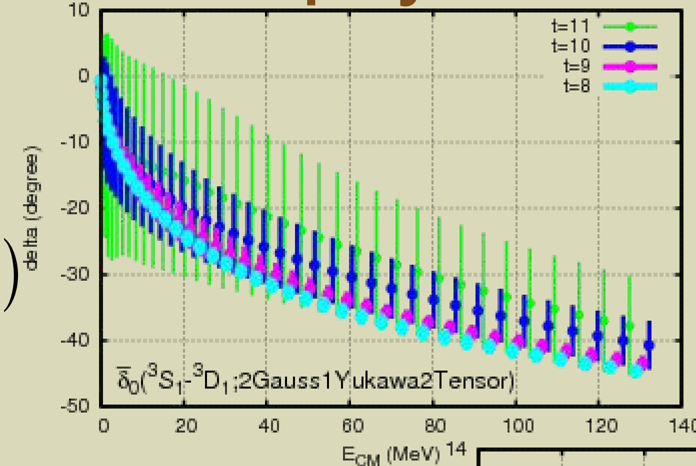


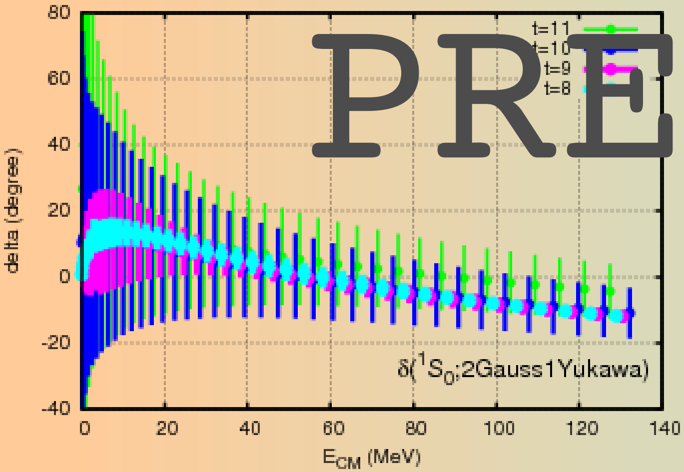
$V_C(\,^1S_0)$

$V_T(\,^3S_1 - \,^3D_1)$

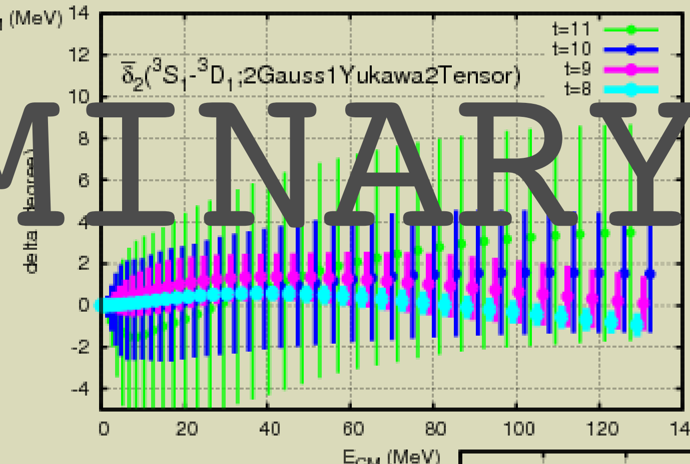
Very preliminary results of the SN(I=3/2) phase shift at the physical point

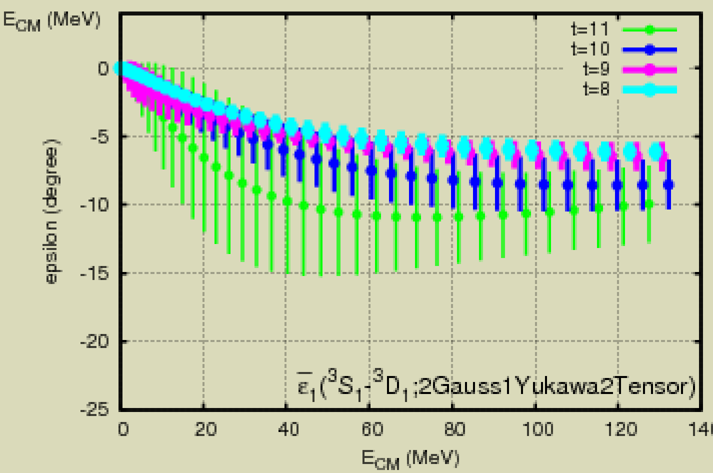
$\Sigma N\,(I=3/2)$





PRELIMINARY





More or less qualitatively similar to (recent) phenomenological approaches:
Fujiwara, et al., PRC54(1996)2180,
Arisaka, et al., PTP104(2000)995,
Haidenbauer et al., NPA915(2013)24.

Summary

(I-1) Latest results of LN-SN potentials at nearly physical point. (Lambda-N, Sigma-N: central, tensor)
Statistics approaching to 1.96 => 2.0 (=present/scheduled in 2015)

Increasing statistics, the result at large t converges to the result at next-to-smaller t (with poor statistics).

Signals in spin-triplet are relatively going well smoothly.

The channels that the quark model predicts strongly repulsive have relatively poor signals; Simultaneous calcs (NN to XiXi) is the point we have to take into account for the comprehensive perspective as well as energy-computing-resource efficiency.

(I-2) Effective hadron block algorithm for the various baron-baryon interaction

Comput.Phys.Commun.**207**,91(2016) [arXiv:1510.00903(hep-lat)]

The algorithm will be applied to more wide range problems.

Future work:

(II-1) Physical quantities including the binding energies of few-body problem of light hypernuclei with the lattice YN (and NN) potentials