Computing Nucleon Electric Dipole Moments in Lattice QCD

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References:
M. Abramczyk, S. Aoki, T. Blum, T. Izubuchi, H. Ohki, S. Syritsyn, Phys.Rev. D96 (2017) no.1, 014501
N. Yamanaka, S. Hashimoto, T. Kaneko, H. Ohki (JLQCD), PRD 98, 054516
S. Syritsyn, T. Izubuchi, H. Ohki, 1901.05455, and work in progress

Frontiers in Lattice QCD and related topics, April 26, 2019

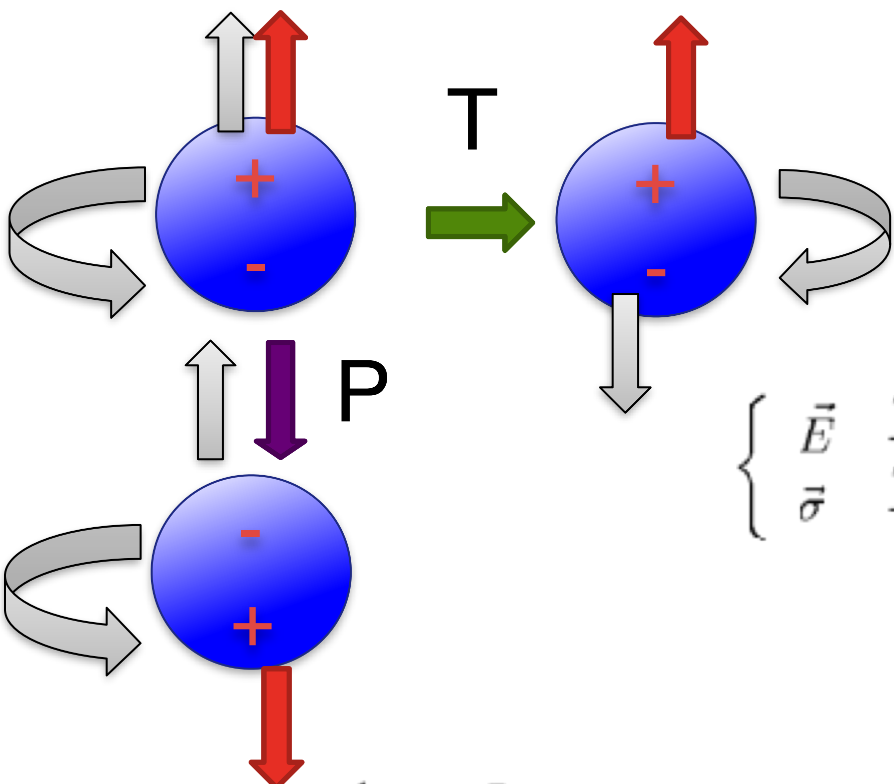
outline

- Introduction
- Parity mixing problem on lattice EDM calculation
- Lattice Study
  - old formula v.s. new formula (on lattice)

  numerical check using chromo-EDM
- Implication to the θ-EDM
- Noise reduction for θ-EDM
- quark EDM
- Summary

Introduction

- Electric Dipole Moment d
  Energy shift of a spin particle in an electric field
- Non-zero EDM : P&T (CP through CPT) violation



$$H_{\mathrm{EDM}} = -d \langle \vec{\sigma} \rangle \cdot \vec{E}$$

$$\begin{cases} \vec{E} \xrightarrow{\mathrm{T}} \vec{E} \\ \vec{\sigma} \xrightarrow{\mathrm{T}} -\vec{\sigma} \end{cases}$$

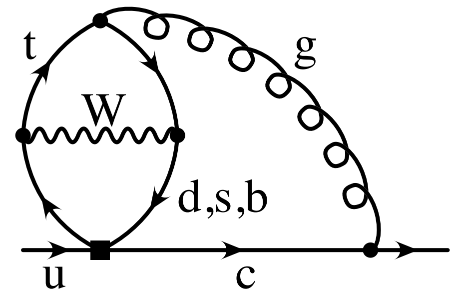
→ ***H_{EDM}* is CP-odd !**

$$\begin{cases} \vec{E} \xrightarrow{\mathrm{P}} -\vec{E} \\ \vec{\sigma} \xrightarrow{\mathrm{P}} \vec{\sigma} \end{cases}$$

→ ***H_{EDM}* is P-odd**

▪ Origin of EDM: CP-violating (CP-odd) interactions

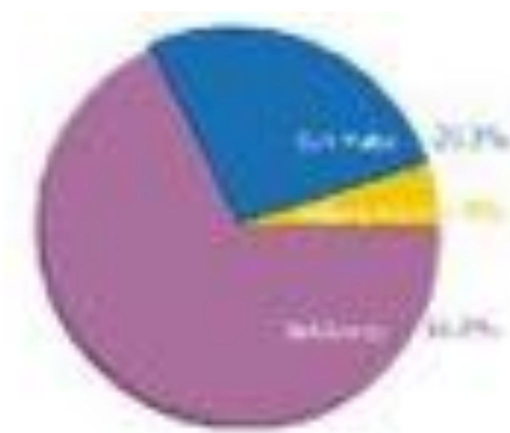
CKM: CP violating interaction in SM
But, electron and quark EDM's are zero at 1 and 2 loop level.
at least three loops to get non-zero EDM's.
EDM's are very small in the standard model.



SM contribution (3-loop diagram)
Ref: [A. Czarnecki and B. Krause '97]

nucleon EDM from CKM : ~ 10^{-32} [e cm]

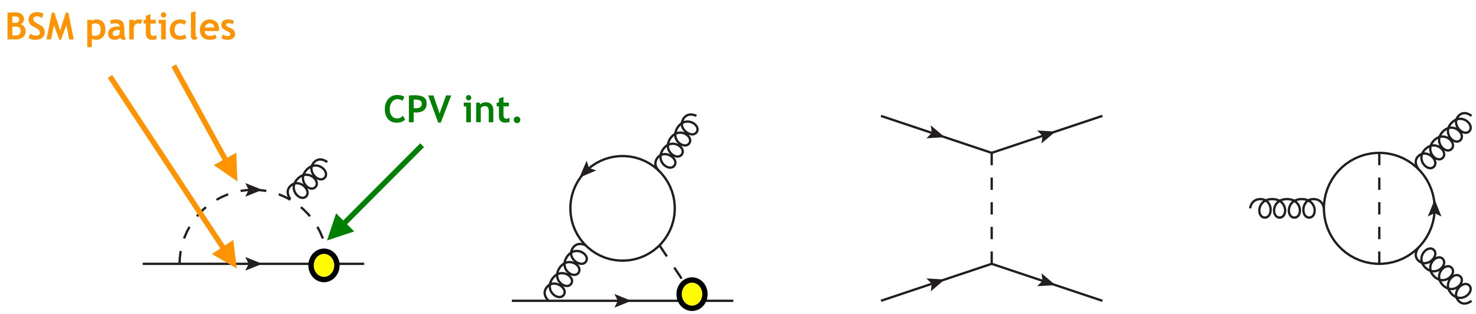
CP violation (CPV) in SM is not sufficient to reproduce matter/antimatter asymmetry. Large CPV beyond SM is required. (Sakharov's three conditions)



•http://www.esa.int/ESA

photon: matter	SM prediction	10^{20} : 1
	Observation	10^{10} : 1

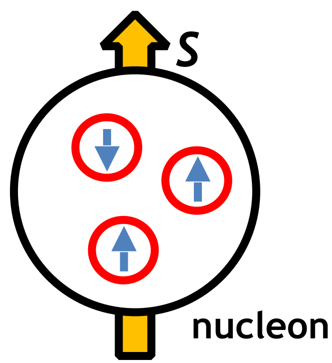
Origin of EDM: CP-violating (CP-odd) BSM physics



CP-odd four-quark Weinberg op.

BSM may induce EDM in lower loop level: a good probe of new physics

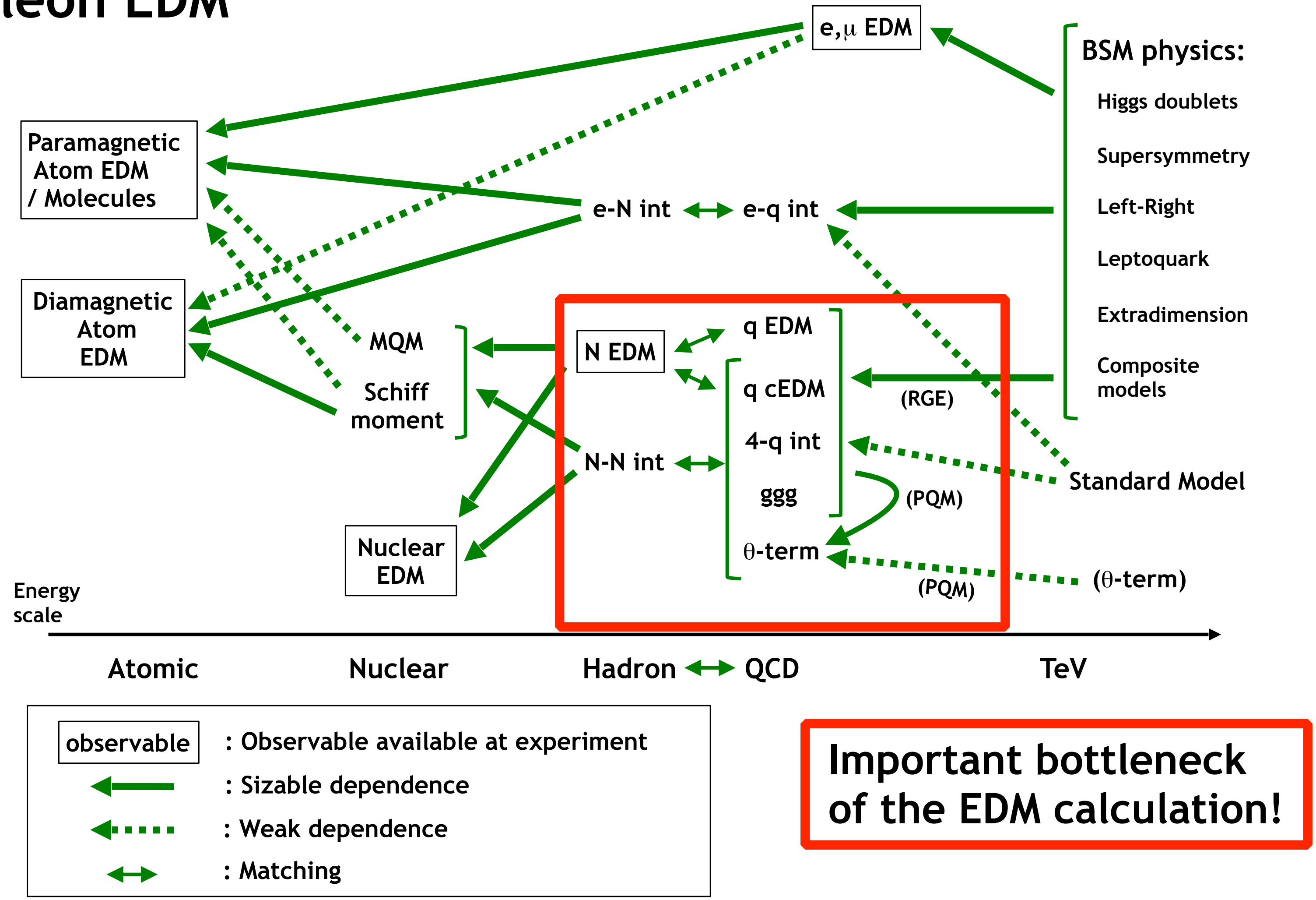
EDM is usually measured using composite particles (neutron, atoms, etc)



EDM effects may be enhanced in the composite system.

[N. Yamanaka, et al. Eur. Phys. J. A53 (2017) 54, Ginges and Flambaum Phys. Rep. 397, 63, 2004]

•Nucleon EDM

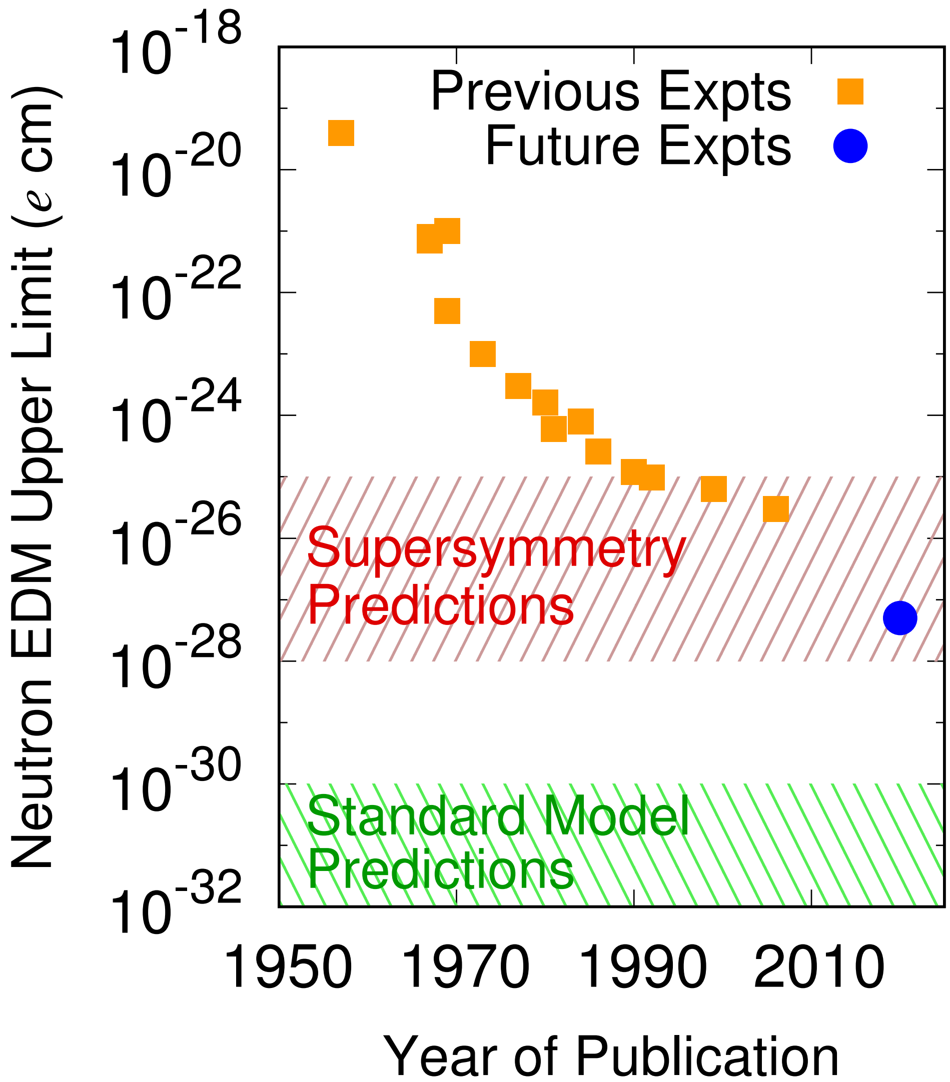


[N. Yamanaka, et al. Eur. Phys. J. A53 (2017) 54, Ginges and Flambaum Phys. Rep. 397, 63, 2004]

Role of (lattice) QCD : connect quark/gluon-level (effective) operators to hadron/nuclei matrix elements and interactions (Form factor, dn)

Non-perturbative determination is important → Lattice QCD calculation!

•Nucleon EDM Experiments



[B. Yoon, talk at Lattice 2017]

Current nEDM limits:

^{199}Hg spin precession (UW) [Graner et al, 2016]
Ultracold Neutrons in a trap (ILL) [Baker 2006]

$$|d_{Hg}| < 7.4 \times 10^{-30}\ \mathrm{e \cdot cm}$$
$$|d_n| < 2.6 \times 10^{-26}\ \mathrm{e \cdot cm}$$

SM nucleon EDMs expectation is

much smaller than the current bound.

- Several experimental projects are on going.

nucleon, charged hadrons, lepton,
PSI EDM, Munich FRMII, SNS nEDM, RCNP/TRIUMF, J-PARC etc…

•Effective CPV operators

$$\mathcal{L}_{eff}^{\cancel{CP}} = \frac{g_s^2}{32\pi^2}\bar{\theta} G_{\mu\nu}\tilde{G}^{\mu\nu}$$ dim=4, θ_{QCD}

$$-\frac{i}{2}\sum_{i=u,d,s}\tilde{d}_i\bar{\psi}_i G\cdot\sigma\gamma_5\psi_i$$ dim=5, chromo EDM

$$-\frac{i}{2}\sum_{i=e,u,d,s}d_i\bar{\psi}_i F\cdot\sigma\gamma_5\psi_i$$ dim=5, e, quark EDM

$$+\,\omega f^{abc}G_{\mu\nu,a}G^{\mu\beta,b}G^{\nu,c}_{\beta}$$ dim=6, Weinberg three gluon

$$+\sum C_i^{(4q)}\mathcal{O}_i^{(4q)}$$ dim=6, Four-quark operators

$\bar{\theta} \leq \mathcal{O}(10^{-10})$: Strong CP problem

Dim=5 operators suppressed by m_q/Λ^2 -> effectively dim=6,

quark EDM … the most accurate lattice data for EDM (~5% for u,d)

Others are not well determined. cEDM, Weinberg ops just started.

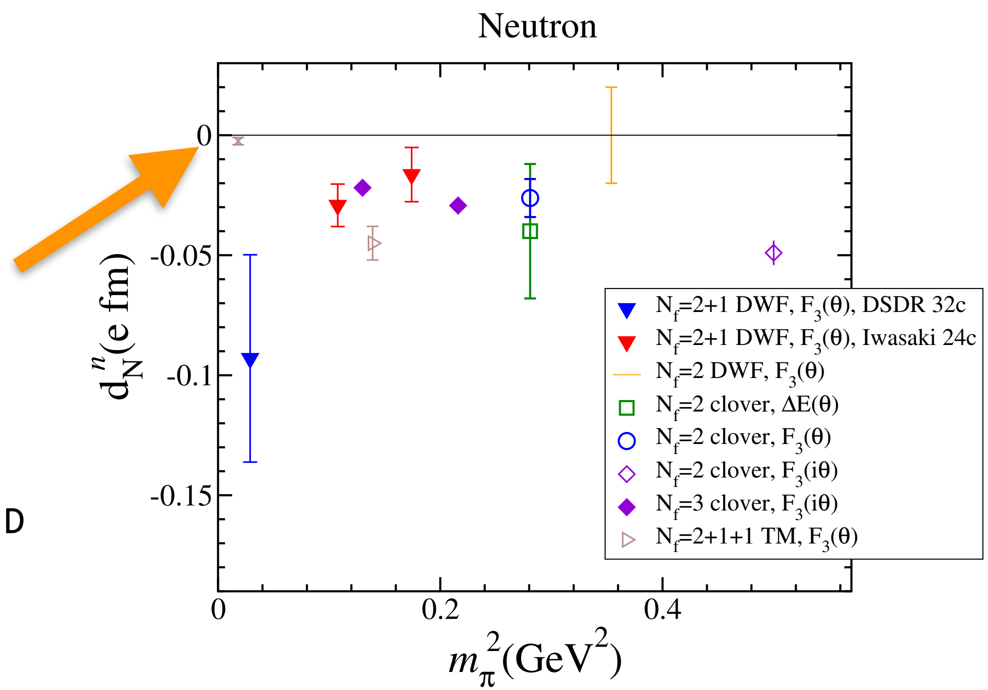
θ_{QCD} induced Nucleon EDMs

Phenomenological estimates

method	value
ChPT/NDA	~ 0.002 e fm
QCD sum rules [1,2]	0.0025 ± 0.0013 e fm
QCD sum rules [3]	$0.0004^{+0.0003}_{-0.0002}$ e fm

[1] M. Pospelov, A. Ritz, Nuclear Phys. B 573 (2000) 177,
[2] M. Pospelov, A. Ritz, Phys. Rev. Lett. 83 (1999) 2526,
[3] J. Hisano, J.Y. Lee, N. Nagata, Y. Shimizu, Phys. Rev. D 85 (2012) 114044.

Lattice calculations



[E. Shintani, T. Blum, T. Izubuchi, A. Soni, PRD93, 094503(2015)]

Phenomenology: |dn| ~ θ_{QCD} 10^{-3} e fm -> |θ_{QCD}| < 10^{-10}
Lattice : |dn| ~ θ_{QCD} 10^-2 e fm -> severer constraint on |θ_{QCD}|

Problem: a spurious mixing between EDM and magnetic moments in all previous lattice computations of nucleon form factor.

Parity mixing problem
on
the CP-violating nucleon form factors

M. Abramczyk, S. Aoki, T. Blum, T. Izubuchi, H. Ohki, and S. Syritsyn,
Lattice calculation of electric dipole moments and form factors of the nucleon
Phys.Rev. D96 (2017) no.1, 014501, selected editor's suggestions

Definition of nucleon form factors

Nucleon form factor in C, P-symmetric world (CP-even)

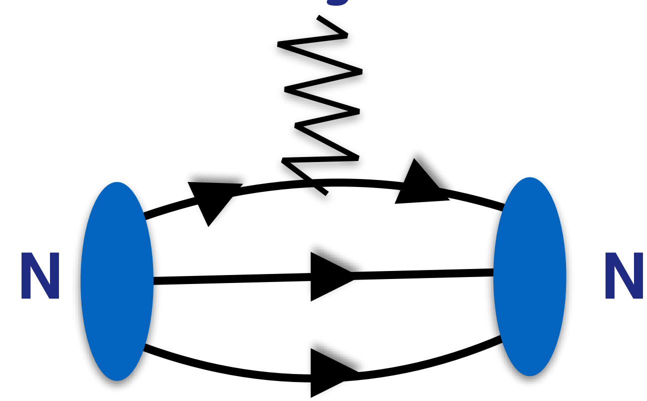
$$\langle p', \sigma' | J^\mu | p, \sigma \rangle = \bar{u}_{p',\sigma'} \left[F_1(Q^2)\gamma^\mu + F_2(Q^2) \frac{i\sigma^{\mu\nu} q_\nu}{2m_N} \right] u_{p,\sigma}$$

$$(q = p' - p, \quad Q^2 = -q^2)$$

u_p : spinor wave function for the nucleon ground state |p,σ>

$$(\not{p} - m_N) u_p = 0$$

J : electromagnetic current



Definition of nucleon form factors

Nucleon form factor in CP-broken world

$$\langle p', \sigma' | J^\mu | p, \sigma \rangle = \bar{u}_{p',\sigma'} \left[\underbrace{F_1(Q^2)\gamma^\mu + F_2(Q^2)\frac{i\sigma^{\mu\nu}q_\nu}{2m_N}}_{\text{P, T even}} - \underbrace{F_3(Q^2)\frac{\gamma_5\sigma^{\mu\nu}q_\nu}{2m_N}}_{\text{P, T odd}} \right] u_{p,\sigma}$$

Refs. [many textbooks, e.g. Itzykson, Zuber, “Quantum Field Theory“]

- The same spinor u_p (F1, F2 are same as CP-even case.)
- Non-zero F3 is a signature of the CP violation (F3= 0 -> CP-even)
- permanent EDM:

$$d_n = \lim_{Q^2 \to 0} \frac{F_3(Q^2)}{2m_N}$$

All previous lattice studies (prior to 2017) use a different spin structure for the form factors.

revisit of the nucleon CP-odd (EDM) form

Nucleon 2 point function in CP-conserving theory

$N = u[u^T C\gamma_5 d]$ Lattice nucleon operator for sink and source

$\langle 0|N|p,\sigma\rangle_{CP-even} = Z u_{p,\sigma}$ Nucleon ground state in CP-even vacuum

u_p is a solution spinor of the free Dirac equation: $(\not{p} - m_N)u_p = 0$

$$
\begin{aligned}
C_{2pt}(\vec{p};t)_{CP-even} &= \langle N(\vec{p};t)|\bar{N}(\vec{p};0)\rangle_{CP-even} \\
&= \langle N(\vec{p},t)\left[\sum_{k,\sigma}\frac{|k,\sigma\rangle\langle k,\sigma|}{2E_k}\right]\bar{N}(\vec{p};0)\rangle_{CP-even} + (\text{excited states}) \\
&\xrightarrow[t\to\infty]{} |Z|^2\frac{e^{-E_p t}}{2E_p}\Big(\sum_\sigma u_{p,\sigma}\bar{u}_{p,\sigma}\Big) \\
&= |Z|^2 e^{-E_p t}\frac{m_N - i\not{p}}{2E_p}
\end{aligned}
$$

Completeness condition for free Dirac spinor

(From now on excited states are omitted.)

Nucleon 2 point function in CP-broken theory

$$\langle 0|N|p,\sigma\rangle_{\not{CP}} = Z\tilde{u}_{p,\sigma}$$ Nucleon ground state in CP-broken vacuum

$\tilde{u}_p$ is a solution spinor of the free Dirac equation: $(\not{p} - m_N e^{-2i\alpha\gamma_5})\tilde{u}_p = 0$

Asymptotic state is modified: (CP-violating) γ5 mass is allowed in general.

$$\begin{aligned} C_{2pt}(\vec{p};t)_{\not{CP}} &= \langle N(\vec{p};t)|\bar{N}(\vec{p};0)\rangle_{\not{CP}} \\ &= |Z|^2 \frac{e^{-E_p t}}{2E_p} (\sum_\sigma \tilde{u}_{p,\sigma}\bar{\tilde{u}}_{p,\sigma}) \\ &= |Z|^2 e^{-E_p t} \frac{m_N e^{2i\alpha\gamma_5} - i\not{p}}{2E_p} \end{aligned}$$

Completeness condition for free Dirac spinor

$\tilde{u}_p = e^{i\alpha\gamma_5} u_p$ is a solution to the above Dirac equation.

$$\sum_\sigma \tilde{u}_{p,\sigma}\bar{\tilde{u}}_{p,\sigma} = e^{i\alpha\gamma_5}(\sum_\sigma u_{p,\sigma}\bar{u}_{p,\sigma})e^{i\alpha\gamma_5} = m_N e^{2i\alpha\gamma_5} - i\not{p}$$

[Completeness condition for free Dirac spinor with γ5 mass]

Calculation of 3 point function in CP-broken theory

$$C_{3pt}(\vec{p'},t;\vec{p},\tau)_{\cancel{CP}} = \sum_{\vec{y},\vec{z}} e^{-i\vec{p'}\cdot\vec{y}+i\vec{p}\cdot\vec{z}} \langle N(\vec{y},t) J^\mu(\vec{z},\tau) \bar{N}(0)\rangle_{\cancel{CP}}$$

$$= |Z|^2 \frac{e^{-E_{p'}(t-\tau)-E_p(\tau)}}{4E_{p'}E_p} \sum_{\sigma,\sigma'} \underbrace{\langle N(p')|p',\sigma\rangle_{\cancel{CP}}}_{①} \underbrace{\langle p',\sigma|J^\mu|p,\sigma'\rangle_{\cancel{CP}}}_{②} \underbrace{\langle p,\sigma'|N(p)\rangle_{\cancel{CP}}}_{③}$$

① & ③: $\langle 0|N|p,\sigma\rangle_{\cancel{CP}} = Z\tilde{u}_{p,\sigma}$

②: $\langle p',\sigma'|J^\mu|p,\sigma\rangle_{\cancel{CP}} = \bar{\tilde{u}}_{p',\sigma'} \left[\tilde{F}_1(Q^2)\gamma^\mu + \tilde{F}_2(Q^2)\frac{i\sigma^{\mu\nu}q_\nu}{2m_N} - \tilde{F}_3(Q^2)\frac{\gamma_5\sigma^{\mu\nu}q_\nu}{2m_N} \right] \tilde{u}_{p,\sigma}$

Refs: original works since 2005

"All" previous (prior 2017) lattice studies:

$\tilde{F}_1, \tilde{F}_2, \tilde{F}_3$: defined in the rotated spinor basis $(\tilde{u})$

Two form factors are different!

$$\begin{cases} F_2(Q^2) \neq \tilde{F}_2(Q^2) \\ F_3(Q^2) \neq \tilde{F}_3(Q^2) \end{cases}$$

(u) $\qquad$ $(\tilde{u})$

Relation between two spinor basis

$$\bar{\tilde{u}}_{p',\sigma'}\left[\tilde{F}_1\gamma^\mu + (\tilde{F}_2 + i\tilde{F}_3\gamma_5)\frac{i\sigma^{\mu\nu}q_\nu}{2m_N}\right]\tilde{u}_{p,\sigma} = \bar{u}_{p',\sigma'}\left[\tilde{F}_1\gamma^\mu + e^{2i\alpha\gamma_5}(\tilde{F}_2 + i\tilde{F}_3\gamma_5)\frac{i\sigma^{\mu\nu}q_\nu}{2m_N}\right]u_{p,\sigma}$$

[conventional “lattice” parametrization since 2005]

$$\equiv \bar{u}_{p',\sigma'}\left[F_1\gamma^\mu + (F_2 + iF_3\gamma_5)\frac{i\sigma^{\mu\nu}q_\nu}{2m_N}\right]u_{p,\sigma}$$

[textbook]

Relation between $\{F_1, F_2, F_3\}$ and $\{\tilde{F}_1, \tilde{F}_2, \tilde{F}_3\}$

$$(F_2 + iF_3\gamma_5) = e^{2i\alpha\gamma_5}(\tilde{F}_2 + i\tilde{F}_3\gamma_5), \Leftrightarrow \begin{cases} \tilde{F}_2 = \cos(2\alpha)F_2 + \sin(2\alpha)F_3 \\ \tilde{F}_3 = -\sin(2\alpha)F_2 + \cos(2\alpha)F_3 \end{cases}$$

$$\Rightarrow \quad \begin{aligned} [F_2]_{\text{correct}} &= \tilde{F}_2 + \mathcal{O}(\alpha^2) \\ [F_3]_{\text{correct}} &= \tilde{F}_3 + 2\alpha F_2 \end{aligned}$$

There is a spurious contribution of order (α F2) to the previous lattice results.
In other words, CP violation effects come from both tilde{F3} and α, not only tilde{F3}.

This mixing angle α has to be calculated, and rotated away to get “net” CP-violation effect.

Similar issues in the ChPT (perturbative) calculations? (α may appear in the mass correction.)

Numerical check using the chromo EDM operator

Form factor method

vs

Energy shift method

Computational resources : ACCC HOKUSAI greatwave, Fermilab, JLab [USQCD project]

How to calculate CP-odd interaction on a lattice

Linearization of CP-odd interaction (e.g. : θ-EDM)

$$e^{-S_{QCD}-i\theta Q} = e^{-S_{QCD}}\left[1 - i\theta Q + \mathcal{O}(\theta^2)\right]$$

$$\langle\mathcal{O}\rangle_{\cancel{CP}} = \langle\mathcal{O}\rangle_{CP-even} - i\theta\langle Q\cdot\mathcal{O}\rangle_{CP-even} + \mathcal{O}(\theta^2)$$

(CP-even) (CP-odd)

CPV operator : Q, cEDM, etc..., θ << 1

Original (CP-even) gauge configurations can be used. No sign problem.

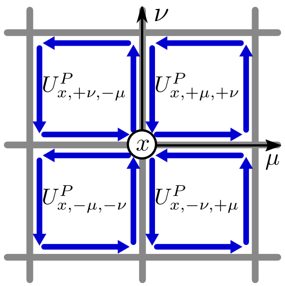
c.f. Dynamical simulation including CP-odd interactions

$$\langle\mathcal{O}\rangle_\theta \sim \int \mathcal{D}U(\mathcal{O})e^{-S_{QCD}-\theta_{imag}Q}$$ [R. Horsley et al. (2008); H. K. Guo, et al., 2015)]

Non-perturbative treatment of CP-odd interactions.
Analytic continuation to imaginary θ.
Need additional simulation for ensemble generations to get non-zero topological sector.
Check linearity of θ (ensemble generation for various imaginary θ)

•Chromo EDM operator

$$\mathcal{L}_{cEDM} = \sum_{q=u,d} \frac{\delta_q}{2} \bar{q}[G_{\mu\nu}\sigma^{\mu\nu}\gamma_5]q$$



✱Dimention 5 CP violating operator, mixing with dim-3 pseudo scalar operator.

✱Beyond standard model origin

✱Chiral symmetry is important.
The clover term in Wilson-type action = Chromo-magnetic dipole moment (chromo-MDM).

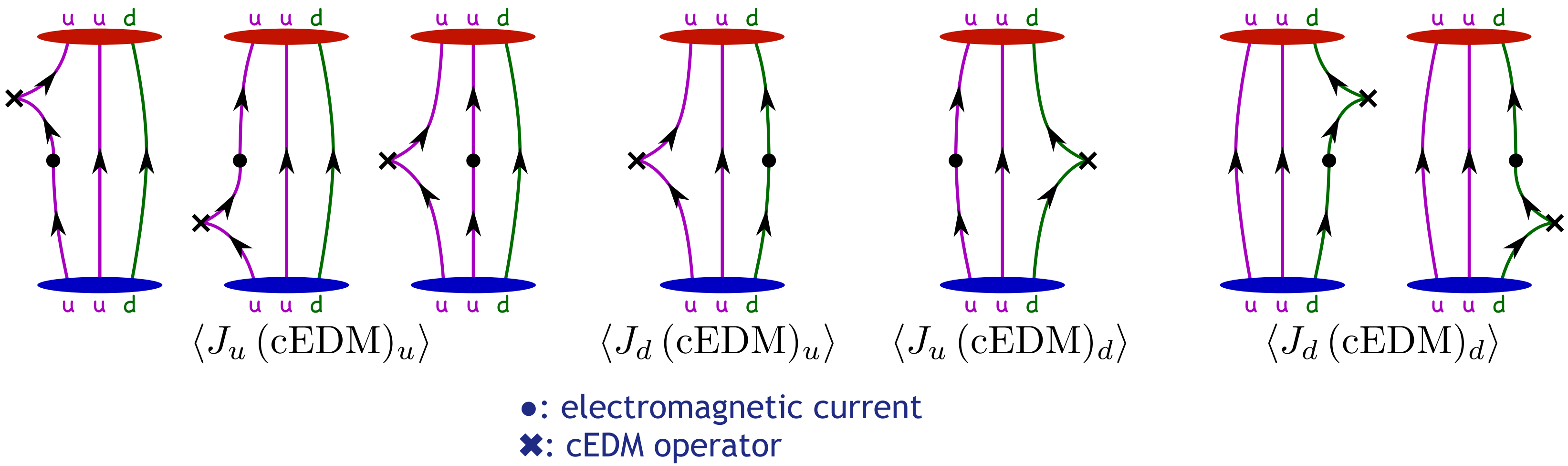
$$\mathcal{L}_{clover} = a\bar{q}\left[G_{\mu\nu}\sigma^{\mu\nu}\right]q$$

In presence of CPv, additional operator mixing of chromo-MDM appears.

➡We use chirally symmetric domain wall fermion
(gauge ensemble by RBC-UKQCD

Lattice calculation of chromo EDM operator

✱Our study: only connected diagrams (without renormalization, subtraction)



✱Simulation parameters

$L_x^3 \times L_t \times L_5$	a [fm]	am_l	am_s	m_π [MeV]	m_N [GeV]	$\mathcal{E}_0$ [GeV^2]	conf	stat	N_{ev}	$N_{ev}^{\mathcal{E}=1,2}$	N_{CG}
$16^3 \times 32 \times 16$	0.114(2)	0.01	0.032	422(7)	1.250(28)	0.110	500	16000	200	150	100
$24^3 \times 64 \times 16$	0.1105(6)	0.005	0.04	340(2)	1.178(10)	0.0388	100	3200	200	200	200

1. Form factor method

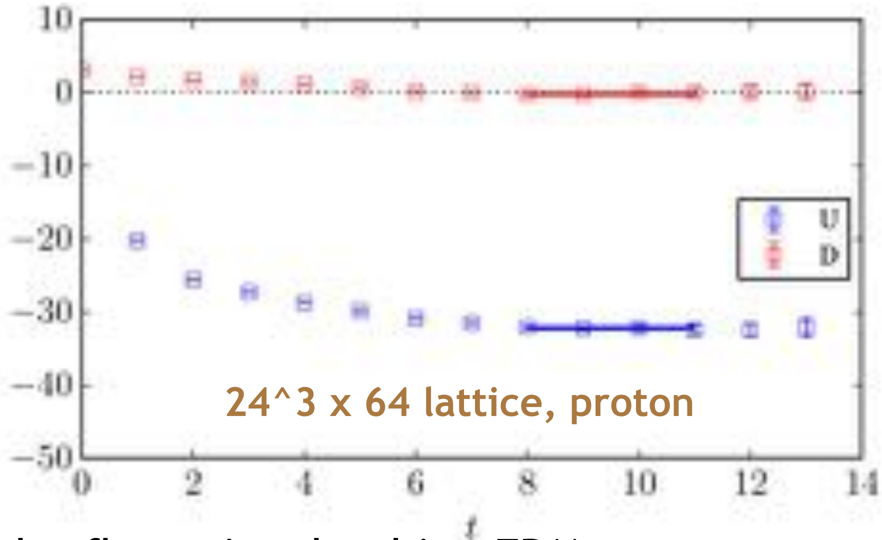
Mixing parameter induced by cEDM

$$C_{2pt}^{CP-odd}(t) = \langle N(t)\bar{N}(0)\sum_x \mathcal{O}_{cEDM}(x)\rangle$$

$$\begin{aligned} C_{2pt}(\vec{p};t)_{\cancel{CP}} &= |Z|^2 e^{-E_p t}\frac{m_N e^{2i\alpha\gamma_5} - i\not{p}}{2E_p} \\ &= |Z|^2 \frac{e^{-E_p t}}{2E_p}\left[(m_N - i\not{p}) + 2i\alpha m_N\gamma_5\right] + \mathcal{O}(\alpha^2) \end{aligned}$$

(CP-even) (CP-odd)

$$\alpha^{eff}(t) = -\frac{\mathrm{Tr}\left[T^+\gamma_5 C_{2pt}^{CP-odd}(t)\right]}{\mathrm{Tr}\left[T^+ C_{2pt}(t)\right]}$$



Mixing angle α depend strongly on the flavor involved in cEDM.

For proton, its strength for U-cEDM is large, no signal for D-cEDM.

For nucleon, no signal for U-cEDM.

Result of F3 form factor (L=24)

$$C_{3pt}^{CP-odd}(T,t) = \langle N(T) J^{\mu}(t) \bar{N}(0) \sum_x [\mathcal{O}_{cEDM}(x)] \rangle$$

a standard plateau method:

$$R(T,t) = \frac{C_{3pt}^{CP-odd}(T,t)}{c_{2pt}(t)} \sqrt{\frac{c'_{2pt}(T) c'_{2pt}(t) c_{2pt}(T-t)}{c_{2pt}(T) c_{2pt}(t) c'_{2pt}(T-t)}}$$

"correct" F3 :

$$\boxed{(1+\tau) F_3(Q^2) = \frac{m_N}{q_z R} \mathrm{Tr}\left[T^{+}_{S_z} \cdot R(T,t)^{\mu=4} \right] - \alpha G_E(Q^2)}$$

R: kinetic factor

projection operator : $T^{+}_{S_z} = \left[\frac{1+\gamma^4}{2} (-i\gamma^1\gamma^2) \right]$

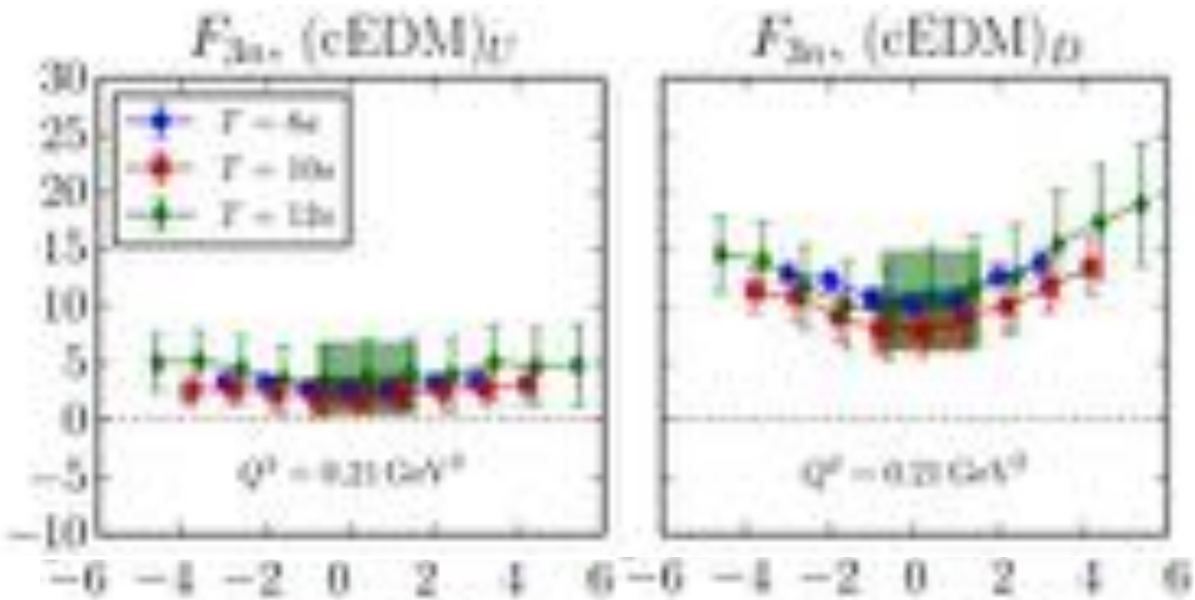
GE: Sachs electric form factor $G_E(Q^2) = F_1(Q^2) - \tau F_2(Q^2), \quad \tau = \frac{Q^2}{4m_N^2}$

Recall the 3 pt functions:

$$C_{3pt}(\vec{p}', t; \vec{p}, \tau)_{\cancel{CP}} = \sum_{\vec{y},\vec{z}} e^{-i\vec{p}'\cdot\vec{y} + i\vec{p}\cdot\vec{z}} \langle N(\vec{y}, t) J^{\mu}(\vec{z}, \tau) \bar{N}(0) \rangle_{\cancel{CP}}$$

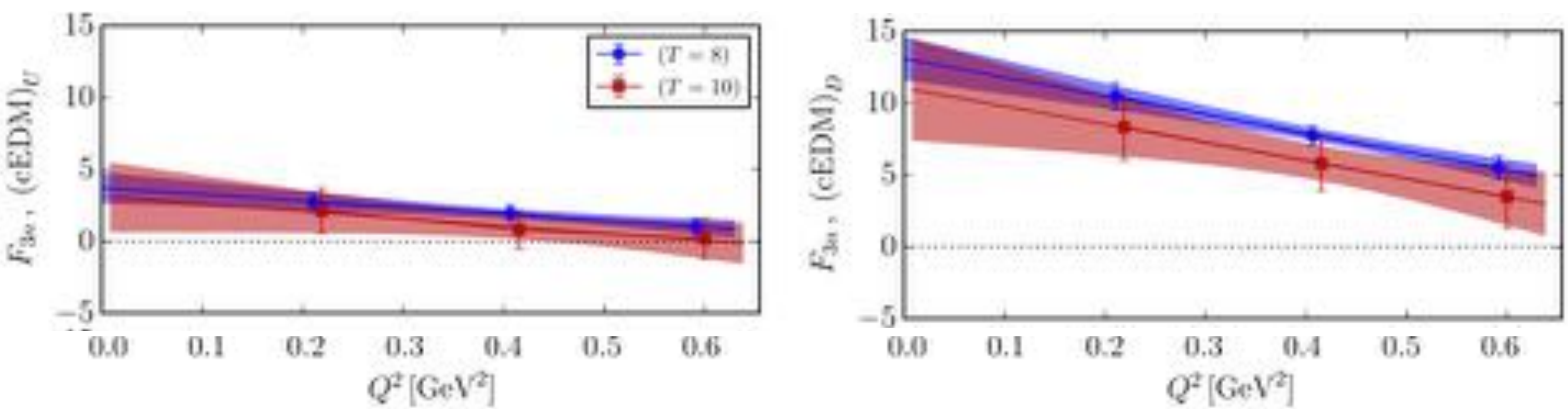
$$= |Z|^2 \frac{e^{-E_{p'}(t-\tau) - E_p(\tau)}}{4 E_{p'} E_p} \sum_{\sigma,\sigma'} \langle N(p') | p', \sigma \rangle_{\cancel{CP}} \langle p', \sigma | J^{\mu} | p, \sigma' \rangle_{\cancel{CP}} \langle p, \sigma' | N(p) \rangle_{\cancel{CP}}$$

Result



Neutron, u-cEDM t-T/2 Neutron, d-cEDM

$m_\pi = 340[\mathrm{MeV}]$



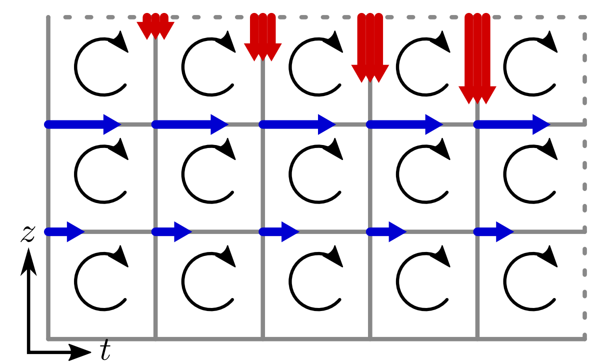
Linear Q^2 fit to nucleon F3 form factor

2. Energy shift method

Lattice QCD with background constant electric field

- Uniform electric field preserving translational invariance and periodic boundary conditions on a lattice (Euclidean imaginary electric field)
- used for the nucleon polarizability [W. Detmold, Tiburzi, and Walker-Loud, (2009)]
- First applied to the CPV effects.
- No sign problem: Analytic continuation of CP-odd interaction

Charge quantization due to finite volume.



$$U_\mu \to e^{iQ_q A_\mu} U_\mu$$

$$A_t(z,t) = \mathcal{E}_n z$$

$$A_z(z,t) = -\mathcal{E}_n L_z t \delta_{z=L_z-1}$$

strength of E field $\mathcal{E}_n = n\dfrac{6\pi}{L_z L_t}, \quad (n = \pm 1, \pm 2, \cdots)$

charge quanta $Q_q \mathcal{E}_n L_z L_t = 2\pi m, \quad (m : \text{integer})$

$(Q_u = 2/3, \quad Q_d = -1/3)$

24^3x 64 lattice minimal value of E (|n|=1) $\mathcal{E}_0 = \dfrac{6\pi}{L_z L_t} \sim 0.037 \text{ GeV}^2$

$\sim 186 \text{ MV/fm}$

Nucleon 2 point function with a constant Ez-field

$$C_{2pt}^{\cancel{CP}}(\vec{p}=0,t,\mathcal{E}) = |Z|^2 e^{i\alpha\gamma_5}\left(\frac{1+\gamma_4}{2}\right)\left[\frac{1+\Sigma_z}{2}e^{-(m+\delta E)t}+\frac{1-\Sigma_z}{2}e^{-(m-\delta E)t}\right]e^{i\alpha\gamma_5}+\mathcal{O}((\kappa,\mathcal{E})^2)$$

$$\sim |Z|^2 e^{-mt}\left[\underbrace{\frac{1+\gamma_4}{2}}_{\text{(CP-even)}}+\underbrace{i\alpha\gamma_5-\Sigma_z\delta E t}_{\text{(CP-odd)}}\right]$$

(t >> 1)

spin dependent interaction energy

Energy shift : $\delta E = -\frac{\zeta}{2m}(i\mathcal{E})$

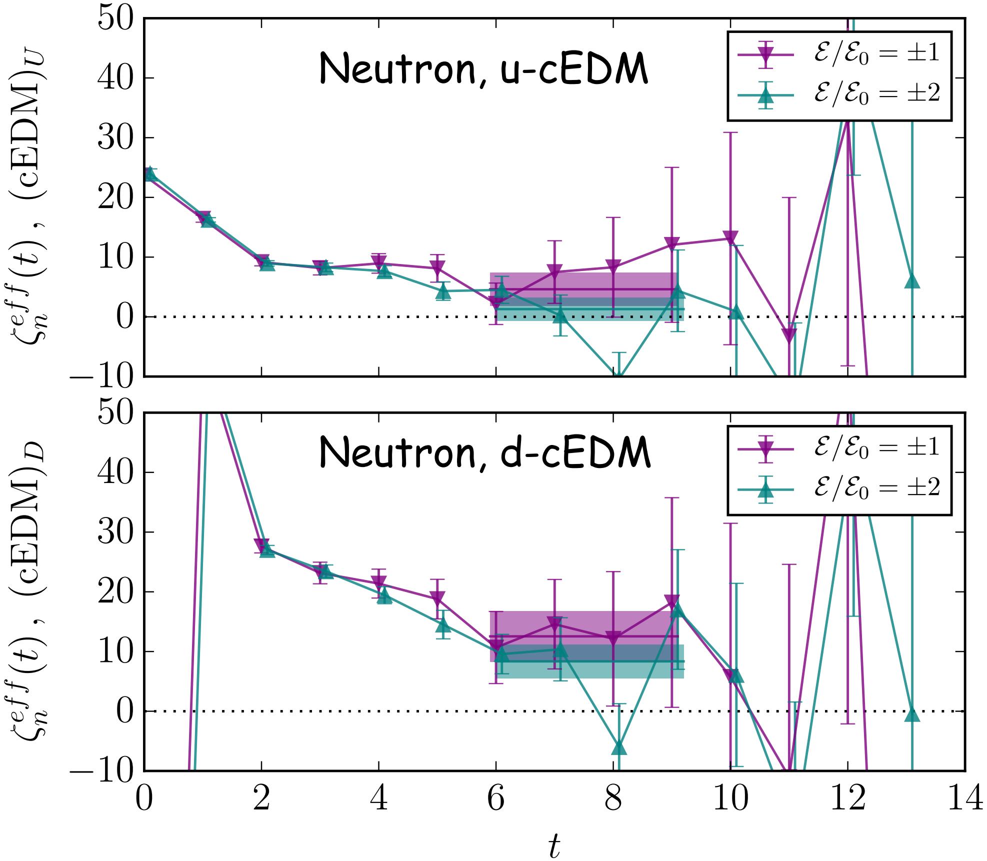
"Effective" energy shift (extraction of the term proportion to linear-time)

$$\zeta^{eff} = 2mF_3^{eff}(0) = -\frac{2m}{\mathcal{E}_z}[R_z(t+1)-R_z(t)],$$

$$R_z(t) = \frac{\mathrm{Tr}[T_{S_z}^{+}C_{2pt}^{CP-odd}(t,\mathcal{E})]}{\mathrm{Tr}[T^{+}C_{2pt}(t,\mathcal{E})]}$$

$$C_{2pt}^{CP-odd}(t,\mathcal{E}) = \langle N(t)N(0)\sum_x[\mathcal{O}_{cEDM}(x)]\rangle_{\mathcal{E}\neq 0}$$

Effective energy shift for Neutron (L=24)



$m_\pi = 340[\mathrm{MeV}]$

Only neutron is considered. (Analysis of charged particle propagators is more complicated.)

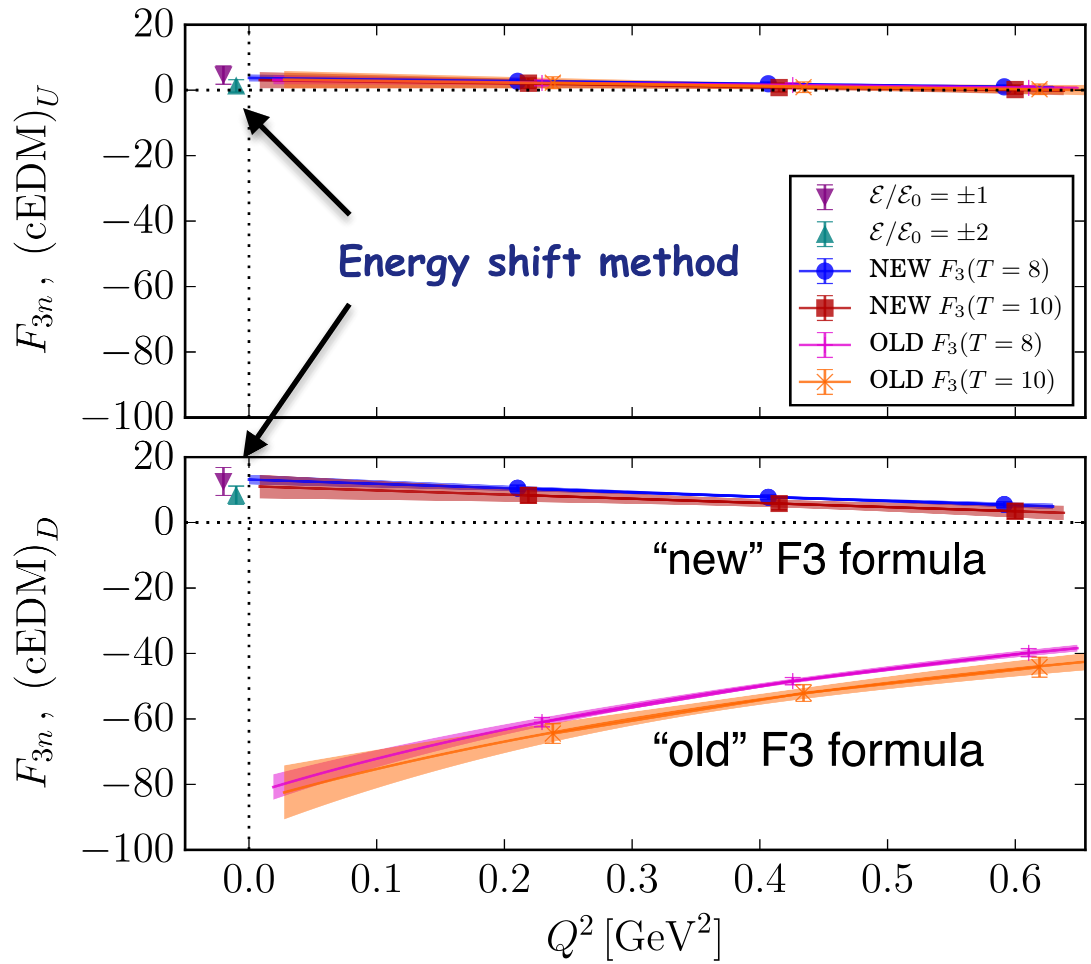
Non-zero signal for spectator d-cEDM.

Effective energy plateau around t = 6~10.

Results for |Ez|=1, |Ez|=2 are consistent. -> Higher order effects of E-field can be neglected.

New formula vs. Old formula

$m_\pi = 340[\mathrm{MeV}]$



Neutron, u-cEDM

F2 mixing effect is tiny.

$\alpha_u \sim 0$

Neutron, d-cEDM

large spurious mixing.

$\alpha_d \sim 30$

u-cEDM: New and Old formula results give similar value consistent with energy shift method.

d-cEDM: "new" formula result is consistent with the energy shift method.

"old" F3 has a sizable mixing due to large α (cEDM mixing α ~ 30) [c.f. α for topological charge]

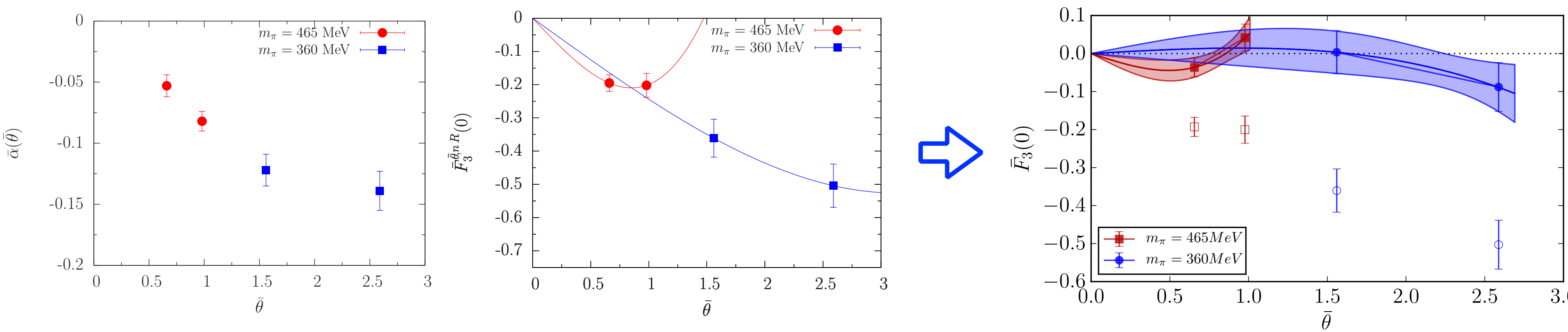
Implication of new formula for the theta induced EDM

Reanalysis of “lattice” θ induced EDM

Correction is simple: $$[F_3]_{\text{correct}} = \tilde{F}_3 + 2\alpha F_2$$

[F. Guo et al., PRL 115, no.6, 062001 (2015)] Form factor method

Dynamical calculations with finite imaginary θ angle



Correction made by ourselves

[C. Alexandrou, et al. (ETMC)]: Form factor method

dn = -0.045(06) e fm (7.5 σ) -> +0.008(6) e fm (1.3σ)

[E. Shintani et al, D78:014503(2008)]

Lattice with uniform Minkowski-real background electric field -> Energy shift method not affected by the spurious mixing.

dn=-0.040(28) e fm (1.4σ), the result is not sufficient to see disagreement with the form factor method.

Reanalysis of “lattice” θ induced EDM

Correction is simple: $$[F_3]_{\rm correct} = \tilde{F}_3 + 2\alpha F_2$$

Correction made by ourselves

		m_π [MeV]	m_N [GeV]	F_2	α	$\tilde{F}_3$	F_3
Ref[1]	n	373	1.216(4)	−1.50(16)	−0.217(18)	−0.555(74)	0.094(74)
Ref[2]	n	530	1.334(8)	−0.560(40)	−0.247(17)	−0.325(68)	−0.048(68)
	p	530	1.334(8)	0.399(37)	−0.247(17)	0.284(81)	0.087(81)
Ref[3]	n	690	1.575(9)	−1.715(46)	−0.070(20)	−1.39(1.52)	−1.15(1.52)
	n	605	1.470(9)	−1.698(68)	−0.160(20)	0.60(2.98)	1.14(2.98)
Ref[4]	n	465	1.246(7)	−1.491(22)	−0.079(27)	−0.375(48)	−0.130(76)
	n	360	1.138(13)	−1.473(37)	−0.092(14)	−0.248(29)	0.020(58)

Ref[1] : C. Alexandrou et al., Phys. Rev. D93, 074503 (2016),
Ref[2] : E. Shintani et al., Phys.Rev. D72, 014504 (2005).
Ref[3] : F. Berruto, T. Blum, K. Orginos, and A. Soni, Phys.Rev. D73, 054509 (2006)
Ref[4] : F. K. Guo et al., Phys. Rev. Lett. 115, 062001 (2015).

The lattice results are consistent with phenomenological estimates.

After removing spurious contributions, no signal of EDM.

How to improve the signal?

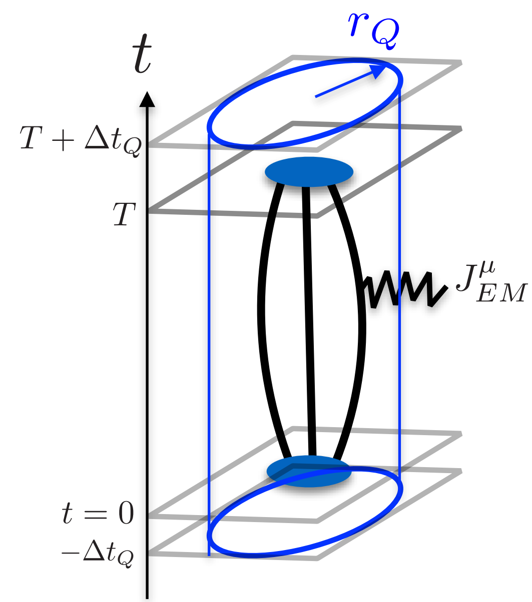
Noise reduction for θ-induced EDM

Noise reduction for θ-induced EDM

Statistical error $\sim$ V_4

Topological charge: $Q \sim \int_{V_4} G\tilde{G}, \quad \langle Q^2 \rangle \sim V_4$

nucleon EDM: $F_3 \sim \langle Q \cdot (N J^{\mu}_{EM} \bar{N}) \rangle$



Constraining to the fiducial volume for Q

$$Q \sim \int_{V_Q} d^4 x q(x)$$

- 4d spherical [K.-F. Liu, et al, 2017]

$$|x_Q - x_{sink}| < R$$

- truncation in t-direction [Shintani et al 2015, Guo et al 2019]

$$|t_Q - t_J| < \Delta t$$

- 4d “cylinder” (new)

$$V_Q : |\vec{x}| < r_Q, \quad -\Delta t_Q < t_0 < T + \Delta t_Q$$

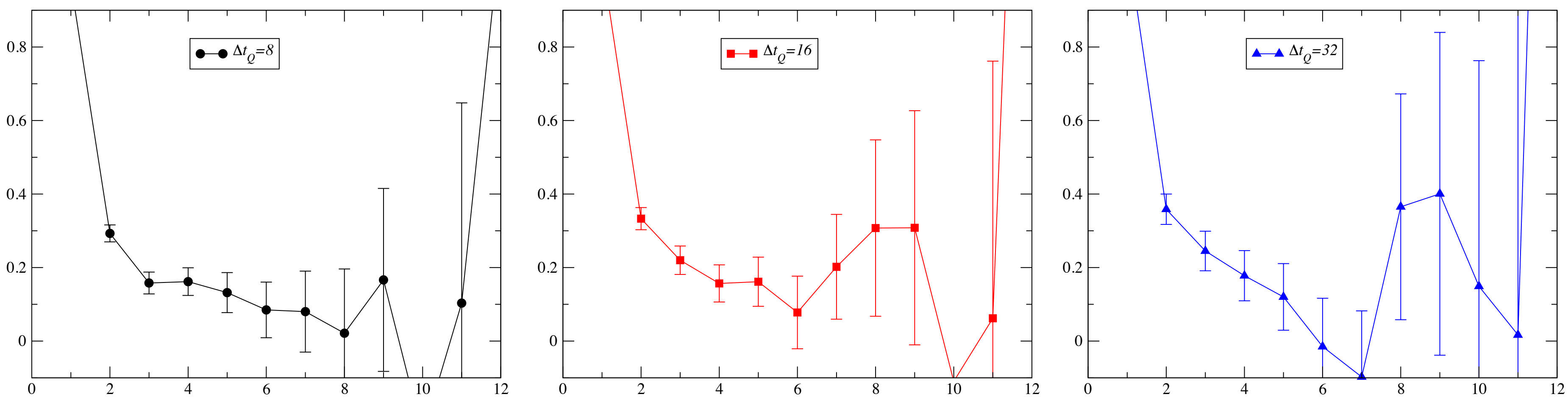
Truncation in t-direction

F3 from energy shift method

- Nf=2+1 Domain wall fermion, 24^3x64, a = 0.11 fm
- mπ=340 MeV
- 700 configurations, (32sloppy + 1exact samples)
- Three different electric background fields with x, y, and z-directions
  → 67200 k statistics
- reduced topological charge Q : truncation in t-direction $|t-6| \leq \Delta t_Q$

F3 : energy shift from θ-term

$F_3^{eff}(0)$ $\Delta t_Q = 8$ $\Delta t_Q = 16$ $\Delta t_Q = 32$

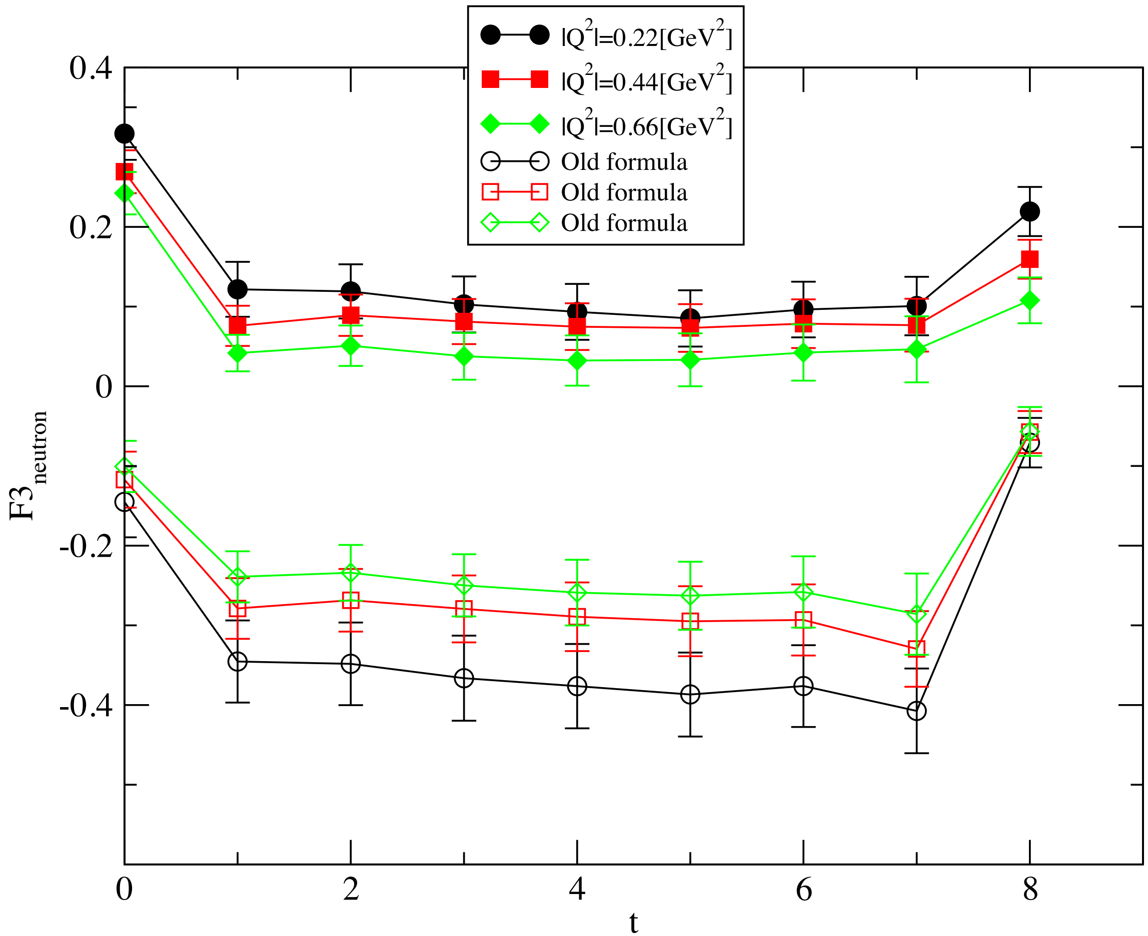


t t t

$$\zeta^{eff} = 2mF_3^{eff}(0) = -\frac{2m}{\mathcal{E}_z}[R_z(t+1) - R_z(t)],$$

F3 from form factor method

- reduced topological charge Q : truncated in t-direction $|t - T/2| \leq \Delta t_Q$
- Nf=2+1 Domain wall fermion, 24^3x64, a = 0.11 fm
- mπ=340 MeV
- 700 configurations (32sloppy + 1exact samples) → 22400 statistics

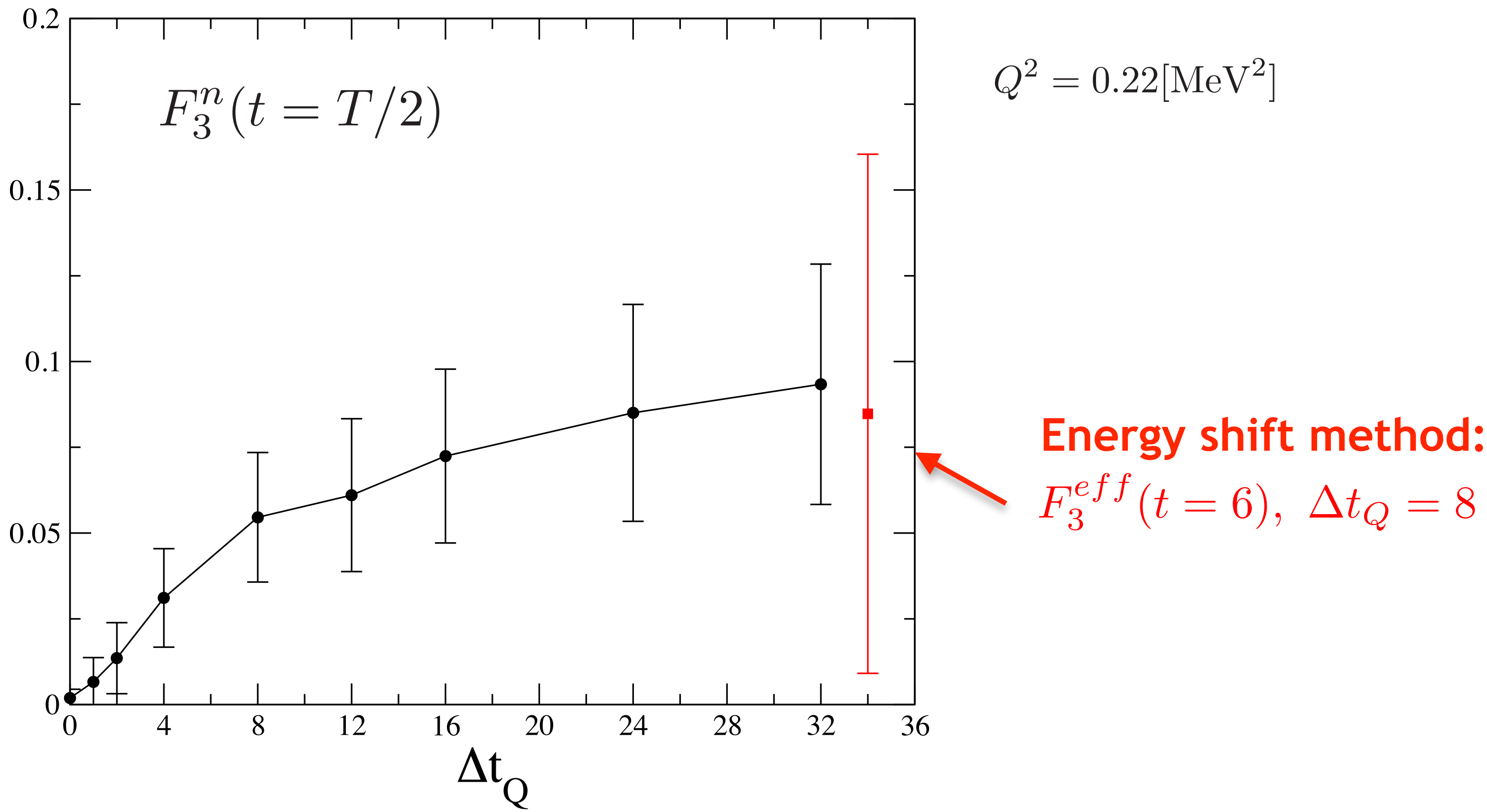


$m_\pi = 340[\mathrm{MeV}]$

$\Delta t_Q = 32$

Comparison of two methods for θ-EDM

$m_\pi = 340[\mathrm{MeV}]$



$$|2m_n d_n| = |F_3^n(0)| \simeq 0.05 \cdot \theta$$

Truncation method works for both methods.
"New" formula : consistent with energy shift.
Form factor method has better accuracy.

Dim=5 : qEDM

$$-\frac{i}{2}\sum_{i=e,u,d,s} d_i\bar{\psi}_i F\cdot\sigma\gamma_5\psi_i$$

quark EDM operator

$$\langle N|\frac{\delta(\bar{\psi}\sigma\cdot\tilde{F}\psi)}{\delta A_\mu}|N\rangle \propto \epsilon_{k\lambda\mu\nu}q_k\langle N|\bar{\psi}\sigma_{\lambda\nu}\psi)|N\rangle$$

$$\langle N|\bar{\psi}\sigma_{\lambda\nu}\psi|N\rangle = g_T\bar{u}_N\sigma_{\lambda\nu}u_N$$ (nucleon tensor charge)

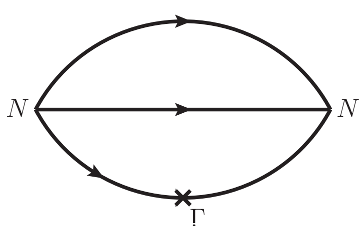
⇨ $$\frac{F_3}{2m_N} \equiv d_N \propto g_T \qquad d_N = d_u g_T^u + d_d g_T^d + d_s g_T^s$$

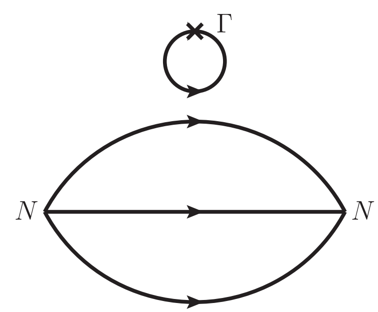
- Dimension 5 CP violating operator
- No need for CP-odd form factor

→ No spurious mixing problem in quark EDM

- dq ~ mq in most models, $m_s/m_d \sim 20$

→ strange quark contribution (disconnected diagram) is important.





Strange contribution : disconnected diagrams only (noisy)

Result of nucleon tensor charge

[N. Yamanaka, et al. for JLQCD Collaboration, PRD 98, 054516 (2018)]

Simulation parameters:

N_f = 2+1 QCD using overlap quarks + Iwasaki gauge action

Lattice spacing : a = 0.112(1) fm   Fixed topology Q = 0

16^3 x 48 lattice, m_π = 540, 450 MeV, 24^3 x 48 lattice, m_π = 380, 290 MeV

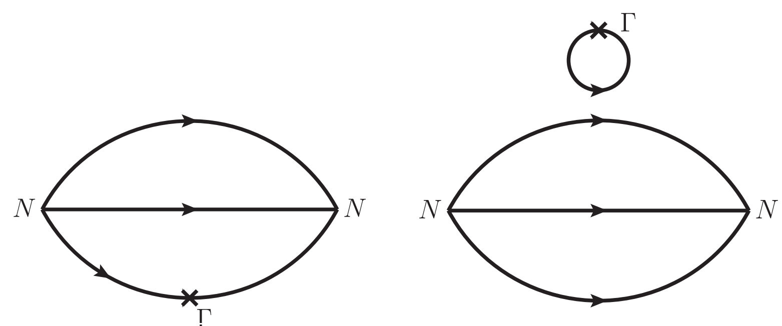
(50 Configurations for each quark mass)

All-to-all propagators:

- Low and high mode contributions: $$D^{-1} = \sum_k^{160} \frac{1}{\lambda_k} v_k v_k^\dagger + (\text{“high modes”})$$
- 160 (for 16^3x48), 240 (for 24^3x48) exact low Dirac eigenmodes
- High mode contribution with noise method

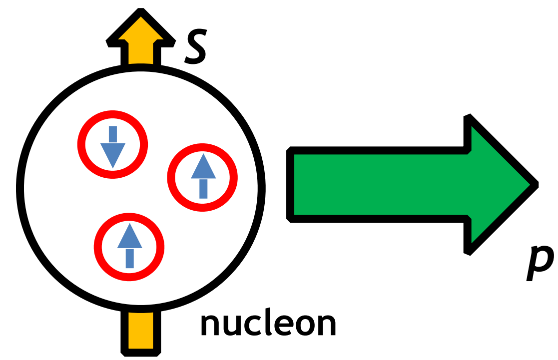
Truncated Solver Method (c.f. AMA) for high modes:

$$\langle O \rangle_{TSM} = \langle O \rangle_{Str} - \langle O \rangle_{Rel} + 1/N_G \, \Sigma_G \langle O_G \rangle_{Rel}$$



Improvement of disconnected diagrams with x,y,z directions

Nucleon tensor charges have spatial directions:



$$\langle N(p,S)|\bar{q}i\sigma^{\mu\nu}\gamma_5 q|N(p,S)\rangle = 2(S^\mu p^\nu - S^\nu p^\mu)\delta q$$

average axial and tensor charges over x, y, z polarizations
for the disconnected diagram,
which is computationally efficient for the calculation of disconnected diagrams,
since the quark loop is calculated independently

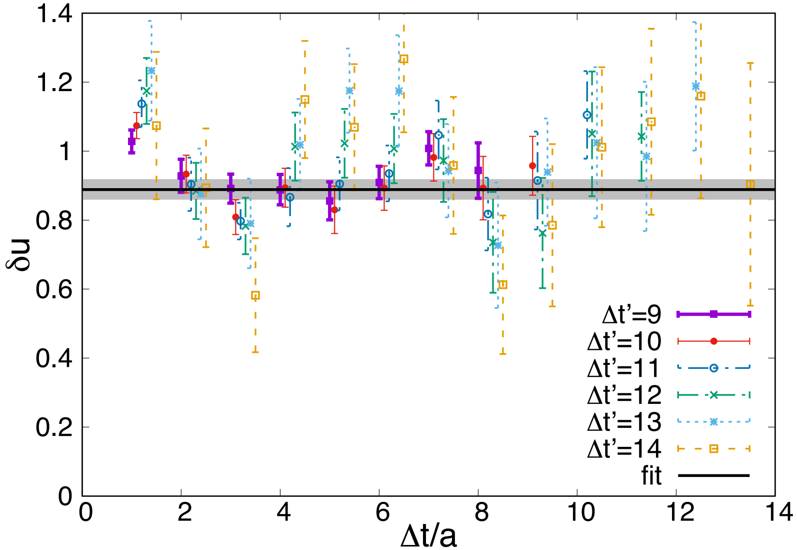
$$\delta q \equiv \frac{1}{2m_N}\langle N(s_z=+1/2)|\bar{q}i\sigma_{03}\gamma_5 q|N(s_z=+1/2)\rangle$$

$$C_{3\mathrm{pt}}^{(\mathrm{disc})}(t_{\mathrm{src}},\mathbf{y}_{\mathrm{src}},\Delta t,\Delta t') =$$

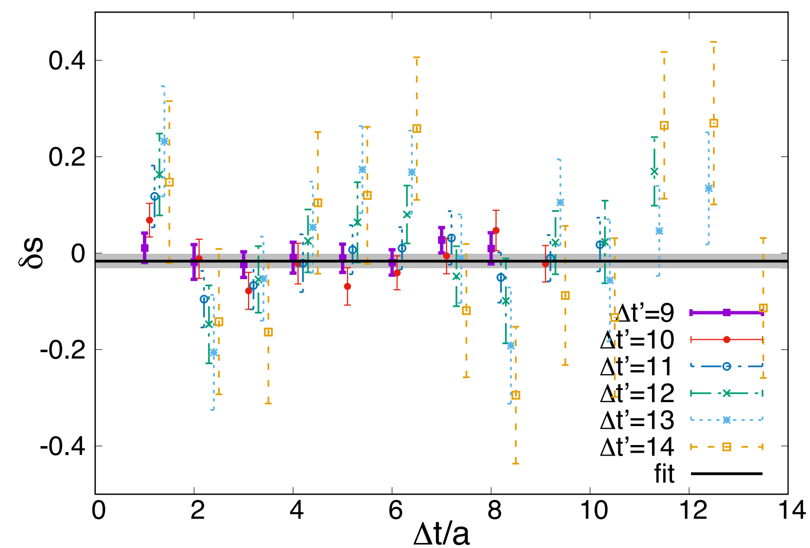
$$-\frac{1}{3}\sum_{i=x,y,z}\frac{1}{N_s^6}\sum_{\mathbf{x},\mathbf{z}}\left\langle \mathrm{tr}_s\left[\mathcal{O}_{\Gamma_i}(\mathbf{z},t_{\mathrm{src}}+\Delta t)D^{-1}(\mathbf{z},\mathbf{z})\right]\mathrm{tr}_s\left[\Gamma_+ P_i N(\mathbf{x},t_{\mathrm{src}}+\Delta t')\bar{N}(\mathbf{y}_{\mathrm{src}},t_{\mathrm{src}})\right]\right\rangle$$

where $\{O_{\Gamma_i},P_i\}$ = $\{A_1,\gamma_5\gamma_1\}$, $\{A_2,\gamma_5\gamma_2\}$, $\{A_3,\gamma_5\gamma_3\}$, $\{T_{01},\gamma_5\gamma_1\}$, $\{T_{02},\gamma_5\gamma_2\}$, $\{T_{03},\gamma_5\gamma_3\}$,

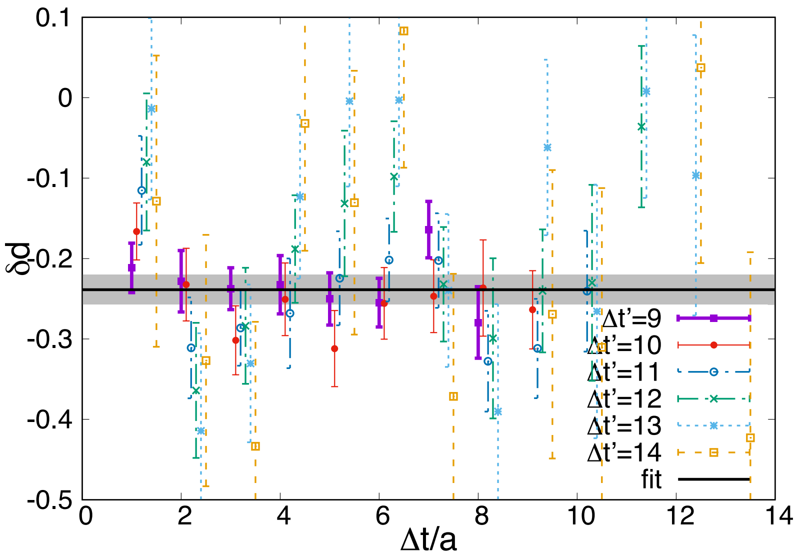
δu, m=0.015:



δs, m=0.015:



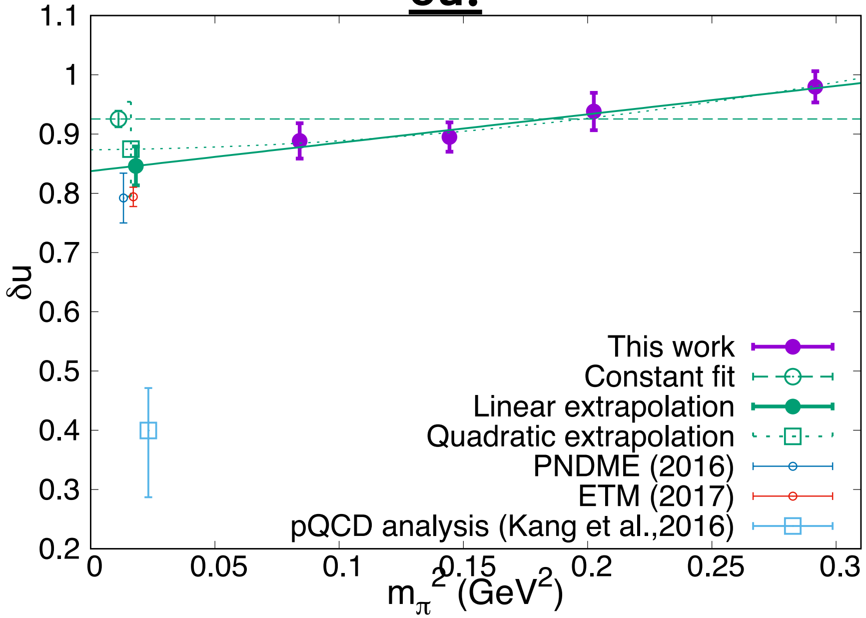
δd, m=0.015:



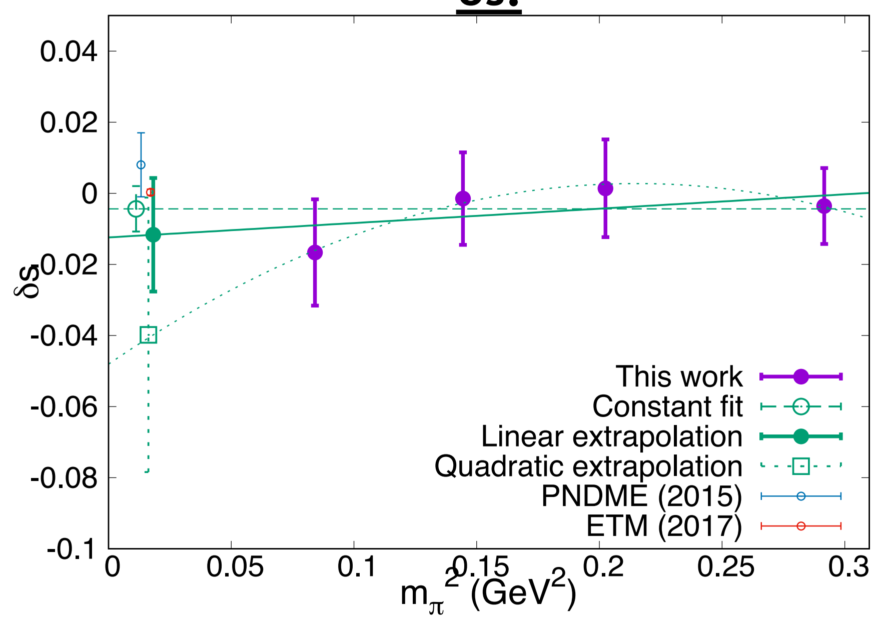
δs(disconnected contribution) is very small (consistent with zero)

chiral extrapolation

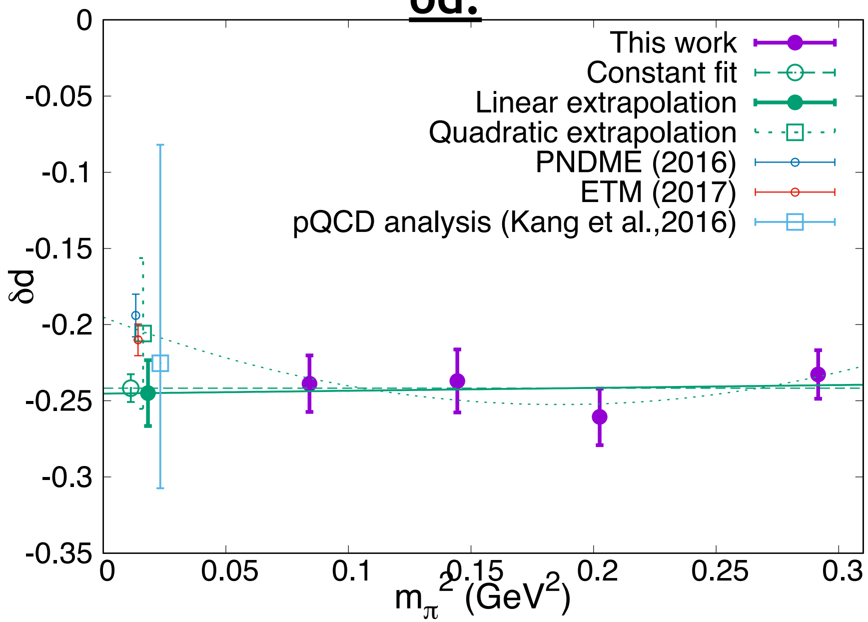
δu:



δs:



δd:



Our result:

$\delta u = 0.85(3)_{stat}(2)_{x}(7)_{a\neq 0}$

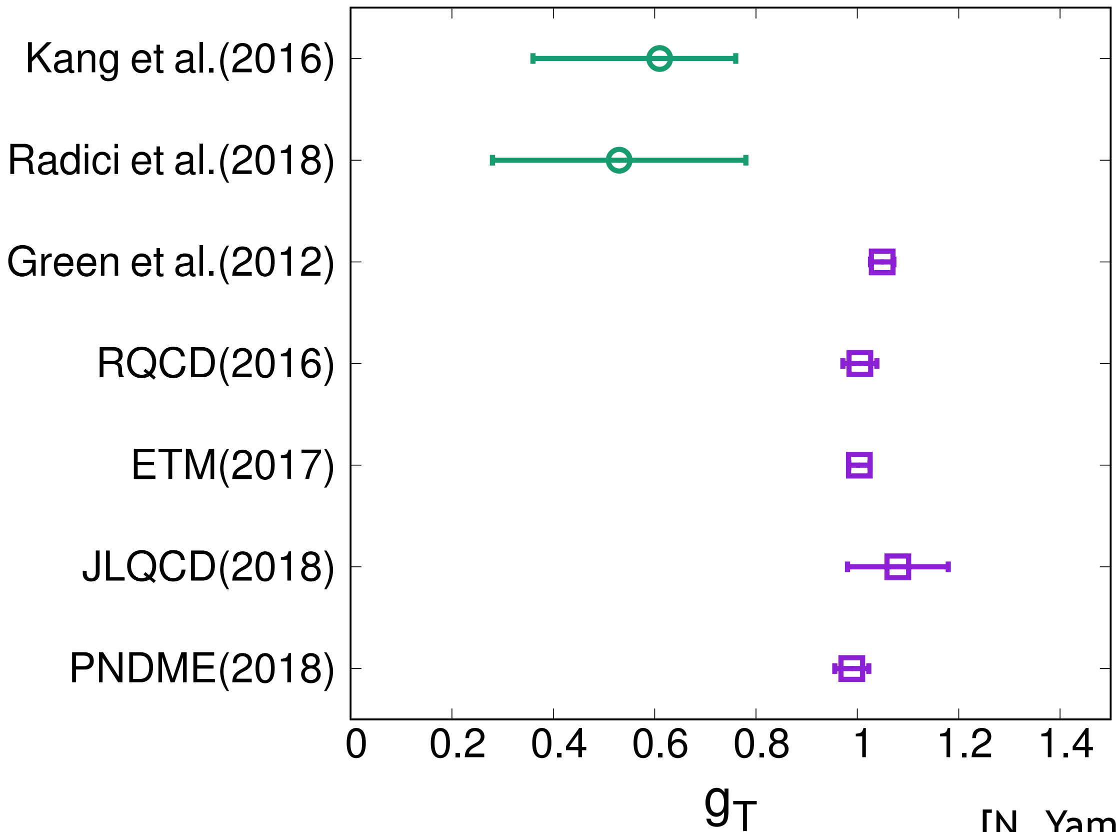
$\delta d = -0.24(2)_{stat}(0)_{x}(2)_{a\neq 0}$

$g_T^s = \delta s = -0.012(16)_{stat}(8)_{x}$

Consistent with other previous results.

Recent results: the isovector tensor charge

$$g_T \equiv \frac{1}{2m_N} \langle p | \bar{u} i \sigma_{03} \gamma_5 u - \bar{d} i \sigma_{03} \gamma_5 d | p \rangle = \delta u - \delta d,$$



[N. Yamanaka, 1902.00527]

All lattice results are very accurate and show consistency among them.
The lattice error is much smaller than phenomenological estimates.
Lattice : important input for nEDM

Current status of lattice EDMs

* θ-EDM

Many lattice results: after correcting spurious mixing, results 50-100% error.

For m_π =340 [MeV], $|2m_n d_n| = |F_3^n(0)| \simeq 0.05 \cdot \theta$

Assuming a scaling $|d_n| \sim m_q \sim (m_\pi)^2$

An extrapolated value at physical point: $F_3^n(0) \sim 0.01 \cdot \theta, \quad |d_n| \sim 0.001 e\mathrm{fm} \cdot \theta$

* chromo-EDM

Exploratory studies started.

Nonzero signals for bare operators. Need to calculate operator mixing and renormalization -> position space renormalization.

(c.f. RI-MOM: Bhattacharya, et al., “15)

* quark-EDM

u,d quark: ~ 3-5 % error,   s-quark: need better precision

* Weinberg operator

100 % error

* 4 quark operators

Not explored yet.

Summary

Precision study of EDM is important.

- Beyond the Standard model physics searches using nuclei are competitive and complementary to the energy frontier new physics searches.

Lattice computation of EDM

- Reanalysis of the lattice method to compute the (CP-odd) nucleon form factors.
  - There exists a spurious mixing between MDM and EDM form factors on lattice.
- Lattice numerical confirmation of “new” form factor formula
  - proposal to calculate EDM on a lattice using energy shift, that is not affected the mixing problem.
  - cEDM operator is used to check the consistency between “new” form factor method and the energy shift method.
- All the previous lattice θ-EDM results using the form factor method must to be corrected.
  - Resulting EDM form factor |F3| are reduced, become one σ signal or less.
  - High precision computation is more important.

- Various nucleon EDM computations on lattice are ongoing.

Thank you