$\mathrm{SU}(2)_{CS}$ and $\mathrm{SU}(4)$ symmetries of high temperature QCD

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April 18th, 2019

PRD 96 (2017) no.9, 094501
1902.03191

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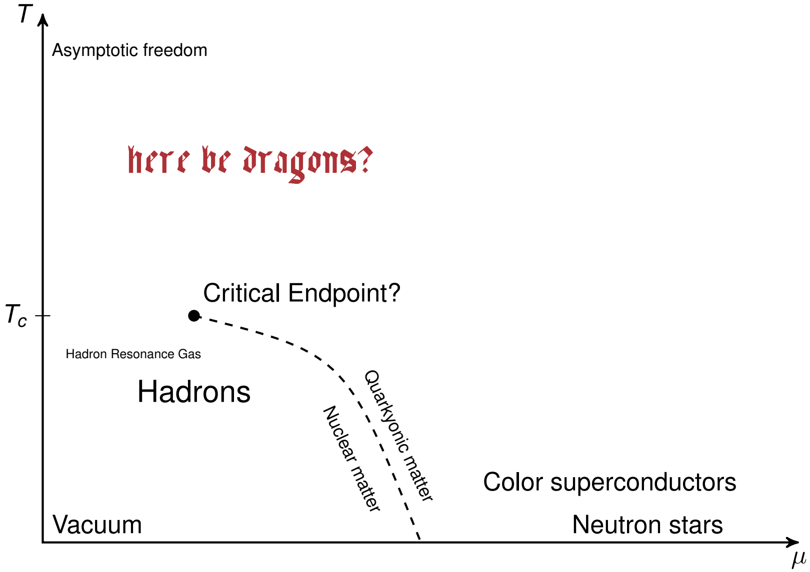
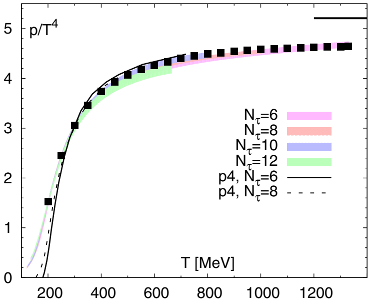
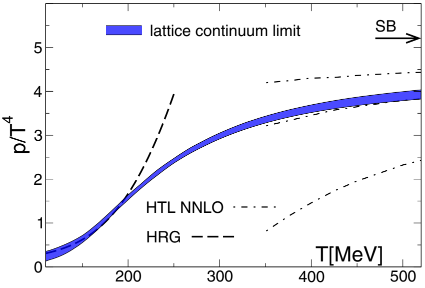
Conjectured phase diagram of QCD



The high temperature phase of QCD

- Experimental access by Heavy Ion Collisions (LHC, RHIC, FAIR, NICA)
- Theoretical access through Lattice QCD:
  - High T thermodynamics turn to precision measurements
  - *Sign problem* for finite chemical potential
  - Critical temperature $T_c \simeq 154$ MeV



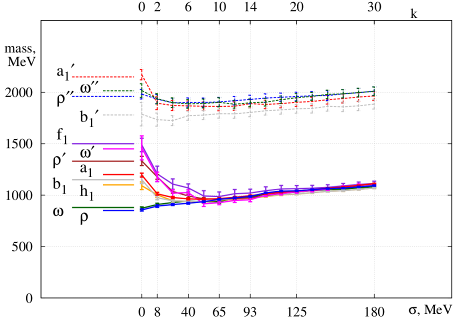


left: A.Bazavov *et al*, Phys.Rev. D97 (2018) no.1, 014510
right: S.Borsanyi *et al*, Phys.Lett. B730 (2014) 99-104

An experiment: modifying the Dirac spectrum

Numerical studies of Hadron spectrum upon Dirac low-mode truncation

$$\langle \bar{q}q \rangle = \pi \rho(0) \qquad Q_{top} = n_- - n_+$$



Chiral spin $SU(2)_{CS}$ and $SU(2n_f)$ symmetries derived

similarity due to suppression of low modes in high T QCD?

M.Denissenya,L.Glozman,C.B.Lang, Phys.Rev.D91, 034505

High temperature lattice ensembles

- $n_f = 2$ Möbius DW fermions, Symanzik gauge action
- $N_s = 32$ lattices, $T_c = 175\text{MeV}$
- L_s is set between $10 - 24$ for good chirality
- spatial correlations in z-direction: $zT = (n_z a)/(N_t a) = n_z/N_t$
- Temperatures between $1.25T_c - 5.5T_c$:

T [MeV]	$32^3 \times 12$	$32^3 \times 8$	$32^3 \times 6$	$32^3 \times 4$
$\beta = 4.10$		220		
$\beta = 4.18$		260		
$\beta = 4.30$	220	330	440	660
$\beta = 4.37$		380		
$\beta = 4.50$		480		960

JLQCD collab. (G.Cossu *et al*), Phys.Rev. D93 (2016) no.3, 034507
A.Tomiya *et al*, Phys.Rev. D96 (2017) no.3, 034509

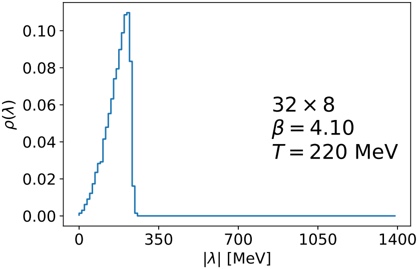
Eigenvalue distribution at high T

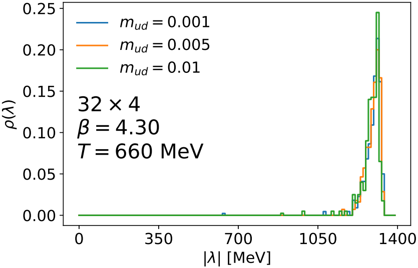
Spectral density $\rho(\lambda)$
for high T ensembles

40 eigenmodes / configuration

$\sim$ 15 configurations

Strong suppression of low modes!





Operators and the Dirac algebra

Measure local isovectors $\mathcal{O}_\Gamma(x) = \bar{q}(x)\Gamma q(x)$

Fix direction of propagation (*z-direction*):

$$C_\Gamma(n_z) = \sum_{n_x, n_y, n_t} \langle \mathcal{O}_\Gamma(n_x, n_y, n_z, n_t) \mathcal{O}_\Gamma(0,0,0,0)^\dagger \rangle$$

Using $\partial_\mu j^\mu = \partial_\mu j_5^\mu = 0$ the Gamma structures for the Vectors are:

$$\mathbf{V} = \begin{pmatrix} \gamma_1 & = Vx \\ \gamma_2 & = Vy \\ \gamma_4 & = Vt \end{pmatrix} \quad \mathbf{A} = \begin{pmatrix} \gamma_1\gamma_5 & = Ax \\ \gamma_2\gamma_5 & = Ay \\ \gamma_4\gamma_5 & = At \end{pmatrix}$$

$$\mathbf{T} = \begin{pmatrix} \gamma_1\gamma_3 & = Tx \\ \gamma_2\gamma_3 & = Ty \\ \gamma_4\gamma_3 & = Tt \end{pmatrix} \quad \mathbf{X} = \begin{pmatrix} \gamma_1\gamma_3\gamma_5 & = \gamma_2\gamma_4 & = Xx \\ \gamma_2\gamma_3\gamma_5 & = \gamma_4\gamma_1 & = Xy \\ \gamma_4\gamma_3\gamma_5 & = \gamma_1\gamma_2 & = Xt \end{pmatrix}$$

γ_3 & $\gamma_3\gamma_5$ no propagation due to current conservation! + Pion, Scalar

What to expect from two-flavor $\mathcal{L}_{QCD}$ and χS?

Symmetry of massless $\mathcal{L}$:

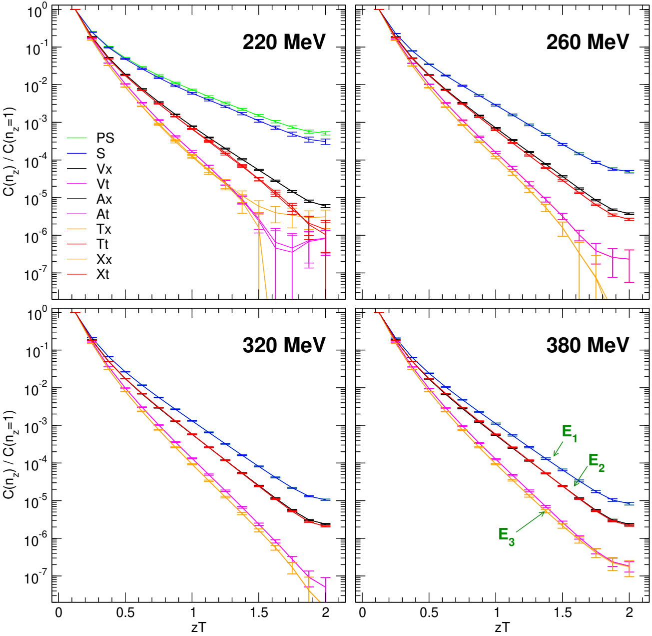
$$\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_A \times \mathrm{U}(1)_V$$

PS	*Pseudoscalar* $\bar{q}(\vec{\tau} \otimes \gamma_5)q$	$U(1)_A$ $\longleftrightarrow$	**S**	*Scalar* $\bar{q}(\vec{\tau} \otimes \mathbb{1}_D)q$
V	*Vector* $\bar{q}(\vec{\tau} \otimes \gamma_k)q$	$SU(2)_A$ $\longleftrightarrow$	**A**	*Axial Vector* $\bar{q}(\vec{\tau} \otimes \gamma_5\gamma_k)q$
T	*Tensor Vector* $\bar{q}(\vec{\tau} \otimes \gamma_3\gamma_k)q$	$U(1)_A$ $\longleftrightarrow$	**X**	*Axial Tensor V.* $\bar{q}(\vec{\tau} \otimes \gamma_5\gamma_3\gamma_k)q$

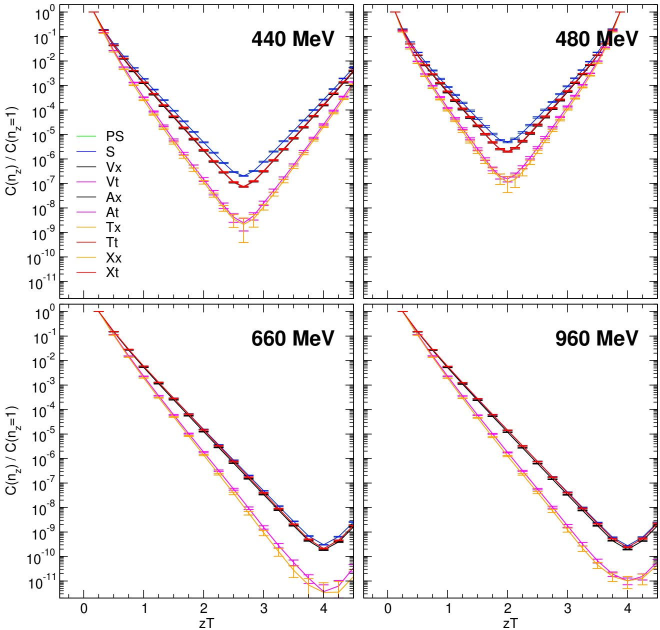
$U(1)_A$ *broken by* $\langle\bar{q}q\rangle$ *and axial anomaly*

$SU(2)_L \times SU(2)_R$ *broken by* $\langle\bar{q}q\rangle$

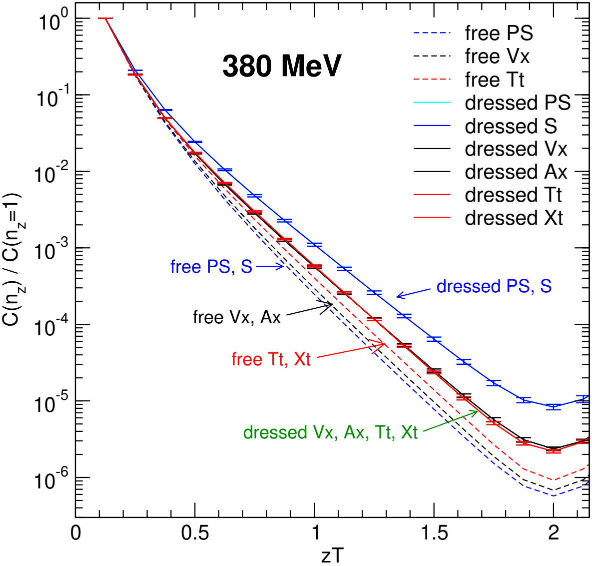
Spatial correlations for $T \leq 2T_c$



Spatial correlations for $T > 2T_c$



E_1 and E_2 multiplets at $2T_c$



free ($U(x)_\mu = \mathbb{1}$), non-interacting quarks: ***chiral symmetry***

$U(1)_A: \quad S \leftrightarrow PS$

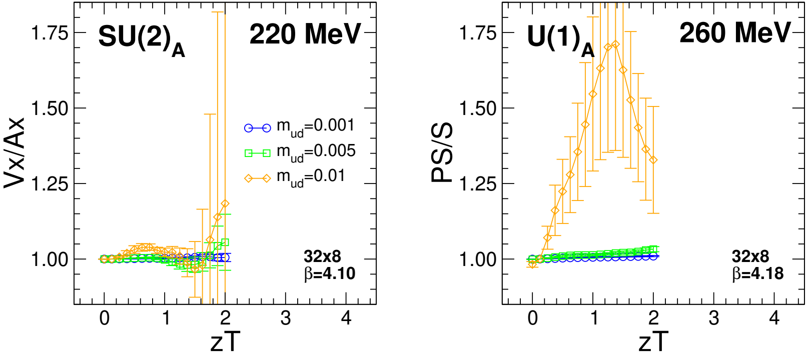
$SU(2)_A: \quad V_x \leftrightarrow A_x$

$U(1)_A: \quad T_t \leftrightarrow X_t$

dressed meson correlators: ***larger symmetry***

$\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ and $\mathrm{U}(1)_A$ symmetries

- $\langle \bar{q}q \rangle$ and topological susceptibility* suggest *'good'* symmetries
- Use ratio of *'connected'* operators as measure



*previous talk,
JLQCD collab. (K.Suzuku *et al*), EPJ Web Conf. 175 (2018) 07025

$SU(2)_{CS}$ chiral spin and $SU(4)$ symmetries

$$\Psi \xrightarrow{SU(2)_{CS}} e^{i\vec{\Sigma}\vec{\theta}/2}\Psi \qquad \vec{\Sigma} = \{\gamma_k, -i\gamma_5\gamma_k, \gamma_5\}$$

◇ Physical interpretation: $\begin{pmatrix} u_L \\ u_R \\ d_L \\ d_R \end{pmatrix}$ *all components of fundamental vector mix!*

◇ for spatial $z-$correlators generated by representations:

$$\begin{array}{lll} R_1: & \{\gamma_1, -i\gamma_5\gamma_1, \gamma_5\} & \\ R_2: & \{\gamma_2, -i\gamma_5\gamma_2, \gamma_5\} & \end{array} \Rightarrow \begin{array}{l} A_y \leftrightarrow T_t \leftrightarrow X_t \\ A_x \leftrightarrow T_t \leftrightarrow X_t \end{array}$$

◇ Minimal group containing $SU(2)_{CS}$ and χS is $SU(4)$:

$$\left.\begin{array}{l} V_x \leftrightarrow T_t \leftrightarrow X_t \leftrightarrow A_x \\ V_y \leftrightarrow T_t \leftrightarrow X_t \leftrightarrow A_y \end{array}\right\} E_2$$

$$\left.\begin{array}{l} V_t \leftrightarrow T_x \leftrightarrow X_x \leftrightarrow A_t \\ V_t \leftrightarrow T_y \leftrightarrow X_y \leftrightarrow A_t \end{array}\right\} E_3$$

L.Glozman, Eur.Phys.J. A51 (2015) no.3, 27

L.Glozman and M.Pak, Phys.Rev. D92 (2015) no.1, 016001

Symmetries of the Lagrangian

$$\Psi \xrightarrow{SU(2)_{CS}} e^{i\vec{\Sigma}\vec{\theta}/2}\Psi \qquad \vec{\Sigma} = \{\gamma_k, -i\gamma_5\gamma_k, \gamma_5\}$$

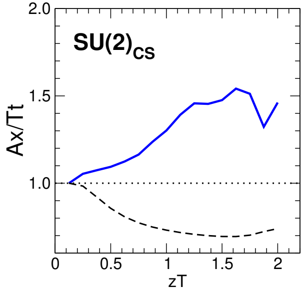
Free, massless Lagrangian: $\mathcal{L} = \bar{\Psi} i \partial\!\!\!/ \Psi$

Covariant derivative: $D_\mu = \partial_\mu - igA_\mu$

Massless (fermionic) Lagrangian: $\mathcal{L} = \bar{\Psi} i D\!\!\!\!/ \Psi = \bar{\Psi} i \gamma^0 D_0 \Psi + \bar{\Psi} i \gamma^i D_i \Psi$

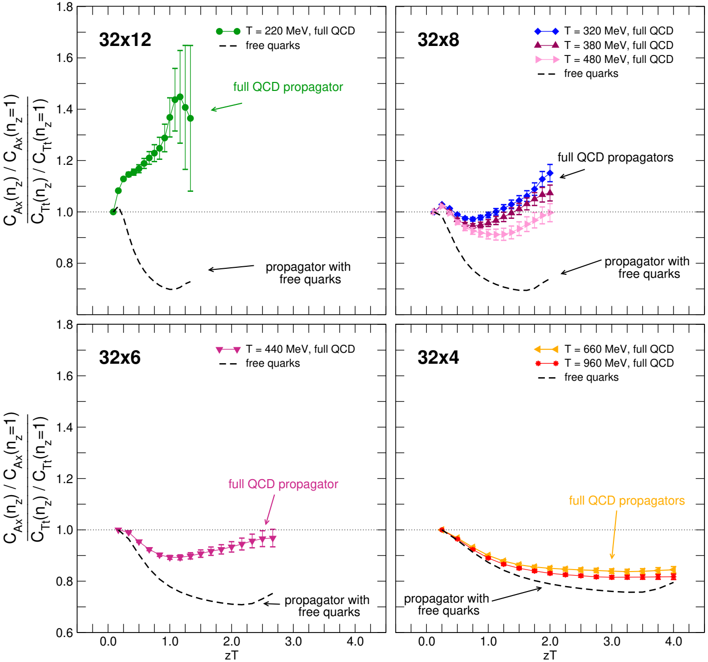
breaks $SU(2)_{CS}$

$SU(2)_{CS}$ invariant



- Kinetic term breaks $SU(2)_{CS}$
- 'Magnetic' term breaks $SU(2)_{CS}$
- **'Electric'** term is $SU(2)_{CS}$ symmetric

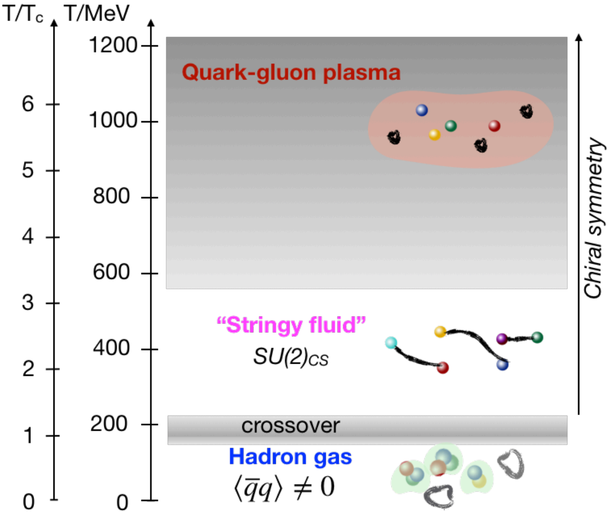
A_x and T_t mix under $\mathrm{SU}(2)_{CS}$
Use ratio to measure breaking!



A_x / T_t ratio measures $SU(2)_{CS}$ breaking

The phase diagram & chemical potential

$$S = \int_0^\beta \int d^3x \;\; \bar{\Psi}[\gamma_\mu D_\mu + \mu\gamma_4]\Psi$$



Conclusions

✓ spatial correlations at temperatures $1.25 - 5.5T_c$

✓ chiral symmetry and effective $\mathrm{U}(1)_A$ restoration above T_c

✓ approximate $SU(2)_{CS}$ symmetric region $\rightarrow SU(4)$

$\Rightarrow$ $SU(2)_{CS}$ a tool to distinguish
color-electric and *color-magnetic* contributions

chiral quarks connected by color-electric field
as elementary objects at high T ***(strings?)***

Thank you for listening!